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ABSTRACT

This document consists of two major sections. The first of these is actually three units that look at applications of arithmetic to business. Basic pre-algebra mathematics is an assumed prerequisite. These units are titled: 367-The Real Numbers; 368-Fractions and Their Uses: Ratio and Proportion; and 369-Exponents: Powers and Roots. Each module concludes with a test. This material is designed for individual study, and seeks to review arithmetic fundamentals and present topics in elementary algebra which have direct application to problems in business and to the use of the computer to solve such problems. The second section of this document is a unit that views applications of decimal arithmetic and area formulas to business. The material was written to give interior design students a trial run through computations which underlie nearly every project in residential design. It is one of a set of modules prepared for a course in business mathematics, but is viewed as also appropriate for inclusion in an interior design course. (MP)

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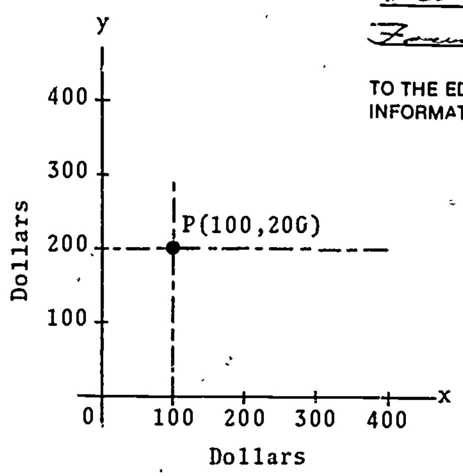
UNITS 367-369

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT

CONCEPTS OF MATHEMATICS FOR BUSINESS: BACKGROUND MATHEMATICS

by Martha F. Kasting



APPLICATIONS OF ARITHMETIC TO BUSINESS

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CONCEPTS OF MATHEMATICS FOR BUSINESS:
BACKGROUND MATHEMATICS

by

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Intermodular Description Sheet: *UMAP Units 367-369*

Title: CONCEPTS OF MATHEMATICS FOR BUSINESS: BACKGROUND MATHEMATICS

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Review Stage/Date: III 5/30/80

Classification: APPL ARITH/BUSINESS

Prerequisite Skills:
1. Basic pre-algebra mathematics.

Output Skills:

Part 1 (Unit 367)
1. Be able to classify numbers as being a) integers, fractions, or neither; b) rational or irrational.
2. Identify numbers with points on a line, use the symbols for less than and greater than, and arrange numbers in an increasing or decreasing sequence.
3. Work with signed numbers.
4. Find the distance between any two numbers on the line.
5. Work with a variable.
6. Use parentheses in writing mathematics expressions.
7. Use the commutative, associative, and distributive laws for addition and multiplication.
8. Write an integer as a product of prime numbers.

Part 2 (Unit 368)
1. Identify equivalent fractions.
2. Reduce fractions.
3. Add or subtract fractions.
4. Multiply or divide one fraction by another.
5. Find a constant ratio of one quantity to another.
6. Solve equations of proportionality with have one variable.
7. Find a constant of proportionality.
8. Graph equations of proportionality which have two variables.

Part 3 (Unit 369)
1. Multiply a number by itself n times and use exponential notation to indicate this operation.
2. Use negative exponents to denote reciprocals of numbers.
3. Find the square root of a perfect cube.
4. Find one cube root of a perfect cube.
5. Solve equations involving squares and square roots.
6. Use the laws of exponents for multiplication and division.
7. Multiply two binomials.
8. Factor a quadratic polynomial.
9. Simplify quotients of polynomials.
10. Work with length, area and volume of square- and cube-shaped objects.

Other Related Units:

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists, and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

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INTRODUCTION

This, the first of a set of modules containing lecture notes for individual study of business mathematics, has as its objectives:

(i) to review the fundamentals of arithmetic.

(ii) to present topics in elementary algebra which have direct application to problems in business and to the use of the computer to solve such problems.

The supplementary reading assignments refer you to Schaum's Outline Series, College Algebra, by Murray Spiegel (McGraw-Hill). Most of the material in Schaum is far more complicated than the mathematics you will encounter here. However, the text is recommended for its many solved problems which demonstrate all the mathematical aspects of the problems encountered in one semester introductory course in business mathematics.

If you seek extra help with the work at hand, you may prefer to consult one of the many textbooks in basic mathematics or introductory algebra available in campus bookstores or libraries.

Each module has a companion test which may be found on the last pages of the module. When you can answer all of a test's questions correctly, you are ready to move on to the next module. When you understand all of the material in these modules, you are ready for the next series of modules in this sequence, "The Mathematics of Finance," which is also available from UMAP.

There is a companion module to these two, "An Introduction to BASIC", which was written to teach you how to use the computer to solve problems in accounting, marketing and finance. It is also available from UMAP.

1. THE REAL NUMBERS

1.1 Sets of Numbers

The *natural* numbers are those we count with. We can collect them in a set, call the set N, and write it thus:

$$N = \{1,2,3,4,5,\ldots,\}.$$

The curly brackets enclose the members of the set N. The three dots "..." indicate that not all the members of the set have been listed. The comma which follows the three dots indicates that this set contains infinitely many numbers.

If we add the number zero to N we get the set of *whole* numbers. If we also add the negative of each natural number to N, we get the set of *integers*, I:

$$I = \{,\ldots,-5,-4,-3,-2,-1,0,1,2,3,4,5,\ldots,\}.$$

The *positive integers* are the natural numbers.

The set of *rational numbers* consists of all numbers which can be written as the quotient of two integers, m/n, with n not equal to zero. These are the fractions.

$$\frac{1}{3} \quad \frac{3}{10} \quad \frac{889}{5} \quad \frac{-72}{73} \quad \frac{0}{4} \quad \text{are rational numbers.}$$

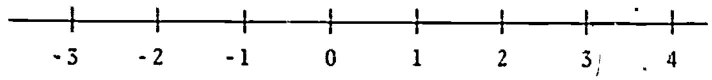
Integers are rational numbers. For example, 6 equals $\frac{6}{1}$.

Numbers which cannot be expressed as the quotient of two integers are *irrational numbers*. Examples are π (pi) and $\sqrt{2}$, the square root of two. We'll limit this study to the rational numbers.

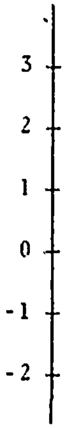
The set of *real numbers* is the collection of rational and irrational numbers.

1.2 The Real Number Line

We can associate each real number with a point on a line. Draw a horizontal line. Choose a point on this line, call it the origin, and assign to it the number zero. Locate the positive integers in order one unit apart to the right of zero and the negative integers in order one unit apart to the left, like this:



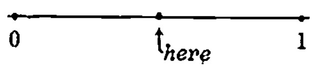
If we choose to place the integers on a vertical line, we locate the negative integers below and the positive integers above the origin.



Simple geometric constructions (or a good ruler) will locate the points on the number line corresponding to rational numbers. It can be proved that corresponding to each real number there is one and only one point on the line and, conversely, to every point on the line there corresponds only one real number.

$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{500}{1000} \quad \frac{-3}{-6} \quad .5$$

are five names for the *same* rational number, which is located halfway between zero and one:



1.3 Ordering the Real Numbers

Any two distinct numbers can be compared by the ordering relation "less than", according to their positions on the number line:

Two is less than five, and we write $2 < 5$.
Zero is less than three, and we write $0 < 3$.
Minus two is less than one, and we write $-2 < 1$.

A given number is less than any other number located to the right of it on a horizontal line.

We reverse the direction of the symbol for "less than" to obtain a symbol for "greater than":

$$3 > 2 \text{ since } 2 < 3 .$$

We may arrange numbers in a chain using these symbols. Suppose, for example, we are given the set $S = \{1,-1,19,0,-43\}$. Using the symbol "<" we form the chain

$$-43 < -1 < 0 < 1 < 19 .$$

When these numbers are ordered by "<" and then listed in this order, they form an *increasing sequence*

$$-43, -1, 0, 1, 19 .$$

If instead, we order the set S using the symbol ">", we obtain the chain

$$19 > 1 > 0 > -1 > -43$$

and corresponding to this chain, the *decreasing sequence*

$$19, 1, 0, -1, -43 .$$

The symbol "$\leq$" means "less than or equal to", the symbol "$\geq$" means "greater than or equal to"

Examples: $2 \leq 3$ since $2 < 3$.
also $3 \leq 3$ since $3 = 3$.

1.4 Arithmetic of Signed Numbers

A. Addition and Subtraction

As you add negative and positive numbers,

Think: (+) credits plus (-) debits
(+) deposits plus (-) checks
(+) profits plus (-) losses

Thus $15 profit + $8 profit = $23 profit
15 + 8 = 23

$15 profit + $-8 loss = $7 net profit
15 + (-8) = 7
15 + (-8) = 15 - 8 = 7
(subtraction)

$-15 loss + $8 profit = $-7 net loss

$-15 + 8 = -7$

$-15 loss + $-8 loss = $-23 loss

$-15 + (-8) = -23$

$-15 - 8 = -23$

B. Multiplication and Division

For now, there are just two rules to remember:

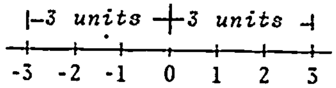
1. The product or quotient of two numbers with the *same* sign is positive.
   $(1)\cdot(10) = 10$ and $(-1)\cdot(-10) = 10$
2. The product or quotient of two numbers with *opposite* signs is negative.
   $(1)\cdot(-10) = -10$ and $(-1)\cdot(10) = -10$

1.5 Absolute Value: Distance on the Number Line

Let's find a method for computing the distance between any two distinct numbers on the number line. *This distance will always be positive.*

Suppose one of the numbers is zero. Each positive number is the same distance from zero as its negative:

[The distance between -3 and 0 equals 3.]



[The distance between 3 and 0 equals 3.]

This common distance is called the *absolute value* of the number, and is denoted in the textbooks by two vertical bars:

$|-3| = 3$ $|3| = 3$ $|0| = 0$

$|-10| = 10$ $|10| = 10$ (the distance between 0 and itself)

In BASIC computer programming language the abbreviation ABS(X) is used to denote "the absolute value of the number X";

ABS(-3) = 3 ABS(3) = 3 ABS(0) = 0

ABS(-10) = 10 ABS(10) = 10

Remember, the absolute value of a number is *never* negative.

Now suppose we wish to find the distance between two nonzero numbers A and B. To do this

a. subtract one number from the other,

b. take the absolute value of the result.

In other words, find either ABS(A-B) or ABS(B-A). They are equal:

Examples: ABS(4-37) = ABS(-33) = 33

or. ABS(37-4) = ABS(33) = 33 .

Find the distance between -2 and 3 on the line:

ABS(-2-3) = ABS(-5) = 5 or ABS(3-(-2))

= ABS (3+2) = 5.

Find the distance between -6 and -8 on the line:

ABS(-6-(-8)) = ABS(-6+8) = ABS(2) = 2 .

The absolute value function has many uses in the money business. Suppose Jim's bank balance was $175 on October 1 and $-17 two days later. How much had been taken from his account? (What is the "distance" between +175 and -17?)

