The Comprehensive School Mathematics Program (CSMP) is a program of CEMREL, Inc., one of the national educational laboratories, and was funded by the National Institute of Education (NIE). Its major purpose is the development of curriculum materials for kindergarten through grade 6. The original purpose of this draft report was to present a comprehensive summary of the mathematics achievement of CSMP students in intensive field trials of the curriculum. However, as the sponsor has discontinued funding, this report is likely to be the only one. Therefore, there is a focus on the main results of the CSMP evaluation, with many secondary issues attended to only in a cursory manner, if at all. Further, there is a considerable amount of data that relates to issues in mathematics education generally which are not presented. Readers are encouraged to scan the table of contents for an idea of what was originally intended, but could not be included. (MP)
Extended Pilot Trial of the Comprehensive School Mathematics Program

Summary of Student Achievement Data, Draft Report

BEST COPY AVAILABLE

Martin Herbert
Knowles Dougherty
Mathematics Research and Evaluation Studies

November 1982

CEMREL, Inc.
St. Louis, Missouri
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Description of Evaluation Report Series

The Comprehensive School Mathematics Program (CSMP) is a program of CEMREL, INC., one of the national educational laboratories, and is funded by the National Institute of Education. Its major purpose is the development of curriculum materials for grades K-6.

Beginning in September, 1973, CSMP began an extended pilot trial of its Elementary Program. The pilot trial was longitudinal in nature; students who began using CSMP materials in kindergarten or first grade in 1973-74, were able to use them in first and second grades respectively in 1974-75, and so on in subsequent years. Hence the adjective "extended".

The evaluation of the program in this extended pilot trial was intended to be reasonably comprehensive and to supply information desired by a wide variety of audiences. For that reason the reports in this series are reasonably non-technical and do not attempt to explore widely some of the related issues. The list of reports for previous years is given on the next page.

Final Reports in the series are:

9-A-1 Summary of Student Achievement, Draft Report
9-A-2 Summary of Implementation Data; Draft Report
9-B-1 Sixth Grade MANS Test Data
9-C-1 Sixth Grade Evaluation: Teacher Questionnaires

The present report, and report 9-A-2, are summary reports describing results from the full nine-year study covering grades K through 6. As of this time (November, 1982), these two reports are only draft versions and many important issues could not be explored with the care that attended the other 48 volumes of this series.
Extended Pilot Trials of the
Comprehensive School Mathematics Program

Evaluation Report Series

1-A-2 External Review of CSMP Materials
1-A-3 Final Summary Report, Year 1
1-B-1 Mid-Year Test Data: CSMP First Grade Content
1-B-2 End-of-Year Test Data: CSMP First Grade Content
1-B-3 End-of-Year Test Data: Standard First Grade Content
1-B-4 End-of-Year Test Data: CSMP Kindergarten Content
1-B-5 Test Data on Some General Cognitive Skills
1-B-6 Summary Test Data: Detroit Schools
1-C-1 Teacher Training Report
1-C-2 Observations of CSMP First Grade Classes
1-C-3 Mid-Year Data from Teacher Questionnaires
1-C-4 End-of-Year Data from Teacher Questionnaires
1-C-5 Interviews with CSMP Kindergarten Teachers
1-C-6 Analysis of Teacher Logs

2-B-1 Second Grade Test Data
2-B-2 Readministration of First Grade Test Items
2-B-3 Student Interviews
2-C-1 Teacher Questionnaire Data
2-C-2 Teacher Interviews, Second Grade
2-C-3 Teacher Interviews, First Grade

3-C-1 Teacher Questionnaire Data, Year 3

4-B-1 Standardized Test Data, Third Grade
4-B-2 Mathematics Applied to Novel Situations (MANS) Test Data
4-B-3 Individually Administered Problems, Third Grade
4-C-1 Teacher Questionnaire Data, Third Grade

Evaluation Report 5-B-1 (1978) Fourth Grade MANS Test Data
5-B-2 Individually Administered Problems, Fourth Grade
5-C-1 Teacher Questionnaire and Interview Data, Fourth Grade

Evaluation Report 6-B-1 (1979) Comparative Test Data: Fourth Grade
6-B-2 Preliminary Test Data: Fifth Grade
6-C-1 Teacher Questionnaire Data: Grades 3-5

7-B-2 Fifth Grade Evaluation: Volume II, Test Data
7-B-3 Fifth Grade Evaluation: Volume III, Non-Test Data
7-B-4 Re-evaluation of Second Grade, Revised MANS Tests
7-B-5 Achievement of Former CSMP Students at Fourth Grade
7-B-6 Student Achievement, Rapid Implementation Model

8-B-2 Evaluation of Revised Second Grade, MANS Blue Level
8-B-3 Evaluation of Revised Third Grade, MANS Green Level
8-B-4 Three Evaluations of Gifted Student Use
8-C-1 Preliminary Study of CSMP "Graduates"

Key to Indexing

Evaluation Reports are labelled m-x-n, where "m" is the year of the pilot study, with 1973-74 as Year 1. "x" is the type of data being reported where A is for overviews and summaries, B is for student outcomes and C is for other data. "n" is the number within a given year and type of data.
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Appendix A: Comprehensive List of MANS Tests

Appendix B: Graphs of Class Means by Grade and Category
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Appendix C: List of School Districts Participating in MANS Testing

Appendix D: Advisory Panel
Since 1973, the Comprehensive School Mathematics Program (CSMP) has been developed, pilot tested and widely used in schools. Adopted by 134 sites across the country, it has been a massive curriculum development and implementation effort involving over $10 million in federal funds. CSMP has been the mathematics program for students in 9,000 classrooms throughout the country and more than 6,000 teachers have been trained to teach CSMP. To assess the program's impact, an independent evaluation has been conducted, including the assessment of the mathematics achievement of 14,000 students nationwide.

The original purpose of this report was to present a comprehensive summary of the mathematics achievement of CSMP students in these intensive field trials of the curriculum. This draft report was to be the first stage in this reporting process. Unfortunately, because of a decision by the sponsor to discontinue funding, the present report is likely to be the only report. Hence, there is a focus on the main results of the CSMP evaluation and many secondary issues must be attended to in only a cursory manner, if at all.

There is also a considerable amount of data that relates to issues in mathematics education generally (not just CSMP), which cannot be presented in this abbreviated report. The interested reader can scan the Table of Contents for an idea of what was originally intended for this report.
Description of the CSMP Curriculum and Its Development Cycle

CSMP began at Carbondale, Ill., with funding from the US Office of Education and Southern Illinois University, and then at St. Louis with funding first from the USOE and then NIE. The Director of CSMP from 1965 to 1979 was Burt Kaufman, and the curriculum is in large measure a tribute to his energy and dedication. Frederique Papy brought many new ideas to CSMP during her time as Associate Director for Research and Development; her influence pervades the entire curriculum.

The CSMP curriculum is the result of a long process of development, field testing, and revision in a wide range of geographical locations. Those sites varied in size and SES of community, and students' ability levels, as measured by standardized achievement tests, ranged from very low ability, including Title I, to upper track students defined as gifted by their districts. During those trials, complete sets of Teacher's Guides and students' books were written for each grade level. Materials, like the story books, the Papy Mini-computer and analysis cards for the String Game, were also provided.

Over the ten years of the project's history a four cycle model of materials development took place, essentially by grade levels:

1. CSMP staff wrote lessons and taught them informally in local classes.

2. A local pilot-test version of the curriculum was prepared from the revised lessons, taught by a few local teachers and observed by the CSMP staff.
3. A "Final Experimental" version of the curriculum, based on a revision of the local pilot version was taught for two years in the nationwide set of schools electing to pilot the curriculum.

4. A publication edition was prepared based on final revisions resulting from the extended pilot trials.

A formal evaluation mechanism was established at CEMREL, which was structurally independent of CSMP and funded under separate contracts. This evaluation group, which came to be known as the Mathematics Research and Evaluation Studies Project (MRES) was responsible for conducting summative evaluation based on the Extended Pilot Tests.

