DOCUMENT RESUME

ED 218 101 SE 038 102

AUTHOR	Merrill, Samuel III; Enelow, James M.
TITLE	Decision Analysis for Multicandidate Voting Systems. Applications of Elementary Decision Analysis to Political Science. [and] An Application of Voting Theory to Congress. Applications of Decision Theory and Game Theory to American Politics. Modules and Monographs in Undergraduate Mathematics and Its Applications Project. UMAP Units 384 and 386.
INSTITUTION	Education Development Center, Inc., Newton, Mass.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	80
GRANT	SED-76-19615-A02
NOTE	61p.
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Answer Keys; *College Mathematics; Game Theory; Higher Education; Instructional Materials; *Learning Modules; *Mathematical Applications; Mathematical Models; *Matrices; *Political Science; *Politics; Problem Solving; Supplementary Reading Materials; Voting
IDENTIFIERS	Decision Models; *Decision Theory

ABSTRACT

This document consists of two modules. The first studies a variety of multicandidate voting systems, including approval, Borda, and cumulative voting, using a model which takes account of a voter's intensity of preference for candidates. The voter's optimal strategy is investigated for each voting system using decision criteria under uncertainty (Savage regret and Laplace criteria) and under risk (expected utility). Voting systems are compared with regard to the relative ease with which the voter can approximate his or her optimal strategy, the relative freedom of the voting system from offering superfluous strategies, and the empirical impact as determined by survey data. The second module is designed to help the user gain an understanding of how a simple theory of voting can be used to analyze strategic voting in Congress. It is noted that voting in the United States Congress is frequently strategic. A model is presented to explain and predict voting on congressional amendments. Both units contain problem sets, and answers to these exercises are provided. (MP)

Finally, we will use survey data to study the impact different voting systems might have had in the 1972 Democratic Presidential primaries, assuming that the voters used optimal strategies.

2. EXAMPLES OF VOTING SYSTEMS

We begin by describing several possible voting systems for single-vacancy, multicandidate elections. In each system considered below, the candidate with the most votes wins. Some of these systems are in current use, others are not. To lend perspective to the analysis, we have deliberately included some which may not be advisable under any circumstances. Our purpose is to take a fresh look at the advantages and disadvantages of each, unfettered if possible by preconceived notions.

Plurality. Each voter casts one vote for one candidate. This is the system most commonly used in the United States, and in parliamentary elections in Canada and Great Britain.

Cumulative Voting. Each voter may apportion a set number of votes (the same for each voter) among the candidates. (When employed at present, this method is normally used when there are several vacancies, as for a corporate board of directors or for a multimember legislative district such as in the Illinois House of Representatives. However, we will consider here its effect were it applied in a single-vacancy race.) (See Brams (1975) for a more detailed discussion of cumulative voting.)

Approval Voting. Each voter is permitted to cast votes for (i.e., approve) as many candidates as he wishes, but he is allowed no more than one vote per candidate. (See Brams (1978: ch. 6) for a description of the recent research on approval voting.)

Cardinal Rating Voting. Each voter rates the candidates on a common scale, say, from 0 to 10, and casts a number of votes for each candidate determined by his rating.

Borda System. Each voter ranks the candidates in order of preference and casts a number of votes for each equal to the number of candidates ranked below him. For example, if there are five candidates, each voter will cast 4, 3, 2, 1, and 0 votes for the various candidates. (See Borda (1781) or D. Black (1958).)

Example. Suppose there are four candidates: Adams, Bianco, Cohen, and Delaney, and five voters J, K, L, M, and N. Table 1 lists possible rankings given by the voters for the four candidates, from most preferred (rank 1) to least preferred (rank 4). Exercises 1-4 refer to this table.

Table 1

Voters	Adams	Bianco	Cohen	Delaney
J:	1	2	3	4
K:	1	3	2	4
L:	4	1	2	3
M:	3	2	1	4
N:	4	2	3	1

Exercise 1. Determine the total vote for each candidate in Table 1 for the plurality system and for the Borda system.

Exercise 2. In Borda voting, each voter ranks the candidates 1st, 2nd, 3rd, and 4th. For the ranks specified in Table 1 determine the sum of the ranks for each candidate. Explain in what sense choosing as winner that candidate whose rank sum is smallest is equivalent to the Borda system.

Exercise 3. Suppose each voter ranks the candidates and then casts a number of votes for each candidate equal to the difference between the number of candidates ranked below that candidate and the number of candidates ranked above that candidate. Calculate the vote totals for the ranks in Table 1 (note that some votes cast are negative). Is this system equivalent to the Borda system?

Exercise 4. The rule for casting votes described in Exercise 3 permits the voter to express indifference between two or more candidates, i.e., to give two or more candidates the same rank. If a voter prefers Adams, is indifferent between Bianco and Cohen, and considers Delaney least desirable, determine the votes he casts

Intermodular Description Sheet: *UMAP Unit 384*

Title: DECISION ANALYSIS FOR MULTICANDIDATE VOTING SYSTEMS

Author: Samuel Merrill, III
Department of Mathematics/Computer Science
Wilkes College
Wilkes-Barre, PA 18703

Review Stage/Date: III 5/8/80

Classification: APPL ELEM DECISION ANALYSIS/POLITICAL SCI

Prerequisite Skills:
1. Handle simple algebraic inequalities.
2. Knowledge of elementary probability.
3. Manipulate finite summations (optional; used in some derivations).

Output Skills:
1. Become familiar with a variety of multicandidate voting systems, including approval, Borda, and cumulative voting.
2. Understand basic concepts in decision analysis, including Savage (minimax) regret and expected utility.
3. Apply these concepts to strategic decisions made by voters in order to compare voting systems.
4. Use survey data to study the possible impact of various voting systems.

Other Related Units:

The author wishes to thank Steven Brams of New York University and Robert Freysinger of Wilkes College for helpful comments during the preparation of this module.

MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists, and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

This module was developed under the auspices of the UMAP Political Science Panel, G. Robert Boynton, Chair. The Project would like to thank Edward Z. Marrinan of the University of Dayton, Manus A. Midlarsky of the University of Michigan, and Edward W. Packel of Lake Forest College for their reviews, and all others who assisted in the production of this unit.

This material was prepared with the partial support of National Science Foundation Grant No. SED76-19615 A02. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF or the copyright holder.

ABSTRACT

The module studies a variety of multicandidate voting systems, including approval, Borda, and cumulative voting, using a model which takes account of a voter's intensity of preference for the candidates. The voter's optimal strategy is investigated for each voting system using decision criteria under uncertainty (Savage regret and Laplace criteria) and under risk (expected utility). Voting systems are compared with regard to the relative ease with which the voter can approximate his optimal strategy, the relative freedom of the voting system from offering superfluous strategies, and the empirical impact as determined by survey data.

1. INTRODUCTION

Often a voter is confronted with an election in which more than two candidates are running for a single office. Current election rules in the United States and several other countries permit each voter to express a preference for only one of the candidates. This system of voting disregards the intensity of preferences felt by the voters for the various candidates, except insofar as voters who are unconcerned may choose not to vote. In particular, it often awards a plurality to a candidate who is the first choice of a minority, while another candidate may enjoy approval by a larger proportion of the electorate and could, if elected, serve with a wider mandate.

For example, in the 1970 New York Senatorial race, there were three candidates: Ottinger (Democrat), Goodell (Republican-Liberal), and Buckley (Conservative). As it turned out, Buckley won with 39% of the vote, followed by Ottinger with 37%, and Goodell with 24%. To a large extent the two candidates perceived to be liberal (Ottinger and Goodell) divided the votes from a common constituency. Many observers have speculated that a majority of the voters preferred Ottinger to Buckley and some have also contended that a majority preferred Goodell to Buckley.

A variety of alternative voting systems have been proposed to determine the winner in a multicandidate

election. For example, the winner might be determined by summing ranks or ratings of the candidates or each voter might be permitted to vote for more than one candidate. For a number of such voting systems, we will investigate optimal strategies for a voter (e.g., how he should rank or rate the candidates or how many candidates he should vote for). This will be done under a variety of assumptions concerning the voter's knowledge of the likely outcome of the election and will be based on a model which takes into account the voter's intensity of preference for the candidates.

First we will consider voter strategies (i.e., decision criteria) under uncertainty, by which we will mean that estimation of the likelihood of the candidates' success is not possible. We will determine optimal strategies based on the Savage criterion, which minimizes regret, and on the Laplace criterion, which maximizes influence on the outcome under the assumption that all candidates are equally likely to contend for first place. These concepts will be described in detail later.

Next we will assume that the voter is capable of estimating the likelihood of the candidates' success based on polls or other information. It will be assumed that the voter wishes to cast his ballot in such a way as to maximize his influence on the outcome of the election. Under this assumption we will show how the voter's optimal strategy in seeking this goal can be determined for a number of voting systems.

An important purpose of computing optimal strategies lies in the evaluation of voting systems. As we will see, the practical determination of an optimal strategy may not be an easy task for the voter. We suggest that an important criterion for society in choosing voting systems is the relative ease with which the voters can approximate their optimal strategies. We will demonstrate a qualitative difference among voting systems in regard to this criterion. Furthermore, it will be seen that, under the assumptions of the model developed, certain voting systems reduce to other known systems when superfluous (non-optimal) strategies are eliminated.

Finally, we will use survey data to study the impact different voting systems might have had in the 1972 Democratic Presidential primaries, assuming that the voters used optimal strategies.

2. EXAMPLES OF VOTING SYSTEMS

We begin by describing several possible voting systems for single-vacancy, multicandidate elections. In each system considered below, the candidate with the most votes wins. Some of these systems are in current use, others are not. To lend perspective to the analysis, we have deliberately included some which may not be advisable under any circumstances. Our purpose is to take a fresh look at the advantages and disadvantages of each, unfettered if possible by preconceived notions.

Plurality. Each voter casts one vote for one candidate. This is the system most commonly used in the United States, and in parliamentary elections in Canada and Great Britain.

Cumulative Voting. Each voter may apportion a set number of votes (the same for each voter) among the candidates. (When employed at present, this method is normally used when there are several vacancies, as for a corporate board of directors or for a multimember legislative district such as in the Illinois House of Representatives. However, we will consider here its effect were it applied in a single-vacancy race.) (See Brams (1975) for a more detailed discussion of cumulative voting.)

Approval Voting. Each voter is permitted to cast votes for (i.e., approve) as many candidates as he wishes, but he is allowed no more than one vote per candidate. (See Brams (1978: ch. 6) for a description of the recent research on approval voting.)

Cardinal Rating Voting. Each voter rates the candidates on a common scale, say, from 0 to 10, and casts a number of votes for each candidate determined by his rating.

