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ABSTRACT

The document opens with an editorial comment discussing the role of mathematics in teaching and learning. Then, 13 reports related to mathematics education are abstracted and analyzed. Three of the reports deal with problem solving, and another three look at aspects of cognitive development. There are two each on mathematics instruction, mathematics achievement, and learning theory. The remaining report deals with spatial perception. Research related to mathematics education as reported in CIJE and RIE between April and June 1982 is also noted. (MP)

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An editorial comment . . .

Mathematics Education Research: The Role of Language in Mathematics Teaching and Learning

Dora Helen Skypek
Emory University

Consider these two statements:

"36 divided into 4 groups is $4\sqrt{36}$."

For the contradiction to be obvious, one has to read aloud, "36 divided into 4 . . . is 4 divided into 36."

"$17 \div 5 = 3r2$."

The statement suggests that "3r2" names a number (that is, "17 + 5," "17 - 5," and "17 x 5" all name numbers -- so why not "17 ÷ 5?"). It also violates the definition of division ($17 \neq 5 \times 3r2$) and confounds a meaningful understanding of the relation "is equal to."

It is not surprising that many students are confused about the meaning of the operation of division and the associated algorithms. The resolution of difficulties in teaching and learning about division is, of course, far more complex than just cleaning up the language, but I do think these and other miscommunications are part of the problem.

To read the oral language responses and comments elicited from students in task-based interviews (Erlwanger, 1974; Davis and McKnight, 1980; Clement, 1982; Skypek, 1974) is to become convinced that, whatever else is going on, communication in the teaching of mathematics is inadequate, incomplete, and sometimes incorrect. The investigators who report remediation moves during the interview -- oral probes, discussions of similar (even simpler) exercises, and other verbal hints about reasonable or appropriate responses -- find, in general, that even with able students "the effect of semantic knowledge on algorithmic behavior" (Davis and McKnight, 1980, p. 75) is little or none. The question for many investigators and teachers then becomes, "what extended remediation experiences can best ensure effective relearning?"

It is, however, a larger and more basic question that I want to

address: What is the role of language in teaching and learning mathematics? In particular, what are the early and continuing language experiences in the mathematics classroom that ensure meaningful learning and that minimize the misconceptions and deficiencies evident in the reported interviews and in the introductory language samples?

Conroy and Healey (1982) speak of a "language-mathematics interface" and the need for "some conceptual framework in which to view the relation between language and mathematics, the development of language skills for learning mathematics and the changing function of language in the development and growth of mathematical understanding." Bauersfeld (1979) addresses the role of language in the context of human interaction through which teaching and learning mathematics is realized. He writes that "mathematical meaning is a construction via social negotiation about what is meant" and asks the question, "How can we expect to find adequate information about teaching and learning when we neglect the interactive constitution of meaning?" (p. 25).

My own examination (Skypek, 1981) of theories and practices in teaching language prompted the following conjectures about the role of oral language in teaching and learning mathematics:

Teachers of mathematics, like teachers of language, have mistakenly assumed that skills in reading and writing coded information are the basic skills. They are, however, merely derived skills. The basic skills are at the levels of thought and oral language, where meaning is involved. It is in making messages for other people from our experiences with things and people and in making sense of messages received from other people that we hone our thinking and our language. The subsequent task of matching oral language with written language (or numerical coding schemes) is easy and requires little intellectual work.

A variety of other writers - practitioners, educational theorists, mathematics educators, and researchers -- have addressed the relatedness of talking, listening, reading, or writing to student performance in mathematics. At least one current textbook on methods in teaching elementary school mathematics spells it out this way:

> Since (1) children cannot reasonably be expected to write what they cannot read, (2) children can-

> not to be expected to read what they cannot say, and (3) children cannot be expected to say what they have not heard, the natural development of vocabularies proceeds from listening to speaking to reading to writing. The initial work and study of quantitative expressions must be in the form of oral communication about real problem situations. Only after children can orally describe the quantitative situation should they encounter or be expected to read even the simplest of number sentences. (Lerch, 1981, p. 14).

But it is not enough to agree that students and teachers should use oral language in the mathematics classroom. Neither is it enough to assume that teachers will know how to motivate and orchestrate the development of oral language about mathematics. Teacher educators in language arts/reading/English are not likely to investigate the issues raised here. Instead, the problems must be addressed by mathematics educators. We need to develop a theoretical framework that recognizes and delineates the role of language in teaching and learning mathematics. We could begin that task with some specific protocols for classroom testing and analyses. Creative longitudinal investigations that incorporate the recommendations of learning theorists, mathematics educators, and classroom teachers would be even better.

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Fall 1982

Battistã, Michael. THE INTERACTION BETWEEN TWO INSTRUCTIONAL TREATMENTS OF ALGEBRAIC STRUCTURES AND SPATIAL-VISUALIZATION ABILITY. Journal of Educational Research 74: 337-341; May/June 1981.

Abstract and comments prepared for I.M.E. by THEODORE A. EISENBERG, Ben Gurion University of the Negev, Beer Sheva, Israel.

1. Purpose

(a) To determine if students who were given a verbal-spatial presentation of the fundamental properties of algebraic structures would grasp the underlying ideas of the structures better than those who only received instruction in a verbal format.

(b) To determine the strength of the correlation between a spatial visualization test and the performance of the two groups taught algebraic structures vis-a-vis the two approaches.

(c) To use (a) and (b) above to affirm an aptitude (for spatial visualization) by treatment (method of presentation) interaction.

2. Rationale

Only very general statements can be made about aptitude-treatment-interaction studies in the area of spatial visualization and mathematics achievement.

> Students with high spatial-visualization ability will learn more than students with low spatial visualization ability in mathematics instruction in which spatial or visual presentations are common. Furthermore, this effect will be in evidence to a much lesser degree in mathematics instruction in which spatial or visual presentations are not used. (p. 338)

The purpose of this study was to test this hypothesis.

3. Research Design and Procedures

Two sections of undergraduate elementary education majors ($n_1 = 15$, $n_2 = 21$) took a course entitled "Number Systems for Elementary Teachers." Both sections were taught by the experimenter.

The topics of binary operations, groups, group properties, and distributive properties were presented during five classroom hours of instruction. Homework assignments were given, collected, and graded after each

classroom period.

The two groups differed as much as possible with respect to spatial illustrations and examples of the concepts being studied. For example, students in the verbal-spatial section saw an example of a group which is generated by the flips and turns of an equilateral triangle. Students in the verbal section also studied this group, but it was generated by looking at all permutations of three letters. Both sections used approximately the same amount of mathematical symbolism.

With the exception of the experimental period, both sections were taught in the same manner at all other times. Each section was given five course exams. These were identical with the exception of the one concerned with algebraic structures. Here, two forms were tailor-made to reflect the type of instruction given. For example, when the verbal section was asked questions on the permutation group, similar questions were asked on the isomorphic group generated by the flips and turns of the equilateral triangle.

Three scores were given to each student: a score on the algebraic structure test, a total score for the three other exams given in the course, and a score on the Purdue Spatial Visualization Test: Rotations.

The data were analyzed by constructing regression equations for the two sections. The spatial visualization score was used to predict the algebraic structure test score. It was expected that the slope of the regression line for the verbal-spatial treatment section would be steeper than that for the verbal treatment section.

4. Findings

The slope of the regression line for the verbal group was higher than that of the verbal-spatial group (.52 vs. .07). In other words, this means that the correlation between the spatial visualization test and the algebraic structures test was much higher for the verbal group than it was for the verbal-spatial group (.31 vs. .03). This is exactly the opposite of what was expected. On the total score for the three other exams, the verbal-spatial group scored higher ($\overline{X}$ = 304, sd = 50) than the verbal group ($\overline{X}$ = 269, sd = 74).

5. Interpretations

The author discusses a number of possibilities as to why his findings are opposite to what he expected. He also discusses why his findings are in conflict with other research studies in this area, including one study he himself conducted.

Abstractor's Comments

This study is plagued by many flaws. For example:

1) Why would anyone think that an instructional treatment of five classroom hours can significantly affect one's performance on subjects as difficult as binary operations, groups, group properties, and distributive properties? These topics are difficult no matter how they are presented, and to think that one can expect significant differences in performance between the two groups, which differed for only five hours of instructional time, borders on the incredulous. (In fairness, the author also cites this as his primary reason for obtaining his non-significant and contradictory results).

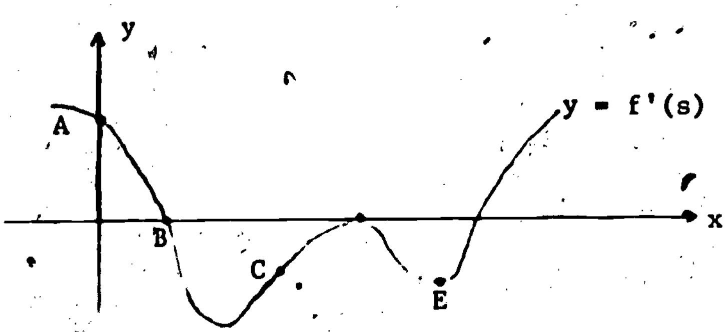
2) There are some standard guidelines for reporting educational research. One of these is to mention the reliability and validity coefficients of the instruments used. The author failed to do this. The .03 correlation between the spatial visualization exam and the structures test is so suspect that it hardly seems worth commenting upon. Validity and reliability coefficients of all measures should have been been stated from the outset. A Buros reference would also have been appropriate for the Purdue Spatial Test -- if one exists. Also, what is the rationale for using this test? The example question taken from this test seemed unrelated to the objective of the experiment.

3) The author, with a pre-stated bias toward the verbal-spatial group, taught both experimental sections. This is unacceptable, at least by my standards.

4) The statistics, as presented in the article, are a hodge-podge of jargon, poorly defined and poorly presented. Subscripts are used without explanation. Indeed, there is no rationale to use a linear regression model of one variable on another. The author should have used more meaningful and powerful statistics, for example, multiple regression

or ANOVA, assuming, of course, that the treatment really can be considered a treatment.

5) It is unclear as to whether or not the author examined the hypothesis he intended to test. It is true that some group structures can be thought of as being generated by rotations of equilateral triangles, squares, etc. But this approach is generally used only to obtain the table of binary operations for the group; once obtained, the underlying rotations are seldom referred to again. In other words, the visualization of the triangle being flipped, turned, etc., is contrived and is not inherent to the group itself. Better illustrations could have been used; e.g., asking the students to determine if the points A, B, C, D, and E on the graph y = f'(x) correspond to max, min or points of inflector on the graph y = f(x).



