This issue contains abstracts and critical comments of 10 articles. Studies included focus on sex differences in mathematical learning, intuitive functional concepts, multiple embodiments of place value concepts, instructional activities of high and low achievers, effective management at the beginning of the school year, cross-age peer tutoring, problem solving, cross-national comparisons, and subtraction. Research reported in RIE and CIJE between April 1983 and June 1983 is also listed. (MNS)
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Center for Science and Mathematics Education
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Abstract and comments prepared for I.M.E. by GRACE M. BURTON, University of North Carolina at Wilmington.

1. Purpose
This five-year longitudinal study was designed to determine what sex differences emerged in a sample of mathematically precocious youth from the time they were identified in grade 7 or 8.

2. Rationale
In six of the Johns Hopkins talent searches, there was a large sex difference in mathematical reasoning ability. Despite lack of differences in SAT verbal scores between the sexes, boys performed at a significantly higher level than did girls. While much of the sex-related literature suggests that differences will not be apparent when course-taking behavior is controlled, the sex differences were significant at the seventh-grade level, when the course-taking history of boys and girls is identical. The authors hypothesized that the male superiority evident at the seventh-grade level would increase during the high school years and would be at least partly accounted for by the early advantage in mathematical reasoning as evidenced on the SAT-M.

3. Research Design and Procedures
Participants in the first three talent searches of the Study of Mathematically Precocious Youth (SMPY) who scored at least 390 on the SAT-M or 370 on the SAT-V as seventh and eighth graders were sent a questionnaire four to five years after they took the tests. Four yearly "waves" of questionnaires were sent, the first in December 1976. The final response rates, after two follow-up procedures were employed, ranged from 90 to 94 percent, resulting in a total sample of 1,966 students. The
sex composition of the final sample (62% male) was not significantly different from that of the population (61% male). While there were no differences in mathematics or verbal scores between male respondents and non-respondents, female non-respondents were significantly less mathematically able than their responding peers. The data were analyzed separately for waves one and two. A third analysis was done on waves three and four combined. A variety of SPSS produces were employed.

Effect sizes were calculated and classified as small, medium, or large (Cohen, 1977). Only large effects were considered important.

4. Findings

At the end of high school the sex difference in SAT-M scores in favor of boys was significant at the .001 level and, as measured by effect size, of importance. A significant difference in SAT-V scores favoring females in the second wave of the talent search had disappeared by the time of the follow-up study. There were no significant sex-related differences in verbal scores by the time of the high school administration of the SAT.

Seventh grade SAT-M was the best predictor with respect to courses in mathematics taken during high school. The significant difference (p < .001) in favor of males, because the effect size was small, was not considered important. Girls reported receiving somewhat better grades (p < .001) than did boys. The effect in this case was deemed important. Boys tended to take courses earlier during their high school careers and took calculus more frequently than did girls. These factors were rated as important. Only small amounts of variance in mathematics course-taking could be accounted for by family background. The best predictor was a retrospective one, having rated mathematics as a favorite course in high school. Liking for mathematics at the seventh-grade level was not a strong predictor, nor was sex or ability.

More boys than girls took the College Board Math Level 1 test. While the difference was significant (p < .01), the effect was small. Male scores, however, on the more difficult Level 2 test were importantly larger. The ratio of males to females taking these or the AP examinations
was sometimes as large as 3 to 1. Differences in participation in mathematics contests during high school were in favor of males, significant, but not important.

No significant differences in reported college major were disclosed. There were significant, but not important, differences in favor of males in number of students electing mathematics in their first college semester.

Asked to rank their liking of mathematics on a single global scale, males and females responded similarly. Girls, however, were more likely than boys to prefer verbal areas in high school.

5. Interpretations

Socialization appears not to be the explanation for sex differences in mathematics. Differences in mathematical reasoning ability are found in mathematically gifted youth as early as the seventh grade, when they could not be the result of differential course-taking. These differences have an effect on mathematics achievement.

Male superiority in mathematics reasoning ability might be due to the fact that mathematically gifted males are developing intellectually at a faster rate than are mathematically gifted females. Mathematics course grade differences in favor of girls may be explainable by the better conduct and demeanor often found in female students. The fact that females take fewer mathematics courses despite these better grades may be due to stronger liking than is true for males of verbal areas.

Twelfth grade appears to be too late for intervention efforts designed to increase female participation in mathematics. Planners of these intervention efforts should take note that mathematical reasoning ability may be a more important predictor of mathematics achievement than is attitude towards mathematics. Factors contributing to the differences in achievement have still not been isolated.

Questions remain as to the degree to which the findings which were disclosed in this select group of mathematically talented youth can be generalized to other populations.
Abstractor's Comments

This ambitious (and expensive) study provides data for considering the mathematical course-taking behavior of a small but special group of students—the mathematically precocious. The study was reported at great length, and extensive tables were included for those who wish to delve more deeply into the responses of the nearly 2,000 students in the sample.

The authors chose to use two criteria for reporting "significance"—first, the usual significance levels expressed as probabilities; second, effect size (Cohen, 1977). Fuller explanation of effect size, since it played a major role in the report of findings, might have been included in the text.

Benbow and Stanley enter, not for the first time, an area of hot debate: with respect to sex differences in mathematical ability, is it Nature or Nurture? They state that a satisfactory answer is not yet possible, but repeat their 1980 conclusion that "putting one's faith in boy-versus-girl socialization processes as the only permissible explanation...is premature" (p. 620). They do seem to accept, however, that the "ability of males developed more rapidly than those of females" (p. 598). It may be a little premature for that conclusion as well.

This abstractor is not convinced that seventh-grade boys and girls, even (especially) in 1972-4, had identical experiences in and out of school. True, up to that time they had taken the same number of mathematics courses, usually from the same teacher and in the same physical classroom. There is, however, no guarantee that the psychological classroom was the same for both. Copious research from that time period (c.f. Casserly, 1975; Marlow and Davis, 1976) would suggest that it was not. Teacher expectations' (Levine, 1976) and behavior (Good, Sikes and Brophy, 1973; Caplan, 1977) were found to vary according to sex of student. Nor was it, at that time, apt to be the case that parents (Helson, 1971; Astin, 1975; Rubin, Provenzano and Luria, 1976) or peers (Luchins, 1976) provided a sex-neutral environment where academic aspirations or leisure activities were concerned. It is unlikely that a girl gifted in mathematics was provided the same encouragement and support...
as an equally gifted boy.

Even, however, if it were proven to be the case that gifted boys are innately more mathematically talented than gifted girls, the question remains: what shall we do with the finding? The fascinating applications of science that allow future parents to control in part the genetic make-up of their children are as yet ineffective in disclosing which babies will develop into scientists and mathematicians or into people who will use these disciplines to enrich the lives of others. Given this lack of data—and the enormous variation in individual capabilities and interests—it would seem as Stephen Gould suggests, that imposing a biological value upon groups is an irrelevant and highly injurious enterprise (1980, p. 159).

We can, on the whole, do little more than nurture talent where we find it, and spend currently-scarce resources developing ways to do this. Since more academically talented males than females take calculus and other advanced courses, could we make these courses more appealing to the female cohort? What could counselors, teachers, principals, or business people do to make non-required courses attractive and available? How can the mystique surrounding the recreational uses of mathematics be neutralized? If social pressures—so important during the adolescent years—impact differentially on the mathematically gifted student according to sex, is this remediable? If indeed boys are better mathematics reasoners than girls and by the seventh grade, when does this superiority, which is not apparent during the elementary school years (Hooper, 1975), begin? If the parental influence on achievement attitudes continues strong (Parsons, Adler and Kaczala, 1982), what can be done about it?

Benbow and Stanley are to be congratulated for helping raise so many questions, the answers to which can guide intervention programs. The task remains to find those answers, develop those programs, and implement them effectively.

References


1. **Purpose**

"This study aimed to assess the intuitive background of junior high school pupils as they developed the concept of function" (p. 361). For the purpose of this study "the term 'intuitions' is taken to refer to mental representations of the facts that appear self-evident" (p. 360).

2. **Rationale**

The authors believe that intuitions play an important role in the understanding of mathematics and should be taken into account by teachers and curriculum developers. They feel that "the teaching process should be based on the intuitive knowledge of the learner, especially at the stage when a new topic is approached" (p. 361). The specific topic chosen for this study was the concept of function and a number of its subconcepts (i.e., image, preimage, extrema, growth, slope). Since intuitions are the result of personal experience, it cannot be expected that all students will have the same intuitions about functions. If instruction is to be intuitively based then "there is a need to assess first the basic intuitions and experiences various student populations have with functions ..." (p. 366).

3. **Research Design and Procedures**

Three versions of a 42-item multiple-choice questionnaire booklet were constructed. Each version contained both a concrete function and an abstract function. The three versions contained the same functional relationships, but differed in setting -
either diagram, table, or graph. The breakdown of items for each was: five image, five preimage, and five extrema questions for both the concrete and abstract functions; five growth questions about the concrete function; five slope questions about the concrete function; and three slope questions about the abstract function. As a validity check, all included questions were classified by subconcept by at least four out of a panel of five high school and college mathematics teachers. KR-20 reliability was estimated at .91 for the full test and at .86 and .81 for the concrete and abstract subtests, respectively.

