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ABSTRACT

This document is intended to provide a standard for assessing the quality of the mathematics program, and is a guide for planning and implementing improvements in a schools program. The instructors and administrators who use the guide are viewed as the critical and final links in a unique chain that connects what is known about high quality mathematics programs with what happens to students in the classroom. It is noted that the handbook is the result of many hours of intense discussion, writing, and reactions of mathematics educators from throughout California. The material is subdivided into the following major parts: (1) Introduction; (2) The Content of the Mathematics Program (What Students Learn); (3) The Methods of Teaching Mathematics (How Students Learn); (4) Support for Implementation of a Quality Mathematics Program; and (5) Planning for the Improvement of the Mathematics Program. A major factor in the development of the guide was the participation of Professor George Polya, whose work was looked upon as the foundation of contemporary mathematics learning. It is noted that many of the concepts Professor Polya shared with the handbook writing committee were incorporated in descriptions of what constitutes high quality programs. (MP)

# HANDBOOK FOR PLANNING AN EFFECTIVE MATHEMATICS PROGRAM





## Kindergarten Through Grade Twelve

## Publishing Information

The *Handbook for Planning an Effective Mathematics Program* was prepared by three writers Carol Iddins, Evelyn Silvia, and Don Walker and by a handbook committee, working under the direction of Joseph Hoffman, Consultant in Mathematics Education, California State Department of Education. (See the Acknowledgments on page vii for a list of committee members.) The handbook was edited by Theodore R. Smith and prepared for photo-offset production by the staff of the Bureau of Publications, California State Department of Education, with artwork and design by Steve Yee and Paul Lee. The document was published by the Department of Education, 721 Capitol Mall, Sacramento, CA 95814; printed by the Office State Printing; and distributed under the provisions of the Library Distribution Act.



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A list of other publications available from the Department may be found on page 69 of this handbook.

*EDITORIAL NOTE ABOUT THE COVER:* The builders of the Golden Gate Bridge created an engineering triumph that wedded mathematics and art in a breathtaking union. The graceful catenary curve of the suspension cables is given by

$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$$

where $a$ is a constant that determines the amount of "sag." The added weight of the vertical support cables and the roadbed structure resulted in a modification of the catenary curve. One should note that the mathematical tools available during the design and construction of the bridge (1937) did not permit a level of accuracy expected in today's computerized world. Nevertheless, none of the precalculated and precut vertical support cables was more than 15 centimetres off a perfect fit.

One of the toughest mathematical problems solved in the design of the bridge involved the stresses and forces acting on the towers. This problem required the solution of 33 simultaneous linear equations in as many unknowns, using paper and pencil only!

For more mathematical information about the Golden Gate Bridge, write to Chief Engineer, Golden Gate Bridge District, P.O. Box 9000, San Francisco, CA 94129.

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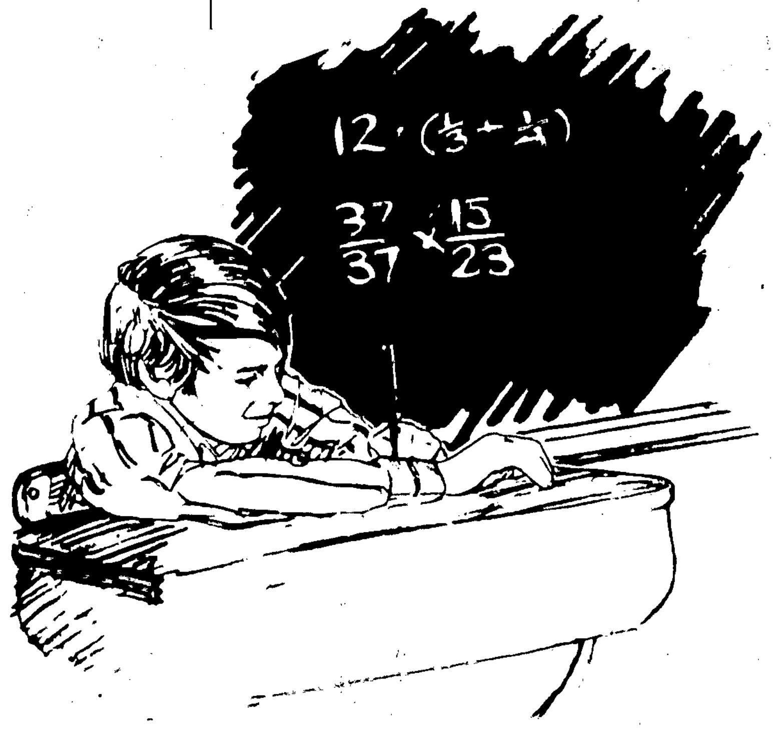
***"I encourage you to use this handbook for assessing, planning, developing, and delivering a high quality mathematics program in order to help today's students face the challenges of tomorrow."***

WILSON RILES



# FOREWORD

You who read and use *The Handbook for Planning an Effective Mathematics Program* in your schools are the critical and final links in a unique chain that connects what is known about high quality mathematics programs with what happens to students in mathematics classrooms. To make full use of the power of this chain, you should be familiar with all its links.

One link we are proud to forge into the chain is the valuable influence of the eminent mathematician and educator, George Polya. His many books and expositions on understanding, learning, and teaching problem solving through mathematical discovery over the past half-century are now recognized as the foundation of contemporary mathematics learning. During the development of the handbook, Professor Polya honored us with several hours of interaction with the handbook writing committee and shared many profound concepts, which the committee incorporated into its description of high quality mathematics programs.

Another link in the chain is the power of community involvement in planning, developing, and implementing the educational program of the local school. If you are concerned with the mathematics program in your local school, then, regardless of your mathematics training and abilities, this handbook was prepared for your use.

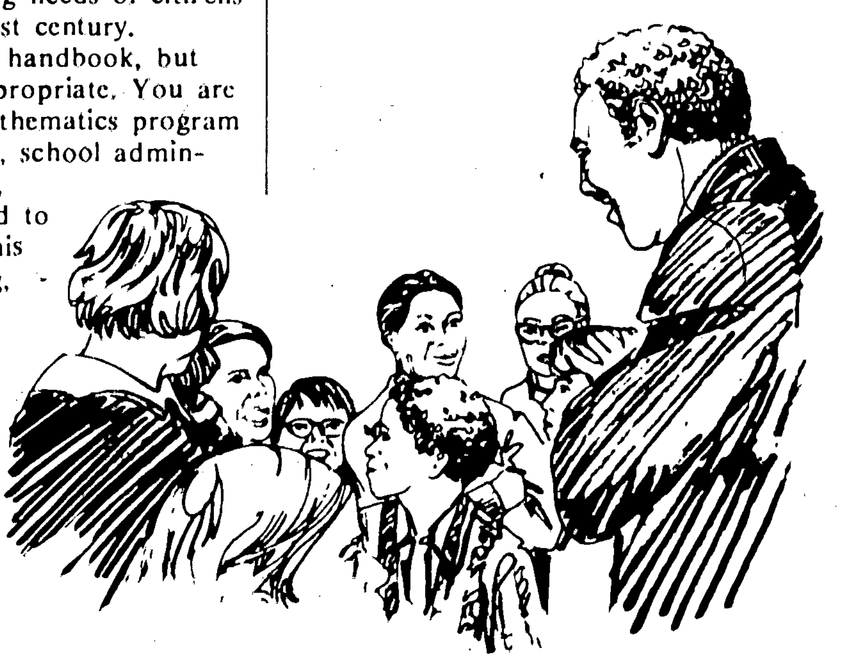
New staff development opportunities, teacher-center programs, and mathematics teachers organizations, such as the California Mathematics Council, are other important links in the chain. They have demonstrated their value to the chain by bringing to teachers of mathematics an awareness of, and the teaching skills necessary to meet, the learning needs of citizens of the technological society of the twenty-first century.

You will find other links described in the handbook, but another word about your involvement is appropriate. You are the final link in the improvement of the mathematics program in your local school. You parents, teachers, school administrators, members of the school community, counselors, and students must be committed to quality education. I encourage you to use this handbook for assessing, planning, developing, and delivering a high quality mathematics program in order to help today's students face the challenges of tomorrow.

Wilson Riles

*Superintendent of Public Instruction*

*You who use the* **Handbook for Planning an Effective Mathematics Program** *are the critical and final links in a unique chain that connects what is known about high quality mathematics programs with what happens to students in mathematics classrooms.*



# PREFACE

*This handbook provides a standard for assessing the quality of the mathematics program, and it is a guide for planning and implementing improvements in the school's mathematics program.*

The California State Department of Education has consistently encouraged all members of the school community to participate in the process for improving school programs. In keeping with that policy, the Department is providing leadership and assistance to school communities in California by preparing a series of handbooks that focus on the curriculum in specific subject areas. This *Handbook for Planning an Effective Mathematics Program* is the fourth in that series. Handbooks in science, writing, and reading are already in print.

This handbook and those in the other curricular areas are addressed to all individuals and groups that wish to review and improve educational programs. However, the documents are addressed more specifically to those persons at school site levels who plan and implement curricula: teachers, school administrators, curriculum specialists, parents and other members of the community, and students. This handbook provides a standard for assessing the quality of the mathematics program, and it is a guide for planning and implementing improvements in the school's mathematics program. We believe that these handbooks are unique in providing assistance without being overly technical; however, we encourage the reader to supplement these handbooks with other documents, such as the state curriculum frameworks, the county superintendents' *Course of Study*, and district curriculum guides.

We sought the valuable assistance and advice of many knowledgeable people in the development of this handbook. Many of them are identified in the acknowledgments, but there were countless others who reviewed preliminary drafts and made valuable suggestions.

We will not know the ways this document has proven its value until many of you have had the opportunity to use it. We sincerely urge those of you who do so to inform us of its strengths and weaknesses. Please direct your response to the Instructional Services Section, California State Department of Education, 721 Capitol Mall, Sacramento, CA 95814.

DAVIS W. CAMPBELL
*Deputy Superintendent for Programs*

RAMIRO REYES
*Associate Superintendent of Public Instruction*

JAMES SMITH
*Administrator, Instructional Services Section*

# ACKNOWLEDGMENTS

The State Department of Education gratefully acknowledges the contributions the following people made to the development of this document. The final draft was written by Carol Iddins, Sacramento, who reorganized the contents and added a literary touch to the writing. Evelyn Silvia, University of California, Davis, was the writer most responsible for capturing and describing the essence of a high quality mathematics program. Finally, Don Walker, Rio Linda Union Elementary School District, was the initial writer who greatly expanded the potential application of this document as a tool for improving school mathematics programs.

The practicality of this handbook is the result of many hours of intense discussion, writing, and reactions by the following people:

Joan Akers, Santee School District
Bonnie Allen, Office of the El Dorado County Superintendent of Schools
Elle Amundsen, Cupertino School District
Jan Berry, California School Boards Association, Sacramento
Barbara Bethel, San Diego City Unified School District
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Virginia Doyle, Tahoe-Truckee Unified School District
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Lyle Fisher, Tamalpais Union High School District
Jane Gawronski, Office of the San Diego County Superintendent of Schools
Ruth Hadley, Lompoc Unified School District

*The practicality of this handbook is the result of many hours of intense discussion, writing, and reactions of mathematics educators from throughout California.*

Leon Henkin, University of California, Berkeley
Tom Lester, San Juan Unified School District
Robert McFarland, Office of the Alameda County Superintendent of Schools
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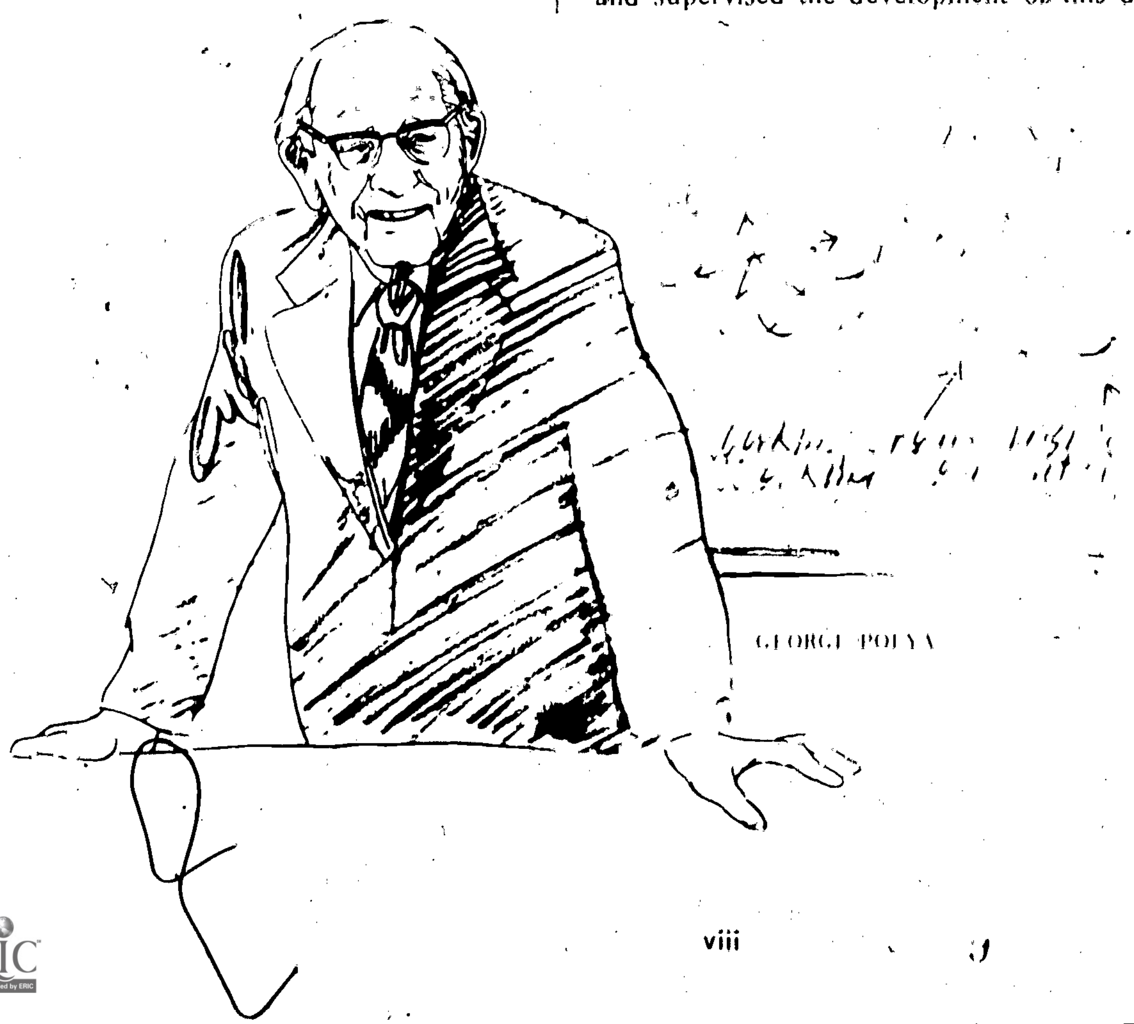
Dale Seymour, Palo Alto
Linda Webster, Office of the Alameda County Superintendent of Schools

The State Department of Education staff who assisted in the development of this document were James Smith, Manager of the Instructional Services Section, and Mae Gundlach, School Improvement Program Consultant. Other staff who provided valuable suggestions were Phil Daro, Fred Dobb, Paul Gussman, Donavan Merck, and Robert Tardif. Joseph Hoffmann, Consultant in Mathematics Education, coordinated and supervised the development of this document.

*George Polya's many books and expositions on understanding, learning, and teaching problem solving through mathematical discovery over the past half-century are now recognized as the foundation of contemporary mathematics learning.*



GEORGE POLYA

PART

# Introduction

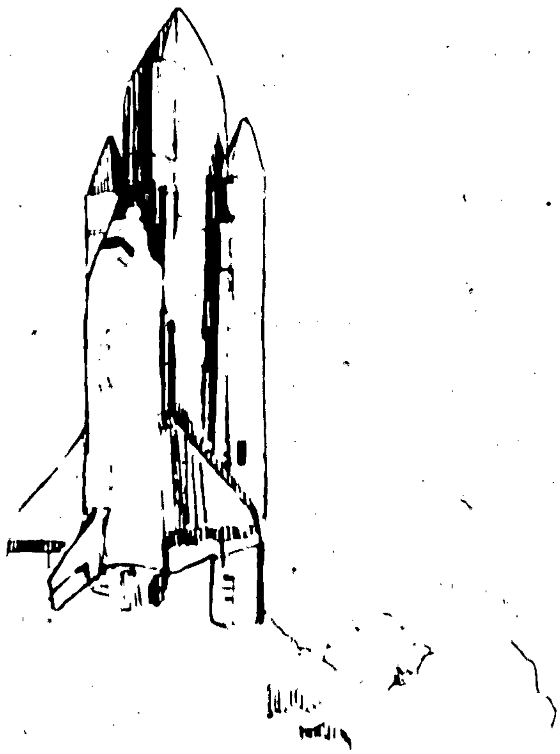
Mathematics, cornerstone of the sciences, is an absolute necessity for those who live in today's society. People use some application of mathematics every day and continually rely on a multitude of human services without ever realizing that those services are made possible through the power of mathematics.

Most jobs now require mathematical skills. Indeed, the mathematics required in many jobs is becoming increasingly sophisticated. It is well known that science and engineering rely heavily on mathematics. What is less well known is that the mathematics of calculus is now required for studying medicine, architecture, business administration, and forestry. Even in accounting, where arithmetic skills have always been fundamental, advanced mathematical techniques, such as the theories of probability and linear algebra, are often necessary.

## The Broad Spectrum of Mathematics

To prepare students for life in today's highly technical society, their mathematical training must include and go far beyond providing training in the simple skills of counting, computing, putting numbers into formulas, and even solving equations. Learning *only* rote mathematical rules ill equips students to apply those rules to solve problems outside the classroom. Instead, the mathematics curriculum must focus on what mathematical concepts mean, how they are related, and where they apply. Most importantly, all mathematics concepts must be taught in such a way that students understand their application in day-to-day living and their value in various careers and vocations.

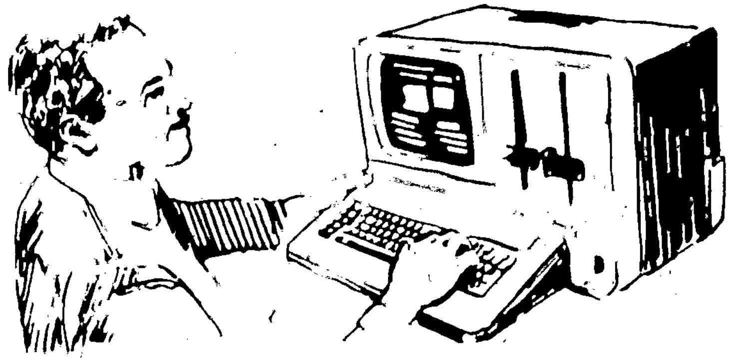
Furthermore, students should develop an understanding of other purposes served by mathematics. Mathematics is not just a tool for solving problems related to science and daily living; mathematics is a science in its own right. It is also one of the humanities one which has captured and stimulated the most creative minds all through the ages; it is the most precise of languages one that is continually growing in order to accommodate new ideas and solve new problems; and it is a form of mental recreation that completely fascinates and absorbs the mind. It is important that all students, both



***Mathematical training must include and go far beyond providing training in the simple skills of counting, computing, putting numbers into formulas, and even solving equations.***

elementary and high school, understand this broad spectrum of mathematics.

Computer technology was born suddenly in this generation and has drawn on the full spectrum of mathematics. Computers have reduced much of the drudgery of life, while



*While it is not necessary for everyone to know how computers work, it is important that students understand what the computer can do for them.*

increasing the amount of needed mathematical knowledge and facility. Anyone who handles money, makes either long-term or short-term purchases, invests in stocks, or uses charge cards must be fully aware of the increasing computerization of society. While it is not necessary for everyone to know how computers work, it is important that students understand what the computer can do for them. Above all, the mathematical processes for solving problems must ultimately prepare students to think rationally in the face of challenging situations.