Here the student must really visualize and understand the meaning of f'(x).

6) The underlying idea of this study is worthwhile -- the study itself, unfortunately, was not.

Bright, George W.; Harvey, John G.; and Wheeler, Margariete Montague. VARYING MANIPULATIVE GAME CONSTRAINTS. Journal of Educational Research 74: 347-351; May/June 1981.

Abstract and comments prepared for I.M.E. by JERRY P. BECKER, Southern Illinois University, Carbondale.

1. Purpose

The study sought to investigate the effectiveness of the game Order Out and differences in achievement between three treatment groups when a) sets of fraction bars, b) pictorial representations of fraction bars, or c) neither physical nor pictorial aids were available to subjects while playing the game.

2. Rationale

In setting the context for the study, the investigators make reference to their earlier research in which four sets of game-related variables that may play a role in achievement of the mathematical content of instructional games were identified (Bright, Harvey, and Wheeler, 1977). One such set of variables includes the variable "required or available game resources". This is the variable the investigators varied in the present study.

3. Research Design and Procedures

Order Out was the game used in the investigation (cf., Developing Mathematical Processes). The objective of the game is for subjects to order common fractions. Subjects were all (N = 85) fifth-grade students in an elementary school and all (N = 177) seventh-grade students in a middle school (four intact fifth-grade classes and eight intact seventh-grade classes). During the year preceding the study each class had a teacher-taught unit on common fractions in a normal classroom situation.

Subjects played Order Out twice a week for five weeks, with each session 20 minutes in duration. Preceding this, teachers were acquainted with the game as well as with experimental design and procedures. Further, teachers were briefed on their role in the study (i.e., not do any teaching on common fractions while the study was underway).

Twenty-minute pre-and posttests were used in the study. In each, subjects were to order forty pairs of common fractions. The items were partitioned equally into four groups: a) both fractions less than or equal to ½ or greater than or equal to ½, and one denominator a multiple of the other; b) both fractions less than or equal to ½ or greater than or equal to ½, and neither denominator a multiple of the other; c) fractions on opposite sides of ½ and one denominator a multiple of the other; and d) fractions on opposite sides of ½ and neither denominator a multiple of the other (p. 348). No pairs of fractions were the same and denominators in each case were unequal and randomly chosen from the numbers 2-12, inclusive. All fractions were proper, with numerators randomly selected.

Four forms of both the pretest and posttest were developed, since fraction bars and fraction bar pages were used with some but not all subjects. This was done as follows:

> First the 40 items were divided into two subsets; Subset 1 and Subset 2, by randomly partitioning each cell into two equal subcells. Then a fraction bars page was printed across from the page containing one of the subsets. Finally, the order of the subsets, either accompanied or not accompanied by a fraction bars page, was randomized. (p. 348)

In this way one test form consisted of Subset 1 with fraction bars page followed by Subset 2. Similarly, another form consisted of Subset 2 with fraction bars page preceded by Subset 1. In carrying out the analysis, the investigators used ANOVA to compare scores on the four pretest/posttest forms in order to determine whether the fraction bars page affected scores and whether the appearance of the fraction bars page before or after appearance of items without the fraction bars page affected scores.

Treatments covered thirteen days, with each subject receiving the pretest on the first day. On the second day, then, the game was described (appropriately) to subjects in each treatment. Game play then followed for five weeks, with game playing groups randomly formed each week within treatment group within each class. Games were usually played by three subjects. Following this, in the seventh week, posttests were given.

4. Findings

At the fifth grade level, pre- and posttest reliabilities on all forms ranged from .85 to .92; at the seventh grade level, they ranged from .76 to .82. No significant differences existed among pretest means or among posttest means; accordingly, pretest and posttest data were each pooled within grade level.

For both grade levels, posttest means were significantly greater than pretest means for all three treatment groups. ANCOVA (using pretests score as covariate) was used to determine whether any differential effects existed (due to treatment, sex, or treatment x sex) on posttest scores. For both grade levels, no significant differences were found due to treatment, sex, or interaction. Data were then examined for possible trends. For grade five subjects, mean gain from pretest to to posttest was consistently greater for girls than for boys. For grade seven subjects, the girls' mean gain score was greater than boys' on the fraction bars treatment; however, for the other two treatments mean gains were nearly the same for boys and girls. Finally, the investigators presented evidence that the increase in percentage of girls reaching mastery (90%) from pretest to posttest was always greater than that for boys.

5. Interpretations

The researchers concluded that each treatment effectively improved achievement of ordering fractions for students, but that the treatments were not differentially effective. Thus, Order Out can be effectively used to improve students' ability to order pairs of fractions. This, the researchers' report, is consistent with findings in their earlier studies. They further are careful to point out that whether or not Order Out is more effective than some other teaching technique is open to question - they did not investigate that question in the present study.

The investigators provide and briefly discuss four explanations for the results (p. 350): (1) students may have had opportunities to practice ordering of fractions while not playing the game; (2) the Hawthorne effect may have been present; (3) administration of pretests and posttests

may have resulted in improvement; (4) student maturation may have played a role in the results. The investigators make no conclusions based on the post-hoc data analysis, but make reference to the trends (sex-related) and relate their thinking to a question raised by Fennema (in other research) to suggest the hypothesis that girls will achieve at least as well as boys when manipulatives are used. Further, they relate their observations to the thinking of Kagan and hypothesize that girls will achieve at least as well as boys when the involvement is the same for both boys and girls. Finally, the researchers identify a number of limitations that must be considered when interpreting results of the study and finish their report by suggesting that (1) further studies similar to this one may not be warranted unless an observation scheme can be implemented to provide information about frequency and quality of the use of manipulatives; (2) the sex-related hypotheses regarding use of manipulatives by girls and equal involvement by boys and girls need further study (p. 351).

Abstractor's Comments

I believe the investigation is an important one for mathematics educators since use of manipulatives has had a pervasive influence on teacher training and classroom teaching. To explore the effectiveness of Order Out, as the investigators have done in such a careful fashion, and to report it in such a concise and objective manner, provides an excellent contribution to the research literature. My feeling is that the problem investigated is an important one and the design, methodology, and analysis of data were impeccably planned and reported. While writing, concisely, the researchers carefully point out what can and what cannot be concluded from the results. There is scarcely anything about the study which is subject to criticism in the view of the reviewer. Further, the researchers set forth a direction for further research that could be very useful, namely, the sex-related hypotheses.

This study investigated the role of Order Out in achievement. Since another important dimension of mathematics learning is attitudes towards mathematics, I wonder whether this variable might be incorporated in future research studies. Perhaps manipulatives may play an

important role in bringing about positive attitudes towards mathematics.

Reference

Bright, George W.; Harvey, John G.; and Wheeler, Margariete Montague. Cognitive Effects of Games on Mathematics Learning. Madison: University of Wisconsin, 1977. ERIC: ED 166 007.

Carpenter, Thomas P.; Hiebert, James; and Moser, James M. THE EFFECT OF PROBLEM STRUCTURE ON FIRST-GRADERS' INITIAL SOLUTION PROCESSES FOR SIMPLE ADDITION AND SUBTRACTION PROBLEMS. Technical Report No. 516. Madison: Research and Development Center for Individualized Schooling. October 1979. ERIC: ED 180 840.

Abstract and comments prepared for I.M.E. by EDWARD C. RATHMELL, University of Northern Iowa.

1. Purpose

The major purpose was to identify and characterize the processes and strategies that young children use to solve different types of addition and subtraction problems presented verbally and with concrete materials. Another purpose was to examine the errors resulting from the application of inappropriate or incorrect strategies.

2. Rationale

Current mathematics curricula are designed to facilitate the development of addition and subtraction by first introducing these operations using joining for addition and take away for subtraction. Then there is an emphasis on mastery of the basic addition and subtraction skills before children are expected to use these skills to solve other problem types. Consequently, formal instruction focuses on a single problem type for each of these operations for all or nearly all of the early work.

However, it has been clearly shown that children use various strategies to solve addition and subtraction problems prior to formal instruction.

> The working hypothesis of the study was that prior to formal instruction many children can solve a variety of different problems involving addition and subtraction operations. Furthermore, they develop different strategies for solving different problems. (p. 2)

The identification of the different strategies children use to solve different problem types will help clarify the understanding that they bring to the formal introduction of addition and subtraction.

3. Research Design and Procedures

One of the main variables involved the structure of the problem. Addition and subtraction problems were characterized by active or static situations and by set inclusion or no set inclusion. Each of the different problem structures, as represented by the cells in the diagram below, were included in the study (p. 7).

	active	static
set inclusion	joining and separating	part-part-
no set inclusion	equalizing	

Another variable involved the mode of presentation. The problems were presented in one of two modes, either concrete or verbal. For the problems involving concrete objects, the experimenter constructed sets and asked the children to count. Then an addition or subtraction question was asked about the objects. The verbal problems were read to the subjects and reread as often as was needed. Concrete objects were available for the subjects to use.

The numbers used in the problems included the ten number families with both addends greater than 2 and less than 10; the sums were between 10 and 16 and the differences between the addends were greater than 1. In each addition problem the first addend was less than the second addend. In each subtraction problem the difference was less than the number being subtracted.

These number families were assigned to the problems in a way such that each subject was presented each number triple once with the verbal problems and again with the corresponding concrete problem. Different number combinations were assigned to different problems for each subject.

The subjects included all 43 first graders at a parochial school located in a middle class neighborhood. Prior to the study they had "no formal instruction in symbolic representation of addition and subtraction..." (p. 25). However, "several lessons had been presented involving joining, separating, part-part-whole and comparison problems" (p. 25).

The students were encouraged to use concrete materials to represent these problems.

Each subject was interviewed in two separate 15-20 minute sessions. One was used for solving verbal problems and the other for concrete problems. The verbal problems were presented first to half of the subjects and the concrete problems were presented first to the other subjects. If the strategies that the children used were not obvious, they were asked to tell how they found the answer.

4. Findings

The number of correct strategies (over 75%) and correct responses (over 50%) was quite high for all problem types except comparison problems involving addition. Very few children used a strategy that would indicate the choice of a wrong operation.

For addition problems the joining and part-part-whole appeared to be treated by children as the same type of problem. This was true both in the type of representation (counters, fingers, or no physical representation) and in the solution strategy (counting all, counting on from the first number, counting on from the larger number, etc.). The comparson situations were not only more difficult, but were represented differently and solved differently.