The three versions of the questionnaire were randomly distributed by teachers within 24 coeducational classes in grades six through nine in 12 different schools "at the beginning of the school year when none of the classes had yet studied the concept of function" (p. 369). Students in grades eight and nine had studied a unit on Cartesian coordinate systems in grade seven. Schools and classes were chosen to ensure homogeneous distribution over grade level and over an ability-social level variable labelled "absolv." (Students were classified high or low-absolv according to their ability level and the percentage of disadvantaged students in the school they attended.) "In summary, each pupil was assigned four characteristics, Grade (6, 7, 8, or 9), Absolv (high or low), Setting (D, G, or T), and Sex (F or H)" (p. 369). 443 students completed at least 90% of the questionnaire and were included in the analysis.

4. Findings

A four-way analysis of variance, using the mean score on the total questionnaire as the dependent variable, yielded the following significant effects: grade, absolv, setting, grade x absolv, absolv x sex, and grade x absolv x sex. Further analyses revealed the following:
1. Although in general performance increased with grade, there was a significant decrease between grades seven and eight.

2. High-absolv students outperformed low-absolv students at all grade levels, but the main progress for the high-absolv students came between grades six and seven, while that for low-absolv students came between grades eight and nine.

3. The diagram setting presented more difficulties than the other settings at all levels of grade and absolv. The high-absolv students outperformed the low-absolv students on all settings, but the high-absolv students preferred the graph setting while the low-absolv students preferred the table setting.

4. Even though the overall performance difference between boys and girls was non-significant, boys outperformed girls in grades six and seven while girls outperformed boys in grades eight and nine. Boys' (mainly low-absolv) performance dropped considerably between grades seven and eight, while that of girls remained relatively constant. Overall, high-absolv boys outperformed high-absolv girls while low-absolv girls outperformed low-absolv boys. However, in grade nine, high-absolv girls outperformed high-absolv boys.

5. Performance on the concrete and abstract subtests paralleled performance on the full test. All significant effects on the full test carried over to the concrete test, while all except setting and grade x absolv x sex carried over to the abstract test. The trends observed on the full test carried over to both of the subtests.

6. Image questions were answered best, while slope questions were answered worst. For slope questions, both high and low-absolv students preferred the graph setting, but for
all other questions the high-absolv students preferred the graph setting, while the low-absolv students preferred the table setting.

5. **Interpretations**

The authors reached the following conclusions:

1. Pupils' intuitions on functional concepts do grow with their progress through the grades.
2. No differences in the intuitions between boys and girls in junior high school were observed. However, there are indications that girls tend to develop their intuitions at a different rate from boys.
3. High-absolv pupils demonstrate correct intuitions more often than low-absolv pupils.
4. It is not true that intuitions in concrete situations are more often correct than in abstract ones.

**Abstractor's Comments**

For years scientists and mathematicians have written about how their intuitions have contributed to the discovery and development of many of their ideas. Indeed, the history of science is filled with episodes where intuitions have led to significant findings. Mathematicians and philosophers of mathematics have discussed the relationship between intuitions and mathematical reality and many great mathematicians have advocated an intuitive approach to mathematics instruction. Experienced mathematics educators and teachers are well aware that many topics are best taught through an approach that builds upon students' intuitive and common sense ideas rather than through a more formal approach. Unfortunately, it is the case that what is intuitively evident to one student may not be to another. Also, it is not clear how intuitions develop nor how they can best be utilized. Research on the nature, development, and role of intuitions is needed if we are going to capitalize on them in our teaching.
Unfortunately, the notion is even more difficult to study than it is to define. In this study the researchers have only indirectly tapped the intuitions they set out to assess. Multiple-choice tests do not seem to be the most appropriate way to study mental representations of self-evident facts. It seems that what the researchers found out was whose representations led to more correct answers. It would have been more interesting and fruitful if they had: (1) looked at how students' pre-instructional interpretations of the diagrams, graphs, and tables led to their making sense and extracting information from them and (2) related these ideas and behaviors to the concept of functional relationship. In this way, explanatory information might have been gathered bearing on questions such as: "Why did the high-absolv students prefer the graph setting while the low-absolv students preferred the table setting?" and "Why did the performance of low-absolv boys drop so drastically between grades seven and eight?", etc.

In any assessment like this one, it is very important to look at group differences, but in this regard the variable "absoiv" has some shortcomings. Not only does it lack a clear analog in other settings, but some of its classifications are troublesome. For example, high-level students in schools with over 80% disadvantaged students were classified "low" while low-level students in schools with under 20% disadvantaged students were classified "high". If possible, it would have been preferable to classify students on the basis of some standard measures of mathematical and other cognitive performance to allow for clearer interpretation and generalization.

The authors hypothesized that performance on the concrete function questions would be better than on the abstract function. I would have hypothesized the same. I wonder whether the inclusion of both on the same test had any affect on the end results. Did the concrete questions serve as hints to the abstract questions?
If a follow-up study is done, it would be interesting to look at concrete/abstract differences if there were separate tests. This could be done by giving each student only one of the two versions or by giving all students both versions with the order of administration counterbalanced.

It is very helpful for teachers to have information concerning their students' pre-instructional background on various topics. This study revealed some beneficial findings concerning students' untutored knowledge about functions.

Abstract and comments prepared for I.M.E. by LOYE Y. "MICKEY" HOLLIS, University of Houston-University Park.

1. Purpose
   The purpose of the study was to determine if using multiple embodiments rather than a single embodiment of concepts related to three-digit numbers resulted in greater understanding of selected place value concepts.

2. Rationale
   Educators and learning psychologists typically recommend and encourage the use of manipulative materials in teaching mathematics. This theory is supported by a number of research studies.

   Questions concerning the use of more than one material to teach selected mathematical concepts are often raised. Some mathematics educators believe presenting multiple embodiments of a concept will increase the student's ability to generalize the concepts and to not associate the concept with any one particular embodiment. Research studies on the use of multiple embodiments do not agree on their value.

3. Research Design and Procedures
   The subjects were selected from 50 middle-class students enrolled in the second grade. One student was initially eliminated and the remaining 49 were randomly assigned to one of two treatment groups. The final sample was two groups of 21 students, due to some students being absent too much and the need for equal-sized groups.

   This study employed a 2 x 4 factorial design, with repeated measures on the time dimension. Measures were taken on Days 6, 10, 13, and on Day 20 following a seven-day retention period.
Both groups were taught by the principal investigator. Coffee
stirrers (prebundled in hundreds, tens, and ones), Dienes' base-ten blocks (flats, longs, and units) and chips from "Chip Trading Activities" (green, blue, and yellow) were used with the multiple embodiment treatment. The only physical model used with the single embodiment treatment was the coffee stirrers.

There was a total of thirteen treatment periods of 30 minutes. The order in which the two groups were taught was rotated so that each group was taught first on alternate days.

4. Findings

There was no difference between the overall means of the two treatment levels. The time factor was significant beyond the 0.01 level and indicates there was a significant trend over time with the repeated measures dimension across both treatment levels. The interaction of Time-by-Group, however, was not significant, an indication that the growth pattern over time of the two treatment groups was roughly the same.

5. Interpretations

One possible interpretation of these results is that it does not make any difference whether one uses three concrete exemplars or only one to teach these numeration concepts to second-grade pupils.

In summary, given the possible interpretations of the results of this study, it cannot be stated categorically that one should or should not use multiple embodiments to teach selected decimal numeration concepts to children in Grade 2. What must be noted, however, is that this study, like several others, does call into question the advantage of using multiple embodiments of a mathematical concept in the instructional process.
Abstractor's Comments

The researchers noted that sensitivity of the measuring instruments, instructor enthusiasm, length of treatment, or previous experience of the subjects might have affected their results. One thing that was not noted was the number of subjects. An "N" of 21, even with repeated measures, is small when there may be other factors at work.

A question could be raised about the choice and/or number of physical models used with the multiple embodiment treatment group. The prebundled coffee stirrers and the Dienes' base-ten blocks are very similar, especially when used as they appear to have been used in this study. The use of one of these combined with the "Chip Trading Activities" might have proved more profitable.

Abstract and comments prepared for I.M.E. by JOHN ENGELHARDT, Southern Oregon State College.

1. **Purpose**
   Both higher- and lower-achieving classes of junior high English and mathematics teachers were studied in order to observe instructional strategies which differed between the achievement groupings. Within this study a subset of teachers was selected and the present article focused on the narrative data as well as inferential statistical data of this subgroup.

2. **Rationale**
   The author points to lack of specific research-based suggestions on how to differentiate instruction for ability groups. Given that classroom management is related to classroom composition (Doyle, 1979), it would be helpful to know which specific techniques worked for different class compositions.

3. **Research Design and Procedures**
   A sample of 51 teachers (25 English, 26 mathematics) from eleven junior high schools in a large southwestern urban district were observed in two of their class sections. Observers were trained in writing narratives focusing on management and organization, in rating student engagement (on task, off task, and shades in between), in rating specific components of the overall classroom behavior (44 specifics), and in maintaining time logs of the various classroom activities. Teachers were observed from 14-20 hours in each of two classes during the school year, with roughly half the observations in the first three weeks of school. A total of 1400 observations of one-hour duration was taken.
California Achievement Tests (CAT) from the previous year were used as covariates in measuring academic progress, with specially constructed achievement tests in mathematics and English administered at the conclusion of the school year after all observations were completed. Student attitudes toward school, instructor, and class were assessed prior to achievement testing using a form adapted from the Student Rating Scale of Instructors (Stallings, Needels, and Stayrook, 1979).

