This paper presents an overview of a program of research analyzing teaching as a cognitive skill and exploring the cognitive information carried by classroom students. The research, which focuses on cognitive skills required for performing a complex interactive task in an ill-structured, dynamic task environment, is being conducted in the specific context of the presentation of expert elementary school teachers. A review is presented of previous work by educational psychologists and cognitive scientists who have explored epistemological theories as they apply to the act of teaching. A description is given of the sequence of mental processes an expert teacher uses in planning and teaching a lesson, the knowledge base from which the teacher operates, and how it determines the structure of activity episodes for the lesson. Also discussed are empirical studies that provide detailed information regarding characteristics of knowledge that constitute cognitive skill in teaching, including general schematic knowledge of teaching activities, construction of plans, knowledge of the subject matter and related pedagogy, and detailed sequences of events that occur during presentation of lessons. Figures illustrating general models of instruction and cognition are appended. (JD)
Overview of a Program of Research on Teachers' and Students' Routines, Thoughts, and Execution of Plans

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Learning Research and Development Center
University of Pittsburgh
June 1983

As presented at the annual meeting of the American Educational Research Association, Montreal, April 1983.

The research reported herein was supported by the Learning Research and Development Center, supported in part by funds from the National Institute of Education (NIE), United States Office of Education. The opinions expressed do not necessarily reflect the positions or policy of NIE, and no official endorsement should be inferred.
Overview of a Program of Research on Teachers' and Students' Routines, Thoughts, and Execution of Plans

This paper presents a brief overview of a program of research the purpose of which is to analyze teaching as a cognitive skill and to begin to map the array of cognitive information carried by students in the classroom.

The research focuses on the cognitive skill required for performing a complex interactive task in an ill-structured, dynamic task environment. The task is performed regularly by skilled individuals, with variation in specific activities and informational content from one performance to another. We characterize general knowledge structures, especially knowledge of generic procedures, that enable the task to be performed skillfully with significant variations in its informational content. We are investigating properties of this general knowledge that enable formulation of specific plans, which are then integrated with knowledge of generic procedures during execution. These plans specify activities that are chosen to accomplish goals related to specific informational material. We are also focusing on knowledge that is required to adapt to circumstances encountered in the task environment that were not anticipated when the plan was formulated.
The specific context in which we are studying cognitive skill is the presentation of mathematics lessons by expert teachers at the elementary school level. To understand the nature of knowledge and skill that enables effective performance of this task, we are observing and analyzing performance of expert teachers, and comparing their performance with less experienced and less effective teachers. To understand the nature of the information carried and used by children, we are observing and interviewing a sample of children from the classrooms of experts.

We chose the domain of mathematics teaching for several reasons. First, the task of teaching has a number of properties that make it potentially productive for the study of plan execution. A plan for each day's activity, called the "lesson plan," is commonly constructed by the teacher, and while we consider this written plan to be only a fragment of the cognitive plan that is operative, it provides a tangible basis for analyzing the relationship between task performance and planning. Second, observation of experienced teachers reveals a large amount of activity that occurs regularly and effectively, but which is not included in teachers' explicit plans. This activity provides an opportunity for study of implicit or tacit knowledge of generic procedures that experienced teachers have acquired. A third favorable property of the teaching task is that it is moderately, although not extremely, ill-structured. Enough unforeseen circumstances arise to require the plan to be formulated in a flexible way and to provide significant adjustments and improvisations for analysis, but in most cases the main course of events in the presentation of a lesson is under the teacher's control and can be carried out according to plan. Thus,
we anticipate that the task of teaching provides a domain in which cognitive analysis is feasible, but in which a cognitive analysis will raise significant theoretical issues. These theoretical issues will provide significant extensions beyond our current understanding of knowledge and skill for problem solving and performance of complex cognitive tasks.

The subject of mathematics provides an especially favorable domain in which to conduct a cognitive analysis of teaching. Since mathematics is a well-defined and structured discipline, the objectives, tasks, and sequence of instruction are specified quite clearly. It is also relatively easy for teachers to determine when their instructional goals have been met successfully, since students' performance on math problems can be observed and provide relatively clear information about their progress in learning.

In approaching this research we are drawing heavily on the work of educational psychologists who have long been working in the area. We are also incorporating the work of cognitive scientists. A major contribution was made by Sacerdoti (1977), who developed the concept of a procedural network to represent knowledge for planning solutions of problems in the blocks world. In Sacerdoti's system, called Network of Action Hierarchies (NOAH), knowledge units are schemata corresponding to actions at different levels of generality. Each action schema includes information about consequences of the action, and about conditions that are prerequisites for performing the action. Plans can be formed by choosing actions whose consequences match goals that arise, and whose prerequisites either are satisfied in the situation or become goals that can be achieved by further planning. Stefik (1980) developed MOLGEN, a
system that constructs plans that conform to constraints. MOLGEN can formulate plans in situations where global actions interact; that is, where choices made about one component of the plan imply constraints on other components. Stefik developed a process of constraint posting to enable this planning to occur. Stefik also allows for inter-segment linkages through pointers that indicate one action's relation to another even when those actions are quite distant from each other.

Psychological analyses have been provided to illustrate the usefulness of these concepts in characterizing knowledge that enables human performance in complex tasks. Greeno, Magone & Chaiklin (1979) used a hierarchical knowledge structure and top-down planning based on Sacerdoti's (1977) ideas in a model that simulates high-school students' performance in geometry problems requiring construction of auxiliary lines. Atwood, Polson, Jeffries, and Ramsey (1978) concluded that processes of top-down, breadth-first planning like those in NOAH comprise a major component of the cognitive activity of individuals working on problems of software design.