## The Purpose of the Handbook

The *Handbook for Planning an Effective Mathematics Program* was designed as a tool for assessing and improving a school's mathematics program. It identifies and explains the essential components of a high quality program. In each of the following parts of the handbook, one of the major components of program planning is analyzed: the content of the mathematics program (*what* students learn); the methods of teaching mathematics (*how* students learn); and support for implementation of a quality mathematics program (the necessary preparation for learning to take place).

Each component is further divided into essential program elements; that is, the characteristics of exemplary mathematics programs. The effects of these characteristics on student understanding, attitudes, and achievement are discussed, thus establishing a guide for judging the quality of mathematics programs. Any program that is considered to be exemplary will exhibit all these characteristics.

# The Use of the Handbook

The handbook should be read to gain an understanding of the characteristics of exemplary mathematics programs and for an overall perspective on how these should be combined to provide a well-integrated and well-designed program. The handbook includes illustrative examples of the characteristics, which are followed by a series of questions that indicate what to look for in assessing the quality of a school's mathematics program.

The questions that appear throughout the handbook should not be answered simply yes or no. If the answer is "yes," it is important to identify "to what degree." If the answer is "no," it is important to know "why." The major questions are outlined in a checklist at the conclusion of the handbook (page 53). This "Checklist for Assessing the Quality of a School's Mathematics Program" challenges observers and planners to analyze a mathematics program and to discuss the steps that need to be taken next. In order to respond adequately to the items in the checklist, it will be necessary for users of the handbook to observe classroom activities, interview students, teachers, and others; and review textbooks and other instructional materials.

Although the handbook will be useful for mathematics specialists, it was designed specifically for groups or individuals, such as school site councils and school personnel who have the responsibility for assessing, improving, developing, or making decisions about mathematics programs. One of the principal goals of the handbook is to make mathematics education a schoolwide concern, not just the province of those who are most experienced in mathematics instruction. Therefore, because the intended audience includes teachers from different disciplines, parents, community members, and students, the writers of the handbook avoided, whenever possible, technical language.

*John von Neumann and others of the Institute for Advanced Study discovered that base-two numerals representing instruction codes and also other data could be stored electronically. Because of the work of this brilliant mathematician and his colleagues in 1945, it was possible to eliminate masses of special wiring that had been required to solve mathematical problems electronically. Thus, the stage was set for designing modern computers.*

JOHN VON NEUMANN
1903–1957

*School planning teams for improving mathematics programs will find useful information in the analyses provided by the California Assessment Program.*

California Assessment Program

## Other Resources

School planning teams for improving mathematics programs will find useful information in the analyses provided by the California Assessment Program (CAP). Through the CAP, the attainment of students in the basic skills of reading, language, and mathematics is measured annually in the public elementary and secondary schools in California. CAP focuses on the effectiveness of school-level programs and provides information to program planners about the relative strengths and weaknesses of each school's basic skills programs. CAP was not designed to assess the progress of individual students.

Program planners should also consult the *Mathematics Framework and the 1980 Addendum for California Public Schools,* the county superintendents' *Course of Study,*[1] and the Department of Education's school improvement publications. (See page 69 of this handbook for a list.)

Another valuable source of information is the National Council of Teachers of Mathematics. One of its publications, *An Agenda for Action: Recommendations for School Mathematics of the 1980's,*[2] provides a valuable perspective on new directions in mathematics education.

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[1] *Course of Study: A Program Planning Guide for Grades Kindergarten Through Twelve, 1981-84.* Hayward, Calif.: Office of the Alameda County Superintendent of Schools, 1981.

[2] *An Agenda for Action: Recommendations for School Mathematics of the 1980's.* Reston, Va.: National Council of Teachers of Mathematics, 1980.

# The Content of the Mathematics Program (What Students Learn)

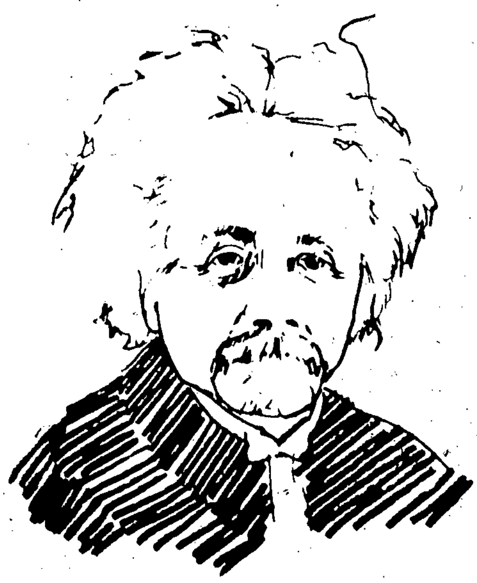
## Overview

If a school's goal is to help students acquire the ability to function effectively in today's rapidly changing society, a quality mathematics program will include activities that build the students' confidence in dealing with situations requiring mathematical skills. Furthermore, in a high quality program, students will be taught certain mathematical processes and learn why and under what conditions they should use each process.

The understanding of mathematics begins with a development of the skills a person needs to communicate mathematical ideas, so this discussion of the content of mathematics begins at that point. Mathematics specialists have identified the following elements as essential to developing a comprehensive mathematics program that will provide students with the skills, knowledge, and values they need to understand and use mathematics successfully:

- *The language of mathematics.* Students develop fluency in using the language of mathematics so that they are able to apply their mathematical skills in other settings and to communicate with others using mathematical terminology.
- *A comprehensive mathematics curriculum.* Through a comprehensive curriculum, students are given opportunities to develop mathematical skills and concepts that build on one another, relate to possible career and life situations, and meet their diverse learning needs.
- *Computing skills.* Students acquire computing skills in the context of their day-to-day experiences and through interesting activities.
- *Problem solving and application.* Students are provided regular practice in solving problems so that they learn problem-solving strategies and critical thinking skills while developing an understanding of the practical uses of mathematics.

*Perhaps the most well-known mathematical formula of modern times was the result of Albert Einstein's discovery that the amount of energy contained in an object is related to its mass:* $\mathbf{E = mc^2}$.



ALBERT EINSTEIN
1879 -1955

# The Language of Mathematics

Students should never learn computing skills as unrelated facts or be unable to use those skills to solve real problems. A student's inability to use mathematical skills comfortably is probably because mathematics does not really "make sense"; it is merely a series of unrelated rules. A good mathematics instructor creates the "sense" by giving students opportunities to see that mathematical symbols are not just things to be manipulated according to mysterious rules. Rather, students become fluent in mathematics when they learn to use the symbols and terms to record and communicate the ideas of mathematics.

*"What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises."*
FROM APHORISMEN BY G. C. LICHTENBERG

The language of mathematics helps students translate the elements and the relationships of those elements in a problem situation into mathematical symbols that yield a solution through mathematical procedures. In an effective program, teachers take the mystery out of mathematics by demonstrating that every step in a mathematical procedure has clear justification and meaning.

## Fluency in the Language of Mathematics

When learning a foreign language, students study vocabulary and grammar and practice translating from the familiar to the foreign and back again. The eventual goal is to be fluent in the foreign language and to be able to think in it. Students may follow a similar process as they become fluent in the language of mathematics.

The "grammar" of mathematics involves the use of symbols and terms. A sentence in mathematics may include combinations of numbers, operational symbols, parentheses, and defined terms. It is as important for students of mathematics to understand and use these terms as it is to learn the syntax of a language. For example, elementary students should be able to explain what "½ of something" means. In high school algebra, students should be able to explain that $2x + 1 = 5$ is not just a mathematical expression, but that it represents a series of operations performed on some number; namely, a number was multiplied by two, then one was added to the product, and the sum was equal to five.

It is also crucial that group discussion lead to the class's "discovery" of any mathematical rule. The extra time taken for discussion and discovery can help give meaning to an otherwise meaningless rule. For example, exercises should be designed so students discover why common denominators are used in the addition of fractions, rather than being told to memorize a sequence of "magical" steps.

Learning the language of mathematics should continue throughout the school's mathematics program. At every grade level, the use of appropriate terminology should be a natural part of mathematical experiences.

## 1.3 Do students use mathematical terminology fluently?

**What to look for:**

a. Does the teacher use correct terminology whenever it is appropriate?

b. Can the students read and give the meaning for symbols and terms?

c. Do students explain terms, symbols, and rules to each other and the teacher?

d. Does the teacher provide the students with a variety of experiences to work with symbols and terms? For example:
   (1) Matching symbols with the terms
   (2) Exchanging mathematical expressions with equivalent expressions
   (3) "Finding what's missing" exercises

e. Do students get involved in discussions that lead to a discovery of mathematical relationships?

### Skills of Communicating in Mathematics

To develop fluency in a language, teachers must require students to do more than translate the language or learn the grammar. And it is also true in mathematics. Teachers must involve their students in activities that help them learn to communicate with mathematical expressions. For example, students should be able to communicate with diagrams, symbols, equations, and other mathematical expressions. If they know and understand that there are different representations for the same ideas, they are more likely to be able to work with the ideas successfully in different settings. Students who are fluent in the language of mathematics can tell where they are in a mathematical process, why a mathematical process works, what mathematical task they are trying to accomplish, and when, perchance, they need help.

Many learning experiences can help students improve their skills of communicating in mathematics. For example, students can participate in games, invent mathematical puzzles, and give project reports to their classmates. As another example, the teacher might ask one student to describe a geometric shape to the other students, who must then draw the shape as it is described. The students then share their pictures and discuss the descriptions.

Another way to build skills in communication is to ask the students to help clarify or correct a point on which the teacher pretends to be confused. This activity increases student attention as well as improves their abilities to use mathematical terms. For this technique to be successful, the teacher must write *exactly what* the students say. When students are held accountable for what they say, it is amazing how precise they become in using mathematical terminology.

**Describe the objects that cast these shadows.**

***Teachers must involve their students in activities that help them learn to communicate with mathematical expressions.***

*"The very essence of [mathematics] is the prevention of waste of the energies of muscle and memory."*

FROM THE NATURE OF MATHEMATICS BY PHILLIP E. B. JOURDAIN

## 2. Can students communicate in mathematics?

**What to look for:**

a. Do teachers help students express their thoughts when they are doing mathematics?

b. Are students asked how concepts are related? For example:
   (1) How is addition related to sets of objects?
   (2) When should multiplication instead of addition be used?

c. Are there regular class discussions on subjects that involve comparing sizes, numbers, areas, and so forth?

d. Can students tell each other why they are following certain steps?

e. Can students explain the steps for solving an equation?

f. Are discussions of mathematical reasoning a natural and regular part of the classroom activity? For example:
   (1) Can students explain how percentages are derived, how to use them, and the relationship of percents to fractions and decimals?
   (2) Can students discuss how to compute with fractions?

g. Do students give verbal reports on homework and projects they have completed?

h. Are students encouraged to develop independent study projects in which they explore applications of concepts?

# A Comprehensive Mathematics Curriculum

Because mathematics is such an integral part of everyday life in today's society, mathematical competence is one of the ingredients of a satisfying and productive life. Therefore, a well-designed curriculum in which students are prepared for careers and day-to-day living should include a wide range of mathematics topics in which mathematical concepts and skill areas are developed thoroughly. Appendix C to the *Mathematics Framework and the 1980 Addendum* is the California State Board of Education's 1980 "Criteria for Evaluating Instructional Materials in Mathematics," and the criteria represent the core of instruction for any good mathematics program in kindergarten through grade eight. The county superintendents' *Course of Study* is another good reference for developing comprehensive mathematics programs.

## The Breadth of Mathematical Skills

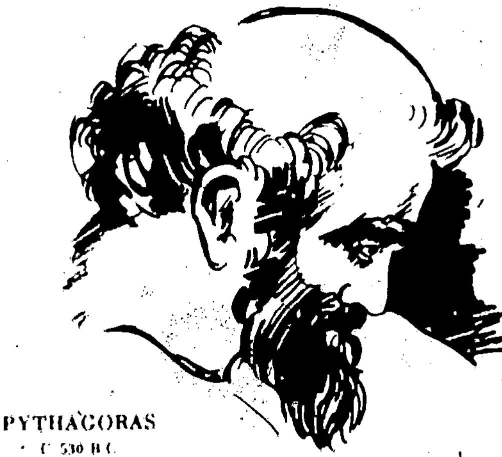
The breadth of mathematics refers to the scope of essential skills and concepts that students should master. For example, by the end of the ninth grade, every student should have developed an appreciation for the subject of mathematics and should have acquired at least the mathematical skills and concepts that are necessary for day-to-day living. These include: (1) proficiency in computing; (2) the ability to read graphs and charts; (3) understanding of percentages; (4) skills of measurement; (5) a facility with geometric concepts; (6) development of logical thinking processes; (7) a facility for discussing mathematical concepts; and (8) the ability to use mathematics to solve a variety of problems. (See the Appendix, page 61, for the breadth of mathematics expected of eighth graders in California public schools.)

For students who do not reach the required level of proficiency by the ninth grade, an effective school mathematics program provides for the acquisition of such skills through remedial programs available in grades nine through twelve.

Once students have reached the required level of proficiency, a school's mathematics program should help them prepare for their career choices through a variety of courses. The options available for students should include: (1) mathematics for business and basic accounting; (2) mathematics preparatory for college or university training (See page 15 of this handbook for a more thorough discussion of the requirements of colleges and universities for their entering freshmen.); (3) mathematics for vocational choices; and (4) mathematics for consumer needs. It is recommended that all students take some mathematics course in their senior year.

Mathematics instruction can also play an important role in helping students improve their reading and writing skills. Teachers can help students meet local proficiency standards by using mathematics content and providing mathematics-based experiences which require students to use those basic skills regularly.

*Mathematics instruction can play an important role in helping students improve their reading and writing skills.*



PYTHAGORAS
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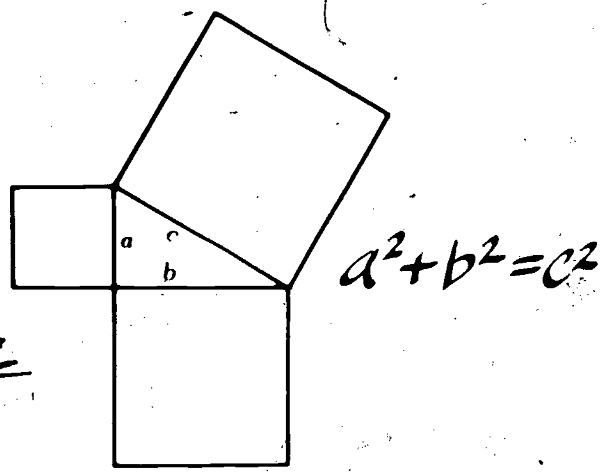



"*The beautiful has its place in mathematics for here are triumphs of the creative imagination....*"
W. F. WHITE

## 3. Does the mathematics program cover the breadth of required mathematical skills?

**What to look for:**

a. Are opportunities provided for all students to gain an understanding and to use all essential skills and concepts? Those are:
   (1) Arithmetic numbers and operations
   (2) Geometry
   (3) Measurement
   (4) Calculators and computers
   (5) Probability and statistics
   (6) Relations and functions
   (7) Logical thinking
   (8) Algebra

b. Do students demonstrate facility with problem-solving skills by drawing diagrams, looking for patterns, forming equations, and so forth?

c. Do proficiency standards in mathematics include reading graphs and charts, computing restaurant bills, and naming geometric shapes?

d. Are a wide range of courses available at the secondary level? For example:
   (1) Basic mathematics courses
   (2) Consumer and career courses
   (3) Computer literacy courses
   (4) College preparatory courses of both technical and general college programs

e. At the secondary level, do students have an opportunity to take a different mathematics course every year, and do titles of courses clearly identify the content?

f. Are all twelfth-grade students who plan to attend college enrolled in a mathematics course?

g. Are mathematics teachers familiar with the district-adopted proficiency standards that their students must meet?

h. Do mathematics teachers make assignments in mathematics that help students improve their reading and writing skills?

## The Depth of Mathematics Instruction

In a comprehensive mathematics curriculum, skills and concepts are woven together and developed into a hierarchy for better understanding. Many of the mathematics skills that students acquire have common characteristics that should be used creatively. For instance, students encounter the concept of regrouping many times by many different names. When learning addition and subtraction, students may call the concept "carrying, borrowing, or regrouping." When learning to use money, the students may call it "making change." When learning fractions, they may call the concept "finding equivalent fractions." At the secondary level, the teaching of prime factorization of numbers leads to factoring of algebraic expressions.

Thus, whenever possible, the teacher illustrates that the new concept is really an old friend. On the other hand, for a student who has been unsuccessful in learning the concept by a previous name, the teacher uses different strategies to build the desired skill. Furthermore, the teacher builds and reinforces cognitive skills by stressing understanding, application, analysis, synthesis, and evaluation of concepts. At the highest level, the students develop an appreciation of the beauty and elegance of mathematics.

*Archimedes, one of the greatest mathematicians of all time, was killed by a Roman soldier after the fall of Syracuse. According to some historians, the soldier found Archimedes drawing circles in the sand and became angry when Archimedes yelled, "Don't spoil my circles!"*

ARCHIMEDES
287 212 B.C.

## 4. Does the mathematics program have depth?

**What to look for:**

a. Do the skills to be acquired contain a core of common learnings and minimum competencies every student is expected to learn?

b. Do the students know that the skill they are learning is built on a previously acquired skill?

c. Does the teacher make use of puzzles, posters, and student projects that require the use of several skills?

d. Do students use bridging phrases like "It works just like . . . ."?

e. Do teachers plan learning tasks that are designed to build the higher levels of cognitive understanding beyond knowledge and comprehension? For example: application, analysis, synthesis (formulation of relationships between concepts), and evaluation

f. Does the teacher stress awareness of the universal applications of many mathematics concepts, such as the use of mathematics in music and the uses of geometric patterns in art?

## Priority for Mathematics in the School-Level Plan

A mathematics program should reflect a schoolwide commitment to mathematics education. One of the indicators of such commitment is the development of a school-level plan that places a high priority on mathematics education, allocates specific time for it in the learning program, provides for the coordination of mathematics with other subjects, and ensures financial support for the program. The school mathematics program should be designed so that students can progress toward clearly stated goals without unnecessary repetition. This can be achieved by careful planning through all grade levels. The school-level mathematics program must be an integral part of any other school-level plan, such as the school improvement plan or the compensatory education plan. It should never be viewed as something separate from or in addition to such plans. The mathematical skills and concepts taught in classes for bilingual, compensatory, and special education students should be the same as those taught in the regular program.

The development of a schoolwide mathematics program should involve students, teachers, counselors, administrators, and parents; and it should be coordinated among classrooms, grade levels, and feeder schools as much as possible. Thus, when more than one teacher in a school is teaching the same grade level or course, students will be taught the same skills, and upon completion of the grade or course, comparable rankings will indicate comparable proficiency. Further, students will benefit from a systematic comprehensive mathematics program that includes all of the important concepts and skills without unnecessary repetition. The *Mathematics Framework and the 1980 Addendum for California Public Schools* provides an excellent basis for mathematics curriculum development and assessment within a school plan.

*The development of a schoolwide mathematics program should involve students, teachers, counselors, administrators, and parents.*

A comprehensive mathematics curriculum plan or guide should:

- Include a statement of the school's basic goals for mathematics development.
- Have an identified rationale for relating course content to students' developmental levels.
- Be organized into clearly described levels that indicate where concepts and skills are expected to be taught by each teacher and how the concepts and skills taught at one grade level fit into those that follow.
- Have an effective and thorough procedure for examining and adopting instructional materials in mathematics.
- Include a plan for enrichment activities.
- Include a plan for continuous evaluation and improvement.

Other indicators of a strong schoolwide and districtwide plan are identified in Part IV of this handbook.