For the verbal subtraction problems the problem structure seemed to determine the solution strategy. Separating problems led to a separation strategy. Joining problems led to an adding on strategy. Comparison and equalizing problems led most often to a matching strategy. The strategies used for part-part-whole problems were mixed between separation and adding on. For concrete subtraction problems the dominant strategy was separation for most of the problem types. This is in contrast to addition, where there was little difference between concrete and verbal problems.

5. Interpretations

The students demonstrated a high level of success. Over two-thirds used a correct strategy for at least 8 of the 10 verbal problems. There were also few instances of children who chose a solution strategy

representing the wrong operation. Furthermore, they were successful at "interpreting action or relationships implied in the problems" and in using "different models of addition and subtraction when convenient" (p. 59).

Since children have a rather rich repertoire of processes available to them in their problem-solving activities, it may be that verbal problems are the appropriate context in which to introduce addition and subtraction.

> Perhaps by basing our introduction of operations on verbal problems and integrating verbal problems throughout the mathematics curriculum rather than using them only as an application of previously taught algorithms, we can allow children to develop their natural ability to analyze problem structure and to develop a broader concept of basic operations. (p. 64)

Abstractor's Comments

The researchers raise an important curriculum question in this study. How should children be introduced to an operation? Typically we develop meaning for addition from a single problem type, namely joining. Similarly, the meaning of subtraction is developed from take away or separation situations. Only after considerable practice developing symbolic computational skills are other problem types even introduced. By this time the connections between the operation and the single problem type that was used to introduce it are so strong that children often seem to have difficulty recognizing another problem type as being an instance of the operation. This is evidenced by a common response to verbal problems that are not obviously joining or separating, that being, "Do I plus or minus?"

The results of this study indicate that children have the capabilities to deal with a variety of problem types involving addition and subtraction even prior to formal instruction. Furthermore, they use a variety of solution strategies to solve those problems. The solution strategy often seems to be determined by the structure of the problem. However, they apparently do not understand that different types of problems can be represented by a single operation.

Should we attempt to take advantage of this problem-solving ability

that children have and present many different problem types to develop meaning for an operation early in the instructional sequence? If so, consideration needs to be given to the problem of representing these different situations by a single operation.

Since one of the stated objectives of the study was to "characterize processes and strategies children use in solving selected addition and subtraction problems" (p. 3), it seems unusual that the number sizes would be restricted so as to discourage the use of counting on and the use of doubles in some of the heuristic strategies. A more realistic representation of the solution strategies that children actually use would have been obtained if there had not been these restrictions. The researchers admitted that the choice of numbers may have affected the choice of strategies that the children used, but made the decision because the children would be less likely to know the facts and the strategies would be more likely to be observed. Still, there should have been no restriction.

While the subjects are described as having the ability to solve a variety of problems prior to formal instruction, "several lessons had been presented involving joining, separating, part-part-whole and comparison problems" (p. 25). Since the tasks of the study were of the same type of problems, those "several lessons" may have affected the outcome of the study. It is conceivable that the children learned some of the strategies exhibited. Even so, if they were learned so easily, they are worthy of serious consideration in the early mathematics curriculum.

Fuson, Karen C.; Geeslin, William E.; Damarin, Suzanne E.; and Jansson, Lars C. EXPLORATIONS IN THE MODELING OF THE LEARNING OF MATHEMATICS. Columbus, Ohio: ERIC/SMEAC. March 1979. ERIC: ED 173 113.

Abstract and comments prepared for I.M.E. by LEN PIKAART, Ohio University.

One of the ten working groups of the Georgia Center for the Study of Learning and Teaching Mathematics is concerned with "Models for Learning Mathematics." The eleven papers in this volume were selected through an anonymous review process conducted by the Center. A brief introductory chapter by William Geeslin indicates that the papers "...represent an attempt to clarify theory and formulate models of mathematical learning... [They] also represent an attempt to clarify the very meaning of the terms 'theory' and 'model'" (p. 2).

Following are abstracts of each paper taken from the table of contents.

John Richards. "Modeling and Theorizing in Mathematics Education." Modeling is an activity, a purposive behavior, whose significance is determined by the theoretical basis of the research framework. Building a model, or employing an already available model, must occur within the structure of a theory. Through a survey of the development of models in the nineteenth centry, this paper distinguishes several essential features of models. Models establish a partial analogy which depends equally on a clear similarity, and an obvious difference, between the model and what is being modeled. A model typically is a temporary explanatory device which allows the researcher to simplify, visualize, and idealize what is being modeled. This paper places the use of models in mathematics education within a broader research framework which provides for deeper understanding of the methodological benefits and limitations of modeling.

Leslie P. Steffe, John Richards, and Ernst von Glaserfeld. "Experimental Models for the Child's Acquisition of Counting and of Addition and Subtraction."

Two experimental models, one for acquisition of counting and the other for acquisition of the relationship between addition and subtraction by six-,

seven-, and eight-year-old children, are presented in this paper. The models are based in (1) a constructivist epistemological framework and (2) teaching experiments. The constructivist framework is presented and a three-stage model posited for acquisition of structural knowledge in mathematics. The two experimental models presented reflect the three stages of the more general model.

Harold W. Mick and Gerald D. Brazier. "A Logical-Cognitive View of Learning and Teaching Mathematical Concepts."

The purpose of this paper is to present a logical-cognitive point of view concerning the relationships among mathematical concepts, the psychology of the learner, and instruction. The developmental psychology of Piaget is the basic background from which this model emerges. Two classifications of mathematical concepts are identified: figural concepts and operational concepts. Various combinations of these concepts are hypothesized to form internalized conceptual structures. The study of the ways in which these structures are formed and function is assumed to be the very essence of the psychology of learning mathematical concepts --to comprehend or understand a concept means to assimilate it into an appropriate structure. An analogy is drawn between mathematical (conceptual) structures and Piaget's operational structures, and their corresponding acquisitions. In particular, simple abstraction is applied to figural concept acquisition and reflective abstraction is applied to operational concept acquisition. Piaget's periods of mental development are outlined and used as motivation in proposing three learning-instructional phases of concept acquisition. These phases are referred to as exploration, assimilation, and formalization.

Diana B. Mierkiewicz. "Instructional and Theoretical Implications of a Mathematical Model of Cognitive Development."

A non-mathematical summary and analysis of a mathematical model of cognitive development created by Saari are presented. The Saari model provides a representation of the qualitative aspects of cognitive growth and consists of two major components--the cognitive structures and the processes by which they change. Instructional and theoretical implications also are drawn from the model. The area of the teaching of rational

number concepts is used to illustrate the kinds of instructional implications derivable from the model. Research literature from this area is examined as a possible source of empirical support for these implications. At the theoretical level, the Saari model seems to encompass some of the qualitative aspects of the theories of Piaget, Gagné, and Ausubel. The characteristics modeled in each theory are discussed. Suggestions for future research are drawn from the model.

Nicolas Herscovics. "A Learning Model for Some Algebraic Concepts."

The article introduces a teaching-learning model (didactic reversal) for constructing the concept of first-degree equations in one unknown and for the concept of linear equations in two variables.

When taught formally, the acquisition of these concepts represents for the student a problem of accommodation through an assimilative process by building meaning for the new algebraic forms on the basis of the pupil's existing cognition. For the concept of equations in one unknown, this is achieved by introducing arithmetic identities and transforming them into equations. For the concept of linear equations in two variables, this implies formalizing the student's concept of the straight line in the Cartesian plane. It is only when the algebraic forms have acquired meaning that reversal is encouraged. That is, starting now from meaningful algebraic forms, the student is asked to find their arithmetic or geometric representations.

These learning schemes are based on the assumption that there are different modes of understanding mathematics and that their integration is essential to any pedagogical presentation. It is for this purpose that the article reviews three models of understanding which describe types of understanding of mathematics.

Thomas R. Post and Robert E. Reys. "Abstraction, Generalization, and the Design of Mathematical Experiences for Children."

This paper suggests a model for conceptualizing mathematical concepts. It emphasizes two important aspects of conceptual development: abstraction and generalization.

The variability principles suggested by Dienes form the conceptual framework from which the ideas in this paper emanate. Perceptual

variability was hypothesized by Dienes to promote abstraction of a mathematical concept and mathematical variability was similarly hypothesized to promote generalization of that same concept. The ideas developed are an attempt to provide a conceptual framework within which these psychological principles can be utilized in the design and implementation of instructional activities within the classroom setting. As will be seen, the individual exemplars are topic or concept specific, although the procedures for their development remain consistent throughout. These developmental procedures can be applied to a wide variety of mathematical concepts.

The first part of this paper deals with several observations and concerns regarding the current disparity between learning theories and the design of instructional settings for children.

Karen C. Fuson. "Towards a Model for the Teaching of Mathematics as Goal-Directed Activity."

A preliminary model of mathematics learning and teaching that is useful to elementary school teachers is described. The theoretical perspective is that early mathematics learning consists of goal-directed activity sequences which the teacher gradually helps the child to do. This perspective is drawn from Soviet psychologists, particularly Vygotsky, Leontiev, and Gal'perin. After the static model is discussed, four types of change across time within the learner--mastery, abbreviation, generalization, and internalization--are analyzed. Finally, general implications for instruction are outlined.

James G. Greeno. "Preliminary Steps Toward a Cognitive Model of Learning Primary Mathematics."

A cognitive theory of learning should have three components: a theory of the knowledge that students need before they can learn, a theory of the knowledge that they have after they have learned successfully, and a theory of the process of transition. This paper describes the current state of progress toward a cognitive theory of learning elementary addition and subtraction concepts and operations. Results thus far include a model of preschool children's knowledge for counting sets of objects and models of the procedures that children learn constituting their

knowledge of basic addition and subtraction facts. An analysis of semantic structures required to understand basic quantitative relations in addition and subraction word problems has been developed. The theoretical problem of learning is discussed, emphasizing the distinction between formal language of arithmetic and semantic models of the formal language.

A. Edward Uprichard and E. Ray Phillips. "Intraconcept Analysis of Rational Numbers Addition and Subtraction: Indirect Validation Studies."

The purposes of this paper are threefold. First, an analysis of hierarchy validation strategies is presented along with recommendations for alternative validation procedures. Second, the authors discuss two studies employing the use of an indirect validation strategy. In both studies an intraconcept analysis technique was used to generate an initial hierarchy for rational number addition and subtraction. Third, hunches for sequencing and teaching these two skill areas derived from the analysis of the results are discussed.