Hayes-Roth and Hayes-Roth (1978) studied performance in a task of planning a series of errands, and developed a model of interacting knowledge and decisions that includes opportunistic, bottom-up use of components of the situation. Hayes-Roth and Hayes-Roth's model includes a form of posting of inferred information like that included in the HEARSAY model of language understanding; these are consistent with Wilkins's (1980) use of posted inferences in the knowledge-rich domain of chess. An important discussion of planning and problem solving in an ill-structured domain was contributed by Flower and Hayes (1980), who focused on the interaction of multiple constraints functioning at
different levels.

While psychological investigations and AI simulations have examined the intricacies of planning and problem solving in somewhat constrained settings, educational researchers have worked on teachers' lesson plan development and on interactive teaching (Jackson 1966; Peterson, Marx & Clark, 1978; Yinger, 1977). The background for the work on the cognitive processes of teachers has been work in teacher effectiveness. Studies of teacher effectiveness usually focus on teacher behaviors and classroom processes that affect student achievement. Recently, there have been some major breakthroughs in this area based on refined conceptual models and the ability to more precisely compare variables.

Research on teacher cognition is less well developed and, so far, less successful than the teacher effectiveness work. The most commonly examined teacher behaviors are preactive, before teaching, behaviors such as lesson plan construction and interactive behaviors which deal with actual in-class teaching (Jackson, 1966). This work has assumed that lesson plan production is a profitable avenue for pursuit and that lesson flexibility or change is desirable (Peterson, Marx & Clark, 1978; Yinger, 1977). The work on preactive planning, as well as the work on interactive teaching, tends to ignore the total context in which these events take place (Berliner, 1981). For example, planning in mid-year is different than at the beginning of the year, not only because the teacher's behavior is stabilizing (Yinger, 1977; Clark & Yinger, 1980), but also because the teachers are operating in different contexts.
Yinger's (1977, 1978) ethnographic work on preactive planning revealed that teachers did not follow a linear sequence in lesson plan construction but rather that the teacher was mainly concerned with the planning of activities, which typically became routinized halfway through the school year. Behavioral objectives were not found to be a major part of the teacher's planning, rather action segments typifying plans. This finding lends support to the Morine studies (Morine & Vallance, 1975; Morine, 1976). Morine's work built on Yinger's and proposed that a cyclical interaction occurs between the planning and interactive phases (Morine, 1976).

In the last few years, several teams have begun research on teachers' cognitions while teaching. This body of research has treated the teacher as a decision-maker (Shavelson, 1976; Shavelson & Stern, 1981), and/or as an information processor (Clark, 1978; Clark & Joyce, 1981). The decision making work has been helpful in advancing the methodology of the research by using videotapes to stimulate recall. However, it has tended to restrict the types of questions asked to the teacher. Every time a teacher calls on a pupil for an answer or switches examples, s/he is clearly deciding between alternatives; however, the visible decision may be quite trivial in terms of lesson success or teachers' perceptions of lesson success. The information processing approach has tended to treat decisions as outcomes of a complete process of information filtering, as opposed to rational decision making. Our conceptualization considers the overall goal for a teacher to be the successful completion of a lesson, or completion of the agenda, rather than the execution of a series of perfect tutorials or critical decisions.
The three most relevant efforts on teacher thinking while teaching are by Morine and Vallance (1975), McNair (1978-79), and Anderson-Levitt (1981). Both Morine's and McNair's work shared Shavelson's view of the teacher as decision-maker (Shavelson & Stern, 1981). In a continuation of the planning studies, Morine and Vallance (1975) examined the in-class decision making of high and low gain teachers. Morine's work strongly suggests that teachers focus on the total teaching task, not on the partitioned tutorial task. Morine and Vallance's work, as well as that of Shavelson and Stern (1981) and Peterson, Marx and Clark (1978), tended to look for alternatives and "responsiveness" in the teaching plan. When responsiveness (that is, changes in reaction to students) was not found they tended to view the teaching as rigid. This interpretation is confounded by the fact that less effective teachers are more likely to attend to student cues, both academic and social, than are effective teachers. An alternative explanation is that the task formulation of experts is clearer, they know how to achieve it, namely, by not getting overloaded or bogged down in distracting details, and that their changes are more subtle and less disruptive.

McNair (1978-79) investigated the "in-flight" decisions which occur during teaching. Through the use of stimulated recall of classroom interaction, McNair attempted to identify what cues teachers attend to and use in decision making. The results indicated that most of the teachers based their minor alterations of behavior on student cues, that is, on their perceptions of what was happening with the student. McNair considers teachers as "fine tuners" of an initial plan, rather than decision-makers. That is, through their interactions with the students, teachers pick up cues related to students' involvement/interest in the
classroom activities and are continually making "fine adjustments" in order to keep the classroom running smoothly.

In a related study of teachers' interpretation of student language, Anderson-Levitt interviewed a teacher in France using videotaped stimulated recall (Anderson-Levitt, 1981). The most significant feature of her work is the schematic representation which demonstrates the flow of thinking between current actions, known past opinions about students, and theories of student behavior. The analysis tends to treat more lightly the overt objective of a given lesson — namely, what the teacher was trying to teach.

An insightful analysis of expert tutoring by Collins and Stevens (1982) has provided relevant information about teacher thinking in the framework of cognitive psychology. Their work analyzes several laboratory teaching events (tutorials) into formal processes with goals and subgoals, and shows the tutor weaving through multiple alternative paths in a chess-like pattern of lead and block. What Collins and Stevens build is an excellent model of knowledge exchange between a learner and an expert in subject matter and pedagogy. Additional questions arise when knowledge exchange is considered in the more traditional setting of the classroom, where multiple distinct and occasionally competing goals may be operating.

Research on teacher cognitions has emphasized the construction of written plans, the lack of flexibility in altering plans, the teacher as a decision-maker or as a tutor of twenty children. Our research seeks to explicate the full agenda beyond the written plans, to view the teacher as having a reasonable goal of accomplishing the agenda, and
will view the teacher as managing a complex multifaceted system.