Borda System. Each voter ranks the candidates in order of preference and casts a number of votes for each equal to the number of candidates ranked below him. For example, if there are five candidates, each voter will cast 4, 3, 2, 1, and 0 votes for the various candidates. (See Borda (1781) or D. Black (1958).)

Example. Suppose there are four candidates: Adams, Bianco, Cohen, and Delaney, and five voters J, K, L, M, and N. Table 1 lists possible rankings given by the voters for the four candidates, from most preferred (rank 1) to least preferred (rank 4). Exercises 1-4 refer to this table.

Table 1

Voters	Adams	Bianco	Cohen	Delaney
J:	1	2	3	4
K:	1	3	2	4
L:	4	1	2	3
M:	3	2	1	4
N:	4	2	3	1

Exercise 1. Determine the total vote for each candidate in Table 1 for the plurality system and for the Borda system.

Exercise 2. In Borda voting, each voter ranks the candidates 1st, 2nd, 3rd, and 4th. For the ranks specified in Table 1 determine the sum of the ranks for each candidate. Explain in what sense choosing as winner that candidate whose rank sum is smallest is equivalent to the Borda system.

Exercise 3. Suppose each voter ranks the candidates and then casts a number of votes for each candidate equal to the difference between the number of candidates ranked below that candidate and the number of candidates ranked above that candidate. Calculate the vote totals for the ranks in Table 1 (note that some votes cast are negative). Is this system equivalent to the Borda system?

Exercise 4. The rule for casting votes described in Exercise 3 permits the voter to express indifference between two or more candidates, i.e., to give two or more candidates the same rank. If a voter prefers Adams, is indifferent between Bianco and Cohen, and considers Delaney least desirable, determine the votes he casts

according to the rule in Exercise 3. This system is called the Adjusted Borda System.

Suppose that the voters also rate the four candidates listed in Table 1 on a scale from 0 to 10 (where 10 indicates most desirable), as given in Table 2. Note that these ratings are consistent with the rankings given in the former table. Exercises 5 and 6 refer to Table 2.

Table 2

Voters	Adams	Bianco	Cohen	Delaney
J:	10	8	7	0
K:	10	2	8	0
L:	0	10	8	7
M:	2	3	10	0
N:	0	9	6	10

Exercise 5. Assuming that these ratings are used in the cardinal measure system, determine the vote totals for each candidate. Now under approval voting, assume that each voter votes for (approves) each candidate he rates above 5. Determine vote totals.

Exercise 6. Suppose an election were conducted under cumulative voting to fill a single vacancy. Assuming that each voter has 10 votes at his disposal, use the data in Table 2 to decide how you feel each of the five voters should apportion his votes. Determine the vote totals for your apportionments.

There are, of course, other voting systems which take account of voter preferences among the candidates. Ranked preferences may be used to seek a Condorcet winner (if one exists), i.e., a candidate who would win a majority against each of the other candidates. This concept appeals to the sense of justice of many analysts, including in particular D. Black (1958, p. 66), who recommends use of the Condorcet method, with the winner to be chosen by the Borda system if no Condorcet winner exists.

The Copeland method attempts to resolve this problem by awarding victory to the candidate who can win the most pairwise contests, thus assuring the election of the Condorcet winner if one exists. However, in the case of either 3 or 4 candidates, if no Condorcet winner exists (and no two candidates receive exactly the same number of votes) the Copeland method fails to determine a unique winner. For example, when there are 3 candidates, there are 3 pairwise contests. If there is no Condorcet winner, no candidate can win as many as 2 of them. Hence each wins one contest and the Copeland rule is inconclusive.

Exercise 7. Show that for four candidates, if no Condorcet winner exists (and no two candidates receive exactly the same number of votes), the Copeland method fails to determine a unique winner.

Another attempt to retain part of the Condorcet principle is to hold a runoff between the top two contestants. However, this is more expensive than a one-stage voting system and may overlook a compromise candidate who stands third on the first ballot. As a case in point, most observers believe that Congressman Richard Bolling, a centrist who failed to make the runoff for U.S. House Democratic Majority Leader in 1976, could have defeated any of his opponents in a two-man race.

Still another modification of the Condorcet principle is called preferential voting (or the single transferable vote or Hare system when there is more than one vacancy). (See Brams (1979).) After the voters rank the candidates, the candidate with the fewest first-place votes is dropped. His second-place votes are given to the remaining candidates. The process is repeated until one candidate has a majority. Preferential voting, versions of which have been used in Ireland (Rae (1972)) and in Ann Arbor, Michigan (Brams (1979)), has disadvantages similar to those of the runoff.

Whatever the merits of these Condorcet-type alternatives, we will focus on the five voting systems described at the beginning of this section, as they are amenable to study through the model we are about to develop.

3. THE MODEL

We first express the model in terms of approval voting and then generalize it later to encompass a number of voting systems including the five specified above. We fix a particular voter (to be called the focal voter) and address the question: How should the focal voter rationally select the subset of candidates to vote for under approval voting? Assume there are K candidates $c_1, \ldots, c_K$, and that:

(A) The focal voter assigns a numerical rating f_i to candidate c_i so that the quantity $f_i - f_j$ is intended to represent the utility or payoff to that voter of having candidate c_i elected instead of candidate c_j.

(B) The number of voters is large enough that the probability of an m-way tie (m > 2) is negligible, relative to that for a 2-way tie.

(C) The voter can exercise power only if his votes are decisive. By this we mean that for some pair of candidates c_i and c_j, he can break a tie for first place (or produce such a tie) which would occur had he abstained. In this case, the voter receives a payoff of $(f_i - f_j)$, provided that he votes for c_i but not c_j. The contingencies for which the focal voter has a chance to be decisive will be specified as the pairs (c_i, c_j).[1]

[1] If the total vote received by candidates c_i and c_j from the other voters is even, say 100, then the focal voter can be decisive only if the 100 votes are split 50-50. In this case, if he votes for c_i but not c_j, c_i is elected; if he votes for c_j but not c_i, c_j is elected. If for example, he chooses the former, his payoff is $(f_i - f_j)$. If the total vote from the other voters for c_i and c_j is odd, say 99, then the focal voter can be decisive if the 99 votes are split either (a) 49 for c_i and 50 for c_j or (b) 50 for c_i and 49 for c_j. Assume that, in case of a tie in the final vote, a procedure is used which selects either candidate with equal probability. In case (a), if the focal voter votes for c_i but not c_j, then c_i is elected with probability ½; if he votes for c_j but not c_i, c_j is elected. If he chooses the former, his payoff is $\frac{1}{2}(f_i - f_j)$. For case (b), a similar analysis leads to the same payoff. Thus his total expected payoff is $(f_i - f_j)$, just as before. For convenience, the language in the remainder of the module will reflect the case when the total vote from the other voters for c_i and c_j is even. See Brams and Fishburn (1979) for an alternative development leading to the same result as this note.

4. DECISIONS UNDER UNCERTAINTY: THE SAVAGE REGRET CRITERION

There are several commonly used decision criteria on which the voter can base his decision concerning which candidates to vote for. Some apply to decisions under uncertainty (where nothing is assumed known about the relative likelihood of the various contingencies). Others apply to decisions under risk (where probabilities can be assigned to the relative likelihood of contingencies). We begin by considering two criteria for decisions under uncertainty: the Savage regret method and the Laplace method. (Luce and Raiffa (1957: 280 and 298).)

The Savage regret method chooses that decision which minimizes the maximum regret over all contingencies which might be suffered for the given decision. Regret is computed relative to the best payoff that could be achieved for a particular contingency.

For example, suppose there are 3 candidates denoted simply as A, B, and C, and that the focal voter rates them 10, 7, and 0, respectively. We will refer to the set S of candidates voted for as a strategy. We need only consider contingencies in which the focal voter is potentially decisive--that is, he can make or break a tie among the other voters. Such contingencies occur when a pair of candidates would be tied for first place or differ by one vote had the focal voter abstained. If, in this example, the pair consists of candidates A and B, the contingency will be denoted by the symbol AB.

A payoff matrix (See Table 3) is constructed in which each row corresponds to a strategy and each column to a contingency. For example, if strategy (A) is chosen, and contingency AB occurs, the focal voter assures by his ballot a win for A instead of B, so that his payoff is (10 - 7) = 3. Similarly, had he chosen the strategy (B,C) and the same contingency occured, he would have assured a win for B instead of A (his vote for C would have no effect), so his payoff would be (7 - 10) = -3.

Exercise 8. Construct a payoff matrix if the candidate ratings are 10, 3, and 0, respectively.

Table 3

Payoff matrix (Contingency)

Strategy	AB	AC	BC
(A)	3	10	0
(A,B)	0	10	7
(A,C)	3	0	-7
(B)	-3	0	7
(B,C)	-3	-10	0
(C)	0	-10	-7

Regret matrix (Contingency)

Strategy	AB	AC	BC	Maximal Regret
(A)	0	0	7	7
(A,B)	3	0	0	3
(A,C)	0	10	14	14
(B)	6	10	0	10
(B,C)	6	20	7	20
(C)	3	20	14	20

Next we construct a regret matrix (see Table 3): each entry gives the regret suffered relative to the best payoff that could have been attained for the contingency corrresponding to the entry. For example, if contingency AB occurs, 3 is the best possible payoff so the regret is 0 for strategy (A) or (A,C), but for strategy (A,B) it is (3 - 0) = 3 and for (B) it is 3 - (-3) = 6.

In the final column of the regret matrix we place the maximal regret for each strategy. The Savage method of minimal regret then chooses that strategy for which the maximal regret is smallest, in this case strategy (A,B).

For an arbitrary 3-candidate election, we may assume, without loss of generality, that the focal voter rates candidates A, B, and C as 1, r, and 0 where $1 \geq r \geq 0$. (This is possible since strategic decisions do not depend on changes of scale or position of the ratings.) Payoff and regret matrices for this situation are given in Table 4. The strategies have been numbered for convenience.

Table 4

Payoff matrix (Contingency)

Strategy	AB	AC	BC
1: (A)	1-r	1	0
2: (A,B)	0	1	r
3: (A,C)	1-r	0	-r
4: (B)	r-1	0	r
5: (B,C)	r-1	-1	0
6: (C)	0	-1	-r

Regret matrix (Contingency)

Strategy	AB	AC	BC	Maximal Regret
(A)	0	0	r	r
(A,B)	1-r	0	0	1-r
(A,C)	0	1	2r	2r, r ≥ .5 1, r < .5
(B)	2(1-r)	1	0	1, r ≥ .5 2(1-r), r<.5
(B,C)	2(1-r)	2	r	2
(C)	1-r	2	2r	2

Exercise 9. Compute the regret matrix and determine the Savage regret strategy for the data of Exercise 8.

