This study guide is specifically designed for individuals preparing to take the Georgia Teacher Certification Test (TCT) in mathematics. The test covers seven subareas: (1) sets, numbers, numeration, operations, etc.; (2) geometry; (3) algebra; (4) trigonometry; (5) analysis and calculus; (6) probability and logic; and (7) measurement, relations, and computers. The guide contains a listing of content objectives and selected references for each subarea. (JD)
National Evaluation Systems, Inc., has prepared for distribution by the Georgia Department of Education the set of content objectives found in this Study Guide. These objectives have been verified as important content requirements for initial certification. Not all of the listed objectives have had test items written for them. The selected objectives have not been identified. All objectives which appear here are certification requirements and a sampling of them will be tested.

When the project to develop the Georgia Teacher Certification Tests (TCT) was begun in November 1976, an Ad Hoc Committee composed of Georgia educators was appointed to work with NES on each TCT. The function of these Ad Hoc Committees was to review all NES-generated materials with a goal of making the materials more reflective of Georgia education needs. The first step in the test development process was that of content domain specification. Educators identified all content knowledge that an applicant would need to know to function effectively in a Georgia school. This content was further defined into content objectives, which were sent to currently practicing Georgia educators for verification. These educators provided actual ratings of the "job-relatedness" of the content objectives. At that point, it was possible to identify, from the original domain specification, the extent of essentiality of specific content skills for successful performance on the job. Test items were written for the most essential objectives which spanned the content of the field.

The purpose of providing objectives is to explicitly define the content required of an applicant for certification in this field. Further, the statement of these objectives should assist in preparing for the criterion-reference content knowledge test. We encourage applicants to study these materials, which will enhance their understanding of the content field and alleviate any unnecessary concerns about the nature of the Georgia Teacher Certification Tests.

Along with these materials go hopes for a rewarding career in education.

If you have questions or desire further information, contact:

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STUDY GUIDE FOR TCT IN MATHEMATICS
Georgia Teacher Certification Testing Program
Field 05: Mathematics

INTRODUCTION

This Study Guide was specifically designed for persons preparing to take the Georgia Teacher Certification Test (TCT) in Mathematics. The mathematics test was developed by the National Evaluation System, Inc., and educators in the state of Georgia. The test covers seven subareas: Sets, Numbers, Numeration, Operations, etc.; Geometry; Algebra; Trigonometry; Analysis; Probability and Logic, and Measurement, Relations and Computers.

This study guide contains:
1. A listing of the content objectives for each subarea.
2. An alphabetical listing of selected reference materials with suggested subareas identified.

This listing of numerous sources does not mean that all are needed to grasp a particular concept or meet a given objective. Some examinees will have better access to certain sources than to other sources. Additionally, in some cases several subareas have been referenced to the same readings because those readings cover several topics. The references given are suggested references only and are not intended to be an exhaustive or complete listing.

In addition to the content objectives and readings that will follow, you should be aware that:
1. The TCT items are multiple choice with four possible answers.
2. There are no penalties for guessing when unsure of an answer.
3. While examinees are given 3 1/2 hours of actual test time, they may request up to an additional hour if needed.
4. In order to pass the TCT one does not have to pass each subarea. Your total score is determined by the number of correct answers.

Examinees wanting specific help with test-taking skills should ask for assistance from their college/university counseling center and/or refer to one or more of the references listed below:


MATHEMATICS

CONTENT AREAS

I. Sets, Numbers, Numeration, Operations, Their/Properties and Number theory

II. GEOMETRY

III. ALGEBRA

IV. TRIGONOMETRY

V. ANALYSIS AND CALCULUS

VI. PROBABILITY AND LOGIC

VII. MEASUREMENT, RELATIONS AND COMPUTERS
GEORGIA TEACHER CERTIFICATION TESTING PROGRAM

CONTENT OBJECTIVES

FIELD 05: MATHEMATICS

I. SETS, NUMBERS, NUMERATION, OPERATIONS, etc.

Use set notation and Venn diagrams, to describe relationships between sets, and identify sets which result from basic operations such as union, intersection, and Cartesian products.

Name rational numbers in fractional or decimal form (include exponential form of expanded notation).

Identify equivalent expressions of absolute value (including equalities and inequalities).