ANSWER: ABS(175-(-17)) = ABS(175+17) = ABS(192)

= 192 dollars.

1.6 Variables

Math textbooks commonly label sets with capital letters and members of a set with small letters. If we say that n is a member of the set $I = \{,\ldots,-3,-2,-1,0,1,2,\ldots,\}$ we mean that n is an integer. n is called a *variable*, since it might represent any number in the set.

Equations involving n are statements which restrict the values n may assume. To solve an equation for n, find the value(s) n must have in order to make the statement true.

If $n + 5 = 15$ then $n =$ ____.
If $n + 1 = 1$ then $n =$ ____.
If $n + 1 = 0$ then $n =$ ____.
If $3n + 1 = 28$ then $n =$ ____.

ANSWERS: 10, 0, -1, 9

Let's solve the last equation for n step by step:

$$3n + 1 = 28$$
$$3n + 1 - 1 = 28 - 1 \quad \text{(subtract 1 from each side of the equation)}$$
$$3n = 27$$
$$\frac{3n}{3} = \frac{27}{3} \quad \text{(divide each side by 3)}$$
$$n = 9$$

1.7 A Role for Parentheses in Mathematics

What is wrong with the expression

$n = 4 - 3 + 2$?

The answer is that without some prior agreement you cannot be sure what value n has. See for yourself:

a. Subtract 3 from 4 and add 2 to the result: ____. Your answer should be 3.

b. Add 2 to 3 and then subtract this sum from 4: ____. Your answer should be -1.

The value you get for n depends upon the order in which you add and subtract. You can specify this order if you use parentheses.

Parentheses indicate that you perform the enclosed operation first.

a. $(4-3) + 2 = 1 + 2 = 3$
b. $4 - (3+2) = 4 - 5 = -1$

There is no ambiguity in either of the expressions after the parentheses have been inserted.

The computer must have unambiguous instructions in order to function, so we use parentheses liberally in communicating with the machine. If parentheses are absent, the computer performs additions and subtractions in parade order from left to right.

Problems for you. Compute x, y, and z.

$x = (16-8) + 10$ $\qquad x =$ ____
$y = 16 - (8+10)$ $\qquad y =$ ____

Now play computer:

$z = 16 - 8 + 10$ $\qquad z =$ ____

ANSWERS: 18, -2, 18

When more than one set of parentheses appears in an expression, remember this rule:

Always work from the inside of the innermost parentheses out.

Examples:

$$\begin{aligned} 2 - (3+((4\cdot5)-6)) &= 2 - (3+(20-6)) \\ &= 2 - (3+14) \\ &= 2 - 17 \\ &= -15 \end{aligned}$$

Problem: Verify that

$6 - (3+((8/2)-1)) = 0$.

$6 - (3+((8/2)-1)) =$ ____

You can connect the "ears" for clarity; then work from the inside out.

1.8 Rules of Addition and Multiplication

The operations of addition and multiplication are governed by the following important rules.

Let a, b, and c be any real numbers.

A. The *commutative laws* state that

$a + b = b + a$ and $a \cdot b = b \cdot a$

Examples:

$2 + 4 = 4 + 2$ and $2 \cdot 4 = 4 \cdot 2$

Note:

Subtraction and division are *not* commutative:

$2 - 4$ does not equal $4 - 2$

$2 \div 4$ does not equal $4 \div 2$.

B. The *associative laws* state that

$a + (b+c) = (a+b) + c$ and $a(bc) = (ab)c$

Notes:

(i) Subtraction and division are *not* associative:

$4 - (3-2)$ does not equal $(4-3) - 2$

$12 \div (6 \div 2)$ does not equal $(12 \div 6) \div 2$

(ii) We may choose one of three symbols or no symbol at all to indicate multiplication:

$a \times b$, $a \cdot b$, $a * b$, ab, all mean "a times b".

(iii) We may choose one of three ways to indicate division:

$a \div b$, a/b, $\frac{a}{b}$, all mean "a divided by b"

The commutative and associative laws for addition and multiplication enable us to make a list of numbers and add them together or multiply them in any order without affecting the result. The parentheses may be dropped. However, parentheses are essential to give unambiguous meaning to problems involving subtraction, division or mixtures of operations.

C. The *distributive law* is the major rule for multiplying and factoring a sum of numbers. It states that:

$a(b+c) = ab + ac$

Examples:

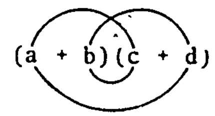
$2(3+4) = 2 \cdot 7 = 14$ and
$2 \cdot 3 + 2 \cdot 4 = 6 + 8 = 14$

$x(3+4) = x \cdot 3 + x \cdot 4$	Distributive Law
$= 3x + 4x$	Commutative Law
$2z + 3z = (2+3)z$	Distributive Law
$= 5z$	Addition
$3ab + ac = a(3b+c)$	Distributive Law (factoring)
$6ab - 8a = 2a(3b-4)$	more factoring
$14\,abcd - 21ab = 7ab(2cd-3)$	again more factoring

Example: Multiply one binomial by another:

$(a+b)(c+d) = (a+b)c + (a+b)d$
$= ac + bc + ad + bd$

or, make a face:



Example: Multiply the binomials $2 + a$ and $3 + b$.

$(2+a)(3+b) = 2 \cdot 3 + a \cdot 3 + 2 \cdot b + a \cdot b$
$= 6 + 3a + 2b + ab$

1.9 Primes and Composite Numbers

Perhaps you noticed we sneaked the procedure called *factoring* into the preceding paragraph. You may want some practice with factoring before beginning Part 2.

This section is intended to induce you to practice, and incidentally to explore the interesting structure of the positive integers.

First, a definition or two:

(1) In the expression $ab = c$ the number c is the *product* of a and b, the numbers a and b are *factors* of c.

(2) If p is any positive integer greater than 1, then p is a *prime* number if the only positive integer factors of p are itself and 1.

$3 = 3 \cdot 1$ is a prime. 1 and 3 are its only factors.

$11 = 11 \cdot 1$ is also a prime.

$6 = 6 \cdot 1$, but also $6 = 2 \cdot 3$, so 6 is *not* a prime.

(3) Positive integers that are not primes are called *composites*.

$6 = 2 \cdot 3$ $\quad$ $39 = 3 \cdot 13$

$81 = 3 \cdot 3 \cdot 3 \cdot 3$ are composites.

Next, an ancient game. About 200 B.C., a Greek astronomer named Eratosthenes devised a number "sieve" to sift out the composites from the primes. Here is the procedure to apply to the table below.

Cross out the 1. 1 is not a prime. Do not cross out 2, but cross out all larger numbers divisible by 2. Go to 3. Do not cross out 3, but cross out all larger numbers divisible by 3. Go to 5 (4 has been crossed out.) Do not cross out 5, but cross out all other numbers divisible by 5. Continue this process for every integer in the table that has not been crossed out. The numbers that remain uncrossed will be the primes less that 100. The crossed-out integers are the composites.

Count the primes in your table. Do you have 25 of them?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Finally, the factoring practice. A composite number can be factored into prime factors, that is, it can be written as the product of primes. Write the following composite numbers as the product of primes. Use intermediate steps. The first two are worked out for you.

$48 = 4 \cdot 12 = 4 \cdot 4 \cdot 3 = \underline{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$

$75 = 3 \cdot 25 = \underline{3 \cdot 5 \cdot 5}$

10 = ____________________

36 = ____________________

64 = ____________________

87 = ____________________

100 = ____________________

108 = ____________________

144 = ____________________

343 = ____________________

Partial Answers:

$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

$343 = 7 \cdot 7 \cdot 7$

ASSIGNMENT:

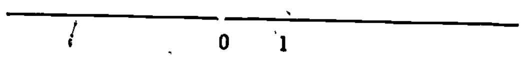
If you have digested this material, you are ready to take Test 1. For further study and many examples, read Schaum's Outline: College Algebra, Chapter 1, except for exponents and powers, and fractions.

TEST 1

Work Test 1 without a calculator.

1. On the number line below the location for zero and one is indicated. Locate and label points corresponding to the numbers:

   3, -1, 3.5, -2, 1.5 .



2. Compare each pair of integers by inserting the correct symbol, <, =, or > between them.

-546	-204	-638	-422
575	-268	-371	-720
-908	697	243	387

3. Arrange the following numbers in an increasing sequence:

   60, -69, -34, 86, 6, 1, 36, 21

4. Add or subtract:

-23 + 35	71 + (-24)
-37 - 69	62 - (-24)
60 + 31	-70 + (-26)
66 - 42	-82 - (-26)

5. Find the final bank balance after the transactions tabled below:

Checks	Deposits	Balance
		1500.00
-71.40		
	72.41	
	14.88	
-57.38		
	50.61	
	21.65	
-5.50		
-52.60		
	55.57	
	55.83	

6. Find the distance on the number line between the points a and b:

a	b	Distance
22	28	______
-12	-11	______
-12	22	______
28	-11	______

7. Find abs(a-b):

a	b	abs(a-b)
-38	-16	______
33	30	______
33	-38	______
-16	30	______

8. Find n:

-12 + n = 27	n = ______
12 ÷ n = 3	n = ______
-4n + 27 = 39	n = ______
13 + n = 7	n = ______
-13 - n = 5	n = ______
-6n + 7 = 37	n = ______

9. Write the following numbers as products of primes:

171	320
104	245

2. FRACTIONS AND THEIR USES: RATIO AND PROPORTION

2.1 Equivalence of Fractions

A fraction represents the quotient of two integers:

$\frac{m}{n}$ means "m divided by n", or $m \div n$.

The numerator (or dividend) is the integer above the fraction line. The denominator (or divisor) is the integer below this line. Division by zero is never permitted. Hence, *the denominator must never be zero.*

$\frac{0}{8} = 0$ *but* $\frac{8}{0}$ *does not exist.*

Textbooks and computers "one-line" fractions, printing

m/n to mean $\frac{m}{n}$.

Thus, 2/3 means "two-thirds" or "two divided by three"

A parenthetical reminder:
18/(6-3) does not equal 18/6 - 3 .
18/(6-3) = 18/3 = 6 while
18/6 - 3 = 3 - 3 = 0 .

Always follow the computer procedure when you compute "one-liners":

Step 1: work within parentheses.
Step 2: multiply or divide, left to right.
Step 3: add or subtract, left to right.

Problems:

(i) Which of the following expressions equals 8?

36 + 12/6, 48/12 - 6, 48/(12-6)

(ii) Find x if

x = 4 + 21/7 - 5 · 6

ANSWERS: (i) only the last one
(ii) -23

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The fundamental principle of fractions states that:

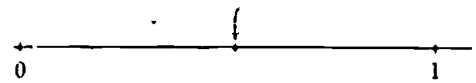
> If both numerator and denominator of any fraction are multiplied by the same nonzero number, the value of the fraction remains unchanged.

Examples:

$$\frac{1}{2} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \qquad \frac{1}{2} = \frac{1(-4)}{2(-4)} = \frac{-4}{-8}$$

The fractions 1/2, 3/6 and -4/-8 are *equivalent.* Equivalent fractions all represent the *same* rational number.