We note that strategy 1 dominates strategy 3 in the sense that the payoff for strategy 1 is as good as or better than that for 3 for every contingency, and strictly better for at least one. In fact strategy 1 dominates the 3rd, 5th, and 6th strategies in Table 3, and strategy 2 dominates the 4th, 5th, and 6th strategies. Hence we can restrict attention to the first two strategies, which are not dominated by any others. (A strategy that is undominated is called admissible.) The maximum regret for strategy (A) is r. The maximum regret for strategy (A,B) is 1-r. Since under the Savage criterion we wish to minimize maximum regret, the rational voter should vote for only A if r < .5, and for A and B if r > .5. (He should be indifferent between these two options if r = .5.)

We may rephrase this result as follows: according to the Savage regret criterion, if a voter participating in an approval voting election rates the candidates on a scale from 0.0 to 1.0 with the extremes of the scale used for the least preferred and most preferred candidates, respectively, he should cast votes for those candidates rated above .5. It turns out that this result remains true for any number K of candidates (see Appendix A for the derivation). Equivalently, the rule prescribes that the voter vote for each candidate

whose rating exceeds $(f_1 + f_K)/2$, where the ratings are $f_1 \geq f_2 \geq \dots \geq f_K$.

5. DECISIONS UNDER UNCERTAINTY: THE LAPLACE CRITERION

We turn now to the second method for decisions under uncertainty: The Laplace method treats all contingencies as <u>equally likely</u> and determines the <u>expected value</u> of the payoffs for each possible strategy under that assumption. This means that for each strategy we average the payoffs over all contingencies and then choose that strategy for which this average is largest.

For example, using the payoff matrix in Table 3, this expected value for strategy (A) is $(3+10+0)/3 = 4.33$. The expected value for strategy (A,B) is $(0+10+7)/3 = 5.67$. The other expected values for this matrix are -1.33, 1.33, -4.33, and -5.67, respectively. Clearly, the value is largest for strategy (A,B), so that is the Laplace strategy for this payoff matrix.

In general, if there are 3 candidates, we see from the payoff matrix in Table 4 that the expected value for strategy (A) is $(2-r)/3$, and for strategy (A,B) is $(1+r)/3$. The expected values for the other four strategies are $(1-2r)/3$, $(2r-1)/3$, $(r-2)/3$, and $(-r-1)/3$. Each of the last four strategies is dominated by either the first or the second strategy. Furthermore, strategy 1 is better than strategy 2 when $(2-r)/3 > (1+r)/3$, i.e., when $r < .5$. Note that for <u>three</u> candidates, this is the same result we obtained using the Savage regret method.

We now apply the Laplace criterion to the case of K candidates. It will be convenient to drop our assumption that the candidates $c_1, \dots, c_K$ are listed in order of the ratings by the focal voter. Also it will be sufficient simply to total the payoffs in each row of the matrix, since the expected values are obtained by dividing these totals by the number of contingencies, which is the same for each strategy. For strategy (c_1), the total is

$$(f_1 - f_2) + (f_1 - f_3) + \dots + (f_1 - f_K).$$

For any strategy S (recall that S is simply a subset of $(c_1, \dots, c_K)$), the total is

$$U(S) = \sum (f_i - f_j), \qquad (4)$$

where the summation is over all $c_i \in S$ and $c_j \notin S$. We will call U(S) the <u>total utility</u> for the strategy S.

Suppose that the focal voter has decided to vote for the candidates in set S and wishes to know if he could improve his total utility (<u>i.e.</u>, obtain a better Laplace strategy) by also voting for another candidate c_i. He observes that U(S) and $U(S \cup \{c_i\})$ have the same summands in (4) except for those involving c_i. Thus

$$U(S \cup \{c_i\}) - U(S) = \sum_{c_j \notin S} (f_i - f_j) - \sum_{c_j \in S} (f_j - f_i)$$
$$= Kf_i - \sum_{j=1}^{K} f_j.$$

Hence he will improve total utility by voting for c_i precisely if

$$f_i > \frac{1}{K} \sum_{j=1}^{K} f_j. \qquad (5)$$

It follows that the focal voter achieves maximal total utility by voting for the set of those candidates c_i for which (5) holds.

Thus the Laplace criterion tells the rational approval voter to vote for those candidates whom he rates above the average of all the candidates. The Savage regret criterion tells him to vote for those candidates who rate above the average of his first choice and his last choice. In many cases (and always for $K = 3$) both criteria will lead to the same conclusion. For example, if there are four candidates, rated 10, 8, 7, and 0, then the average of all four is 6.25, while the average of the first and last choices is 5.0. Using either criterion, the voter should vote for the top three. If the candidates are rated 10, 3, 2, and 1, then by either criterion the rational voter should vote only for his first choice. Note that the voter does not in general increase his power by voting for as many candidates as possible. Rather, his greatest power occurs when he votes for somewhere in the

vicinity of one half of the candidates. (See Merrill (1980), Brams and Fishburn (1979), and Weber (1977).)

Exercise 10. Determine the optimal strategy for both the Laplace and Savage regret criteria for each voter in Table 2. Determine the winning candidate in each case if optimal strategies are used.

---

6. DECISIONS UNDER RISK: EXPECTED UTILITY

We now turn to decisions under risk and will seek that strategy which maximizes the expected value of the payoff when a subjective probability is assigned to each contingency. We will refer to this criterion as the method of *expected utility*.

For example, suppose that there are 3 candidates and the focal voter estimates (on the basis of polls or other information) that c_2 and c_3 are the stronger candidates. Let us say he estimates the probability of contingency (c_2,c_3) to be twice that of either (c_1,c_2) or (c_1,c_3). In general we denote by p_{ij} the probability that in the voter's estimation there would be a tie for first place between c_i and c_j given that there is such a tie between some pair of candidates (if the focal voter abstains). For convenience, let $p_{ii} = 0$. Thus in our example, $p_{12} = p_{13} = .25$ and $p_{23} = .5$. The expected values for strategies (c_1) and (c_1,c_2) are $(2-r)/4$ and $(1+2r)/4$, respectively (see the payoff matrix in Table 4). The strategy (c_1) will be better if $(2-r) > (1+2r)$, i.e., when $r < 1/3$. Hence the voter should vote only for c_1 if $r < 1/3$.

In general we define the expected utility for a strategy S by:

$$\text{(6)} \qquad U(S) = \sum (f_i - f_j)p_{ij}$$

where again the summation is over all $c_i \in S$ and $c_j \notin S$. (The corresponding definition for plurality voting appears in McKelvey and Ordeshook (1972). Extension of formula (6) to other voting systems can be found in Merrill (1979 and 1980)). The Laplace criterion is, of course, the special case in which all p_{ij} are equal. By an argument similar to that used before, we can show that a voter should include a candidate c_i in his

strategy if

$$\text{(7)} \qquad f_i > \sum_{j=1}^{K} q_{ij}f_j$$

where

$$q_{ij} = p_{ij}/\sum_{m=1}^{K} p_{im}.$$

***Exercise 11.** (Exercises marked with an asterisk (*) are intended for students with more mathematical background.) Derive Formula (7).

Note that, generally speaking, the larger values of q_{ij} will correspond to the stronger candidates, at least if more than one has a good chance to win. Hence the voter's rule of thumb in this setting would be to vote for candidates whom he rates above the average of all of his ratings with that average being weighted according to the strength of the candidates.

7. COMPUTATION OF OPTIMAL STRATEGIES FOR GENERAL VOTING SYSTEMS

We are now ready to apply the decision criteria we have developed to systems other than approval voting. To extend the model, we assume that the voter casts v_i votes for candidate c_i for $i = 1,\ldots,K$, where the v_i must satisfy constraints peculiar to the voting system in question. We will treat in detail only the method of expected utility, leaving the application of the Savage regret criterion to the exercises.

In this setting the definition of expected utility for a strategy $S = (v_1,\ldots,v_K)$ for the focal voter is:

$$\text{(8)} \qquad U(S) = \sum_{v_i > v_j} (f_i - f_j)\,(v_i - v_j)p_{ij},$$

where the probabilities p_{ij} depend on the voting system in question. Note that $(v_i - v_j)p_{ij}$ represents the probability that the focal voter will be decisive, while $(f_i - f_j)$ represents his payoff if he is decisive.

Our purpose is to show that for each of the five voting systems introduced in Section 2, the optimal strategy under the criterion of expected utility can be expressed in terms of a single index called the strategic value for a candidate. For a particular voting system and focal voter, we define the strategic value $E(c_i)$ for candidate c_i by

$$(9) \qquad E(c_i) = \sum_{j=1}^{K} (f_i - f_j) p_{ij}.$$

The strategic value $E(c_i)$ represents the expected payoff accruing to one incremental vote for candidate c_i.

Proposition 1. If $S = (v_1, \ldots, v_K)$ is a permissible strategy for the focal voter and voting system under study, and U is the expected utility function given in (8), then

$$(10) \qquad U(S) = E(c_1)v_1 + E(c_2)v_2 + \ldots + E(c_K)v_K = \sum_{i=1}^{K} E(c_i)v_i.$$

(See Appendix B for the proof.)

The following table gives the optimal strategy in terms of the strategic values $E(c_i)$ for each of the five voting systems described earlier.

TABLE 5

Voting System	
Plurality	Vote for the candidate for which $E(c_i)$ is largest.
Cumulative Voting	Cast all votes for that candidate with the largest $E(c_i)$.
Approval Voting	Vote for c_i if and only if $E(c_i) > 0$.
Cardinal Rating	Give the highest permitted rating if $E(c_i) > 0$ and the lowest permitted rating if $E(c_i) < 0$.
Borda System	Rank the candidates in order of the values of $E(c_i)$.

To verify, for example, the optimal strategy for approval voting, note that by (10), if $E(c_i) > 0$, voting for c_i increases U(S). If $E(c_i) < 0$, voting for c_i decreases U(S), and if $E(c_i) = 0$, voting for c_i has no effect on U(S).

Exercise 12. Verify the optimal strategies for the other four voting systems given in Table 5.

Exercise 13. For the candidates in Table 2, assume that $p_{12} = p_{13} = p_{24} = p_{34} = 1/6$, $p_{23} = 1/3$, and $p_{14} = 0$ (this represents a situation in which c_2 and c_3 are thought to be the strongest candidates). Determine the optimal strategies for each voting system for each voter according to the data in Table 2. Assuming that optimal strategies are used, determine the winning candidate for each voting system. For cumulative voting, assume ten votes per voter. (Hint: First work out a table of values $E(c_i)$ for the five voters and four candidates.)