## 5. Is a high priority given to mathematics instruction both in the school-level plan and in practice?

**What to look for:**

a. Do teachers, parents, students, and others play a significant role in developing curriculum and planning programs?

b. Does each teacher know his or her role in implementing the plan?

c. Is there a written plan that emphasizes mathematics education and coordinates it with other subjects and other school-level plans?

d. Are the skills and concepts taught in special education, bilingual education, compensatory education, or other special classes consistent with those taught in the regular classes?

e. Are there mechanisms for regular communication about the progress of individual students between teachers in special programs and teachers in the regular classrooms?

f. Is there coordination among classrooms of the same level or courses so that students are provided comparable experiences and skill development so that unnecessary duplication of experiences and skill development are avoided?

g. Are there mechanisms for communication among schools in a district to ensure that students receive preparation needed to advance successfully to the next level?

h. Does a process exist for adopting instructional materials in mathematics and for examining, on a regular basis, the content of the mathematics program?

i. At the secondary level, do elective or enrichment courses in mathematics have as rigorous a curriculum as the required courses?

j. Do students with advanced skill or interest have opportunities for accelerated learning?

### Remediation of Learning Problems

When assessing a school's comprehensive mathematics curriculum, special attention should be given to its programs for remedial instruction. These programs are essential for students who have not attained minimally acceptable levels of proficiency in any of the skills and concepts from computing through calculus.

The remedial program should cover the same content as the regular instructional program, but it should be taught differently. Remedial instruction should not be presented in the same way that resulted in previous unsuccessful learning.

The content of the remedial program should also be organized around clearly stated objectives and provide the students with challenging and interesting mathematical activities. Those responsible for remedial instruction should use practical applications of mathematics with other topics which improve students' attitudes toward the skills being remediated.

### 6. Is remedial instruction available throughout the mathematics program?

**What to look for:**

a. Are objectives in remedial instruction designed to enable the students to advance into the mainstream of the curriculum?

b. Do classes for remedial instruction provide for opportunities beyond the acquisition of computing skills?

c. Do teachers possess the teaching skills for providing remediation, as demonstrated by their use of appropriate materials and techniques?

d. Is unnecessary drill and "more of the same" type of reinforcement avoided by planning reinforcement activities, based on individual student needs and interests?

e. Are optional reteaching and remedial opportunities available to students who fail to master the mathematics skills the first time?

f. As students learn more complex skills, do they receive review practice on previously learned skills, as needed?

g. Are students shown how previously learned skills are useful for learning more complex skills?

*In many instances, the requirements for entry into certain areas of study will require high school students to take mathematics courses beyond those needed for general admission to the university.*

## Preparation for College

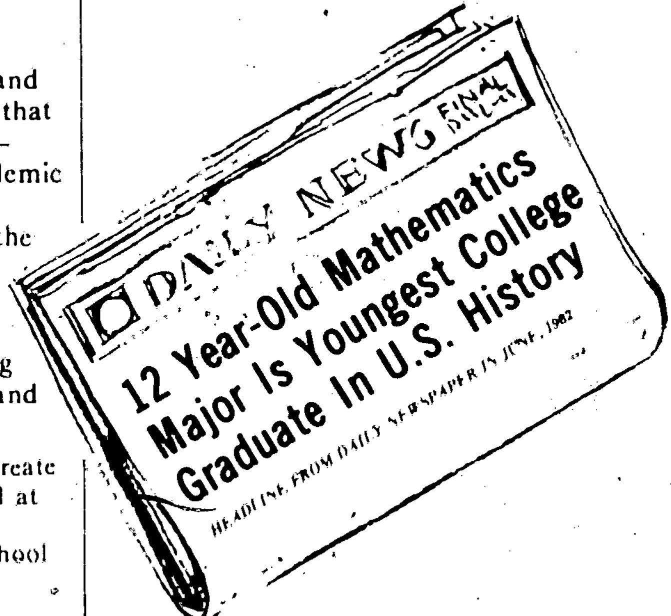
A comprehensive school program should include the necessary courses for those planning to go to college. And in many instances, the requirements for entry into certain areas of study will require high school students to take mathematics courses beyond those needed for general admission to the university. For example, all majors in the natural and life sciences, engineering, and mathematics require calculus. Many social science majors require either statistics or calculus or both. Careers in environmental sciences, dentistry, medicine, optometry, pharmacy, and biostatistics also require calculus for undergraduates. Many students are not aware that large numbers of fields outside the natural and mathematical sciences often require calculus or statistics as prerequisites.

In some high schools, it may be difficult to offer a full range of regular and advanced mathematics courses each year. However, the problem may be resolved in a number of ways if it is given adequate attention and planning. Some options include: offering advanced mathematics on a two-year cycle; providing for independent study, home tutors, or correspondence courses; and permitting students to enroll in a nearby college program, or cooperating with a nearby high school to offer courses in advanced mathematics.

Recently, the academic senates of the California Community Colleges, the California State University, and the University of California declared in a position statement that the minimum proficiencies in mathematics and English now required for high school graduation are insufficient to provide students with the foundation they need to be successful in college and university course work. The academic senates pointed out that there are "varied and complex causes (for the) underpreparation of entering college freshmen." However, as the academic senates pointed out, one of the problems "is a lack of understanding among students, parents, and educators of the competencies expected of entering college students." Recognizing their responsibility for identifying such competencies, the members of the academic senates recommended the following as being necessary for ensuring that college freshmen are adequately prepared in English and mathematics:

1. The curriculum for students planning to pursue a baccalaureate education should include at least four years of English and at least three years of mathematics.
2. The academic program taken in the senior year of high school should include one year of English and one year of mathematics.
3. Diagnostic examinations to assess student competencies in English and mathematics should be given no later than the junior year in high school. The results of these examinations should be used to counsel students concerning their study in the senior year.
4. The results of competency assessment in English and mathematics of entering students at the colleges and universities should be made available to the students' respective high schools so that appropriate evaluation of instructional programs can be made.
5. Counseling of students and their parents concerning college preparation should occur as early as possible to provide a foundation for successful college and university study and to broaden the spectrum of career choices. Early counseling is especially needed for groups which are now underrepresented in California colleges and universities.
6. At all levels of education, from elementary school through college, grades in English and mathematics should be based upon achievement rather than upon effort or attendance so that students will receive accurate assessment of their competencies.

In their position statement, the academic senates also outlined "the core of the necessary skills in English and mathematics needed by entering college freshmen, regardless of intended major or the specific admission requirements of the institution the student plans to attend." The senates' specific recommendations on mathematics appear in Appendix B to this handbook.



## [illegible]

**What to look for:**

a. Do teachers and counselors have the latest information about college preparatory requirements and the courses which prepare graduates for many of the major fields they will be entering?

b. Do college preparatory students know the course requirements for their intended majors?

c. Are college preparatory students given opportunities to study advanced mathematics, including trigonometry and other advanced courses in mathematics?

## Skills in Computing

*A good mathematics program will be most effective when teachers present skills in computing as enjoyable, challenging, and necessary for the achievement of other goals that students wish to attain.*

Computing skills are essential in day-to-day living, and they require the person to have a thorough understanding of whole numbers, fractions, and decimals, as well as speed and accuracy in adding, subtracting, multiplying, and dividing whole numbers. A good mathematics program will be most effective when teachers present skills in computing as enjoyable, challenging, and necessary for the achievement of other goals that students wish to attain.

### The Teaching of Computing Skills

Skills in computing must be taught carefully at all levels, and at the beginning the underlying concepts of the basic skills must be emphasized. As students learn new skills, teachers must reinforce previously learned skills through a program of carefully planned practice that is closely related to, but different from, the way the skills were learned initially. In an effective mathematics program, the teacher finds a balance between learning experiences in which drill and memorization are emphasized and interesting reinforcement activities that will challenge students to use their newly acquired computing skills. The drills are short and are given daily as a complement to other forms of practice. Some of the short drills are "thought problems," informal timed tests, and games involving computation.

However, much of the needed practice can and should be provided through practical problem solving (which is related to different curricular activities), purposeful games, puzzles, and mechanical or electronic calculators. For example, the teacher may use newspapers and magazines to stimulate an "If-we-had-some-money day." Students are asked to determine the price of their ideal kitchen, house, or car from the prices listed in a newspaper or a home builder's guidebook. As another example, the teacher may ask the class to choose between receiving $1,000 each day for 31 consecutive days or receiving 1 cent for the first day and doubling the amount each consecutive day for 31 days. The activity provides valuable computational review, and the conclusion is fascinating. Exercises such as these provide practice while relating to the types of situations people encounter in real life.

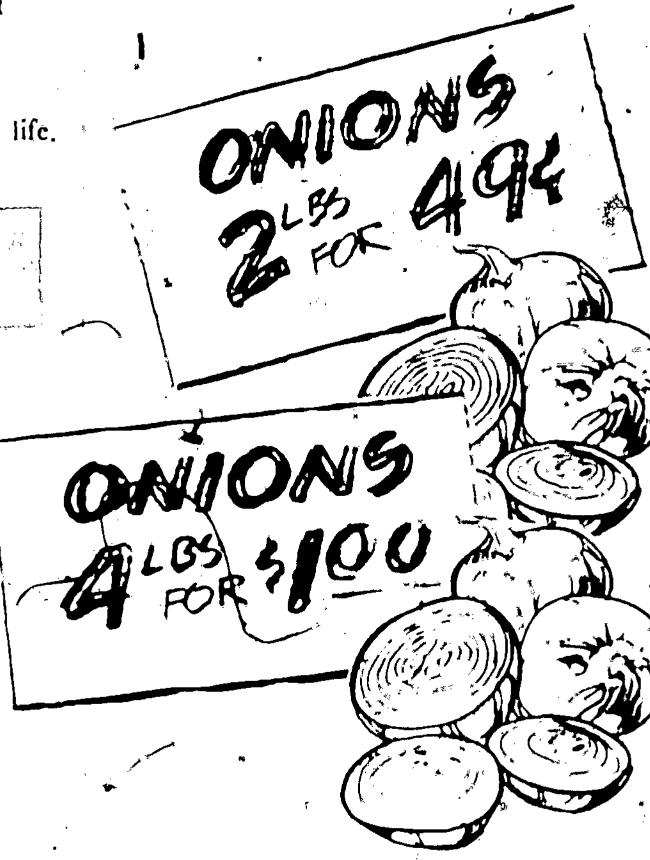
**8. Are computing skills taught through interesting and challenging activities?**

**What to look for:**

a. Are group activities used daily for students to practice computing skills; e.g., nongraded timed drills and chalkboard contests?

b. Do students play purposeful games of computing; e.g., dominoes, cribbage, and teacher-constructed games?

c. Do students have interesting individual practice? For example
   (1) Completing or constructing magic squares
   (2) Completing number sequences with hidden patterns
   (3) Decoding hidden messages in problem sets
   (4) Figuring totals on hypothetical restaurant menus
   (5) Computing, with decimals, a tax table or a debt amortization table
   (6) Using fractions to compute earnings from hourly, weekly, or monthly wage scales
   (7) Making attendance reports, inventory reports, and other numerical reports for the school
   (8) Constructing scale drawings or maps that involve multiplication




*A good mathematics program will include many varied activities in which students must use skills of estimating reasonable answers to problems and doing arithmetic mentally.*

## Estimation and Mental Computation

A good mathematics program will include many varied activities in which students must use skills of estimating reasonable answers to problems and doing arithmetic mentally without pencil and paper. The current growing use of calculators increases the need for good estimation ability so that students can catch "calculator" errors. (Additional discussion of calculators is given on pages 36–38.) Students who have mastered the skill of estimation can determine whether an answer is reasonable, and they are more likely to check the accuracy of their work than they would have if they had not acquired the skill. At all levels of mathematics, students should be taught several strategies for estimating answers, and they should be given much practice in using the strategies. In assigning problems to students, the teacher should alternate between expecting students to check their answers and expecting them to show estimates for the answers.

## 9. Do students learn to estimate and to check their answers?

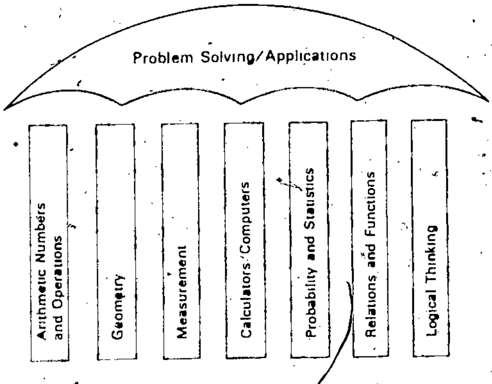
**What to look for:**

a. In oral drill, do students get feedback and appropriate reinforcement regarding how reasonable their answers are?

b. Are calculators used by students to check their estimates of answers to complex problems?

c. Are estimates made by rounding the original numbers to one or two significant (nonzero) digits?

d. Are students shown how to use visual representations to check the reasonableness of their answers? For example:
   (1) For multiplication/division, the visual representation may be used for the area concept, jumps on a number line, or scale factors.
   (2) For addition/subtraction, students may refer to an abacus, length units of measurement, counting tiles, and fraction circles.




*FROM THE* MATHEMATICS FRAMEWORK AND THE 1980 ADDENDUM FOR CALIFORNIA PUBLIC SCHOOLS

## Problem-Solving Skills in Mathematics

The authors of the *Mathematics Framework and the 1980 Addendum* identified four essential problem-solving/application skills:

- Formulating the problem
- Analyzing the problem
- Finding the solution
- Interpreting the solution[1]

*Formulating problems* is crucial for day-to-day living, because problems encountered outside school are usually not packaged neatly in textbook language. Students must learn to ask questions, clarify relationships, and determine what information is needed. For instance, a real-life situation, such as planning a sprinkler system for a yard, will challenge students to describe the problems inherent in doing that job.

*Analyzing problems* involves identifying the features that are significant to the central problem and planning strategies to deal with them. Planning might involve guesswork, estimating, drawing diagrams, creating concrete models, listing similar elements, or breaking the problem into manageable parts. The last and most crucial step in problem analysis is translation of the problem into mathematical symbols, because it demands an explicit definition of the problem and selection of an appropriate strategy for the solution. It also demands risk taking.

[1] *Mathematics Framework and the 1980 Addendum for California Public Schools.* Sacramento: California State Department of Education, 1982, p. 62.

*Finding solutions* requires mathematical skills beginning with an understanding of number properties and operations. Students should learn that some problems have several solutions and others may have none. Estimation should be used regularly with all problem-solving exercises so that students will learn to check their results.

*Interpreting the solutions* should occur at all levels of instruction. Students must learn to review the problems and solutions, to judge the validity of their translations to mathematical symbols, and to check the accuracy of their use of mathematical rules. Further, students should learn to make correct generalizations from their solutions and to apply the results to solving more complex problems.

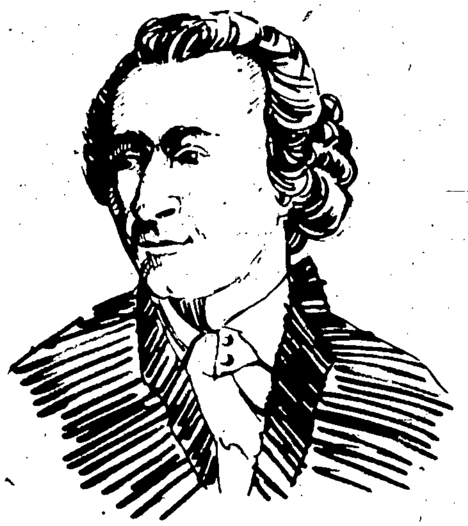
## *10. Are the skills of formulating and analyzing problems and finding and interpreting solutions emphasized?*

**What to look for:**

a. Do students have practice formulating problems? For example:
   (1) Does the teacher include discussion of real-life jobs and problems as a regular part of the curriculum?
   (2) Are students expected to complete homework exercises that are not identical to the examples given in class?
   (3) Are students encouraged to formulate solvable problems?
   (4) Are conditions and numbers in word problems changed to create new problems from old ones?

b. Do students have practice analyzing problems? For example:
   (1) Are problems assigned that require students to draw diagrams, create concrete models, list similar elements, break a problem into parts, discover patterns and similarities, seek appropriate data, and experiment with the models of a problem?
   (2) Do students translate verbal expressions into mathematical symbols and terms?
   (3) In class discussions and homework assignments, does the teacher use concrete problem situations that are not already clearly defined and translated into mathematical equations?
   (4) Do the teachers encourage students to work in small groups to dramatize problems, construct models, list elements, and so forth?

c. Do students have practice finding the solution to problems? For example:
   (1) Do teachers give assignments that require students to use a variety or combination of the basic skills and mathematical processes; i.e.,



LEONHARD EULER
1707—1783

*Leonhard Euler is considered by many to be the most prolific mathematician of all time. He published material in every branch of mathematics, and his contributions to the calculus are universally recognized.*

some problems in a set require subtraction, some require addition, and others require both?

(2) Do teachers use problem-solving activities to help students develop all skills and concepts?

(3) Is the use of estimates and guesses encouraged to test the reasonableness of answers?

d. Do students practice interpreting the solutions to problems? For example:

(1) Do students demonstrate and explain to each other how they found the solutions?

(2) Do class discussions focus on interpreting the solutions with questions such as the following: Was the problem solved? What does the solution mean? Was the best approach used? Would another approach work? Can the solution be used in solving another problem?

(3) Are students encouraged to look for different ways of thinking about a problem that may sometimes result in a different answer?

(4) Are students given opportunities to defend their solutions rather than being told flatly that they are wrong?

*Students are given opportunities to defend their solutions rather than being told flatly that they are wrong.*

PART II

# The Methods of Teaching Mathematics
## (How Students Learn)

### Overview

Ultimately, the quality of a mathematics program is only as good as the teachers in that program. Each student should be presented with exciting and successful experiences in mathematics, and no one method or approach will work for all students. Each teacher must therefore make use of a full range of strategies and devices that can be matched to the students' learning needs, to the students' expressed interests, and to the content of the mathematics program.

In assessing the methods used in a school's mathematics program, one should consider carefully the following three elements:

- *Learning styles and teaching strategies.* The teacher employs a variety of strategies and instructional processes to enable each student to learn successfully the mathematics content.
- *The effect of attitudes on achievement.* The teacher motivates the students to learn mathematics by providing them with daily opportunities to feel that they are both successful and challenged in mathematics. The students are engaged in meaningful and productive learning tasks during the entire time that is allocated for daily mathematics instruction.
- *Calculators and computers.* Mathematics teachers make use of calculators and computers creatively to lead students to a better understanding of mathematical processes and problem solving while supplementing and reinforcing other instructional activities.

*Each teacher must make use of a full range of strategies and devices that can be matched to the students' learning needs, to the students' expressed interests, and to the content of the mathematics program.*

### Learning Styles and Teaching Strategies

In a high quality mathematics program, the teacher sets the stage for students to explore, discover, and learn mathematics concepts in meaningful ways. To be an effective teacher of mathematics, the instructor must know mathematics; the students' mathematical abilities, interests, and learning styles; and the ways to teach that make appropriate use of a variety of materials and strategies.

## Differences in Learning Styles

Students vary widely with respect to experiences, feelings, interests, capabilities, rates at which they learn, and ways in which they prefer to learn. Because of these differences, the instructional materials and processes should be equally diverse. Over a period of time, students should be involved in a wide variety of activities, including teacher-led discussions; assignments from textbooks; student-led discussions; individual or small group work on projects; experiments using manipulatives; activities that involve collecting data and making graphs; use of audiovisual materials; involvement in meaningful games; and outdoor experiments. However, it is

*Students vary widely with respect to experiences, feelings, interests, capabilities, rates at which they learn, and ways in which they prefer to learn.*

essential that every activity be purposeful and designed to help students learn specific skills or concepts. Diversity of activities just for the sake of variety is of limited value to students.

Students should learn how to use available resources for extending their knowledge both in and out of the classroom. Because the teacher's knowledge of the subject is usually the best learning resource in the classroom, care must be exercised to avoid student dependence on the teacher's authority and knowledge. Teachers should make a conscious effort to seek out and make available sources of information that will help students learn to formulate their own questions in such a way that they can find the answers independently.

