These modules develop two prevalent explanations among political scientists of who gets what in the budgetary process. Specifically, the problem of how an agency's level of appropriations changes over time is addressed. It is noted that budgeting is a political process in the classic sense, in that it elicits and embodies patterns of conflict and compensation centering on who gets what, when, and how and revolves around the allocation of limited resources to various recipients. The documents are designed to provide some applications of first-order linear difference equations. (MP)
THE BUDGETARY PROCESS: INCREMENTALISM

by Thomas W. Likens

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THE BUDGETARY PROCESS: INCREMENTALISM

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Author:

Review Stage/Date: III 2/25/79

Classification: APPL FIRST ORD LIN DIFF EQ/POL SCI

Suggested Support Materials:

References: See Section 7 of text.

Prerequisite Skills:
1. Have a knowledge of high school algebra.

Output Skills:
1. Better understanding of the budgetary process and its outcomes.
2. Gain a better perspective on modeling political decision-making.
3. Use first order linear difference equations to generate time series.
4. Use the solution of a difference equation to explore the time-dependent character of budgetary expenditures.

Other Related Units:
Exponential Models of Legislative Turnover (Unit 296)
The Dynamics of Political Mobilization I (Unit 297)
The Dynamics of Political Mobilization II (Unit 298)
Public Support for Presidents I (Unit 299)
Public Support for Presidents II (Unit 300)
Laws that Fail I (Unit 301)
Laws that Fail II (Unit 302)
The Diffusion of Innovation in Family Planning (Unit 303)
Growth of Partisan Support I (Unit 304)
Growth of Partisan Support II (Unit 305)
Discretionary Review by Supreme Court I (Unit 306)
Discretionary Review by Supreme Court II (Unit 307)
The Budgetary Process: Competition (Unit 333)

The Project would like to thank Paul Nugent and R. Bruce Mericle for their reviews and all others who assisted in the production of this unit.

This material was prepared with the support of National Science Foundation Grant No. SED 76-19615 A02. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF, nor of the National Steering Committee.
The budgetary process involves the allocation of limited resources to various recipients. Government agencies, retired government workers, the poor, the aged, soldiers, farmers, and students are among those who compete for the resources distributed through the federal budget. Budgeting is thus a political process in the classic sense: it elicits and embodies patterns of conflict and competition centering on "who gets what, when, and how."

This module develops one prevalent explanation among political scientists of the problem of who gets what in the budgetary process. More specifically, we will address the problem of how an agency's level of appropriations changes over time. How does the agency decide how much to ask for each year? How does the Congress decide what to give? And, what are the consequences of these decisions for the change in appropriations over time?

2. BUDGETARY INCREMENTALISM

The process of budgeting is one form of policy making. It has been argued that the process of making policy decisions consists of a series of choices that are only marginally different from the status quo. Man's limited capacities for problem solving, the pathologies of information processing and transmission in organizations, and the costliness of careful planning and data analysis, all severely alter what might normally be thought of as "rational" policy behavior. Policy makers, rather than making innovative changes, tend only to make small policy adjustments of a serial and remedial nature. Policy making, therefore, is often described as an incremental process (Lindblom 1959; Braybrooke and Lindblom 1959; Simon 1957; Cyert and March 1963).

It has been argued by many political scientists that budgetary decision making is also an incremental process. Studies by Fenno (1962, 1964) and Wildavsky (1964, 1974) reveal that, indeed, both the members of appropriations committees and subcommittees in Congress, and agency administrators, think in incremental terms. Fenno has observed that House Appropriations Committee members, proud of their ability to guard the federal purse, do in fact make marginal adjustments in many budgetary appropriations each year. Rather than reconsidering basic policy choices each year, these Congressmen tend to adjust incrementally the budget by giving agencies a little less than they asked for, but more than they received last year.

Similarly, Wildavsky has observed that agency officials, when faced with the problem each year of deciding what to request of the Congress, usually think of marginal gains over what their agency is already receiving. That is, the agency's current level of appropriations is usually thought of as its "base," and the agency officials usually seek some "fair share" increase over this base each year.

To summarize, then, the theory of budgetary incrementalism asserts that in order to minimize the uncertainties and costs of making budgetary decisions, both the Congress and the federal agencies make marginal adjustments to the status quo. The incremental strategy of the agencies is to request a bit more each year than they received last year. And the incremental strategy of the Congress is to appropriate a little less than the agency is requesting, but still to give enough of a "fair share" of the budget to keep the agency happy (Wanat 1974; Davis et al. 1966; Crecine 1969).
3. FORMALIZATION

How can we express these ideas mathematically? What will be the result of these incremental decisional processes? The following discussion combines several formalizations of an incremental decision-making strategy, in a very simple way. The reader is referred particularly to Davis et al. (1966) and Wanat (1974) for more extended analyses.