Exercise 14. (This exercise may involve outside reading.) A strategy is called sincere if it reflects the true rankings of the voter for the candidates, i.e., if $v_i \geq v_j$ whenever $f_i > f_j$. For which voting systems is a voter more likely to find his optimal strategy in conflict with his sincere strategy? In which systems is the choice of the winner most sensitive to replacement of sincere strategies by optimal strategies? Use the example in Exercise 13 and/or any other examples to aid in your discussion. See Brams (1975) and Brams and Fishburn (1978) for further discussion concerning these points.

---

8. COMPARISON OF VOTING SYSTEMS WITH REGARD TO OPTIMAL STRATEGIES

To assess the relative difficulty of determining optimal strategies under different voting systems, we first note an algebraic rearrangement of Formula (9) for strategic value:

$$(11) \qquad E(c_i) = \sum_{j=1}^{K} (f_i - f_j) p_{ij} = f_i \sum_{j=1}^{K} p_{ij} - \sum_{j=1}^{K} f_j p_{ij} = p_i \left(f_i - \sum_{j=1}^{K} \frac{p_{ij}}{p_i} f_j\right),$$

where

$$p_i = \sum_{j=1}^{K} p_{ij}$$

is a rough measure of the strength of candidate c_i (a strong candidate is more likely to get into ties for first place than a weak candidate).

Thus according to the criterion of Table 5, under approval voting, the voter should vote for c_i if

$$f_i > \sum_{j=1}^{K} \frac{p_{ij}}{p_i} f_j \tag{12}$$

whereas a voter under the plurality system should choose that candidate for whom the entire expression in (11) is a maximum. Thus implementation of the optimal strategy is qualitatively different and more difficult for the voter to follow under plurality voting than under approval voting. In particular, the voter's decision under approval voting requires only that, for each $i = 1,\ldots,K$, he express a preference between candidate c_i and a gamble or lottery involving the other candidates (see (12)). The weights (probabilities) for this lottery are related (but not exactly proportional) to the expected electoral strength of the candidates.

The optimal decision for plurality voting requires that the voter attach a numerical quantity to his intensity of preference between candidate c_i and the lottery mentioned above, multiply that quantity by the measure p_i of expected electoral strength, and then choose that candidate for whom this product is maximal. Thus it seems likely that loss of voting power for the individual due to deviation from the optimal strategy through ignorance or misunderstanding of that strategy may be more severe under plurality voting than it would be under approval voting. Applying the same reasoning to the other criteria in Table 5, we conclude that determination of the optimal strategy under the Borda system or under cumulative voting is at least as difficult as under plurality voting.

One desirable feature of a voting system is that it not confuse the voter with superfluous options. We

now observe that certain voting systems reduce to other known systems when non-optimal strategies are eliminated. These latter systems permit no non-optimal strategies other than abstentions.

From Table 5, note that all optimal strategies under cumulative voting consist of casting all one's votes for a single candidate. Since all voters have the same number of votes to cast and none finds it in his interest to divide his vote between two or more candidates, we may assume, without loss of generality, that each has only one vote. This leaves each voter with precisely the options available under plurality voting, the additional options of cumulative voting are superfluous.

Similarly, all optimal strategies under cardinal rating voting use only the highest and lowest permitted ratings. Since the range of ratings permitted each voter is the same, we may assume that the range is [0,1]. Hence the only useful options are casting 0 or 1 vote per candidate, precisely the options available under approval voting. It can be shown (see Merrill (1980)) that all strategies under approval voting are uniquely optimal for some ratings f_i and probabilities p_{ij}, with the exception of the strategies of voting for all or none of the candidates. These last two strategies amount to abstention. Finally, it can be shown that the adjusted Borda system (see Exercise 4) reduces in a similar way to the Borda system.

9. THE CHOICE OF DECISION CRITERIA

Considerable study has been directed to the question of whether voting behavior is best described as a decision under uncertainty (using, e.g., the Savage regret criterion) or as a decision under risk (using expected utilities). If such an analysis is to be descriptive of the real world, it should be based on empirical studies. On the other hand, one objective of political science is to carry out prescriptive analysis, which is in this case the determination of what decision criterion ought to be used by the rational voter.

Mayer and Good (1975) argue prescriptively that the Savage regret criterion in its pure form is inappropriate since the voter usually has some information about the likely outcome of the election and because he is not contending against an intelligent opponent. (The step in the Savage procedure of computing the maximal regret for each possible strategy would suggest that some opponent is attempting to reduce the voter's influence by exploiting his weaknesses. Such an assumption seems unjustified.) These writers suggest that for most voters the true situation lies intermediate between the Savage regret and the expected utility models.

Tullock (1975) points out that the Savage regret criterion implies that one should vote (under the plurality system) for a candidate with only an infinitesimal chance of winning (for example, himself) as long as that candidate is his first preference. Tullock believes most people would consider this implication of the Savage regret criterion to be unreasonable.

Ferejohn and Fiorina (1975) consider descriptive behavior with regard to voter turnout rather than voting strategy. Their analysis, based on U.S. election data, suggests strongly that the Savage regret criterion is a better model for the decision of whether to vote at all than is the expected utility model. This module is, of course, concerned with voting strategy, not with voting turnout.

J. Black (1978), using data from Canadian elections and surveys of voter intensity of preference, finds support for the expected utility model in determining voting strategies. Specifically he analyses the tendency of a voter to cast a plurality ballot for a party other than his first preference under appropriate circumstances (see Exercise 14), a phenomenon which is predicted by the expected utility model but not by the Savage regret model. Cain (1978), in a similar analysis of the 1970 British General election, also finds support for the expected utility model, especially when the election is very close.

10. EMPIRICAL IMPACT ON THE OUTCOME OF MULTICANDIDATE ELECTIONS

In our final section we provide some empirical data concerning the effect various voting systems might have had on an election, assuming voters used their optimal strategies for each system. (See Merrill (1980: Section 6).) To do this we employ the "thermometer ratings" (on a scale of 0 to 100) by a national sample (CPS 1972 American National Election Study) of 1017 Democrats for the four candidates most active in the 1972 Democratic Presidential primaries (Humphrey, McGovern, Muskie, and Wallace).

The candidate ratings made by each respondent were interpreted as voter utilities f_i, $i = 1,\ldots,4$. Optimal strategies under the Laplace criterion were determined for the plurality, approval, and adjusted Borda voting systems. Vote totals for the resulting hypothetical elections are presented in Table 6, along with the results of (sincere) cardinal rating voting. (The latter totals were divided by the number of voters (1017) so that the values given represent the average rating for each candidate.)

Table 6

Voting system	Humphrey	McGovern	Muskie	Wallace
Plurality	299	335	128	255
Approval	652	590	461	371
Adjusted Borda	1829	1742	1357	1174
Average Cardinal rating	62.0	59.5	54.3	46.3

We note that McGovern is the winner under plurality voting, followed by Humphrey, Wallace, and Muskie, in that order. Under each of the alternative voting systems, the centrist candidates Humphrey and Muskie run stronger, each moving up one position in the order of finish.

The Laplace criterion used to obtain Table 6 assumes in effect that all probabilities p_{ij} are equal. Similar vote totals were obtained using the expected utility criterion for a variety of possible values of the p_{ij}. Most of these scenarios resulted in the same

orders of finish for the respective voting systems as in Table 6.

11. CONCLUSION

We have investigated a voter's optimal strategy under a variety of voting systems using both criteria under uncertainty (Savage regret and Laplace criteria) and criteria under risk (the expected utility criterion). We have argued that a voter's task of estimating his optimal strategy is least difficult under approval voting. It was found that cumulative and cardinal rating voting reduce to plurality and approval voting, respectively, when superfluous strategies are eliminated.

Finally an empirical comparison of voting systems assuming use of optimal strategies suggests that approval and Borda voting can produce results very similar to one another (and to that of sincere cardinal rating) but very different from that of plurality voting. These alternative systems tend to benefit centrist candidates, while still permitting voters to express support for more extreme candidates.

12. ADDITIONAL EXERCISES

*__Exercise 15.__ Construct a payoff matrix for plurality voting, where there are K candidates. (There are K strategies, one for each of the K candidates. Assume as before that $1 = f_1 \geq f_2 \geq \ldots \geq f_K = 0$. Show that the maximum regret for strategy (c_k) is the larger of x and y where

$$x = \max_{i,j \neq k} (f_i - f_j)$$

and

$$y = 2 \max_{i<k} (f_i - f_k)$$

and $y = 0$ if $k = 1$. Show that the maximum regret is minimized when $k = 1$.

*__Exercise 16.__ Show that the Laplace procedure applied to plurality voting leads to the same conclusion as the Savage regret method.

*__Exercise 17.__ Assume that a voter under cumulative voting has 1.0 vote at his disposal which he can apportion among the candidates, i.e., he can give v_i votes to candidate c_i where $v_i \geq 0$ and

$$\sum_{i=1}^{K} v_i = 1.$$

Also assume that $1 = f_1 \geq f_2 \geq \ldots \geq f_K = 0$. For example, if K = 3 he might choose $v_1 = .5$, $v_2 = .4$, and $v_3 = .1$. Denoting by $P(v_1,\ldots,v_K;\ c_i,\ c_j)$ the payoff for strategy $(v_1,\ldots,v_K)$ and contingency (c_i,c_j), show that

$$P(v_1,\ldots,v_k;\ c_i,c_j) = (v_i - v_j)(f_i - f_j)$$

and that the regret is given by

$$R(v_1,\ldots,v_K;\ c_i,c_j) = (1 - v_i + v_j)(f_i - f_j).$$

*__Exercise 18.__ For the cumulative voter of Exercise 17, and for K = 3 candidates with $f_2 = r$, show that the optimal Savage regret strategy is:

$$v_1 = 1/(1 + r),\ v_2 = r/(1 + r),$$

and

$$v_3 = 0 \text{ if } r \geq .5$$

and

$$v_1 = 2(1 - r)/(2 - r),\ v_2 = r/(2 - r),$$

and

$$v_3 = 0 \text{ if } r < .5.$$

*__Exercise 19.__ Assume that a cardinal rating voter must cast votes v_i so that $0 \leq v_i \leq 1$, and that $1 = f_1 \geq \ldots \geq f_K = 0$. For K = 3, show that the optimal Savage regret strategy is to set $v_i = f_i$ for i = 1, 2, and 3. Thus for K = 3, the Savage regret strategy is not only sincere but reflects ratings as well as rankings. (In fact, for cardinal rating voting, the Savage regret strategy is $v_i = f_i$ for any number K of candidates.)

13. REFERENCES

Black, D. (1958). The Theory of Committees and Elections. Cambridge University Press.

Black, J. (1978). "The Multicandidate Calculus of Voting: Application to Canadian Federal Elections." American Journal of Political Science 22 (August): 609-638.

Borda, J. (1781). "Memoire sur les Elections au Scrutin." Histoire de l'Academie Royale des Sciences, Paris.