Effective teachers use direct teaching to the entire class when it is appropriate; for example, introducing a new topic, clarifying a concept about which most of the class appears to be confused, and explaining an example. Teachers may also use direct teaching for demonstrating a skill and, in so doing, model the desired behavior that the students are trying to learn. For example, in solving a problem which no one in the class can do, the teacher may say, "I wonder if this problem can be split into easier parts?" or "Shall I draw a diagram?" or "Should I make a table?" Then the teacher thinks aloud while making a decision.

In a high quality mathematics program, teachers can be observed using student interests as a way of reinforcing the usefulness of mathematical concepts. For example, students who discuss cars may be challenged to develop charts or graphs comparing the cost and efficiency of various models.

Students' interests in career information may be used to stimulate new areas of study. For example, tours of local businesses and industries, presentations by speakers who use mathematics in their work, and participation in work-related programs can all serve to reinforce learning and motivate study.

## 11. Do teachers provide for differences in students' learning styles?

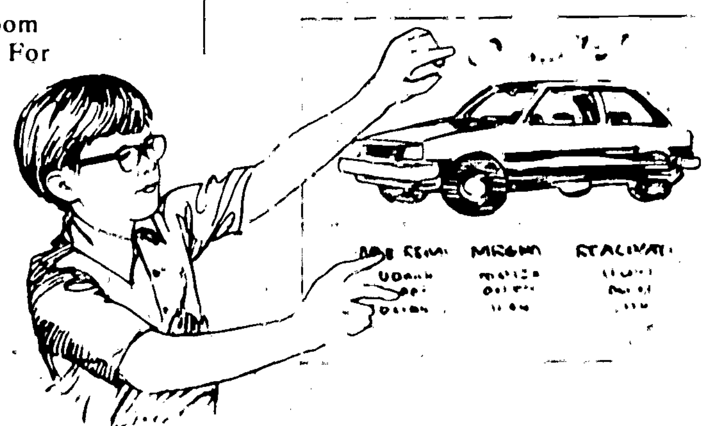
**What to look for:**

a. Does the teacher know each student's background and interests?

b. Are classroom activities and materials diverse and selected to meet the range of the students' abilities, language skills, interests, and needs?

c. Do the teachers use direct teaching to the entire class when it is appropriate and when it will help them achieve the instructional objective?

d. Does the teacher in his or her regular classroom instruction use a wide variety of approaches? For example:
   (1) Large and small group instruction
   (2) Lecture or expository method
   (3) Media presentations
   (4) Mathematics laboratories
   (5) Computer-assisted instruction
   (6) Role playing
   (7) Group work and peer instruction
   (8) Programmed instruction
   (9) Scientific inquiry
   (10) Drill
   (11) Individual instruction



e. Do students receive instruction in a language they understand?

f. Does the teacher explain the purpose of each activity and relate it to the needs, strengths, interests, and learning styles of the students?

g. Does the regular instructional plan provide students with opportunities to pursue special interests in the classroom setting? For example, self-selection activities, choice among alternative activities, and individual projects.

h. Does the curriculum provide for special interest activities that are related to learning objectives? For example, a project to construct three-dimensional

*In a high quality mathematics program, teachers can be observed using student interests as a way of reinforcing the usefulness of mathematical concepts.*

geometric models aids in the understanding of geometric figures studied in solid geometry.

## Student Assessment

Before an effective instructional program can be designed, the teacher must assess the individual learning characteristics of the students, determine the students' previously acquired abilities, and discover the special interests that can be tapped for motivational purposes. This assessment should include the use of a wide variety of diagnostic assessment tools that are available at all grade levels. These tools range from standardized, nationally normed tests to less sophisticated verbal measurements, such as oral interviews and discussions. In any case more than one form of assessment should be used to ensure accuracy of information for each student. (In assessments of limited-English-proficient [LEP] students, care should be taken to ensure that it is the students' mathematical skills being assessed rather than the students' abilities to understand English.)

Teachers should make diagnostic assessments continually to provide important information about student understanding. Based on the assessment information, teachers decide on the use of alternative instructional approaches, variations in student groupings, the need for remediation, or advancement to a new topic. Each teacher should have a number of imaginative ways to assess accurately and effectively the entire class quickly and frequently (in less than ten seconds every few minutes). These include "thumbs up or thumbs down" in silent response to a yes-or-no question; each student responding on a slate-like board which he or she holds up at a given signal; and color-coded cards for students to hold up to signal their mental state red for "I'm confused," yellow for "now I get it," and green for "hurry up, I'm ready to do the assignment."

*Each teacher should have a number of imaginative ways to assess accurately and effectively the entire class quickly and frequently.*

Another effective assessment strategy is one that not only provides for students to respond but also encourages students to internalize or "fix" a concept in their minds. For this strategy, the teacher asks everyone to concentrate on a specific idea for 30 seconds eyes shut, no pencils, no talking, no reading. After 30 seconds, students are called on or volunteer to explain the concept in their own words, to give an

application of the concept, to tell how the concept is related to some other concept, or to do some similar task. For example, when a geometry class has been introduced to the notions of point, line, and plane, the teacher says, "Now concentrate on what you have just heard about points, lines, and planes, and in 30 seconds I will ask some of you to tell us how they are alike and how they are different." When this strategy is used regularly and frequently, as it should be, it will become increasingly effective.

**12. Are** [illegible]

**What to look for:**

a. Are there established assessment procedures for diagnosing students' needs prior to the placement of students in courses or the use of instructional materials?

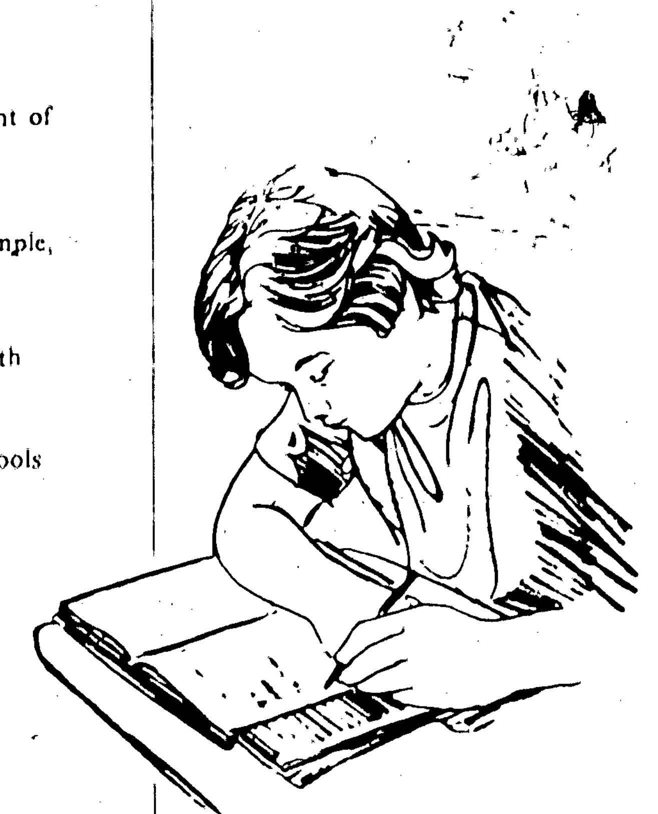
b. Do teachers have established procedures for determining students' areas of interest? For example, through:
   (1) Observation techniques
   (2) Interest inventories
   (3) Discussions on hobbies, careers, and so forth
   (4) Informal conversations
   (5) Analysis of student questions

c. Are a broad range of performance assessment tools used regularly which allow for differences in students' learning styles? For example:
   (1) Regular homework assignments
   (2) Oral demonstrations
   (3) Teacher observations
   (4) Commercially prepared tests
   (5) In-text tests
   (6) Teacher-prepared tests
   (7) Checklists
   (8) Interest/attitude inventories
   (9) Attitude-behavior-discipline reports
   (10) Interviews, class or small group discussions
   (11) Criterion-referenced tests tied to performance objectives
   (12) Student projects

d. Are limited-English-proficient students assessed in a language they understand, and do they receive instruction through the use of materials in their primary language, bilingual teachers, aides, tutors, or peers?

e. Are assessments of progress free of language and cultural biases?

f. Do teachers analyze student errors to assist them in diagnosing? For example:
   (1) Basic facts or errors in computing procedures



(2) Errors that occur repeatedly and indicate a misunderstanding of concepts

g. Does the teacher frequently use whole-class assessment strategies that determine whether every student is following the lesson or whether a change in teaching strategy is needed?

## Keeping Records of Student Progress

Students need to be kept aware of their progress toward the mastery of prescribed mathematical skills and concepts. Progress records should be based on many different forms of assessment and may vary to accommodate teachers' preferences.

Assessment records should include a determination of which skills and concepts have been mastered and which need more study. By keeping the records up-to-date, the teacher may use them for giving positive reinforcement when students show good progress and for assigning appropriate learning activities.

*At age ten, the famous German mathematician, Karl Gauss, astounded his teacher by discovering a short cut for adding the whole numbers to 50 an assignment meant to keep the precocious student occupied.*

KARL FRIEDRICH GAUSS
1777 1855

### 13. *Is the progress of every student assessed and recorded?*

**What to look for:**

a. Are progress records kept for each student and checked frequently by that student and the teacher?

b. Are progress charts maintained, and can students explain what they mean?

c. Are parents informed regularly of their child's progress in learning mathematics skills and concepts?

d. Are both speed and accuracy of the student's skill in computing included in the progress record?

e. Does the amount and level of practice assigned to each student vary according to the individual's progress?

f. Do students take timed tests to assess their retention of computing skills learned previously?

g. When a student's retention falls below the acceptable level on speed or accuracy, are practice activities provided?

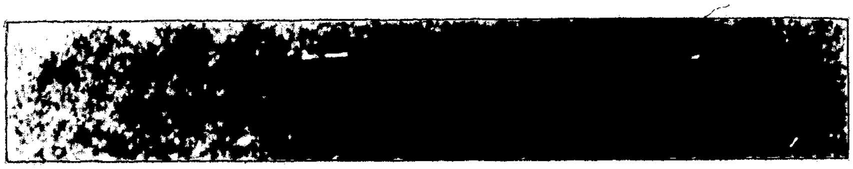
## The Use of Manipulative Materials

Of all the planning that sets the stage for a good learning experience, none is more central nor crucial than the teacher's plan for presenting the lesson. Even here, the selection of the topic, the objectives, the assessing of the students' understanding, and the organizing of the materials and classroom may overshadow the concern for how the students will build the bridge from the familiar to the unknown; in other words, from what they already know to what they should learn. Teachers should be familiar with current research in education on effective bridging strategies in order to plan their lessons effectively. One particularly propitious example of the research is Jerome Bruner's three learning stages a widely accepted, but often ignored, theory that supports classroom use of "hands-on" materials.

In his book, *Toward a Theory of Instruction*, Mr. Bruner identifies these stages of learning: concrete, representational, and symbolic. That is, students should learn a mathematical concept through experiences with concrete, three-dimensional objects (manipulations); then with representations of the physical objects (for example, pictures); and finally, with symbols to refer abstractly to the concept.[1]

Teachers should have a commitment to using manipulatives to the students' best advantage and should demonstrate creative uses of manipulative materials. In the early grades, some possible activities in which manipulatives may be used include: (1) identifying likenesses and differences; (2) classifying and categorizing objects by their characteristic features; (3) comparing objects by size; (4) grouping objects; and (5) making conjectures based on the manipulations.

The use of manipulatives should also have a place in intermediate and higher grades, both in discovering new concepts and skills and in providing remedial help. Colored rods, base-ten blocks, and graph paper should be used to illustrate properties of whole numbers, the concepts of fractions, and the relationships between fractions and decimals. Paper folding can also be used to illustrate geometric concepts and some area formulas (for example, parallelograms and circles). Pipe cleaners and a sheet of cardboard can be used effectively to illustrate three-dimensional concepts introduced in a high school geometry course.



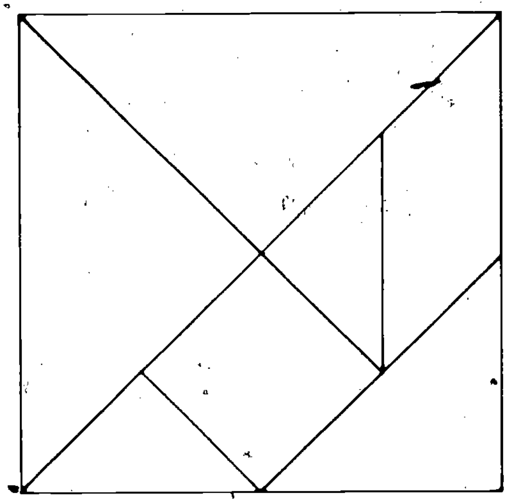
**What to look for:**

a. Do students have access to three-dimensional models or familiar objects as a regular part of their mathematics instruction? For example, engines, model kits, mathematical manipulatives, objects to measure, and geometric forms.

b. Do teachers ask sequences of questions that lead students in making the connection between the concrete and the abstract?

c. In the primary grades, is the teaching of place value and addition of multi-digit numbers developed

[1] Jerome Bruner *Toward a Theory of Instruction* New York W W Norton and Co., 1966



TANGRAM PUZZLE

***Teachers should have a commitment to using manipulatives to the students' best advantage and should demonstrate creative uses of manipulative materials. For example, teachers could ask their students to cut out the pieces of the tangram puzzle (above) and use the five small pieces to form a square. Then the students could be asked whether they could place the two large pieces around the square to form a triangle, a parallelogram, a trapezoid, and finally a rectangle.***

through the use of manipulatives? For example:
(1) Students represent two-digit numbers with place value materials, such as bean sticks or base-ten blocks.
(2) Students play "trading" games.
(3) Students add two-digit numbers by manipulating place value material, regrouping, or trading when appropriate.
(4) Materials are available for students to use, as needed, in computing standard problems.

d. Is the concept of fractions developed through activities based on concrete objects? For example:
(1) Fraction "pies," strips, or squares
(2) Parts of sets or groups of objects
(3) Number lines or "clock" circles
(4) Colored rods

e. At the junior and senior high school levels, do students have opportunities to reestablish previously learned skills and concepts through concrete physical or visual models?

f. Is the relationship of mathematics concepts to "real life" situations continually emphasized and demonstrated visually?

g. Are topics taught so that students "discover" or "see" mathematics concepts before the teacher introduces a mathematics rule?

*Peer discussions reinforce learning by challenging students to exchange viewpoints and analyze possible strategies or solutions.*

### Grouping of Students

Group work, often a useful approach to learning, should be flexible and should be based on the interests, needs, and learning styles of students. Generally, the options for grouping students in a class include discussion with the whole group, small groups, and individuals. Each option is more appropriate than the others for certain students. Peer group work often provides the teacher with time for individual or group remediation. More importantly, perhaps, peer discussions reinforce learning by challenging students to exchange viewpoints and analyze possible strategies or solutions, thus developing and sharpening their logical or critical thinking skills while increasing their abilities to communicate with the language of mathematics.

## *15. Are students grouped in a variety of ways to reinforce learning?*

**What to look for:**

a. Does the teacher use groupings that are based on the assessment of the students' learning styles and related to the lesson objectives? For example:
(1) Small groups in the regular classroom
(2) Tutorial programs
(3) Use of resource teachers

(4) Mathematics labs
(5) Instructional aides
(6) Cross-age tutors
(7) Alternative classes
(8) Programmed texts or materials
(9) Mini-courses

b. Do students receive instruction with the whole class at appropriate times, such as when a concept is being introduced or when groups want to share information or results from a small group or individual project?

c. Do students work in small groups when it is appropriate? For example:
(1) To prepare for debates and panel discussions
(2) To solve a problem through brainstorming, discussion, and an exchange and analysis of ideas
(3) To collect, organize, and represent data or a graph or report
(4) To receive instruction that is appropriate to their level
(5) To do skill reinforcement activities, such as peer teaching and games
(6) To receive remedial instruction

d. Do students work individually when it is appropriate? For example:
(1) A follow-up assignment for a difficult concept
(2) Practice for needed skills
(3) A report on an area of interest

e. Are students encouraged to work together to exchange and analyze various problem-solving strategies or solutions?

*In a high quality mathematics program, the teacher presents challenging projects, asks stimulating questions, and poses meaningful problems with equal frequency to students of all ability levels.*

## The Effect of Attitudes on Achievement

Learning does not "just happen" through the application of a few learning theory principles about achievement and concept development. By creating an expectation of maximum achievement, providing for success and challenges, providing for productive learning time, and assigning homework, the teacher motivates students to reach their potential achievement levels.

### Effect of High Expectations on Motivation

Studies consistently show that student achievement levels are noticeably affected by teacher expectations. Teachers must honor each student's right to work up to his or her maximum potential. In a high quality mathematics program, the teacher presents challenging projects, asks stimulating questions, and poses meaningful problems with equal frequency to students of all ability levels.

In addition, teaching is not effective in an undisciplined atmosphere. The creation of a productive and effective learning environment requires an uncompromising understanding from the first day: the teacher has the responsibility to create such an environment. Then, step by step, the teacher and the students build the desired environment in the classroom. Teachers can convey high expectations for achievement while building an effective learning environment in a variety of ways:

- Set and maintain classroom standards for discipline, punctuality, time on task, completion of work, and maximum effort applied to the tasks.
- Set an example for effort by beginning lessons on time and being prepared.
- Reinforce expected behavior and take time to discuss below-level (or inadequate) effort in a positive attempt to bring students up to expected levels of achievement.

To be motivated, students must be active participants in the learning process. Even in remedial, basic, and enrichment instruction, a high level of rigor is often the challenge that students need to become active participants. Encouraging students to identify the steps in their thinking processes also increases participation. To ensure continued involvement, teachers should focus on the positive progress the student makes, not on the unsuccessful attempts. For example, a struggling student may be encouraged and motivated by being reminded how far he or she has progressed rather than how he or she compares with the top student in the class.

[illegible]

**What to look for:**

a. Do teachers expect all students to achieve? For example, are the concepts, values, skills, and knowledge acquired by students in special classes, such as special education, compensatory education, and bilingual education, the same as those acquired by students in the regular classes?

b. Does the teacher demonstrate a belief in the students' abilities to do the work?

c. Do teachers call on low achievers as frequently as on high achievers, and do they allow sufficient time for responding?

d. Does the teacher work with low achievers to determine the reason for lack of progress and to redesign their study programs?

e. Do the teachers set a standard for excellence that they themselves model by being prepared for every lesson and beginning every class on time, by promptly correcting and returning quizzes and homework, and by spending the entire class period actively teaching?

f. Is there a policy for uniform and fair enforcement of behavioral standards?

g. Are there written schoolwide class standards for workmanship, punctuality, and behavior?

h. Is there a uniform schoolwide policy on grading, and do students and parents know the amount and quality of work necessary for a student to receive a specific grade?

i. Do teachers, parents, and students know what behavior is expected of them?

j. Does the administration support teachers in their efforts to enforce the adopted standards?

k. Does the teacher encourage students to make suggestions and to ask questions, and are students' ideas incorporated in lectures and discussions?

l. Are students involved in the learning process? For example, do they discuss problem-solving strategies, pose questions, and encourage each other?

## Success and Challenge in Stimulating Learning

High expectations are not the only ingredients needed for creating motivation. Teachers must also design their lessons and teaching strategies so as to facilitate student success. For example, exams or assignments should be returned with written comments that indicate what was done well or commendably. Another helpful strategy is discussing with a student the accuracy and rationale of his or her verbal responses.

Lessons should also be sequenced so that every student obtains some level of success on some of the problems. For example, a geometry problem should require the use of more than one skill. Some students might have success by illustrating the problem; others, by posing questions. At the high school level, a geometry problem might be a proof with sections to be completed or a proof with an error to be found and corrected.

Arranging for each student to have opportunities for success may not be always easy. However, such opportunities are more likely to occur if there is a structure that provides for objective measurement of student achievement, timely diagnosis of need when students fail to achieve, and, if necessary, appropriate alternative instruction for remediation.

