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ABSTRACT

A "doubly-centered" raw data matrix is one for which both columns and rows have both unit variance and means equal to zero. The factor scores from one analysis are the same as factor pattern coefficients from the other analysis except for a variance adjustment. This study explored an extension of the reciprocity principle which may have applications in some research situations. Analysis of an actual data set was used to illustrate the technique. The Multiple Teachers Factor Survey measured perceptions of four "teacher-types" by undergraduate students. A procedure was presented for calculating factor scores when subjects have rated several referents or the researcher wishes to compare factor scores for different subject groups. The procedure was the application of a conventional factor score coefficient matrix to a doubly-centered matrix of standardized scores. Tables illustrate two-mode techniques, a doubly-centered data matrix, varimax rotated pattern coefficients, cosines among factor axes, means and confidence intervals, and first subject's factor scores. (DWH)

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FACTOR ANALYSIS BASED ON
"DOUBLY-CENTERED" RAW DATA MATRICES

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Paper presented at the annual meeting of the Southwest Educational Research
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ABSTRACT

The paper presents a procedure for calculating factor scores when subjects have rated several referents or the researcher wishes to compare factor scores for different subject groups. The procedure involves application of a conventional factor score coefficient matrix to a "doubly-centered" matrix of standardized scores. The paper discusses the procedure by presenting an analysis of 145 education students' perceptions of four types of teachers.

The merits of few research techniques have been discussed as heatedly as have the merits of factor analysis (Royce, 1980; Tryon, 1979). The primary criticism of factor analysis has been an argument that knowledge regarding the dimensions underlying variables is not per se valuable. This criticism has led to the view that the techniques have been overutilized. For example, several researchers have factor analyzed ratings of the credibility of information sources; the researchers have postulated that credibility is an important factor which influences the use of research and evaluation information by policy makers (Thompson, 1981a). Cronkhite and Liska (1980, p. 102) voice the following criticism of these studies:

Apparently, it is so easy to find semantic differential scales which seem relevant to [information] sources, so easy to name or describe potential/hypothetical sources, so easy to capture college students to use the scales to rate the sources, so easy to submit those ratings to factor analysis, so much fun to name the factors when one's research assistant returns with the computer printout, and so rewarding to have a guaranteed publication with no fear of nonsignificant results that researchers, once exposed to the pleasures of the factor analytic approach, rapidly become addicted to it.

Sax (1979, p. 80) similarly notes that "the relative simplicity of this discriminative [i.e., correlational] approach has appealed to many graduate students [and to at least some faculty] who, once they have mastered the technique of computing correlations or other statistics, have used these methods

without due regard for the value of their research proposals."

Notwithstanding the severity of these criticisms, however, it is clear that some factor analytic applications have proven very helpful to social scientists. For example, the methods are very useful in evaluating instrument validity (Thompson & Pitts, in press). Thus Nunnally (1967, p. 100) notes that some researchers have referred to construct validity as "factorial validity." Factor analysis can also be usefully applied as a prelude to analysis of variance procedures and their analogues (Morrow & Frankiewicz, 1979). For example, McCulloch and Thompson (1981) report a study in which factor analysis was performed partly in order to conserve degrees of freedom in a multivariate analysis of variance. Finally, factor analysis can also be a potent aid to theory building, as Guilford (1959) has demonstrated.

However, factor analysis may be most helpful if viewed as a procedure which protects the measurement integrity of social science instruments. This last consideration has typically been overlooked in the literature, perhaps because the logic of the argument requires the integration of several, albeit well-known, statistical theorems. As Thompson (1980a, p. 547) notes, "factor analysis represents an attempt to identify the factors embedded in a matrix of indices of association, i.e., either a correlation or a variance-covariance matrix." What syllogism dictates that the factors extracted from a matrix of indices of association must be built from "true score" variance?