$\frac{1}{2} = \frac{3}{6} = \frac{4}{8}$ which is located *here*



on the number line.

2.2 Reducing a Fraction to Lowest Terms

The fundamental principle in the box above allows us to simplify a fraction by deleting any factors which the numerator and denominator have in common. This procedure is called *cancelling.* When the only factor common to both numerator and denominator is 1, and when the negative sign appears, if at all, in the numerator, the fraction is in reduced form, or *reduced to lowest terms.*

Examples:

(i) Reduce $\frac{60}{-105}$ to lowest terms.

$$\frac{60}{-105} = \frac{12 \cdot 5}{-21 \cdot 5} = \frac{4 \cdot 3 \cdot 5}{-7 \cdot 3 \cdot 5} = \frac{4}{-7} = \frac{-4}{7}$$

(ii) Reduce $\frac{-1}{-2}$ to lowest terms.

$$\frac{-1}{-2} = \frac{\cancel{-1} \cdot 1}{\cancel{-1} \cdot 2} = \frac{1}{2}$$

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Now you try it: reduce the following to lowest terms.

(i) $\frac{13}{52}$ (ii) $\frac{-64}{-60}$

(iii) $\frac{105}{-120}$ (iv) $\frac{121}{162}$

ANSWERS: 1/4, 16/15, -7/8, 121/162

2.3 Addition and Subtraction of Fractions

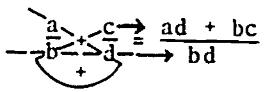
To add or subtract fractions is simple as long as the fractions have the *same* denominator. You merely add the numerators and put the result over the common denominator.

$$\frac{3}{5} + \frac{4}{5} - \frac{2}{5} = \frac{3 + 4 - 2}{5} = \frac{5}{5} = 1$$

The quickest way to add two fractions with different denominators is to use the hard-and-fast rule:

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$	Rule for Adding Fractions

Just follow the arrows:



To subtract a positive fraction, or add a negative fraction (same thing), tag the numerator with the minus sign:

$$\frac{4}{9} - \frac{1}{2} = \frac{4}{9} + \frac{-1}{2} = \frac{4 \cdot 2 + 9(-1)}{9 \cdot 2} = \frac{8 - 9}{18} = \frac{-1}{18}$$

Try it:

$\frac{3}{7} + \frac{6}{5} =$

$\frac{3}{7} - \frac{6}{5} =$

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To add a lot of fractions together without a computer you can use the equivalence property to give all the fractions a common denominator. One that always works is obtained by multiplying together all of the different denominators.

Example:

Add $\frac{5}{6}$, $\frac{1}{2}$, $\frac{-4}{3}$, $\frac{2}{5}$.

A common denominator is $6 \cdot 2 \cdot 3 \cdot 5 = 180$

$$\frac{5}{6} + \frac{1}{2} + \frac{-4}{3} + \frac{2}{5}$$

$$= \frac{5 \cdot 30}{6 \cdot 30} + \frac{1 \cdot 90}{2 \cdot 90} + \frac{-4 \cdot 60}{3 \cdot 60} + \frac{2 \cdot 36}{5 \cdot 36}$$

$$= \frac{150 + 90 - 240 + 72}{180}$$

$$= \frac{72}{180} = \frac{2}{5}$$

Note:

The *least common denominator* of the above fractions is 30, and if you can find it you can save labor on the arithmetic. The least common denominator of a set of integer denominators is the smallest integer that is divisible by each member of that set.

Your problem: Add $\frac{1}{3}$, $\frac{-2}{5}$, $\frac{3}{4}$, $\frac{1}{6}$.

ANSWER: 17/20

2.4 Multiplication and Division of Fractions

The product of two fractions is a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example: $\frac{2}{5} \cdot \frac{21}{9} = \frac{42}{45}$

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To divide one fraction (the dividend) by another (the divisor), we invert the divisor and multiply:

Example: $\frac{3}{4} \div \frac{-11}{2} = \frac{3}{4} \cdot \frac{2}{-11} = \frac{6}{-44} = \frac{3}{-22} = \frac{-3}{22}$

This procedure is justified by the following little true story:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}} = \frac{\frac{ad}{bc}}{\frac{cd}{dc}} = \frac{\frac{ad}{bc}}{1} = \frac{ad}{bc} = \frac{a}{b} \cdot \frac{d}{c}$$

2.5 Ratios

Let a and b be two numbers, with $b \neq 0$. One way to compare a with b is to look at the *ratio of a to b*, which is the fraction

$$\frac{a}{b}.$$

Sometimes this ratio is written a:b.

Example:

A finance class has 28 male and 12 female students.

a. What is the male to female ratio for the class?

The ratio is 28 to 12, or 28:12, or $\frac{28}{12} = \frac{7}{3}$

b. What is the female to male ratio?

This ratio is 12 to 28, or 12:28, or $\frac{12}{28} = \frac{3}{7}$

Note that ratios, expressed as fractions, can be reduced. Thus 12:28 is the same ratio as 3:7, since

$$\frac{12}{28} = \frac{\cancel{4} \cdot 3}{\cancel{4} \cdot 7} = \frac{3}{7}$$

Example:

Company A reported that each share of stock, with selling price \$40, earned \$5 in one year.

Company B reported that each share of its stock, with selling price \$12, earned \$2 the same year.

What is the price to earnings ratio of Company A's stock?

40 to 5, or 40:5 or $\frac{40}{5} = \frac{8}{1}$.

. . of Company B's

12 to 2 or 6:1 .

2.6 Proportions

A *proportion* is a statement of the equality of two ratios. a:b = c:d or $\frac{a}{b} = \frac{c}{d}$ are two ways of writing the same proportion. Proportions obey the rule for equality of fractions.

$\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$

This rule states that if we cross-multiply the terms in the proportion, the two products are equal:

$\frac{2}{3} = \frac{4}{6}$ since $2 \cdot 6 = 3 \cdot 4$.

$\frac{12}{20} = \frac{3}{5}$ since $12 \cdot 5 = 20 \cdot 3$.

If any three of the four terms in a proportion are known, the fourth term can always be found.

Example:

Suppose we know that $\frac{5}{4} = \frac{x}{8}$. What is x?

$5 \cdot 8 = 4 \cdot x$	cross-multiply
$40 = 4x$	
$\frac{40}{4} = x$	divide both sides of the equation by 4
$10 = x$	

Example:

A grocery store charges \$4.80 for 5 dozen oranges. How much will it charge for 18 dozen oranges? \$4.80 is to 5 dozen as x is to 18 dozen, or

$$\frac{4.80}{5} = \frac{x}{18}$$

$$5x = (4.80)(18)$$

$$x = \frac{(4.80)(18)}{5}$$

$$x = 17.28$$

Since the ratio is dollars:dozen, we have that x gives us the number of dollars for 18 dozen, or x = \$17.28.

2.7 The Constant of Proportionality

The work "proportional" is often used in a way that is related to the problems we have been working. When we say "the distance traveled is proportional to time" we mean that the ratio of distance traveled (d) to time (t) is constant:

If d is proportional to t, then $\frac{d}{t}$ is constant. If we let k represent this constant value, we may write

$$\frac{d}{t} = k$$

then

$$d = kt .$$

Example:

An automobile is cruising along at a constant speed. In this case, the distance traveled will be proportional to the time. Suppose the auto travels 5 miles in 15 minutes. Can you find the constant speed?

$$\frac{d}{t} = \text{constant (speed)}$$

$$\frac{5 \text{ miles}}{15 \text{ minutes}} = \frac{5 \text{ miles}}{1/4 \text{ hour}} = \frac{4 \cdot 5 \text{ miles}}{4 \cdot (1/4) \text{ hours}} =$$

$$\frac{20 \text{ miles}}{1 \text{ hour}} \quad \text{or 20 miles/hour.}$$

2.8 The Straight Line Graph of a Proportion

Pat invests \$4000, while Mike invests \$2000 in a business partnership. They agree to split the profits in proportion to their initial investments.

If at the end of a year Pat receives y dollars in profits while Mike receives x dollars ($x > 0$), then the ratio y/x must equal the ratio 4000/2000 = 2. In symbols:

$$\frac{y}{x} = 2$$ where 2 is the constant of proportionality.

If we multiply both sides of this equation by x,

$$\frac{y}{x} \cdot x = 2 \cdot x$$

and cancel on the left,

$$\frac{y}{\not x} \cdot \not x = 2x$$

we obtain the equivalent equation,

$$y = 2x .$$

We can use this relationship between Pat's share (y) and Mike's share (x) of the profits to find the dollar value of y for any positive dollar value of x.

Make a data box for different values of x:

Mike's profits:	x =	\$100	\$200	\$300	\$400
Pat's profits:	y =				

Compute y, using y = 2x:

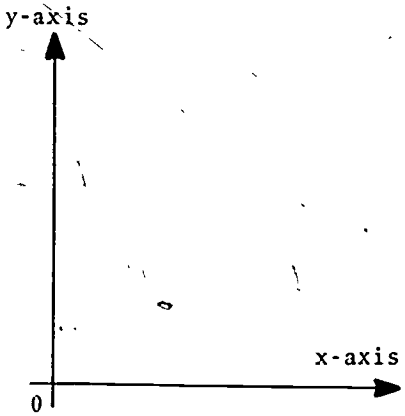
Mike's profits:	x =	\$100	\$200	\$300	\$400
Pat's profits:	y =	200	400	600	800

Now let's use the information from the data box to draw a graph of the equation y = 2x.

Draw a horizontal line and call it the *x-axis*.

Draw a vertical line and call it the *y-axis*.

Call the intersection of these two lines the *origin*, 0.



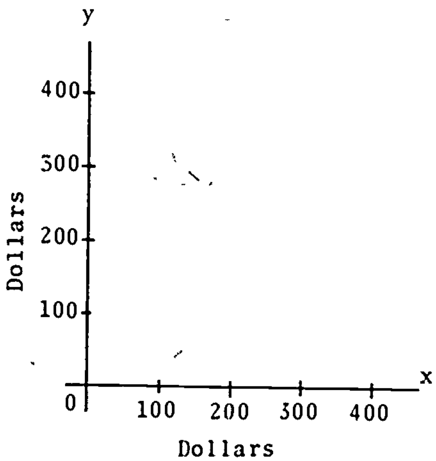
To the right of the origin on the x-axis, mark off equal units. Let each unit represent $100.

Above the origin on the y-axis, mark off equal units of $100.

Now look at the data box:

x =	$100	$200	$300	$400
y =	200	400	600	800

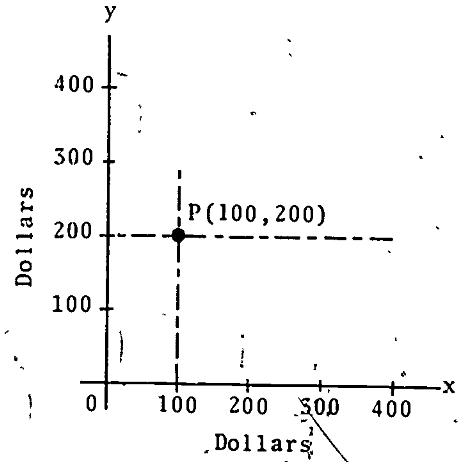
If x = $100, y = $200.