To assess observer reliability, pairs of observers were sent to classrooms on 23 occasions and complete data sets were checked. The only problem encountered seemed to be some deletion of narrative material. Regular meetings were held with observers to maintain consistent understandings. Between-observer agreements of component ratings were reported at \( p < .12 \) and student engagement ratings at \( p < .001 \). Complete information regarding these assessments can be found in Evertson et al. (1980).

A subset of the 51 teachers was selected for further study. These teachers (6 mathematics, 7 English) were determined by the fact that their two classes differed by two or more grade levels in mean entering achievement (based on the CAT). Teachers' low-ability classes had an entering mean of 2.8 grade levels below placement, while high-ability classes were 0.4 above grade placement on the average.

Two-way ANOVA with subject matter a between-groups factor and ability level a within-groups factor was used to examine differences between higher- and lower-ability classes.

4. Findings

Subject matter differences revealed \( (p < .05) \) that English teachers were rated higher in occurrence of verbal class participation, nurturing student's affective skills, and relating content to pupil interest and background. Ability differences revealed \( (p < .05) \) that higher-ability teachers were rated as nurturing
affective skills and maintaining a task-oriented focus. Lower-ability teachers had significantly more disruptive and/or inappropriate behavior and had more conferences to stop this behavior. The author reported other results with higher p values (.06 < p < .11).

Higher-ability classes across both subjects had a larger percentage of students on task (p = .05). Higher-ability classes also had more transitions than lower-ability classes, but the average length and total transition time were greater (p < .05) in lower-ability classes.

Evaluation of narrative data on mathematics classes revealed that teachers did not vary much in activity pattern either among themselves or between ability levels. There was no real differentiation of instruction across higher- and lower-ability classes. The pattern observed was essentially the following: opening (mostly procedural), checking and grading, lecture/discussion, seatwork, close. Additionally, the time allocated for these components did not differ significantly across teachers or levels of ability.

The author reported on a case study of the management techniques of two mathematics teachers in their lower-ability classes. Teacher B was labeled as "reactive" and teacher F as "proactive." Teacher B was plagued by inappropriate behavior which disrupted her attempts to help students. Teacher F, whose lower-ability class had the highest residualized achievement scores of the six mathematics teachers, differed in how he structured activities. He allowed more time for checking and discussion (13.7 minutes-F, 6.5 minutes-B) and lecture and seatwork introduction (14.4 minutes-F, 8.7 minutes-B) with substantially less time for seatwork (22.5 minutes-F, 35.1 minutes-B). He incorporated seatwork practice into his lecture, thereby distributing student involvement with the material and allowing for more immediate feedback. This resulted in higher task orientation for the class, as noted by observers.
5. **Interpretations**

Analysis of quantitative and qualitative data revealed that:

1) Lower-ability classes, although smaller in number, are harder to manage and keep on task than higher-ability classes.
2) Teachers did not differentiate instructional approaches for ability levels.
3) There are ways of providing instruction in low-ability classes to increase productive use of time and student involvement.
4) Active teaching (proactive) can be a useful way to view instruction.

**Abstractor's Comments**

This study contained a number of elements of thorough classroom research. The sample size was quite large, the data collected were quite detailed, and the length of study was of sufficient duration to justify belief in the findings. However, no mention was made as to how the sample was selected, so assumptions of randomness and implications of results for a population larger than the sample are questionable.

A small point, but one worth mentioning, was the consistent use of ability groups when in fact the students were grouped by achievement. Correct usage was made in the title and promptly abandoned. Little mention was made of the posttest, which was specially constructed. It is interesting to note that of the 12 mathematics classes in the substudy, five of the six lower-achievement classes and three of the six higher achievement classes had negative residual achievement scores. It is not clear how to interpret this. In addition, no units were reported for the posttest. It also seems that the CAT was used both as a covariate measure and as a high/low grouping measure which is not entirely appropriate statistically.
The richness of the narrative data was much appreciated by this reviewer, who found the case study report quite interesting. It laid some foundation for further exploration in search of effective instructional strategies for achievement groupings. What struck this reviewer was the surprising qualitative differences between teachers B and F, along with the corresponding lack of quantitative differences. Teacher B's residual achievement score was -.09, with 75 percent on-task academically and only 5 percent off-task. Teacher F had a residual achievement of .06, with 85 percent on-task academically and 6 percent off-task. This perhaps points up the significant value of narrative data in trying to ascertain what goes on in the mathematics classroom aside from the typical quantitative measureables.

Aside from the criticisms mentioned above, this reviewer thought highly of the study. Several thought-provoking and disturbing questions arise. Are we as mathematics teachers so set in our pattern that we fail to differ not only from each other, but for the students we teach? Are we so inclined to mold the student to our style rather than looking for ways to adapt?

Proactive teaching or direct instruction has research grounding (Good and Grouws, 1978, 1979) as an effective way to teach mathematics, especially to lower-achieving students at the intermediate grade level, but has not been as pronounced in success at the junior high level. Perhaps more study will bear out Teacher F's style as an appropriate way to differentiate instruction. Clearly something needs to be done to effect more academically productive time for students in these classes.

Abstract and comments prepared for I.M.E. by CHARLEEN M. DERIDDER, Knox County Schools, Tennessee.

1. Purpose

The purpose of this year-long study was to identify and assess beginning-of-the-year management practices of groups of junior high school teachers of English and of mathematics that were selected and categorized from the data gathered as more effective and less effective classroom managers. The study sought to answer the question of how more effective managers differed, if at all, from less effective managers in their management procedures during the first three weeks of school.

2. Rationale

The authors quote educators who point out the importance of management skills and note the correlation of management variables with student achievement gains. Several studies were cited that have contributed to the body of information on this subject. Most such studies have been short-term and cross-sectional. It was conjectured that a longitudinal study, such as this one, might provide a more adequate base for suggesting how to initiate classroom behavior to promote long-term management effectiveness.

3. Research Design and Procedures

Essentially, the study first accumulated data on 26 mathematics and 25 English teachers for a three-week period. These teachers were volunteers from 11 different junior high schools in the southwest. The study included three-fourths of eligible experienced teachers and one-half of eligible first-year teachers. These data were set aside, and then data were continued to be gathered on these teachers. At the end of the school year, four subsets of teachers (six more effective, six less
effective managers among mathematics teachers and seven more effective, seven less effective managers among English teachers) were determined on the basis of the data collected after the first three weeks. Analysis was then made of the data collected on these groups during the first three weeks.

The data gathered involved the comparison of means of more effective and less effective managers with respect to percent of students on-task, percent of students off-task, observer management factor, residual student achievement, and student rating of teacher. Selection of the subsample of the four teacher groups was based on computing and summing across these criteria which provided a composite management effectiveness ranking. These teachers taught classes that had similar average achievement levels.

The procedure of the study began with the work of 18 trained observers in the classrooms of these 51 teachers.

The first three weeks. Each teacher was observed on first, second and fourth day, then three or four times the next two weeks in one certain class. Each was also observed four or five times in a second class during the second two weeks.

The remaining school year. Observers were reassigned to observe different teachers. They observed each of two classes per teacher every three to four weeks.

The observation data included classroom narrative records based on a set of 42 guideline questions. Observers dictated a record from their notes into audiocassettes; a typical narrative was seven to ten pages long. A time use log was compiled on each teacher. A record of Student Engagement Rates (SER) was kept which described students as on- or off-task in academic or procedural activities for each class session. After each observation, the observer rated, on a five-point scale, selected managerial, instructional, and behavioral characteristics. These data are labeled Component Ratings (CR), and consist of 36 items which describe teacher and student behavior.

Narrative Ratings (NR) were compiled for a teacher's first three weeks based on the classroom narratives. The project staff made summary ratings of 29 behaviors and characteristics based on procedures used in
an earlier study of elementary teachers.

California Achievement Test score data from the preceding spring were used to determine class means as a basis of entering achievement levels. These means were also used as a predictor when computing residual student achievement. At the end of the year, students were tested in English and mathematics by tests which reflected the content of the district-wide adopted textbooks.

Once the subsample was identified, the focus of the study was an attempt to determine management behaviors which characterize more effective managers as compared to less effective managers during the first three weeks of school. The data gathered during the first three weeks were then analyzed in a variety of ways.

Student Engagement Rates of the four groups were compared using two-way analyses of variance. In terms of Component Ratings, the average rating on each variable was computed across observations and a series of two-way ANOVAs (more vs. less effective, mathematics vs. English) was run. Next the Narrative Ratings were analyzed using a series of two-way ANOVAs.

Additional analyses were performed to address several questions, such as, were there initial differences in student behavior in the classrooms of the more and less effective managers?

Reliability checks of the observation variables were performed using both between-observers agreement and between-periods stability coefficients. The reliability of the achievement and attitude measures were determined using internal consistency coefficients.

There was also a summary made of the correlations between each criterion and CR or NR variables which shows consistency across assessments, residual achievement, observer management factor, off-task, and academic on-task. One exception was the residual achievement criterion in English which showed weak correlations.

4. Findings

The answers sought by this study were to the questions of whether and how more effective and less effective managers differed in their behavior
at the beginning of the year. The researchers identified significance of results at the 10%, 5%, and 1% levels. The results of the SER variables indicated that more effective managers in both English and mathematics had high on-task rates at the 5% level; lower off-task, unsanctioned behavior rates and less dead time at the 10% level.