Suzanne K. Damarin. "An Organizational Model of Abilities Related to Inference in Mathematical Contexts."

For some time researchers and mathematics educators have been concerned by the poor performance of preservice elementary teachers on reasoning tasks. This paper argues that it is necessary to examine reasoning within the context of elementary mathematics, and that errors attributed to reasoning may be errors in interpretation of statements.

Statements about mathematical concepts are classified and organized by a model in which equivalent statements are linked by translations. Two types of understanding of statements (constructive and interpretive) are identified. Research related to the model is summarized, and implications for both further research and curriculum design are examined.

Lars C. Jansson. "Logical Reasoning Learning Hierarchies."

An explanation and definition of ordering theory, a deterministic measurement model, are presented. This theory is then applied in a replication study and an extension study in order to construct logical reasoning learning heirarchies based upon Piaget's sixteen binary combinations. The first study, of 50 grade nine subjects, attempted to

replicate an earlier study of Airasian, Bart, and Greaney. The second study extended the same investigation to subjects in grades seven and eleven. Logical reasoning hierarchies are presented for all three grade levels and comparisons are made with the replicated study.

Abstractor's Comments

There are several very impressive aspects of this publication. It reflects the cooperative efforts of the ERIC Clearinghouse for Science, Mathematics, and Environmental Education at The Ohio State University, where it was published as a "Mathematics Education Report," and as previously noted, The Georgia Center for the Study of Learning and Teaching Mathematics. The volume theme of models for learning mathematics is certainly important, but seldom addressed. The reader will find thoughtful analyses of the ideas of leading scholars like Bruner, Dienes, Gagné, and Piaget. Almost all of the proposed models are insightful and provocative. Authors describe the foundations, interpretations, and consequences of their models. Most impressive is the fact that each author has attempted to corroborate the proposed model by examining research evidence. Even though the proposed models, which are content-specific, are tentative and may change in time, the authors in many cases have developed studies to support or refute their models.

On the negative side, the work suffers from uneven quality, weaknesses in editing, and a lack of cohesion. Outstanding are the papers by (1) Richards, who sets the stage for the entire volume, (2) Steffe, Richards, and von Glaserfeld, (3) Mierkiewicz, (4) Fuson, (5) Uprichard and Phillips, and (6) Jansson. At the other extreme are those by (1) Post and Reys and (2) Damarin. Post and Reys propose a two-dimensional model which focuses on perceptual variability or multiple embodiments as one dimension and mathematical variability as the other. The value of the model may be obscured by the unimpressive choices for the elements of mathematical variability. For example, the mathematical components of area are the rectangle, parallelogram, triangle, trapezoid, and other shapes (p. 132)--hardly an adequate partitioning of the mathematical concept of area. The cells of the matrix are not clearly identified. Sometimes they are lessons, and sometimes activities, but they

are always identified simply as topics. These cells form a partially ordered set so that some should be accomplished before others; for example, teachers should plan to do multi-shape cutouts of rectangles before using a geoboard. The model becomes even more confusing when cells are not drawn (p. 134), the ordering arrows are omitted (pp. 132, 134, 135), and notes are not connected to a referent (p. 132).

Damarin has explored the use of logic by preservice elementary teachers. Her model is a pictorial indication of the equivalence of mathematical information in the form of a mathematical statement, a sorting of the replacement set, or a logical statement. She examines the translation from one form to another. Judging a model is a highly subjective activity, but this reviewer found Damarin's model weak because it ignores other forms of mathematical information and because it emphasizes special forms. Pictorial representations such as graphs are not considered. Also, she does not distinguish between symbol and verbal forms. The point is that a major strength of mathematics is that statements, expressions, and concepts have many different forms or representations. At a given time one particular form may be more useful than another. Also, her discussion of logic appears restricted to symbolic logic with an emphasis on truth tables. Apparently ignored is the notion that the same ideas could be learned without the calculus of symbolic logic. Overall, there is a question of how the model explains mathematical learning in the sense adopted by the other authors.

On balance, the importance and value of the document far outweigh any weaknesses. Typographical errors crept into the document which are not typical of ERIC publications and a final chapter reviewing and summarizing the several approaches to models of learning would have been welcomed. But the important idea is that a major volume has been produced which carefully describes models for learning mathematics. Several viable alternatives have been presented and the focus is on research corroboration of the proposed models. Every graduate program in mathematics education would find the volume of interest and useful.

Goldin, Gerald A. and McClintock, C. Edwin (Eds.). TASK VARIABLES IN MATHEMATICAL PROBLEM SOLVING. Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics and Environmental Education, November 1979. ERIC: ED 178 366.

Abstract and comments prepared for I.M.E. by RICHARD E. MAYER, University of California, Santa Barbara.

TASK VARIABLES IN MATHEMATICAL PROBLEM SOLVING is an outgrowth of a conference held in May of 1975 in Athens, Georgia, and sponsored by the Problem Solving Project of the Georgia Center for the Study of Learning and Teaching Mathematics. According to the editors of this collaborative volume, two distinct groups were formed at the conference and are represented in this book: (1) the Task Variables Group, chaired by Gerald Kulm, focused on the development of a classification system for task variables in story and word problems; (2) the Heuristics Group, chaired by J. Philip Smith, focused on the development of a coding system for thinking-aloud protocols of students' solutions to story and word problems.

The book consists of 14 main papers (presented in 10 chapters) and two reaction papers. Unlike most edited volumes, TASK VARIABLES IN MATHEMATICAL PROBLEM SOLVING is organized around a unifying theme. The theme is a search for consensus among researchers concerning the important dimensions along which algebra word problems may differ. The book begins by listing the relevant categories of task variables (in chapters 1 through 5), and then suggests applications to research (in chapters 6 through 8), and applications to teaching (in chapters 9 and 10). The book has benefited from a great deal of collaborative effort and communication among contributors, so that it truly represents a consensus.

The first five papers present the categories of task variables in mathematical problem solving. The first paper, by Gerald Kulm, provides an excellent overview for the entire book, including a review of previous systems for classifying task variables (by Kilpatrick, Polya, Wickelgren, Krutetskii, and others) and an outline of the system for classifying task variables that is described throughout this volume. According to Kulm, a task variable is "any characteristic of problem tasks which assumes a

particular value from a set of possible values." Thus, Kulm shows that a task variable may be "numerical" (e.g., the number of words in a problem) or "classificatory" (e.g., problem content area). Furthermore, Kulm argues that research and teaching in mathematical problem solving would benefit if there were a "standardization of vocabulary" or common language used to describe mathematical word and story problems.

Kulm outlines five major categories of task variables that are discussed more fully in subsequent chapters: (1) syntax variables, such as problem length (e.g., measured by the number of words in the problem); (2) content variables, such as type of mathematical expression (e.g., measured as nonomial, quadratic, linear, etc.); (3) context variables, such as degree of practicality (e.g., as measured as applied versus concrete); (4) structure variables, such as complexity of state-space (e.g., as measured by number of blind alleys or number of alternative first moves); and (5) heuristic behavior variables, such as the type of strategy that is required to solve a problem (e.g., as measured as "working backwards" or "trial and error" or the like).

The second chapter, by Jeffrey Barnett, provides an informative review of prior research on syntax variables, and provides a more detailed definition and listing of the major categories of syntax variables. The major categories are: (1) length variables, including 18 variables such as number of words, number of numerals, number of punctuation marks, average word length, number of words per sentence, and so on; (2) grammatical structure variables, including 19 variables such as number of verbs, number of nouns, noun to verb ratio, number of subordinate clauses, number of prepositional phrases, and so on; (3) numeral and mathematical symbol variables, including four variables, in word form and so on; (4) question sentence variables, including four variables such as number of words in question sentence, whether the question sentence appears before or after the data, and so on; and (5) sequence variables, including three variables such as whether or not the numbers in the problem appear in exactly the same order as needed for problem solving, and so on. Examples and recommendations for research and instruction are provided.

The third chapter, by Norman Webb, provides an informative literature review, and provides detailed definitions and listings of the major

categories of content and context variables. The major categories of content variables are: (1) mathematical topic, based on subject area such as ratio, binomial, quadratic, or based on traditional problem types such as rate, age, mixture, etc.; (2) field of application, such as biology, chemistry, physics, etc.; (3) semantic content, based on key words such as "greater than" or "reduced by" or "altogether," or based on mathematical vocabulary such as "average" or "root of an equation," etc.; (4) problem elements, based on goals such as "to find" or "to prove" or based on givens that are either "conjunctive" or "disjunctive"; and (5) mathematical equipment, such as calculator, compass, protractor, etc. The major categories of context variables are: (1) problem embodiments, such as manipulative, pictorial, symbolic, verbal, etc.; (2) verbal context, including distinctions between familiar versus unfamiliar, applied versus theoretical, concrete versus abstract, factual versus hypothetical, etc.; and (3) information format, such as whether or not there are hints, or whether the problem is multiple-choice or free-answer. Examples of these variables are provided.

The fourth chapter, by Gerald Goldin, introduces the reader to state-space analyses of problems, and offers a classification system for structural variables. State-space analyses involves breaking a problem down into a given state, goal state, all legal operators, and all possible intervening states. Most of the examples of state-space analyses in this chapter come from puzzle problems such as 'missionaries and cannibals', or from algebra equations. However, the author suggests that the structural variables that are listed may be applied to algebra story and word problems. The major categories of structural variables are: (1) problem complexity variables, including 11 variables such as total number of states, number of blind alleys, number of possible first moves, and so on; (2) algorithm or strategy variables, including six variables such as length of solution path generated by a particular algorithm or number of times a particular loop in an algorithm is transversed; (3) initial state variables, including three variables such as number of parentheses, number of occurrences of any particular operation, and number of equations and unknowns; (4) symmetry and subproblem variables, including six variables such as number of elements in the symmetry group; and (5) problem

relationship variables, including two variables such as existence of an isomorphism between problem state spaces.

Chapter 5, by C. Edwin McClintock, provides a review of research on heuristic processes including an examination of protocol scoring systems by Kilpatrick, by Lucas, by Kantowski, and by Blake, as well as a list of heuristics. The list of heuristics includes: (1) techniques for understanding the problem, (2) techniques for selecting a problem representation, (3) techniques for exploiting a problem representation, and (4) techniques for utilizing alternative representations. The paper includes many examples of these heurisitics with respect to algebra word and story problems.