The following description of the CSMP curriculum has been excerpted from materials prepared by the developer for promotional purposes:

DESCRIPTION An underlying assumption of the CSMP curriculum is that children can learn and enjoy learning much more math than they do now. Unlike most modern programs, the content is presented not as an artificial structure external to the experience of children, but rather as an extension of experiences children have encountered in their development, both at the real-life and fantasy levels. Using a "pedagogy of situation," children are led through sequences of problem-solving experiences presented in game-like and story settings. It is CSMP's strong conviction that mathematics is a unified whole and should be learned as such. Consequently, the content is completely sequenced in spiral form so that each student is brought into contact with each area of content continuously throughout the program while building interlocking experiences of increasing sophistication as the situations become more challenging. A feature unique to CSMP is the use of three nonverbal languages that give children immediate access to mathematical ideas and methods necessary not only for solving problems, but also for continually expanding their understanding of the mathematical concepts themselves. Through these languages the curriculum acts as a vehicle that engages children immediately and naturally with the content of mathematics and its applications without cumbersome linguistic prerequisites. These languages include: the Language of Strongs (brightly colored strongs and dots that deal with the fundamentally useful and important mathematical notion of sets); the Language of Arrows (colored arrows between pairs of dots that stimulate thinking about relations between objects); and the Language of the Papy
Minicomputer. The Minicomputer, a simple abacus that models the positional structure of the numeration system, is used both as a computing device and as motivation for mental arithmetic. Its language can be used to represent all decimal numbers, positive or negative, and encourages creative thinking about the nature and properties of numbers. CSMP is flexible enough to facilitate whole-group, small-group, and personalized instruction, and is appropriate for all children from the "gifted" to the "slow learners." It recognizes the importance of affective as well as cognitive concerns and has been developed and extensively tested in classrooms nationally.

IMPLEMENTATION REQUIREMENTS School system signs cooperative agreement with CSMP and appoints local coordinator who undergoes 3-10 days of training (depending on highest grade level adopted) in St. Louis during spring or summer prior to first year of implementation. Coordinator trains all teachers new to CSMP before start of school. Smallest adoption unit is one teacher in one classroom. No training charge, but system pays expenses of attending training. Teachers and coordinators are required to buy training kits: K-3, $10, 4-6 $10. Optional adopter-site training is available; there is a fee for this service.

FINANCIAL REQUIREMENTS Start-up costs for one teacher and 30 students: Kindergarten, $140; first grade, $150; second grade, $200; fourth grade, $220; fifth grade, $220. Replacement costs for 10 students: kindergarten, $10; first grade, $26; second grade, $34; third grade, $47; fourth grade, $47; fifth grade, $48.

It is clear from the above that CSMP is quite different from most traditional programs. The innovative program characteristics of CSMP form a double-edged sword with great educational potential but also with corresponding problems in implementation, independent of students' achievement. Some of these are the following:

a. The curriculum contains new mathematical content particularly in probability, statistics, geometry, and other areas of applied mathematics, and this content is often new to the teachers themselves.

b. The curriculum contains new pedagogical techniques, especially the "languages" of the minicomputer, arrow diagrams, and string pictures, blended into a detailed and integrated sequence of lessons. These techniques require a considerable perservice training program for teachers to learn how to use them effectively, and they make the
curriculum more complex to explain to parents and school administrators.

c. CSMP often uses stories or situations in a discovery approach so that lessons tend to be longer than mathematics lessons usually are and place more emphasis on whole-group discussion. This requires more daily preparation for the teacher, at least the first time through the materials, and possible a change in teaching style.

d. CSMP places great emphasis on the spiral approach. This requires, as an article of faith on the part of the teacher, the acceptance of the premise that in many lessons some students will not master all of the lesson on that day, but will eventually learn the material if undue pressure on immediate mastery is avoided. This is in direct contrast to the pressures put on schools for mastery learning, criterion referenced testing, and minimum competencies -- all of which mandate absolute levels of achievement at fixed points in time regardless of the curriculum being used. The use of the CSMP curriculum in the context of these externally mandated requirements may require considerable skill on the part of local administrators, as well as faith that this approach will eventually lead to satisfactory achievement.

e. Because of the highly integrated and interdependent nature of lessons within a year and the content from year-to-year, the CSMP elementary curriculum cannot be implemented on a casual basis without classes eventually getting far behind in the recommended schedule. This means that, logistically, it is difficult to maintain CSMP if the level of implementation drops below a certain minimum, and that it is particularly important that all CSMP teachers in a school follow, at
least minimally, the intended curriculum. (It also raises questions concerning the transfer of individual students into and out of the program.)

f. CSMP is somewhat more expensive in yearly replacement costs than most regular textbook programs. Though various monies are available for implementation assistance, knowledge of their availability and requirements for approval (including the access to state approved lists) can become difficult for school administrators.

The data reported here are from districts and classes which for the most part were able to take advantage of these innovative characteristics and where CSMP was clearly the main source of mathematics instruction, albeit with countless adaptations. The implementation issues described above are reported more fully in report 9-A-2.

The Evaluation of CSMP: Goals and Plans

The evaluation of CSMP has been carried out by an organizationally separate division of CEMREL called Mathematics Research and Evaluation Studies (MRES). This group operated independently from the CSMP developers under the guidance of an Advisory Panel composed of five nationally recognized experts in the fields of math evaluation and education (see Appendix D). The MRES group produced an extensive Evaluation Report Series, 50 volumes in all, dealing with a wide variety of topics concerning CSMP. These are listed on pages iii and iv.
In the first evaluation report in this series (Evaluation Report Overview, Design and Instrumentation), the goals of the evaluation of CSMP were spelled out. Essentially three major issues were to be addressed.

1. **Intrinsic Merit.** In the opinion of qualified reviewers, is the CSMP program and its mathematical content sound and relevant?

2. **Practicality.** In terms of comparative costs, supervisory and instructional personnel requirements, management of students, etc., can school systems adopt the program with relative ease?

3. **Outcomes.** Do students learn the traditional skills and concepts, and the special skills and concepts of the CSMP curriculum, and can they transfer these skills and concepts more effectively (than Non-CSMP students) to unfamiliar mathematical contexts?

The issue of intrinsic merit was addressed early in the evaluation and reported in Evaluation Report 1-A-2, External Review of CSMP Materials. The issue of practicality was addressed each year of the evaluation and reported at various points in the Evaluation Report Series (see titles numbered n-C-n, page iv). A draft of a summary of these and other findings was prepared in November of 1982 and appears as Evaluation Report 9-A-2, Summary of Implementation Data, Draft Report. The outcomes issue was addressed each year of the evaluation and reported at various points in the Evaluation Report Series (see titles numbered n-B-n, page iv). This report is the draft of the summary of these reports on outcomes.
The main thrust of the testing of student outcomes was coordinated with the Extended Pilot Test (E.P.T.) of the Experimental Version of the CSMP curriculum. That version became available usually one grade level at a time, at intervals of a year or two. During the first year of the E.P.T. at any one grade level, preliminary testing took place with about a dozen intact CSMP classes and usually two or three dozen individual CSMP students and a comparable number of Non-CSMP classes and students from school districts close to St. Louis. The main purpose of the preliminary testing was to pilot the test instruments themselves. Then in the second year of the E.P.T. at any one grade level, the revised test instruments were used to test larger numbers of CSMP and Non-CSMP classes from school districts throughout the United States.

The content of the tests initially matched the three aspects of the "outcomes" evaluation goal. At the beginning, the testing involved standard content, CSMP content, and novel content. The CSMP curriculum differs significantly enough from the traditional curricula to make comparisons of achievement on content specific to either CSMP or Non-CSMP rather meaningless. As time went on, it became apparent that the most valid test and comparison of real student achievement involved content unfamiliar to either group of students. Hence, the importance and utility of the MANS Tests, (novel content) increased.

Figure 1 shows the schedule of testing in the Extended Pilot Test. The entry "Individual" in Figure 1 refers to tests which were administered individually to students and which contained content novel to both groups.
Figure 1

Schedule of Actual Testing Indicating Test Content By Year and By Grade

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Content</td>
<td>(Report Numbers in Parentheses)</td>
<td>Test Content</td>
<td>Test Content</td>
<td>Test Content</td>
<td>Test Content</td>
<td>Test Content</td>
<td>Test Content</td>
<td>Test Content</td>
</tr>
<tr>
<td>Individual</td>
<td>(120) Individual Content</td>
<td>Individual</td>
<td>Individual</td>
<td>Individual</td>
<td>Individual</td>
<td>Individual</td>
<td>Individual</td>
<td>Individual</td>
</tr>
<tr>
<td>Group</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
</tr>
<tr>
<td>Individual</td>
<td>Individual Content</td>
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<tr>
<td>Group</td>
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<td>Group Content</td>
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<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
<td>Group Content</td>
</tr>
</tbody>
</table>

(Continued on following page)
II. THE MANS TESTS

The MANS Tests (Mathematics Applied to Novel Situations) are a collection of short tests that assess how well students in grades 2 to 6 can use mathematical thinking and skills to solve problems that are new or unfamiliar to them.