Preliminary Studies

Consider, as an example, a fourth grade mathematics teacher leading a lesson on two digit multiplication within the constraints of a 40 minute class period. We can examine some of the resources and taxations related to the task. There may be 25 to 30 children, while a standard curriculum series is used, the order of presentation may be rearranged. The teacher's agenda consists of a series of action goals for both the teacher and the student. Proceeding through the agenda, s/he is bombarded with massive amounts of information: social and management information carried by student behaviors; temporal concerns carried by the fit, or lack of it, between the clock and the sequence of actions; students' grasp of the academic presentation, reflected by the errors and correctness of student responses and questions; random external interruptions such as supply deliveries, principal's announcements, etc.

The skilled teacher filters and selects carefully from this rich array of information in order to maximize the chance of accomplishing the agenda; the newer teacher is easily distracted and pulled away.

Figure 1 represents the global model of the teaching-learning exchange. In this conceptualization, student learning behaviors are considered to influence and be influenced by student cognitions. Teacher behaviors are likewise influenced. Figure 1 shows the cognitions of the teacher and student during instruction and forms a loop. Teaching behaviors are taken as the starting point. These behaviors influence student behaviors; student behaviors influence teacher cognition, which in turn influences teacher behaviors. The
teacher is viewed as "reading" the actions of the class, students, and self, and making finely tuned adjustments in the working agenda. Student cognitions are influenced by the student's own behavior and by the teacher's behavior. Both sets of cognitions are influenced by prior knowledge, beliefs, plans, and the actions of the other students.

Currently we hypothesize that, as Figure 2 displays, teachers are reading information from: the social cues of the environment, verbal cues from students, or his or her own sense of/or concern with time, prior information on these particular students and students in general, the agenda, and subject matter knowledge. Their thoughts are organized around the goal of achieving the agenda, or as Mehan (1979) has put it, on getting through. Thus, child and subject matter knowledge is drawn on for agenda formulation, responding to questions, and selecting students; whereas social and verbal cues are only used if they further or threaten the agenda, not as information in themselves (unless, of course, the agenda contains social or linguistic elements, which in some cases it does).
Before class a good teacher has some ideas about the nature of the lesson to be taught. These ideas are probably not well-reflected in lesson plans. Rather, they are contained in something we refer to as the teacher's agenda. An agenda refers to the total goal and action sequence constructed by the teacher for a given time period, not just the written plans. It includes the purpose of the lesson, the organizing structure for the lesson, the activities for both teacher and student, the goals for each element, anticipated problems with their concomitant solutions, and the connections between the current lesson and prior and future lessons. The agenda is constrained by or structured by the teacher's belief system (children should be kept busy), routines in place (homework collection, seatwork management, etc.), and the curriculum in use. How teachers go about constructing the agenda is important, but so is the complete content of it.

Teachers' knowledge for planning and teaching lessons includes (1) general schemata and beliefs that are capable of incorporating specific lesson materials and activities, (2) global constraints that may be related to general beliefs about children's learning, classroom practices, and general knowledge and understanding of mathematics, and (3) standard routines and problem-solving strategies used in performance of specific teaching activities. Operating plans, which focus on activities involving specific material in the daily lessons, identify activities that vary substantially from day to day. In performance of teaching, the operating plan provides little more than a set of titles for a few of the activities that will be included in the lesson. These titles are analogous to brief verbal rules for cognitive procedures or instructions that partially specify a series of steps for accomplishing
some task. Recent research on verbal rules (Chaiklin, 1982) and on instructions (Smith, 1982) emphasize the need for significant background knowledge for effective performance of the tasks that the rules or instructions are intended to elicit.

We conceptualize teaching of a lesson as a number of episodes or activity structures, each containing an identifiable and partially predictable set of actions for both the teacher and the students. Episodes include such units as having the students check their homework, explaining a concept to the class, working example problems for the class, having students work problems publicly as an extension of the lesson, and having students work privately as a rehearsal of the presentation. Preliminary evidence indicates that this kind of episode or activity structure provides a useful characterization of the events that occur in the teaching of a lesson.

(activity schemata. A major component of our hypothesis about teachers' background knowledge is a set of activity schemata, which are similar to the schemata for global actions in Sacerdoti's (1977) system, and to the Knowledge Sources in Wilkins's (1980) system for planning in chess. Each schema in the system includes two kinds of information: information about requisite conditions and consequences of the activity, and information about internal structure of the activity, including components of lower-level activity that are included in performance of the activity. The components of an activity schema may themselves be schemata that contain information about requisite conditions and internal structure. The requisite conditions constitute subgoals that are required for the activity to be successful, and the component subactivities correspond to if-then strategies that are available for
achieving subgoals. Some components of schemata are well-rehearsed routines for standard activities, such as distributing materials that students need for work at their seats.

A second schema that is operating during the lesson is an information schema. This schema helps to collect, post, and recall critical elements of information during the execution of action plans. The expert teacher is in fact capable of generating manageable packets of information that can be used anywhere in the lesson. It is the management of information that permits appropriate flexibility in lessons and insightful responsiveness to students.