The next two papers summarize research reports concerning the role of context, content, and syntax variables. Chapter 6, by Gerald Goldin and Janet Caldwell, reports a large-scale study that compares the performance of children in grades 4 through 12 on solving four types of problems: (1) abstract-factual problems, (2) concrete-factual problems, (3) abstract-hypothetical problems, and (4) concrete-hypothetical problems. Abstract word problems describe symbolic objects such as, "The number 33 is given..."; concrete word problems describe real situations such as, "Jane has 32 gumdrops...". Factual word problems use definite descriptions such as, "Jane has five more than twice as many, so she has 17 dolls"; hypothetical word problems use tentative descriptions such as, "If Jane had five more than twice as many, she would have 17 dolls." The results indicated that for all age groups, concrete problems were easier to solve than abstract problems, and for the older age groups, factual problems were easier than hypothetical problems. This provides strong support for the claim that syntax, content, and context variables exert an influence on students' problem-solving performance. Similarly, Chapter 7A, by William Waters, reports a concept learning study in which the concreteness of the stimuli influenced problem-solving performance.

The next four papers report research studies involving protocol analyses. Chapter 7B, by Harold Day, compared the problem-solving protocols of students solving problems with simple structure versus problems with complex structure. In general, systematic trial-and-error was used more often on complex than simple problems, whereas deductive algorithmic

approaches were used on more simple problems than complex problems. Chapter 7C, by George Luger, examined subjects' performance on the Tower of Hanoi problem, and on transfer between isomorphs of this problem. Chapter 8A, by Fadia Harik, examined subjects' protocols involving textbook-like algebra story problems. One major finding was that subjects tended to employ trial-and-error strategies rather than deductive strategies. Chapter 8B, by John Lucas, Mary Grace Kantowski, Nicholas Branca, Howard Kellogg, Dorothy Goldberg, and J. Philip Smith, presents a detailed coding system for thinking-aloud protocols. The coding system has been field-tested, has been developed by consensus among a group of researchers, and even seems to be used with some reliability. The system consists of a list of 14 heuristic processes, such as "draw a diagram" or "test special cases," and approximately two dozen actions, such as "reads problem" or "summarizes information"; in addition, the coding system includes symbols concerning outcomes, questions, errors, and punctuation. The system allows a researcher to take any problem-solving protocol as input, and to generate a list of symbols (based on the coding dictionary) as output.

The final three papers in the book explore instructional applications of the task variables approach to mathematical problem solving. Chapter 9, by Janet Caldwell, suggests some instructional objectives for solving single-operation word problems (i.e., sixth-grade level) and for solving ratio or proportion word problems (i.e., Algebra I Level). Examples of objectives include: "recognizing two problems having different grammar and syntax as mathematically the same" (syntax variables), "recognizing key word and stating the associated arithmetic operation" (content variables), "recognizing two problems having different contextual embodiments as mathematically the same" (context variables), and "recognizing that essential information is missing" (structure variables). Chapter 10A, by George Luger, distinguishes between "routine problems" (such as river, money, or age problems), and "non-routine problems" (such as tick-tack-toe or Tower of Hanoi). In addition, the author provides examples of how instruction for each type of problem can serve to highlight the structure of the problem. Chapter 10B, by Alan Schoenfeld, reviews some techniques for teaching problem-solving heuristics to mathematics students.

The book closes with two thoughtful "reaction papers." The first, by Max Jerman, states that the objective of this book is to help researchers reach a consensus on a standard vocabulary and definition of problem categories. Jerman agrees that this volume has been successful in distinguishing task variables based on syntax, content, and context, but he seems to have reservations concerning the useability of the state-space analysis of problem structure or the protocol coding system for problem-solving heuristics. The second reaction, by Jeremy Kilpatrick, commends the books' contributors for helping to develop a taxonomy of problem types and a system for describing how different types of problems are solved.

Abstractor's Comments

TASK VARIABLES IN MATHEMATICAL PROBLEM SOLVING represents my ideal of what an edited volume should be. The book is unified around a common theme, rather than being a fragmented collection of individual papers. The book involves much discussion and collaboration among authors, rather than a collection of independent, unrelated statements. The book attempts to provide a common language for researchers in the field, rather than having some nebulous, vaguely stated purpose. The authors and the editors are to be commended for their success in producing a useful edited volume.

The goal of TASK VARIABLES IN MATHEMATICAL PROBLEM SOLVING was to reach consensus among the mathematics education community concerning how to categorize or describe word and story problems, and how to describe students' problem-solving protocols. While the book has made some progress, including fine reviews of previous categorization systems, it remains to be seen whether researchers will actually embrace the system spelled out in this book.

Readers may find that the variables concerning problem representation (i.e., syntax, content, and context variables) are more understandable than the variables concerning problem solution (i.e., structure and heuristic variables). In particular, in this book heuristic variables are portrayed as potentially independent variables in problem-solving research (i.e., a characteristic of the task), whereas heuristics are more commonly seen as dependent variables (i.e., characteristics of the

problem solver's behavior). Similarly, the analysis of structure variables makes assumptions concerning how the problem solver will represent and search through the problem space; as such, this analysis goes beyond a straight-forward analysis of the task. Readers may also have some difficulty in applying the protocol scoring system or structure analysis to their own problems, since many details are left out. A "how to" manual is needed.

Another general problem is that the book is based largely on a logical task analysis that does not take full advantage of empirical work in the psychology of problem solving. For example, in describing categories of problem types, more advantage could have been taken of empirical work by Hinsley, Hayes and Simon (1977), or by Riley and Greeno (1978), or by Mayer (1981). Riley and Greeno's work has shown that arithmetic word problems that share the same arithmetic form may differ psychologically to the student depending on whether two sets are merged, one set is added to another, or two sets are compared. Similarly, Mayer's analysis of story problems in high school textbooks indicated that there are many psychologically distinct varieties of "DRT" problems or "river current" problems, etc.

An additional problem is that the book does not take advantage of theoretical work in the psychology of problem solving. All authors, with the exception of those discussing state-space analysis, focus largely on the logical task of building a sort of "non-theoretical" framework. An ultimate goal, however, should be to integrate the role of problem representation (e.g., syntax, context, and content) and the role of problem-solving processes and algorithms (e.g., structures and heuristics) into a unified theory of mathematical problem solving. For example, Resnick and Ford's (1981) The Psychology of Mathematics for Instruction calls for a unified theory.

Finally, in addition to proposing a common language for task variables, this volume suggests implications for research and instruction in mathematical problem solving. My fear is that the framework discussed in this book will be applied directly to research and instruction, without benefit of any underlying theoretical thought. In particular, I fear that research in the future will aim for describing how each of the major

task variables affects problem-solving performance. We may see research questions such as, "Does concreteness affect solution time?" or "Does field of application of problems affect proportion correct responses?" While such research is a useful component, the future focus of research should be on how task variables affect performance (i.e., what is going on inside the student's head). I also fear that the framework in this volume will be directly applied to the design of instruction, without appropriate supporting research. For example, one of the suggested instructional objectives based on the current analysis is to teach students to recognize "key words" such as "altogether" means "add," etc. It is exactly this senseless approach to representing problems that lies at the heart of many errors in problem solving. Research is needed to determine the effects of instructional objectives that focus on teaching students how to understand problems as compared to short-cut tricks that emphasize correct answers. In summary, if we could understand the cognitive processes involved in mathematical problem solving, we would be better able to understand why problem representation influences performance and to understand how to teach subjects to deal effectively with a variety of problems.

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Hiebert, James. COGNITIVE DEVELOPMENT AND LEARNING LINEAR MEASUREMENT. Journal for Research in Mathematics Education 12: 197-211; May 1982.

Abstract and comments prepared for I.M.E. by ALAN R. OSBORNE, The Ohio State University.

1. Purpose

The intent of the study was to examine whether certain cognitive developmental capacities are required to learn basic concepts and skills in measurement. The two capacities examined were logical ability, specifically conservation and transitivity, and information processing ability.

2. Rationale

Task analysis of the learning associated with linear measurement suggests that children need to acquire control of conservation and transitivity prior to learning linear measurement. Hiebert uses the failure of numerous studies to demonstrate this necessary condition to justify examination of the learning in the setting of a teaching experiment. He further notes that a possibly significant variable is the amount of information that a learner must retain and use in dealing with conservation and transitivity tasks; hence, information concerning the learner's capability to process information in an instructional setting should be examined for the teaching experiment to reveal whether conservation and transitivity are necessary readiness conditions.

3. Research Design and Procedures

Hiebert pretested 137 first-grade children in order to select 32 having appropriate characteristics for the 2x2 design matrix for the two variables of high/low logical ability and high/low information processing capacity. A linear measurement pretest was used to eliminate students who already exhibited any of the behaviors to be taught in the experiment. Logical ability was measured using conservation tasks adapted from Inhelder and Sinclair and transitivity tasks adapted from Smedslund. High-ability children had to exhibit both conservation and transitivity in linear settings; low-ability children, neither.

Information processing ability was measured by asking subjects to repeat in reverse order a series of digits that they had just heard. Children who could accurately respond for a least seven out of ten tasks for three-digit spans were classified as high ability. Pretesting was concluded when eight subjects were found for each cell.

Four lessons were taught individually to each child. The lessons concerned: (1) comparing and ordering two lengths, (2) constructing a discrete representation of a given length, (3) iterating units and representing a length numerically, and (4) the inverse relationship between unit size and unit number. The 10- to 15-minute instructional period for each lesson consisted of two or three problems on the topic. Each problem provided an assessment measure for the measurement learning. In addition each lesson had a post-instructional task that required integration of the single concepts or skills that had served as the basis for each problem. The investigator and one of two trained observers evaluated the performance of each child with suitable levels of inter-rater agreement on performance.

The hypotheses concerning the variables of logical ability and of information processing capacity were analyzed using different partitionings of the same performance data and, thus, recognized as not statistically independent. Two-way analysis of variance was used to test for main effects as well as interactive effects for the variables.

4. Findings

When tasks that had been identified as specifically dependent on logical reasoning ability (eight of thirteen) were used as the basis of analysis, the data indicated that logical reasoning accounted for 23% of the variance in performance, but neither information processing ability as a main effect or interaction was significant. For a second variable concerned with measurement techniques not possessing an apparent dependence on conservation or transitivity, the information processing variable was significant, with the low group performing better than the high group. No main effect for logical ability or interaction effect was observed.