Begin by defining:

\[ R_t = \text{any particular agency's request for dollars at time } t \]

\[ A_t = \text{the appropriations granted to that agency at time } t \text{ by the Congress.} \]

Note here that budgeting is taken as a series of discrete events. This is certainly a reasonable approach, since requests and appropriations occur once each year at the federal level. Although the process of budgeting is almost continuous, its outcome—a particular set of requests by the agency and a set of appropriations by the Congress—occurs only once each year.

For example, the Department of Health, Education and Welfare has its own budget office which is continuously concerned with how money is obtained and spent. It develops a specific request for money appropriated by the Congress, over $100 billion for fiscal year 1979. Similarly, the Congress examines the specific request by the Department of Health, Education and Welfare, and grants a final appropriation of funds to that agency for fiscal year 1979. The result of this request-review-appropriation process for all agencies is reported yearly in The Budget of the United States Government, Fiscal Year, a document which gives detailed information about how much the government spends each fiscal year and for what purposes.

The agency's incremental strategy, recall, is to ask the Congress each year for a little more than it received last year. We can formalize this process by writing:

\[ R_t = p_1 A_{t-1} \]

Equation (1) simply asserts that an agency's request in fiscal year \( t \) will be some fixed proportion over its last year's appropriation. Empirically, we would expect to find in most instances that for different agencies, \( p_1 \) would range between 1.0 and 1.20. If \( p_1 = 1.10 \), for example, the agency is usually asking for a ten-percent increase in its appropriations each year.

Why should \( p_1 \) range between 1.0 and 1.20? Because the magnitude of \( p_1 \) reflects the fact that the process we are describing is an incremental one. If, for example, \( p_1 \) were found to be 2.0 for some agency, it would be hard to conclude that an incremental process was at work. Marginal adjustment in the status quo will not cause an agency to seek to double its size each year (as \( p_1 = 2.0 \) would imply).

The decisional strategy of the Congress, again, is to cut back on an agency's request, but not so severely that major conflicts are produced (Wildavsky 1964; Fenno 1964). Usually the Congress accepts the agency's current appropriations as a "base" which is safe from major cuts, while looking at the agency's increases with a more severe eye. A simple way of formalizing this strategy is to write:

\[ A_t = p_2 R_t \]

In words, the Congress makes appropriations each year which are some fixed proportion of the agency's request. Since the agency rarely ever gets all it wants, we would expect to find empirically that \( p_2 \) would range between, say, 0.80 and 1.00.

There are, of course, a variety of short-run political forces which will also enter into the budgetary process. Wars, for example, may temporarily boost the appropriations which are requested and granted for the Department of Defense. Economic factors may also cause the size of an
agency's annual budgetary increment to shrink or grow. For our purposes, we will treat these short-term influences as randomly distributed errors. In other words, the equations expressing the decisional strategies of the agencies and the Congress ought really to be written as:

(3) \[ R_t = p_1 A_{t-1} + \text{error} \]

(4) \[ A_t = p_2 R_t + \text{error} \]

But assuming that these errors are not systematic, a good first approximation of the budgetary system may be had from the purely deterministic Equations (1) and (2).

4. ANALYSIS

What will be the consequence of an incremental decision strategy in budgeting? An answer may be obtained by using Equations (1) and (2) to obtain a dynamic equation of the form:

(5) \[ \Delta A_t = f(A_t) \]

That is, we desire an expression which will predict the change in appropriations (\( \Delta A_t \)), from a knowledge of the current level of funding which an agency receives \( f(A_t) \).

Rewrite Equation (2) as:

(6) \[ R_t = \frac{A_t}{p_2} \]

Substitute Equation (6) into Equation (1) to obtain:

(7) \[ \frac{A_t}{p_2} = p_1 A_{t-1} \]

Multiplying through Equation (6) by \( p_2 \) thus produces:

(8) \[ A_t = p_1 p_2 A_{t-1} \]

Although proof is beyond the scope of this module, it can be shown that we may advance the time subscript of Equation (8) without violating the rules of algebra. That is, the equivalency still holds if we write Equation (8) as:

(9) \[ A_{t+1} = p_1 p_2 A_t \]

(For the more advanced reader, we have simply applied the linear operator \( E \), an advancement operator, to Equation (8), thus obtaining Equation (9): See Cortes, Przeworski and Sprague (1974), or Goldberg (1958) for an extended discussion. A brief discussion of linear operators may be found in UMAP module "Discretionary Review by the Supreme Court: Part Two, Analysis of the Model" by Likens.)