When a lesson is taught, schematic knowledge is retrieved and determines the structure of activity episodes for the lesson. Activity components that are specific to the current lesson are integrated with the schematic knowledge to form the sequence of actions in the lesson. We refer to the structure formed by integrating general activity schemata with specific lesson components as the agenda of the lesson. An investigation of properties of lesson agendas is described in the study of teachers' planning, in which we interview teachers before they present lessons, to obtain information about the degree to which they explicitly plan various components of teaching (beyond the sketch in the lesson plan) and the degree to which they anticipate contingencies that can arise during teaching. We assume that the general activity schemata are relatively stable, but this implies that they can be combined flexibly with a wide variety of specific lesson activities. An important theoretical issue that needs to be addressed involves characteristics of general schematic knowledge that support this flexibility.
**Global constraints.** A second general component of knowledge that we hypothesize is a set of global constraints. A very significant constraint in classroom teaching, which is less salient in tutorial teaching, involves the time that is available for the lesson. A classroom teacher must complete the agenda within a specified total amount of time while maximizing the learning and understanding of the students. In addition to constraints of time, teachers also include monitoring and diagnostic activities related to general constraints. Teachers monitor the student audience more or less continuously to satisfy constraints involving student attention or activity relevant to the instructional agenda. They also include diagnostic activities to determine whether specific students have reached acceptable levels of understanding or learning.

A theoretical possibility is that global constraints of teaching are at least partially determined by teachers' belief systems about children's learning and desirable classroom methods. Constraints also may be generated by the teachers' general knowledge and understanding of mathematics, involving relationships among different mathematical topics or general principles that should be reflected in the method of presenting material and in the students' understanding. In another study we are investigating characteristics of teachers' general mathematical knowledge and their beliefs about children's learning and classroom practice in order to provide information about this factor in teaching performance.
Performance: Teaching a Lesson. Teaching a lesson can be conceptualized as the execution of a plan. The plan structure is the agenda, formed from general activity schemata and specific lesson activities from the lesson plan, along with global constraints of teaching and local constraints that are represented in the teacher’s schemata for activities. Note that this plan structure may be largely implicit, specified by knowledge structures that are retrieved during execution rather than explicitly represented in advance of the teaching activity.

Performance of teaching involves retrieving schematic information from the plan structure, identifying subgoals that correspond to requisite conditions of the schema, determining that the requisite conditions are satisfied or performing other actions that achieve them, and finally performing the activities specified as components of the schema. Activities are performed under global constraints and other local constraints that correspond to requisite conditions of schemata.

Empirical Studies

A series of empirical studies are being conducted that are designed to provide detailed information regarding characteristics of knowledge that constitute cognitive skill in teaching, including general schematic knowledge of teaching activities, construction of plans, knowledge of the subject matter and related pedagogy, and detailed sequences of events that occur during presentation of lessons. In addition to records of the activities of teachers as they present lessons, we are obtaining retrospective protocols. These data will be used in the
development of a model that characterizes teachers' cognitive processes. This model will focus on teachers' responsiveness to circumstances that arise in the classroom and relate their actions to the goals and expectations delineated in their agendas. In the future, selected aspects of this model will be implemented as computer simulations, when this seems useful to clarify principles or to demonstrate the sufficiency of a set of hypotheses regarding some phenomena. We are also collecting data on the knowledge level and thinking processes of students.

Activity structures and routines. The purpose of this study is to provide basic data for the development of hypotheses about the major components of knowledge that constitute skill in teaching elementary mathematics. Videotaped records of the teaching of mathematics lessons are being gathered and the translated into systematic descriptions that we call activity structure maps. These maps provide a basis for inference of underlying knowledge structures and processes which can be developed in the framework of activity schemata and constraints described above. A goal of the analysis is to account for the major components of the activity structure maps, in the sense of showing that our hypotheses about knowledge structures and processes can generate the activities that occur. The study is designed to establish the basic structure of a typical math class for a given teacher and to place the math class in the context of an entire day, and also within a sequence of math lessons. We expect variation in the number and type of activities in which teachers engage, as well as the amount of time spent in those activities. The observed structure of any given lesson, day, and week will also be a reflection of the time constraints and agenda
under which the teacher is operating.

Figure 3 shows the activity structures for a specific math lesson - there are eight activities identified. For each activity there are a number of actions to be carried out by both the teacher and the student. Each action contains some scripted or routine elements and some spontaneous or unknown elements. The degree to which the routine elements frame and guide the spontaneous ones is significant; it appears, for example, that these frames operate as effective guides for experienced teachers and more as obstacles for novice teachers.

**Agenda content and formation.** The objective of the second study is to understand the agenda for any given lesson, how it is constructed, and how it is changed during instruction. Since we view the agenda as one of the major cognitive inputs that affect teacher behavior, it is critical that we understand what it contains, in what detail, and the flexibility built into it (if any). Each teacher is being interviewed prior to each of six to eight mathematics classes. The information from the first study is important in contextualizing the information obtained. The interview is open-ended and semi-structured. The interview is used to further establish and confirm the sequence of activity structures for the class, for example: review, drill, transition, presentation, transition, and practice. The interview is also used to elicit the teacher's detailed description of goals,
examples, games, anticipated problems, and what s/he is looking for in the students' actions (for example, Figure 4). A class observation is used to note additions, deletions, alterations, etc. Following class, the teacher is again interviewed to clarify what went on, how the teacher viewed the lesson and how discrepancies are interpreted (Figure 5).

Figure 4 provides an absolutely clear line of action statements: the topic is fractional parts of a number (do a presentation), work through 32 items on two pages (shared presentation, public practice, guided practice), extend the lesson with the use of counters (public practice), do additional work sheets (guided or monitored practice). What is not transparent are the routines to be used or the information gathered at each stage and used at later stages. Those must be developed from direct observation of the task performance. Figure 5 shows a change in sequence of the action segment (counters before workbook instead of after) which comes as a consequence of the information obtained in the presentation, namely that more concrete practice is needed before written performance demands are placed on the students.

The analysis is aimed at understanding the nature of the teacher's planned and actual agenda, how agendas are formed, what the critical knowledge sources are, and how management routines are established and used. For any given teacher, the entire set of interviews is examined to identify recurring action elements, presence or absence of routine, descriptors, and any constraint "tags" that are included. The teachers' productions are relatively short, almost mental notes for the lesson, and our task is to learn the meaning of each component of the notes.
This is accomplished in part by matching the observed action sequence to the planned one and in part through the teacher's discussion of the lesson in the post interview. The more long-range goals, such as those concerned with attitudes toward mathematics, appropriate peer respect, etc. are revealed through action.