Brams, S. (1975). Game Theory and Politics. New York: Free Press.

Brams, S. (1978). The Presidential Election Game. New Haven: Yale University Press.

Brams, S. (1979). "Approval Voting: A Practical Election Reform for Multicandidate Elections." National Civic Review, forthcoming.

Brams, S. and P. Fishburn (1978). "Approval Voting." American Political Science Review 72: 831-847.

Brams, S. and P. Fishburn (1979). "Efficacy, Power and Equity in Aproval Voting." To appear.

CPS 1972 American National Election Study. Ann Arbor, Michigan: Inter-university Consortium for Political Research.

Cain, B. (1978). "Strategic Voting in Britain." American Journal of Political Science 22 (August): 639-655.

Ferejohn, J. and M. Fiorina (1975). "Closeness Counts only in Horeshoes and Dancing." American Political Science Review 69: 920-925.

Luce, D. and H. Raiffa (1957). Games and Decisions. New York: Wiley.

Mayer, L. and I. J. Good (1975). "Is Minimax Regret Applicable to Voting Decisions?" American Political Science Review 69: 916-917.

McKelvey, R. and P. Ordeshook (1972). "A General Theory of the Calculus of Voting." In J. F. Herndon and J. L. Bernd, eds., Mathematical Applications in Political Science VI. Charlottesville: University Press of Virginia, pp. 32-78.

Merrill, S. (1979). "Approval Voting: A "Best Buy" Method for Multicandidate Elections?" Mathematics Magazine 52 (March): 98-102.

Merrill, S. (1980). "Strategic Decisions under One-stage Multicandidate Voting Systems." Public Choice 35 (Summer).

Rae, D. (1971). The Political Consequences of Election Laws. New Haven: Yale University Press.

Tullock, G. (1975). "The Paradox of Not Voting for Oneself." American Political Science Review 69: 919.

Weber, R. (1977). "Comparison of Voting Systems." Forthcoming.

14. ANSWERS TO EXERCISES

1.

Candidate:	Adams	Bianco	Cohen	Delaney
Plurality:	2	1	1	1
Borda:	7	10	9	4

2. For a four candidate race, the number of Borda votes a voter casts for a candidate is obtained by subtracting the candidate's rank from the number 4. Thus a small rank sum corresponds to a large Borda vote. In fact, the total Borda vote for a candidate is obtained by subtracting the rank sum from 4n, where n is the number of voters.

3.

Candidate:	A	B	C	D
Vote total:	-1	5	3	-7

Yes. Using the method suggested in this exercise, the number of votes cast by a voter drops by two between each rank, whereas it drops only one in the Borda system. This expands the candidate totals but does not alter the relative position of those totals (compare vote totals in Exercises 1 and 3).

4. A: 3, B: 0, C: 0, D: -3.

5.

Candidate:	A	B	C	D
Cardinal rating:	22	32	39	17
Approval:	2	3	5	2

7. For 4 candidates, if there is no Condorcet winner, no candidate can win more than 2 of the 6 pairwise contests. For the number of victories to add to 6, at least two candidates must each win 2 contests, so the Copeland rule is inconclusive.

8.

	AB	AC	BC
(A)	7	10	0
(A,B)	0	10	3
(A,C)	7	0	-3
(B)	-7	0	3
(B,C)	-7	-10	0
(C)	0	-10	-3

9.

	AB	AC	BC	Maximal Regret
(A)	0	0	3	3
(A,B)	7	0	0	7
(A,C)	0	10	6	10
(B)	14	10	0	14
(B,C)	14	20	3	20
(C)	0	20	6	20

Maximal regret is least for strategy (A).

10.

Voter	Laplace	Savage
J	(A,B,C)	(A,B,C)
K	(A,C)	(A,C)
L	(B,C,D)	(B,C,D)
M	(C)	(C)
N	(B,D)	(B,C,D)

Cohen wins under either criterion. The only difference between the results is that voter N votes for Cohen under the Savage regret but not under the Laplace crierion.

12. Plurality:

By (10), $U(S) = E(c_i)$ where c_i is the candidate voted for.

Cumulative:

Again by (10), any vote counts more toward total utility if placed on the candidate for whom $E(c_i)$ is largest.

Cardinal rating:

If $E(c_i) > 0$, the higher the rating, the higher the contribution to total utility. If $E(c_i) < 0$, the reverse is true.

Borda:

Placing the most votes on the candidates with the highest strategic value maximizes total utility.

13.

	A	B	C	D
J	5	8	2	-15
K	10	-18	18	-10
L	-18	17	5	-4
M	-9	-10	32	-13
N	-15	14	-4	5

(Divide cell entries by 6 to obtain the values of $E(c_i)$ for each voter.)

Results:	A	B	C	D	Winner
Plurality:	0	3	2	0	B
Cumulative:	0	30	20	0	B
Approval:	2	3	4	1	C
Cardinal rating:	20	30	40	10	C
Borda:	6	10	10	4	B-C (tie)

14. Optimal strategies are more often sincere under approval or cardinal rating voting (see the references cited in Exercise 14, and the proposition stated in Exercise 19).

15. APPENDICES

Appendix A. Derivation of the optimal strategy under the Savage regret criterion for a K-candidate race.

Suppose there are K candiates and, without loss of generality, assume that $1 = f_1 \geq f_2 \geq \ldots \geq f_K = 0$. First, in seeking an optimal Savage regret strategy, we may restrict our attention to strategies which include a vote for c_1 but not for c_K. (For if S is any strategy not including a vote for c_1, it is dominated by the strategy consisting of voting for the same candidates plus c_1. This follows since adding a vote for c_1 increases the payoffs by $(1 - f_j)$ for contingencies (c_1, c_j) and has no effect for the other contingencies. A similar argument shows that c_K need not be included in an optimal strategy.)

Next we note that each column in the payoff matrix (and hence the corresponding column in the regret matrix) contains only three distinct entries. In fact if we consider the column for contingency (c_i, c_j), the payoffs are:

(i) $(f_i - f_j)$ if $c_i \in S$ but $c_j \notin S$, or

(ii) 0 if c_i and c_j are either both in or both not in S, or

(iii) $-(f_i - f_j)$ if $c_i \notin S$ but $c_j \in S$.

The corresponding regrets are then

(i') 0

(ii') $(f_i - f_j)$

(iii') $2(f_i - f_j)$

for the same three conditions, respectively.

Hence for a particular strategy S, the maximum regret for S is the larger of A_S and B_S where

$A_S = \max(f_i - f_j)$ under condition (ii), and

$B_S = 2 \cdot \max(f_i - f_j)$ under condition (iii).

We claim that the strategy of voting precisely for those candidates c_i for which $f_i > .5$ minimizes the

maximum regret. This strategy is described analytically by

$$S_O = \{c_i : f_i > .5\}.$$

Note that (maximum regret for S_O) = $A_{S_O} \leq .5$. If T is any other strategy, then either

Case 1: There exists $c_k \notin T$ with $f_k > .5$, or

Case 2: There exists $c_k \in T$ with $f_k \leq .5$.

For case 1,

$$\begin{aligned}(\text{Maximum regret for } T) &\geq A_T \\ &\geq \max_{c_i, c_j \notin T} (f_i - f_j) \\ &\geq f_k - f_K \\ &= f_k - 0 > .5, \\ &\text{since } c_K \notin T.\end{aligned}$$

For case 2,

$$\begin{aligned}(\text{Maximum regret for } T) &\geq A_T \\ &\geq \max_{c_i, c_j \in T} (f_i - f_j) \\ &\geq f_1 - f_k \\ &= 1 - f_k \geq .5, \\ &\text{since } c_1 \in T.\end{aligned}$$

Hence the maximum regret for S_o is less than or equal to the maximum regret for any other permissible strategy for approval voting. (Any candidate for which $f_i = .5$ can be included in the strategy without altering the maximum regret.) Thus S_o is the optimal Savage regret strategy.

Appendix B. Proof of Proposition 1.

First observe that

$$\begin{aligned}2U(S) &= \sum_{i=1}^{K} \sum_{j=1}^{K} (f_i - f_j)(v_i - v_j)p_{ij} \\ &= \sum_{i=1}^{K} \sum_{j=1}^{K} (f_i - f_j)v_i p_{ij} - \sum_{i=1}^{K} \sum_{j=1}^{K} (f_i - f_j)v_j p_{ij}.\end{aligned}$$

But interchanging i and j, we may write

$$\begin{aligned}- \sum_{i=1}^{K} \sum_{j=1}^{K} (f_i - f_j)v_j p_{ij} &= - \sum_{j=1}^{K} \sum_{i=1}^{K} (f_j - f_i)v_i p_{ij} \\ &= + \sum_{i=1}^{K} \sum_{j=1}^{K} (f_i - f_j)v_i p_{ij}\end{aligned}$$

Hence $2U(S) = 2 \sum_{i=1}^{K} \sum_{j=1}^{K} (f_i - f_j)v_i p_{ij} = 2 \sum_{i=1}^{K} E(c_i)v_i$,

which completes the proof.

STUDENT FORM 1

Request for Help

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name____________________________ Unit No.______

Page______		Section______		Model Exam Problem No._____
○ Upper	OR	Paragraph______	OR	Text Problem No._____
○ Middle				
○ Lower				

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature________________________

Please use reverse if necessary.

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

STUDENT FORM 2

Unit Questionnaire

Name______________________________ Unit No.______________ Date______________

Institution______________________________ Course No.______________________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much Longer	Somewhat Longer	About the Same	Somewhat Shorter	Much Shorter
____	____	____	____	____

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain______________________________

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain______________________________

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)

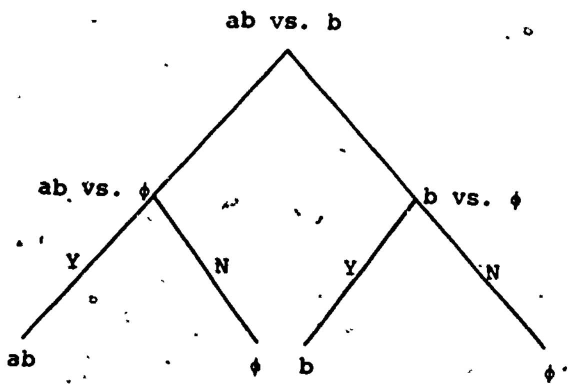
umap

UNIT 386

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT

AN APPLICATION OF VOTING THEORY TO CONGRESS

by James M. Enelow



APPLICATIONS OF DECISION THEORY AND GAME THEORY TO AMERICAN POLITICS

edc/umap/55chapel st./newton.mass.02160

AN APPLICATION OF VOTING THEORY TO CONGRESS

by

James M. Enelow
Department of Political Science
State University of New York
Stony Brook, New York 11794

TABLE OF CONTENTS

Intermodular Description Sheet: *UMAP Unit 386*

Title: AN APPLICATION OF VOTING THEORY TO CONGRESS

Author: James M. Enelow
Department of Political Science
State University of New York
Stony Brook, New York 11794

Review Stage/Date: III 4/25/80

Classification: APPL DECISION THEORY & GAME THEORY/AMER POL

Prerequisite Skills:
1. High school algebra.
2. Elementary probability theory.
3. Elementary utility theory.
4. Ability to understand tree diagrams.