Equation (9) thus demonstrates that next year's appropriations may be predicted from this year's appropriations if we know, on average, how much the agency requests \( p_1 \) and how much the Congress tends to cut this request \( p_2 \). In our example, if a particular agency typically requests a 20-percent increase each year, then \( p_1 = 1.20 \). If the Congress tends to cut this agency's requests each year by 10 percent, then \( p_2 = 0.90 \). Substituting this information into Equation (9) produces

(10) \[ A_{t+1} = (1.20)(0.90)A_t \]

(11) \[ A_{t+1} = (1.08)A_t \]

As a consequence of the incremental strategies of the agency and Congress, therefore, the agency will expand by eight percent each year. The rate of growth, in general, may be ascertained if we write Equation (9) in the form of Equation (5).

Subtract \( A_t \) from both sides of Equation (9):

(12) \[ A_{t+1} = A_t = p_1 p_2 A_t - A_t \]

But recall that, by definition,

(13) \[ \Delta A_t = A_{t+1} - A_t \]

so we may rewrite (13) as:

Equation (8) without violating the rules of algebra.
\[ \Delta A_t = (p_1 p_2^{-1}) A_t \]

The expression \( (p_1 p_2^{-1}) \) thus provides the agency's rate of growth each year. So long as this term is positive, the agency will exhibit geometrically increasing appropriations over time.

For example, if an agency requests an average increase of 12 percent \( (p_1 = 1.12) \) and the Congress grants on average about 93 percent of this request \( (p_2 = .93) \), then the agency's growth rate is

\[ (1.12)(.93) - 1 = 0.0416 \]

If an agency finds itself in this situation, its average growth over time will be 4.16 percent per year. For example, if the agency begins with $1,000,000, its appropriations for the next five years will be

<table>
<thead>
<tr>
<th>Year</th>
<th>Appropriations (in Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,000,000</td>
</tr>
<tr>
<td>1</td>
<td>((1.0416)(1,000,000) = 1,041,600)</td>
</tr>
<tr>
<td>2</td>
<td>((1.0416)(1,041,600) = 1,084,931)</td>
</tr>
<tr>
<td>3</td>
<td>((1.0416)(1,084,931) = 1,130,063)</td>
</tr>
<tr>
<td>4</td>
<td>((1.0416)(1,130,063) = 1,177,074)</td>
</tr>
<tr>
<td>5</td>
<td>((1.0416)(1,177,074) = 1,226,040)</td>
</tr>
</tbody>
</table>

In fact, so long as the agency's growth rate is positive, the time path for appropriations will grow exponentially over time and will look, in general, similar to the trajectory illustrated below in Figure 1. This predicted pattern of change in agency appropriations does in fact occur very frequently in the budgetary process. Figure 2 below illustrates the appropriations for the Department of Health, Education and Welfare for fiscal years 1952 to 1975. The pattern is strikingly similar to the curve predicted by our incremental theory.

![Graph showing growth in spending predicted by incrementalism.](image)

**Figure 1.** Growth in spending predicted by incrementalism.

![Graph showing appropriations to the Department of Health, Education and Welfare.](image)

**Figure 2.** Dollar appropriations to the Department of Health, Education and Welfare, fiscal years 1952-1975.

The theory of budgetary incrementalism, then, predicts that agencies will experience a smooth-pattern of growth in appropriations over time. Notice that if all agencies grow in this fashion, the federal budget will also exhibit a similar pattern of change.

We conclude with two questions. First, will this growth ever stop? And second, how can we find out how long it will take the budget to grow by some specified amount (for example, how long will it take the budget of an agency to double in size)?

Both questions may be answered quite simply, once one knows that all linear difference equations with constant coefficients of the form

\[ Y(t+1) = cY(t) \]

have a solution given by

\[ Y(t) = c^t Y(0). \]

Thus, if one knows the value of the constant \( c \) and the initial value of \( Y(t) \), \( Y(0) \), then one can immediately ascertain the value of \( Y(t) \) at any point in time. For example, if \( c = 2 \) and \( Y(0) = 1 \), \( Y(t) \) at time \( t = 3 \) is given by:

\[ Y(3) = 2^3(1) = 8. \]

The reader should verify this by generating the first three values of \( Y(t) \). (See Goldberg 1953, pages 121-153 for a more detailed discussion.)