Figure 4 & 5 Here

Studies of teachers knowledge bases. The purpose of this study is to assess teachers' knowledge of math and their belief systems about students and teaching. The knowledge and beliefs are analyzed in terms of the global constraints that we postulate on the basis of observations of their performance. In order to assess the more stable aspects of the teacher's knowledge about student ability, the curriculum, and subject matter, we are asking each teacher to engage in several non-teaching tasks. First, they estimate the overlap between what the students have learned and what is tested on a standardized test. Pilot work in this area suggests substantial differences between experts and novices in both the accuracy and depth of analysis teachers use to complete this task. These differences are a function of the kinds of information a teacher sees as relevant, attends to, and remembers. Using either the standardized test that is administered by the school district in the spring of each year or the district-mandated criterion-referenced math tests, the teachers are asked to predict student performance on each mathematics item and to describe how that decision was made.
Second, we tap teachers' knowledge of the subject matter, and students and the expectation the teacher has about learning the subject matter. This aspect of teacher cognitions is assumed to be relatively stable within teacher and variable across teachers, especially at various stages of experience. We do not assume that an elementary mathematics teacher must be a mathematical theoretician in order to be effective, but rather must have a thorough understanding of the structure, hierarchical nature and underlying principles and concepts of the content. Further, we assume that this understanding of the subject matter, in conjunction with an understanding of pedagogical issues, allows the teacher to have more alternative instructional paths open to him/her and, therefore, more flexibility in instructional behaviors.

Using modified items from the mathematics curriculum in use in the district (Heath), teachers are asked to construct and label categories of items. They then are asked to discuss the properties of those categories with respect to instructional demands. In addition, we are interviewing the teacher in a semi-structured way about one topic, fractions, to probe the depth of their underlying knowledge. Understanding the specific knowledge that teachers have about a focused topic will let us see how that knowledge actually interacts with lesson presentation. For example, in teaching equivalent fractions, children are taught to multiply the top and the bottom (numerator/denominator) by the same number. The text does not explain why, nor do most teachers. The question then is - Do teachers realize that that is the same as multiplying by one, and if they do or don't, does that affect their teaching?
Fractions was chosen because in the fourth grade text it is the topic that has the most new material presented. The fractions interview will also be used as a basis for examining teachers' beliefs about children's learning—specifically the differences (if any) between what teachers believe about mathematics and what they think children can learn and understand.

Information and problem solving during the execution of plans. The purpose of this study is to provide detailed evidence regarding the internal structure of our characterization of teachers' knowledge and thinking. We do not believe that teachers can identify the internal structure of their cognitive activities in any complete way. However, by obtaining detailed retrospective protocols of teachers viewing videotapes of their own performance, we have an opportunity to discover alternative hypotheses to those we have developed on the basis of our observations of the teachers' performance.

One of the most complex and least understood aspects of teachers' cognitions is the information processing that occurs during the instructional act itself, the interactive stage. It is at this stage that teacher cognitions are exhibited in instructional behaviors and it is those behaviors that impact upon student cognitions and behaviors. It is also the stage in which the most information is available to be received or attended to by the teacher. The teacher, however, must act and cannot process such large amounts of information on-line. Therefore, it is at this stage that we will look for how information is filtered, what information is attended to and what is ignored, and how these processes drive teacher behaviors.
Each teacher is videotaped while teaching mathematics from three to five times. As soon as possible after the taping, and after the researcher has seen the tape, the videotape is played back to the teacher to stimulate recall. The protocols obtained during this phase are analyzed and used to test the basic model of cognitive skills required for teaching. The model is being refined based on discrepancies and omissions noted from both the stimulated recalls, the actions in the tapes, and interview responses.

Insert Figures 6 & 7 Here

Figures 6 and 7 show the first stage of analysis of two activity segments from a single lesson: An expert’s homework check and an expert’s lesson presentation. The first clear-cut goal is to get the homework corrected and handed in. Within this there are three subgoals: to establish who did or did not do their work (indolents post their names on the board); to correct the work to assess the general success rate.

The expert attains her goals by using two schemas, one action schema and one information schema. The action schema follows three goals closely. First the teacher calls attendance to which each child responds yes or gives an explanation. If the child has not done the work, s/he puts his/her name on the blackboard. An informational schema is also being activated (I-SCHEMA) which now has a list of students who did not do their work. Second, she calls the problem out loud with the students giving the answer chorally. If the chorus weakens, the teacher
learns that that item is difficult. I-SCHEMA is used to record the key item features. Last, she calls out numbers of items missed starting from zero and stopping when there are consecutively no hands raised. I-SCHEMA records those students who miss a lot of items.

For each activity structure shown the basic goal is identified and the subgoals or components are listed. Many of the activities and routines are initiated by a very short verbal phrase that alerts students to either change ("Erase, be seated.") or the action itself ("Page 169."). For each subgoal the actions used to accomplish the goal are identified and the functions and or outcomes are reported. A function is an inferred consequence of an action that is not identical to the goal or subgoal but may meet known or inferred constraints. An outcome or product is listed only if the consequence of an action produces something that must be carried forward into another goal or subgoal, in some cases these outcomes are themselves goals. Thus, an individual child's failure to perform in one action may produce a goal to tutor that child continuously throughout all other activities. The basic goals for the lesson do not stand alone but both receive and produce products from other activity structures.

