ABSTRACT

This report is a pilot effort to understand how knowledge information is effectively imparted in a classroom. The work is built on a study of the cognitions of effective teaching. Expert teachers were identified by monitoring the gain scores of students in classrooms over a 5-year period. All teachers were interviewed, observed, and videotaped over a 3-month period. The collected data were analyzed by first developing basic activity structures for each teacher, then analyzing each lesson on which notes, direct observation or videotapes were made, and building an activity structure analysis. The substantive topics covered are: student perceptions of class structure and organization, student-teacher perception of in-class actions and events, and student growth in competence as well as performance. The methodological results deal with the improvements needed in the mechanism by which relevant data can be gathered and interpreted. These results indicate two areas for improvement: test construction and analysis. Testing can be improved by adding items that are drawn more directly from textbooks and from teacher presentation examples. Analysis can be improved by a closer fit to instruction. Data can be analyzed for points of instructional confusion and mapped onto weaknesses in student performance. (PN)
Student Cognitions During Instruction

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In the context of studying teachers' cognitions during instruction, work has also been carried out to understand student cognitions in classroom settings. Research on teachers' thinking has revealed the significance of routines that act to reduce the cognitive complexity confronting the teacher at any one time and simplify the execution of plans. (Bromme, 1982; Leinhardt, 1983). Teachers are seen as drawing on stored, easily accessible knowledge about curriculum, subject matter and students, while constructing the agenda for teaching, and as reading the environment for information to adjust the agenda and execution of plans. Teachers are seen as operating with two intertwined knowledge systems during instruction: an action system and an information system. The action system constitutes the skeleton sequence of actions to be engaged in and the routines used to implement the actions. The action system also carries space into which unique specific lesson material can be inserted or space for information obtained currently or from other parts of the system to be used. The information system is the procedure by which experts stay aware of learner status and lesson status, it is

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what makes their particular lesson development efficient and easy to execute.

The comparable work on students addresses issues such as the students' awareness and labels for routines (what are the rules for board work, etc.), the students' predictive capabilities with respect to agenda (what's going to happen today), the students' information processing and understanding during instruction in relationship to the information being imparted (what does that mean, how do you know), the rules for performance (when do you answer), and the actions of other students (what is 'so and so' doing there). By fitting both pieces of work together, we hope to understand just how knowledge information is effectively imparted in a classroom. The current report is a pilot effort of work very much in progress. Where line of analysis are to be continued, they are indicated; where some beginnings of results are understood, they are stated.

Research on student cognitions during instruction is still in its infancy. The primary perspective of the research to date has been a structural, meta-cognitive, learning to learn one (Anderson, 1981; Peterson and Swing, 1982; Peterson, Swing, Braverman and Buss, 1982; Winne, 1982; Winne and Marx, 1980; Winne and Marx, 1981; Winne and Marx, 1982). Anderson's (1981) work focussed on an important aspect of elementary school lessons, that of students' perceptions of seat work. A principle finding and one that is certainly consistent with what we have also been seeing is that a primary concern and attitude of young students toward seatwork is to do it - to complete it. Further Anderson's work points out how surprisingly willing children seem to be to do things that make little sense to them and how unaware the students
are of the content or objective of the lesson or seatwork.

Peterson et al. (1982) in laboratory settings has specifically probed for students' meta-cognitive activities. She discovered that older (10,11) children can, when asked, describe the strategies used during instruction. High achieving students appeared to try to relate new information to old and to figure out the problems on line. Low achieving students were lost or confused more often and seemed to have competing thoughts during instruction. The rate of presentation of new material and the learning from the repeated interviews is probably quite different in the simulation than in real classrooms, but the findings are most suggestive.

The research of Marx and Winne (1981) and Winne and Marx (1982) has addressed the fit between what teachers want students to be thinking and what they are thinking. One feature of their findings is especially salient in the current work, students are strategically active, students can access effective strategies more easily when the routine for the strategy is known and when the segment to be understood is small.

What is to some extent unclear from this work is the way it fits into the totality of the lesson. We know students are spending only some of the time attending to the presentation or rehearsal portions of the lesson, that they tune in and out. Further, we know that only a small fraction of the input over a week represents "new" material. Much of the time in a lesson is spent dealing with review, practice, management, etc., and very little if any is spent thinking about thinking. If meta-cognition training is to fare better than other elemental learning to learn repairs that have been offered, it must be
tailored and well fitted to the natural mental ecology of a student.