Students draw conclusions about their chances to succeed from the actions and remarks of teachers. When teachers have clear instructional objectives and share them with the students, everyone has a better perception of when success is near or has been achieved. At the elementary level, teachers may begin a lesson by communicating to the students the purpose of a new exercise, for example, "Today you are going to learn how to use a number line to help you see that multiplication is

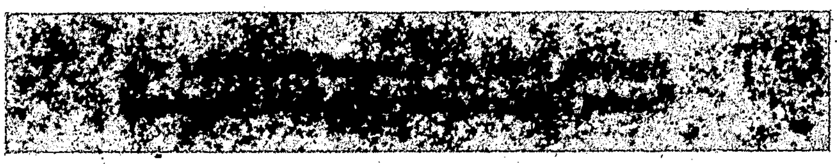
GEORGE POLYA

*"Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place."*

FROM MATHEMATICAL DISCOVERY BY GEORGE POLYA similar to repeated addition." At the secondary level, the teacher may remind the students of how the day's assignment relates to an ongoing achievement objective. When students know why they are performing a certain operation or practicing certain exercises, it is more likely that they will attend to the task conscientiously and thereby increase the likelihood of success.

Also, in high quality programs, teachers use a variety of resources for giving students challenging and interesting experiences. For example, teachers may use the skills, backgrounds, and interests of other faculty or other community members to show students the rewards of learning and to model high achievement and enjoyment of mathematics. For example, a parent who has traveled to Egypt may share his or her knowledge of the engineering of the pyramids with a geometry class, or a woodshop teacher who has worked as a carpenter may discuss career options and uses for mathematics in the building industry. These presentations might then be followed by assignments that require independent study or group projects that capitalize on high interest levels.

In order to develop a positive attitude toward learning, a student needs to know that successful performance is a legitimate, hard-earned achievement. High scores on easy examinations are not likely to build a sense of competence.



**What to look for:**

a. Does the teacher take advantage of correct responses to build self-confidence and success?

b. Does the teacher treat students' incorrect responses with sensitivity?

c. Do students receive regular assessments of their achievements and progress toward mastery?

d. Do the students find out whether their answers or statements are right, almost right, or wrong?

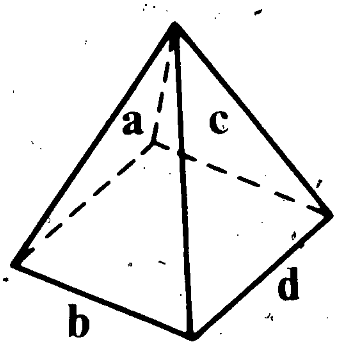
e. Are assignments graded and returned promptly?

f. Are students made aware of the goals they are trying to achieve?

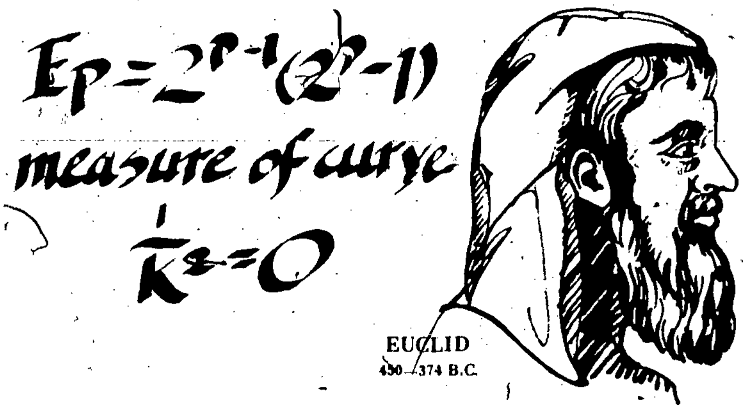
g. Are students encouraged to set personal goals related to the mathematics curriculum?

h. Is a large portion of the students' time spent working on tasks that lead to success and legitimate feelings of competence?

i. Do assignments have graduated levels of challenge (some success is possible for all, and everyone is challenged)?




*A parent who has traveled to Egypt may share his or her knowledge of the engineering of the pyramids with a geometry class.*




*Although often quoted as, "There is no royal road to learning," the original, as written by Euclid, read, "There is no royal road to geometry."*

j. Are students encouraged to work on projects for extra credit and enrichment?

k. Are tests designed to cover and reflect the level of the instruction?

l. Are the objectives of each lesson clearly communicated to students?

m. Are community resources identified and utilized to extend classroom learning and to model achievement and enjoyment of mathematics?

n. Are community interests tapped to promote student growth and achievement; e.g., donation of computer time by public agencies, private industry, universities, and colleges?

## Adequate and Productive Learning Time

A crucial component of the mathematics program that directly affects student attitudes and achievement is the amount of time students spend studying mathematics. However, a clear difference exists between the time allocated to mathematics and the time students spend working productively on the subject. It is understandable that there may be discrepancies in the time devoted to learning mathematics among different school sites, grade levels, and even classrooms at the same grade level. It is essential that differences in time allocations be carefully scrutinized; and those differences should be minimized by having schoolwide agreement on allotments of time and by setting an equitable school schedule.

An important part of learning mathematics is doing it. In an effective mathematics program, students are actively and positively involved in the learning process. The time spent watching and listening in class should be balanced with the time spent asking questions, responding to questions, interacting, and writing or recording. Drill and practice, a daily necessity, should be provided with a clear emphasis on both developing skill and maintaining interest. In review exercises, students should have to think about mathematical concepts rather than operate in a mechanical way.

More important still, to make the time given to mathematics productive, teachers should rarely or never assign whole pages of essentially the same problem; for example, a page of addition of two-digit numbers or a set of word problems whose solutions require that the smaller number be subtracted from the larger number. Students should learn to select the appropriate arithmetic operations rather than follow examples blindly.

## *18. Is a substantial amount of time allocated daily for studying mathematics?*

**What to look for:**

a. Are students actively involved in mathematics study for the entire allotted time period, or do they spend time waiting for assistance or information, copying material, preparing for study, or engaging in off-task activities?

b. At the elementary level, is there an agreed-upon time allocation for mathematics at each grade level?

c. Do teachers honor the agreed-upon time allocation for mathematics?

d. At the secondary level, are class schedules adjusted to minimize the loss of instructional time because of scheduled interruptions, such as assemblies, fire drills, sporting events, and school plays?

e. Are students assigned interesting tasks related to the instructional topics while the teacher takes attendance, signs passes, and so forth?

f. Is the teaching well organized, sequential, and direct?

g. Does the teacher give clear directions about the goals of the lesson and set clear standards for the students' role in cooperating to achieve the goals?

h. Is lecture time balanced with the time spent asking questions, responding to questions, interacting, and writing or recording?

i. Does direct instruction and student participation predominate over drill and desk work?

*An important part of learning mathematics is doing it.*

j. Is a reasonable time limit set on achieving objectives and accomplishing tasks after initial instruction has been completed?

k. Are transitions from one subject or period to the next accomplished with speed and a minimum loss of instructional time?

l. Do the teacher's assignments and behavior emphasize the development of good study habits for the students? For example:
   (1) Students read and discuss directions.
   (2) Home study assignments call for the use of a variety of skills.

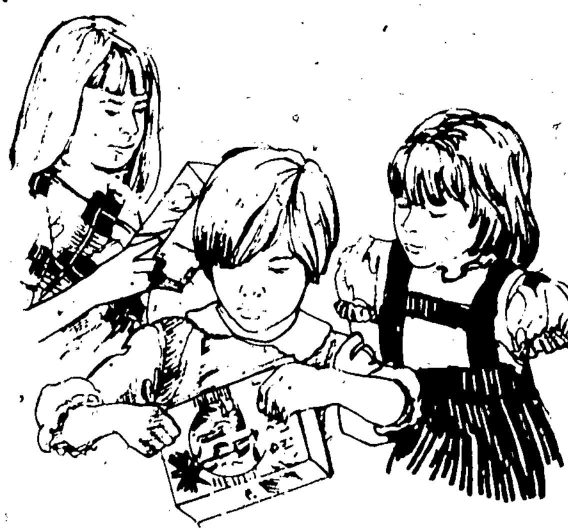
(3) The teacher draws on the students' prior learning; e.g., when asked, "How do I do this?" the teacher asks the student, "Tell me what you have already tried," or "Identify where you are having trouble."

## The Role of Home Study

Home study is another important component of the mathematics program, as it extends the amount of time students spend in learning and applying mathematical concepts. It also prepares students for the demands that will be made upon them to think and work independently in jobs or in universities. Homework should reinforce the concepts learned in class and should be completed before class. In a high quality program, class time should be devoted primarily to interactions between the teacher and students or among peers; so homework should be used in class only as a discussion tool for solving problems and enhancing the use of the language of mathematics.

Home study is an important way of extending productive learning time. Thus, homework assignments should be designed carefully to relate to in-class lessons and to reinforce concepts and solving skills. This means that every student should have a copy of the textbook to take home for individual study. If students are to develop good study habits that will transfer later to good career choices, it is important that those habits be reinforced in school. For example, teachers should encourage students to explain assignments and ask students to restate written and oral directions in their own words.

There should be schoolwide agreements on the purpose of homework, the amount expected, and the consequences of not completing the homework on time. Parents should be informed of these agreements and how they can help make them work.



*Homework assignments should be designed carefully to relate to in-class lessons and to reinforce concepts and solving skills.*

**What to look for:**

a. Is most of the homework related to content being learned in the classroom and designed to reinforce the skills and concepts taught there?

b. Is homework an integral part of the teaching-learning process? For example, is it assigned in such a way that allows students to discuss and respond to what is expected, rather than as a "parting shot" as students leave class?

c. Do students understand the homework assignment?

d. Does every student have a textbook which can be used at home?

e. Do students complete homework problems, as assigned, and discuss them when necessary?

f. Are parents provided with suggestions and guidelines on how to supervise and assist their children with homework?

g. Are there well-known and uniformly applied consequences for not completing homework on time?

h. Are parents contacted when students fail to complete homework assignments on time?

i. Is there schoolwide agreement on the quantity of homework assigned?

## Calculators and Computers

Calculators and computers offer a rich source for mathematics learning experiences. When these tools are used creatively and in a timely fashion, they can enhance students' appreciation for mathematics as a process while facilitating the accomplishment of a variety of learning objectives.

*Computer literacy includes knowledge of a computer language, mastery of computer programming skills, and the ability to understand and organize computer data.*

Teachers in the school's mathematics program should be responsive to the fact that the use of calculators and computer literacy will increasingly become a condition for employment in both nonprofessional and professional jobs. Computer literacy includes knowledge of a computer language, mastery of computer programming skills, and the ability to understand and organize computer data.

### Use of Calculators and Computers

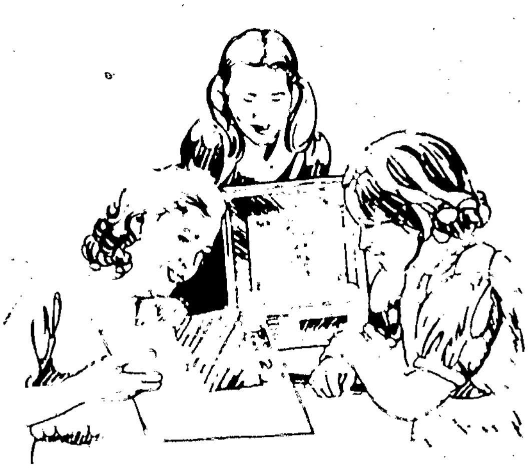
Great care must be taken to ensure that calculators and computers are employed to supplement and motivate student learning. In a high quality mathematics program, their purpose and need will be thoroughly discussed and agreed upon before substantial purchases of hardware and software are made. Their use should be based on clear educational goals rather than on an assignment of repetitive problems to "punch" into a calculator or of nonproductive "playtime" on a computer. Calculators and computers may be used effectively for reinforcement activities if the teacher plans the activities to generate interest and curiosity. For example, students may learn about order of operations from the problem of multiplying (*18075 + 35792*) by 2. The answer to the problem on a calculator, 107734, when turned upside down, is "HELLO!" This type of exercise can motivate students to work on a wide range of mathematical skills and to make up problems of their own.

107734 hELLOI

It is imperative that the use of calculators and computers not supplant the learning of the essential computing skills. Neither calculators nor computers are substitutes for recall and understanding of the operations and processes they are designed to perform.

In addition, the instructional materials (software) selected for the mathematics program should provide opportunities for calculators and computers to be used as an integral part of mathematics instruction. Other specific uses for calculators and computers in the classroom may be found on pages 75—83 of the *Mathematics Framework and the 1980 Addendum for California Public Schools*. In a high quality mathematics program, calculators and computers are used for:

- Reinforcing computing skills
- Providing immediate feedback on mathematical processes
- Checking the accuracy of answers and locating errors
- Discovering concepts and relationships
- Providing enrichment opportunities
- Applying problem-solving skills



## 20. Are calculators and computers used in a variety of ways to stimulate and reinforce learning?

**What to look for:**

a. Are calculators and computers and accompanying software selected and purchased on the basis of their potential for enhancing the objectives of the mathematics instructional program?

b. Are there established criteria for textbook and media selection that refer to the use of calculators and computers at all levels?

c. Does the program provide opportunities for students to use calculators and computers in all mathematics skill areas? For example:
   (1) Motivation
   (2) Enrichment
   (3) Problem solving
   (4) Student assessment
   (5) Drill and practice
   (6) Application
   (7) Reinforcement and review
   (8) Development of new concepts

d. Do students demonstrate their knowledge of the usefulness of calculators and computers by using them in practical ways; e.g., checking the accuracy of previously completed computations, working on the data from newspapers, and checking bills from utility companies?

e. Are direct learning activities on calculators and computers, such as programming, student assessment, and problem solving, given more emphasis than games and playtime?

f. Is there a well-designed curriculum that leads to computer programming at the upper grade levels?

g. Do the advanced mathematics courses offer students the opportunity to develop programming abilities?

h. Are the uses of computers in society presented by community persons with technical knowledge?

## Skills Required by the New Technology

With increased use of calculators and computers, it is essential that estimation and mental arithmetic be established as high priority skills that students need to build regularly and consistently. For example, most calculators do not supply a printout of what was entered; therefore, it is most important for the user to have a sense for the approximate size of the answer. Thus, it is just as important for students to learn to estimate how big an answer should be as it is to learn to use the calculator to get the answer.

To use calculators and computers effectively, students must learn to use mathematical symbols precisely, and they must learn that a direct relationship exists between mathematical thought processes and electronic processes. For example, students who do not use parentheses properly are made keenly aware of the need for such precision when they discover that punctuation affects what the calculator or computer will do. In fact, if the users do not follow the required steps or misspell words, the device simply refuses to respond. Such inaction can be very effective at improving the students' desire to be precise.

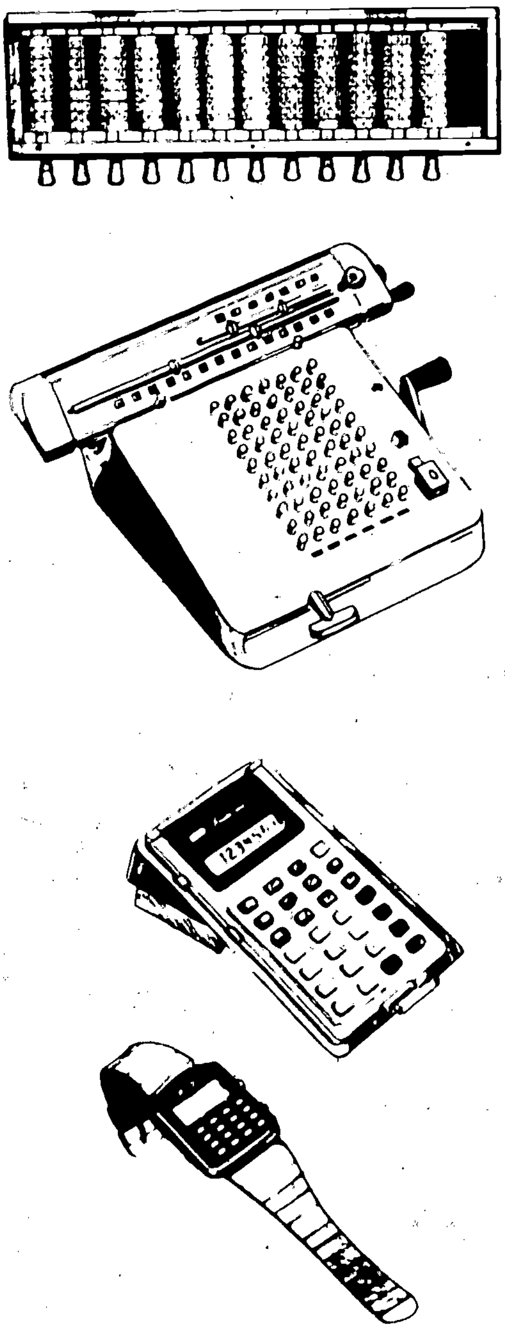
24. [illegible]

**What to look for:**

a. Does each student have opportunities for developing a sense of the approximate size of answers?

b. Are students given experiences that enable them to see what calculators and computers will or will not do if proper operations are not observed?

c. In courses or units dealing with calculators and computers, do the teachers stress uses, capabilities, limitations, misconceptions, misuses, and dangers of the equipment?



EVOLUTION OF THE CALCULATOR
FROM NAPIER'S RODS TO THE
WRISTWATCH CALCULATOR

# PART IV

# Support for Implementation of a Quality Mathematics Program

## Overview

In parts II and III of this handbook, the emphasis has been on the actions in the classroom the part of the mathematics program that serves the students directly. This part describes the support system needed to enhance that service. However, in the development of a support system for any instructional area, there are common considerations, and in this handbook, the precepts that seem to be most needed in mathematics instruction will be presented and illustrated with mathematics-related examples. Mutual respect and support among teachers, administrators, parents, and community form the foundation of an effective mathematics program. For example, as students learn that mathematics skills are necessary to everyday life, they should see that this skill development is expected and supported at home, in the community, and in all areas of the school program.

An assessment of the support for learning in a school's mathematics program should include careful consideration of the following elements:

- *The school climate.* It is important that administrators, teachers, parents, community members, and students work together as a team to coordinate resources and to improve continually the quality of the mathematics program.
- *Staff development.* Each school site should have, and have maintained, an ongoing process in which instruction is improved through creative and stimulating teacher-training experiences.

*Administrators, teachers, parents, community members, and students must work together as a team to coordinate resources and to improve continually the quality of the mathematics program.*

## The School Climate

The climate of a success-oriented mathematics classroom is enhanced by the involvement of the many resources available to a school, including faculty, parents, district and school administrators, local businesses, local institutions of higher learning, and professional organizations. What happens in the mathematics classroom is not independent of either what is happening in the school or in the students' homes.

HYPATIA OF ALEXANDRIA
370 415

*Although regarded as one of the great philosophers in history, Hypatia of Alexandria, Egypt, was also highly regarded as a mathematician and astronomer. She was a very popular lecturer and became the most well-known figure of the Neoplatonic school in Alexandria.*

## The Climate of Achievement at the School Site

At the school site, the cooperation of the teaching staff and administration is absolutely essential in creating a high quality program. In mathematics cooperation is particularly important for: articulation among grade levels; attention to productive learning time; establishment and maintenance of classroom control; provision of sufficient and appropriate instructional materials; assurance of ongoing contact with parents; and maintenance of communication with various community resources.

Regular meetings between the administrators and all or part of the faculty are important for discussing special problems related to the teaching of mathematics. One of the issues the faculty and administration should address together is the establishment and maintenance of schoolwide standards for attendance, punctuality, classroom behavior, and academic work. Procedures to ensure uniform, just enforcement of standards should be regularly and systematically applied.

Planners of an effective school program will also see that teachers are enthusiastic and interested in each others' teaching experiences. The entire school staff will reflect an attitude that academic achievement is the primary purpose of the school and that students have the right to learn in an atmosphere where they are respected as people.

The school counselor is an important member of the school site team, especially at the junior and senior high school levels. Counselors, for example, can assist parents and students in making the best possible choices of classes; however, to do that type of counseling, they should be knowledgeable in these areas: (1) the content of the mathematics program; (2) career and academic opportunities; (3) the needs, interests, and talents of the students; and (4) the mathematics that is needed for various career choices.