The MANS Tests are normally contained in two student booklets at each grade level. The booklets contain several tests. Each test has its own directions which a specially trained tester follows in explaining the task and the sample items. Students then complete the items in that test on their own. A flexible time limit, usually about 5 or 6 minutes, allows almost all students to finish.

Each MANS Test takes up one or two pages in a booklet so that diagrams and illustrations are large, words are easy to read and there is ample space for students to do scratch work. For most tests, students produce their own answers instead of selecting one of several given alternatives. Answers are to be written in the booklet and can be erased or crossed out; no special pencil is required.

The MANS Tests were developed by the MRES staff to evaluate CSMP. The MANS Tests evolved from a need to develop tests which could be used to compare the progress of CSMP and Non-CSMP students. The tests had to reflect the emphasis CSMP places on generalized thinking skills and problem solving. However, to be fair to Non-CSMP students, they could not contain any of the representational languages or activities associated with CSMP.
To meet the need for tests that could be used to evaluate the effectiveness of CSMP as a program that develops mathematical thinking and problem solving skills, an extensive process of test development and revision was undertaken. The development occurred sequentially, one grade level at a time. At each grade level, the Mathematics Research and Evaluation Studies (MRES) staff first developed prototype tests. Sometimes the ideas for the tests were adapted from ideas in previous research in mathematics education; most times the ideas were original. The Advisory Panel (whose members are listed on the front cover) independently reviewed all of the test prototypes. Occasionally, teachers, math supervisors, and researchers also reviewed the tests. If the tests survived these reviews, they were pilot tested in a few local classes. On the basis of results from these pilot classes, tests were revised, in some cases eliminated.

The original version of the MANS tests resulted from this continuing process of development, review, testing, and revision. This version was used in the first Extended Pilot Test involving 15 to 20 local classes. After further refinement of the tests, they were used in the second Extended Pilot Test at that grade level. This evaluation study involved from 60 to 60 classes in several states, and is the basis for the main data of this report.

At each stage in this process of development, review, testing, and revision, the work was guided by the Advisory Panel. Some of the important considerations in the review and revisions of these tests, were the following:

- Intrinsic Merit: importance of the mathematical skill required;
- Curricular fairness; student interest in the novel problem context.
Administration: clarity and brevity of directions; student understanding of the task; low reading level; attractive format; unspeeded.

Technical: item analysis including range of difficulty levels, error analysis, discrimination coefficients; test analysis including ceiling and floor effects, ability level differences, reliability (KR 20 studies of internal consistency), analysis of class means and evidence of construct validity.
III. THE EXTENDED PILOT TESTS: MAIN RESULTS

The main results to be reported here are those obtained in the second year of the E.P.T. at each of grades two through six. This report will concentrate on the CSMP/Non-CSMP comparisons of student achievement on standard and novel content.

The main results reported here have the following characteristics. They compare the performance of CSMP students to that of Non-CSMP students of similar ability. They are based on results obtained from tests that are technically sound. They are based on the performance of CSMP and Non-CSMP students who (by and large) have had at least three years of formal instruction in CSMP or a more traditional program, respectively, and who are eight or more years of age. They are based on classroom experiences which are close to typical for both CSMP and Non-CSMP. Finally, for each grade level the data is collected in a similar fashion.

Design. School districts have used many strategies in adopting CSMP. One strategy never adopted is the kind of random assignment of teachers and students to experimental (CSMP) and control classes that would satisfy the conditions required for a true experimental comparison. Thus, the design of the Extended Pilot Test has had to be less rigorous. Generally, the strategy has been to first select a representative group of CSMP classes from districts using CSMP at that grade level. Then, in cooperation with local districts, appropriate NON-CSMP classes were selected in such a way as to minimize as much as possible factors other than curriculum which might cause differences in achievement between the two groups.
Through third grade, CSMP classes were selected from the local area and from districts with several CSMP classes (i.e., at least six). Comparison classes were usually selected from the same schools as CSMP classes, with teachers and students as similar to their CSMP counterparts as possible.

From fourth to sixth grade, school and district wide adoption of CSMP became more common and comparison classes came to be selected from other, similar schools in the district, or from other districts using CSMP at lower grade levels. Most districts with at least two CSMP classes were included in these later E.P.T.'s.

An average of 50% of the available CSMP classes participated in these Extended Pilot Tests. The curriculum used by the comparison classes was the usual mathematics curriculum for the district, almost always one of the common elementary textbooks. They were only minor differences between these textbooks, and in the analysis of class means, these comparison classes were combined into a single "Non-CSMP" category.

Table 1 gives the distribution of classes participating in the E.P.T. second year testing, by sites and grade level. During the five years covered in this testing, over 300 classes (half CSMP and half Non-CSMP) participated from 24 districts (sites). Very few of these 300 classes were in fact the same class counted twice because there were two 12-month gaps in the one-grade-per-year schedule of the E.P.T. and because the participating districts varied somewhat from year to year. With the exception of some classes in sixth grade, all the classes were regular in that they hadn't been grouped by ability. However, as can be seen by the last line in Table 1, the
### Table 1

Distribution of Classes Participating in the E.P.T. Testing

(Number of CSMP classes given first, Non-CSMP second)

<table>
<thead>
<tr>
<th>Site No.</th>
<th>Type of Community</th>
<th>Section of Country</th>
<th>Second Grade</th>
<th>Third Grade</th>
<th>Fourth Grade</th>
<th>Fifth Grade</th>
<th>Sixth Grade</th>
<th>All Grades</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Medium City</td>
<td>North Central</td>
<td></td>
<td></td>
<td>0-3</td>
<td>0-6</td>
<td>0-9</td>
<td></td>
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<tr>
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<td>Exurban</td>
<td>Central East</td>
<td></td>
<td>2-2</td>
<td>2-2</td>
<td>2-2</td>
<td>6-6</td>
<td></td>
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<tr>
<td>3</td>
<td>Medium City</td>
<td>South Central</td>
<td>6-6</td>
<td></td>
<td></td>
<td></td>
<td>6-6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Large City</td>
<td>North Central</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6-6</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Large City</td>
<td>Central East</td>
<td></td>
<td></td>
<td></td>
<td>2-2</td>
<td>2-2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Suburb</td>
<td>Central</td>
<td>3-3</td>
<td>4-5</td>
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<tr>
<td>7</td>
<td>Small City</td>
<td>West</td>
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<td>8</td>
<td>Suburb</td>
<td>North Central</td>
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<td></td>
<td>1-6*</td>
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<td>North Central</td>
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<td></td>
<td></td>
<td></td>
<td>7-0*</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Suburb</td>
<td>North Central</td>
<td>3-2</td>
<td>4-4</td>
<td>6-6</td>
<td>6-6</td>
<td>8-6</td>
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<td>Large City</td>
<td>Central</td>
<td></td>
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<td>0-6*</td>
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<td>0-6</td>
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</tr>
<tr>
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<td>Medium City</td>
<td>North Central</td>
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<td>0-5*</td>
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<td>Suburb</td>
<td>Central</td>
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<td>6-0</td>
<td></td>
<td>12-0</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Small City</td>
<td>North Central</td>
<td>1-1</td>
<td></td>
<td></td>
<td></td>
<td>1-1</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Suburb</td>
<td>North East</td>
<td>2-2</td>
<td>6-0</td>
<td>6-0</td>
<td>6-0</td>
<td>20-2</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Large City</td>
<td>South</td>
<td></td>
<td></td>
<td>5-3</td>
<td></td>
<td>5-3</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Suburb</td>
<td>Central</td>
<td>6-5</td>
<td>4-3</td>
<td>2-2</td>
<td>2-2</td>
<td>14-12</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Large City</td>
<td>Central East</td>
<td></td>
<td>3-4</td>
<td></td>
<td></td>
<td>3-4</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Small Town</td>
<td>South East</td>
<td></td>
<td>15-12</td>
<td></td>
<td></td>
<td>15-12</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Medium City</td>
<td>North East</td>
<td>6-6</td>
<td>6-12</td>
<td></td>
<td></td>
<td>12-18</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Large City</td>
<td>Central</td>
<td>3-3</td>
<td>1-1</td>
<td>1-2</td>
<td></td>
<td>5-6</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Suburb</td>
<td>Central</td>
<td></td>
<td></td>
<td>3-0</td>
<td></td>
<td>3-0</td>
<td></td>
</tr>
</tbody>
</table>