Locate 100 on the x-axis; dash a vertical line through x = 100.

Locate 200 on the y-axis; dash a horizontal line through y = 200.

The point, P, where the two dashed lines intersect represents x = 100 *and* y = 200.



We write

P = (100,200) = (x,y).

Each point (x,y) in the data box may be located in the same manner. However, the y-axis must be extended to accommodate the largest value of y, which is 800. The collection of all points (x,y) from the data box is the *graph* of the data.

These points lie on a straight line. Connect them. *The line is the graph of the equation*

y = 2x .

The sketch shows that portion of the line corresponding to values of x from zero to 400. With reference to our original problem, each point on this line segment represents

(Mike's profits, Pat's profits)

The graph is very useful. For example, you can read from the graph the answer to such questions as:

What are Pat's profits if Mike's profits are $250?

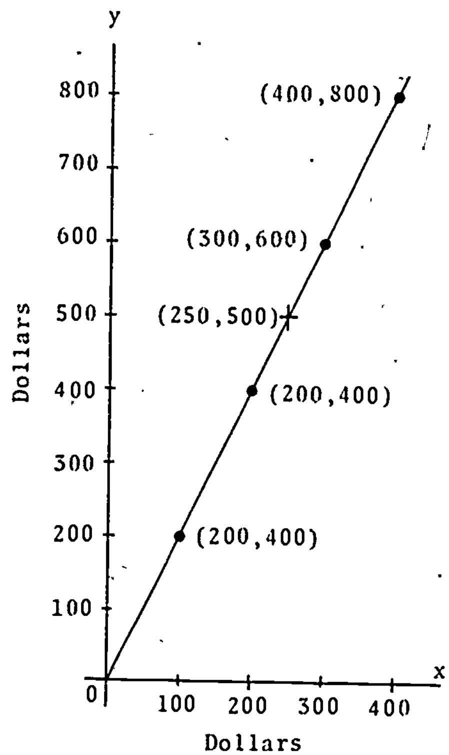
ANSWER: $500.
(Find the value of y corresponding to x = 250.)

Whenever the ratio of two quantities y and x is constant,

$$\frac{y}{x} = k$$

the points (x,y) will lie on the line whose equation is

$$y = kx .$$



ASSIGNMENT:

If you understand the material in this chapter, you should be ready to take Test 2.

For further study, Schaum's Outline: College Algebra has examples (some of which are difficult) of:

factoring: Chapter 4
fractions: Chapter 5
graphing: Chapter 10
ratio and proportion: Chapter 15.

TEST 2

1. Compute: 37(31 + 35) - 11/2 = ______
   (35 + (31 - 37)/2) + 11 = ______
   (11 - 31) - 39 = ______
   35 - (11/2 - 37) = ______

2. Reduce each fraction to lowest terms. Circle all fractions equivalent to 2/7.

   12/42   8/28

   70/-245   70/280

3. Add or subtract. Write your answer as a reduced fraction m/n.

   2/7 + 9/5 =
   5/2 - 7/9 =
   9/3 + 7/6 =
   6/9 - 3/7 =

4. Write a fraction equal to:

   51/25 multiplied by 16/44
   51/44 divided by 16/25
   36/45 multiplied by 27/50
   36/50 divided by 27/45

5. The men's department of Sam's Shoe Shop has average monthly sales of $858 while the women's department has average monthly sales of $2376.

   The ratio of men's sales to women's sales is ______.

   Expressed as a reduced fraction, this ratio equals ______.

   The ratio of children's shoe sales to men's shoe sales is 93 to 33.

   Monthly sales of children's shoes average $______.

6. A supermarket charges $5.84 for 4 dozen oranges. How much will 42 dozen oranges cost?

   ______.

7. Find x.

$x/9 = 15/45$ $\quad$ $45/x = 5/15$

$x/7 = 15/21$ $\quad$ $21/x = 3/15$

8. A drapery fabric has a selling price of $10.95 per yard. If x = the number of yards purchased and y = the total price of x yards, fill in the data box below:

x =	5/8	7/8	9	25
y =				

9. Use the values of x and y given in the data box below to find the constant of proportionality, k where $y/x = k$.

x =	-2	2	4	6
y =	-6	6	12	18

Locate the points (x,y) on the coordinate system. Draw a line through these points. The equation for the line you have drawn is

____

3. EXPONENTS: POWERS AND ROOTS

3.1 Powers of a Number and Positive Integral Exponents

Suppose you wish to find the product of 2 multiplied by itself five times. You could write this as

$$2 \times 2 \times 2 \times 2 \times 2 = 32 \quad \text{or}$$
$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32.$$

but you would save effort if you used *exponential notation:*

$$2^5 = 32 \quad \text{(two to the fifth equals 32)}$$

In this last expression the factor to be repeated, 2, is called the *base* and the raised integer 5 is the *exponent*. The exponent tells us the "power" to which we "raise" the base two, or in plain English, how many times we multiply 2 by itself.

With exponents you can simplify the products

$$5 \cdot 5 \text{ to } 5^2 \qquad x \cdot x \cdot x \text{ to } x^3$$
$$5 \cdot 5 \cdot x \cdot x \cdot x \text{ to } 5^2x^3$$
$$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \text{ to } 10^6$$

(which equals one million.)

"Three to the fourth power" is written 3^4:

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 = 81$$

"Six squared" is written 6^2 and is equal to 36. In order to be consistent we agree that "a to the first power" equals a, or $a^1 = a$ for any number a.

3.2 The Reciprocal of a Number and the Negative Integral Exponent

We use a negative exponent to denote the *reciprocal* of a number. Since the reciprocal of 2 is 1/2, we write $2^{-1} = 1/2$. Since the reciprocal of 4 is 1/4, we write $4^{-1} = 1/4$.

For any nonzero number, a, we define
$a^{-1} = 1/a$ where $1/a$ is the reciprocal of a.

Furthermore, if n is any positive integer, we define

$$a^{-n} = 1/a^n$$

Examples:

$$3^{-2} = 1/3^2 = 1/9 \qquad 10^{-3} = 1/10^3 = 1/1000$$

"Ten to the minus sixth power" is written 10^{-6} and equals 1/1,000,000.

The definition also works for fractions if we remember that the reciprocal of the nonzero fraction p/q is the fraction q/p. That is,

$$(p/q)^{-1} = q/p \qquad (2/3)^{-1} = 3/2$$

3.3 All the Integer Exponents

We have meanings for all the integer exponents except zero. To give consistent meaning to exponents we must define

$a^0 = 1$ for all nonzero numbers a.

Thus $\quad 10^0 = 1 \qquad 2^0 = 1$

$1^0 = 1 \qquad (3/4)^0 \quad 1$ and on and on.

We can summarize the meaning of integral exponents with a table.

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TABLE I

$2^4 = 16$	$5^4 =$ ____	$10^4 = 10,000$
$2^3 = 8$	$5^3 =$ ____	$10^3 = 1,000$
$2^2 = 4$	$5^2 =$ ____	$10^2 = 100$
$2^1 = 2$	$5^1 =$ ____	$10^1 = 10$
$2^0 = 1$	$5^0 =$ ____	$10^0 = 1$
$2^{-1} = 1/2$	$5^{-1} =$ ____	$10^{-1} = 1/10$
$2^{-2} = 1/4$	$5^{-2} =$ ____	$10^{-2} = 1/100$
$2^{-3} = 1/8$	$5^{-3} =$ ____	$10^{-3} = 1/1,000$
$2^{-4} = 1/16$	$5^{-4} =$ ____	$10^{-4} = 1/10,000$

The powers of 5 are left to you.

3.4 Roots and Radicals

If two numbers, a and b, satisfy the equation

$$a^n = b$$

with n a positive integer, then a *is an* n^{th} *root of b*. The n^{th} root of 0 is always 0.

A. Square Roots

The 2th root, fortunately, has a special name: square root. A positive number always has *two* square roots, while a negative number has *none* at all.

Examples:

(i) Since $3^2 = 9$, 3 is a square root of 9. Since $(-3)^2 = 9$, -3 is also a square root of 9.

(ii) Since $4^2 = 16$ and $(-4)^2 = 16$, 4 and -4 are both square roots of 16.

(iii) The two square roots of 25 are ____ and ____.

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(iv) Since $1/2 \cdot 1/2 = 1/4$ and $(-1/2) \cdot (-1/2) = 1/4$, $1/2$ and $-1/2$ are square roots of $1/4$.

(v) The two square roots of $1/9$ are ____ and ____.

ANSWERS: (iii) 5, -5

(v) 1/3, -1/3

Any positive number, b, has a positive square root and a negative square root. The positive, or principal square root of b is called a *radical*, and is denoted by $\sqrt{b}$. Let's use the radical symbol in examples i-v above.

(i) $\sqrt{9} = 3$ and $-\sqrt{9} = -3$

(ii) $\sqrt{16} = 4$ and $-\sqrt{16} = -4$

(iii) $\sqrt{25} =$ ____ and $-\sqrt{25} = -5$

(iv) $\sqrt{1/4} = 1/2$ and $-\sqrt{1/4} =$ ____

(v) $\sqrt{1/9} =$ ____ and $-\sqrt{1/9} =$ ____

Radicals appear often in equations. If we write "$y = \sqrt{16}$" then we mean y has the single positive value 4.

If we write "$y = \pm\sqrt{16}$" then we mean y has two possible values, +4 or -4.

B. Cube Roots

If $a^3 = b$, then *a is a cube root of b.*

Every number has exactly *one* real cube root. The cube root of a negative number is negative; the cube root of zero is zero; the cube root of a positive number is positive.

Examples:

(i) Since $5^3 = 125$, 5 is the cube root of 125. We write: $\sqrt[3]{125} = 5$.

(ii) Since $(-5)^3 = -125$, [that is, $(-5) \cdot (-5) \cdot (-5) = -125$], -5 is the cube root of -125. We write: $\sqrt[3]{-125} = -5$.

(iii) The cube root of 8 is ____.
The cube root of -8 is ____.

3.5 Fractional Exponents

We use an exponent to denote the *powers* of x; this exponent is always an integer:

"x squared" is written x^2

"x cubed" is written x^3

We can also use an exponent to denote the *roots* of x; but this exponent will always be a *fraction*, the reciprocal of an integer.

"The square root of x" becomes "x to the one-half power".

$$\sqrt{x} = x^{\frac{1}{2}}$$

"The cube root of y" becomes "y to the one-third power".

$$\sqrt[3]{y} = y^{\frac{1}{3}}$$

and if n is any positive integer, the nth root of *a* satisfies:

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Here is a way you can exploit factorization to find roots of a "nice" number:

If you can factor a positive number into two identical positive factors, then one of these is the square root.

$$64^{\frac{1}{2}} = \sqrt{64} = \sqrt{8 \cdot 8} = 8$$

$$(4/25)^{\frac{1}{2}} = \sqrt{4/25} = \sqrt{\frac{2 \cdot 2}{5 \cdot 5}} = 2/5$$

$$(49x^2)^{\frac{1}{2}} = \sqrt{49x^2} = \sqrt{7 \cdot 7 \cdot x \cdot x} = \sqrt{7x \cdot 7x}$$
$$= 7x$$

$$(a^2 + 2ab + b^2)^{\frac{1}{2}} = \sqrt{(a+b) \cdot (a+b)} = a + b$$

Use this technique to find:

$169^{\frac{1}{2}} =$ ________

$(1/144)^{\frac{1}{2}} =$ ________

$(81y^2)^{\frac{1}{2}} =$ ________

$(x^2 + 4x + 4)^{\frac{1}{2}} =$ ________

ANSWERS: 13, 1/12, 9y, x + 2.