Output Skills:
1. To gain an understanding of how a simple theory of voting can be used to analyze strategic voting in Congress.

Other Related Units:

MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists, and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

The Project would like to thank Thomas E. Elsner of General Motors Institute, Flint, Michigan, William A. Hansen of DELTAK, Inc., Oak Brook, Illinois; Harold Hanes of Earlham College, Richmond, Indiana, and members of the UMAP Political Science Panel for their reviews, and all others who assisted in the production fo this unit.

This material was prepared with the partial support of National Science Foundation Grant No. SED76-19615 A02. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF or the copyright holder.

1. INTRODUCTION

It is a common observation that voting in the United States Congress is frequently strategic. This observation is usually interpreted to mean that a congressman's vote on a legislative proposal may be guided by strategic considerations and not strictly by his own preferences regarding the matter. For example, suppose a Senator prefers the originally negotiated Salt II treaty to no treaty at all, but would like to see the number of missiles allowed under the provisions of the treaty reduced. Assume, now, that such an amendment were offered. Should the Senator necessarily vote for it? If he perceived that adoption of the amendment would bring about almost certain rejection of the treaty by the Soviet Union, he might vote against it. On the other hand, if he perceived that adoption of the amendment would lessen but not destroy the chances of the treaty's acceptance by the Soviets, he might accept the risk of rejection and vote for the amendment.

The above example captures many features of the voting model that will be developed in this module. The purpose of this model will be to explain and predict voting on congressional amendments. We shall focus our attention on two types of amendments--those which are seen as increasing and those which are seen as decreasing the chances of a bill's passage. The first type will be called a "saving" amendment and the second type a "killer" amendment. We will assume that a congressman's vote on either of these two types of amendment is based on two factors--his preferences regarding the possible outcomes once the final vote on a bill is taken and his assessment of how the amendment in question will affect the likelihood of the bill's passage. These two factors will allow us to construct a lottery theory of strategic voting that has elsewhere been called expected utility sophisticated (EUS) voting.[1] After showing how this theory works, we will apply it to an actual case of a saving amendment--the Mathias amendment to the 1966 Civil Rights bill. We shall also discuss killer amendments to the Salt II treaty.

2. AN EXPECTED UTILITY THEORY OF SOPHISTICATED VOTING

Let us initially assume a simple situation--an amendment to a bill is voted on by n voters followed by a vote on the bill itself. The following tree diagrams the structure of these two votes.

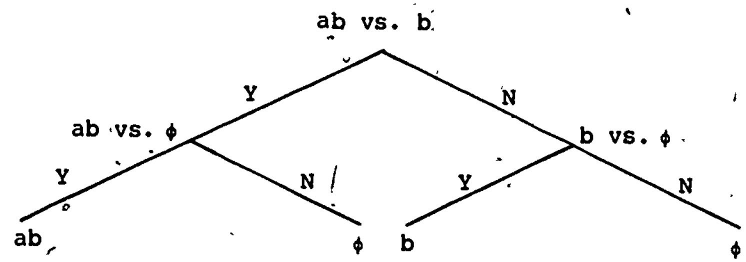


Figure 1.

Initially, the amendment is voted on, providing the voter with two choices: "yea" (Y) or "nay" (N). If a majority of the n voters vote Y (which may be a special majority, such as (2n)/3n), the next vote is a contest between the amended bill and no bill and, again, the voter can vote either Y or N. On the other hand, if a majority of the n voters vote N on the amendment, the next vote is a contest between the unamended bill and n bill, and, once again, the voter can vote Y or N. Therefore, after the second vote is taken, three outcomes may result--the amended bill (ab), the unamended bill (b), and no bill (ϕ).

This description outlines the bare structure of the voting process as seen by the voters. We assume that each voter can rank order the three possible outcomes we have described from best to worst. This ranking will be termed his preference order. Assume that no voter is indifferent between any two of these three outcomes. This means that the voter can rank these outcomes in 3! = 6 possible ways. This listing is given in Figure 2.

Preference Type #

1	2	3	4	5	6
b	b	ab	ab	ϕ	ϕ
ab	ϕ	b	ϕ	b	ab
ϕ	ab	ϕ	b	ab	b

Figure 2.

Let us now make another assumption--that each voter's preferences are sufficiently "consistent" to be represented by cardinal utility numbers. These numbers measure the "strength" of an individual's preferences, cannot be compared across individuals, and can always be normalized so that an individual's first ranked outcome can be assigned the number "1" and his worst outcome the number "0". These numbers are also assumed to satisfy the "expected utility hypothesis," which will be explained shortly. The utility of the i^{th} voter ($i = 1, \ldots, n$) for the three outcomes of the voting process is, then, $u_i(ab)$, $u_i(b)$, and $u_i(\phi)$, these numbers all being contained in the interval $[0,1]$.

The final piece in the model has been alluded to earlier. This is each voter's subjective probability estimates of two events--that the amended bill will pass and that the unamended bill will pass. In the case of our Salt II example, the term "pass" could be replaced by some other term denoting acceptance by the treaty's signatories. However, we shall keep matters simple for now and consider the term "pass" to apply to majority acceptance by the n members of the voting body. We are assuming, then, that each voter forms an estimate of the likelihood that the amended bill will pass and an estimate of the likelihood that the unamended bill will pass. These estimates may vary from one voter to the next and are based on whatever information each voter possesses concerning the preferences of other voters and his assessment of their voting intentions. Such information is assumed to be imperfect. Further, voters may share whatever information they possess, but they are assumed to make their voting decisions independently of one another. In other words, the voting game is noncooperative.

Let us now see how these probability estimates can be incorporated into the tree diagram of Figure 3, where p_i denotes the estimate of the i^{th} voter ($i = 1, \ldots, n$) that ab will pass ($0 \leq p_i \leq 1$) and q_i the estimate of the i^{th} voter that b will pass ($0 \leq q_i \leq 1$). The estimates of the i^{th} voter that ab and b will fail are $1 - p_i$ and $1 - q_i$, respectively. We are now ready to state the fundamental hypothesis of our model. We assume that the i^{th} voter sees his choice of voting Y or N at any

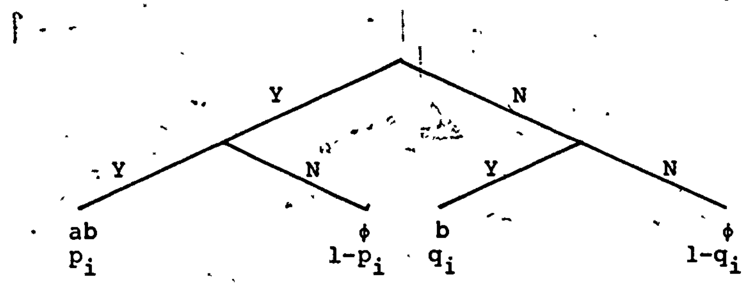


Figure 3.

point in the voting process as a choice between the two <u>lotteries</u> associated with passage or failure of the issue being voted on at that point. Thus, if $L_1 = (p_i ab, (1-p_i)\phi)$ and $L_2 = (q_i b, (1-q_i)\phi)$, then the i^{th} voter votes Y on the amendment if he prefers L_1 to L_2 and N otherwise, where L_1 is a gamble that yields ab with probability p_i and ϕ with probability $1 - p_i$, and L_2 is interpreted likewise. To determine which lottery he prefers, we employ the expected utility hypothesis which states that he prefers the lottery whose utility expected value is greater. More formally, the i^{th} voter votes Y on the amendment if and only if

$$u_i(L_1) > u_i(L_2) \rightarrow$$

$$(1) \qquad p_i u_i(ab) + (1-p_i)u_i(\phi)$$

$$> q_i u_i(b) + (1-q_i)u_i(\phi).$$

We shall explore the implications of expresion (1) momentarily but, first, note that voting on final passage simply involves a preference comparison of the two remaining possible outcomes. On the other hand, if two amendments were being voted on serially, voting on the first amendment would involve the comparison of two compound lotteries, i.e. each lottery would be a lottery between two lotteries. An example of a compound lottery will be given later.

Let us now draw out some implications from expression (1) for voting on "saving" and "killer" amendments. We shall define these two types of amendments as follows. If $p_i > q_i$, then voter i sees the amendment as "saving" the bill, while if $q_i > p_i$, voter i sees the amendment as a "killer." This reflects the idea that a saving amendment is one which is seen as increasing the chances of a bill's passage, while a killer amendment is seen as decreasing these chances. Under what circumstances, then, will a voter of some given preference

type vote Y or N on a "saving" or "killer" amendment? Let us start with voters of preference type #1. For simplicity, we shall sometimes use a double index to indicate a voter's preference type. Thus, a generic member i of preference group #1 will be labelled i1.

Under what conditions, then, will voter i1 vote Y on a saving amendment? It is a straightforward exercise to demonstrate that if

$$u_{i1}(ab) > \frac{p_{i1}}{p_{i1}}$$

then voter i1 will vote Y, while if the inequality is reversed he will vote N.

Exercise 1. Prove this.

Suppose, now, that $p_{i1} = 2q_{i1}$. Then, since $q_{i1}/p_{i1} = .5$, $u_{i1}(ab)$ must exceed .5 for i1 to vote Y on the amendment. If $p_{i1} = 3\ q_{i1}$, then $u_{i1}(ab)$ must exceed .33. In other words, the more the voter thinks the amendment increases the chances of the bill's passage, the more likely he is to vote for it. Of course, the reverse also holds. If $p_{i1} = 1.2\ q_{i1}$, then u_{i1} (ab) must exceed .83 for i1 to vote Y on the amendment.

Under what conditions will voters of preference types #2 - #6 vote Y or N on a saving amendment? Interestingly, algebraic manipulation reveals that voters of preference types #2 and #5 will <u>always</u> vote N on a saving amendment, while voters of preference types #3 and #4 will <u>always</u> vote Y. For example, for a voter of preference type #2 to vote Y on a saving amendment, $(1-p_{i2})\ u_{i2}\ (\phi)$ must exceed $q_{i2} + (1-q_{i2})u_{i2}\ (\phi)$. But, since $p_{i2} > q_{i2}$, $(1-q_{i2}) > (1-p_{i2})$, so this is impossible. Only voters of preference type #6 are like voters of preference type #1 in being able to vote either way. If

$$u_{i6}(ab) > 1 - \frac{q_{i6}}{p_{i6}}$$

then voter i6 will vote Y on a saving amendment, while if the inequality is reversed he will vote N.