**TOTAL CLASSES**  
36-34 33-36 30-21 31-25 26-37 156-153

**Percentile Rank on Covariate**  
56-54 55-56 64-62 61-60 77-78

*Upper track classes, i.e. students specially grouped by ability*
classes tended to be somewhat above average in ability; this was particularly true in sixth grade due to the presence of upper track classes. At each grade level, approximately 60 classes were tested, about 30 CSMP and 30 Non-CSMP. Again, from the last line of Table 1, the ability level of the CSMP classes matched that of the Non-CSMP classes at each grade. Finally, by scanning the second and third columns of Table 1, the reader can see that nearly every type of community and section of the country is represented. Rural districts and the Far West part of the country are not well represented, however.

Categories of MANS Tests. Altogether some 111 tests were given during the five years of testing reported here. Appendix A gives a description of these tests. Many of the tests were used at more than one grade level, with items of different levels of difficulty; only 54 truly different tests are described.

For purposes of reporting, the tests have been categorized. Ninety-five of the 111 tests emphasize one of seven mathematical processes. These seven processes are named and described on the following pages and an example showing a few items from a test in that process category is given.

The other 16 of the 111 tests emphasize one of five special topics in mathematics: Algebra, Geometry, Logic, Organizing and Interpreting Data, and Probability. Again, examples of each can be found in Appendix A. These special topics were tested mostly in grade six but to some extent in grades five and four.
Computation

Straightforward calculation with basic facts and algorithms.

SUBTRACTION

Examples from a third grade test.

\[
\begin{array}{ccc}
11 & -5 & 64 \\
73 & -5 & -28 \\
\end{array}
\]

MULTIPLICATION

\[
5 \times 8 = 40 \\
\times 2
\]

Estimation

Rapid calculation of approximate answers. For this category only, fixed time limits are prescribed.

Examples from a sixth grade test. Short time limit.

\[
\begin{array}{c}
\frac{7}{12} + \frac{5}{12} \\
\frac{27}{12} \div 3 \\
\end{array}
\]

Mental Arithmetic

Solution of numerical problems that emphasize an understanding of numbers and operations, but do not require great mental computational facility.

Examples from a sixth grade test. No "scratch work" is allowed.

\[
\begin{array}{c}
7,001 - 6,999 = \\
98,001 - 98,000 - 5,000 = \\
12 \times 500 = \\
-250 = 150 \\
101 \times 43 = \\
7 \times 43 = 301 \\
14 \times 43 =
\end{array}
\]
Number Representations
Recognition, or production of different ways of representing numbers, including place value, number lines, measurement.

Example from a second grade test. For each of the first group of items (A through F), the tester says aloud a number for students to write in the blank.

Relationships and Number Patterns
Recognition or application of given patterns, orders, or relationships in sets of numbers.

Examples from a fourth grade test. Three sample items, explaining how the "machines" work, are done previously.

Elucidation of Multiple Answers
Production of many correct answers to a given problem.

Examples from a sixth grade test.
Word Problems

Solution of word problems requiring low levels of reading comprehension and computation and classified according to type of problem.

Examples from a second grade test. Tester reads the items aloud, frame-by-frame.

Apples cost 5¢ each and bananas cost 2¢ each.

Sally buys 3 apples and 1 banana.

Method of Analysis. Each of the participating classes was given some test of general academic ability. In the earlier grades, the actual test used varied from site to site and year to year but was usually either an academic intelligence test or a test of reading comprehension or vocabulary. In the later grades, classes uniformly took an appropriate level of the Gates-McGinitie Vocabulary Test.

For the main results reported here, the analysis of student achievement was done each year on class means. For each test, a mean score was calculated across all the students in the class who took all the achievement tests in the battery and who also took the test of general ability. The corresponding mean score on the test of general ability was also calculated. In both cases raw scores were used. Then an analysis of covariance procedure (with general
ability as the covariate) was used on each test to compare the mean score for the CSMP classes that year versus the mean score for the comparable Non-CSMP classes. From this analysis of covariance can be determined the probability that the observed difference in mean scores could have occurred by chance. (If that probability is less than 5%, the difference is often deemed to be "statistically significant".)

For each of these achievement tests, the mean scores for CSMP and Non-CSMP classes were compared in another and much simple way: the percentage difference in mean scores between the two groups of classes. (Previous to this the mean raw scores had been statistically adjusted to take into account differences between the two groups on the test of general ability. Since these two groups were invariably quite similar in general ability, such adjustments usually were quite small—less than 2%.)

Results By Grade and Test Category. In Table 2 there is a summary of the results of the testing, by grade level. For each grade, the table gives the number of achievement tests that were administered, the number of tests for which the analysis of covariance indicated a statistically significant difference in favor of CSMP or Non-CSMP, and the average percent difference in the adjusted mean scores. The bottom line of the table shows that 111 tests were administered over all grades. Of those 111 tests, 69 produced a statistically significant difference in favor of CSMP, two in favor of Non-CSMP, and an average percent difference in the adjusted mean scores of 13.4 in favor of CSMP.

1 Normally less than 15% of the students in a class had to be excluded due to this requirement. In addition, normally a class lost another 5% due to students joining the class late in the year or being labeled "special" in some handicapping way.
<table>
<thead>
<tr>
<th>Grade Level of E.P.T.</th>
<th>Number of Tests Administered</th>
<th>Number of Times Significant</th>
<th>Average Percent Difference</th>
<th>In Favor of:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second Grade</td>
<td>11</td>
<td>3</td>
<td>13.4</td>
<td>CSMP</td>
</tr>
<tr>
<td>Third Grade</td>
<td>15</td>
<td>10</td>
<td>12.9</td>
<td>CSMP</td>
</tr>
<tr>
<td>Fourth Grade</td>
<td>25</td>
<td>18</td>
<td>18.9</td>
<td>CSMP</td>
</tr>
<tr>
<td>Fifth Grade</td>
<td>26</td>
<td>15</td>
<td>12.9</td>
<td>CSMP</td>
</tr>
<tr>
<td>Sixth Grade</td>
<td>34</td>
<td>23</td>
<td>10.1</td>
<td>CSMP</td>
</tr>
<tr>
<td>Total Across All Grades</td>
<td>111</td>
<td>69</td>
<td>13.4</td>
<td>CSMP</td>
</tr>
</tbody>
</table>

The results at each grade reflect the overall results rather consistently with two minor exceptions: in second grade, not as large a proportion of the tests produced statistically significant differences in favor of CSMP, and in fourth grade the average percent difference in adjusted mean scores was much larger.