Methodology

This pilot work is built on a study of teacher cognitions. Expert teachers were selected for study by monitoring the gain scores of students in classrooms in the Pittsburgh Public Schools over a five year period. Those teachers whose students showed an unusual gain (in the top twenty percent) over a five year period, and whose students had high final locations were identified and asked to participate in a study of the cognitions of effective teaching. Two of the fourth grade teachers from the larger study were asked to participate in a study of students' cognitions as well. All teachers were interviewed, observed, and videotaped over a three month period. This rather large collection of data was analyzed by first developing basic activity structures for each teacher, then analyzing each lesson on which we had notes, direct observation or videotapes, and building an activity structure analysis. From the activity structures we then began a much more fine-grained analysis that looked at each goal and the subgoals used to accomplish the goals, the consequences of a variety of actions and the interrelationships of actions and goals carried throughout the lesson.

From the two teachers that were selected for participation, one set of data was used. (The tapes of the other teacher were unfortunately stolen.) Eight students were selected for the in-depth study from Ms. Day's class, four girls and four boys: two high performing, two low and four middle performing students. Their level of performance was determined by teacher nominations followed by a confirming check of their previous city-wide math test, MAP test (the tests are given every six weeks; for a description of the MAP program see Salmon-Cox 1982,
1983, and LeMahieu, 1983). Five types of data were collected on each student: a pre and post fraction knowledge interview; and their pre and post MAP tests; a class structure interview; an in-class think aloud; and an after class stimulated recall. These data were used to assess three basic areas: student knowledge, student processing during instruction, and student perception of activity structures. In addition we have stimulated recalls from the teacher for the same days.

The fraction knowledge interview, administered before the fractions instruction, consists of topics with several sub-questions on each topic. It is designed to form a core of fraction concepts around which discussion can take place. It covers such topics as what is and is not a fraction, how to operate with fractions and whole numbers, etc. The issues of interest are the type and extent of understanding, not merely the items correct.

The class structure interview was administered early in the study and was designed to assess the awareness the students have about activity boundaries in the typical lesson and the rules and routines for communications and actions in the class. In the in-class think aloud students were interviewed in class during class to find out what they were thinking at particular times during the lesson. For the good students especially, this seemed to be an intrusion, so not all students were interviewed in this way. The stimulated recalls using videotapes were done with all students and the teacher on the same day as the lesson. The analysis of data from the students' stimulated recalls is built around the frame of our teacher analysis. That is, we are building a skeleton for each given lesson and comparing the location and content of children and teacher commentary.
The pre/post interview data is used to help locate the specific lesson or series in which the information was transmitted to see whether what was being taught was picked up by the student. The pre and post interviews will be revised so that they reflect text material more completely. The other two types of data, structural and process interviews will continue to be modified somewhat. The structure interview assesses the degree to which the child has a sense of activity structures, a sense of routines and how things are done. For instance, when is homework handed in, how it is handed in, what happens? How do questions get answered? What are you (the student) supposed to do, how? Do you always do it that way? We will try to build maps that link the teacher's information with the students'.

Results

There are two aspects to the results, substantive and methodological. The substantive topics that will be covered are: student perceptions of class structure and organization, student-teacher perception of in class actions and events, and student growth in competence as well as performance. The methodological results deal with the improvements needed in the mechanism by which relevant data can be gathered and interpreted.

Structure. There are three sources of information on student sense of class structure: the structure interview, in-class think alouds and stimulated recalls. The principle source of information is the interview -- the think alouds and recalls are used to support or contradict and fill in the interview material. The structure interview was given at the beginning of the study. The interview was conducted by
a single white female interviewer. The structure interviews took approximately 15-20 minutes and were open ended semi-structured. The core question was what goes on in math class; how does it proceed; what are the rules for action. Regardless of level of skill all eight children were very capable of telling us the main activity segments for the lesson. In the following summary the number of students volunteering the information is in parenthesis after the generic form of the statement. Math follows lunch (8). Students take out their own math books (5). Students take out pencil and paper (4). The teacher identifies the pages to be covered (8). The teacher explains the topics (3) and the teacher does sample problems on the board (4). The students do their work (6) and if a student has trouble she/he gets help from the teacher (7). The work is corrected either by self check at the back of the room if you finish early enough or by choral answers or by the teacher with you (7) (it's more fun to do it yourself). When a student is done s/he goes on to a ditto or to read a book (4) and the period ends when one is finished (7) or when the bell rings (3) or when language begins (2). Homework is given out last period of the day (1) is given four days a week (4) and is corrected right after bell work in the morning (2).