The four post-instructional tasks that required integration of

ideas and skills treated separately in the problems of each lesson were analyzed as a performance measure that would be more likely to be related to the information processing variable. This variable was not significant in accounting for differences in performance; however, the logical ability variable accounted for 31% of the difference in performance.

5. Interpretations

Hiebert observed that "There are too many children in this study who failed the Piagetian tasks and still acquired appropriate measurement strategies to advocate using these as readiness tasks." Observing that children's solution procedures do not match the structural logic of problems, he discusses types of simple skills and techniques that allow children to bypass the logical structure of some measurement tasks.

The failure to find significant effects of the information processing variable was discussed in terms of the lack of similarity between the digit span task and the measurement activities, with the suggestion being made that information processing measures should be made in a context of greater similarity with the learning tasks. Although considering the possibility that the learning tasks and assessment measures were not of sufficient complexity for information processing to represent a significant variable affecting performance, Hiebert argues that context-specific measures of information processing capacity are needed for future research.

Abstractor's Comments

Identification of information processing capacity as a major confusing factor in understanding how young children cope with using conservation and transitivity defines a research problem of difficulty and significance in understanding how measurement is learned. Information processing capacity is at best nebulously defined to involve factors of encoding, storage, manipulation, and retrieval of information. These factors all seem to be involved in the logic inherent in the use and understanding of conservation and transitivity. A major difficulty in using the reverse digit span tasks as the measure of the information

processing capacity variable in the context of this study is that these same factors only appear to be involved in the digit span memory task. That is to say, the ineffable involvement of information processing in measurement is matched by the lack of specificity and precision in describing its involvement in repeating digits in reverse order. Is there a general ability or capacity of information processing? Or are we simply looking for evidence of how memory operates when we want to know whether (a) encoding has been successful and (b) retrieval of single ideas or ideas in combination is a factor in using conservation and/or transitivity? I have the feeling that we do not know what the digit span task measures; it seems more of a symptom than a direct measure of information processing capacity.

The study was well-conceived and is clearly described. Expensive in terms of time because of the individual teaching and assessment interviews, it represents a very careful analysis of how young children deal with linear measurement. Even with the heavy investment of time, I would trust the results more if more time had been invested in instruction. Ten to fifteen minutes on each of the four measurement topics is quite limited given the nature of the skills and understandings involved, although the data do indicate the children learned.

Of particular note is the quality of Hiebert's analysis of the inconsistencies between children's solution procedures and the logic inherent in the structure of measurement tasks. The analysis carries an important message for those who would use inappropriate readiness tasks to protect children from learning situations.

Lean, Glen and Clements, M. A. (Ken). SPATIAL ABILITY, VISUAL IMAGERY, AND MATHEMATICAL PERFORMANCE. Educational Studies in Mathematics 12: 267-299; August 1981.

Abstract and comments prepared for I.M.E. by J. LARRY MARTIN, Missouri Southern State College.

1. Purpose

The purpose of the study was to determine i) whether a person's "preferred mode of processing mathematical information" could be operationally defined and reliably measured and ii) the nature of the relationships among this preferred mode, spatial ability, and mathematical performance.

2. Rationale

Among mathematicians and mathematics educators there is some expectation that spatial ability, mental imagery, and mathematical performance are interrelated. The paper begins with several illustrations of why one might have such expectations, but reports that research results are not definitive. However, the main thrust of this study is toward information processing style and its bearing on mathematical performance. Can students be categorized on a 'preferred mode of processing mathematical information' scale from verbal-logical to visual? If they can and if style affects performance, then, assuming teachers can influence thought processes, implications for further research and ultimately teaching would be numerous.

3. Research Design and Procedures

The sample consisted of 116 first-year engineering students at the University of Technology, Lae, Papua New Guinea. Their mean age was 19.6 years. All but two of the sample were male. The basis for selection into the sample is not stated.

Each participant received a battery of five spatial tests during the first two weeks of their course of study in two two-hour sessions. During the third week another two-hour testing session consisted of a mathematics test and an associated questionnaire developed by Suwarsono

and modified for this investigation. The mathematical problems are deemed suitable for secondary pupils in Australian schools. The associated questionnaire describes different methods commonly used by students. After attempting solutions in the mathematics test, students indicate which (if any) of the commonly used methods they used. It is this modified questionnaire which was used to operationalize the construct 'preferred mode of processing mathematical information' on an 'analyticality-visuality' scale.

Subsequent to the mathematics test and preference questionnaire, ten students were individually interviewed to determine their preferred methods of solving the problems on the mathematics test. Interviewers were not cognizant of the students' previous written responses. Interview results were compared to the questionnaire results.

Finally, during their regular course of study participants received two additional tests, one a 'Pure' Mathematics test and the other an 'Applied' Mathematics test. The 'Pure' Mathematics test required manipulation of algebraic, trigonometric, and vector expressions. The 'Applied' Mathematics test contained problems from elementary mechanics.

Multiple regression analysis used these last two measures (i.e., 'pure' and 'applied' scores) as dependent variables and the five spatial tests together with the modified Suwarsono instrument as possible predictor variables. For the regression analysis with the 'Applied' Mathematics as the dependent variable, 'Pure' Mathematics was also included as a possible predictor.

4. Findings

As mentioned earlier, preferred mode of processing mathematical information was judged for ten students by interviewers, independent of the Suwarsono results. Four of the five students classified as 'analytic' by the interviewers ranked 1, 2, 3, and 4 on the analyticality-visuality scale as assessed by the Suwarsono instrument. The one student classified as visual by the interviewers was ranked 10 on the Suwarsono instrument.

Chi-square tests for normality were applied to each predictor variable to determine whether distributions were sufficiently close to normal

to justify use in the regression analysis. Each proposed predictor proved satisfactory in this regard. Because a correlation of .70 between two of the spatial tests posed possible multicollinearity problems, the two were collapsed into one variable defined as the sum of the two scores. This left, then, six predictor variables.

These six predictors together contributed 22% of the variance in the 'Pure' Mathematics test scores. The strongest single contributing factor was the 'preferred mode of processing mathematics information' obtained from the modified Suwarsono questionnaire. It contributed 9% of the variance and was the only predictor variable whose estimated standardized coefficient was different from zero.

The 'Pure' Mathematics test score was added as a possible predictor of performance on the 'Applied' Mathematics test, making seven predictors for the dependent variable 'Applied' Mathematics. Together these predictors contributed to 39% of the variance in 'Applied' Mathematics test scores. 'Pure' Mathematics was the heaviest contributor, accounting for 29%. The next heaviest contributor was one of the spatial tests with only 4%. 'Pure' Mathematics was the only predictor variable with a standardized coefficient differing significantly from zero.

As a result of a factor analysis of all the variables, predictor and dependent, the author identified four factors. He identifies them as a 'mathematics' factor, a 'spatial' factor, a 'mathematical processing' factor, and a 'reasoning' factor.

5. <u>Interpretations</u>

Both the multiple regression analysis and factor analysis indicate that 'preferred method of processing mathematical information' is a distinct component of cognition and that the Suwarsono instrument provides a promising means of measuring it. Students preferring verbal-logical means of processing mathematical information tended to outperform those preferring more visual approaches on the mathematics tests and, surprising enough, even on the spatial tests. Spatial ability and knowledge of spatial conventions had little influence on mathematical performance. The present study is in apparent conflict with other studies which indicate positive relationships between spatial ability and mathematical

performance and/or the desirability of visual processes for solving mathematics problems. The investigator proposes the conflict could be due to the routine nature of the 'Pure' and 'Applied' Mathematics tests. Other relevant studies have mostly emphasized non-routine problems. The author cautions, also, that many non-mathematical variables "such as student motivation, work habits, teaching, and language competence" (p. 296) could contribute to mathematical performance.

<u>Abstractor's Comments</u>

This investigation is well-planned and clearly reported. However, there are a few questions that one could ask. For example, the participants "were 116 entrants into the Engineering foundation year at the University ..." (p. 278). Did these 116 comprise the entire entering class or were they selected from a larger group? If so, how?

Other questions revolve around the investigator's interpretation of why students preferring verbal-logical modes of processing outperformed students preferring visual modes, especially considering that these results were not expected nor consistent with previous studies (Moses, 1977, 1980; Webb, 1979). The investigator proposed that the discrepancy was due to the routine nature of the problems. The difference in the difficulty of the problems certainly could have an effect on the results. The investigator suggests that the more verbal-logical student is able "to cast away ... unnecessary 'concrete' details" (p. 295). The more visual student "tends ... to retain as part of his thinking, unnecessary 'concrete' details' (p. 295). Retaining these "unnecessary" concrete details, then, actually hinders abstraction when abstraction would provide the most efficient means to the solution. This interpretation seems feasible. However, if it is true, wouldn't one expect the verbal-logical student also to outperform the visual on non-routine problems as well? Wouldn't "unnecessary" concrete details still be present on non-routine problems? But the previously cited studies reported that students preferring visual processing modes outperformed the verbal-logical students. Thus, the investigator's interpretation is not very satisfying.

Most questions stimulated by studying this investigation deal with

issues outisde its scope. This is characteristic of a good study. A natural follow-up study would be one using non-routine problems. Students in this study had a mean age of around 20. Can we isolate and measure a 'preferred mode of processing mathematical information' at earlier ages? Is that preferred mode stable across age? Is it or can it be influenced by teachers? As the author states, further research is needed to clarify the characteristics of the 'preferred mode' trait.

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Lemoyne, Gisele and Favreau, Mireille. PIAGET'S CONCEPT OF NUMBER DEVELOPMENT: ITS RELEVANCE TO MATHEMATICS LEARNING. Journal for Research in Mathematics Education 12: 179-196; May 1981.

Abstract and comments prepared for I.M.E. by DOUGLAS T. OWENS and ROSITA TAM, University of British Columbia.

1. Purpose

In this study, the authors related Piaget's definition of operational thinking to the numerical strategies of children. The purpose of the study was to "specify the relationship between logical and numerical structures by examining the process used by operational and preoperational children in solving simple addition and subtraction problems" (p. 180).

2. Rationale

The authors state that in spite of attempts to apply developmental and learning theories to arithmetic teaching and curriculum, many children still fail arithmetic as early as first grade. This failure has usually been attributed to competence and performance factors. The authors cite several studies to support their view that there exists a strong relationship between arithmetic achievement and the logical concrete operational stage. Since the operational stage has usually been defined as the ability to perform conservation tasks in these studies, the authors contend that an investigation is needed to establish the operational stage of a child not only in conservation tasks, but also in seriation and classification tasks.