The major premise of the syllogism is that error variance is randomly distributed, by definition. As Kerlinger (1973, p. 445) notes, "any obtained score is made of two components, a 'true' component and an error component." The

error component can be defined as one minus the reliability of the measure. That is, it "is some increment or decrement resulting from several of the factors responsible for errors of measurement." As Kerlinger (1973, p. 443) explains, "random or error variance is self-compensating: scores tend now to lean this way, now that way. Errors of measurement are random errors."

The minor premise of the syllogism is the error variance, since it is randomly distributed, attenuates indices of association. As Allen and Yen (1979, p. 75) suggest, "since a test score cannot correlate more highly with any other variable than it can with its own true score, the maximum correlation between an observed test score and any other variable is $\sqrt{r_{xx'}}$ [i.e., the square root of the test's reliability]." Thus social scientists must be concerned that measurement error will attenuate statistical results:

Conceive of the relation between two variables, intelligence and competence. We want to know, say, what role intelligence plays in a certain kind of competence. If one or both of the measures of intelligence and competence are not reliable, then it is not possible to determine accurately how they are related--or "correlated," as it is said. The magnitude of the [true] relation may be high, but if one or both of the measures are not reliable, the calculated relation will be low simply because of lack of reliability. (Kerlinger, 1979, p. 137)

The fact that random variables are uncorrelated can be heuristically demonstrated by correlating repeated rolls of different pairs of dice; the fact that social scientists sometimes correct calculated correlation coefficients

(Guilford, 1965, p. 480) also testifies to the existence of this phenomenon.

The conclusion of the syllogism is that since the "common variance" represented by indices of association tends to represent reliable variance (see Kerlinger, 1973, p. 665), and since it is from these indices that factors are extracted, it follows that factors tend to be constructed from the "true score" components of variables. As Gorsuch (1974, p. 94) puts it, "the unreliable portion of the [variable's] variance is, by definition, random error and therefore could not be related to any factor whether common or specific." This does not mean that factors are sacrosanct from a measurement viewpoint; other strategies can be taken even in a factor analytic context to protect measurement integrity (Thompson, 1981b, 1981c; Thompson & Frankiewicz, 1980). Still, even less erudite factor analytic methods do tend to protect measurement integrity, and this should be of some solace to social scientists, since abstract social science constructs usually can only be indirectly measured and thus tend to be that much less than perfectly reliable.

Overview of Two-Mode Methods

Whether factors are extracted from a matrix of indices of association with correlation coefficients or covariances off-diagonal, the most frequently performed analyses construct the association matrix from a two dimensional raw data matrix. The two dimensions which define this raw data matrix have come to be called "modes." Social science research can involve some combination of three modes: individuals from a population of people, variables from a universe of variables, or occasions from the universe of points in time. Cattell (1952) has summarized permutations of these modes as a series of factor analytic

"techniques." Table 1 presents studies in which the techniques have been applied or discussed.

INSERT TABLE 1 ABOUT HERE.

Purpose of the Paper

Just as today psychometricians struggle to understand recently developed confirmatory factor extraction (Joreskog, 1969) and confirmatory factor rotation (Frankiewicz & O'Sullivan, 1980) procedures, the 1930s were years during which researchers debated the merits of various exploratory factor analytic methods. For example, at about the same time Stephenson (1935) and Thomson (1935) developed what is now commonly referred to as Q-technique factor analysis. The years which immediately followed saw a lengthy interchange of views (Stephenson, 1953, p. 13) about the nature and relative merits of Q as against R technique analysis (Burt & Stephenson, 1939; Cronbach & Gleser, 1954).

Burt (1941) finally clarified the relationship between R and Q solutions by advancing what has been termed the "reciprocity" principle. Burt demonstrated that R and Q solutions from a "doubly-centered" raw data matrix could be readily related. A "doubly-centered" raw data matrix is one for which both columns and rows have unit variance and means equal to zero. Table 2 presents an example of a "doubly-centered" raw data matrix.