If you can factor a number into three identical factors, then one of these is the cube root:

$$8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2$$

$$64^{\frac{1}{3}} = \sqrt[3]{64} = \sqrt[3]{4 \cdot 4 \cdot 4} = 4$$

$$(27y^6)^{\frac{1}{3}} = \sqrt[3]{27y^6}$$
$$= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}$$
$$= \sqrt[3]{3y^2 \cdot 3y^2 \cdot 3y^2} = 3y^2$$

Try for yourself,

$125^{\frac{1}{3}} =$ ________

$(1/8)^{\frac{1}{3}} =$ ________

ANSWERS: 5, 1/2

One more definition: If m and n are positive integers, we define

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$$

the n^{th} root of the m^{th} power of a.

Examples:

$$9^{\frac{3}{2}} = \sqrt{9^3} = \sqrt{9 \cdot 9 \cdot 9} = \sqrt{(3\cdot3\cdot3)(3\cdot3\cdot3)}$$
$$= \sqrt{27 \cdot 27} = 27$$

$$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{8 \cdot 8} = \sqrt[3]{(2\cdot2)(2\cdot2)(2\cdot2)}$$
$$= \sqrt[3]{4 \cdot 4 \cdot 4} = 4$$

Try a couple:

$16^{\frac{3}{2}} =$ ________

$27^{\frac{2}{3}} =$ ________

ANSWERS: 64, 9

3.6 Powers and Roots in Equations

Examples:

(i) Solve the following equation for x:

$x^2 = 25$

ANSWER: $x = \pm\sqrt{25} = \pm 5$ *two* values for x.

(ii) Solve the following equation for y:

$y = \sqrt{25}$

ANSWER: $y = 5$. *one* value for y.

(iii) Solve the following equation for x:

$x^3 = 8$

ANSWER: $x = 2$. *one* value for x.

(iv) Solve the following equation for y:

$y = \sqrt[3]{8}$

ANSWER: $y = 2$.

3.7 The Laws of Exponents

(i) When you multiply two factors having the *same* base, you *add* the exponents.

$$a^n \cdot a^m = a^{n+m}$$

Convince yourself:

$$2^3 \cdot 2^2 = (2\cdot2\cdot2)(2\cdot2) = 2^5 = 2^{3+2}$$

$$2^{-1} \cdot 2^1 = (1/2) \cdot 2 = 1 = 2^0 = 2^{-1+1}$$

(ii) $\frac{a^n}{a^m} = a^{n-m}$

Cancel and verify

$$\frac{2^4}{2^3} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2 = 2^{4-3}$$

$$\frac{2^3}{2^4} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{2} = 2^{-1} = 2^{3-4}$$

(iii) $(a^n)^m = a^{nm}$

Recall the use of parentheses; work from the inside out:

$$(2^3)^2 = (8)^2 = 64 = 2^6$$

$$(10^3)^{-2} = (1000)^{-2} = \frac{1}{(1000)^2} = \frac{1}{10^6} = 10^{-6}$$

(iv) $a^n b^n = (ab)^n$

Here the powers are the same, but the bases differ:

$$2^3 \cdot 3^3 = (2\cdot2\cdot2)(3\cdot3\cdot3)$$
$$= (2\cdot3)(2\cdot3)(2\cdot3)$$
$$= (2\cdot3)^3$$

The commutative and associative laws allow this.

3.8 Some Polynomials and Their Factors

An algebraic expression is a combination of numbers and letters which represent numbers.

$3a$,	$2xy^2$,	$-1/2z^3$	are monomials (one term)
$a + b$,	$2x^2 - y$,	$x - 3$	are binomials (two terms)
$ax^2 + bx + c$,	$2x - y + 7$,		are trinomials (three terms)

Let's look at a general expression of the form

$$a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$$

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where n is some nonnegative integer and $a_0, a_1, \ldots, a_n$ are real numbers. Such an expression is called a *polynomial* in x of *degree n* with real coefficients. The subscripted constants $a_0, a_1, \ldots, a_n$ are the coefficients of the corresponding powers of x.

Examples:

Polynomial	Degree
$x - 3$	1
$2x^2 + 4x - 7$	2
$x^3 - 8$	3
$x^{10} - 1$	10

The commutative and associative laws for addition and multiplication of real numbers hold for polynomials. The distributive law, which also holds, provides the justification for the process of *factoring*:

$x^2 + x = x(x+1)$ — $x + 1$ and x are each factors of $x^2 + x$.

$3x^3 + 3 = 3(x^3+1)$ — 3 and $x^3 + 1$ are factors of $3x^3 + 3$.

Since $(x-a)(x-b) = x^2 - ax - bx + ab$
$= x^2 - (a+b)x + ab$,

$x - a$ and $x - b$ are factors of the quadratic polynomial $x^2 - (a+b)x + ab$.

3.9 Quotients of Polynomials

A rational algebraic fraction is an expression which can be written as the quotient of two polynomials.

Examples:

$$\frac{x - 1}{x^2 - 1} \qquad \frac{x^2 + 2x}{x^2 + x - 2}$$

The rules for manipulation in algebraic fractions are the same as for fractions in arithmetic.

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Example:

$$\frac{(x+2)(x-1)}{(2x-1)(x-1)} \text{ is equivalent to } \frac{x+2}{2x-1}$$

$(x \neq 1/2,\ x \neq 1)$

We may cancel nonzero *factors* common to both numerator and denominator.

Example:

$$\frac{x^2+4x+4}{x^2-4} = \frac{(x+2)(x+2)}{(x+2)(x-2)} = \frac{x+2}{x-2} \quad (x \neq \pm 2)$$

$$\frac{x^2-x-2}{x^2-4} = \frac{(x+1)(x-2)}{(x+2)(x-2)} = \frac{x+1}{x+2} \quad (x \neq \pm 2)$$

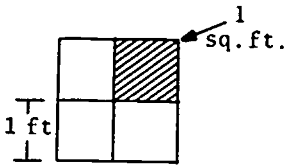
$$\frac{x^2-x-2}{x^2-2x} = \frac{(x+1)(x-2)}{x(x-2)} = \frac{x+1}{x} = \frac{x}{x} + \frac{1}{x}$$

$$= 1 + \frac{1}{x} \quad (x \neq 0,\ x \neq 2)$$

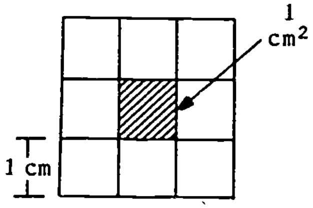
3.10 Squares and Cubes; Their Length, Area and Volume

Let's take a parting shot at exponents by using them in measurements of real-world sidewalk squares and sugar cubes, or some such square or cubical shaped objects.

(i) A 2-foot square has an area of $2^2 = 4$ square feet



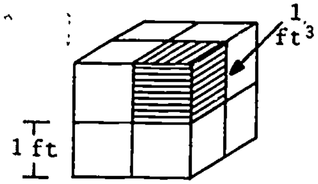
(ii) A square whose side is 3 centimeters long has an area equal to $3^2 = 9$ square centimeters This area is often written as 9 cm^2.



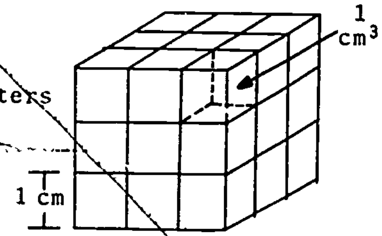
(iii) A square whose side has length of x units has an area equal to x^2 units.

area of a square	$A = x^2$

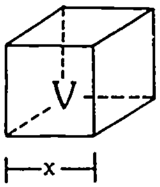
(iv) A 2-foot cube has a volume of $2^3 = 8$ cubic feet $= 8\ ft^3$.



(v) A 3-centimeter cube has a volume of $3^3 = 27$ cubic centimeters $= 27\ cm^3$ or 27 cc or 27 ml.



(vi) A cube whose side has length of x units has a volume equal to x^3 cubic units.



volume of a cube	$V = x^3$

ASSIGNMENT:

When you have studied this chapter, you are ready to try Test 3.

The following problems in Schaum's Outline: College Algebra may help you through this unit:

Chapter	Problems
1	11, 23
2	all
3	1, page 22
4	1-5, pp. 28-30
5, 6, 7	all

TEST 3

These questions test your knowledge of powers and roots. Remember, the computer uses the arrow, ↑, to indicate an exponent. For example:

"five-squared" is written "5↑2",
"x-cubed" is written "x↑3",
"the cube root of x" equals "x↑(1/3)".

The computer used "SQR(x)" to denote the square root of x. For example:

SQR(81) = 81 (1/2) = 9 .

1. A calculator problem. Compute: write without exponents:

9↑2 + 2↑9 = ______ (9↑2)(2↑9) = ______

6↑3 + 3↑6 = ______ (6↑3)(3↑6) = ______.

2. Compute; write without exponents:

9↑4 = ______
9↑3 = ______
9↑2 = ______
9↑1 = ______
9↑0 = ______
9↑-1 = ______
9↑-2 = ______
9↑-3 = ______
9↑-4 = ______.

3. Find the following square roots:

SQR(16) = ______ SQR(81/16) = ______

SQR(25) = ______ SQR(36/25) = ______

SQR(4) = ______ SQR(49/4) = ______.

4. Find the following cube roots:

-216↑(1/3) = ______

125↑(1/3) = ______

729↑(1/3) = ______

-64↑(1/3) = ______

(-512/27)↑(1/3) = ______.

5. Find all values of x which satisfy:

x↑2 - 36 = 0	x = ______	or x = ______
125 - 5x↑2 = 0	x = ______	or x = ______
x↑2 - 1x - 56 = 0	x = ______	or x = ______
x↑2 - 2 = 79	x = ______	or x = ______.

6. Solve for x:

SQR(x) = 9	x = ______
SQR(x) = 8	x = ______
SQR(x) = 12	x = ______.

7. Multiply:

x(x + 8) = ______

(x - 3)(x - 7) = ______

(x - 6)(x + 6) = ______

(x + 7)(x - 8) = ______.

8. Simplify the following expressions by factoring and cancelling:

(x + 6x)/x = ______

(x↑2 + 14x + 45)/(x + 9) = ______

(x↑2 - 4)/(x - 2) = ______

((x + 5)↑6)/((x + 5)↑9) = ______.

9. A merchant has a salesroom that measures 22 by 41 feet. How many square feet of floor space has he? ______.

How many square yards is this? ______.

Find the minimum cost of tiling the floor with tile priced at $16.81 per square yard. ______.

The merchant's office is a square room with 196 square feet of floor space. What is the length of the room? ______.