In terms of Component Ratings, there was a significant difference between more effective and less effective managers with respect to 16 of 36 items. Four of those were at the 10% level, seven at the 5% level, and five at the 1% level. These items included clarity in giving directions, stating desired attitudes, presenting clear expectations for work standards, and consistency of response to inappropriate behavior. More effective English teachers, (but not mathematics teachers) were rated higher than less effective teachers on the variables of describing objectives clearly, using materials that effectively supported instruction, and using and encouraging analytic processes.

Results obtained in the comparison of Narrative Ratings indicated significant differences in 22 to 29 items. One was at the 10% level, 13 at the 5% level, and eight at the 1% level. These items included instructional clarity and coherence, regular academic feedback to students, effective monitoring of student work, effective intervention to stop students from avoiding tasks, frequency of unsolicited call-outs (less for the more effective managers), and social talk among students during seatwork and lecture. Only a few subject matter effects and interaction affects were noted.

5. **Interpretations**

There are several broad themes indicated by clusters of variables differentiating more or less effective managers. More effective managers were more successful in teaching rules and procedures to students. These teachers were more attentive to and more immediately responsive to undesirable student behavior. They were more consistent in maintaining the rules of classroom procedure. More effective managers rated considerably higher than less effective managers in their ability to maintain student responsi-
bility for productive use of time. More effective managers were more successful in those variables related to communicating information clearly to students. Another major area of difference was that of organization of instruction. More effective managers had less wasted time in their activities and more time on task.

Although certain behaviors were identified as antecedent conditions in effectively managed classrooms, the conclusion cannot necessarily be made that they are causal factors. However, common factors between this study and other management research suggest that these behaviors contribute to year-long management effectiveness.

Abstractor's Comments

This detailed, in-depth, comprehensive study was supported in part by the National Institute of Education. The amount of time and effort expended in this study is certainly impressive. The work of the classroom observer required that he/she make notes based on 42 guideline questions, tabulate time use by students every 15 minutes, and complete a 36-item rating of teacher and student behavior on a scale of 1 to 5 for each class. Computation produces the estimate that each of 51 teachers was observed 32 times. The narrative record alone for each observation was 6 to 7 pages which means some 13,000 to 14,000 pages of data.

Great care was taken on the part of the researchers to determine the reliability of the assessment procedures and to guard against bias on the part of the observers. The fact that this study followed and, to some extent, was based on previous work provides continuity and reinforcement of significant variables with respect to effective classroom management. Also, the researchers are to be commended for the way selection of more effective vs. less effective manager subjects were identified for the study.

There are some questions and concerns, however, that might be mentioned.

1. Although data tables in the study indicated at which of the three levels variables were significant, the authors' discussion of the findings was generalized and did not differentiate among results with respect to
levels of significance.

2. Was there any disparity in the socio-economic levels of the students in the classes observed in the eleven different schools? If so, was there any correlation between this and the identification of the more vs. less effective management teachers?

3. Mention was made that the content of the end-of-year English test assessed mainly usage, punctuation, and spelling, while writing skills, literature objectives, and other communication skills were not addressed. It would appear that only lower-level cognitive skills were assessed in terms of student achievement. This might suggest that the identification of these classroom management skills characteristic of effective managers be qualified as those capable of producing student achievement of lower-level cognitive English skills.

4. In a similar way, the nature of the mathematics test requires description. Was it primarily a test of computational skills? Were there items involving concepts and applications of mathematics or use of problem-solving skills? It is conceivable that a teacher in the process of teaching problem solving might have students in the Polya phase of trying to "understand the problem". Such students tend to exhibit apparent off-task, non-purposeful behavior while mulling over the problem. Such behavior might be labeled "off-task" by the observers in this study.

5. In compiling the data, the authors list the mean scores of the four groups, indicating the differences between more effective and less effective management in mathematics and in English. Comparisons are then made by grouping the mathematics and English more effective managers together and the mathematics and English less effective managers together. It can be observed from the data in the tables that in four of the 16 variables found significant, the means of the more and less effective managers in mathematics differed by only 0.3 on a scale of 1 to 5, differed by 0.1 on one of these variables, and were identical on still another on the CR instrument. On five variables, where no significance was found, the mean scores of the mathematics subjects differed from 0.3 by as much as 0.7. On the NR instrument, there were two of the significant variables which
indicated the means of the mathematics subjects were identical. It might be useful to examine the data for the mathematics subjects separately.

In terms of the findings of the study, it would be interesting to know:

a. Were any of the more effective managers first-year teachers?
b. Was the number of years of experience a factor?
c. Was there a maximum of management effectiveness at any given number of years of experience?
d. Was there a greater incidence of more effective managers from any one or more of the eleven schools?
e. In the same vein, was there a greater incidence of less effective managers from any one or more of the eleven schools?

In light of the current tremendous concern for teacher accountability, a study such as this certainly has merit. While research appears to indicate that certain classroom management techniques are essential to student achievement, perhaps researchers should also give attention to that which is being achieved, or that which is recommended to be achieved by the educational community. Such findings might suggest a classroom climate that would add a different dimension to the management variables identified as significant in this study.
Abstract and comments prepared for I.M.E. by ROY CALLAHAN, State University of New York at Buffalo.

1. **Purpose**

The study examined the cross-age peer tutoring process, with special consideration being given to the social dynamics in the tutoring situation. Particular attention was focused on attitudes between tutor and tutee, and the motivating effect of the process on the students involved in the tutoring process.

2. **Rationale**

Studies have pointed to increased increments of achievement by tutors and tutees involved in the tutoring process. To what are such increments attributable? An obvious response is that it provided increased time on task. However, a number of studies attribute the change to the social dynamics involved in the tutoring situation. This study drew heavily from the works of Lippett (1976) and Sarbin (1976), who suggested that the tutor-tutee relationship is the primary reason for the beneficial impact of the tutoring process on achievement. This study attempted to describe the attitudinal and motivational factors at play in the dynamic tutor-tutee interactions.

3. **Research Design and Procedures**

The study took place in a university K-8 laboratory school. Three multi-age groupings were made: primary, intermediate, and middle school. It was a two-phase study: (1) remedial mathematics instruction and (2) computer literacy instruction. Three male and three female middle school tutors worked with 11 male and one female tutees from the primary and intermediate groups in phase 1 (remedial mathematics). Six male and no female middle school tutors worked with four male and two female inter-
mediate tutees in phase 2 (computer literacy). Tutors were selected from volunteers on the basis of interest in tutoring and proficiency in mathematics.

Tutors received four 30-minute training sessions prior to their first tutoring session. These sessions introduced them to the mathematics content and material of the tutoring lessons and also focused their attention on diagnosing tutees' problems, finding alternate instructional explanations, and providing encouragement to tutees to complete lessons. A role-playing procedure was used in these training sessions.

In phase 1 (remedial mathematics), six tutors were randomly assigned to work with two tutees, one at a time, for 30-minute sessions twice a week for eight weeks. In phase 2 (computer literacy), six tutors worked with six tutees (one-on-one), again for 30-minute sessions twice a week for eight weeks.

During phase 1 verbatim verbal protocols of verbal interactions between tutor and tutee were collected. Four ten-minute observations were made by two trained observers during the middle six weeks of the study. A blind rater checked protocols recorded independently by the two observers. During phase 2, the tutoring sessions were tape-recorded and then transcribed to obtain four ten-minute observations similar to phase 1.

Independent variables examined were: (1) affective dimensions of verbal behavior categorized as to (a) locus of initiative, (b) influence, (c) verbal reinforcement; (2) instructional verbal behavior dimensions such as explanations, providing examples, or asking questions; (3) student learning progress as measured by task completion rates in mathematics, and performance on a 20-item multiple choice test on computer literacy; (4) tutor-tutee attitudes and perceptions based on interviews of these sets of people and their teachers.

4. Findings

In regard to the affective dimensions of verbal behavior, there were no significant differences between verbal interactions initiated by tutors or tutees. This result indicated that both tutor and tutee take active
roles in the tutoring process. In regard to influence, verbal behaviors considered "directive" accounted for about 31% of the tutor behavior; however, about 50% of the tutee behavior was "directive." This result indicated that the tutor-tutee relationship was a give-and-take situation with assumption of the "directive" role quite evenly distributed. Approximately 20% of the tutor behaviors were classified as positive reinforcement while 12% were classified as negative reinforcement.

In regard to instructional verbal behaviors, tutors tended to limit most of their instructional behaviors to explaining directions, asking questions, confirming correct responses, or pointing out incorrect responses. A majority of the tutees' instructional verbal behaviors were related to answering questions from the tutor or instructional materials. These results indicated that tutors employed a very restricted number of instructional behaviors in the tutoring process.

As a further descriptive refinement, the study examined the extent to which interactions between tutors and tutees are related to age and sex of the tutoring dyads as well as the nature of subject matter taught. Regarding age, it appeared that with older tutee groups more verbal behaviors were initiated by the tutee than the tutor, and there was also significantly greater frequency of tutee responses to tutor questions in dyads with younger tutees. Regarding sex influences, it was found that a significantly greater proportion of verbal behaviors was initiated by the tutee rather than the tutor in same-sex dyads when compared with different-sex dyads. Different-sex dyads indicated a significantly greater frequency of tutee responses to tutor questions and statements than did same-sex dyads. No differences were found in any of the behaviors when the dyads were divided according to differing subject matter.