3. Research Design and Procedures

The experiment was conducted for a five-week period. The sample was selected from 38 children who had completed the first grade. Criteria for selection were based on results from classification, seriation, conservation, and ordinal correspondence tests. "The final sample consisted of eighteen middle class children, ranging in age from 6.7 to 7.6 years" (p. 180). The preoperational group included nine children classified at Stage 1 in all four tests; the operational group included

nine children classified at Stage 3 in all four tests.

Six types of problems involving addition and subtraction were assigned:

1. $a + b = \_\_\_\_$
2. $a + \_\_\_\_ = c$
3. $\_\_\_\_ + b = c$
4. $a - b = \_\_\_\_$
5. $a - \_\_\_\_ = c$
6. $\_\_\_\_ - b = c$

The arithmetic testing was given individually. The experimenter intervened and provided cues or assistance in the case of failure. Every good answer was considered valid if it was maintained a week later when similar problems of each type were given to all children.

In order to identify the processes used, the method of Newell and Simon (1972) was applied to the behavior of the children solving the various addition and subtraction problems. The formalism of the production system was used to define these processes. The protocol analysis focused on a study of the main characteristics of the numerical strategies to establish a production system for each child. These strategies "included the use of the properties of the sequence of natural numbers, the physical representation of numbers and counting, and the recourse to addition and subtraction tables" (p. 183).

No statistical test was applied to the data. The authors contend that "it did not seem relevant to apply statistical tests to the data" (p. 184) due to the fact that help was given to all children during the problem solving and subsequent correct responses were included in the data.

4. Findings

In addition and subtraction problems, the performance of the operational group was superior to that of the preoperational group. Eight of the operational children were capable of analyzing the relationships between the terms of an operation in order to select an efficient strategy in five out of the six types of problems. Almost half the preoperational children showed this ability in four out of the six types of problems. The majority of the operational children and almost half the preoperational children were able to create more than one efficient production for each type of problem.

"Some children in the two groups extended their comprehension of the ordinal and cardinal aspects of numbers by associating addition and subtraction operations with the movement from one position to another [counting] along the sequence of natural numbers" (p. 190). "Preoperational children ... resort to external memory [e.g., fingers or blocks] more often than their counterparts. Operational children show a better knowledge of addition and subtraction tables and seem to make more efficient use of them than preoperational children" (p. 195). "In both groups, some children revealed a good comprehension of the additive composition of numbers" (p. 190), such as the meaning of addition or subtraction as the inverse. Operational children checked their answers more frequently, which proved to be very useful where one of the terms a or b was to be found.

5. Interpretations

The authors concluded that "concrete operational thought, as defined by the ability to classify, to seriate, to conserve number and to establish ordinal correspondence between two series, is sufficient for success in basic operations on numbers" (p. 195). The fact that almost half of the preoperational children in the sample used strategies showing operational characteristics led the authors to conclude that "numerical structures are constructed before or at least concomitantly with class and relation structures" (p. 194). Moreover, since almost half of the preoperational children performed well in addition and subtraction problems, the authors "question the relevance of concrete operational thought as a competence factor related to mathematical learning" (p. 195).

Abstractor's Comments

The authors are to be commended for a most thorough study of the relationship between Piaget's claims about number development and arithmetic operations on natural numbers as seen in school curricula. Many studies have shown global correlations between Piagetian conservation and arithmetic achievement. This study revealed actual thinking processes, as determined by individual interviews with minimal intervention, of children at the preoperational and concrete operational stages.

To be in the former group a child had to be preoperational on four different tasks and to be in the latter group a child was required to exhibit operational thinking on all four tasks. This is a well-designed carefully executed study of an interesting question.

I found the report of the study difficult to read. For example, the meanings of such phrases as "additive composition of numbers" and "movement from one position to another along the sequence of natural numbers" were difficult to discern. The thorough report of the analysis (a strong point) was encumbered with fairly technical symbolism which was not clearly explained. The authors used the term "problems", an ambiguous term, to describe the six types of addition and subtraction equations.

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Noelting, Gerald. THE DEVELOPMENT OF PROPORTIONAL REASONING AND THE RATIO CONCEPT. PART II - PROBLEM-STRUCTURE AT SUCCESSIVE STAGES: PROBLEM-SOLVING STRATEGIES AND THE MECHANISM OF ADAPTIVE RESTRUCTURING. Educational Studies in Mathematics 11: 331-363; August 1980.

Abstract and comments prepared for I.M.E. by JUDITH THREADGILL-SOWDER, Northern Illinois University.

1. Purpose

In Part I the author sought to determine and confirm hierarchical stages in the development of the concept of proportional reasoning. Having succeeded, he devoted Part II to the description and analyses of these stages.

2. Rationale

From the perspective of work on advanced organizers, establishing a relationship between concepts of hierarchical construction and adaptive restructuring is necessary. Extensive previous work at Laval University has allowed the working out of various stages of development of the concept of proportional reasoning, with interpretation of strategies at each level. This work is summarized through the research presented in these articles.

3. Research Design and Procedures

Part I: The Experiment

A 23-item test was given to 321 students, ages 6 to 16. For each item, the students were shown two boxes, A and B, in which there were plastic cups, some filled with orange juice and some with water. The experimenter pretended to mix the orange juice and water into a large container beside each box. Students were to tell which mixed drink (if either) would have the stronger orange flavor and give an explanation for the answer. For example, in Item I, (3,1) vs. (1,3), three glasses of orange juice were to be mixed with one glass of water in A, and one glass or orange juice was to be mixed with three glasses of water in B.

Students either passed or failed each item.

4. Findings

Part I

Frequency of success for each item was used to order the items according to difficulty. A scalogram was made and analyzed. The resulting "perfect" hierarchical scale of items was grouped into categories of items of the same kind, according to defined criteria. Subjects who passed one item of a category but failed items in the next category were grouped and the groups compared for age distribution using the Kolmogorov-Smirnov test. Operational levels were then assigned to the stages, following the Piagetian chronology of development. Three intuitive stages, two concrete operational stages, and two formal operational stages, were found. (Typical protocols for each stage are illustrated in the original article.) Finally, a factor analysis was performed on the overall results, yielding six factors, verifying six of the seven stages found in the structural analysis of items.

Part II: First-Order Treatment of Results

The particular problem structure at each stage was determined and expressed in mathematical form, and the problem-solving strategies used at each stage were characterized. Two alternate strategies occurring at each stage were "between" and "within" strategies. In a "between" strategy, a ratio is complicated or simplified in order to compare it with the other ratio. This ultimately leads to the common denominator algorithm. In a "within" strategy, division is used to form a quotient for each ratio which can be compared to 1. This strategy ultimately leads to percentages.

Example: Stage II B, higher concrete operational, is determined by success at problems expressed as "Equivalence class of ratio (a,b)." A student given the problem expressed as (1,2) vs. (2,4) said that the drinks would taste equally of orange juice "because both the proportion of water and juice have been doubled." This is considered a "between" strategy, mathematically expressed as $m(a,b) = (ma, mb)$. A second student, given the problem (4,2) vs. (6,3), chose equality because "In A,

4 glasses of juice for 2 glasses of water, it is equal to B when there are 6 glasses of juice for 3 glasses of water." This is a "within" strategy symbolized as $\frac{a}{b} = \frac{c}{d}$.

In the final part of the first-order treatment, the strategies at each stage were analyzed and shown to be embedded in strategies at successive levels.

Part III: Second-Order Treatment of Results

In this epistemological treatment, mechanisms for passing from one stage to another were examined. Two periods of restructuring were described, each containing four phases or stages. Period I is characterized by between stage mechanisms as follows: differentiation of first and second terms, differentiation of within and between relations, and finally the integration of relations, giving rise to concepts of ratio and quantity. Between stage mechanisms in Period II are: differentiation of terms inside a pair, differentiation of the operations of addition and multiplication (leading to the formal operations stage), and differentiation and integration of algebraic and logical operations.

It is noted that development between phases is qualitative, involving restructuring of a strategy, whereas development within a phase is quantitative, involving extension of a scheme to quantitative variations. Conceptual development is thus bidimensional rather than linear, corresponding to the two aspects of training, namely understanding and exercise.

Abstractor's Comments

Part I of this fine pair of articles is all one could expect of a report on a first-rate research study. Part II is lagnaippe.

The study is firmly based on the author's knowledge of and involvement in research on the development of the concept of proportion, and on his acknowledged background with the Genevan tradition. It is unusual to find a study so thoroughly Piagetian but which employs a variety of empirical research techniques to arrive at and substantiate claims made in Piagetian terms. The protocols included, although a bit lengthy and repetitious, assist the reader in understanding the characteristic behaviour of children at each stage.

Part II, particularly the last half, requires concentrated reading.

The reader is rewarded, however, with a greater insight into the development of children's understanding of an important mathematical concept, and of the hierarchy of mechanisms and strategies used during this development.

Two points made by the author are worth special mention. First, the methodology for this study has been specified in detail, because in the past there has been no clearly defined methodology specific to developmental research. It is my opinion that this work will serve as a model to others interested in developmental research.

The second point made is that, from a pedagogical point of view, the "within" strategy is more effective than the "between" strategy. This point bears more study, since the "between" strategy is more commonly taught in the U.S.

Owston, Ronald D. SYSTEMATIC COMPUTATIONAL ERRORS AND ACHIEVEMENT IN ELEMENTARY MATHEMATICS. Alberta Journal of Educational Research 27: 114-120; June 1981.

Abstract and comments prepared for I.M.E. by CAROL A. THORNTON, Illinois State University.

1. Purpose

The purpose of the study was to study the relationship between systematic computational errors and achievement in addition, subtraction, and multiplication.

2. Rationale

Computational errors occur at all levels of achievement in elementary mathematics (Ashlock, 1976), yet no research appears to have been done which relates the kind of computational error pupils make to the level of achievement.

3. Research Design and Procedures

Subjects in a sample of 60 fifth-grade Indian children, living on three reserves in New Brunswick, were given the Algorithm Assessment Test used by the Provincial Department of Education. This is a 200-item diagnostic test assessing computational skill in addition, subtraction, and multiplication. It covers a range of 11 skill levels for addition, 14 for subtraction, and 15 for multiplication, with five assessment items for each level. The test itself has ten subtests of items from all three operations, and is administered on ten consecutive school days. Although no time limit is imposed for subtests, most students in the present study found 30 minutes sufficient for each.