Figure 6 shows the actions related to achieving the first goal - homework. The cue is given by "Ok, set 43". Attendance is rapidly called - each child answers yes or writes their name on the blackboard - time to complete, approximately 30 seconds. The routine is well rehearsed and universally known. The action provides information and exerts a monitoring and public control function. An outcome is that the teacher knows who has not done the work.
The second subgoal is to correct the homework. The students take colored pencils out and respond with the correct answer in lowest terms chorally as the teacher calls the problem "1/12 + 1/12" "2/12 or 1/6". Time to completion 106 seconds. The teacher's calling out the problem serves the function of pacing the class through and reinforcing the pairing of problem and answer. A second function is to note if any of the items produced problems for the group as a whole. This is determined by the situation in which multiple answers are shouted. Thus, at this point in the lesson through the use of two of the three homework checking routines the teacher knows which children she doesn't know about (namely the ones who didn't do their homework) and which problems, if any, create difficulties. The last subgoal is to discover which of the children had difficulties in general with the assignment. This is done in 30 seconds by calling out the number of problems missed and having children raise their hands. The homework (or classwork) activity structure accomplishes a lot in a little time and produces information that can be easily carried forward into the rest of the lesson. The routines used are attendance response, choral response, and hand raising. The teacher has reduced the amount of potential processing and has kept a simple component of the lesson simple. In teachers who are less successful we often see large amounts of time and intellect expended on just such a simple component.

Figure 6A shows an analysis of the homework segment using a production system to display the goals (rectangles), test (diamonds), and actions (hexagons). The unanalyzed routines are described in triangles.
In contrast, a novice teacher doing the homework check activity behaves somewhat differently. The homework check activity for one novice was an extended activity (6 minutes) in which the goal was reached somewhat indirectly and without the type of teacher control we saw in the previous example. The homework activity is made up of the same two subgoals, the first identifying who did homework, and second, orally correcting the problems. For the first subgoal, the novice stood up at the front of the room and said "who doesn't have their homework?" The students did one of the following: stayed seated and held up completed work, stood up and walked to the teacher and said either they had it or did not have it, called out that they didn't have it. The novice teacher responded that homework is important and there are no acceptable excuses, and marked on a posted sheet whether work is completed or not. She included no summary action, thus, she did not know if she knew about the homework status of everyone. The I-SCHEMA is not in place and not working. The novice uses a less effective question, does not have a routine to obtain the information, and is not maintaining control of the flow of information. The students, in an attempt to comply with the somewhat unclear request, respond in a variety of confusing ways. The total time to accomplish the first subgoal is 85 seconds. It is also the case that she is unable to retain the information in memory to carry it forward or has incomplete information as we see in the next section.

The second goal is to correct the problems. This can be done as the expert did it, or by the teacher collecting the work and correcting it and returning it, or some other combination. The second segment of the homework check is answering the problems. The novice calls out a
set of problem numbers (1-10) and assigns a child to call out the
answers as the teacher calls the problem number. The student slowly
calls out the answers in order. (The first child chosen is the lowest in
the class, does not have her work done and is doing it in her head.)
Thus for the first 10 problem answers the teacher has lost control of
pace and correctness of answer, however, it is only when the child fails
on the sixth problem that the novice realizes the student has not done
her homework. (To get to the seventh problem took 105 seconds). The
novice then calls on four separate children each of whom gives the
answer. The rest of the class is checking the work off at their desks.
The novice then picks her main "trouble maker" to do the next block of
10 problems. The rationale for choosing the child was that it was the
first time the child had volunteered for anything. He misses one
problem but then continues - going through ten problems in 70 seconds.
The last child chosen goes through the sequence quickly but the sound of
the child saying the problem number and answer next to each other is
confusing, (e.g. 24, 27; 28, 64).

The novice teacher clearly has the beginning of a strategy for
getting homework checked. First she does it (she did not earlier in her
teaching and another novice did not even check the homework which leads
to another set of difficulties). Second she realizes that she should
have some structure and time is a constraint. During each cycle she
starts by having the child pace it, then she takes over the pacing.
Figure 7 shows the analysis of a presentation of new material - changing a mixed number to a fraction. There are three subgoals: The first is to review the labels (vocabulary) needed, the second is to present the task, and the third is to demonstrate the algorithm. Overlaid on the subgoals are several systems of constraints which themselves help construct the solution: keep the lesson moving (Footnote 1), get through the task (Footnote 2), call on different children (Footnote 3), watch for the stragglers and help them (Footnote 4), keep interest and action up (Footnote 5), don't embarrass children (Footnote 6).

To review the labels the teacher asks for a definition. She selects one of the weakest children, Connie, to answer. This is both to encourage Connie and to do a bottom level check - if the weakest students can get it, the lesson can move rapidly. Connie does not get it and produces by her failure another goal 2A1, to check on her for the rest of the period. The teacher then moves to one of her strongest students for the definition - she also fails. The teacher tries again with a middle level - he fails. The teacher then calls on a top child and repeats the definition and has the students rehearse it chorally. This is an analysis of the action schema. Clearly the I-Schema is being used to construct subgoal 2A; but further analysis is need of this segment.

In the time constraint system Ms. Longbranch is behind for the second goal of the lesson, namely to define how to change mixed numbers to fractions. She now must move ahead and up the pace but maintain involvement. She does this by having choral reading of the rule from the rule cards at the front of the board. So within 1 1/2 minutes she
has reviewed the definitions introduced an algorithm and rehearsed it. (It probably should be pointed out that the prior lesson involved extensive work with drawing mixed numbers and talking about 1 1/2 of a sandwich and 3/2 of a sandwich, etc.) Ms. Longbranch is now ready to use a routine of public practice where a problem can be put on the board and a child is called on to orally guide the teacher through the operation. Ms. Longbranch shares control slightly by permitting volunteers but calls on one child at a time to do each of the three steps of the algorithm.