Exercise 2. If

$$\frac{q_{i1}}{p_{i1}} = \frac{q_{i6}}{p_{i6}}$$

for some i1 and i6, is it possible for them to both vote the same way on a saving amendment?

It is not difficult to see that the more an amendment increases the chances of a bill's passage, the less likely i6 is to vote for it. For example, if $p_{i6} = 3\ q_{i6}$, then u_{i6} (ab) must exceed .67 for i6 to vote Y on the amendment.

Let us now discuss voting on killer amendments. It is again a straightforward algebraic exercise to establish the conditions under which members of each preference group will vote Y or N. Now, members of preference groups 1 and 2 invariably vote N and members of preference groups 4 and 6 invariably vote Y. Again, to compute one example, for a voter of preference type #1 to vote Y on a killer amendment, $p_{i1}\ u_{i1}$ (ab) must exceed q_{i1}. But $q_{i1} > p_{i1}$ and $1 > u_{i1}$ (ab), so this is impossible. However, members of preference groups 3 and 5 can vote either way. If

$$\frac{p_{i3}}{q_{i3}} > u_{i3}(b)$$

voter i3 will vote Y on a killer amendment, and if the inequality is reversed he will vote N. Likewise, if

$$1 - \frac{p_{i5}}{q_{i5}} > u_{i5}(b)$$

voter i5 will vote Y on a killer amendment, and if the inequality is reversed he will vote N.

Exercise 3. Assume $p_{i3} = p_{i5} = 0$. How will i3 and i5 vote on the amendment?

---

3. THE MATHIAS AMENDMENT TO THE 1966 CIVIL RIGHTS BILL

We will now see how well our theory can predict and explain voting on a real example of a saving amendment. The 1966 Civil Rights bill (HR 14765), as reported by the House Judiciary Committee, contained a controversial open housing provision, known officially as Title IV. The intent of the Mathias amendment offered on the House floor by Rep. Mathias, was to weaken this section of the bill by allowing a homeowner to provide a real estate broker with discriminatory instructions, if the broker did not solicit them.

This amendment was offered in an attempt to save Title IV from being stricken from the bill. The two votes that fit the requirements of our model are, therefore, the vote on the Mathias amendment and the vote on a motion by Rep. Moore to recommit the 1966 Civil Rights bill to the Judiciary Committee with instructions to delete Title IV. A Y vote on the motion to recommit is a vote to <u>delete</u> Title IV and a N vote is a vote to leave Title IV in the bill. The voting tree, therefore, looks like Figure 4.

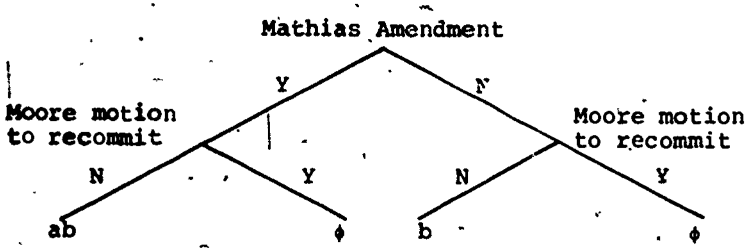


Figure 4.

The three outcomes are: Title IV with the Mathias amendment (ab), Title IV without the Mathias amendment (b), and the 1966 Civil Rights bill without Title IV (ϕ).

We will assume that all voters saw the adoption of the Mathias amendment as increasing the chances that Title IV would be saved, thus enabling us to apply our earlier predictions about voting on a saving amendment to all members of the House. In order to do so, however, we need some measure of each Representative's preferences regarding the three outcomes shown in Figure 4. The measure we will use (employed in Table 1) is the % "right" votes cast by each Representative according to Americans for Democratic Action (ADA) on 17 selected votes in 1966. ADA is a liberal interest group particularly identified with advocating a stronger federal role in domestic areas such as housing and civil rights.

We shall assume that type 1 voters have the highest ADA scores and type 6 voters the lowest. This follows from the rankings given the three possible outcomes by these voters. Type 1 voters rank the stronger open housing section first, the weaker open housing section second, and no open housing section last. Type 6 voters rank the three outcomes in reverse order. Thus, type 1 voters are in complete agreement with ADA's preferences and type 6 voters are in complete disagreement. Type 2 voters should also have high ADA scores, although their attitude is "all-or-nothing." Type 3 and 4 voters should have intermediate scores, since they rank the weaker open housing section first, and type 5 voters should have low ADA scores, like type 6 voters.

The following is a list of how we expect the members of each preference group to vote. Since the Mathias amendment passed, the second vote was a contest between ab and ϕ (recall that a Y vote on recommittal is a vote for ϕ).

Predicted Voting on
Mathias Amendment
and Moore Motion

Preference Type #

1	2	3	4	5	6
YN	NY	YN	YN	NY	NY
or					or
NN					YY

Figure 5

Table 1 presents our findings. Only type 1 voters are predicted to vote NN and only type 6 voters are predicted to vote YY. Interestingly, of the 26 voters who voted NN, 19 had scores between 80 and 100, while of the 40 voters who voted YY, 24 had scores between 0 and

Table 1

ADA Scores

(% "Right" votes on 17 issues of 1966)

Votes on Mathias Amendment and Moore motion to recommit HR 14765*

	100-90	89-80	79-70	69-60	59-50	49-40	39-30	29-20	19-10	9-0	Totals
#NN	12	7	2	0	0	0	1	0	2	2	26
#YN	23	71	30	12	11	11	5	8	11	13	195
#NY	0	0	4	5	0	4	3	10	14	109	149
#YY	0	1	5	1	2	2	3	2	3	21	40
Totals	35	79	41	18	13	17	12	20	30	145	410

* Only members who voted on both are included

19. This corresponds well with our predictions. It is also interesting to note that the dispersion of amendment votes is greatest for voters with high and low ADA scores. This conforms with our prediction that only type 1 and type 6 voters can vote either way on the amendment.

Note also that if type 1 voters are assumed to have ADA scores of 80-100 and type 6 voters ADA scores of 0-19, only 29 out of 289 voters with such scores voted contrary to the predictions of our model. Our model, therefore, has a 90% success rate with these voters.

As for voters in the 20-79 range, 64% of them voted YN. This leads us to believe that most of them were type 3 or type 4 voters. If the three outcomes were arrayed on a single horizontal dimension from left to right in the order b-ab-ϕ and a second vertical dimension were used to measure strength of preference from last to first, it would be possible to represent each voter's preference order by 3 points in a two-dimensional coordinate system. If these points were then connected in left to right order each preference order would correspond to a preference curve.

__Exercise 4.__ Draw a graph of all 6 preference curves.

A preference curve is __single-peaked__ if, looking from left to right, it always rises or falls, or it rises to a point and then falls, doing so no more than once. Based on this definition, only type 1, 3, 4, and 6 voters have single-peaked preference curves. But, our data suggests that most voters held one of these 4 preference types, so our conclusion is that most Representatives held single-peaked preferences with respect to an underlying dimension that would seem to measure degree of federal control over private housing.

An interesting finding emerges from reading the floor debate on the Mathias amendment. Among self-identified type 1 and type 6 voters, some offered voting justifications based on __substantive__ considerations and others offered justifications based on __tactical__ considerations. Clearly, a substantive jus-

tification is one based on the magnitude of u_i (ab), while a tactical justification is based on the ratio of q_i to p_i. Thus, a type 1 vote such as Rep. Albert (D-Okla., ADA-82%) justified a Y vote on the Mathias amendment by calling the amended Title IV "an important step forward."[2] Recalling that a type 1 voter casts a Y vote on a saving amendment if and only if

$$u_{i1}(ab) > \frac{q_{i1}}{p_{i1}}.$$

Albert's justification is clearly consistent with our model. On the other hand, a tactical justification by a type 1 voter for a Y vote is provided by Rep. Diggs (D-Mich., ADA-82%), who termed the amendment a "tactical concession." Diggs makes clear his limited enthusiasm for the Mathias amendment, but recognizes that there are "not enough affirmative votes"[3] for Title IV without it. Thus, for Diggs, q_{i1}/p_{i1} would appear to be near zero.

Exercise 5. How would you interpret the justificatory intent of Rep. Poff (R-Va., ADA-0%), a type 6 voter, who stated "that any liberal who votes for the Mathias amendment will be indicted by liberals for having "gutted" Title IV,..."[4]

Other examples of substantive and tactical justifications given by type 1 and type 6 voters for Y and N votes on the Mathias amendment are easy to come by. Thus, we find not only a good rate of predictive success for our model, but also a striking degree of verisimilitude with the actual pronouncements of the congressmen themselves.

4. THE SALT II TREATY

In this final section of the module we shall employ our model to discuss killer amendments to the Salt II treaty. We shall consider a killer amendment to the treaty to be one which all Senators see as decreasing the chances of the treaty's acceptance--not by the Senate, however, but by the Soviet Union. This case is similar to the one which occurred with respect to the Panama Canal treaties, where a host of amend-

ments was offered not to bring about Senate rejection of the treaties, but to bring about rejection of the treaties by Panama. If anything, such amendments increase the chances of acceptance by the Senate since they typically involve changes that favor United States interests. However, for now, we will only consider the effect of a Salt II treaty amendment on the chances of the treaty's acceptance by the Soviets.

Before beginning, it is important to distinguish an amendment to the treaty from a reservation or understanding. An amendment changes the actual text of the treaty, while a reservation or understanding does not. Thus, the Soviet warning in the summer of 1979 (immediately after the Salt II treaty was signed), that changes in the treaty would bring about "a fantastic situation"[5] was aimed at preventing treaty amendments.

Let us now analyze the strategic environment surrounding a killer amendment to the Salt II treaty. Recalling our earlier results, Figure 6 lists the expected votes for members of each preference type.

Voting on Killer Amendment

	Preference Type					
	1	2	3	4	5	6
Vote	N	N	Y or N	Y	Y or N	Y

Figure 6

Recall also that if

$$\frac{p_{i3}}{q_{i3}} > u_{i3}(b)$$

voter i3 will vote Y, while if the inequality is reversed he will vote N. On the other hand, if

$$1 - \frac{p_{i5}}{q_{i5}} > u_{i5}(b)$$

voter i5 will vote Y, while if the inequality is reversed he will vote N. The example give in the introduction to the module is clearly one of a type 3

voter faced with the problem of how to vote on a killer amendment to the Salt II treaty.