Most students (6) know what topic of the next math lesson will be (division with remainder, adding fractions, etc). This latter result is different from previous work (see Anderson, 1981) the reason may be the level - "What is the topic for the next class" vs "What is the objective of this set of exercises". Most students would tell new students how to do the topics of math not how the class is structured. Elements which are school wide (folding paper into 16ths) are unreported, while
elements unique to a teacher are reported. There was no systematic
differences between high level children and lower level performers on
their awareness of the rules of the class or on their perception of the
main activity segments.

While some children needed considerable prompting in order to
elicit information; others, such as Cecil, responded to a simple
statement, "Tell me about your math class", with the following
description of his math class (CS,12/7/82,S): (footnote 1)

Well, um, we start math after lunch. When we come up we
t - well she don't tell us to get our math book out
because we know right away that after lunch we get our um
math book. The she tell us what page and sometimes she
write it on the board and um - and um we turn to that.
And after something that - that's knows - we - we do she
just say um she just says the directions and tells us to
do that and that whatever something that um like - like
we gettin' ready to start fractions like after we start
she - she tells us um - she go over some with us and
then she just says how to do it. And um she does three
three up on the board and show us how to do it. And then
after that we just do the problems, get out our pencil
and eraser and start - and get out a plain piece of paper
and start doing our math right. And - and we write our
name and our - our number. She - she gives us numbers
like if we forget to write our name um we write our
number. Like for twelve - my number's twelve. And we do
it - and just start doing our work.

Okay. And then what happens after that?

Um, at - when - when we get finished um we raise our hand
to tell her that - that we finished. Sometimes
-sometimes she says when we get finished we don't have to
um raise your hand, just start reading a book or
something. And um you know close up your math book. And
when she looks up she'll know that you're finished. You
know - with your math. And she has these dittos
like - after - sometimes after we do our math or, after we
do anything um she give us those dittos. Is over there
by the globe. And um - and then um.
Students responded to questions of math lesson structure in very similar ways regardless of ability level. Some middle students were very shy and needed considerable prompting to describe their views of the lesson. Others went on at a great rate with little interviewer prodding. As previous work has shown (Leinhardt, Bar Tal, and Walker, 1976) students' perceptions of the class and articulations are often independent of skill level in the subject.

The rules for small contingencies are well known and revealed during the think alouds. If seatwork is not done in class it becomes homework, or if a few items are not completed then they are marked as zeroes. If, on the other hand, work is completed early then the extra sets are done, followed by dittos or reading a book or sitting. The teacher goes to each child in turn, skipping a problem or using manipulations is encouraged, help can be obtained by hand raising. Hand raising during presentation seems to the children to be irrelevant to being selected, but they do it anyway. Below are some brief comments describing several routines and activity segments: homework and practice.

Homework assignment and checking are both routines about which the students have a fair degree of awareness. One girl, Sue describes the assignment of unfinished class work as homework (ME, 01/13/83, SR).

What happens if you don't get your problems done during class?

She says like if you only get two problems done, you would have to take your book home and your paper to finish your whole page.

Oh. So, then it would be homework?
Do you also get other homework specifically for homework that she gives you especially for homework?

If you don't do your math, she tells you to do that plus your other homework.

She is also quite aware of when homework is checked and how the teacher handles it (ME, 01/13/83, SR).

And when do you check your math homework?
In the morning right after bell work.
And then what happens if you don't have it all right?
Then she puts it in her record book.

Adam, a competent middle student gives his view of the practice time which follows presentation (GS, 01/07/83, SR).

When Ms. Day tells you to start your practice, what do you do? ... can you tell me about what you do when she says start your practice now?

You get our - you get out a paper and our pencils and we write down the problem on the um paper and the answers of the problem.

Okay. Okay, and then when she - usually she walks around the room, and what does she do then?