INSERT TABLE 2 ABOUT HERE.

To be specific, it has been shown (e.g., Jones, 1967) that except for a variance adjustment, the factor scores from one analysis are the same as the factor pattern coefficients from the other analysis. The relationship applies,

however, only to scores and coefficients derived from unrotated and thus orthogonal factors. Strategies for linking R and Q results in other cases have also been discussed (Holley, 1970; Thompson, Frankiewicz & Ward, 1981).

The purpose of this paper is to explore an extension of the reciprocity principle which may have important applications in some research situations. Specifically, the paper discusses analysis of doubly-centered data matrices with a view toward supporting inquiry regarding both variable dimensions and types of persons. This strategy is responsive to the apparently "irresistible but inappropriate impulse of some researchers to generalize information about persons' scores on item dimensions [from R solutions] to statements about types of persons" (Thompson, Frankiewicz & Ward, 1981, p. 4). Analysis of an actual data set will provide a concrete basis for this discussion.

Heuristic Application

In 1975 Miller, Thompson, and Frankiewicz reported development of an instrument, the Multiple Teachers Factor (MTF) Survey, which measures perceptions of four "teacher-types:"

- 1) "Of all the teachers I have ever had, the one teacher whom I thought was the best teacher....,"
- 2) "Myself as teacher....,"
- 3) "Of all the teachers I have ever had, the one teacher whom I thought was the worst teacher....," and
- 4) "Of all the teachers I have ever had, the one teacher from whom I learned the most content material..."

Subjects are asked to rate each referent using 24 semantic differential scales.

For each scale, subjects respond on "unnumbered graphic scales" and these are subsequently scored using a relatively large number of scale intervals in order to maximize reliability (Thompson, 1981b).

For the purposes of this study, 145 undergraduate education students completed the MIF Survey. These data were factor analyzed in a conventional R-technique principal components analysis. The factors from the analysis are presented in Table 3. The factors are very similar to those derived in previous studies (e.g., Thompson & Miller, 1978). The factors appear to measure, respectively, perceptions of teacher warmth, teacher rigor, teacher intellect, and teacher assertiveness.

INSERT TABLE 3 ABOUT HERE.

The Table 3 results actually represent factors based upon analysis of a raw data matrix in which the columns have been standardized into Z scores. Standardizing columns has no effect upon calculated correlation coefficients, since correlation coefficients are entirely insensitive to linear transforms, including subtracting column means from each column entry and then dividing each result by column standard deviations. However, many computer programs routinely perform this standardization because it simplifies the calculation of the matrix of interrow correlation coefficients.

The same raw data matrix was also "doubly-centered" and then factor analyzed. That is, the means and standard deviations of the rows were computed. Then each row mean was subtracted from each row entry and the results were divided by the row's standard deviation. After the factor analysis of this "doubly-centered" raw data matrix, the cosines of the angles among the factors

from the "singly-centered" solution and from the "doubly-centered" solution were computed. They are presented in Table 4.

INSERT TABLE 4 ABOUT HERE..

The diagonal of the cosine matrix suggests that some of the factors from the two solutions correspond, and some do not. This result is not surprising. The row standardization represents an area transform of the columns of the raw data matrix which does change the interrow correlation coefficients. This, of course, tends to result in different factors.

The problem, then, is to identify a procedure which respects the integrity of the "singly-centered" factors while attempting to produce results which have something of the character of a "doubly-centered" solution. The problem can be resolved by modifying the conventionally-employed algorithm for computing factor scores. Although many researchers unfortunately are not aware of it, several algorithms for computing or estimating factor scores are available, and each has somewhat different properties (Frankiewicz, 1971). However, the most commonly employed algorithm (Horst, 1965, p. 479) computes least squares estimates of factor scores from an orthogonal principal components solution via:

$$Y_{N \times F} = Z_{N \times V} S_{V \times F} (S'_{F \times V} S_{V \times F})^{-1}$$

where N is the number of persons,

V is the number of variables,

F is the number of factors,

Z is a matrix of Z scores,

S is the matrix of factor structure coefficients, and

Y is the resulting matrix of standardized factor scores.