The first problem (Subgoal 2C) is 2 1/2, a relatively easy problem; the first child (a middle level anxious child) is called on to perform the first part of the algorithm (multiply the whole number by the denominator), the teacher follows the rule for the second step while the student dictates to her (add the numerator). These actions are carefully watched by the students both because it is the first real demonstration and because of the relative excitement of watching a student tell the teacher what to do. The second problem that is given to the students is 3 2/5 to be done by a top level child. He goes through all the steps smoothly, thus rehearsing publicly the algorithm. To check how the lesson has landed she goes to the weakest child (from goal 2A1) and rehearses the steps while the child gives the answers and Ms. Longbranch writes the answer on the board, e. g. for 2 3/4. The actions produce a third example on the board, rehearsal of the algorithm, and check the weakest child. She is caught up and is ready to begin public practice on the blackboard with groups of students.
Student Cognitions. The purpose of this study is to assess the thinking of students as they are being taught mathematics. We are interested in students' conceptions of class structure as well as students' on-line thoughts. There are three types of interviews: a knowledge/understanding interview of fractions; an interview of classroom routines, structures and rules; and an interview during class and from stimulated recalls. In the future this will be expanded to include in-depth observation of student behaviors and productions. We hope to be able to build action/goal maps similar to those developed for the teacher and then to fit them together with type of teacher. We are also trying to develop a picture of when certain critical ideas are sent (covered in class or homework).

This type of analysis of routines and activity segments is a useful way of starting to understand how teachers and students deal with a dynamic ill-structured task setting. They constrain some of the elements by making them more or less static, and transform some of the tasks into highly standard elements calling up entire repertoires of mutually understood behaviors. The next exciting aspect to examine is how these individuals deal with the dynamic and substantive elements of a lesson.

Outcomes

The outcome of the set of work will be an explication at a rather micro level of teacher's and student's actions, routines, and thoughts. Another outcome will be a deeper theoretical understanding of task framing and accomplishment in complex dynamic environments. We hope to be able to develop a theoretical understanding of both the action
schemata and the informational schemata utilized by expert teachers.
References


Footnotes

1. Ms. Longbranch's concern about keeping the lesson moving is exemplified in the following excerpts: Interviewer: "What are the advantages of using choral check that you did for homework & for this?" Ms. Longbranch: "It's quick. It's very quick." (12/14/81; lines 401-403) Ms. Longbranch: "...it seems the way I have math scheduled I only have that 40 minutes so I really have to know what I'm doing. I have to have my 40 minutes organized." (11/19/81; lines 201-205) Ms. Longbranch: "...my math is 40 minutes...I can never drag math out for a couple of extra minutes." (1/6/82; lines 244-245).

2. Ms. Longbranch's underlying constraint of getting through the task is expressed in this quote of 12/3/81, lines 342-347, "...I don't have this written down anywhere, but in my mind I have it. I'm going to be finished with fractions before Christmas. I have to be, you know, to get on. So I'll just pace myself now so that I will get finished.

3. Ms. Longbranch tries to call on different children: "...everyone doesn't get to the board every day. But most of them do." (1/6/82; lines 291-292) "I think I was trying to get all the children to the board that I thought would have any difficulty at all." (12/14/81; lines 151-153)
4. One important concern is to watch for stragglers and give them additional help: "...I can tell (the ones that have trouble), they're always the last ones to stand up. So I know they need special attention." (11/24/81; lines 38-41). "But usually the ones who have trouble will get to the board that day. The better ones will get turns, you know, every day, or three..." (12/11/81; lines 447-451). Then when you see the same person is always the last one getting up, well you know he or she is really having a tough time of it." (12/14/81; lines 358-361).

5. Ms. Longbranch is operating within the constraint of keeping the children interested and action moving: "There's no specific reason why I have them stand - just to keep them moving,...". (1/6/82; lines 462-464) "I feel like if I don't have them keep moving constantly, or doing something constantly, their attention span, I don't care how good they are, it just floats away." (1/6/82; lines 475-478)

6. Ms. Longbranch always avoids embarrassing children: "I don't always pick out the poor ones, or else they'll know for sure ... you know, & I feel bad and then they won't want to go to the board. This way, everyone wants to go to the board." (12/11/81; lines 439-441;443-445). "And they hate to be the last ones sitting down. But I never point that out that they're the last ones." (12/14/81; lines 351-355)
<table>
<thead>
<tr>
<th>TEACHER ACTION</th>
<th>STUDENT ACTION</th>
<th>ACTIVITY STRUCTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Tells them to get out homework papers.</td>
<td>1. Get out papers.</td>
<td>Homework Check</td>
</tr>
<tr>
<td>2. Reads problems.</td>
<td>2. Choral responses, check own work.</td>
<td>(2½ Minutes)</td>
</tr>
<tr>
<td>3. Asks for # correct.</td>
<td>3. Raise hands to indicate # correct.</td>
<td></td>
</tr>
<tr>
<td>5. Requests books away.</td>
<td>5. Put books away.</td>
<td>(1 Minute)</td>
</tr>
<tr>
<td>6. Asks questions about steps in changing mixed numbers to fractions.</td>
<td>6. Chorally read steps from cards.</td>
<td></td>
</tr>
<tr>
<td>7. Asks for steps in solving two problems.</td>
<td>7. Individual response.</td>
<td>Presentation</td>
</tr>
<tr>
<td>8. Sends some children to board.</td>
<td>8. Designated children go to board.</td>
<td>(4 Minutes)</td>
</tr>
<tr>
<td>9. Gives children one problem at a time to work.</td>
<td>9. Children at board do problem, others do it at seat.</td>
<td>Shared Presentation (4½ Minutes)</td>
</tr>
<tr>
<td>10. Asks child to verbalize steps in solving each problem.</td>
<td>10. Individual responses about steps in problem solving.</td>
<td></td>
</tr>
<tr>
<td>12. Passes out paper.</td>
<td>12. Fold papers.</td>
<td>(1 Minute)</td>
</tr>
</tbody>
</table>

Figure 3. Activity Structure of Expert Lesson
<table>
<thead>
<tr>
<th>TEACHER ACTION</th>
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</tr>
</thead>
<tbody>
<tr>
<td>13. Gives children problems one at a time initially, then three at a time.</td>
<td>13. Several work problems at board.</td>
<td>Guided Practice (16½ Minutes)</td>
</tr>
<tr>
<td>15. Checks individual children's completed problems.</td>
<td>15. Stand when problem is finished.</td>
<td></td>
</tr>
<tr>
<td>17. Passes out homework.</td>
<td>17. Look at homework paper.</td>
<td></td>
</tr>
</tbody>
</table>
February 11, 1982

G. Right now they're doing newspapers and reading. It's the end of the second period. What time are you teaching math today?