She checks what we do um answers and then when everybody's finished, we check all the answers together.

Students at all ability levels know how the classroom works. They have actually been seen to prompt the teacher if some segment is missing. The structure they describe is very similar to the structure the teacher describes with the exception of the patterning of the travel routine.
In terms of methodology a direct open-ended interview is most efficient in getting at the issue of structure. However, the interview must be constructed around the observed in-class rules, not simply a list of topics. Previous work has suggested that children respond to fanciful situations well. However, ours bombed. We asked students how they would know they were in math class if they were blindfolded. They answered either that they wouldn't know or they would know by the teacher's talk. We also asked them what they would tell a new child about the system and all said they would tutor him or her in the subject matter. There are three possible reasons: (a) the children found the question strange and uninteresting; (b) the interviewer herself was less than enthusiastic; (c) there was insufficient rapport established with the children over time. We are fairly convinced that (c) is definitely true but the other two may be true as well.

Instructional Processes. The second area of student thought of interest to us involved the way in which the teacher (Ms. Day) actually ran a lesson and how the processes of any given lesson are characterized by both teacher and student. A single sample lesson will be discussed. Figure 1 shows the action segments of this somewhat typical lesson and the basic goals. In brief, the lesson, which is on subtracting fractions with unlike denominators, starts with the teacher putting the pages to be worked on the board. She moves into a presentation of four problems, and works through subtracting fractions with unlike denominators. The presentation consists of four goals. First there are two reviews (a) finding common denominators; (b) subtracting fractions; and then (c) the presentation of subtracting fractions with unlike denominators. The final goal of public practice (d) consists of doing
three problems on the board in steps with a different student responding to each step. A global constraint operating on this portion of the lesson is the belief that weaker children should be queried to determine their understanding of the lesson. The consequence is that the presentation is often arduous. Following the presentation is an extensive independent practice period during which the teacher tutors children one at a time. The sequence of tutoring is dictated by a preplanned pattern of movement combined with student need. Described below are a set of five segments from the lesson, and the teacher and two students' responses are noted. The two students are both middle level.

Figure 1 Here

Figure 1 represents our analysis of the goal structures of the lesson from the teachers perspective. The first general goal, presentation, is divided into five subgoals each of which has activities attached to it and sometimes functions as well as outcome. In this particular case the subgoals are not intertwined at this level of analysis, but multiple parallel goals are possible. Stars on the figure indicate the portions of the lesson in which the teacher and students made comments.

First Segment: Transition. At the beginning of the class the activity is transition, the children are returning from lunch. The teacher looks to see if all children have their books and notices one, Sal, who does not. Sal is rummaging through his desk and Ms. Day pauses to wait.
Ms. Day, in the stimulated recall, comments that Sal is about to change schools and has stopped working. She also comments on the fact his desk is a mess. Cecil and Tina notice the incident as well. (CS, 01/10/83, SR)

Cecil

Do you have to wait for somebody to get a book a lot?

You shouldn't—if it takes them too long, then they umm, should go down to the office, but, his desk shouldn't be a mess like that.

Tina

Does Mrs. Diskin usually have to wait for somebody to get their book?

Not usually. Not usually, it just happened today. (CC, 01/10/83, SR)

Segment 2: Presentation. The second segment occurs within the presentation and contains the first subgoal, that of reviewing common denominators. The teacher puts the problem on the board \(\frac{5}{8} - \frac{1}{4}\) and calls on Cecil to respond. He jumps ahead and gives the answer \(4 \times 2\), rather than identifying the common denominator. The teacher who is using him as a model for the class wants the step boundaries clearly identified, so she stops Cecil and has him backtrack. Ms. Day's comment is that students tend to get confused with many steps. She thus emphasizes the procedural goal of getting the steps down. Cecil’s goals are different—first and foremost to get to the place where the problem can be done and only secondarily to be the model. (CS, 01/10/83, SR)

Okay, you knew right away that the common denominator was going to be eight.

Mmmhmm. Because, she already said that if both of 'em both of the denominators are the same it just stays that way.

Mmmhmm.
And all you have to do is -put over the number that you had to subtract or add, whatever.

Okay. Umm, today she called on you to help show that first problem, umm does she call on you a lot for that? Do you know the answers a lot?

Yah.

Pretty good?