After the testing sessions, answers were scored correct or incorrect and the total score for each of the three skill areas was found for each subject. Then all incorrect errors for each skill level were examined for error patterns. If a pattern was evident in at least three out of the five answers for a given skill level, the error was classified as systematic. When error classifications were complete, the papers were divided into high and low achievers for each operation,

with high achievers scoring above the median score for the operation and low achievers falling below.

4. Findings

The investigator found that the greatest proportion of systematic errors occurred in subtraction (38.5% of the total), followed by multiplication (27.7%) and then by addition (12.4%). Application of a chi-squared test (with Yates correction factor) resulted in a significant relation between error type and achievement in multiplication, with high achievers tending to make nonsystematic errors and low achievers making systematic errors. No significant relation was found between achievement and error type for either addition or subtraction.

5. Interpretations

The findings of the study support those of Roberts (1968) to the extent that errors involving incorrect application of an algorithm occur at all achievement levels. The present study extended these results and found a relationship between achievement and error type for multiplication, though not for addition and subtraction. The number of errors made by the high group for each operation were small, so one cannot generalize to a larger sample. Given the limitation, perhaps nonsystematic errors exceeded systematic ones for high achievers because these students made more "sophisticated" types of errors which may have gone undetected by scorers. If individual follow-up interviews had been possible, more nonsystematic errors might have been classified as systematic for high achievers. Despite the tentative relationship existing between multiplication achievement and error type, the study does suggest that systematic computational errors of the type identified by Ashlock (1976) and others contribute substantially to poor achievement. This being the case, teachers can (1) try to identify systematic errors among low achievers and prescribe appropriate remediation and (2) employ preventive teaching techniques so common systematic errors, such as those identified by Ashlock (1976), do not occur.

Abstractor's Comments

Overall, the present study was well done. The investigator carefully constructed the design and built in reliability checks where possible to avoid misinterpretation in the error diagnosis. In the discussion of the report, the researcher is cautious to point out limitations and does not generalize beyond the sample.

One problem, which may influence a reevaluation of the relationship between achievement and error type for each operation, is the lack of demarcation between "high" and "low" achievers. One wonders whether focusing on scores made by students in the upper vs. lower thirds might have been preferable to the contrast selected, that above and below median. Notwithstanding this possible clarification in delineating achievement groups, the study contributes to the diagnostic-remediation literature in its support of careful analysis, remediation, and prevention of common error patterns, and for this reason is educationally significant.

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Standifer, Charles E. and Maples, Ernest G. ACHIEVEMENT AND ATTITUDE OF THIRD-GRADE STUDENTS USING TWO TYPES OF CALCULATORS. School Science and Mathematics 81: 17-24; January 1981.

Abstract and comments prepared for I.M.E. by DOUGLAS B. MCLEOD, San Diego State University.

1. Purpose

This study evaluated the effectiveness of instruction supplemented by hand-held calculators or programmed-feedback calculators (the "Little Professor"), as opposed to traditional instruction without calculators.

2. Rationale

Research on the use of calculators in mathematics classes has concentrated on the intermediate grades. This study extends that research to the primary level (Grade 3), and also tests whether "immediate feedback enhances the effectiveness of calculator use in mathematics instruction" (p. 18).

3. Research Design and Procedures

Nine classrooms, chosen at random from eleven schools, were randomly assigned to treatments using hand-held calculators, programmed-feedback calculators, or no calculators (a control group). All three groups followed the regular curriculum. In addition, the hand-held calculator group spent eight to ten minutes each day checking results, drilling on basic facts, and doing calculator activities, while the other calculator group spent the same amount of time using the "Little Professor" to practice the basic facts. In all three groups, instruction emphasized the basic facts for addition, subtraction, and multiplication.

The sequence of events in the study included an orientation meeting for the teachers during the first week of the semester, pretests during the second week, and eleven weeks of instruction followed by the posttests and, four weeks later, the retention tests.

The SRA Assessment Survey was used to measure achievement for the pretest (Form E), posttest (Form F), and retention test (Form E). Dutton's Attitude Toward Arithmetic Scale was also administered on all three

occasions. Data were analyzed using analysis of covariance. For each criterion variable (computation, concepts, total achievement, and attitude), the pretest was used as the covariate for the posttest, and the posttest was used as the covariate for the retention test.

4. Findings

The hand-held calculator group scored higher than the other two groups in computational skills (and total achievement) on both the posttest and retention test. The programmed-feedback calculator group performed better than the control group on the computational skills posttest, but not on the retention test. No significant differences were found on tests of concepts or attitudes.

5. Interpretations

The evidence suggests that "the supplementary daily use of the hand-held calculator was more effective in promoting acquisition and retention of computational skills" (p. 23) than either programmed-feedback calculators or traditional instruction. The somewhat surprising superiority of the hand-held calculator to the programmed-feedback calculator may be due to the flexibility of the hand-held calculator. "Consequently the basic design of the two types of calculators appears to have been more important than immediate feedback in determining the differences found between the two calculator groups" (p. 23).

Abstractor's Comments

The study used traditional methods to provide evidence that the hand-held calculator is useful in third-grade classrooms, even when the goals of instruction are restricted mainly to learning basic facts. The results also provide evidence about the limited usefulness of programmed-feedback calculators, even for very limited goals. The study does not deal in depth with theoretical issues, and the psychological issue of immediate feedback does not appear to be a major focus of the work.

A number of technical questions come to mind. For example, there was no evidence that analysis of covariance was the proper statistical technique; perhaps there are aptitude-treatment interactions lurking in

the data somewhere! Also, using the posttest as a covariate for the retention test tells us more about what was forgotten than what was learned during the eleven weeks of instruction. Finally, one has to hope that a more appropriate measure of attitude could have been found.

The paper was generally well written, although the use of the term "algorithm" was confusing. As usual, limited journal space makes it difficult to present detailed information about the training for teachers, the organization of the treatments, or the extent to which teachers presented the treatments as intended. Also, readers should keep in mind the long delay that frequently occurs between data gathering and journal publication. When this study was conducted back in 1977, the questions would have seemed much more timely than they do now.

Zweng, Marilyn J.; Geraghty, Jean; and Turner, Jonathan. CHILDREN'S STRATEGIES OF SOLVING VERBAL PROBLEMS. August 1979. ERIC: ED 178 359.

Abstract and comments prepared for I.M.E. by MARY GRACE KANTOWSKI, University of Florida.

1. Purpose

The purpose of the study was to identify factors that result in the successful solution of single-step and multiple-step word problems, verbal problems dealing with rate, and verbal problems containing extraneous data.

The overall question posed by the author was whether it was most beneficial to successful problem solving for the child to focus on the problem (the question, the information given, the relationships of parts to whole, the size of the answer); on tools used to solve the problem (drawings, manipulatives, calculators); or on transformation of the problem (restatement with smaller numbers, restatement in less verbal form, restatement in a less formal setting).

2. Research Design and Procedures

A clinical methodolgy was employed in the study. In all, 162 students in grades 3 through 6 in schools in Iowa City and Des Moines participated in the study. Each child was asked to solve fifteen "textbook type" problems in, at most, two 45-minute sessions. Interviews with the students were recorded and relevant behaviors noted on a printed form. Except for the first and last problems, the problems were randomized by type and difficulty. Problems included single-step problems (one operation), multiple-step problems (2 operations), problems with extraneous data, and problems involving finding the unit rate. The problem sets were different for each grade level with some overlap. Students were given calculators and popsicle sticks and told that they could use drawings to aid in solution.

Hints were developed to assist students who had difficulty solving the problems immediately. Hints included lower verbal content, use of smaller numbers, action of the data, part-to-whole relationships, focusing on the question and the data given, focusing on the size of the

answer, and "personalization" of the problem. Hints for the rate problems directed students to use drawings. The first three hints given to the student were randomly selected from among all hints. If these were ineffective, the investigators selected further hints based on their judgment of what would be most helpful to the student. Percentages of correct solutions with and without use of hints were computed by problem, grade level, and ability level.

3. Findings

The results included many interesting findings which will be selectively summarized here. Most students at all grade levels could solve the single-step problems. Ability to solve single-step problems improved as grade level increased. This was not true for multiple-step problems. Moreover, in the multiple-step problems there was a marked increase in the ability to solve the problems between the low and average ability levels and the average and high ability levels. Problems containing extraneous information were more difficult than other problems at all ability levels. With respect to the hints, all ability levels found helpful the "personalize" hint, the hints to use manipulatives and drawings, and the hints to decrease the quantity size in the problem. The low and average ability students used the calculator more often than the high ability students. As might be expected, the calculator was used most often in problems involving division. In independent solution of the rate problems, techniques involving many-to-many correspondence were used about nine times as often as strategies involving unit rate. Interested readers are encouraged to read the entire report for a more complete discussion of the findings.

4. Implications for Instruction

The author suggests that textbooks should include more multiple-step problems, hints to students to aid them in the solution of problems, and more emphasis on strategies for solution.

Abstractor's Comments

Zweng has conducted a carefully planned and well-executed study

that provides researchers with valuable information and hypotheses for further study, practitioners with suggestions for instructional techniques, and developers with a direction for needed change. The study was certainly comprehensive. Several aspects of problem solving were addressed. One technique employed in the study that is becoming popular in research and instruction was that of providing hints to students who were initially unable to solve the problems independently. Although some of the data support the effectiveness of the use of hints, some of the effectiveness could have been obscured by the randomization of the first three hints. It would be intersting to look at differences in the effectiveness of the hints if students had been allowed to select their own hints or if all hints had been selected by the examiners as those after the first three were. Suggesting randomized hints could have interfered with the students' thinking instead of providing the assistance they were designed to provide. The author mentioned that for the trial problems the third- and fourth-grade students were encouraged to use calculators, popsicle sticks, and drawings to help them solve the problems, while the fifth- and sixth-graders were given only the calculators. It is not clear why the older students were not also encouraged to use drawings, since the use of drawings and diagrams is such a powerful problem-solving tool at all levels, a fact substantiated by the results of the study. These criticisms are minor and do not detract from the value of the study. Clear and important questions were asked concerning attributes of problems, transformations of problems, and tools used in problem solving that have an effect on successful problem solving at several grade levels, and responses to the questions were supported by the data collected.

Studies such as this one demonstrate the potential for the clinical methodology to get to the heart of practical questions in mathematics education.

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