Find the minimum cost of covering this floor with carpet priced at $25.52 per square yard. ______.

4. REFERENCES

TEXT

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5. ANSWERS TO SELECTED TEST QUESTIONS

Test 1

3. -69, -34, 1, 6, 21, 36, 60, 86

4. 12, 47, -106, 86, 91, -96, 24, 18

5. 1583.94

6. 6 1 34 39

7. 22

8. 39, 9, -8, -6, -18, -5

9. Partial answers: 9(19), 10(32), 8(13), 7(35)

Test 2

1. 2436.5 43 -59 42.5

2. Equivalent fractions: 12/42 8/28

3. and 4. (Decimal form): 2.086 1.722 4.167 .238 .742
   1.811 .432 1.2

5. Men/Women = 26/72 Children's = $2,418

6. $61.32

7. 3, 135, 5, 105

8. $6.84 $9.58 $98.55 $273.75

9. y = 3x

Test 3

1. 593 41472 945 157464

2. 6561, 729, 81, 9, 1, .11111, .01235, 1.37E-03, 1.5E-04

3. 4 9/4 = 2.25
   5 6/5 = 1.2
   2 7/2 = 3.5

4. -6, 5, 9, -4, -8/3 = -2.66666667

5. 6 or -6, 5 or -5, 7 or -8, 9 or -9.

6. 81, 64, 24

7. $x^2 + 8x$
   $x^2 - 10x + 21$
   $x^2 - 36$
   $x^2 - 1x - 56$

8. 7, x + 5, x + 2, $(x + 5)^{-3}$

9. 902 sq. ft.
   100.2 sq. yd.
   Tile: $1684.36
   Length 14. Carpet: $555.77

STUDENT FORM 1

Request for Help

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name____________________ Unit No.______

Page____ ○ Upper ○ Middle ○ Lower	OR	Section______ Paragraph______	OR	Model Exam Problem No.____ Text Problem No.____

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature____________________

Please use reverse if necessary.

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

STUDENT FORM 2

Unit Questionnaire

Name____________________________________ Unit No._____________ Date_______________

Institution________________________________ Course No._____________________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally toc detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

____Much Longer ____Somewhat Longer ____About the Same ____Somewhat Shorter ____Much Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain____

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain____

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)

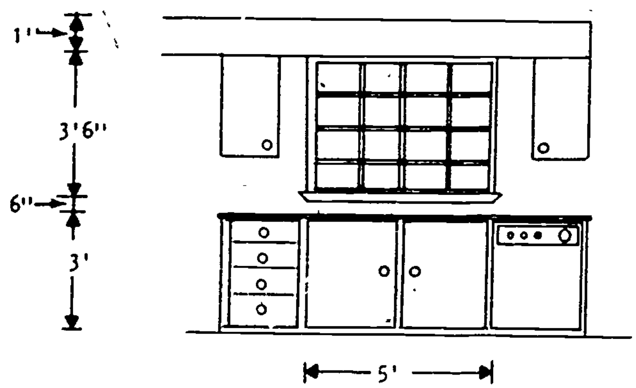
umap

UNIT 375

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT

INTERIOR DESIGN: PREPARING AN ESTIMATE

by Martha Kasting



APPLICATIONS OF DECIMAL ARITHMETIC AND AREA FORMULAS TO BUSINESS

edc/umap / 55chapel st./newton, mass 02160

INTERIOR DESIGN:

PREPARING AN ESTIMATE

by

Martha Kasting
Department of Mathematics
University of Louisville
Louisville, Kentucky 40208

TABLE OF CONTENTS

Intermodular Description Sheet: *UMAP Unit 375*

Title: INTERIOR DESIGN: PREPARING AN ESTIMATE

Author: Martha Kasting
Department of Mathematics
University of Louisville
Louisville, Kentucky 40208

Review Stage/Date: III 3/4/80

Classification: APPL DEC ARITH & AREA FORMULAS/BUSINESS

Prerequisite Skills:
1. Some acquaintance with simple area formulas and basic decimal arithmetic.

Output Skills:
1. Preparation of professional cost estimates.

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MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists, and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

The Project would like to thank Joseph Malkevitch of York College (CUNY) and Richard G. Montgomery of Southern Oregon State College for their reviews, and all others who assisted in the production of this unit.

This material was prepared with the support of National Science Foundation Grant No. SED76-19615 A02. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF, nor of the National Steering Committee.

INTERIOR DESIGN: PREPARING AN ESTIMATE

INTRODUCTION

This exercise was written to give interior design students a trial run through computations which underlie nearly every project in residential design. It is one of a set of modules prepared for a course in business mathematics, but is also appropriate for inclusion in an interior design course.

For her assistance I wish to thank Assistant Professor Nancy Pearcy, director of the Interior Design program at the University of Louisville, and a professional member of the American Society of Interior Designers.

Martha Kasting

INTERIOR DESIGN: PREPARING AN ESTIMATE

In this exercise you are asked to prepare estimates (specifications) for the various decorative materials used for those surfaces that enclose space: floors, walls, ceilings, and windows. The problem will involve three rooms in a residential interior. The floor plan to be used is found on page 18. Disregard the hatched area within the plan.

You will consider the budgetary requirements of two "clients"; the first able to afford expensive furnishings (Brand A), the second requiring more moderately priced items (Brand X). Only two brands of each item will be considered here. You will be given wholesale prices (cost to you), of materials for floor, wall and window coverings. Labor costs will be beyond your control, so will be considered only at the last.

Keep a record of the time you spend on this project. Your fee for services should pay you well for the time you put into this type of work, but only if you learn to work very quickly.

1. WALL COVERING: PAINT

Paint is the most common treatment for walls since it is by far the least expensive. A gallon of enamel, chosen for its washability, will cover walls, woodwork and ceiling of the kitchen with one coat. (Two coats, and therefore two gallons, will be needed to cover dark surfaces with a light color. Assume one coat here.)

Two gallons of latex will cover all other ceiling areas in this project. The living room will require two gallons of latex wall paint, the dining room another gallon. One gallon of enamel will treat the woodwork in both the living and dining areas. Only one brand of paint will be considered, good paint.

PAINT WORKSHEET

Room	Type of Paint	Wholesale Cost/Gallon	Number of Gallons	Total Cost Brand A
Kitchen	Enamel	$12.85		
Living-Dining				
Ceiling	Latex Flat	$ 7.14		
Woodwork	Enamel	12.85		
Living Walls	Latex Flat	7.14		
Dining Walls	Latex Flat	7.14		

(i) Total bill if all surfaces are painted: ____

(ii) Total bill if dining room walls are not painted: ____

2. WALL COVERING: PAPER

There are brochures available for estimating the number of rolls of wallcovering needed for a given size room. Brand X will be estimated this way. But since this is a math exercise, we will work out a precise estimate for papering the dining-room with Brand A. The latter estimate is the type you might do before ordering a very expensive covering (parchment, imported masterpieces, special restoration projects, etc.) when the unused portion of the order could not be returned. You would also do this before applying a complicated design, or one with animals whose heads might otherwise be cut off at the ceiling, or if you were very anxious to save a few dollars.

Wallpaper is packaged in double rolls, 14 yards long and (often, not always) 20 inches wide. Just to confuse you, it is priced by the single roll and may be purchased that way. A single roll is 7 yards long.

We will consider two types of paper for the dining room: Brand X with a 4" repeat and Brand A with a 27" repeat. This design repeat must be matched from panel to panel, so there will be waste in cutting.

Brand X Estimate

This paper, with a negligible 4" design repeat and a modest price tag, can be estimated quickly. The procedure follows:

1. Measure the perimeter of the dining room. Ignore the added inches from the bay window.

   $P = 21 + 2w =$ ____ feet.

2. Measure the ceiling height, h. Use 8 feet.

3. Compute the total number of square feet of wall area:

   $A = P \times h =$ ____ square feet.

4. Estimate that a single roll of wallpaper will cover 30 square feet. Allowance for some waste is included. Find the number of rolls, R.

   $R = A/30 =$ ____ (round up to the next whole number).

5. Subtract from R one single roll for every two wall openings (doors, windows). Count the bay as three windows. The result, N, is the number of single rolls to order.

   N = ____ single rolls of Brand X paper. (Did you get 12?)

The wholesale cost of Brand X paper is $7.85 per single roll. Cost of N single rolls = $____.

Brand A Estimate

This wallpaper, which has a 27" pattern repeat and cannot be returned to the manufacturer, will be estimated panel by panel.

1. Measure the perimeter of the room, including the extra inches for the bay window.

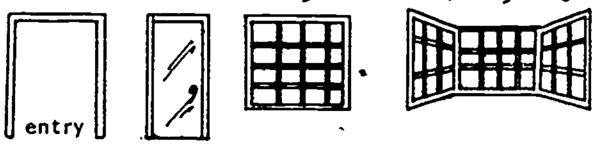
   perimeter, P = _____ feet, or _____ inches.

2. Divide P by the 20-inch roll width to find the number of panels of paper needed. (Be sure P is in inches.)

   number of panels, N = P/20 = _____ .

3. Separate N into short panels and full-length panels: On page 17 measure the total width, W, across the two doorways, the single window and the bay.

   $W_1 + W_2 + W_3 + W_4 + W_5 + W_6 = W$



   W = _____ in.

   Divide W by 20" to find S, the number of short panels.

   number of short panels = W/20 = _____ = S.
   (Round down to the lower integer.)
   number of full length panels F = N - S = _____.

4. Measure the ceiling height, here 8 feet. To allow for waste in matching patterns add half the pattern repeat length to the ceiling height to get a full-length panel, FL.

   FL = _____ inches.

   Subtract 6 feet from the full panel length, FL, to get the short panel length, SL:

   SL = _____ inches.

5. Find the total length of paper required.

   (number of full-length panels) x (full length)
   = F x FL = _____."

   (number of short panels) x (short panel length)
   = S x SL = _____."

   Total length of paper = _____ inches
   = _____ yards.

6. Divide the total number of yards required by 7 (yards per single roll) to get the number of single rolls required. Round up to the nearest whole number. Rolls required = _____.

The wholesale cost of Brand A wallpaper is $25.00 per single roll. Total cost of Brand A wallpaper is $_____.

3. WINDOW TREATMENT: CAFE CURTAINS

We will consider three types of window treatment: cafe curtains, draw draperies and decorative wooden blinds. The living and dining areas will always receive the same treatment.

Cafe curtains can be worked into projects where budget is a critical item. Fabrics are inexpensive and the area to be covered is less than the entire window. Assume all fabric is 45 inches wide.

The width of the curtain should be at least twice the width of the window (three times this width if the fabric is sheer.)

For cafe curtains, measure 1/2 the length of the window. Add 1/4 yard for hems and casing and 1/3 yard for the valance. This will give total fabric length per window. Double the width of each window to get the width of the curtain. Divide this doubled width by 45" to find the number of panels per window (round to the nearest larger whole number.) The number of panels times the desired length, rounded to the nearest quarter of a yard gives the amount of fabric needed for each window. Treat the dining room bay as one window.

Enter your calculations on the drapery worksheet on the following page.