Mmmhmm. Oh, yah. Yah, she calls on me a lot, that's if nobody else don't -have their hand up.

Cecil is driven by wanting to do the item and has learned that as soon as the denominators "match", you can "put over the number", namely you can work with the numerators.

Segment 3: Presentation. The third segment represents a response to a general question to the two children: What are you (the others) supposed to be doing while a problem gets worked on the board? Both children say, "paying attention". Cecil adds -

If they don't understand they're supposed to raise their hands. The teacher will tell them how to do it. And that's why before we started, she said is there anybody else now who do not know how to do it. And, you know a few people raised up their hand. So, she's supposed to tell them like she did. And, uh go over a few problems. (CS, 01/10/83, SR)

Neither child really seems to feel that you work problems out along with or ahead of the presentation. However, consistent with the Peterson findings, at least one of our high performing students seems to actually work or rehearse the problems. As soon as the sample goes up they try to get the answer. (TL, 12/10/82, TA)
Segment 4: **Public Practice.** The problem is $1/2 - 1/3$, and Tina is called on to work the problem by first identifying the common denominator (6). Tina, looking at the first denominator, answers "3", (i.e. $2 \times 3 = 6$). Ms. Day says, "What will I multiply times 2 to get 3?" Tina answers, "2", (i.e. $3 \times 2 = 6$). Ms. Day then realizes that Tina has been out of town as the interviewer probes further. (LD, 01/10/83, SR)

Do you think she was giving you the multiplier?

She might, but, uhm, I think she missed that day.

Oh.

Cecil's assessment of the exchange is that it's just one more example of what a good teacher Ms. Day is - even though he had responded with a similar type of error the first time - that is, giving the number to be used to multiply by rather than the process.

Tina, when viewing the exchange, seems bewildered by the miscommunication. (CC, 01/10/83, SR)

Okay, yah. When - when you said it the first time, when you said the two, what were you thinking about? When you said, she asked you what's the number that - that um - that you would put beside the three...

I thought it was 2 times 3.

Oh, you were thinking of 2 times 3, and then you were thinking of 3 times 2. (Tina says three times two simultaneously.) Okay. When um - when did you - when did you um figure out that it should be six?

It's 3 times 2.

Is what?

Six.
Unfortunately we can not be positive that Tina was thinking $2 \times 3$, rather than make the 2 a 3, as Ms. Day thought. The speed with which she does the seatwork suggest that she did understand the process but could not label the steps as the teacher wanted.

**Segment 5: Monitored Practice.** Segment five is taken from the monitored practice activity of the lesson. Ms. Day travels around a select group of children (a different selection each day) and tutors.

(LD, 01/10/83, SR)

You decided not to go on and do any more problems today. Sometimes you add problems on the board, you put some additional ones up; why did you decide not to?

I just decided to get them started and, ah, I figured enough of them would know what they were doing today, that I could just hit the ones that didn’t know very much about it.

So you decided that you’d pick the kids who—did you decide beforehand which kids you were going to go with then?

No, I decided to quickly check Tina, and not because you said she was going to be first, because she was out of town, I figured she’d get it real quick and I wouldn’t have to spend too much with her. ‘Cause even though she’s ahem, slow in other things, in math, she’s fast. So she did catch on real quick. Then I went to Frank ‘cause he was right between me and Cecil (inaudible). Well, I’d swear Cec— that Frank was playing dumb, you know. (CC, 01/10/83, SR)

All of a sudden I think he wanted to reduce them the, ah, fractions that were being added or subtracted to lowest terms before he did the, ah, answer. So I told him he had to get the answer first and then reduce it. (LD, 01/10/83, SR)
Tina commenting on the practice segment during which Ms. Day visited is noncommittal. (CC, 01/10/83, SR)

Right. How many problems did you do while Ms. Day was there? Do you remember?

Three.

Oh, you did three. She watched while you did three? Does she usually — does she usually watch while you do a few problems?

Nuhu.

What does she usually do?

She usually just watch one and then she leaves.

Both Cecil and Tina understand the system well — and it concurs with Ms. Day's assessment. Traveling meets the goal of social control — maintaining on task behavior; it meets the goal of extensive tutoring when needed and/or answering questions; and finally, it meets the informational goal of learning who is in trouble and what types of items are difficult. Each of these segments was selected to give a glimpse of the actions involved in a lesson and to examine what goes on in them. Once all of the segments have been analyzed, we can contrast high and low performers as well as understand the frame into which the substantive thoughts fit.