In regard to student learning progress, the tutees' task completion rates in mathematics were lower than the class rates before the tutoring began, surpassed them during the program period, and maintained these gains even after the program was completed. With the computer literacy phase, there was a significant difference in gain score for the tutee group when compared to a comparison group on age and mathematics achievement.
In regard to attitude and perceptions, tutor interviews indicated that a majority expressed a positive attitude toward the program and a desire to stay with the same tutee. There was some slight tendency indicated to work with a different student—mostly female tutors who requested a change to a same-sex tutee. Tutee interviews indicated that a large majority of tutored students had positive feelings about the experience. Both "receiving" (teachers of the tutees) and "sending" (teachers of the tutors) teachers viewed the program as an effective way to provide remedial instruction to slow students.

5. Interpretations

This descriptive study of the interpersonal dynamics at play in a classroom cross-age peer tutoring situation involving remedial mathematics and computer literacy instruction tends to provide evidence that the tutor's role is based more on friendship than on teacher-like authority. The data suggested a situation that could be characterized as a give-and-take friendship between peers rather than one where the tutor dominates and directs; initiating and directing roles interchanged between tutor and tutee with relative equality. The data indicated that the student tutors used a very restricted range of instructional techniques, which suggests that the tutoring process may be most useful for practicing or reviewing tutee's skills rather than developmental work. The data also suggest that age and sex of students in the tutoring dyad may affect the character of the interactions in the tutoring process. Same sex and similar age dyads may provide an instructional situation based on give-and-take friendship; opposite sex and dissimilar age dyads may tend toward more of a teacher-like authority characteristic in the tutoring situation.

Abstractor's Comments

For a number of reasons, this is not a very important study. However, it does make some minimum contribution to the accumulation of knowledge about cross-age peer tutoring as an instructional procedure.

The formal use of older and more knowledgeable students to assist
younger, less knowledgeable students in schools has quite an extensive history. Monitorial schools gained much popularity in this country during the first half of the 19th century, in great measure due to the limited funding for education. Necessity being the mother of invention, schools began to utilize students (mostly boys) who knew a little in teaching the others (again mostly boys) who knew less. In a fit of hyperbole, the English educator, Lancaster, whose name the system popularly assumed, wrote, "The system spread from Thames to Ganges; it has encircled the equator; it has encompassed the poles" (p. 9).

Although the formal monitorial school faded from the educational scene around the mid-19th century, there has been some continued interest in the use of students as tutors in the schools. A strand of research has developed that has not only tried to assess the impact of the procedure on the achievement of the tutor and the tutee, but also to understand better the dynamics at work in the tutoring session. It was the latter that was addressed by this piece of research.

Great care must be taken in interpreting this research because of the small and selective number of students involved. This shortcoming is somewhat mitigated by the fact that the study was an intensive examination of affective factors at play in the tutor-tutee situation. Yet care should still be taken when generalizing from the data.

Probably the main contribution of the study is the additional support it provides for Sarbin's (1976) contention that the contribution of tutor to tutee may come from that person's role as friend that may be beneficial in the situation, and not the fact that more teaching time is provided the student. This study also suggests that where there is a sex difference, and greater age differentials between tutor and tutee, the role of tutor may take on more of a teaching character and less a give-and-take friendship character. The friendship role appears to be optimized when the tutor and tutee are of the same sex and have little age differentiation.

Another limiting factor in the study was the measures of student progress used in the remedial mathematics phase. It appeared that tutees increased their pace of going through instructional materials when working
with a tutor, and upheld the pace after tutoring was withdrawn. But nothing is mentioned of the quality of the learning taking place as students go through the instructional materials. It may be that faster is not necessarily better for the slower students who served as tutees.

And finally, the data suggested that tutors were not particularly adept in the instructional process, and were most effective for practicing or reviewing the tutees' skills. This tends to bring us back to the monitory role played by older and more knowledgeable students in the Lancasterian scheme of instruction 100 and 75 years ago. If there were nothing beyond this instructional role, then the monitoring might be better done by a CRT attached to a microcomputer. But this study, along with others, suggests that there may be a more critical dynamic at play between tutor and tutee that makes a contribution to the tutee (and perhaps the tutor). Naisbitt (1982) has used the terms "high tech" and "high touch" when describing the need for counterbalancing human responses (high touch) with new technology (high tech) that is introduced into a society. This study provides an additional glimpse of high touch at play in an instructional setting.

References


Lee, Kil S. **FOURTH GRADERS' HEURISTIC PROBLEM SOLVING BEHAVIOR.**

Abstract and comments prepared for I.M.E. by JACK EASLEY, University of Illinois at Urbana-Champaign.

This study assesses the results of teaching a group of eight fourth-grade children how to use four of Polya's heuristics on story problems, most of which involve combinatorics or proportionality. Heuristics were demonstrated for five sessions and practiced for 15 more, with one story problem to a session. When given similar story problems in individual interviews afterwards, the group instructed performed phenomenally better than a group of similar children who went to regular mathematics classes in school during the same time. Since nothing is said about what they did, we may presume that they did not study combinatoric or proportionality story problems, since these are rather unusual in fourth grade.

Another comparison was made between two sub-groups of the eight children who were taught in the unusual way. One group of four were average students who had also performed at level IIA on the Inhelder-Piaget pendulum and balance tasks and the other group were four above-average students who performed at level IIB on those two tasks. It was found that, on four of the six post-instruction story problems where multiplication was appropriate, all of the above-average, level IIB students used multiplication, and two of them used multiplication on each of the other two problems where it was appropriate. (There were just two problems on which multiplication was inappropriate.) Among the four children who were judged of average ability and who performed at level IIA, only two multiplied once each — they added much more often.

**Abstractor's Comments**

The research design confounds training in heuristics with training in working combinatoric and proportionality problem, and it confounds
The teacher's judgment of student's ability with the level on Inhelder-Piaget tasks. The author concludes the study with five "hypotheses based on the observed results and the theoretical rationale of the study". Two of them appear, because of the design, not to be based on the results and it is unclear what their theoretical basis is. They are: "1. Specific heuristics adapted from Polya can be effectively incorporated into the problem-solving experience of fourth graders."

... Hypothesis 4: In a problem-solving situation where multiplication is appropriate, IIA children use addition procedures primarily, and IIB children use multiplication as well as addition procedures." Possibly, by calling these and other conclusions "hypotheses," it is intended to protect them from criticism.

There are also two pages devoted to summarizing the strategies children used on particular problems in the interviews. Because the problems are unusual, the errors tend also to be unfamiliar. A more descriptive account of the thought processes of children during instruction, as well as during interviews, would have been helpful. This study is a mixed type, which leaves out much that would be expected from both experimental and clinical perspectives. Perhaps that is the fate of mixed studies.
1. **Purpose**

   In order to better understand cross-national differences in mathematics achievement that have been found at the secondary school levels, relationships among elementary school curricula and mathematics achievement at grades 1 and 5 in Japan, Taiwan, and the United States were investigated.

2. **Rationale**

   Such cross-national investigations are seen as valuable for understanding the influence of social, cultural, and educational factors on students' learning. In order for meaningful interpretations to be possible, differences in curricula must be taken into account in the construction of tests to be used in the research.

3. **Research Design and Procedures -- Part I**

   First an analysis of the most recent, popular textbook series used at each of the sites [Sendai, Japan: *New Mathematics* (1978); Taipei, Taiwan: *Public Elementary School Mathematics* (1978); and Minneapolis: *Mathematics Around Us* (Scott-Foresman, 1978)] was conducted. This analysis consisted of the construction of a list containing each concept and skill presented and the grade level and semester in which they were first introduced.

4. **Findings -- Part I**

   Of the 320 topics listed, 64% appeared in all three curricula, 91% appeared in the Japanese series, 81% in the Taiwanese series, and
78% in the American series. Thus Japanese textbooks expose children to more topics than do textbooks from Taiwan or the United States.

In terms of school year and the introduction of concepts/skills, Taiwan was behind the other two countries at the middle of grade one and the middle of grade five. The American curriculum kept pace with that of Japan through the first grade but was behind by the middle of the fifth grade. Throughout the six years of the elementary school curricula, only 26 of the topics were introduced during the same semester in all three countries.

The curricula analysis provided a base on which to build a 70-item achievement test designed for individual administration. Items were ordered according to the mean grade level at which the underlying concepts or skills were introduced. A combination of native and bilingual speakers was used to ensure comparability of items across the three language groups.

5. Research Design and Procedures -- Part II

Random samples of children from 40 classrooms in 10 schools chosen to "represent a random sample of elementary schools" in each of the three locations were selected. Two boys and two girls were randomly selected from the upper, middle, and lower thirds of the distribution of reading scores obtained in each classroom, resulting in a total of 240 first graders and 240 fifth graders from each country. Sendai and Minneapolis were cited as comparable in size and general economic and cultural status. Taipei was noted as comparable in size.

First graders started at Item 1 and continued until four successive items were missed. Fifth graders began with Item 35, which had a lower than fifth-grade level of difficulty, and continued until four successive items were missed. If a fifth grader missed any of Items 35–38, the child was taken back to a lower level item.