Given any set of integers, place them in order from the smallest to the largest.

Identify characteristics of important subsets of complex numbers (e.g., counting, integers, rational, irrational, real) in relation to such properties as closure, order, density, and completeness.

Given a set (finite or non-finite) and two operations defined on the set, identify the group and field properties.

Use prime factorization to determine the least common multiple or greatest common factor of a set of numbers.

Complete construction of a flowchart from a computational algorithm.

II. GEOMETRY

Identify conditions for incidence, parallelism, and perpendicularity of lines and planes.

Identify the properties of point sets which are invariant under transformations such as reflections, rotations, translations, dilations, and shears.

Recognize congruent figures and identify the isometry which maps one onto the other. In particular, identify the group properties associated with rotation and reflection of such given geometric figures as squares and equilateral triangles.

Identify examples which exhibit properties of similarity of geometric figures either by common proportionality of corresponding parts or by the properties that are preserved by dilation transformation.

Identify point sets such as half planes, rays, segments, angles, closed curves (including points inside, on, and outside closed curves), and polyhedra.

Analyze the roles of undefined terms, definitions, and axioms in geometry.

Identify and/or prove theorems about parallel lines and their transversals including the contribution of the parallel postulate.

Apply theorems on circles, and lines related to circles, in proofs and problems.

Use basic constructions to determine the locus of points satisfying one or more conditions.

Apply geometry in the solution of problems in everyday life, such as scale drawings, tesselations, inaccessible distances, indirect measurements, and designs.
Identify a reflection as the basic isometry of a plane (distance-preserving, one-to-one mapping of the plane onto itself).

Identify rotation, translation, or glide-reflection as the composition of reflections; include use of matrices to represent transformations.

Find distance between points, lines, and planes in a coordinate system.

Choose a suitable set of axioms and use a coordinate proof for appropriate theorems.

Represent vectors as ordered n-tuples and define addition of vectors.

Apply vectors as a resolution of forces in a physical setting.

Identify properties of an inner product and its application to relationships between vectors.

Identify properties of a vector space as a system under the operation of addition and with scalar multiplication.

### III. ALGEBRA

Given a set of whole numbers, perform a given operation (addition, subtraction, multiplication, or division).

Using a diagram, show the relation between the real number system and its principal subsystems:

<table>
<thead>
<tr>
<th>Real</th>
<th>Rational</th>
<th>Irrational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integers</td>
<td>Non-integers</td>
<td></td>
</tr>
<tr>
<td>Negative</td>
<td>Whole numbers</td>
<td>Positive integers</td>
</tr>
</tbody>
</table>

Express in simplest form an algebraic expression containing real coefficients.

Give the prime factors of a polynomial for a specific domain in the form of:
- a) the sum or difference of two perfect squares;
- b) the sum or difference of two perfect cubes;
- c) a trinomial of second degree.

Given a linear equation or inequality in one or two variables, identify the solution set.

Given a linear equation or inequality in algebraic form, find the graph; additionally, given the graph of a linear equation or inequality, determine the algebraic expression.

Given a linear equation or inequality in two variables, find the equation of:
- a) a line parallel to the given line;
- b) the line perpendicular to the given line and containing a specific point.

Given a system of linear equations and inequalities with two variables, determine the solution set by graphs and by using addition or subtraction.

Given a system of linear equations in two variables, determine whether the system is dependent, independent, or inconsistent.

Given several graphs of different linear systems of inequalities in two variables, determine the one which is a polygonal convex set.

Given a quadratic equation that has rational roots, find the roots by factoring.

Given a quadratic equation, find the solution set by using the quadratic formula.
Given a graph of a parabola, determine the equation that describes the graph.

Given a quadratic equation, describe the nature of the roots.

Given a circle containing center \((h,k)\) and radius \(r\), find the equation of the circle in the form 
\[
(x - h)^2 + (y - k)^2 = r^2
\]
Given the center, \((h,k)\) and foci on \(y = m\) or \(x = n\), find the equation of the ellipse described.

Given the equation of a hyperbola, find the equation of the asymptotes, the coordinates of the vertices, or the coordinates of the foci.