Differences Between High and Low Performers.

The better students are quite aware of what the lesson will be about and anxiously await the signal that their part in the action system is to be played out. That's not to say they enjoy doing large numbers of problems, but they do want to complete them. They like the control of checking their work, something they can do only if they
finish early enough. The weaker students often seem lost both in terms of the substance of the lesson and in terms of how they should proceed. The lower children report using the teacher for help much more frequently but seem unaware that the teacher will often use other students to bail out a weaker student in oral recitation. The good students are quite aware of their roles, however. The high performers report using the "short-cuts" or other tricks as soon as they are introduced - the lower students do not.

Lower children get more teacher attention from this teacher during the presentation and during seatwork. They are slower in doing their work so they get fewer items of practice. They seem to have less of a sense of what is going on in class and more important, less of a sense of what will happen. They are aware of the global segment boundaries in a lesson, their expected role and that of the teacher. Higher children clearly wait in anticipation to discover what it is they are supposed to do - as soon as that is discovered they try to do it. They are less interested in the non-productive lessons involving manipulatives - in part it seems because the 'what to do' is less obvious and somehow doesn't count.

In terms of improving the depth and importance of the data we can get from a student's reports of instruction, several decisions must be reached. First, are in-class think alouds more productive or are stimulated recalls. Stimulated recalls "feel" better, they are easier, more popular and less disruptive. However, they are somewhat productive in terms of length and richness of protocols. Table 1 shows the number of lines and starts under different conditions. What seems an appropriate strategy to pursue is to combine the methods and greatly
extend the number of interviews each child has so they "learn" to tell us what is going on.

Insert Table 1 Here

Performance. The assessment that we used of children's knowledge of fractions consists of a 10 item interview, with approximately five probes per item. Some of the probes are nested and only used when the global item is missed. The interview was conducted in school by a single white female interviewer. All eight students were interviewed before and after the section on fractions was taught. All eight students also had a MAP (a minimum competency type test given six times a year in math) pre and post test and a unit post test.

Before instruction the students performed on the MAP test in the following way: Out of four fraction items the two highs missed 0 and 1; the four middles missed 2, 3, 2, and one did not take the test; and the two lows missed 2, 3. In the second MAP test one low child missed one and others got all the fraction items right. Thus, at the most primitive level the students gained some proficiency in item performance. At another level, that of a 28 item chapter post test, the results are less outstanding. The two highs and one middle missed no items, the three other middle students missed 1, 1, and 2 items, while the two lows missed 3 and 14 items. Unlike denominators and changing mixed numbers to fractions were the common sticklers.
Ten core items and their prompts were used to assess their understanding of fractions: identifying a fraction, using a regional model, irregularly divided regions, whole as fraction, fractions greater than one, discrete models, equivalent fractions of a region, fractions of a number, fractions of different wholes and number lines.

The answers to questions on representation of improper fractions indicate that most students can create a representing fraction, but do not always recognize equivalence. The answers to questions on discrete representation of fractions indicate this is difficult for most all the students. It is also difficult for teachers. They have trouble if the denominator is not equal to the number of objects in the set. The book spent 10 pages on this but there was little change from the pre to the post interviews. The number line questions were also extremely difficult, and the book spent only three pages on number lines.

Dick (Hi) had not remembered much about fractions on the pre-interview. He had difficulty recognizing and defining fractions. Some memory of fraction concepts came back during the test and he was able to answer some of the last questions better. On the post-interview, he did well on the region representations, but could not handle fractions of sets of objects unless the denominator was equal to the number of objects in the set.

Rose (Hi) was very reluctant to answer any questions on the pre-interview. She missed identifying several numbers as fractions. The interview was stopped as she seemed distressed. On the post interview she was able to complete the interview easily. The only gaps were in giving examples of numbers that were not fractions and in the
use of the number line. Rose is the type of child that has helped drive out pretests from some programmed instruction texts and in her case appropriately so. She simply will not attempt what she does not recognize as having been taught. However once taught she shows considerable stamina.