In our budgetary model, then, appropriations \( (A_t) \) may be deduced for any time \( t \) if we know an agency's rate of growth \( (p_1p_2-1) \) and its initial funding level \( (A_0) \). Using Equation (17) yields:

\[ A_t = (p_1p_2-1)^tA_0. \]

Will the agency's appropriations ever stop growing? Clearly, from Equation (19) the answer is no, so long as \( (p_1p_2-1) \) is greater than unity. Empirically, this quantity has been estimated for many different agencies for several years. In most cases it has been found that

\[ 1 < (p_1p_2-1) < 1.20. \]

In other words, if incremental decision making prevails in the budgetary process, we can expect an ever-increasing federal budget over time. As Figure 3 reveals, this predicted pattern of geometric growth has certainly occurred over the last several years (and, in fact, over the last several decades).

![Figure 3. Total federal spending, 1967-1975.](source)

**Source:** The Office of Budget and Management, The Budget of the United States Government, Fiscal Year 1975.

How long will it take an agency to increase in size by some specified factor? For example, how long will it take an agency to double its appropriations? We can reformulate this latter question by asking "how long will it take for \( A_t \) to equal \( 2A_0 \)?" Substituting into the solution for \( A_t \) (Equation (19)) yields:
Dividing by $A_0$ gives
\[ 2 = (P_1P_2-1)^t, \]
which may be solved for $t$ by taking logarithms:
\[ t = \frac{\log (2)}{\log (P_1P_2-1)}, \]
And in general, if the increase is given by some factor, $X$, the required time is given by
\[ t = \frac{\log (X)}{\log (P_1P_2-1)}. \]

We may conclude with a specific example. Suppose that a particular agency grows at a rate of five percent per year. Such a growth rate reflects the incremental strategies of the agency in asking for appropriations and of the Congress in granting them. The process results in a marginal increase each year of the agency's appropriations by a modest increment. How long will it be before the agency's appropriations have doubled in size?

By Equation (24), where $X = 2$,\[ t = \frac{\log 2}{\log (1.05)} \]
\[ t = \frac{0.301}{0.021} = 14.2 \text{ years}. \]

Incrementalism means an expanding budget, a growth in governmental expenditures over time. It is an uncertainty-reducing strategy but not an efficiency-maximizing one for budgetary decisions.

5. **QUESTIONS**

1. Suppose an agency is very aggressive and typically asks to increase its budget by one-fourth each year. What is the value of $p_1$ in this instance? Suppose that Congress responds to this very aggressive agency by cutting 20 percent out of the agency's request. What is $p_2$?

2. Which agency grows more rapidly—the one in Question 1 or one which asks for a 10 percent increase and receives 97 percent of its request?

3. Suppose an agency requests an increase of 15 percent each year and that the Congress cuts by 5 percent each year.
   a. Write the dynamic equations which express the agency's incremental strategy, the incremental strategy of the Congress, and the outcome.
   b. Assuming that the agency starts with $1 million, write the next three years' appropriations it will receive.
   c. What will be the agency's appropriations in 100 years?

4. How long will it take the agency in Question 3 to triple in size?

5. What would happen if an agency requested an annual increase of 10 percent each year and the Congress cut this request by 30 percent each year? Could this be called an incremental process? Why?

6. **ANSWERS TO QUESTIONS**

1. $p_1 = 1.25$, $p_2 = .80$

2. For the first agency $(P_1P_2-1) = 0.20$. For the second, 0.097. The first grows slightly more than twice as quickly as the latter.

3. a. For the agency: $R_t = (1.15)A_{t-1}$
   For the Congress: $A_t = 0.95R_t$
   Result: $A_{t+1} = 1.0925A_t$
   b. Year 1 = $1,000,000(1.0925) = 1,092,000$
      Year 2 = $1,092,500(1.0925) = 1,193,550$
      Year 3 = $1,193,550(1.0925) = 1,303,953$
   c. $A_{100} = 1.0925^{100}(1,000,000) = 6952.56(1,000,000) = $6,952,560,000

4. 12.4 years, or 13 fiscal years.
   \[ 3A_0 = 1.0925^tA_0 \]
   \[ t = \frac{\log (3)}{\log (1.0925)} = 12.4. \]
5. The equation for the agency's growth would be

\[ \text{At+1} = (0.77) \text{At} \]

The agency, therefore, would receive less and less each year, asymptotically decaying to zero.