M. 10:25.

G. In five minutes.

M. Right.

G. OK. So, tell me what your plans are today.

M. We're going to find a fractional part of a number. We're going to work the exercises 1 to 32 on pages 142-143. We're also going to be working with individual counters.

G. mhm.

M. Having them laying out like twelve counters, finding half of them.

G. mhm.

M. OK. And, they will have worksheets due.

G. Uh-huh. And, is that the order that you're gonna do it in?

M. Right.

Figure 4. Preclass Interview
February 11, 1982

G. (inaudible)?
M. Certainly. Just tell me about it.
G. (giggles) Just tell me again what you want (inaudible) ...
M. Oh, right. I went out of sequence. I decide to do the counters before the exercises in the book.
G. Why?
M. I think, to make it clearer to them.
G. mhm.
M. Manipulating some things that they could understand this.
G. So the first thing you did was present the algorithm.
M. Right.
G. And then the counters as a ...
M. Right.
G. As a manipulative. And then the exercises.
M. And then the exercises.
G. OK.
M. Orally, and then the worksheet for independent work.

Figure 5. Postclass Interview
Constraints: Reinforce doing homework
Keep pace moving
Keep attention
Watch for Ryan & Connie

Expert - 12/14/81

Goal 1  Homework Check
Subgoal 1A - Who has it? Time ~ 30 seconds.
  action: T call attendance - * Cue
           S say yes or put name on board
  function: monitors
  outcome: knowledge of who has not done work - carry forward

Subgoal 1B - Correct work. Time 106 seconds.
  action: T calls out problems
           Ss call out answers - correcting pencils
  function: Paces, both groups have information, keep attention

Interruption - "How many reduced it to 1/6?"

Subgoal 1C - How many got how many correct? Time 22 seconds.
  action: T calls number perfect
           then number incorrect (2,3,4,...)
           Ss raise hands
  function: monitors, summarizes
  outcome: Students who got several wrong noted and carried forward.

Conclusion Cue: *Pass to the front, put your books in your desk.

Figure 6. Expert Homework
**Figure 6A. Procedural scheme for homework**

- **Attendance**
  - All students checked
  - Check students until any more students?
  - Call students names one by one, post negatives

- **Oral correction**
  - All items checked
  - Check each item in order until any more items?
  - Call out item content, listen for choral answer and interpret

- **Oral summary survey number right**
  - All combinations of errors checked
  - Check number wrong until several (?) have no responses
  - Call out number wrong, have students raise hands and note totals and who misses many.
Goal 2 Presentation - Time 4 1/2 minutes

Subgoal 2A: Define a mixed number

action: T asks for definition
weak child is selected - fails

function: involves students,
check on first child

outcome: Goal 2A1 - Check on Connie

action: T calls on Tracy
Child fails - confused

action: T calls on Chris Brown
Child fails - confused

action: Tiffany called on - gets it, ...
T repeats definition, writes 2 1/2 on board.
Ss choral repeat

function: Get definition across
Don't waste time

Outcome: Time is lost - make it up
Goal 2A2

Subgoal 2B: Define operation of changing a
fraction to a mixed number.

action: Teacher leads choral reading of rule

function: Clearly state algorithm, sacrifice
student involvement for time

outcome: Time is caught up - goal 2A2 is met

Figure 7. Expert Presentation
Subgoal 2C: Demonstrate Rules: Select student, select problem - 1st iteration

Problem 1 - 2½

action: T puts 2½ on board
  says rule - part 1 - calls on strong student, Terry
  Ss misspeaks but says it correctly
  T executes

function: T controls fit between rule and action and involves students

action: T says rule - part 2 - calls same student
  Ss adds numerator, states answer
  T executes

function: Same

action: T says rule - part 3 - and pauses
  Ss chorally respond, incorrectly

Interruption: Teacher calls to order and reprimands

function: Keep students obedient

Subgoal 2C: Demonstrate rules: 2nd iteration

Problem 2 - 3 2/5

action: T puts problem on board 3 2/5
  Calls on middle child (Everett)
  Ss says rules and executes
  T writes, pacing through each step

function: 2nd clean demonstration, mid level check, more independence

outcome: Success means can try on a lower student

Subgoal 2C: Demonstrate rules: 3rd iteration

Problem 3 - 2 3/4

action: T puts problem on board
  calls on Connie
  T calls for rule (step by step)
  Connie executes
  T writes reinforces last step

function: Check weakest child, check for success of rule presentation

outcome: Success - Move on.
Figure 7A. Procedural scheme for presentation

1. **Present lesson**
   - Define terms
   - Define algorithm
   - Demonstrate rules

   - **Obtain clear definition of terms**
     - Call on student volunteer
     - Until term defined correctly?

   - **Present rules for "doing" algorithm**
     - Read rules from card and demonstrate
     - Until done once?
     - Call for choral reading; demonstrate each step

   - **Public rehearsal of match between verbal rule and action**
     - Select problem
     - Call on student
     - Until go through 3 cycles
     - Call on student to recite each component of rule, then execute