Adam (mid) had some ready knowledge of fractions on the pretest, but he confused numerator and denominator functions. He did not recognize equivalent fractions and had difficulty with discrete sets unless the denominator was the number of objects. In the post-test he utilized his past knowledge, interpreted region representations and handled equivalent fractions. He showed a limited ability to use the number line.

Cecil (mid) was able to give a very good initial description of a fraction and when it could be used. He could handle most concepts related to region representations, but forgot that fractions are made up of equal pieces. He also had problems representing numbers larger than one. With the discrete sets he was only able to solve the problems if the denominator was the number of the set. At the post-interview he kept all his previously displayed knowledge, was more creative in use of his region model, and could now use a number line, if not perfectly, better than most.

Tina (mid) was very confused by the word "fraction" in the pre-interview. She kept identifying it with "factor". However, she could draw region models of fractions and was able to name fractions correctly at a later point in the interview. With discrete sets she could only answer correctly if the denominator was the number in the
set. She could name the fractions made with equivalent regions but did not show any understanding of equivalence. There was some gain on the post-interview. She could now identify fractions, but did not show understanding of equivalence. She was still having trouble with fraction of a number and could not use a number line.

Sue (mid) showed some previous knowledge in the pre-interview. Her drawings of fractions had the wrong number of parts and shaded parts. She was able to answer questions about given regions. With small sets of discrete objects, she could find fractional sets, but with the large sets responded, for example, $1/4 = 4$. She could not use the number line. On the post-interview she was able to draw the correct picture, but still did not recognize the constraint of equal areas for a fraction. For fractions of small sets of discrete objects she showed more understanding.

Frank (low) on the pre-interview showed many gaps in his fraction knowledge. He had trouble classifying numbers as fractions, drew an incorrect picture of a fraction, and was unconcerned about having the parts equal. Frank also missed all the items on sets of objects and the number line. His post-interview showed some improvement. His pictures were correct, but many other concepts were confused. He could handle fractions of a set only if the denominator of the set was the number of objects in the set and still had no idea of the number line.

At the pre-interview Nicki (low) showed some elementary knowledge of fraction concepts. She was one of the few who was able to define it. She had a little trouble generating drawings of fractions at first and she could not extract all the information given a drawing. With
discrete objects she made the following type of mistake, \( \frac{1}{3} \text{ of } x = 3 \), no matter what the set was. With the number line she could use it as a ruler to get the nearest unit. The post-interview showed she was still missing all the problems with discrete objects and was missing some of the finer points of the region model, especially with fractions greater than 1. She was no longer able to use the number line at all.

What we have learned has in some sense told us more about policy decision in the school district than the development of fraction knowledge in children. When a monitoring testing program is introduced, and it is used for teacher assessment as well as student assessment, it drives the curriculum and may influence levels of knowledge (LeMahieu 1983). The students easily mastered the material covered by the district tests, but showed less grasp of the material covered by the text. Every page in the text was covered, but intensive tutoring and extra lessons were scheduled only for the subset of math material covered in the district text.

In terms of concepts, all children seemed to have a firmer grasp of what a fraction was or was not. The children have little grasp of representations of fractions, other than as parts of regions — they do not map the number line onto their knowledge of rulers, nor can they see the divisional notions of fractions captured by one-third of eighteen.

The work suggests two methodological areas for improvement, test construction and analysis. The interview can be improved by adding items that are drawn more directly from text and from the examples generated during presentations. That is keying the extended knowledge estimates to familiar concepts. In the future we will construct a
content map that covers all material and its presented emphasis and matches that to items mastered and concepts induced. Analyses too can be improved by a closer fit to instruction. The tapes and observations can be analyzed for points of instructional confusion and mapped onto weaknesses in student performance. We suspect that the higher students are more resistant to lack of emphasis, non-routinized presentation, and lack of clarity. But the pre-interview suggests that even good students generalize very little beyond their exposure level.

Conclusion

The naturalistic study of student thought is in its infancy. Work should continue to understand better the structure in which education takes place; the content presentation and response; and growth of competence in mathematics. The structure interview will be focused to tap more directly into the areas of activity segments, routines, and rules used in any particular class.