The test showed high internal consistency, with Cronbach's alphas ranging from .93 to .95.
6. **Findings -- Part II**

Differences between boys and girls were not statistically significant at the .05 level. For both grade levels, children in the United States had significantly lower scores than did children in Taiwan and Japan, even when the topics were known to have been covered in the American textbooks. On both story problems and computational skills, Japanese children scored higher than the children from Taiwan at grade 5 but not at grade 1.

**TABLE 4**

<table>
<thead>
<tr>
<th>Country</th>
<th>Boys M</th>
<th>Boys SD</th>
<th>Girls M</th>
<th>Girls SD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grade 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>20.7</td>
<td>5.7</td>
<td>19.5</td>
<td>4.6</td>
</tr>
<tr>
<td>Taiwan</td>
<td>21.2</td>
<td>5.4</td>
<td>21.1</td>
<td>5.6</td>
</tr>
<tr>
<td>United States</td>
<td>16.6</td>
<td>5.5</td>
<td>17.6</td>
<td>5.2</td>
</tr>
<tr>
<td><strong>Grade 5</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>53.0</td>
<td>7.5</td>
<td>53.5</td>
<td>7.5</td>
</tr>
<tr>
<td>Taiwan</td>
<td>50.5</td>
<td>6.4</td>
<td>51.0</td>
<td>4.9</td>
</tr>
<tr>
<td>United States</td>
<td>45.0</td>
<td>6.5</td>
<td>43.8</td>
<td>5.9</td>
</tr>
</tbody>
</table>

It was found that more classroom time was devoted to mathematics instruction in both Japan and Taiwan than in the United States. American first-grade students reported, by far, the least amount of mean number of minutes spent each week on homework: Sendai, 233 minutes; Taipei, 496 minutes; Minneapolis, 79 minutes. The respective means at grade 5 were 368, 771, 256. American parents also spent the least amount of time assisting their children in homework. Average class sizes reported were: Taiwan, 47; Japan, 39; the United States, 21.
7. Interpretations

The superior performance of Japanese children is partly related to Japan's advanced curriculum; however, factors other than curriculum appear to be critical in accounting for American children's lagging behind children from Taiwan and Japan. Factors suggested by the data include amount of instruction time, amount of homework, and amount of parental involvement.

Abstractor's Comments

The authors have appropriately noted that differences in curricula must be taken into account in order to begin to understand cross-national differences in achievement. They further noted that methodological limitations in cross-national studies often hinder interpretations. Both assertions address issues critical to the study that they themselves have reported.

An analysis of textbooks served as the main method of acquiring an understanding of the content of the mathematics curriculum. Classroom observations to study the curricula were seen as too costly. As a result, the curriculum analysis did not include any significant details on actual implementation in the classroom. Such a shortcoming severely limited the study.

Although many educators stereotype teachers as relying almost solely on the textbook to formulate lessons, we do not know if indeed this was the situation for the teachers at the three sites. Furthermore, by using a procedure in which coders merely checked whether or not a concept or skill was present, the study implicitly assumed equivalent quality in the writing of the textbooks. [The authors use the term "quality" in a different sense — basically to mean "higher level" or "more advanced" (p. 317)]. Another problem, which the authors did indicate, was the fact that no attempt was made to determine the relative importance of concepts or skills in each curriculum.
These concerns, combined with the questionable representativeness of the three cities in the study, dictate that the study be treated as tentative, perhaps more so than did the authors.

Some positive points in the method used include:

a) random sampling with stratification by sex and achievement level (but by reading level rather than mathematics achievement);

b) the strategy used to eliminate children with IQ's below 70; and

c) the manner in which the test was constructed based on the curriculum (i.e., textbook) analysis.

Intriguing but not fully reported are the parent interviews and classroom observations. It was reported, for example, that observations showed that the American children spent less class time in mathematics; however, data on the children's on- and off-task behaviors were collected but not reported. Benjamin Bloom (1981) has stated that research from the international study reported by Husén (1967) showed Japanese students having a substantially higher engagement rate (on task) than did American students. Bloom asserted that the observed differences in engagement rate were large enough to fully account for the achievement differences. Given these earlier research findings, it would seem desirable to analyze the achievement differences in terms of time on task.

Despite its methodological shortcomings, the study has provided evidence that there are notable differences in curricula as well as achievement among the sites studied in the three countries. In order to determine more precisely the reasons for these differences, it would be necessary to make the following improvements (albeit costly) in the design:

a) a curriculum analysis that includes an investigation of the quality and emphasis of the various concepts or skills as written in the textbooks;

b) an observational study of the implementation of the curriculum;

c) a covariate measure of students' mathematical ability; and

d) a sampling from other geographical areas in the three countries.
It would appear that the differential amount of time spent (including both instruction and homework) on mathematics would again turn out to be the leading candidate for "causing" the achievement differences. On the other hand, an analysis of the quality of instruction [e.g., in terms of the use of active teaching behaviors that have been shown to be related to student achievement (Brophy, 1979)] may reveal other explanations for the differences in achievement.

Finally, a major reason for conducting such cross-national studies is the ultimate improvement of instruction in each of the countries. Although there were differences in test scores, it would seem reasonable to expect that all of the countries could benefit by getting a better understanding of what was being done in the other countries as well as their own.

References


1. Purpose

"The purpose of this study is to investigate sex differences in the achievement on mathematical competencies among college students in pre-calculus mathematics courses" (p. 295).

2. Rationale

Previous research relating to sex differences in mathematical competencies has been done at the high school level, indicating that boys achieve at a higher level than girls when mathematics course background is not considered, but that such differences disappear when course background is considered (Davis, 1950). On the other hand, Rust (1964) found that consideration of course background did not eliminate higher male achievement.

3. Research Design and Procedures

The test used to measure mathematical competence was the Beckmann-Beal Mathematical Competencies Test for Enlightened Citizens which contains 48 items sub-categorized into 10 scales, with one item relating to each of the competencies identified in a 1972 NCTM report (see Edwards, 1972). The test was administered to 1046 students who in the first semester of the 1976-77 school year were enrolled in 38 mathematics classes which could be categorized as "College Algebra" or "Mathematics for Elementary Teachers" or "Applied Mathematics" at four state and six community colleges. Students were categorized into five strata according to mathematical background, using a "Years of Math" variable (see Table 1).
Table 1
Classification of Students Based on High School Mathematics Background

<table>
<thead>
<tr>
<th>Group</th>
<th>Years of Math</th>
<th>Description of Mathematics Courses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-5</td>
<td>Vocational Math, General Math, Business Math</td>
</tr>
<tr>
<td>2</td>
<td>0.5-1.0</td>
<td>Minimal College Preparatory</td>
</tr>
<tr>
<td>3</td>
<td>1.5-2.0</td>
<td>Average College Preparatory</td>
</tr>
<tr>
<td>4</td>
<td>2.5-3.0</td>
<td>Above Average College Preparatory</td>
</tr>
<tr>
<td>5</td>
<td>3.5-5.0</td>
<td>Strong College Preparatory</td>
</tr>
</tbody>
</table>

Sex differences were compared using "two-sample T-tests" (p. 296) for each of the ten subcategories and for the test as a whole for all of the students, and again for each of the five mathematical background strata.

4. Findings

When mathematical background was not considered, males scored significantly higher (0.01 level) than females on three subcategories: Geometry, Measurement, and Probability and Statistics. Females scored significantly higher (0.05 level) on one subcategory, Mathematical Reasoning. On the test as a whole, there were no significant sex differences.

When mathematical background was considered, "a clearer picture is obtained." Table 2 gives the means for each sex for each background strata on each of the ten subcategories.
Table 2

MEANS FOR MALES AND FEMALES OF COMPETENCY SUB-CATEGORY SCORES FOR DIFFERENT MATH BACKGROUNDS

<table>
<thead>
<tr>
<th>Math Background</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>F</td>
<td>M</td>
<td>F</td>
<td>M</td>
</tr>
<tr>
<td>Numbers and Numerals</td>
<td>4.36</td>
<td>3.86</td>
<td>4.76</td>
<td>4.76</td>
<td>5.26</td>
</tr>
<tr>
<td>Operations and Properties</td>
<td>4.96</td>
<td>4.38</td>
<td>5.39</td>
<td>5.48</td>
<td>6.18</td>
</tr>
<tr>
<td>Mathematical Sentences</td>
<td>1.54</td>
<td>1.62</td>
<td>1.80</td>
<td>1.91</td>
<td>2.13</td>
</tr>
<tr>
<td>Geometry</td>
<td>2.96</td>
<td>1.95**</td>
<td>3.03</td>
<td>2.81</td>
<td>3.63</td>
</tr>
<tr>
<td>Relations and Functions</td>
<td>1.64</td>
<td>1.38</td>
<td>1.79</td>
<td>1.61</td>
<td>1.96</td>
</tr>
<tr>
<td>Probability and Statistics</td>
<td>1.82</td>
<td>1.14**</td>
<td>2.08</td>
<td>1.69*</td>
<td>2.17</td>
</tr>
<tr>
<td>Graphing</td>
<td>2.21</td>
<td>1.67</td>
<td>2.65</td>
<td>2.63</td>
<td>2.95</td>
</tr>
<tr>
<td>Mathematical Reasoning</td>
<td>1.75</td>
<td>1.67</td>
<td>2.00</td>
<td>2.24</td>
<td>2.00</td>
</tr>
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<td>Business and Consumer Math</td>
<td>3.82</td>
<td>3.57</td>
<td>3.94</td>
<td>4.06</td>
<td>4.61</td>
</tr>
<tr>
<td>All Categories</td>
<td>28.21</td>
<td>24.24</td>
<td>30.75</td>
<td>30.51</td>
<td>34.66</td>
</tr>
</tbody>
</table>

* P < .05  ** P < .01

On the 50 mathematics background-subcategory combinations, males scored significantly higher on 8 (16%) of the comparisons, while females scored significantly higher on 2 (4%). "Therefore, when mathematics background is taken into consideration, there is no real difference in mathematics competency due to sex in 80% of the background-subcategory combinations" (p. 297).