In Table 3 a summary similar to that of Table 2 is given according to test category.
Table 3
Summary of E.P.T. Test Results
All Grades Combined
Given by Test Category

<table>
<thead>
<tr>
<th>Test Category</th>
<th>Number of Tests Administered</th>
<th>Number of Times Significant CSMP</th>
<th>Non-CSMP</th>
<th>Average Percent Difference</th>
<th>In Favor of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Processes:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computation</td>
<td>8</td>
<td>2</td>
<td>1</td>
<td>3.8</td>
<td>CSMP</td>
</tr>
<tr>
<td>Estimation</td>
<td>13</td>
<td>9</td>
<td>0</td>
<td>8.5</td>
<td>CSMP</td>
</tr>
<tr>
<td>Mental Arithmetic</td>
<td>21</td>
<td>19</td>
<td>0</td>
<td>19.1</td>
<td>CSMP</td>
</tr>
<tr>
<td>Number Representation</td>
<td>12</td>
<td>5</td>
<td>0</td>
<td>8.6</td>
<td>CSMP</td>
</tr>
<tr>
<td>Relationships and Number Patterns</td>
<td>22</td>
<td>16</td>
<td>0</td>
<td>20.8</td>
<td>CSMP</td>
</tr>
<tr>
<td>Elucidation</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>16.7</td>
<td>CSMP</td>
</tr>
<tr>
<td>Word Problems</td>
<td>13</td>
<td>8</td>
<td>0</td>
<td>15.1</td>
<td>CSMP</td>
</tr>
<tr>
<td>Special Topics:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Algebra</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>11.0</td>
<td>CSMP</td>
</tr>
<tr>
<td>Geometry</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3.0</td>
<td>Non-CSMP</td>
</tr>
<tr>
<td>Logic</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
<td>---</td>
</tr>
<tr>
<td>Organizing and Interpreting Data</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
<td>Non-CSMP</td>
</tr>
<tr>
<td>Probability</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>11.2</td>
<td>CSMP</td>
</tr>
<tr>
<td>Total Across</td>
<td>111</td>
<td>69</td>
<td>2</td>
<td>13.4</td>
<td>CSMP</td>
</tr>
</tbody>
</table>

All Categories
Most of the tests were in one of the Mathematical Process categories. Of those, the results were strongly in favor of CSMP in Mental Arithmetic and Relationships and Number Patterns. In those two cases the CSMP advantage showed up in both the number of times the tests produced statistically significant differences and in the average percent difference. The results were almost as strongly in favor of CSMP in Elucidation and Word Problems, and slightly in favor of CSMP in Estimation and Number Representations. In the Computation category there was essentially no overall advantage to either group.

In terms of Special Topic categories, the reader is reminded that the majority of these tests were given only in sixth grade. Only in two categories (Algebra and Probability) was there strong evidence for a difference, and in both cases it was in favor of CSMP.

Graphs of Class Means. On the following pages, graphs of class means for Total MANS are shown for each of grades four through six to illustrate the data of Table 2. For each graph, an entry on the graph represents a class; the position on the graph is determined by its "ability" score (IQ, reading, vocabulary—whatever happened to be used that year) and by its Total MANS score.

CSMP classes are shown by x's, Non-CSMP classes by o's. A regression line has been drawn in each graph to show the best prediction of MANS score for a given ability score. One does not need a test of significance to determine that there is a clear pattern for CSMP classes to be above the regression line and Non-CSMP classes to be below the line.
Figure 2, Fourth Grade Class Means
(x=CSMP class, o=Non-CSMP class)

Figure 3, Fifth Grade Class Means
(x=CSMP class, o=Non-CSMP class)
Figure 4, Sixth Grade Class Means
(x=CSMP class, o=Non-CSMP class)
IV. THE EXTENDED PILGRIM TEST: SECONDARY RESULTS

Note to the reader: This chapter has not been completed for this draft report. Some illustrative data from each section is given below.

Analysis of School and District Data

The graphs shown on pages 26 and 27 show entries for each class. In each case, the CSMP classes are doing better than Non-CSMP classes and the differences are statistically significant. When the data are aggregated by schools, instead of classes, the differences remain significant, but the pattern of mean scores becomes clearer.

Notice in Figure 4 that there are six CSMP classes which are scoring well below the regression line. Figure 5, below, is the graph of school means; the six low scoring classes were concentrated in two or three schools rather than being scattered randomly.

![Figure 5, Sixth Grade School Means](x=CSMP School, ●=Non-CSMP School)
Figure 5 illustrates two points. First, at higher grades it is school rather than teacher that may be the more effective factor in achievement. This may be because the MANS Tests are cumulative in nature rather than testing specific sixth grade content, because of the influence of the principal, or because there is cohesion among the teachers in how to teach the math curriculum. The second point is that the CSMP effect is more evident in school level analysis; in Figure 5 there were 16 CSMP schools, 14 of them did better than all but one Non-CSMP schools while the other two did worse than all Non-CSMP schools.

Analysis of district level means provides similar findings—still significant differences and clearer interpretations.

Analysis of Item Data

Extensive item analyses were done each year on all items in the MANS Tests. There were several kinds of items which produced large differences year after year between CSMP and Non-CSMP students. For example CSMP students consistently did better than Non-CSMP students in multiplication of whole numbers, fractions, and decimals. This advantage was largest in the years in which these topics were being introduced in mathematics and reflected the early CSMP emphasis. The differences decreased in later grades. Similarly CSMP students did not initially do as well in whole number subtraction and long division, again reflecting the later (or lesser) emphasis of these topics in the CSMP curriculum.
Analysis of Student Level Data

In order to analyze the effect of the CSMP curriculum according to ability level of student, the following procedure was followed at each grade level. Students were grouped according to percentile rank on the covariate test—reading, vocabulary, or IQ. There were four groups corresponding to highest quarter in ability, second highest, etc. Then comparisons on MANS Tests between CSMP and Non-CSMP students were made for each ability quartile.

Figures 6 and 7, from sixth grade, illustrate the different results which were found in the categories Number Representations and Elucidation (both of which had significant differences in favor of CSMP in Analysis of Covariance on class means).

![Graph showing scores on Number Representations by Ability Level of Students](image-url)
On Elucidation, the low ability CSMP students had just as large a superiority over their non-CSMP counterparts as did the high ability students. This is what happened most of the time, i.e. the results were consistent at different ability levels. But on Number Representations, the low ability CSMP students did not share in the otherwise consistent advantage of CSMP students. This result happened occasionally, most often in the less hard tests; such as Computation.
Appendix A

Description of MANS Tests

Comprehensive List

Note: This is the comprehensive list for the most current versions of the MANS Tests. There are minor discrepancies between these tests and the ones used in the E.P.T.
Process Categories:

C: Computation
E: Estimation
M: Mental Arithmetic
N: Number Representations
R: Relations & Number Patterns
U: Elucidation
W: Word Problems

Special Topic Categories:

A: Algebra
G: Geometry
L: Logic
O: Organization of Data
P: Probability
Category C: Computation

C1 Whole Number Computation

Abstract: Given straightforward computation problems involving whole numbers, produce exact answers (by calculating on paper if necessary). The items do not have the multiple choice response format but are similar in range and difficulty to those found in the standardized achievement tests of the appropriate grade level.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 4):

\[ \begin{align*}
352 + 683 &= 1035 \\
675 - 469 &= 206 \\
143 \times 5 &= 715 \\
\end{align*} \]

C2 Fraction Computation

Abstract: Given straightforward computation items involving simple fractions, produce exact answers (by calculating on paper if necessary). Though the items do not have the multiple choice response format, they are similar in range and difficulty to those found in the standardized achievement tests of the appropriate grade level.

Grade Levels: 4, 5, 6

Examples (from Grade 5):

\[ \begin{align*}
\frac{3}{5} - \frac{1}{5} &= \frac{2}{5} \\
\frac{1}{2} + \frac{1}{2} &= 1 \\
\frac{1}{2} \times \frac{1}{2} &= \frac{1}{4} \\
\end{align*} \]

C3 Decimal Computation

Abstract: Given straightforward computation items involving one and two place decimals, produce exact answers (by calculating on paper if necessary). Though the items do not have the multiple choice response format, they are similar in range and difficulty to those found in the standardized achievement tests of the appropriate grade level.

Grade Level: 6

Examples:

\[ \begin{align*}
0.5 + 0.25 &= 0.75 \\
5 - 1.5 &= 3.5 \\
0.5 \times 0.5 &= 0.25 \\
\end{align*} \]
Category E: Estimation

E1 2 or 5 or 10 Times

Abstract: Given two numbers, quickly estimate whether the first is about 2 or 5 or 10 times as large as the second. A sample is worked collectively.

Grade Levels: 3, 4

Examples (from Grade 3): 65 is about ___ times as large as 12
98 is about ___ times as large as 51

E2 Estimating Intervals: Addition

Abstract: Given a computation problem involving whole number addition, and 5 fixed intervals (0-10, 10-50, 50-100, 100-500, 500-1000), determine which interval contains the answer to the problem, and put an x in the interval. By instruction, format and short time limits, students are discouraged from computing exact answers. Two or three sample items are done collectively.