With respect to in-class thinking, it is necessary to anticipate the "learning" lessons as distinct from representation and review lessons and structure interviews around those. We will use in-class think alouds and stimulated recalls so that we can pinpoint the places that changes, puzzles and insights are likely to occur and focus on them. Children like teachers can be interviewed before and after class to get their own sense of what is about to happen in math. Finally, children need to be observed so that goal-based analytic descriptions of their actions comparable to those of teachers can be built.
In terms of the growth assessments, the point will be to fit those more tightly to the presentation of material. We will revise the interview but try to keep the global items embedded so as to move towards assessment of "understanding".
References


Winne, P. H. Minimizing the black box problem to enhance the validity of theories about instructional effects. Instructional Science, 1982, in press.

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<th>Student Name</th>
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<td>33</td>
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<td>34</td>
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<td>44</td>
<td>47</td>
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<td>38</td>
<td>94</td>
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<td>44</td>
<td>202</td>
<td>61</td>
</tr>
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</table>

34
Figure 1.
Lesson on Mixed Numbers

Ms. Day 1/10/83

Transition: Time 45 seconds

Goal 1 Presentation: Subtracting fractions with unlike denominators - work through 4 problems - Time 9½ minutes.

Subgoal 1A - Review: Changing fractions with unlike denominators to fractions with like denominators - with 1st problem

action: T points out 4 examples on board, reminds them about finding like denominators, gives page numbers (164 & 165)

1ruption:

1. “Wait for Sal to get his book out”

action: T walks to Sal’s desk

outcome: Sal told to clean up desk

2. T points to first example (5/8 - 1/4) asks what is the first thing needed for solution. (Find common denominator)

action: calls on competent middle child (Cecil)

outcome: Cecil answers, gets denominators same

function: accurate rehearsal of 1st step

outcome: 1st step in finding common denominators reviewed

Subgoal 1B - Review: Subtracting fractions with like denominators

action: T asks for second step - subtracting fractions with like denominators

1ruption: calls on low child (Sal) - fails

action: T reminds Jerry (child with hearing loss) to look at board

outcome: 32
L action: Jerry answers correctly at same time Sal says wrong answer
L outcome: misunderstood cue
L action: T explains Jerry’s misunderstanding
L function: encourage listening, monitor
L outcome: Jerry’s comprehension checked; Sal’s not

Subgoal 1C - Review: Format for subtraction
L action: T states rules for writing problems vertically cues Sal to watch
L function: monitor form used
L outcome: reinforce accuracy

Subgoal 1D - Public practice of all steps
★3 action: T asks what’s going to be common denominator in 2nd problem - calls on middle child, (Bob) 1/2 - 3/8 =
S gets denominator with prompt - fails with numerator
T reviews steps again, and prompts child through answer
function: reinforce learning
outcome: check another middle student - weak understanding

★4 action: T call on middle student (Tina) 1/2 - 1/3
S three tries, three fails to get common denominator
T checks child’s absence when concept taught
function: pinpoint difficulty
outcome: Subgoal 1D1: Check Tina
T calls on low child (Ann) hesitates but succeeds
function: check low child
outcome: Subgoal 1D2: Check again
T back to Tina
Goal 2 Practice: Time 26 minutes

action:

"Let's get started", start with problem 1

function:

starts students on independent work

outcome:

T moves into tutorial mode

interruption -

S: - Do we have to write them up and down?

T - Yes, with explanation (Goal 1C)

Subgoal 2A - Tutor, answer questions

action:

T checks Tina - asks what's this 3, can't do anything until have denominator, watches S do second (T is on 1/2 - 3/8, then 5/8 - 1/2)

function:

monitor understanding

outcome:

subgoal 1C1 met

T check Frank

T rips up messy paper, and writes out problem for him

S starts again

T helps student with prompts, questions on which number is denominator, and reteaches vocabulary

S responds to teacher questions, writes wrong numbers (extensive tutoring)

T asks Ann to start working and continues tutoring

T leans over to Tina's desk and checks another problem continues tutoring on 3rd problem (5/6 - 1/3)

Cecil asks how to reduce fraction

action:

T answers Cecil's questions

S continues to ask for explanation on reducing until he understands

T states: will teach process needed in a few days

T tells Ann she will be over

T repeats tutoring with other students

END HERE    CLASS CONTINUES
Subgoal 1E - Check if students understand material

action:

T "Who thinks they can do these now?"

S raise hands if can

T checks Jerry, Sal, Tina

function: monitor

outcome: move into practice