5. Interpretations

"It is of interest to note" (p. 297) that at the subcategory level females did significantly better than males on Mathematical Sentences and Mathematical Reasoning, while males did significantly better on Geometry, Measurement, Probability and Statistics, and Business and Consumer Mathematics. There is a contrast in these subcategories: the ones in which males did better are ones "in which knowledge of specific course content was important" (p. 297), while the ones in which females did better are ones "in which the
ability to reason mathematically was important, but in which specific course content was not critical" (p. 297). In commenting further on the above conclusion, the authors close their report with this statement:

These results tend to reinforce the notion that there is actually no difference in mathematics ability due to sex, but in filling the role society has created for males and females that males may put more effort in mastering the traditional courses encountered in high school. (p. 299)

Abstractor's Comments

At the outset, it must be noted that, because the Stones, Beckmann, and Stephens (1982) study is reported in such abbreviated form, the fidelity between the report itself and the study it purports to represent is problematic to an uncomfortable degree. It would have been unnecessary to raise many of the issues below if the authors had taken (or were allowed) more space to report their study. In this review I have taken the opportunity to focus on the problems that are created and aggravated when a research report suffers a large credibility gap in representing the research actually done.

In the paragraphs to follow, the various issues raised are issues that become problematic when the concerns listed below are neglected.

1. Need for explicitness in a report as regards
   a. a rationale,
   b. a design,
   c. a set of focusing questions or hypotheses.
2. Appropriateness of the analysis.
3. The connection between data analysis and conclusions.
4. Attention to critical aspects of methodology.
Need for Explicitness

At first reading, this study seems deceptively simple and straightforward: the administration of an achievement test to 570 male and 476 female college students categorized into five strata of mathematical background, followed by 66 t-tests presumably to determine whether mathematical background can account for the sex differences. The study looks to be a standard two-factor quasi-experimental status study with sex as one factor, mathematical background as the other, and the Beckmann-Beal as the dependent variable. However, the simplicity begins to dissolve into complexity when the one and only conclusion stated refers not at all to the difference in results obtained when mathematical background is considered and when it is not, thus raising doubts about the intended purpose of the design of the study. In hopes of clarifying these doubts, I returned to the statement of the purpose of the study but found a statement so general as to be of no help at all. The purpose simply announces that sex differences in regard to mathematical competence will be "investigated". Further, since no statement of questions or hypotheses is provided, the intent of the study remains completely ambiguous. I next looked again at the introduction, hoping to find a rationale that might shed light on the explicit focus of the study. Again, there really is no rationale; there is simply a brief review of three studies with no explicitly stated connection to the study. What connection there is is by implication: the three studies were done at the high school level; this study is done at the college level; apparently then, since the other studies focused on mathematics background as the variable of concern, so might this study. Thus, if any hypotheses were to have been stated they might have related to the expectation that, when mathematics background is considered, a considerable portion of the variance on the Beckmann-Beal due to sex would be accounted for. If this is so (and, of course, from the report there is no way of knowing if this is so), why did the authors not state any conclusions relative...
to this unstated hypothesis that was apparently tested using 66 t-tests? Did they think the t-test analysis was inconclusive? To be fair, the authors did state in the "Results" section that "therefore, when mathematics background is taken into consideration, there is no real difference in mathematics competency due to sex in 80% of the background-subcategory combinations" (p. 297). However, they make no comment on the degree of difference in mathematics competence due to sex when mathematics background is NOT taken into account, and, by omission, apparently leave it up to the reader to draw the inference that, on the basis of the analysis done when mathematical background was not considered, there was more difference in mathematical competence between males and females. Unfortunately, the analysis does not support such an inference.

In fact, on the basis of the t-test analysis presented, and with special reference to the differential power of the t-test in the two settings, a strong case can be made for the conclusion that consideration of mathematics background does NOT diminish the differences between the sexes.

What conclusion, if any, is to be drawn from the study? I suspect that the authors realized there were no strong conclusions supported by the data, so that, when they came to write something in the "Conclusions" section, they chose to begin with the words "It is of interest to note" (p. 297). With this rather parenthetical beginning, it is difficult to accept the conclusion (as indicated in the abstract) as anything more than an afterthought, a simple post hoc inference that, while interesting and insightful and perhaps serendipitous, is presumably not the intended product of the design of the study. I believe it is incumbent upon the authors of any research report to state explicitly in question form or other specific form just what the study intended to find out. Without such a statement, and given the complete nonspecificity of the purpose, the design, and the rationale, I am led to assume that the authors have seized upon an apparent serendipitous finding and presented it as the conclusion.
Appropriateness of the Analysis

Doing 66 t-tests is certainly a very gross way of analyzing data from a study, which in purpose is apparently similar to the three studies (particularly the Davis (1950) and Rust (1964) studies) cited in the introduction. That purpose was to assess the degree to which mathematics background, when taken into consideration, could account for the sex differences in mathematics competence. It would seem that a series of simple two-way analyses of variance on the subcategories of the Beckmann-Beal (or a multivariate two-way analysis of variance) would have been much more appropriate. But then, perhaps the purpose of the study was not similar to the studies reviewed in the introduction.

Connection of Analysis with the Conclusions

The authors close their report with a rather bold explanatory statement (quoted in the abstract) concerning what they believe to be a significant aspect of taking mathematics courses in high school: "males may put more effort in mastering the traditional courses encountered in high school." Now while this may be a true statement, it by no means follows from the analysis of the data. There are no data in the study that suggest that it is "effort" that differentiates males from females in traditional mathematics courses in high school. There are even less data to support the conclusion that males put forth more effort than females in high school mathematics courses. In fact, in recent studies (as, for example, the NAEP results released at the 1983 NCTM Annual Meeting at Detroit), if females do do better than males, it is only at the Knowledge level as opposed to higher levels (in direct contrast to the results of this study) suggesting that, to use the reasoning of the authors, females may put forth more effort in mathematics courses, effort that likely has to do with memorization as opposed to understanding. In summary, the statements made by the authors as conclusions might at best function as hypotheses or as topics for further study, but definitely not as conclusions of this study.
Methodology of the Study

There are a number of other shortcomings in the report, some due directly to its brevity, that raise doubts as to the credibility of the research.

1. The methodology is primarily that of a survey done with cluster sampling on a stratified population. A carefully prepared sampling plan is key to the validity of a survey, but there is no indication whatsoever in the report that any aspect of survey methodology was followed or was of concern. For example, nothing is mentioned in regard to (a) how the 10 colleges were sampled from the colleges available (were there any others available?), (b) were the 38 classes sampled the only ones eligible in the 10 colleges? (c) were all students in a given class tested or were some excluded and if so on what basis? (d) who administered the tests (the researcher, the classroom teacher, the principal)? (e) does the total number of students tested (1046) represent 100% return or is it closer to 50%, and so on. As is often the case in educational surveys, the study is dominated by a conception of quasi-experimental design, when the more critical paradigm is that of a survey. For example, in the study, diverse populations were combined (preservice elementary school teachers, presumably mostly female, lumped together with students taking applied mathematics courses presumably at community colleges, presumably mostly males) to get a single population that would contain approximately equal numbers of both sexes. Question: are the females and males comparable in this amalgamated population, or might it be that the females are predominantly teachers-to-be and might represent a slice of the female population which is academically more talented than the population represented by the males in the study? This is a very serious sampling issue. Again, there is insufficient information in the report to assess adequately issues such as this. However, it is interesting to note that the major conclusion of the study could be substantially explained in terms of the alternate hypothesis that
the females are actually from an academically more talented population than are the males in the study. If so (and there is nothing in the report to suggest otherwise), then the males would represent the less able student who would find it more difficult to see the wider significance of the mathematical content that he is learning, and thus not do so well on general mathematics reasoning items. The female, being representative of the more able student, would perhaps not pay any more attention to the content than the less able male, but would possess general strategies and superior reasoning skills which would serve well on items that are less content-specific. While hypotheses such as these are purely speculative, the point in raising them here is to stress the importance of an adequately described sampling plan. What description is provided does little to dispell the possible validity of such speculation.

2. No psychometric specifications are provided for the mathematical competencies test (Beckmann-Beal).

There are some less important matters of reporting that are bothersome but not critical, such as the use of "T-test" as opposed to "t-test", particularly in a situation in which there are multiple dependent variables which could easily and perhaps more appropriately be compared using a multivariate analysis, in which case an uppercase T is involved in the notation referring to the test derived by Hotelling. Also, the use of "subcategory" and "sub-category" in different places in the report should have been picked up (both versions are also used in the abstract in an attempt to remain true to the original). There are other trivial inconsistencies not worth mentioning, but such trivialities tend to be taken as indicative of the care with which the study was done when larger issues of critical import are problematic.