Grade Levels: 2, 3, 4, 5

Examples (from Grade 2): 51 + 53 0 10 50 100 500 1000
189 + 273 0 10 50 100 500 1000

E3 Estimating Intervals: Subtraction

Abstract: The scale is similar to E2 (except that it involves whole number subtraction) and follows it directly in the test booklets.

Grade Levels: 2, 3, 4

Examples (from Grade 3): 93 – 86 0 10 50 100 500 1000
147 – 99 0 10 50 100 500 1000

E4 Estimating Intervals: Multiplication

Abstract: The scale is similar to E2 and E3 (but is devoted to multiplication with whole numbers for the most part) and follows them in the test booklets.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 4): 40 x 10 0 10 50 100 500 1000
4 x 29 0 10 50 100 500 1000
E5 Estimating Intervals: Division

Abstract: The scale is similar to E2, E3 and E4 (but is devoted to division with whole numbers for the most part). There are only four fixed intervals (0-1, 1-10, 10-20, 20-100) in the response format. It follows E4 in the test booklets.

Grade Level: 5, 6

Examples:

\[
\begin{align*}
1 \div 15 & \quad 0 \quad 1 \quad 10 \quad 20 \quad 100 \\
101 \div 9 & \quad 0 \quad 1 \quad 10 \quad 20 \quad 100
\end{align*}
\]

E6 Estimating Fractions <, *, >1

Abstract: Given a calculation (+, -, or :) of two numbers (at least one of which is a fraction or mixed number), quickly estimate whether the answer would be less than, equal to or more than 1. Students are encouraged to work quickly and not to compute exact answers before making their choices. A completed sample item is provided.

Grade Level: 6

Examples:

<table>
<thead>
<tr>
<th>(1\frac{5}{8} - \frac{1}{128})</th>
<th>Less than 1</th>
<th>CHECK ONE</th>
<th>More than 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2\frac{1}{2} \div 3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Category M: Mental Arithmetic

M1 Whole Number Open Sentences

Abstract: Given an open sentence, where the box may be either on the right or the left of the equal sign, where the numbers are large and easy to work with, and where only one operation is used, put the number in the box which makes the sentence true. By instruction and prompting, students are discouraged from "computing the long way" and are not allowed to do any figuring on paper.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 3)

- $500 + \_ = 800$
- $\_ - 150 = 50$
- $2 \times 200 = \_$

M2 Above and Below Zero

Abstract: Given a starting score (which could be above or below zero), and how much the score went up or down, select the correct final score (multiple choice).

Grade Levels: 2, 3

Examples (from Grade 3)

Score at the start: 3 below zero
Then: Lost 4
Score at the end: 7 below zero, 1 below zero, 1 above zero, 7 above zero

Score at the start: 2 above zero
Then: Lost 4
Score at the end: 6 below zero, 2 below zero, Zero, 2 above zero
M3 Negative Hits and Misses

Abstract: Given the description of a "game" with two rules (a) each hit means a gain of 5 points and b) each miss means a loss of 1 point) and partial information on the outcome of turns, the student must deduce the missing information. Two sample items are completed collectively.

Grade Levels: 4, 5, 6

Examples:

<table>
<thead>
<tr>
<th>Started with</th>
<th>Number of Hits</th>
<th>Number of Misses</th>
<th>Ended with</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pam:</td>
<td>4 above zero</td>
<td>6</td>
<td>3 above zero</td>
</tr>
<tr>
<td>John:</td>
<td>2</td>
<td>0</td>
<td>15 below zero</td>
</tr>
</tbody>
</table>

(provided, but not mentioned in instructions)

M4 Fraction Open Sentences

Abstract: Given an open sentence involving at least one fraction, and one of the four arithmetic operations, complete the sentence.

Grade Level: 6

Examples: \[
\frac{3}{5} \div \frac{\square}{\square} = 1 \\
1 - \frac{\square}{\square} = \frac{3}{4}
\]

M5 Decimal Open Sentences

Abstract: Given an open sentence involving at least one decimal number and one of the four arithmetic operations, complete the sentence.

Grade Level: 6

Examples: \[
0.5 \div \frac{\square}{\square} = 1 \\
0.75 - \frac{\square}{\square} = 0.5
\]
Category N: Number Representations

N1 Writing Whole Numbers

Abstract: Part I: The student must write numbers as they are read aloud by the tester.

Part II: Given a number, written in the test booklet, the student must write the number which is 1 (or 10 or 100) more than it. A sample item is worked collectively.

Grade Level: 2

Examples: Part I: Tester says, "Eight hundred twenty" (repeats)
Tester says, "Seven thousand sixty five" (repeats)

Part II: What number is 1 more than 999?
What number is 10 more than 495?

N2 1, 10, 100 or 1000 More

Abstract: Given two numbers, decide whether the first number is about 1, 10, 100 or 1000 more than the second number. (None is exactly right.) Two sample items are worked collectively.

Grade Level: 3

Examples:

1
10
4,265 is about 100 more than 4,254
100
1000

1
10
1,001 is about 100 more than 998
100
1000
Constructing Numbers

Abstract: Given the use of only four digits (2, 5, 7 and 8) and the rule that no digit be used more than once, construct numbers like the smallest (or largest), the second smallest (or largest) or the closest to a given number. The constructed numbers are to be of either 2, 3 or 4 digits and sometimes restricted to a given range of numbers. Collectively, to clarify the rules, two incorrect answers and the correct one are examined for two sample problems.

Grade Level: 4

Examples: What is the second largest four digit number?

What is the smallest three digit number between 730 and 850?

What four digit number between 2,000 and 3,000 is closest to 2,800?

Representing Fractions

Abstract: The scale has five short subsections each containing one of two kinds of items: a fraction or mixed number is given in standard form and must be represented in another specific way or else that process is reversed and the response format is multiple choice. Instruction is largely in the form of a written question or command at the beginning of each subsection.

Grade Level: 4

Examples: Put an arrow at $\frac{1}{4}$ inches.

How much is shaded?
N5 Representing Fractions and Decimals

Abstract: The scale has five short subsections each containing one of two kinds of items: either a mixed number or decimal is given in standard form and must be represented in another specific way or else that process is reversed and the response format is multiple choice. Instruction is largely in the form of a written question or command at the beginning of each subsection.

Grade Level: 5, 6

Examples: Put an arrow at 1.35 inches.

How much is shaded? \[\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \text{ none of these}\]

(A completed sample was given.)

N6 Equivalent Fractions and Decimals

Abstract: Given a fraction (or decimal) determine which members of a set of fractions (or decimals) are equivalent to it. A sample set of four completed items is shown.

Grade Level: 5, 6

Examples: Circle all the fractions that are equal to the one in the box.

\[\frac{2}{3}, \frac{9}{12}, \frac{4}{6}, \frac{3}{2}, \frac{10}{15}\]
Category R: Relationship & Number Patterns

R1 Solving Number Rules

Abstract: Given 3 clues (i.e., pairs of numbers) in a game, determine what the secret method is (i.e., the unique rule relating each of the pairs of numbers) and then use the rule to calculate the missing number from the fourth pair.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 3):

<table>
<thead>
<tr>
<th>Maria's Game</th>
<th>Jim's Game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td>Maria's</td>
</tr>
<tr>
<td>First clue:</td>
<td>5</td>
</tr>
<tr>
<td>Second clue:</td>
<td>7</td>
</tr>
<tr>
<td>Third clue:</td>
<td>8</td>
</tr>
<tr>
<td>Question:</td>
<td>2</td>
</tr>
</tbody>
</table>

R2 Using Number Machines

Abstract: Given labelled "number machines" in sequence and either the initial or the terminating number, determine the other number. There is an introduction showing that "number machines" take in numbers; add, subtract, multiply or divide by a fixed quantity; and give out the resultant number. There are three sample items (each with a "number-machine" sequence) to be worked collectively.

Grade Levels: 3, 4, 5, 6

Examples (from Grade 4):

R3 Sequences

Abstract: Given an incomplete portion of an additive sequence of numbers, determine the missing number. One sample item is worked collectively.

Grade Level: 2

Examples: 28, 25, ___, 19, 16, 13, 1, 1½, 2, ___, 3, 3½, 4
R4 Which Result is Larger

Abstract: Given two quantities (usually similar computation problems using +, -, or x) mark the one which yields the larger result, or mark them both if they are equal. By instruction, format and time limits, students are discouraged from computing exact answers. The correct response should be more easily determined by inspection than by computation. Two sample items are worked collectively.