The matrix applied to the Z scores has also been termed a "factor scores coefficient" or "factor score weight" matrix. The algorithm for deriving this matrix, W, is simply:

$$W_{VXF} = S_{VXF} (S'_{FXV} S_{VXF})^{-1}$$

An analog to factor scores from a "doubly-centered" solution, which, however, honors the integrity of the "singly-centered" solution, can be derived by applying the W matrix from a "singly-centered" solution to a "doubly-centered" Z matrix. This will produce a set of standardized factor scores which may be designated Y'.

Table 5 presents the means and their 95% confidence intervals for the factor score ratings of each referent on each factor using both factor score methods. The tabled results make clear that in some cases the results are quite comparable across solutions. For example, both sets of scores indicate that the subjects perceived themselves to be more warm than even the best teacher they had ever had. On the other hand, the results also make clear that the two procedures can produce dramatically different results. For example, the conventional Y factor scores suggest that the worst teacher is somewhat lacking in assertiveness. The Y' factor scores suggest more strongly that this may even be the distinguishing feature of this teacher-type.

INSERT TABLE 5 ABOUT HERE.

Discussion

"Doubly-centered" factor scores, i.e., Y' , may support more accurate comparisons of factor scores across referents or subject groupings. Table 6 presents the factor scores from the first subject. Coincidentally, this subject's responses seem to reflect the dynamics of the data set as a whole and thus can be consulted to help explain why the two procedures differ.

INSERT TABLE 6 ABOUT HERE.

Standardizing rows to produce the Y' factor scores has the effect of transforming all the ratings of the different referents into a standardized metric. In a figurative sense, the Y' factor scores can be compared across referents just as regression beta weights can be compared across variables (but regression B weights can not be compared).

The Table 6 results suggest that the subjects' responses had a similar metric for ratings of all referents on the factor, warmth. However, the ratings of the "worst teacher" on the remaining factors were very constricted, i.e., had a small standard deviation relative to ratings of the other teacher-types. This makes some sense--the worst teacher is a superlative or extreme teacher-type which should produce a more constricted response pattern for most subjects on most scales. These differences in the standard deviations of responses across referents or groupings of subjects must be considered if the researcher wishes to compare factor scores across referents or the subject groups.

Summary

The paper has presented a procedure for calculating factor scores when subjects have rated several referents or the researcher wishes to compare factor

scores for different subject groups. The procedure involves application of a conventional factor score coefficient matrix to a "doubly-centered" matrix of standardized scores. The paper discussed the procedure by presenting an analysis of 145 education students' perceptions of four types of teachers.

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Table 1
Two-Mode Techniques

Technique	Mode Factored	Mode Over Which Association Computed	Example
R	Variables	Individuals	Thompson, 1980b
Q	Individuals	Variables	Thompson, 1980c
S	Individuals	Occasions	--
T	Occasions	Individuals	Frankiewicz & Thompson, 1979
O	Occasions	Variables	Jones, Thompson & Miller, 1980
P	Variables	Occasions	Cattell, 1953

NOTE: "Mode Factored" constitutes columns of the raw data matrix; "Mode Over which Association Computed" constitutes the rows of the raw data matrix.