Given a quadratic equation in the form \(ax^2 + bx + c = 0\) (where \(a\) and \(b \neq 0\)) that is to be solved by completing the square, find the term that is to be added to both sides of the equation so as to form a quadratic equation of the form:
\[
(x + \frac{b}{2a})^2 = \left(\frac{-c}{a} + \frac{b^2}{4a^2}\right)
\]
Given two or more complex numbers in the form \((a + bi)\) or \((a,b)\), perform the fundamental operations of addition, subtraction, multiplication, or division, and express the result in \((a + bi)\) or \((a,b)\) form.

Given the roots of a quadratic equation which are in the form of the complex number \((a + bi)\), find the quadratic equation that contains those given roots.

Find the absolute value of the complex number \((a + bi)\).

Given the logarithmic equation \(f(x) = \log_b x\), identify the domain, range, and graph of the function.

Identify the use Laws of Logarithms.

Given an expression for \(n\) in terms of an indicated calculation involving a combination of multiplication, division, power, and roots of a real number, find the value of \(n\) using the table of mantissas of common logarithms (and the rules for operations with logarithms).

Given an exponential equation where both sides cannot be expressed as exact powers of the same base (other than 10), solve the equation by taking logs of both sides (table of mantissas provided).

Perform the operations of addition, subtraction, multiplication, and division on several polynomials in one variable.

Given a polynomial function \(f(x) = a_nx^n + a_{n-1}x^{n-1} + a_2x^{n-2} + \ldots + a_0\) where the real zeroes are rational numbers, find the real zeroes of the function.

Indicate that the fundamental theorem of algebra assures that every polynomial of positive degree over \(\mathbb{C}\) has at least one prime factor over \(\mathbb{C}\).

Given one complex root of a polynomial equation, find another root.

TRIGONOMETRY

Given a real number \(x\), find the point \((u,v)\) it maps onto using the wrapping function \(W: x \rightarrow (u,v)\).

Identify the graph of the sine, cosine, and the tangent functions, and specify the domain and range of the functions.

Given a specific value of a circular function, \(\theta < \theta < 2\pi\), determine the value of the other functions.

Use the Law of Sines to find the measure of the sides or angles of a triangle when given two sides and the angle opposite either side.

Given two sides and the included angle of a triangle, find the measure of the remaining side using the Law of Cosines.
Apply the sum and difference and half and double angle identities to find exact values such as
\[ \sin \frac{\pi}{12}; \cos 22\frac{1}{2}^\circ; \text{ and tangent } 105^\circ. \]

Given an angle in radians or degrees, convert one to the other.

Given a complex number \((a + bi)\) in polar form \((\cos \theta + i \sin \theta)\), find the \(n\)th power of the number \((r, \theta)^n\) given DeMoivre's Theorem.

Identify the basic Pythagorean trigonometric identities:
\[ \sin^2 \theta + \cos^2 \theta = 1; \tan^2 \theta + 1 = \sec^2 \theta; 1 + \cot^2 \theta = \csc^2 \theta. \]

Given the value \(x\), evaluate an arc \(\sin x, \arccos x,\) or an arc \(\tan x\) expression.

Given a work problem involving surveying or any related applied trigonometric area, solve the problem using Heron's Formula or trigonometric concepts of Law of Sine, or Law of Cosine.

V. ANALYSIS AND CALCULUS

Given any three of the following: the first term, \(a\); the difference, \(d\); the number of terms, \(n\); and the sum, \(S\); find the remaining terms.

Given any three of the following: the first term, \(a\); the common ratio, \(r\); the number of terms, \(n\); and the sum of the first \(n\) terms, \(S_n\); find the remaining terms.

Given the binomial expression \((a + b)^n\), find the value of the \(k\)th term using the binomial theorem.

Given \(a\) and \(r\), determine the sum of the infinite geometric sequence \(a, ar, ar^2, \ldots, ar^{n-1}\), where \(|r| < 1\).

Find the limit of a convergent sequence, given the rule for its \(n\)th term.

Given a polynomial function, use theorems concerning limits to find the value of the limit of the function.

Find derivations of a variety of functions which exhibit knowledge of sum, difference, product, quotient, and power formulae as well as ability to find derivatives using the definition of derivative.