The need for counselors and teachers to be knowledgeable about the mathematics requirements for college-bound students is underscored by the vast number of students who are entering colleges and universities without the three years of high school mathematics that is necessary for the study of calculus, a course that is required for over 75 percent of all college majors.

Teachers and counselors should also encourage women and under-represented minorities to pursue more mathematics courses in high school. The EQUALS Project at Lawrence Hall of Science, Berkeley, has dramatically called attention to the way these groups have effectively shut themselves out from careers that require mathematics preparation. The correct information on the mathematics needed for careers is available through the National Council of Teachers of Mathematics and the Mathematical Association of America. Counselors and teachers of mathematics should also seek feedback from graduates regarding career and college preparatory information.

**What to look for:**

a. Does a committee composed of school administrators, faculty, parents, students, and community members meet regularly to monitor and assess the mathematics program?

b. Are there schoolwide standards for punctuality, attendance, classroom behavior, and academic work?

c. Is there evidence that administrators set aside time to maintain regular communication with the teachers regarding instructional needs, available services, and support?

d. Do the aides and volunteers have a clear understanding of the mathematics program and their roles in supporting it?

e. Does the mathematics program receive an equitable share of the school budget?

f. Do counselors and mathematics teachers have current information about career options and university requirements related to the level of mathematics taken, and are students given that information?

g. Do counselors and mathematics teachers seek information from former graduates about how the school's mathematics program met their needs?

*In an effective mathematics program, contact with parents occurs regularly, rather than just when difficulties arise.*

## Parent and Community Involvement

In an effective mathematics program, contact with parents occurs regularly, rather than just when difficulties arise. Teachers must remember that their students do not need to be totally successful in mathematics before positive comments are sent home. For example, teachers should use progress reports to give parents good news about their children or simply to send some general information home that will elicit more support or praise for the students' mathematical progress. At the secondary level, teachers may give students written information on careers that require a knowledge of mathematics and encourage them to share and discuss that information with their parents. At all grade levels, parent-teacher events are an important part of building a support system that reinforces the concept of the school as a positive learning environment in which students are provided with skills they need.

The resources that can be used for instructional support are diverse and numerous. In an effective mathematics program, teachers use resources that enable students to see the relationship of what they are learning to their present and future lives. For instance, many parents have mathematics-related jobs or hobbies and are willing to share their mathematical expertise with their children's classes. Local businesses and institutions of higher learning can be another source of guest speakers or career information. Often they are also willing to provide materials that can be used for mathematics projects or group presentations. In addition, work-learn opportunities for the students in the community often motivate the students to learn mathematical concepts, as they see firsthand how knowledge of mathematics is applicable to career situations.

*Parents and students are made aware of course offerings, major objectives of the mathematics program, and relationships of courses to career and university requirements.*

**HOW MUCH?**

Here are some careers that require more than one year of high school math. Universities differ in specific requirements, of course. However, if you take the amount of math listed here for the career of your choice, you will probably be adequately prepared in mathematics to begin a college program in that field.

Mark those programs which interest you.

| For a program in | take this many years of high school math |
|---|---|
| Agriculture | |
| Agricultural economics | [illegible] |
| Entomology | [illegible] |
| Environmental sciences | 4 |
| Food sciences | [illegible] |
| Forestry | [illegible] |
| Genetics | [illegible] |
| Landscape architecture | [illegible] |
| Plant pathology | [illegible] |
| Rural sociology | [illegible] |
| Wildlife ecology | [illegible] |
| Other areas of agriculture | [illegible] |
| Architecture | [illegible] |
| Art | [illegible] |
| Business | |
| Accounting | |
| Economics | |
| Man[illegible] | |

*FROM THE MATHEMATICAL ASSOCIATION OF AMERICA'S BROCHURE, THE MATH IN HIGH SCHOOL YOU'LL NEED FOR COLLEGE*

## 23. Do parents and community members have many opportunities to become involved in the school mathematics program?

**What to look for:**

a. Do teachers make phone contacts with the parents of their students early in the school year and whenever encouragement of the students' progress at home is necessary?

b. Do teachers maintain contact with parents through progress reports, career information, and suggestions for support of the students' progress in mathematics?

c. Does communication to the parents concerning the program include a mechanism for responding to the school?

d. Are meetings involving parents and administrators, support staff, teachers, and district office liaison personnel held during the year?

e. Are parents and students made aware of course offerings, major objectives of the mathematics program, and relationships of courses to career and university requirements?

f. Are meetings scheduled at convenient times for parents and business representatives to discuss job opportunities, job requisites, and career options?

g. Is someone on the staff designated to establish contact with businesses in the community?

h. Does the staff-community liaison person make the staff aware of work opportunities available for students in the community?

i. Is recognition given to businesses and agencies that are involved in the school's work experience program?

j. Do students have access to the following community resources so they can see how mathematics is used in real life situations?
(1) University personnel
(2) Research and laboratory personnel
(3) Business and industry staff
(4) Paraprofessionals and volunteers
(5) Other faculty members

k. Is the classroom expanded to include the community, when appropriate, so that concepts learned in mathematics classes can be applied to real objects and real situations; e.g., local businesses and industries, parks, banking institutions, newspapers, environmental and recycling centers?

l. Do students have class projects that require them to interview people who use mathematics in their careers, such as plumbers, electricians, accountants, computer programmers, and so forth?

## Coordination at the District Level

An essential support element in a high quality mathematics program is the effective communication among feeder schools (i.e., elementary or junior high) and receiving schools (i.e., junior or senior high schools or colleges). Mathematics teachers, representing different grade levels, should meet regularly to discuss programs, student needs, and strategies for improving continuity throughout the program, kindergarten through grade twelve.

Also, the mathematics representatives of the schools should participate in the design of the district's course of study, proficiency requirements, and other district-level curriculum requirements related to mathematics. Finally, the district office should ensure adequate funding for instructional materials and release time for staff to participate in districtwide meetings, staff development programs, development of materials, and other instructional support functions. For some activities, the office of the county superintendent of schools and the State Department of Education can provide valuable assistance and should be consulted by the district staff.

Another important responsibility of the district administration is support of staff development programs at the district and school site levels. This responsibility is discussed in greater detail on page 47.

*"I am convinced that it is of primordial importance to learn more every year than the year before. After all, what is education but a process by which an able person begins to learn how to learn?"*

FROM DEAR ME, BY PETER USTINOV

### *24. Is there coordination at the district level for the development and implementation of a high quality mathematics program?*

**What to look for:**

a. Does a schedule exist for a district or county mathematics liaison person to meet with the mathematics committees from each school on a regular basis?

b. Is there evidence of district-level and county-level support of efforts to improve the school's mathematics program, e.g., development of needs assessments, assistance with program evaluation, selection of instructional materials, and assistance with staff development activities?

c. Does a districtwide committee of representatives from each school meet on a regular basis to design, monitor, and assess the district's mathematics program; e.g., the design of district mathematics courses of study, mathematics proficiency requirements, and other district-level curriculum requirements?

d. Does the district provide release time for faculty to meet with teachers in other school programs or with teachers at different grade levels?

e. Is a published summary of mathematics information, including services and resources, sent from the district office to all staff on a regular basis?

f. Has the school designated a staff member who will receive information from the district, county offices, and other sources on a regular basis and disseminate it to the rest of the staff?

g. Are new materials put on display in a prominent place, such as the staff lounge?

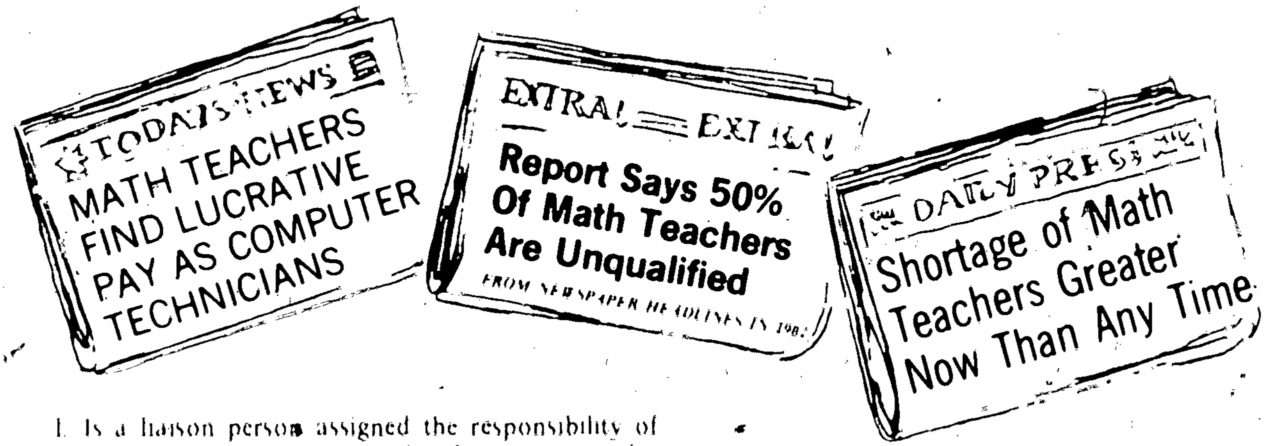
h. Does the district office provide funding support for instructional materials and release time for staff to participate in districtwide meetings, staff development programs, development of materials, and other instructional support functions?

i. Does the district office provide information to schools about funding sources available to support the mathematics program?

j. Do district support staff persons, such as librarians, counselors, and consultants, provide time in their schedules to work with staff members in implementing their mathematics program?

k. Is a liaison person assigned the responsibility of linking resources from universities and colleges with school sites through such programs as field study and graduate degree courses?

*The cooperation of the teaching staff and administration is absolutely essential in creating a high quality program.*




l. Is a liaison person assigned the responsibility of facilitating the communication between secondary school teachers, elementary school teachers, and college and university personnel?

m. Is a staff member responsible for ensuring that categorical support services are coordinated with the regular program?

n. Does the district office have available support mechanisms for assisting schools with program improvement, e.g., use of computer facilities for tabulating questionnaires on program evaluation and needs, consultation or assistance in proposal writing for federal, state, or private agency funding?

o. Is assistance from offices of county superintendents of schools and the State Department of Education solicited and used when available?

p. Is an individual who is well versed in computer technology available to the school as a resource?

## Staff Development

An effective mathematics program grows and changes in response to the growth and changes in uses of mathematics, educational innovations, and in faculty and student needs. It is well known that the number of careers requiring higher levels of mathematics is increasing. Not so well known is the shortage of qualified teachers of mathematics entering the teaching profession and the related need to retrain and recertify teachers in nonmathematics subjects to fill the void. Staff development is the most important activity in the support program for keeping pace with these changes. In a quality mathematics program, the entire school team is committed to work cooperatively to maintain excellence in the existing program while seeking ways to improve the quality of that program.

*In a quality mathematics program, the entire school team is committed to work cooperatively to maintain excellence in the existing program while seeking ways to improve the quality of that program.*

### Criteria for Good Training

Effective staff development is the ongoing process that provides for the continuous growth and development of all the


*Mathematics teachers should have opportunities to develop teaching skills for the nonmathematical learning needs of their students.*

professionals who are directly involved with the education of the students in that school or district, and staff development provides for the growth and interaction of the staff as a unit in a specified effort. It includes a wide variety of activities for the staff; for example, observations of exemplary programs, workshops, institutes, forums, and conferences.

Staff development programs that seek to improve instruction through teacher training must include presentation of relevant theory, demonstrations of desired behaviors, opportunities for practice with feedback, coached applications of what is learned, and frequent occasions for teacher discussions and dialogues. When teachers are able to learn from each other's insights and experiences, their skills are strengthened.

Research has shown that the most successful training programs are conducted during regular working hours or at other times when the trainees are likely to be fresh and enthusiastic about learning. The least successful programs are those conducted immediately after the regular working day when the trainees are likely to be tired and unenthusiastic about being trained. Staff development programs that include follow-up activities are more effective than "one-shot" presentations.

### 25. *Is the staff given opportunities to develop new skills, practice new techniques, and discuss critical issues?*

**What to look for:**

a. Do staff development activities, as well as district curriculum meetings, provide time for discussions concerning curriculum?

b. Do the teachers have opportunities for extending their knowledge of mathematics; e.g., properties of number systems, the metric system of units, computer programming, mathematical proofs, and simple applications of number theory?

c. Are there regular opportunities for planning staff development activities based on group and individual student needs?

d. Do teachers receive training in the use of new mathematics instructional materials adopted by the school or district?

e. Do teachers in subject areas other than mathematics have opportunities to learn ways to reinforce mathematical skills in their courses?

f. Do mathematics teachers have opportunities to develop teaching skills for the nonmathematical learning needs of their students?

g. Do teachers have opportunities for improving their teaching skills through a variety of programs; e.g., teacher exchanges, conferences for teachers of mathematics, and workshops for practicing teaching techniques?

h. Is staff development respected as a high priority activity and offered at times appropriate to teachers' workday?

i. Are teachers encouraged to study and discuss articles, books, or documents which present theories or issues relevant to mathematics instruction?

j. Are staff development activities continued to ensure implementation in the classroom?

k. Do follow-up activities of training programs provide opportunities to practice the prescribed teaching behaviors and to be coached when necessary?

## The Importance of Assessment and Support

Those responsible for planning and implementing effective staff development programs must relate student needs and achievement to the teaching needs of the staff. Teachers must be consulted regarding their professional training needs and their students' learning difficulties. For the plan to be successful, high levels of staff participation also are necessary. The planning and coordination of staff development activities should be the responsibility of teachers and others representing all groups in the school community. The staff development planning should be correlated closely with the mathematics program planning group; for example, if calculators and computers are to be used in the mathematics program, classroom teachers at all levels will need to acquire additional skills. And the school plan should provide opportunities for teachers not only to acquire the new skills but also to increase their knowledge of the wide variety of uses for calculators and computers.

In addition, the staff development plan should be based on information about how the school's mathematics program has met the needs of the students who have completed the program. This information can be collected for an elementary level mathematics program through informal interviews or short questionnaires given to parents of former students and surveys of performance at receiving schools. At the secondary level, information can be gleaned from graduates two, four, and six years after graduation. The information from parents or graduates should be compiled and combined with other relevant information to design and generate interest in staff development programs.

The information gathered on the needs of the faculty and students provides a solid foundation on which a school's staff development committee can build a program. The activities, resources to be used, scheduling of topics, guest speakers, frequency of activities, and so forth should be well planned by the school staff development committee and should include continuous reassessment and feedback obtained from participants, administrators, and others involved.

***"Dear fellow teacher, do not accept any authority except your own well-digested experience and your own well-considered judgment. Try to see clearly what the advice means in your particular situation, try the advice in your classes, and judge after a fair trial."***

FROM MATHEMATICAL DISCOVERY BY GEORGE POLYA

*Staff development is a learning process for individuals that have different needs in different schools at different times.*

It is important that district administrators recognize the importance of staff development and support the efforts of school personnel to learn new skills. It is also important that staff development programs throughout the district be flexible in content and format. District support should be available in the form of funding for staff training, release time for teacher participation, and assistance in making needs assessments and designing training programs. District staff can be particularly helpful in arranging for the use of local resource people, universities, and regional service centers. It is important to remember, however, that there is no single best district staff development program, because staff development is a learning process for individuals that have different needs in different schools at different times.

## 26. Are staff development programs built on assessments of learning needs?

**What to look for:**

a. Does the district and school have an established process for continual assessment of teacher and student needs for constructing a staff development program?

b. Are students who have completed a school's mathematics program interviewed two years or more after completing the program?

c. Has the school designated a staff member who is responsible for staff development planning?

d. Is the school's staff development planning based on support personnel, such as parents, aides, counselors, and volunteers?

e. Are there grade-level and cross-grade-level meetings of teachers to determine staff development program goals and activities?

f. Does the administrative staff participate in school staff development programs?

g. Does a strategy exist for needs assessment to precede the selection of specific staff development activities?

h. Are teachers in subject areas other than mathematics encouraged to become knowledgeable about integrating mathematics in other curriculum areas?

i. Are teachers familiar with professional journals, newsletters, and research bulletins?

j. Does one component of the staff development plan relate to providing staff with knowledge about the appropriate use of available instructional materials?

k. Are district staff training programs flexible enough to be responsive to the needs and concerns of individuals on the staff?

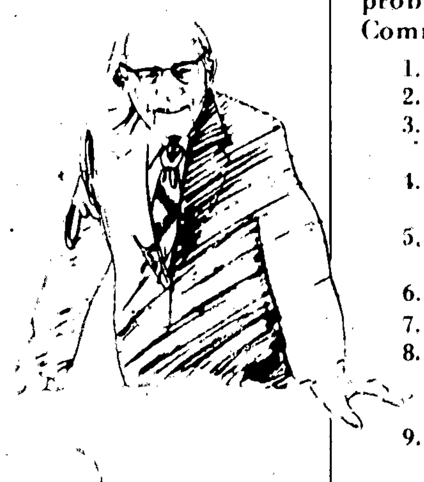
l. Do teachers have access to staff development offerings in computer science and the educational uses of calculators?

m. Are periodicals related to instruction in the use of computers and calculators readily available to the staff?

*Teachers in subject areas other than mathematics are encouraged to become knowledgeable about integrating mathematics in other curriculum areas.*

## Maintenance of Goals

Good staff development programs provide for the continuous monitoring of skills learned or goals met, and the regular staff meeting is an effective starting point for such a maintenance system. Part of each agenda should be devoted to the reinforcement of teaching skills and the promotion of mutual support. It is also important to review commitments to program goals and plans for future training. Such discussions and exercises provide for the development of a support system so that participants will feel they have access to professional help when specific teaching problems arise.



### TEN COMMANDMENTS FOR TEACHERS

When the committee that developed this handbook met with George Polya, the distinguished mathematician, one member asked him what teachers of mathematics should be told about problem solving. He suggested they be given his "Ten Commandments for Teachers":

1. Be interested in your subject.
2. Know your subject.
3. Know about the ways of learning: The best way to learn anything is to discover it by yourself.
4. Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place.
5. Give them not only information, but "know-how," attitudes of mind, the habit of methodical work.
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving the problems to come—try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once—let the students guess before you tell it—let them find out by themselves as much as is feasible.
10. Suggest it, do not force it down their throats.



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Ongoing evaluation of programs is important for the maintenance of skills and the planning of further staff development. The school's staff development council should review the school's mathematics programs annually for the achievement of student learning objectives; continuity from one prerequisite course to another; relationship to new mathematical applications; and appropriateness of instructional materials used.

**[illegible] resources and opportunities [illegible] to [illegible] staff development goals?**

**What to look for:**

a. Does the staff development plan include an evaluation component that has been developed in cooperation with the school's staff development planning committee?

b. Does the staff development evaluation focus on improvement of staff skills and student achievement?

c. Are the activities of teacher centers and mathematics organizations announced as a regular part of the staff meeting agenda?

d. Are publications of professional organizations available at the school site; e.g., the California Mathematics Council, the National Council of Teachers of Mathematics, and their regional affiliate organizations?

e. Has a procedure been established for the routing and posting of materials and information related to local, regional, and statewide staff development training programs?

f. Does the staff development planning make use of community resources when appropriate?

g. Is there a district office person who is responsible for staff development and who meets regularly with representatives from administrative and instructional school staffs to ascertain common areas of need?

h. Is there an established system of communication (e.g., newsletters, meetings) between the central office and all schools regarding individual staff development programs?

i. Does the district office identify and summarize information available outside the district about resources that support staff development programs?

*Publications of professional organizations are available at each school.*

# PART V

# Planning for the Improvement of the Mathematics Program

## Overview

In describing the elements of an effective and exemplary mathematics program, the writers of this handbook have focused on what students learn, how they learn, and what support is needed for that learning to take place. Such a program must be based on careful planning in which teachers, administrators, parents, and students reach agreement on what is expected of students and what the school's program should be in order to meet the students' expectations. It is essential that planning be an all-school activity that takes place continuously throughout the year, rather than an isolated event occurring haphazardly or without direction.