Table 2
Doubly-Centered Data Matrix

Person	Variable			Row	
	I	II	III	Mean	SD
A	1.2	-1.2	0.0	0	1
B	0.0	1.2	-1.2	0	1
C	-1.2	0.0	1.2	0	1
Mean	0	0	0		
SD	1	1	1		

Table 3

Varimax Rotated Pattern Coefficients

Variable	I	II	III	IV
Intelligent	.32	.14	.74	-.22
Undirected	-.40	-.26	.03	.49
Honest	.66	-.02	.40	.00
Scholarly	.12	.26	.75	-.18
Personable	.88	.02	.16	-.02
Easy	.09	-.25	-.21	.68
Distant	-.68	.06	.10	.24
Informed	.40	.15	.74	-.14
Docile	-.09	.05	.00	.68
Caring	.92	.02	.21	.02
Systematic	.00	.68	.21	-.03
Effective	.79	.18	.38	-.13
Profound	.28	.33	.48	.11
Simple	.08	-.03	-.35	.67
Concerned	.90	.10	.20	-.02
Humane	.84	.00	.20	.12
Motivating	.86	.11	.31	-.12
Analytical	.16	.51	.50	-.04
Knowledgeable	.36	.18	.75	-.19
Humorous	.79	.04	.19	.04
Exacting	.02	.81	.20	-.08
Rigorous	-.01	.79	.07	-.14
Enlightened	.68	.16	.48	-.04
Warm	.93	-.03	.15	.00

Table 4
Cosines Among Factor Axes

Solution	Uncentered Solution			
	I	II	III	IV
I	.90	.28	.12	-.31
II	.38	-.76	.17	.49
III	-.20	-.13	.90	-.36
IV	-.02	-.57	-.38	-.73

Table 5
Means and Confidence Intervals

Factor Scores from "Singly-Centered" Data Matrix					
Referent	Warmth	Rigor	Intellect	Assertiveness	
Best	-0.55 (-0.62 to -0.48)	+0.00 (-0.14 to +0.14)	-0.28 (-0.38 to -0.19)	+0.18 (+0.04 to +0.31)	
Myself	-0.63 (-0.68 to -0.58)	+0.08 (-0.06 to +0.22)	+0.20 (+0.09 to +0.31)	-0.22 (-0.37 to -0.07)	
Worst	+1.47 (+1.35 to +1.58)	+0.18 (-0.01 to +0.38)	+0.48 (+0.23 to +0.73)	-0.20 (-0.40 to -0.01)	
Content	-0.28 (-0.37 to -0.20)	-0.26 (-0.43 to -0.10)	-0.40 (-0.50 to -0.30)	+0.24 (+0.09 to +0.40)	

Factor Scores from "Doubly-Centered" Data Matrix					
Referent	Warmth	Rigor	Intellect	Assertiveness	
Best	-0.47 (-0.55 to -0.39)	+0.28 (+0.15 to +0.40)	+0.03 (-0.08 to +0.14)	+0.57 (+0.48 to +0.66)	
Myself	-0.77 (-0.84 to -0.71)	+0.34 (+0.20 to +0.48)	+0.63 (+0.50 to +0.76)	+0.25 (+0.15 to +0.35)	
Worst	+1.38 (+1.25 to +1.50)	-.056 (-0.76 to -0.37)	-0.51 (-0.75 to -0.27)	-1.41 (-1.57 to -1.26)	
Content	-0.14 (-0.23 to -0.04)	-0.05 (-0.21 to +0.10)	-0.15 (-0.28 to -0.03)	+0.60 (+0.51 to +0.69)	

Table 6
First Subject's Factor Scores

<u>Y</u> for Subject 1				
Referent	Warmth	Rigor	Intellect	Assertiveness
Best	-0.58	+0.72	-0.67	-0.61
Myself	-0.40	-0.52	+0.59	-0.29
Worst	+1.52	+2.03	+0.18	+0.85
Content	-0.86	-0.72	-0.18	-0.80

<u>Y'</u> for Subject 1				
Referent	Warmth	Rigor	Intellect	Assertiveness
Best	-0.51	+1.21	-0.50	-0.01
Myself	-0.87	-0.51	+1.03	+0.47
Worst	+1.29	+1.35	-1.90	-1.09
Content	-0.55	-0.27	+0.51	+0.12