Find the absolute value of the complex number represented by:
\[ r = \sqrt{a^2 + b^2}. \]

Given functions \(f(x)\) and \(g(x)\) where \(h(x) = f(g(x))\), find the derivative of \(h(x)\) using the chain rule.

Given a verbal problem, apply the concepts of derivative to find maximum or minimum value of a polynomial function, if such values exist.

Given a curve \(f(x)\) in the first quadrant for the interval \([a, b]\) use an integral to find the area under the curve.

Apply the fundamental theorem of integral calculus \[ \int_a^b f(x)dx = F(b) - F(a) \] to solve problems involving approximation of area between two curves, distance, volume, area of a surface of revolution, and work.

PROBABILITY & LOGIC

Given an "if and only if" statement, identify the hypothesis and conclusion in each case; and given a conditional statement, identify its converse, inverse, and contra-positive.

Recognize examples of conjunction, disjunction, negation, equivalence, and DeMorgan's Law, and their truth value; and complete a truth table to determine whether a given proposition is a tautology.

Given statements containing the universal quantifier and/or existential quantifier, recognize the quantifiers and show how the statements can be negated.
Contrast inductive reasoning and deductive reasoning; in particular show that false statements can result if both parts of the principle of mathematical induction are not satisfied.

Given several samples of proofs, distinguish between those which are direct and those which are indirect.

Determine the patterns of inference used in a given proof or proofs.

Given a graph and a set of questions, answer the questions by interpreting the statistical data presented by the graph.

Determine how certain sampling procedures fail to give consideration to the concept of randomness.

Given specific data, find the mode, the mean, and the median.

Given both specific data and probability distribution of a random variable, determine the mean, the standard deviation, and the variance.

Given a well-defined experiment, use a tree diagram formed by the sample space to describe the possible outcomes to the experiment.

Apply the Fundamental Principle of Counting in a variety of problems, including the use of tree diagrams.

Given a set of n distinct elements, find the number of permutations \( P \) of n elements taken r at a time where n and r are positive integers, and \( r \leq n \), and no elements are repeated, by using the formula:

\[
P_r = \frac{n!}{(n-r)!}
\]

Given a set of n distinct elements, find the number of combinations \( C \) of n elements taken r at a time where n and r are positive integers, and no elements are repeated, by using the formula:

\[
C_r = \frac{n!}{r!(n-r)!}
\]

Relate binomial coefficients to the problem of determining the number of r-element subsets in a set containing n elements.

Given the number of trials \( n \) and the probability of \( p \) success \( (p) \) determine for a binomial distribution the probability of obtaining exactly \( x \) successes in \( n \) trials.

Given a graph of the standard normal curve of a sample space, find the area under the normal curve within a given interval on the x-axis.

Given two events A and B, solve simple probability problems dealing with conditional problems and test for independence; include the special case when A and B are disjoint.

VII. MEASUREMENT, RELATIONS & COMPUTERS

Compute lengths using any standard units, including metric, and identify approximate conversion equivalents from metric units to customary units.

Find areas of plane figures and surface areas of simple space figures.

Measure angles and/or estimate angle measurements.

Select and apply the appropriate formula in finding measures such as perimeter, circumference, area, or volume.

Select appropriate weight (mass) unit, including metric, and apply in problem situations.

Estimate measurements in a variety of problem situations, and choose an appropriate unit to measure a given quantity and state the
greatest possible error of the measurement using that unit.

Identify a relation as a set of ordered pairs, specify the domain and range, and identify the function as a subset of a relation.

Determine whether or not a given relation is an equivalence relation (satisfying the transitive, reflexive, and symmetric properties).

Illustrate how an equivalence relation on a set partitions that set into equivalence classes.

Identify and use properties of inequality relations.

Given a polynomial function, find the inverse function, if it exists, and describe the restrictions necessary for the inverse to be a function.

Determine the Cartesian product of a given set with itself, or another set.

Identify the contributions of such celebrated mathematicians as Gauss, Newton, Archimedes, Euclid, Boole, Cantor, Euler, Descartes, and Pythagoras.

Given a computer algorithm such as divide and average for taking square root, complete a flowchart for the algorithm.

Apply arithmetic computation in consumer and business situations, such as bank records, finance, commissions, taxes, and insurance.
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