Grade Levels: 2, 3

Examples (from Grade 2):

- $585 + 250$ □
- $3 \times 31$ □
- $580 + 290$ □
- $31 \times 3$ □

R5 Labelling Number Lines

Abstract: Given partially labelled number lines, with varying increments, determine certain missing numbers. A sample item is worked collectively.

Grade Levels: 2, 3, 4, 5, 6

Examples (from Grade 2):

![Number Line Example](image)

R6 Multiplication Series

Abstract: Given an incomplete portion of a multiplicative series of numbers, determine the constant multiplier involved in order to complete the portion shown. Portions of several series are shown altogether with one, two or three numbers missing from each. A sample series is examined and completed collectively.

Grade Level: 4

Examples:

- \[ \cdots 1,000 \, 10,000 \, 100,000 \, \cdots \]
- \[ \cdots 2 \, 18 \, \cdots \]
R7 Which Fraction is Larger

Abstract: Given two non-whole numbers written in fractional form (a proper fraction, an improper fraction or a mixed number), circle the larger one. A completed sample item is shown.

Grade Level: 5, 6

Examples: \( \frac{3}{4} \) or \( 1 \frac{1}{4} \)

\( 3\frac{1}{2} \) or \( \frac{5}{2} \)

R8 Which Decimal is Larger

Abstract: Given two non-whole numbers written in decimal form, circle the larger one. A completed sample item is shown.

Grade Level: 5, 6

Examples: .4.999 or 5.1

1.5 or 0.58

R9 Fractions Between Two Others

Abstract: Given two fractions, write another which is larger than the first and smaller than the second.

Grade Level: 6

Examples: \( \_ \_ \_ \) is larger than \( \frac{1}{3} \), but smaller than \( \frac{7}{8} \)

\( \_ \_ \_ \) is larger than \( \frac{1}{4} \), but smaller than \( \frac{1}{2} \)

R10 Decimals Between Two Others

Abstract: Given two decimal numbers, write another which is larger than the first and smaller than the second.

Examples: \( \_ \_ \_ \) is larger than 1.25, but smaller than 2.0

\( \_ \_ \_ \) is larger than 0.42, but smaller than 0.43
Category U: Elucidation

U1 Number Sentences About 8

Abstract: Students are to produce as many different "sentences about 8" as possible, always in the form "8 = ...". Four correct answers to similar exercises about 9 are examined collectively. (9 = 10 - 1, 9 = 1 + 5 + 3, 9 = 3 x 3, 9 = 18 - 2).

Grade Level: 2

Example: My number sentences about 8.
8 = ...

U2 Producing Many Answers

Abstract: Given several different situations each of which poses a problem for which there are many correct solutions, produce as many of them as possible. For each situation, some potential solutions are accepted or rejected for not following the given rules as inappropriate.

Grade Level: 3, 4, 5, 6

Examples (from Grade 3):
Rules: Take out two balls. Add the two numbers to get a score.

What are the possible scores? 6, 2, 35

Rules: Write all the two digit numbers you can. Use only the digits 1, 2, 3.

Give all the numbers that follow the rules. 34, 22

U3 Getting to 12

Abstract: Given a starting point (0), a goal (12) and two rules, invent as many ways of reaching the goal as possible. The rules are that only the numbers 2, 3, 5 & 7 can be used along with addition, subtraction, multiplication or division. Two sample solutions (see below) are worked collectively.

Grade Level: 6

Examples:
Sample 1: 0 + 7 = 7
7 x 2 = 14
14 - 2

Sample 2: 0 + 5 = 5
5 + 3 = 8
8 + 2 = 4
4 x 3 = 12
Category W: Word Problems

W1 One Step Word Problems

Abstract: Solve word problems in which the story (including the question) is read by the tester while the student looks at a series of cartoons and/or follows the story in the captions beneath the cartoons. Seven items require one-step solutions; two items require two.

Grade Level: 2

Examples:

Jill spent 6¢ to buy some bananas.
Bananas cost 2¢ each.

How many bananas did she buy?

Jim found 3 marbles but he lost 4.
And now he has 5 marbles.

How many marbles did he have to begin with?

W2 Two Stage Word Problems

Abstract: Solve word problems in which the solutions require two operations. The numbers in the problems are relatively small; the computational and reading requirements are simple.

Grade Levels: 3, 4, 5, 6

Examples (from Grade 4):

Pam gets 50¢ each week.
She always spends 30¢ and saves the rest.
How much will she save in 4 weeks?

Tom has 3¢ more than Ann.
Tom has 5¢ less than John.
If John has 20¢, how much does Ann have?
W3 Miscellaneous Word Problems

Abstract: Solve word problems which are unusual for third graders in one of several ways: requires three-stage solution, requires working backward from a given final state to an unknown initial state, requires more logical analysis than straight computation, involves proportional ratios, involves extraneous data.

Grade Level: 3

Examples:
At first, Sally had some marbles.  
Then, she lost 3 of them.  
Then, she found 2 marbles.  
After that, she still had 8 marbles left.  
How many did she have at first?

Sam has to move 10 boxes.  
He can carry 3 boxes each trip.  
How many trips will he need to make?

W4 Extraneous Information

Abstract: Solve word problems in which extraneous information is given. Once the relevant information is selected, the solutions are simple one-step problems involving small whole numbers.

Grade Level: 4

Examples:
A belt costs $4.  
A shirt costs $5.  
A hat costs $10.  
How much more does a hat cost than a belt?

Peter has $10.  
He needs 4 pounds of candy.  
Candy is $2 per pound.  
He is buying candy for 6 people.  
How much will the candy cost altogether?

W5 Fractional Sugar

Abstract: Solve word problems each of which start with cups of sugar. The one-step solutions all require simple computations (+, -, x or -) with fractions or mixed numbers.

Grade Level: 4

Examples:
Tina has $4\frac{1}{2}$ cups.  
She buys $5\frac{1}{2}$ more cups.  
How much sugar will she have then?

Kari has $4\frac{1}{2}$ cups.  
She gives away half of it.  
How many cups of sugar will she have left?
W6 Three Stage Word Problems

Abstract: Solve word problems in which the solution requires three operations. The problem is stated in 3 to 5 short sentences and the numbers given in the problems are relatively small.

Grade Level: 5, 6

Examples:

Shirts cost $10 each and ties cost $5 each.
Altogether Joe spent $35 for shirts and ties.
He bought 2 shirts.
How many ties did he buy?

Bill loads 6 boxes in 2 hours.
John loads 4 boxes in 2 hours.
Together, how many boxes do they load in 6 hours?

W7 Decimal Gas

Abstract: Solve word problems each of which start with 6.5 gallons of gas. The one-step solutions all require simple computations (+, -, x, or -) with decimals.

Grade Level: 5

Examples:

Peter has 6.5 gallons.
Then he spills 1.2 gallons.
How much gas will he have left?

Ron has 6.5 gallons.
Next week he will use ten times this much.
How much gas will he use next week?

W8 Novel Word Problems

Abstract: Solve word problems which are novel for sixth graders in one or two of the following ways: involves fractions or decimals, requires more than three-stage solution, answer choices are approximate, requires solving for two unknowns, requires the use of data which is common knowledge but not given in the problem. Response format is multiple choice.

Grade Level: 6

Examples:

Ellen saw pepper plants on sale at 3 plants for 40¢.
She bought 12 plants.
She usually bought 3 plants for 50¢.
How much did she save?

20¢ 40¢ 48¢ $1.60 $2.00

George's father gives him 2¢ for every hour he spends in school.
About how much would he have given George for the month of October?

$.50 $1.00 $3.00 $6.00 $10.00 5.
Category A: Algebra

A1 Algebraic Symbols

Abstract: Given the numerical value of a letter (or letters) produce the numerical value of an expression involving that letter (those letters). In written instructions, two sample items are worked out and implied multiplication (e.g. in 3bc or in d^4) is explained. This scale follows A2 in the test booklet.

Grade Level: 6

Examples: If g = 4 and h = 3 then 5gh = ___

If p = 2 then p^5 = ___

A2 Solving Equations

Abstract: Given simple equations in one unknown, solve for the unknown. Three sample items are worked collectively, including one with a parenthesis.