4. WINDOW TREATMENT: DRAPERIES

For draperies use the same width measurements, but to find the length per window add 1/4 yard to the measured length for hems and headings. For the living room only, figure the cost of lined and unlined draperies in two choices of length: (i) to cover the window and (ii) to reach the floor.

Enter your calculations on the drapery worksheet. Use these costs per yard for curtain fabric:

	Brand A	Brand X
Cafe Curtains	$ 7.14	$ 4.29
Drapery	14.28	7.14
Lining	2.14	2.14

For curtain rods and hardware add $20.00 per window to the final estimate.

5. WOVEN-WOOD WINDOW SHADES

Use the table on page 8 to figure the cost of woven window shades (blinds) for each window in the living, dining and kitchen areas. Enter your calculations on the window shade worksheet, page 7.

WINDOW SHADE WORKSHEET

Room	Window Size	Cost Per Window incl. Valance Brand A	Cost Per Window incl. Valance Brand X	Number of Windows	Total Cost Brand A	Total Cost Brand X
Living						
Kitchen						
Dining						

DRAPERY WORKSHEET Fabric 45" wide. Hardware not included.

Room	Type of Curtain	Window Size Length-Width	Length/panel incl. valance	Number of panels	Number of yards	Cost/Window A	Cost/Window X	Cost/room A	Cost/room X
Living	Cafe								
Kitchen									
Dining									
Living	Draperies Floor length					lined			
						unlined			
Living	Draperies Window length, lined								
Kitchen									
Dining									

Cost of Woven Window Shades

WHOLESALE PRICES-DOLLARS

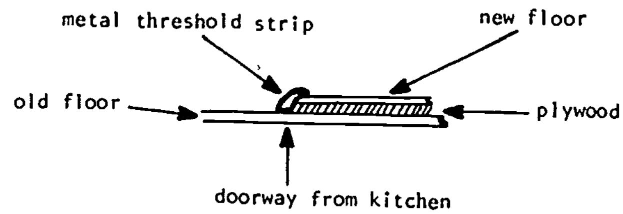
Length Inches	Width-Inches 30 to 36	36⅛ to 42	42⅛ to 48	48⅛ to 56	56⅛ to 63	63⅛ to 69
42⅛ to 48						
Brand A	49.45	54.88	60.30	65.74	79.55	84.97
Brand X	41.21	45.73	50.25	54.78	66.29	70.81
60⅛ to 66						
Brand A	57.10	63.52	69.95	76.37	92.21	98.63
Brand X	47.58	52.93	58.29	63.64	76.84	82.19
66⅛ to 72						
Brand A	68.54	76.46	84.37	92.29	110.35	118.27
Brand X	57.12	63.72	70.31	76.91	91.96	98.56
Valance						
Brand A	23.33	26.66	29.99	33.33	36.66	39.99
Brand X	19.17	21.67	24.17	26.67	29.17	31.67

6. VINYL FLOOR COVERING FOR THE KITCHEN, HALL AND LAVATORY

The price range for good quality floorcoverings is not significant enough to influence a designer's choice. However, the condition of the floor to be covered is very important to consider.

If the floor is new and this is the first covering to be applied, installation costs will run about $2.50 per square yard (1979 prices).

If the floor has a single layer of old linoleum on it, the installer will usually lay plywood on top of it and then add the new covering. This procedure costs about $3.50 per square yard. More importantly, it raises the kitchen floor about 3/4 inch, which causes a break at the doorways.



This is not noticeable if the adjacent rooms are carpeted wall-to-wall. However, a metal threshold across a doorway leading onto shining parquet floors of a Louis Seize salon must be avoided.

To put new vinyl on an old, covered kitchen floor without raising the floor level requires ripping up the old floorcovering, and sanding and repairing the subfloor. The labor cost for this, I am advised, is very high. So consult with an expert before quoting on an old or historic project.

For this exercise assume that both Brand X and Brand A floorcoverings cost you \$10.00 per square yard. However, Brand X will be installed over a new subfloor at \$2.50 per square yard labor cost, while Brand A will be laid on top of old lineoleum at \$3.50 per square yard, (the additional cost is for plywood.)

To find the number of square feet of floor to be covered measure the kitchen, hall and lavatory areas. Subtract the floor area that is shown to be covered by base cabinets.

The area to be covered is _____ square feet,
or _____ square yards
(round up).

Brand A: (cost/sq. yd.) x (number of sq. yds.) = cost

Material _____ x _____ = _____.
Labor _____ x _____ = _____.
Total Cost = _____.

Brand X: (cost/sq. yd.) x (number of sq. yds.) = cost

Material _____ x _____ = _____.
Labor _____ x _____ = _____.
Total Cost = _____.

7. FLOOR COVERING FOR LIVING AND DINING AREAS

Wall-to- wall carpeting is the easy way out. Measure the floor area of the living room and entry and, separately, the dining room.

Use the following costs for materials:

Brand A: \$15.00/sq. yd. plus padding at \$2.10/sq. yd.

Brand X: \$9.25/sq. yd. plus padding at \$2.10/sq. yd.

Room	Area	Cost of Materials Brand A	Brand X
Living-entry	_____	_____	_____
Dining	_____	_____	_____

Rugs are definitely a luxury item. Brand A will be a new oriental (Kirman) while Brand X will be a machine-woven wool. Here are some costs.

TYPE	SIZE 3'x4'6"	5'x7'	8'x10'	9'x11'	11'6"x18'
Brand A	\$420.	985.	1385.	2142.	5963.
Brand X	\$ 85.	195.	424.	550.	1200.

ASSIGNMENT:

(1) Draw your choice of rugs on the plan and then find the cost of

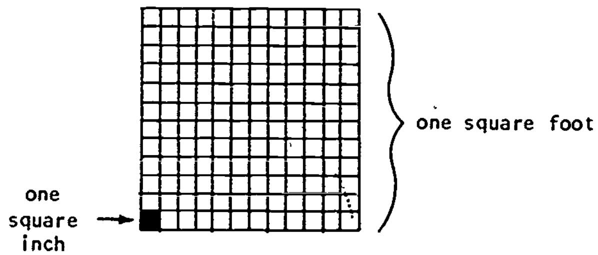
	Brand A	Brand X
Living-entry	_____	_____
Dining	_____	_____

(2) Check you figures to make sure you have used the correct numbers to convert from square inches to square feet, and from square feet to square yards.

The side of a 1-foot square has length L equal to 12 inches. So one square foot has area A given by

$$A = L^2$$

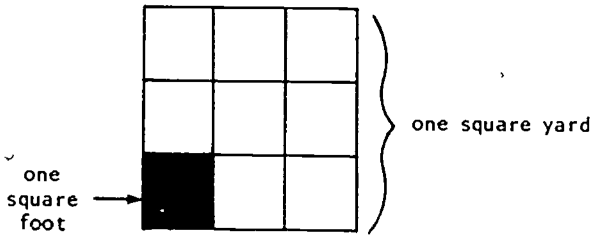
$$1 \text{ square foot} = (12 \text{ inches}) \cdot (12 \text{ inches})$$
$$= 144 \text{ square inches}$$
$$= 144 \text{ in}^2.$$



The side of a 1-yard square has length L equal to 3 feet. So one square yard has area A given by

$$A = L^2$$

$$1 \text{ square yard} = (3 \text{ feet}) \cdot (3 \text{ feet})$$
$$= 9 \text{ square feet}$$
$$= 9 \text{ ft}^2.$$



So to convert square inches into square feet, divide by 144:

$$720 \text{ sq. in.} = \frac{720}{144} = 5 \text{ sq. ft.} = 5 \text{ ft}^2.$$

To convert square feet to square yards, divide by 9:

$$261 \text{ sq. ft.} = \frac{261}{9} = 29 \text{ sq. yd.} = 29 \text{ yd}^2.$$

Check yourself:

$1224 \text{ in}^2 =$ _____ ft^2. $315 \text{ ft}^2 =$ _____ yd^2.

8. PUTTING IT ALL TOGETHER

Here is a table to complete for the Brand X estimate.

BRAND X

Room	Feature	Material	Cost	Labor	Total
Living-entry	Walls	Paint	_____	80.00	_____
	Windows	Cafe curtains	_____	25.00	_____
		Lined short drapes	_____	38.00	_____
		Shades	_____	incl.	_____
	Floor	Carpet	_____	$2.00/ sq.yd.	_____
		Rugs	_____	0	_____
Dining	Walls	Paint (extra)	_____	30.00	_____
		Paper	_____	80.00	_____
	Windows	Cafe curtains	_____	30.00	_____
		Lined short drapes	_____	57.00	_____
		Shades	_____	incl.	_____
	Floor	Carpet	_____	$2.00/ sq.yd.	_____
		Rug	_____	0	_____
Kitchen	Walls	Paint	_____	40.00	_____
	Windows	Cafe curtains	_____	10.00	_____
		Shades	_____	incl.	_____
Kitchen Hall Lavatory	Floor	Vinyl	_____	$2.50/ sq.yd.	_____

Here is a table to complete for the Brand A estimate.

BRAND A

Room	Feature	Material	Cost	Labor	Total
Living-entry	Walls	Paint	____	80.00	____
	Windows	Cafe curtains	____	25.00	____
		Lined short drapes	____	38.00	____
		Shades	____	incl.	____
	Floors	Carpet	____	$2.00/ sq.yd.	____
		Rugs	____	0	____
Dining	Walls	Paint (extra)	____	30.00	____
		Paper	____	80.00	____
	Windows	Cafe curtains	____	30.00	____
		Lined short drapes	____	57.00	____
		Shades	____	incl.	____
	Floor	Carpet	____	$2.00/ sq.yd.	____
		Rug	____	0	____
Kitchen	Walls	Paint	____	40.00	____
	Windows	Cafe curtains	____	10.00	____
		Shades	____	incl.	____
Kitchen Hall Lavatory	Floor	Vinyl	____	$3.50/ sq.yd.	____

Comparing the Cost of Three "Packages".

Combination		Cost	+ 40% Markup	= Estimate
Living-Dining	A carpet, X cafe curtains, paint			
Kitchen-Hall	A vinyl, X cafe curtains, paint	____	____	____
Living	A rug, A shades paint			
Dining	A rug, A shades, A paper			
Kitchen Hall Lavatory	X vinyl, A shades, paint	____	____	____
Living	X rug, A cafe curtains, paint			
Dining	X rug, A cafe curtains, X paper			
Kitchen Hall Lavatory	A cafe curtains, X vinyl, paint	____	____	____

One more thing: for your own musings, estimate the time you have spent on this exercise.

__________minutes.

Does your markup cover your labor as well as your overhead? __________.

9. KEEPING UP WITH THE TIMES

You have just completed an estimate upon which you could rely only if prices today were unchanged from those quoted here in March, 1979. Alas, inflation spoils your estimate.

There are two ways to bring the figures up to date. The first, which a person must follow to survive in business, is to keep all cost data current and to use only the most recent price information.

A second, lazier way is to inflate the estimate you have here by multiplying each figure by a factor which approximates general price increases since March, 1979. During 1979 the rise in the wholesale price index was not far from 12%, that is about 1% per month. If you count the number, N, of months that have passed since March, 1979, and then multiply your cost figures by (100 + N)%, you will have a rough idea of what today's estimate should be.