Planning for change depends on recognizing needs, and need is determined by examining the difference between the existing program of instruction and the desired program. The following checklist of essential program elements will facilitate an assessment of the mathematics program at any school and will help determine how effectively the elements described throughout this handbook are being implemented in that school's program. Frequent referral to the examples and explanations given in parts II, III, and IV of the handbook should be helpful in making these determinations.

In its present form, the checklist provides for an individual or a group to evaluate the level of effectiveness of each program element, and this calls for a subjective judgment to be made; but the four columns in the checklist may be changed to describe the level of implementation of each program element  a more objective judgment.

Once a program has been assessed, priorities should be established for planning the necessary improvements. Planners may find it useful to consider the reasons why particular essential elements or clusters of elements are not "very effective" or not "fully implemented." The setting of priorities should take into account the reasons, such as the following:

1. The element will be implemented later as part of a long-range plan.
2. Much assistance in planning is needed before implementation takes place.
3. Funds are needed for implementation.

*"The most beautiful thing we can experience is the mysterious. It is the source of all true art and science."*

FROM "WHAT I BELIEVE" BY ALBERT EINSTEIN

4. More agreement is needed among the planners.
5. The element was never considered essential to the program.

Before starting to mark this checklist, the users should note well that in some school programs, there may be essential elements that supersede those listed here; such elements deserve the same level of attention and care as those listed. Also, it should be noted that this checklist is not intended as a teacher evaluation instrument. Lastly, the proper use of this checklist requires careful reading of all the preceding text, not merely a scanning of isolated sections here and there.

### WORKING FROM INSIDE, WORKING FROM OUTSIDE

Establishing contacts between the proposed problem and his previous experience is certainly an essential part of the problem solver's performance. He can try to discover such contacts "from inside" or "from outside." He may remain within the problem, examining its elements till he finds one that is capable of attracting some usable element from outside, that is, from his previously acquired knowledge. Or he may go outside the problem, examining his previously acquired knowledge until he finds some element applicable to his problem. Working from inside, the problem solver scans his problem, its component parts, its aspects. Working from outside, he surveys his existing knowledge, and ransacks the provinces of knowledge that are most likely to be applicable to the present problem. The two parts of Fig. 11.2 attempt to give visual expression to "inside" and "outside" work.

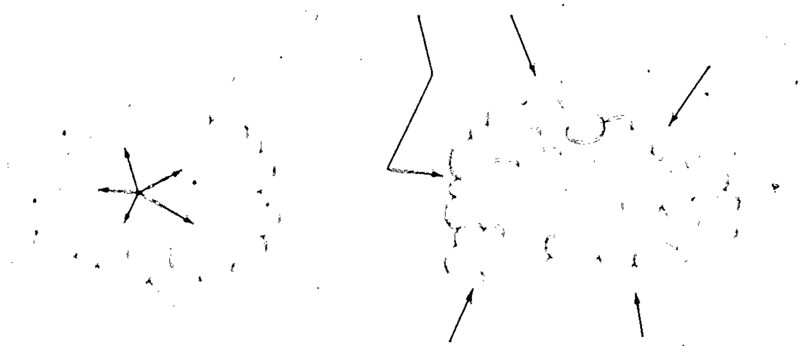


**Fig. 11.2. Working from inside, working from outside—to pierce the clouds.**

# Checklist for Assessing the Quality of a School's Mathematics Program

## The Content of the Mathematics Program
(What Students Learn)

### The Language of Mathematics (See page 6 in the text.)

*The mathematics program provides the following:*

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **1.** Students use mathematical terminology fluently. | | | | |
| a. The correct mathematical terms and symbols are used consistently in the classroom. | | | | |
| b. Students are provided a variety of experiences for learning mathematical terms and symbols. | | | | |
| **2.** Students can communicate in mathematics. | | | | |
| a. Students can use appropriate terms and symbols to communicate what they have learned in mathematics. | | | | |
| b. Students have opportunities to draw logical conclusions and prove conjectures. | | | | |
| c. Students have opportunities to give class reports on independent study projects. | | | | |

### A Comprehensive Mathematics Curriculum (See page 8 in the text.)

*The mathematics program provides the following:*

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **3.** The mathematics program covers the breadth of required mathematical skills. | | | | |
| a. By the end of the eighth grade every student has had opportunities to learn all the essential skills and concepts needed for high school mathematics. | | | | |
| b. By the end of the twelfth grade, every student has had opportunities to learn all the essential mathematical skills and concepts needed for his or her college preparation and consumer needs. | | | | |
| c. Students have ample opportunities in each area of their studies to acquire the skills needed to meet minimum requirements for graduation. | | | | |

## A Comprehensive Mathematics Curriculum—Continued

***The mathematics program provides the following:***

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **4.** The mathematics program has depth. | | | | |
| a. Students can relate newly learned skills and concepts to those learned previously. | | | | |
| b. Learning tasks are provided to build an understanding of mathematics well beyond the level of simply knowing the "facts." | | | | |
| c. Students learn to apply their knowledge of mathematics to problems in the other subject areas. | | | | |
| **5.** A high priority is given to mathematics instruction both in the school-level plan and in practice. | | | | |
| a. The mathematics program is reviewed and planned through mutual agreement on objectives and priorities by representatives from the entire school community. | | | | |
| b. The mathematics program supports objectives and priorities in other subject areas and other school plans. | | | | |
| c. Mathematics instruction at the same grade level or in the same courses provides comparable learning experiences and skill development. | | | | |
| **6.** Remedial instruction is available throughout the mathematics program. | | | | |
| a. At all levels of mathematics instruction, students have opportunities for remediation and assistance on an individual basis. | | | | |
| b. The remediation activities improve attitudes toward learning mathematics and avoid repeating previous learning difficulties. | | | | |
| **7.** Adequate courses and information are available for those students preparing to go to college. | | | | |
| a. Information about the college and career mathematics requirements for the fields students intend to enter is available to the students and parents. | | | | |
| b. College preparatory students have good opportunities for studying trigonometry and other advanced mathematics courses needed for college. | | | | |

## Skills in Computing (See page 16 in the text.)

*The mathematics program provides the following:*

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **8.** Computing skills are taught through interesting and challenging activities. | | | | |
| a. At the elementary level students learn to compute with whole numbers, common fractions, and decimals by first understanding the underlying concepts. | | | | |
| b. Students at all levels practice the computing skills they have acquired with a daily program of interesting reinforcement activities. | | | | |
| c. Students develop an understanding of the practical uses of computing skills in solving real life problems. | | | | |
| **9.** Students learn to estimate and to check their answers. | | | | |
| a. Students are encouraged in a variety of ways to compute without using pencil and paper or other visual references. | | | | |
| b. At all levels of mathematics instruction, students learn many simple and convenient strategies for checking the reasonableness of their answers. | | | | |

## Problem-Solving Skills in Mathematics (See page 18 in the text.)

*The mathematics program provides the following:*

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **10.** The skills of formulating and analyzing problems and finding and interpreting solutions are emphasized. | | | | |
| a. Students learn to formulate and state problems from situations that involve superfluous or vague information and that do not follow simply from examples given in the book or in the class. | | | | |
| b. Students learn a variety of strategies for analyzing mathematical problems and representing them in mathematical terms. | | | | |
| c. Students develop the skills for applying the mathematical principles and processes needed to find the solutions to problems. | | | | |
| d. Students have opportunities for discussing the solutions they have found in terms of the merits of different approaches and possible extensions to other problems. | | | | |

# The Methods of Teaching Mathematics

**(How Students Learn)**

## Learning Styles and Teaching Strategies (See page 21 in the text.)

***The mathematics program provides the following:***

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **11.** Teachers provide for differences in students' learning styles. | | | | |
| a. Students are given opportunities for learning in a wide variety of instructional styles, groupings, and methods. | | | | |
| b. Students are encouraged and taught to use resources outside their classrooms for extending their knowledge of mathematics. | | | | |
| **12.** Many assessment techniques are used in designing the mathematics instruction. | | | | |
| a. Valid assessment procedures are used for placing students in the mathematics courses or providing levels of instruction that are appropriate to their individual abilities and limitations. | | | | |
| b. Students are assessed regularly by means of a wide variety of assessment tools that allow for different learning styles and language abilities. | | | | |
| c. Instruction is continuously moderated by on-the-spot assessments of the students' levels of interest, depths of comprehension, and patterns of errors. | | | | |
| **13.** The progress of every student is assessed and recorded. | | | | |
| a. Careful records are kept on each student's progress toward learning the prescribed mathematical skills and concepts. | | | | |
| b. Progress records are up to date and readily available to the student and his or her parents and counselor. | | | | |
| b. Each student is kept informed of his or her progress and can explain what the records mean. | | | | |
| **14.** Manipulative materials are used to reinforce the learning of concepts. | | | | |
| a. The instructional presentations and materials are designed to lead the students carefully from the familiar to the unknown. | | | | |
| b. Manipulative and other visual instructional materials are used to help students develop a solid understanding of the concepts. | | | | |

## Learning Styles and Teaching Strategies—Continued

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **15.** Students are grouped in a variety of ways to reinforce learning. | | | | |
| a. The grouping of students for instruction is designed to provide the best learning opportunity for each student. | | | | |
| b. Students have frequent opportunities for increasing their understanding of skills and concepts through sharing viewpoints and ideas with other students. | | | | |

## The Effect of Attitudes on Achievement (See page 29 in the text.)

***The mathematics program provides for the following:***

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **16.** Teachers use high expectations to motivate students to learn. | | | | |
| a. The teacher provides a creditable model for the level of performance expected of students through his or her own behavior as a professional person. | | | | |
| b. Every student, regardless of ability level, is challenged equally with meaningful problems and stimulating questions. | | | | |
| c. Teachers demonstrate high expectations for every student. | | | | |
| **17.** Students are stimulated to learn through attainable challenges and justified praise. | | | | |
| a. Lessons are presented in such a way that every student is involved in a learning activity that relates to the lesson objective and that provides a reasonable opportunity for success. | | | | |
| b. Students understand the purpose of each learning task and are fully aware of the amount of progress they have made toward the objective. | | | | |
| c. The teacher uses a variety of resources and people to stimulate a high level of interest and motivation. | | | | |
| d. Students consistently receive positive reinforcement for commendable work, oral or written, and they are sensitively corrected when their work is not satisfactory. | | | | |
| **18.** A substantial amount of time is allocated daily for studying mathematics. | | | | |
| a. A specified time is allotted for the daily study of mathematics at the elementary school level, and mathematics classes meet the allotted number of days per year at the secondary school level, as determined by schoolwide agreement. | | | | |

## The Effect of Attitudes on Achievement—Continued

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| b. Routine classroom procedures are efficient, and unexpected interruptions are handled with minimal disruption. | | | | |
| c. All students are drawn into learning or thinking about mathematics during class discussions. | | | | |
| d. Good study habits are utilized when students are assigned classroom activities. | | | | |
| **19.** Home study is emphasized in the mathematics program. | | | | |
| a. Homework assignments in mathematics extend or reinforce the learning in class and are designed to meet the learning needs of the students on an individual basis. | | | | |
| b. Mathematics homework assignments conform to a schoolwide policy that is understood by students, parents, and all teachers. | | | | |
| c. Mathematics homework is assigned in a way that creates a high level of interest and avoids the appearance of serving as a punishment for not completing the work in class. | | | | |

## Calculators and Computers (See page 36 in the text.)

***The mathematics program provides the following:***

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **20.** Calculators and computers are used in a variety of ways to stimulate and reinforce learning. | | | | |
| a. Calculators and computers are used in the classroom to achieve mathematical learning objectives, including computing skills. | | | | |
| b. Through varied and creative learning activities, students understand the important role of calculators and computers in solving problems and in preparing for careers. | | | | |
| **21.** Students learn to estimate answers and to be precise with the operations on calculators and computers. | | | | |
| a. Students are provided well-planned instruction in the skills demanded for the effective operation of calculators and computers. | | | | |
| b. Instructional materials for calculators and computers are selected on the basis of established learning objectives. | | | | |

# Support for Implementation of a Quality Mathematics Program

## The School Climate (See page 39 in the text.)

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|

***The mathematics program provides the following:***

**22.** There is a climate of achieve[illegible] productivity at the school site.

a The mathematics program needs are reviewed regularly by a local planning committee composed of representatives from all parts of the school community.

b Many sources of information about mathematical skills needed outside the school are used in assessing the effectiveness of the mathematics program.

**23.** Parents and community members have many opportunities to become involved in the school mathematics program.

a Lines of communication are established throughout the school community for finding ways to improve the mathematics program.

b Many opportunities are provided to help parents become informed about and be involved in their child's progress in mathematics.

c Local business people and community leaders are involved in planning and implementing the school mathematics program.

**24.** The district administration provides coordination for the development and implementation of a high quality mathematics program.

a Information about district-level assistance is readily available for planning and implementing the school-level mathematics program.

b The district administration provides opportunities for articulation and coordination of the mathematics program among the schools to ensure continuity of learning experiences for students as they progress through the grades.

c District-level coordination of special programs and funding sources is available to ensure that each student's mathematics program meets his or her learning needs.

## Staff Development (See page 45 in the text.)

*The mathematics program provides the following:*

| | Ineffective | Somewhat effective | Effective | Very effective |
|---|---|---|---|---|
| **25.** The staff is given opportunities to develop new skills, practice new techniques, and discuss critical issues. | ___ | ___ | ___ | ___ |
| a. Staff development activities for mathematics are planned by a widely representative group and given a high priority in the school program. | ___ | ___ | ___ | ___ |
| b. Teachers of all subject areas are encouraged to extend their knowledge of mathematics and to improve their skills in teaching mathematics. | ___ | ___ | ___ | ___ |
| c. Teachers of mathematics at the secondary level are encouraged to improve their teaching skills for the nonmathematical learning needs of their students. | ___ | ___ | ___ | ___ |
| d. Staff development activities include curriculum planning by teachers within and across grade levels and courses. | ___ | ___ | ___ | ___ |
| e. Information about many types of staff development opportunities in mathematics is compiled from many sources; supplemented, as needed, with locally planned activities; and made available to teachers in a timely manner. | ___ | ___ | ___ | ___ |
| **26.** Staff development programs are built on assessments of learning needs. | ___ | ___ | ___ | ___ |
| a. The planning for staff development activities is based on a continual and thorough assessment of the learning needs in mathematics of students, past and present. | ___ | ___ | ___ | ___ |
| b. The staff development activities provide for participant evaluation, and the results of such evaluation are considered when later activities are planned. | ___ | ___ | ___ | ___ |
| **27.** Many resources and opportunities are used to maintain staff development goals. | ___ | ___ | ___ | ___ |
| a. The staff development program includes an evaluation of its effect on teaching skills and student learning. | ___ | ___ | ___ | ___ |
| b. Information and publication from mathematics teachers' organizations and other mathematics programs are made available to all teachers. | ___ | ___ | ___ | ___ |
| c. The district administration provides assistance in planning and implementing staff development programs. | ___ | ___ | ___ | ___ |

# A Sample Continuum of Mathematical Skills and Concepts

This sample continuum is based on the mathematics content section of the "Criteria for Evaluating Instructional Materials in Mathematics (K–8)," which was adopted by the California State Board of Education on March 13, 1980, and appears in the *Mathematics Framework and the 1980 Addendum for California Public Schools*. It includes the mathematical skills and concepts in seven major categories, as prescribed for California public schools, kindergarten through grade eight, by the State Board of Education.

The skills and concepts are not presented here as measurable learning objectives, thereby permitting a broad interpretation of the mathematical level of each skill and concept. Every mathematics program should include these skills and concepts ~~in~~ its "core" of instruction, and, if other factors are constant, the higher the level, the better the program should be.

The suggested level of treatment of these skills and concepts is indicated by the following code:

*A* Introduce the topic, develop awareness, and explore applications of the skill/concept.

*B* Gain a basic understanding or competence, with the skill/concept demonstrated, when some clues and assistance are provided.

*C* Demonstrate comprehension, unaided, through consistent performance (say, 75 percent) on tests or applications of the skill/concept.

| Skill or concept | Suggested level of treatment, by grade level | | | |
|---|---|---|---|---|
| | K-3 | 4-6 | 7-8 | 9-12 |
| **A. Arithmetic, Numbers, and Operations** | | | | |
| **1. One-to-one correspondence, number, counting, and order** | *C* | *C* | *C* | *C* |
| 2. The number line and the coordinate plane | *B* | *B* | *C* | *C* |
| **3. Positive and negative numbers** | *A* | *B* | *C* | *C* |
| 4. Decimal notation and computation with decimals (prior to formal computation with numbers in fraction form) | *A* | *B* | *C* | *C* |
| **5. Memorization and use of basic arithmetic facts of addition and multiplication** | *A* | *C* | *C* | *C* |
| 6. Addition in the development of the operation of subtraction | | *B* | *C* | *C* |
| **7. Subtraction and multiplication in developing the operation of division** | *A* | *B* | *C* | *C* |
| 8. Equality and order relations | *A* | *C* | *C* | *C* |
| **9. Properties of operations in the development of computation skills** | *A* | *B* | *B* | *C* |
| 10. Elementary number theory concepts | *A* | *B* | *C* | *C* |
| **11. Skills of computation with positive and negative numbers** | *A* | *B* | *C* | *C* |
| 12. Selection of the appropriate operations for given situations | *B* | *C* | *C* | *C* |
| **13. Mental arithmetic** | *B* | *C* | *C* | *C* |
| 14. Place value in the decimal numeration system | *B* | *C* | *C* | *C* |

| Skill or concept | Suggested level of treatment, by grade level | | | |
|---|---|---|---|---|
| | K-3 | 4-6 | 7-8 | 9-12 |
| **15. Exponential and scientific notation** | | *A* | *B* | *C* |
| 16. The real number system | | | *A* | *C* |
| **17. Ratio, proportion, and percent** | | *B* | *C* | *C* |
| 18. Rounding off numbers and estimation skills | *A* | *B* | *C* | *C* |
| **B. Geometry** | | | | |
| **19. Intuitive, informal geometry, utilizing environmental models** | *A* | *B* | *C* | *C* |
| 20. Similarity and congruence | *A* | *B* | *B* | *C* |
| **21. Parallelism, perpendicularity, and skewness** | *A* | *A* | *B* | *C* |
| 22. Classification of geometric shapes | *A* | *B* | *B* | *C* |
| **23. Use of geometric instruments** | | *A* | *B* | *C* |
| 24. Construction of three-dimensional models | | *A* | *B* | *C* |
| **25. Length, circumference, perimeter, area, volume, and angle measures of simple geometric figures** | *A* | *B* | *B* | *C* |
| 26. Indirect measurement and the Pythagorean formula | | *A* | *B* | *C* |
| **27. Elementary coordinate geometry** | | *A* | *B* | *C* |
| **C. Measurement** | | | | |
| **28. Measuring familiar objectives through "hands-on" experience** | *A* | *B* | *C* | *C* |
| 29. Arbitrary units for measuring (preceding instruction in standard units) | *B* | | | |
| **30. Standard units as a uniform way of reporting measurements** | *A* | *B* | *C* | *C* |
| 31. Understanding the structure of and using the metric system of units (SI) | *A* | *B* | *C* | *C* |
| **32. Convenient references for metric units without computational conversions between the U.S. customary units and SI units** | *A* | *B* | *C* | *C* |
| 33. Practice with the numerical values of the metric prefixes | | *A* | *B* | *C* |
| **34. Estimating distance, area, volume, mass, and temperature in metric units** | *A* | *B* | *B* | *C* |
| 35. Reading simple measuring instruments and the approximate nature of measurement | *A* | *B* | *B* | *C* |
| **36. Scale drawings and maps** | *A* | *B* | *C* | *C* |
| 37. Formulas for determining perimeter, area, and volume | | *B* | *C* | *C* |
| **D. Calculators and Computers** | | | | |
| **38. Estimation** | *B* | *C* | *C* | *C* |
| 39. Calculating +, −, ×, ÷ | *B* | *C* | *C* | *C* |
| **40. Immediate feedback** | *B* | *C* | *C* | *C* |
| 41. Reinforcement of number relationships | *B* | *C* | *C* | *C* |
| **42. Exploring number patterns** | *A* | *B* | *C* | *C* |
| 43. Motivation and enrichment | *A* | *B* | *C* | *C* |
| **44. Extended application problems to develop concepts that are normally obscured by tedious computations** | *A* | *B* | *C* | *C* |
| 45. Providing integrated curriculum opportunities | | *A* | *B* | *C* |
| **46. Working knowledge of the functions, logic, and mechanics** | | *A* | *B* | *C* |
| 47. Historical perspective | | | *A* | *B* |
| **48. Order properties** | | *B* | *C* | *C* |
| 49. Programming and flow charting | | *A* | *B* | *C* |

| Skill or concept | Suggested level of treatment, by grade level K-3 | 4-6 | 7-8 | 9-12 |
|---|---|---|---|---|
| **50. Examples of use in society** | **A** | **B** | **B** | **C** |
| E. Probability and Statistics | | | | |
| **51. Collecting, organizing, and representing data derived from real-life situations** | **A** | **B** | **B** | **C** |
| 52. Tree diagramming and counting procedures for sample spaces | A | B | C | C |
| **53. Permutations (arrangements) and combinations (selections)** | | **A** | **B** | **C** |
| 54. Making guesses about patterns or trends in data | A | B | C | C |
| **55. Statistical inferences** | **A** | **B** | **B** | **C** |
| 56. Various measures of central tendency and dispersion | | | B | C |
| **57. Elementary concepts of probability** | | **B** | **B** | **C** |
| 58. Reliability of statistical inference | | | B | C |
| **F. Relations and Functions** | | | | |
| 59. Constructing and interpreting tables, graphs, and schedules | A | B | B | C |
| **60. Mappings, correspondences, ordered pairs, and "rules" leading to the concept of a mathematical relation** | **A** | **B** | **B** | **C** |
| 61. The function concept and function notation | | A | B | C |
| **62. A function in mathematical applications** | | **A** | **B** | **C** |
| 63. Patterns and relationships and forming generalizations | A | B | B | C |
| **G. Logical Thinking** | | | | |
| 64. Manipulatives, games, problems, and puzzles which stimulate and develop elementary reasoning patterns | A | B | B | C |
| **65. Trial and error strategies** | **A** | **B** | **C** | **C** |
| 66. Applying reasoning patterns to nonmathematical situations, such as advertising | | A | B | C |
| **67. Direct and indirect reasoning patterns** | | **A** | **B** | **C** |
| 68. Inductive and deductive reasoning patterns | | A | B | C |
| **69. Sentences using: *and, or, not, if . . . then, all,* and *some*** | **A** | **B** | **B** | **C** |