Grade Level: 6

Examples: (7 x h) + 1 = 15, so h = ___

(n + 1) ÷ 3 = 6, so n = ___

A3 Summation Operator

Abstract: Given an open sentence involving one or more summations of consecutive integers, select the answer that completes the sentence. A symbol for such summations (\(\sum\)) is introduced and explained (\(\sum_{i=1}^{6} = 2 + 3 + 4 + 5 + 6\)) and two items are worked collectively.

Grade Level: 6

Examples:

\[1 + 2 = 1 + 3 + \boxed{1}\]

\[1 + 2 + 3 = \boxed{1} + 2 + 3\]

\[1 + 2 = \boxed{1} - 50\]

\[1 + 2 = \boxed{1} + 50\]
Abstract: Given two different transformations (\( \mathcal{T} \) which turns a design clockwise by 90° and \( \mathcal{R} \) which reverses the number of symbols at the top and bottom of a design), the scale consists of two different sections: requiring the application of either \( \mathcal{T} \) or \( \mathcal{R} \) to a design, requiring several applications of \( \mathcal{T} \) and/or \( \mathcal{R} \) to a design. Several sample items are worked collectively in each section.

Grade Level: 6

Examples: Section I: \( \mathcal{T} (\circ \times) = \left(\begin{array}{c} \times \circ \\ \circ \times \end{array}\right) \) 

Section II: 

\[ \begin{array}{c} \circ \circ \\
\times \times \\
\circ \circ \\
\times \times \end{array} \]

do \( \mathcal{T} \) and then \( \mathcal{T} \) 

\[ \begin{array}{c} \times \times \\
\circ \circ \\
\times \times \\
\circ \circ \end{array} \]

do \( \mathcal{T} \) twice
Category G: Geometry

G1 Geometric Loci

Abstract: Determine which picture is described by a given statement, where several pictures are given, each of which has identically placed elements (an 'x', an 'o' and a line) but a different set of dots, determine which picture a given statement describes. First statement is read by the tester.

Grade Level: 4

Examples: A [Diagram] E [Diagram]

In which picture are all the dots the same distance from the x? A B C D

In which picture is each dot just as close to x as to o? A B E F

G2 Geometric Congruencies

Abstract: Given a regular geometric shape divide the shape into a certain number of congruent parts. The word "congruent" is not used. Three correct and three incorrect solutions to a sample problem are examined collectively.

Grade Level: 5

Examples: [Diagram] [Diagram]
Category G: Geometry

G3 Geometric Categories

Abstract: Given nine different geometric figures, identify a set of 2 to 7 figures that are alike in some way, describe the distinguishing characteristic and label the figures accordingly. Go through this process as many times as possible. Two examples are worked collectively.

Grade Level: 6

Examples:

Sample 1 All the figures with "A" have square angles.

Sample 2 All the figures with "B" have only two sides that are one inch long.

All the figures with "C"

All the figures with "D"

etc.
Category L: Logic

L1 Logical Identification

Abstract: Given a specific set of individuals, a specific set of characteristics, the fact that each individual has a distinct combination of characteristics, and several facts about some of the characteristics of some of the individuals, identify the characteristics of each individual. A smaller sample problem is worked collectively.

Grade Level: 6

Example:

These are the 4 boys: Bill, Tom, Ed, Pete.

These are the 4 leagues: indoor soccer, outdoor soccer, indoor hockey, outdoor hockey.

These are the facts: Each boy plays in a different league.

Bill plays indoors.

Tom doesn't play hockey.

Ed doesn't play outdoors and he doesn't play soccer.

What league does each boy play in? (Circle your answers.)

Bill: indoor soccer, outdoor soccer, indoor hockey, outdoor hockey.

Tom: indoor soccer, outdoor soccer, indoor hockey, outdoor hockey.

Ed: indoor soccer, outdoor soccer, indoor hockey, outdoor hockey.

Pete: indoor soccer, outdoor soccer, indoor hockey, outdoor hockey.

L2 Making Sentences False

Abstract: Given a picture of a set of blocks and a true sentence about them, make the sentence false by changing the blocks. In the first two items, three suggested changes in the blocks are given and the student need only mark which ones would falsify the sentence. In the last three items, the student must write a change in the blocks. An item of the first type is worked collectively.

Grade Level: 6

Examples:

JOE'S BLOCKS

"There are triangles above the line and squares below the line."

a. Take away the triangles.

b. Take away the squares below the line.

c. Add squares above the line.

"Triangles go above the line or circles go below the line." (You write what Joe could do to make the sentence false.)
Category 0: Organization of Data

01 Graphing Weight

Abstract: Given a graph in which weight (axis labelled at 10 pound increments for each 5 graph units) is plotted against age (axis labelled at 2 year increments for each 2 graph units), determine age per given weights and vice versa. One sample item is worked collectively.

Grade Level: 5

Examples: How much did Bill weigh at 4 1/2 years of age? __________

How old was Bill when he reached 90 pounds? __________

02 Interpolating from a Table

Abstract: Given a table of prices for pipe of 4 different widths and 4 different lengths, interpolate or extrapolate to obtain the price on a pipe of given dimensions: at least one of which is not shown in the table. Two sample items are worked collectively.

Grade Level: 6

Examples:

<table>
<thead>
<tr>
<th>Width</th>
<th>Cost of Pipe</th>
</tr>
</thead>
<tbody>
<tr>
<td>4&quot;</td>
<td>100'</td>
</tr>
<tr>
<td>8&quot;</td>
<td>$50</td>
</tr>
<tr>
<td>12&quot;</td>
<td>$70</td>
</tr>
<tr>
<td>16&quot;</td>
<td>$90</td>
</tr>
<tr>
<td>20&quot;</td>
<td>$110</td>
</tr>
</tbody>
</table>

How much does it cost to buy pipe which is:

6" x 100' = $_________

20" x 1000' = _______
P1 Choosing the Best Box

Abstract: Given three boxes containing different combinations of 1, 2 and 50-cent balls, determine from which box it would be best to make a blind draw.

Grade Level: 5, 6

Examples:

WHICH BOX WOULD YOU CHOOSE?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

WHICH BOX WOULD YOU CHOOSE?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>50</td>
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<tr>
<td>1</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>50</td>
</tr>
</tbody>
</table>

P2 Dependent Outcomes

Abstract: Given two (or three) spinners and an amount (10) to be achieved or exceeded to win, select (from five standard choices) how often a player would win. Collectively it is shown how a player could win or could lose with a specific set of spinners.

Grade Level: 6

Examples: 9 2 8 2

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>never</td>
<td>less than</td>
</tr>
<tr>
<td>time</td>
<td></td>
</tr>
<tr>
<td>more than</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>never</td>
<td>less than</td>
</tr>
<tr>
<td>time</td>
<td></td>
</tr>
<tr>
<td>more than</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

Graphs of Class Means by Grade and by Category

Note to reader: It was not possible to prepare these prior to the publication of this draft report.
Appendix C

List of School Districts Participating in MANS Testing

Ann Arbor, Michigan
Baltimore County, Maryland
Bedford, Michigan
Bronx, New York
Clarksville, Tennessee
Detroit, Michigan
District of Columbia
Ferguson-Florissant, Missouri
Gillette, Wyoming
Glendale, Wisconsin
Globe, Arizona
Grinnell, Iowa
Guilderland, New York
Harrisonville, Missouri
Hartsdale, New York
Hawaii Department of Education
Janesville, Wisconsin

Ladue, Missouri
Louisville, Kentucky
Madison, Wisconsin
Maplewood-Richmond Heights, Missouri
Marquette, Michigan
Mississippi State, Mississippi
New Hartford, New York
New Orleans, Louisiana
Normandy, Missouri
Philadelphia, Pennsylvania
Polk County, Georgia
Portland, Maine
San Felipe, New Mexico
St. Louis, Missouri
St. Louis Parochial, Missouri
University City, Missouri
Appendix D

Evaluation Panel

Ernest House (Chairman), University of Illinois
Robert Dilworth, California Institute of Technology
Leonard Cohen, Arizona State University
Peter Hilton, State University of New York, Binghamton
Stanley Smith, Baltimore County Schools