For example, estimates for March, 1980, should be roughly 112~~% of~~ those on page 15. Recopy the last column on page 15 on the left below:

March, 1979 Estimate				March, 1980 Estimate
________	x	1.12	=	________
________	x	1.12	=	________
________	x	1.12	=	________

Try this lazy way to update your estimate. Then let Nostalgia for the good old days be your theme in your next design project.

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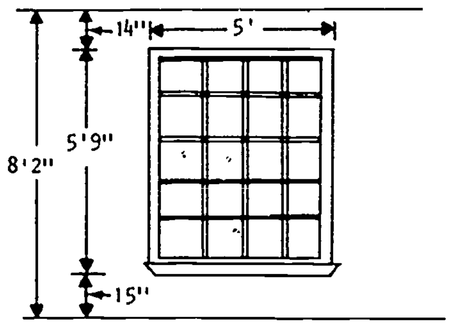
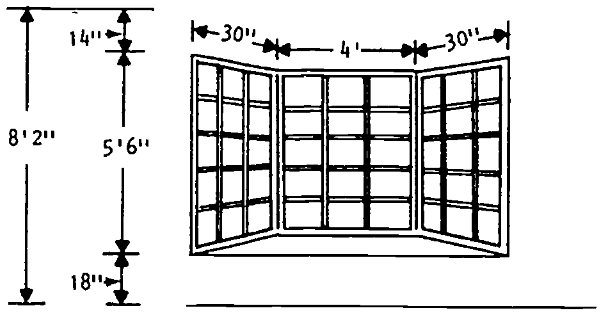
Livingroom (2)

Diningroom (1)

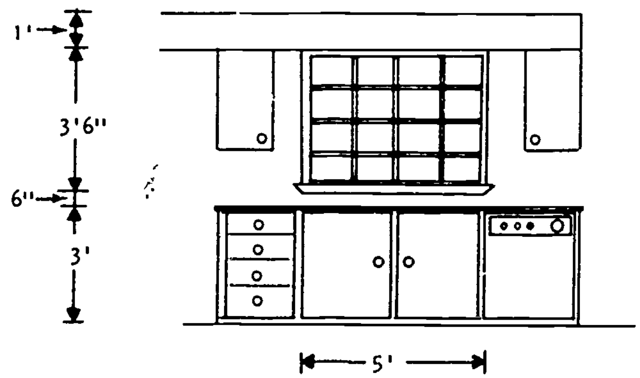


Diningroom

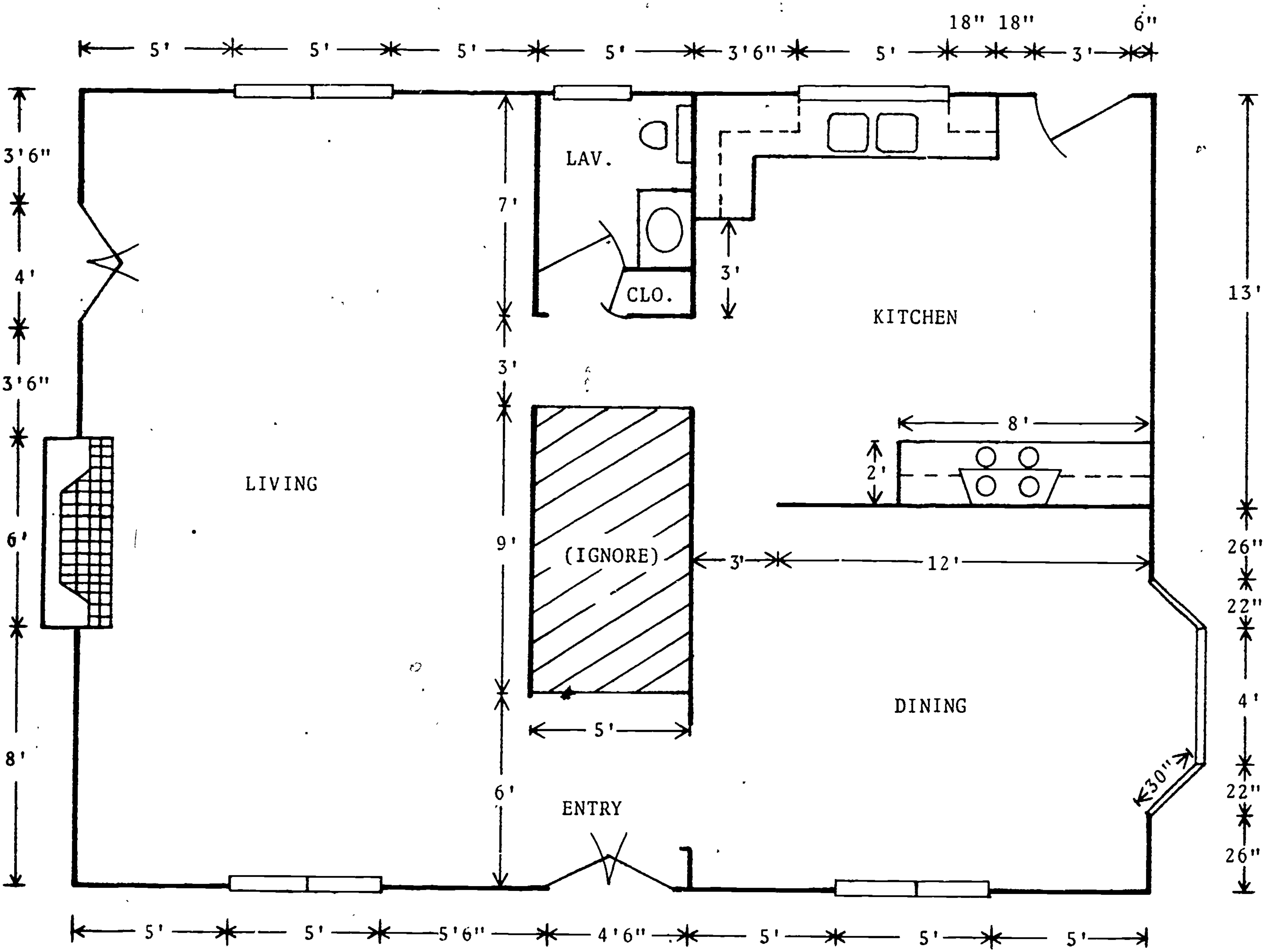
Bay



Kitchen



71



Scale: 1/4" = 1'. Ceiling height = 8'.

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STUDENT FORM 1

Request for Help

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name_______________________________ Unit No.________

Page________		Section________		Model Exam
○ Upper	OR	Paragraph________	OR	Problem No.______
○ Middle				Text
○ Lower				Problem No.______

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature_______________________

Please use reverse if necessary.

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STUDENT FORM 2

Unit Questionnaire

Name____________________ Unit No.________ Date________

Institution____________________ Course No.____________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

____Much Longer ____Somewhat Longer ____About the Same ____Somewhat Shorter ____Much Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain____________________

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain____________________

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)

SOLUTIONS TO SELECTED EXERCISES

Page 2. Paint. (i) \$61.40; (ii) \$54.26.

Page 3. Brand X: P = 54 ft., A = 432 sq. ft., R = 15, N = 12
Cost: \$94.20

Page 5. Brand A: P = 664 in., N = 34, W = 252 in., S = 12, F = 22, FL = 110 in., SL = 38 in., Total length of paper = 80 yds., Rolls required = 12
Cost: \$300.00

Page 7. Drapery Worksheet:

Room	Type of Curtain		Cost/room A	X
L	Cafe		\$ 67.84	40.76
K			24.99	15.02
D			87.47	52.56
L	Draperies	lined	254.51	143.84
	Floor length	unlined	221.34	110.67
L	Draperies		213.46	120.64
K	Window length		60.69	30.35
D	Lined		279.14	157.76

Page 7. Window Shade Worksheet:

Room	Total Cost A	X
L	\$294.02	242.26
K	116.21	95.46
D	407.81	337.09

Page 10-11.

Vinyl for kitchen-hall-lav. Area = 19 sq. yds.
Total cost: Brand A: \$256.50; Brand X: \$237.50

Carpet:	Area sq. yds.	Cost Brand A	Brand X
Liv.-entry	45	\$769.50	510.75
Dining	20	342.00	227.00

Page 12. 1224 $in^2 = 8.5\ ft^2$; 315 $ft^2 = 35\ yd^2$.

Page 13.

BRAND X

Room	Feature	Material	Cost	Labor	Total
Living-entry	Walls	Paint	41.41	80.00	121.41
	Windows	*Cafe curtains	80.76	25.00	105.76
		*Lined short drapes	160.64	38.00	198.64
		Shades	242.26	incl.	242.26
	Floor	Carpet	510.75	\$2.00/ sq.yd.	600.75
		Rugs	(varies)	0	
Dining	Walls	Paint (extra)	7.14	30.00	37.14
		Paper	94.20	80.00	174.20
	Windows	*Cafe curtains	132.56	30.00	162.56
		*Lined short drapes	237.76	57.00	294.76
		Shades	337.09	incl.	337.09
	Floor	Carpet	227.00	\$2.00/ sq.yd.	267.00
		Rug	(varies)	0	
Kitchen	Walls	Paint	12.85	40.00	52.85
	Windows	*Cafe curtains	35.02	10.00	45.02
		Shades	95.46	incl.	95.46
Kitchen Hall Lavatory	Floor	Vinyl	190.00	\$2.50/ sq.yd.	237.50

* \$20.00 per window hardware cost included (\$60.00 for bay).

Page 14.

BRAND A

Room	Feature	Material	Cost	Labor	Total
Living-entry	Walls	Paint	41.41	80.00	121.41
	Windows	*Cafe curtains	107.84	25.00	132.84
		*Lined short drapes	253.46	38.00	291.46
		Shades	294.02	incl.	294.02
	Floors	Carpet	769.50	$2.00/ sq.yd.	859.50
		Rugs	(varies)	0	
Dining	Walls	Paint (extra)	7.14	30.00	37.14
		Paper	300.00	80.00	380.00
	Windows	*Cafe curtains	167.47	30.00	197.47
		*Lined short drapes	186.73	57.00	243.73
		Shades	407.81	incl.	407.81
	Floor	Carpet	342.00	$2.00/ sq.yd.	382.00
		Rug	(varies)	0	
Kitchen	Walls	Paint	12.85	40.00	52.85
	Windows	*Cafe curtains	44.99	10.00	54.99
		Shades	116.21	incl.	116.21
Kitchen Hall Lavatory	Floor	Vinyl	190.00	$3.50/ sq.yd.	256.50

* $20.00 per window hardware cost included ($60.00 for bay).

Page 15. Comparing the cost of three "packages." The second and third package cost will vary, depending on the choice of rugs. Here the estimates are for one 9' x 11' rug for the living room, one 3' x 4'6" rug for the entry, and one 8' x 10' rug for the dining room.

Combination	Cost +	40% markup	= estimate
1	$2023	809	2832
2	5557	2223	7780
3	1778	711	2489

Page 16. March, 1980 estimates using the above data are $3172, $8714, and $2788, respectively.