# Competencies in Mathematics Expected of Students Planning to Enroll in a College or University

*EDITOR'S NOTE: Because of the widespread concern that students enrolling in the colleges and universities in California are underprepared in English and mathematics, the academic senates of the California Community Colleges, the California State University, and the University of California took an official position on the matter early in 1982. In their statement the three senates outlined the problem and made six recommendations, which are cited on page 15 of this handbook. Then they outlined the specific competencies that "represent the core of the necessary skills in English and mathematics needed by entering college freshmen." Their recommendations for mathematics follow.*

*The academic senates outlined the specific competencies that "represent the core of the necessary skills in English and mathematics needed by entering college freshmen."*

## MATHEMATICS

As both the Mathematical Association of America and the National Council of Teachers of Mathematics have noted, "Mathematics is a highly structured subject in which various concepts and techniques are highly dependent upon each other." Reflecting this, the mathematics curriculum in high school and college consists of a sequence of courses, each with specific topics to be learned, to enable students to build upon their skills and understanding of mathematical operations. Students who have not acquired adequate skills and understanding at one course level will find it exceedingly difficult to comprehend the course content in the next.

The amount of recommended high school preparation in mathematics for college-bound students depends on the major field of study to be pursued in college, regardless of any specific college or university admission requirement in mathematics. Students who plan to major in a physical or life science, engineering, pre-medicine, other science-related fields, business, or economics should prepare for the college-level calculus requirement in these majors by taking four years of mathematics in high school and additional mathematics courses if available. Many majors in the social sciences or other professional and pre-professional fields require baccalaureate-level statistics or calculus, sometimes both. Three years of high school mathematics are frequently required as preparation for statistics. To enable students to have full access to college and university programs and career opportunities, it is recommended that all college-bound students master the skills and techniques of high school mathematics through intermediate algebra. This means that all prospective college students should take as a minimum full-year courses in elementary algebra, geometry, and intermediate algebra. Students who must take intermediate algebra in college cannot expect to receive credit toward graduation for this course; it is considered remedial at the baccalaureate level.

Students entering the California State University and many students entering certain programs in the University of California are required to take a diagnostic test in mathematics. Those students who do not demonstrate an acceptable level of proficiency will be required to take remedial course work to overcome their deficiencies.

A mathematics course in the senior year of high school is, therefore, recommended to prepare students for these diagnostic tests.

The following recommendations for college-preparatory programs in mathematics have been developed in cooperation with the UC CSU Workgroup on Diagnostic Testing in Pre-Calculus Mathematics, which includes teachers of mathematics from California high schools and community colleges:

1. Students should begin the study of algebra only after they have mastered arithmetic and the general mathematical skills outlined below in Section I.
2. All college-preparatory students should complete Algebra I, Geometry, and Algebra II. Content recommendations for these courses are contained in Section II.
3. Students who intend to pursue a baccalaureate degree in fields requiring the study of calculus should complete the courses in trigonometry and (in) analytic geometry and mathematical analysis, as outlined in Section III.
4. All college-preparatory students should take a mathematics course during their senior year of high school. This course could be Algebra II or a more advanced course, depending on the student's background. Suggestions for advanced courses are contained in Section III.
5. All mathematics courses should emphasize problem solving. Students should be graded on their ability to solve problems correctly and to display problem-solving processes in a clear, complete, and accurate manner.
6. Computers and hand calculators should be used in imaginative ways to reinforce learning and to motivate students as proficiency in mathematics is gained. Students should develop adequate arithmetic skills in order to avoid reliance upon calculators for simple numerical computations. Calculators should be used to supplement rather than to supplant the study of necessary computational techniques.
7. Each college-preparatory mathematics course should include a comprehensive final examination.
8. All college-bound students should receive diagnostic assessment at the end of their junior year. Examinations used for this purpose for example, those developed by the UC CSU Workgroup on Diagnostic Testing in Pre-Calculus Mathematics or similar instruments should measure achievement levels necessary for success in college mathematics and should provide guidance as to the selection of proper mathematics courses in the senior year.
9. Calculus, when taken by high school students either at their high school or at a nearby college, should be taken only by those students who are strongly prepared in algebra, geometry, trigonometry, and coordinate geometry, and who can demonstrate the mastery of these subjects. The calculus course should be a full-year course and, if offered in high school, should prepare the enrolled students to take one of the College Board's Advanced Placement Calculus Examinations.
10. Understanding the application of mathematics to areas such as the physical, biological and social sciences, and business should be encouraged.
11. To the extent that familiarization with the computer is part of the high school mathematics program, such orientation should emphasize mathematical applications and should not displace essential mathematics topics or courses.

The following constitutes a more detailed specification of the topics to be covered and the skills to be developed in pre-high school and high school mathematics courses.

*All mathematics courses should emphasize problem solving. Students should be graded on their ability to solve problems correctly and to display problem-solving processes in a clear, complete, and accurate manner.*

## Section I. Arithmetic Skills to Be Introduced and Developed Before the Study of Algebra Is Begun

The following basic arithmetical skills should be introduced and developed without the use of a calculator. These skills can then be extended and new mathematics topics learned by effective use of a calculator. The following list highlights major areas. This presentation is not as detailed as that employed for the high school program in Sections II and III.

- Computation with whole numbers, fractions, and decimals
- Understanding the meaning of fractions, decimals, and percent and their relationship to one another
- Translation of situations and verbal problems into mathematical statements
- Facility in rounding, approximation, and numerical estimation; appreciation of reasonableness of numerical answers
- Understanding and use of basic arithmetic properties
- Use and interpretation of graphs and tables
- Computation with positive integral exponents and square roots of perfect squares
- Computation of perimeters, areas, and volumes of simple geometric figures

## Section II. Topics to Be Included in Algebra I, Geometry, and Algebra II

All college-preparatory students should complete courses (here called Algebra I, Geometry, Algebra II) which cover together all the topics listed below. The division of topics among these courses is not meant to be rigid. Certain topics may be introduced earlier or later. Topics introduced in one course should be reinforced in later courses. Applications and problem solving should be emphasized throughout.

*Students should begin the study of algebra only after they have mastered arithmetic and general mathematical skills.*

- *Algebra I*
  - Arithmetic operations and absolute values of positive and negative rational numbers
  - Arithmetic operations with literal symbols
  - Linear equations and their graphs
  - Inequalities
  - Ratio, proportion, and variation
  - Operations with integer exponents
  - Operations with polynomials and rational expressions
  - Systems of linear equations with two unknowns; solutions and applications
  - Special products and factoring
  - Solution of quadratic equations by factoring and formula
  - Solution of elementary word problems
  - Application of formulas for perimeters, areas, and volumes of simple geometric figures
- *Geometry*
  - Extensive reinforcement of the algebraic skills developed in Algebra I
  - Basic postulates of Euclidean geometry, proofs of geometric theorems
  - Angles, parallel lines, congruent and similar triangles, rectilinear figures, circles and arcs, Pythagorean theorem
  - Application of formulas for perimeters, areas, volumes, and surface areas of geometric figures
  - Geometric constructions; loci
  - Coordinate geometry, proofs of geometric theorems by coordinate geometry methods

- Right triangle trigonometry
- Solution of elementary word problems
- Intuitive spatial geometry

- ***Algebra II***
  - Simplification of algebraic expressions
  - Fractional exponents and radicals
  - Absolute value and inequalities
  - Operations on polynomials
  - Quadratic equations, completion of the square, quadratic formula, properties of roots
  - Complex numbers
  - Quadratic inequalities
  - Graphing linear and quadratic functions and inequalities; determination and interpretation of slopes
  - Solutions of equations with rational expressions
  - Systems of linear equations with two and three unknowns: homogeneous, dependent, and inconsistent systems
  - Polynomial equations
  - Binomial theorem
  - Arithmetic and geometric sequences and series
  - Exponential and logarithmic functions and equations
  - The function concept, including compositions and inverse functions, arithmetic operations on functions
  - Solution of word problems, including estimation and approximation

*All college-preparatory students should take a mathematics course during their senior year of high school. This course could be Algebra II or a more advanced course, depending on the student's background.*

## Section III. Advanced Courses in Mathematics

All college-bound students should take a mathematics course during the senior year of high school. The mathematics studied during the senior year should reflect the student's college plans as well as mathematical ability and attainment.

Students who plan to take calculus in college should complete Algebra II prior to the twelfth grade. These students also should take a semester course in trigonometry followed by a semester course in analytic geometry and mathematical analysis. These courses should include the topics listed below.

Students entering fields requiring probability and statistics may elect such a course as an alternative to the course in analytic geometry and mathematical analysis. Computer science is a suitable elective for those planning to enter fields requiring extensive familiarity with computing. For strongly prepared students who have completed analytic geometry and mathematical analysis, a year course in calculus leading to one of the advanced placement examinations of the College Board is recommended. Other electives include linear algebra and integrated courses in science and mathematics. Topic descriptions for elective courses are not included.

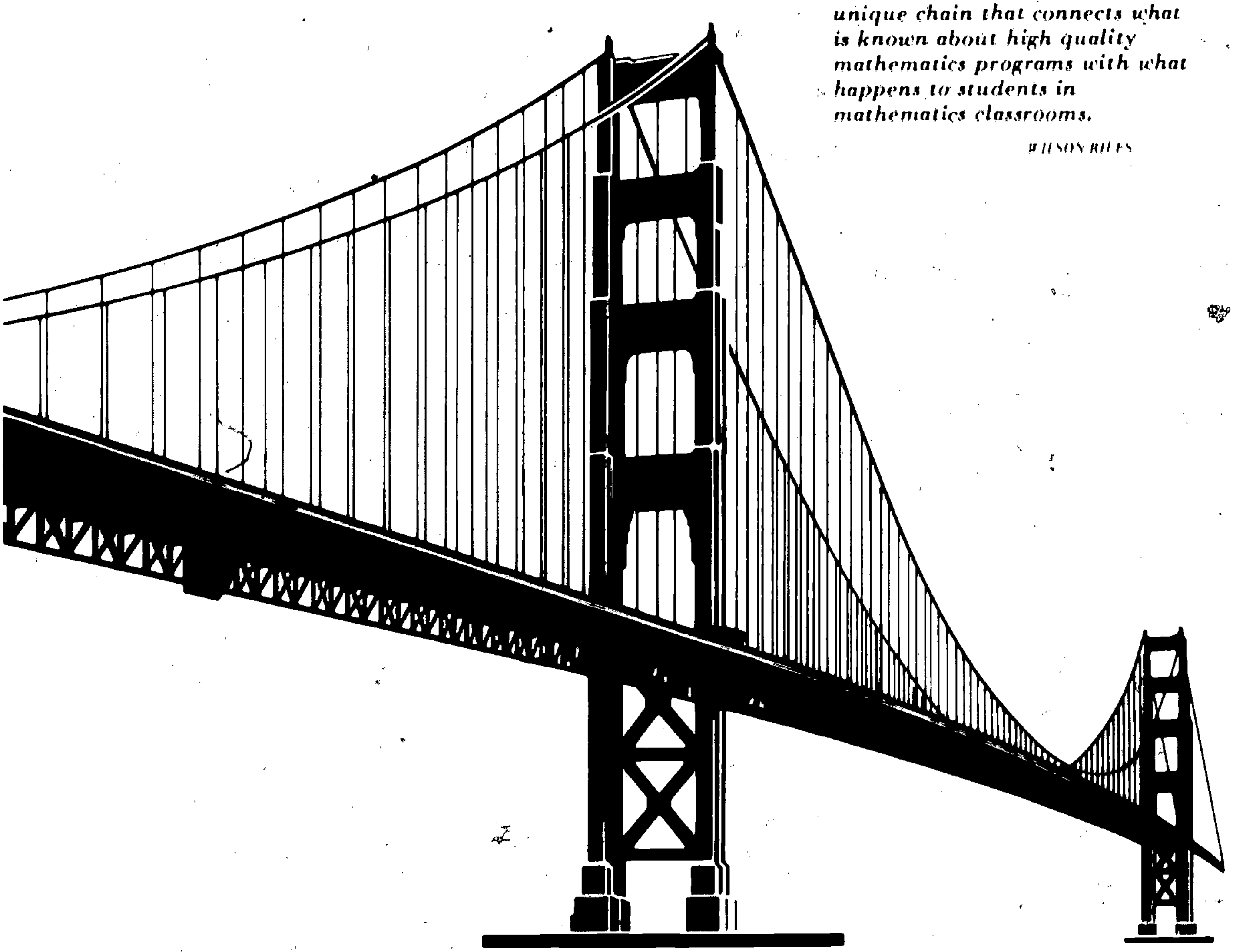
- ***Trigonometry (one semester)***
  - Trigonometric functions as ratios of lengths of sides of triangles and as circular functions
  - Graphical characteristics of trigonometric functions
  - Solution of right triangles
  - Radian and degree measure
  - Trigonometric identities, including double angle, half angle, and addition formulas
  - Laws of sines and cosines, solution of oblique triangles
  - Reinforcement of function concept: exponential, logarithmic, and trigonometric functions
  - Inverse trigonometric functions and their graphs
  - Solution of trigonometric equations

- Polar coordinates and vectors
- Trigonometric form of complex numbers and de Moivre's theorem

- ***Analytic geometry and mathematical analysis (one semester)***
  - Coordinate geometry, including detailed treatment of conic sections
  - Rational functions and their graphs
  - Elementary functions and their inverses, including graphs of these functions
  - Review of polar coordinates and vectors
  - Graphing in polar coordinates
  - Introduction to linear algebra
  - Mathematical induction
  - Parametric equations and their graphs
  - Lines and planes in space; three-dimensional coordinate geometry
  - Introduction to vectors in space

*You who use the* **Handbook for Planning an Effective Mathematics Program** *are the critical and final links in a unique chain that connects what is known about high quality mathematics programs with what happens to students in mathematics classrooms.*

WILSON RILES







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## Other Publications Available from the Department of Education

The *Handbook for Planning an Effective Mathematics Program* is one of approximately 500 publications that are available from the California State Department of Education. Some of the more recent and more widely used publications and those pertaining to mathematics are the following:

| | |
|---|---|
| Accounting Procedures for Student Organizations (1979) | $1.50 |
| Bilingual Program, Policy, and Assessment Issues (1980) | 3.25 |
| California Private School Directory | 9.00 |
| California Public School Directory | 12.50 |
| California School Accounting Manual (1981) | 2.50 |
| California's Demonstration Programs in Reading and Mathematics (1980) | 2.00 |
| Catalog of Instructional Materials in Mathematics (1981) | 2.50 |
| Directory of Exemplary Mathematics Programs in California (1973) | .85 |
| Discussion Guide for the California School Improvement Program (1978) | 1.50*† |
| District Master Plan for School Improvement (1979) | 1.50* |
| Education of Gifted and Talented Pupils (1979) | 2.50 |
| Establishing School Site Councils: The California School Improvement Program (1977) | 1.50*† |
| Foreign Language Framework for California Public Schools (1980) | 2.50 |
| Guide to School and Community Action (1981) | 1.75 |
| Guidelines for School-Based Alcohol and Drug Abuse Programs (1981) | 1.00 |
| Handbook for Planning an Effective Reading Program (1979) | 1.50* |
| Handbook for Planning an Effective Writing Program (1982) | 2.00* |
| History–Social Science Framework for California Public Schools (1981) | 2.25 |
| Improving the Human Environment of Schools (1979) | 2.50 |
| In-service Guide for Teaching Measurement: The SI Metric System (1975) | 1.25 |
| Interim Guidelines for Evaluation of Instructional Materials with Respect to Social Content (1981) | 1.50 |
| Manual of First Aid Practices for School Bus Drivers (1980) | 1.25 |
| Mathematics Framework for California Public Schools, with 1980 Addendum (1982) | 2.00 |
| Mathematics Scope and Sequence Charts (set of four) (1975) | 1.25 |
| Monograph on Staff Development (1980) | 1.50 |
| New Era in Special Education: California's Master Plan in Action (1980) | 2.00 |
| Pedestrian Rules of the Road in California: Primary Edition (1980) | 1.50 |
| Physical Performance Test for California, Revised Edition (1982) | 1.50 |
| Plan for Improving Elementary Mathematics Programs (1976) | 1.25 |
| Planning for Multicultural Education as a Part of School Improvement (1979) | 1.25* |
| Planning Handbook (1978) | 1.50* |
| Proficiency Assessment in California: A Status Report (1980) | 2.00 |
| Proficiency Skill Development Kit (1980) | 7.50 |
| Putting It Together with Parents (1979) | .85† |
| Reading Framework for California Public Schools (1980) | 1.75 |
| Relationship Between Nutrition and Student Achievement, Behavior, and Health (1980) | 4.00 |
| Science Education for the 1980s (1982) | 2.00* |
| Science Framework for California Public Schools (1978) | 1.65 |
| School Improvement: Making California Education Better (brochure) (1982) | NC* |
| Student Achievement in California Schools | 1.75 |
| Students' Rights and Responsibilities Handbook (1980) | 1.50† |
| Toward More Human Schools (1981) | 1.75 |
| Visual and Performing Arts Framework for California Public Schools (1982) | 3.25 |

Orders should be directed to:

California State Department of Education
P.O. Box 271
Sacramento, CA 95802

Remittance or purchase order must accompany order. Purchase orders without checks are accepted only from government agencies in California. Sales tax should be added to all orders from California purchasers.

A complete list of publications available from the Department may be obtained by writing to the address listed above.

*Developed for implementation of School Improvement
†Also available in Spanish at the price indicated

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