Closing Remarks

I have taken a "devil's advocate" role in reviewing this study. This was not my original intention. But the deeper I got into the study the deeper I became embroiled in the ambiguities that compounded
themselves each time I returned to the report to seek further clarification of an issue: the report was at times so vague, so incomplete, so amorphous, so nonspecific, and so unconnected, that not only did it prevent resolution of problems of interpretation, it also prevented any clear identification of the study from ever emerging. Every time I tried to express an issue of concern, I had to make such copious use of terms such as "apparently", "presumably", "if this is so", and so on so that it was difficult ever to make an unconditional statement. However, I would like to end this review on an optimistic note. The authors have identified a significant perspective in their conclusion which when generalized suggests that it may be more productive to view the role of mathematics background not as a control or context variable, but as a process variable: it is the differential way in which the sexes "take" mathematics courses, and student effort may be a part of this, which is significant (as well as the amount of such coursework). More recent research (more recent in the sense of being done later, but perhaps not reported later, such as Becker (1981)), has focussed on coursework as a process variable with important findings. The significance of the present study lies in supporting the validity of the perspective of studies such as Becker's.

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Svensson, Ola and Sjoberg, Kit. SOLVING SIMPLE SUBTRACTIONS DURING THE FIRST THREE SCHOOL YEARS. *Journal of Experimental Education* 50: 91-100; Winter 1981-82.

Abstract and comments prepared for I.M.E. by WERNER LIEDTKE, University of Victoria, Canada.

1. **Purpose**

   The study was designed to create a cognitive process model for the retrieval of selected basic subtraction facts by young children during their first three years in school.

2. **Rationale**

   Process models consider two main categories of cognitive processes in reaching the answer of a simple problem. When an answer is directly retrieved from storage in long-term memory (LTM), the process is labelled *reproductive*. When conscious derivations or manipulations in working memory are required to reach an answer, the process is labelled *reconstructive*. Examinations of existing process models led the investigators to conclude that "relatively little interest has been shown retrieval processes in simple arithmetics." To study the development of cognitive skills, the topic of subtraction facts was chosen.

   Since neither the type of memory nor the counting procedures (up or down) can be revealed in regression analyses of latencies, verbal reports and behaviors were analyzed.

3. **Research Design and Procedures**

   A slide projector was used to present 66 subtraction facts of the form \( M - N \) where \( M < 13, (M - N) > 1, N \neq 0, \) and \( N \neq 1 \).

   Sixteen subjects from two classes, ranging in age from 7.2 to 8.1, were tested individually in the spring of the first school year. During this testing, only reaction time measures were obtained. The subjects were retested in the middle of each semester in grade 2.
By the time the retesting took place in the middle of each semester in grade 3, twelve subjects made up the sample. The data collected about each subject in grades 2 and 3 included reaction time measures and use of memory aids (finger counting), as well as verbal explanations about solution procedures. The counting procedures and the verbal responses were categorized, frequency distributions were constructed, and changes in cognitive strategies over the test periods in grades 2 and 3 were noted.

4. Findings

To accommodate the solution strategies, fourteen categories were created. The main two categories for the reconstructive cognitive processes involved either counting up (U) or counting down (D). The U-categories for \(M - N\) included: counting orally by one from \((N + 1)\) to \(M\) and using the number of counts as the answer; using fingers to count by one from \((N + 1)\) to \(M\) and reading the answer from the fingers; and counting up from \(N\) in steps greater than by one and keeping track of the increments. The D-categories for \(M - N\) included: orally counting down and decreasing \(M\) by one, \(N\) times; orally counting down from \(M\) by one until \(N\) is reached and using the number of counts as the answer; orally counting down in steps greater than by one; and keeping track of counting \(N\) steps down from \(M\) with fingers which record the count as well as show the difference.

Other categories included: counting up all numbers on fingers — i.e., \(M\) is counted, \(N\) is counted, \(N\) is taken away, the remaining fingers are counted; for \(M < 11\), \(M\) is represented without counting, \(N\) is taken away and the remainder is recognized as the answer; additions with equal addends (doubles) are used to find the solution for \(M - N\); and substitution of a simpler problem, i.e., \(11 - 4 = 10 - 3\).

Then there were the categories of: no description of solution; unsolved problems; and of course the immediate recall response or reproductive solution (LTM).
The major observations abstracted from a table showing the distribution of solutions over different strategies for all subjects and all years include the following:

- the proportion of retrieved LTM answers is a little lower than one third of all the answers.
- the use of fingers as external memory aid is frequent (36%).
- common strategies include: counting down without use of fingers (10%), counting down on fingers (15%), and unsolved problems (10%).
- all subjects used different strategies. No one used less than 9 of the 12 different ways of solving the problems.

Line graphs are drawn to show the increase of responses over the testing period in the LTM and the counting up categories. Decreases in the categories involving the use of fingers and in the no answer category are also shown.

Arrow diagrams and calculated proportions support the following conclusions:

- the use of strategies involving the use of fingers and responses in the no answer category decreased.
- increases exist for LTM solutions and strategies involving "counting-up."
- initially the most frequent strategies were LTM, "no answer", and those involving the use of fingers.
- the probability was high that the same strategy would be used for the same problem during following test sessions. This is especially true for LTM solutions.
- changes in strategies from "counting down" to LTM were observed.
- at the final testing session, about two-thirds of the responses fell into LTM and into the counting down categories.
- the major trend for changes in strategies is one from the lower level to a higher level in memory use.
- about one-fourth of all solutions during the last testing utilized external memory aid.
the counting down on fingers strategy is quite common and constant (27% of all solutions in grade 2 and 15% in grade 3). This is almost the only strategy which is quite often preceded by the more advanced LTM strategy. It seems to be the intermediate in the evolution from external memory and strategies to LTM solutions.

5. Interpretations

The study has shown a gradual evolution of children's strategies which begin with no attempt to solve and then include stages that involve: representing all numbers on fingers (external memories); representing only the minuend with fingers; settings where working memory replaces external memories; retrieving the answer from long-term memory. At the time of the final testing, many more problems than a teacher would like were solved with the aid of fingers.

Abstractor's Comments

It is refreshing to read a study that involved the same subjects over a three-year period. One can only surmise that the investigators must have in their possession an abundance of interesting data.

As the report was read and summarized, the following questions and comments came to mind:

(a) How were the subjects sampled from the two classes? Why were these subjects chosen? Were both classes in the same school? Were they taught mathematics by the same teacher?

(b) Why were examples of the type M - 0 excluded from the investigation?

(c) Why was the spring of the first school year chosen for the initial testing? How long had the subjects been in school? What topics in mathematics had been taught prior to the testing?
(d) Verbal responses were collected from each subject. What specific questions were asked? How were the responses recorded? Were they taped or filmed? Were they coded by the investigators? (Did the authors of the report collect the data themselves?)

(e) The verbal responses were classified into solution categories. Were any measures of reliability for this procedure calculated? Are any data on inter-rater agreement available?

(f) The point is made that during the initial testing session in grade 1 no verbal data were collected, only reaction time measures. Yet some of the figures, especially Figure 5, show a classification of responses from this setting which would seem to be impossible to obtain without verbal comments from the subjects.

(g) The numerousness of the solution categories was attributed to the young age of the subjects. Couldn't the assumption be made that the variety of strategies increases as new mathematical skills and ideas are learned? Beattie (1979) identified just as many different strategies for fifth and sixth graders as the authors of this report.

(h) The report includes the observation that "it is interesting to observe that all subjects use many different strategies. ... no one reports the use of less than 9 of the total of twelve ..." Which of these strategies are directly attributable to the curriculum objectives, the pupils' materials, the school, or the teacher? Which of the strategies are a direct result of teaching? Which of the strategies seem to be developed by the subjects? What might some possible reasons be for this "self-development" of strategies? Could this in any way be related to some special personality characteristics or some special behavior patterns?

(i) The increase in percent for LTM solutions, the decrease in percent for solutions obtained by counting on fingers, and the
decrease of responses in the no answer category could be directly related to the objectives for any mathematics program for the early grades. How could the increase in response for the "counting up" category be explained? Could it be that some subjects were taught a counting up procedure for finding missing addends (a + □ = b) and then used this procedure for solving subtraction facts?

The authors claim to have shown a gradual evolution of children's strategies for solving subtraction facts. No attempt is made to identify which part of this evolutionary sequence is directly related to the teaching these subjects have been exposed to. A teaching sequence for subtraction (basic facts) usually involves the following phrases:

1. Understanding
   - identification of subtractive action from experience
   - introduction of symbol
   - use of concrete materials to simulate the action

2. Thinking Strategies
   - properties, patterns
   - relationships among facts

3. Drill Activities
   - practice, problems, games.

Which thinking strategies were these subjects taught during phase 2 of the above sequence? Rather than having discovered the gradual evolution of children's strategies, could it be that the investigators identified the teaching sequence for subtraction (facts)? (Perhaps the author of the mathematics program considered a "gradual evolution" similar to the one identified by the authors as the program was prepared?)
The point is made that "many more problems than a teacher would like, were solved with the aid of the children's fingers as late as in the last term of the third school year." At what age/grade level are these children expected to recall these facts with "reasonable speed and accuracy"?

In general, one is left with the feeling that the authors did not go far enough in the discussion part of their report. No implications for educational settings are stated. Some information is missing and this would make the task of replicating the study a difficult task indeed. However, the results of the study can be used to generate some interesting research questions.

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