Our results indicate, therefore, that Soviet warnings to the Senate against amending the Salt II treaty could only affect the votes of Senators with type 3 or type 5 preferences. The question ~~then~~ becomes: were Soviet threats rational from the standpoint of persuading these Senators to vote N? This is not an easy question to answer. By seeking to imply that p_i was near zero, the Soviets were creating a situation in which type 3 voters would vote N but type 5 voters would vote Y on a treaty amendment. If the Soviets estimated that the type 3 group was larger than the type 5 group, this tactic would appear to make sense. Certainly it would be superior to conveying the impression that v_i was near one. However, our analysis tells us that type 3 votes are not necessarily gained at the expense of type 5 votes. If instead of a tactical approach to influencing Senators, the Soviets had employed a substantive approach, it may have been possible (at least before other events intervened) to persuade both types of voters to vote against any treaty amendments. The way to do this would have been to persuade both types of voters of the attractiveness of the unamended treaty. In this way, $u_{i3}(b)$ and $u_{i5}(b)$ would increase and thus so would the chances of voting N on a treaty amendment for both types of voters.

However, if the Soviets judged that there were very few Senators with type 5 preferences compared to those with type 3 preferences (a not unreasonable assumption since preference type #6 would seem more appropriate for a foreign policy conservative), their approach would have cost them few votes. From this standpoint, therefore, the Soviets were acting in a manner that was clearly purposeful, despite the backlash evidenced in Senator Howard Baker's reply that "the Senate will work its will...without that advice from Russia."[6]

Exercise 6. Assume Senator Baker has type 4 preferences. Would it be rational for him to offer a killer amendment to the Salt II treaty?

On the other hand, if proponents of treaty amendments shared the Soviet perception that type 3 voters were the proper focus of the ratification battle, then they should have been trying to make p_i appear as large as possible. From this perspective the statement by Lieut. Gen. Edward J. Rowny, one of Salt II's negotiators, that amendments would not kill the treaty because "they need it more than we do"[7] was a rational counter-strategy to use against the Soviets.

Thus, our model allows us to understand the battle that took place in the summer of 1979 between the Soviet Union and certain members of the U.S. Senate over amendments to the Salt II treaty. Soviet warnings were much more than a public expression of irritation. Instead, they represented a clear and deliberate plan to influence Senators' votes.

As a final exercise, let us incorporate into our model the statement made at the beginning of this section that a treaty amendment can be a killer with respect to the treaty's signatories but also be a saving amendment from the standpoint of Senate ratification. Figure 7 expands our model to accomodate a "saving-killer" amendment.

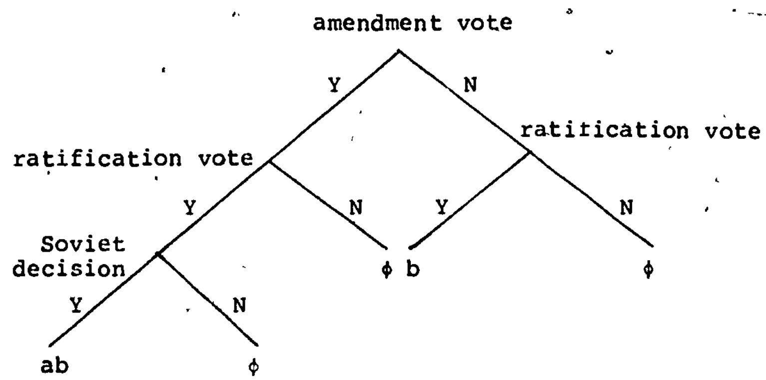


Figure 7.

The new division in the tree represents the Soviet decision to accept or reject the amended treaty. It is assumed that acceptance of the unamended treaty is automatic, since the Soviets signed the treaty in this form. Now, assume p_i and q_i represent the two ratifi-

cation probabilities for the amended and unamended treaties as seen by voter i and that $p_i > q_i$ for all $i = 1, \ldots, n$ (ratification of the treaty requires a two-thirds majority). Let r_i represent the probability as seen by voter i that the Soviets will accept the amended treaty and assume that $q_i > r_i$ for all $i = 1, \ldots, n$.

The two lotteries that each voter must compare before voting on this "saving-killer" amendment are then $(p_i r_i ab, (1 - p_i r_i) \phi)$ and $(q_i b, (1 - q_i) \phi)$. However, since $q_i > p_i r_i$, a "saving-killer" amendment is really just a killer amendment (since $q_i > r_i \geq p_i r_i$) and so needs no special treatment. However, suppose the prospects for ratifying the unamended treaty become suddenly dim and $r_i > q_i$ for all i. Then clearly, $p_i r_i$ may be greater or less than q_i and the analysis becomes more complicated. In any event, the point of this small exercise is to show that our voting model can be expanded to represent more complex decision problems.

Exercise 7. Draw a voting tree to represent the situation where two amendments to a bill are voted on serially followed by a vote on final passage of the bill. What are the two lotteries that each voter must compare before voting on the first amendment?

---

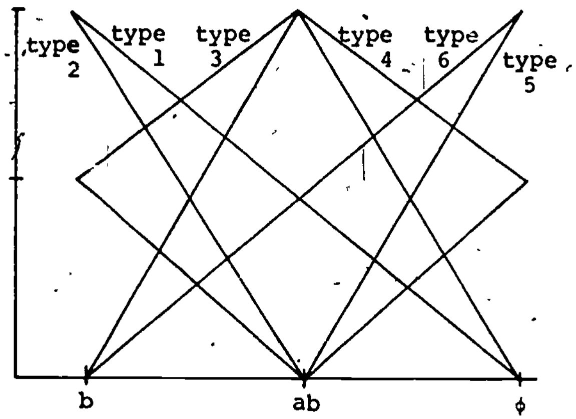
5. CONCLUSION

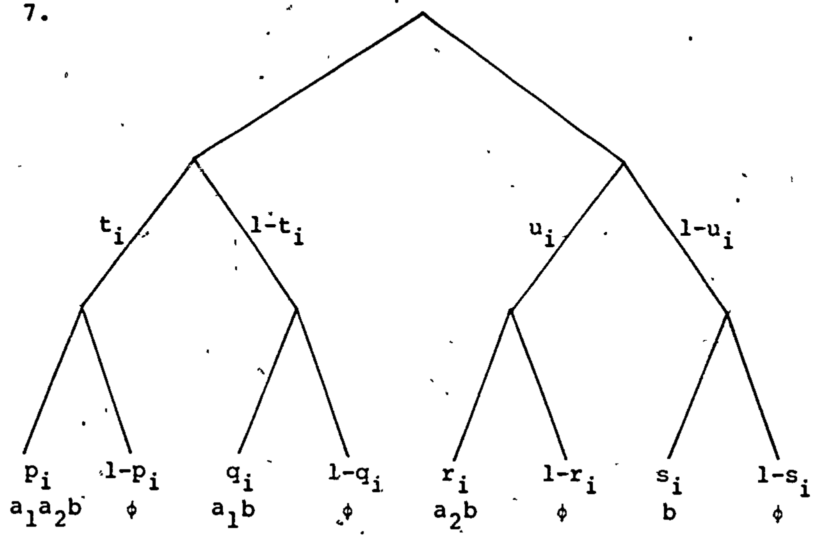
The point of this module has been to develop a simple lottery theory of strategic voting to explain how preferences and subjective probability estimates of how much an amendment can help or hurt a bill combine to determine how a congressman will vote on a legislative amendment. We have focused on two types of amendments--those that a voter thinks will help save a bill and those that a voter thinks will help kill a bill--and showed that on each type of amendment only two of the six possible preference types can vote either Y or N, depending on the values taken by the three variables that determine the voting decision.

We then applied our theory to voting on a real example of a saving amendment, the Mathias amendment to the 1966 Civil Rights bill, and showed that our data agreed substantially with the theory's predictions and also that the model accurately represented the verbal justifications offered by many Representatives. As an example of a killer amendment, we discussed amendments to the Salt II treaty and showed that our model could illuminate the debate carried on between the Soviet Union and some members of the U.S. Senate in the summer of 1979. We also showed how our model could represent the saving and killer aspects of a Salt II amendment. Thus we demonstrated the model's flexibility.

In closing, this module demonstrates that a simple model can capture a great deal of real world complexity, while simplifying reality sufficiently to make straightforward predictions and lay bare the underlying logic of the phenomenon being modelled. This is the purpose of any rigorous scientific investigation and it has been our purpose here.

6. ANSWERS TO EXERCISES

1. Substituting $u_{i1}(b) = 1$ and $u_{i1}(\phi) = 0$, we have from expression (1), $p_{i1}\, u_{i1}(ab) > q_{i1}$. Dividing through by p_{i1} yields $u_{i1}(ab) > q_{i1}/p_{i1}$.
2. Yes. For example, if $q_{i1}/p_{i1} = q_{i6}/p_{i6} = .5$, then i1 and i6 will both vote Y if $u_{i1}(ab)$ and $u_{i6}(ab)$ exceed .5.
3. i3 votes N and i5 votes Y.
4. 

5. His intent is to convince type 1 voters that $u_i(ab)$ is near zero so that q_{i1}/p_{i1} will exceed $u_{i1}(ab)$ and they will vote N, thus increasing the chances of ϕ, Poff's first preference.
6. Yes, since ϕ is preferred to b.
7. 

Assuming the probability estimates are as labelled above, the two lotteries are

$$(t_i\ (p_i\ a_1a_2b,\ (1-p_i)\ \phi),\ (1-t_i)\ (q_i\ a_1b,\ (1-q_i)\phi))$$

and

$$(u_i\ (r_i\ a_2b,\ (1-r_i)\ \phi),\ (1-u_i)\ (s_i\ b,\ (1-s_i)\ \phi))$$

7. NOTES

1. James M. Enelow, "Saving Amendments, Killer Amendments, and a New Theory of Congressional Voting," paper delivered at the American Political Science Association Meetings, Washington, D.C., August 31 - September 3, 1979.
2. Congressional Record (CR), H 18727, August 9, 1966.
3. CR, H 18128, August 3, 1966.
4. CR, H 18124, August 3, 1966.
5. New York Times, July 1, 1979.
6. Ibid..
7. New York Times, July 15, 1979.

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

STUDENT FORM 1

Request for Help

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name_______________________________ Unit No._______

Page_______ O Upper O Middle O Lower	OR	Section_______ Paragraph_______	OR	Model Exam Problem No._____ Text Problem No._____

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

O Corrected errors in materials. List corrections here:

O Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

O Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature_______________________

Please use reverse if necessary.

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

STUDENT FORM 2

Unit Questionnaire

Name______________________ Unit No.____________ Date____________

Institution______________________ Course No.______________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much Longer	Somewhat Longer	About the Same	Somewhat Shorter	Much Shorter
____	____	____	____	____

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain______________________

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain______________________

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)