Note that this does not fit the theory of budgetary incrementalism. In that theory, the "base" is considered safe against cutbacks. Clearly, the problem here is that Congress is not making marginal adjustments but major cuts in spending.

7. BIBLIOGRAPHY


Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name

Page

OR

O Upper

O Middle

O Lower

Section

OR

Paragraph

Description of Difficulty: (Please be specific)

Unit No.

Model Exam

Problem No.

Text

Problem No.

Instructor: Please indicate your resolution of the difficulty in this box.

Corrected errors in materials. List corrections here:

Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor’s Signature

Please use reverse if necessary.
STUDENT FORM
Unit Questionnaire

Name
Institution
Unit No.
Unit Questionnaire
Date
Course No.

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?
   - Not enough detail to understand the unit
   - Unit would have been clearer with more detail
   - Appropriate amount of detail
   - Unit was occasionally too detailed, but this was not distracting
   - Too much detail; I was often distracted

2. How helpful were the problem answers?
   - Sample solutions were too brief, I could not do the intermediate steps
   - Sufficient information was given to solve the problems
   - Sample solutions were too detailed, I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?
   - A Lot
   - Somewhat
   - A Little
   - Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?
   - Much Longer
   - Somewhat Longer
   - About the Same
   - Somewhat Shorter
   - Much Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)
   - Prerequisites
   - Statement of skills and concepts (objectives)
   - Paragraph headings
   - Examples
   - Special Assistance Supplement (if present)
   - Other, please explain

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)
   - Prerequisites
   - Statement of skills and concepts (objectives)
   - Examples
   - Problems
   - Paragraph headings
   - Table of Contents
   - Special Assistance Supplement (if present)
   - Other, please explain

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)
THE BUDGETARY PROCESS: COMPETITION

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Title: THE BUDGETARY PROCESS: COMPETITION

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Review Stage/Date: III 2/25/79

Classification: APPL FIRST ORD LIN DIFF EQ POL SCI

Suggested Support Material:

References: See Section 5 of text.

Prerequisite Skills:
1. Have a knowledge of high school algebra.

Output Skills:
1. Better understand the budgetary process and its outcomes.
2. Gain a better understanding of modeling political processes.
3. Understand the idea of phase-portraits and their use.
4. Understand the difference between a process and its outputs.
5. Understand the difference between incremental and nonincremental decision-making.

Other Related Units:
- Exponential Models of Legislative Turnover (Unit 296)
- The Dynamics of Political Mobilization I (Unit 297)
- The Dynamics of Political Mobilization II (Unit 298)
- Public Support for Presidents I (Unit 299)
- Public Support for Presidents II (Unit 300)
- Laws that Fail I (Unit 301)
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- Growth of Partisan Support II (Unit 305)
- Discretionary Review by Supreme Court I (Unit 306)
- Discretionary Review by Supreme Court II (Unit 307)
- The Budgetary Process: Incrementalism (Unit 332)

The Project would like to thank Paul Nugent and R. Bruce Mericle for their reviews and all others who assisted in the production of this unit.

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THE BUDGETARY PROCESS: COMPETITION

1. INTRODUCTION

In the preceding module, UMAP Unit #332, we examined the consequences of an incremental decision strategy in the budgetary process. In that theoretical approach to budgeting, we assumed:

1. marginal change, and
2. independence in outcomes.

Incrementalism assumes that agencies will receive a "fair share" of the forthcoming budget over their existing "base" level of appropriations. It also assumes that the "fair share" received by any particular agency will be independent of, and have no effect on, the share received by any other agency.

In this module, we will consider an alternative explanation of budgeting outcomes, one which focuses on

1. competition for scarce resources, and
2. interdependence in outcomes.

The approach to budgeting taken here stresses the conflictive nature of politics and the necessary interdependence of budgetary decisions.

There is mounting evidence that interactive, conflictive processes occur in budgeting. In looking at the political strategies used by different agencies in attempting to attain more appropriations, one political scientist (Sharkansky 1965, 1968) has found that agencies vary considerably in the aggressiveness (or "acquisitiveness") with which they seek funds. He also observed that highly aggressive agencies usually grow faster than agencies which are less acquisitive.

In studying different programs within the Atomic Energy Commission, it has been found that the ability of program directors to establish high priorities for their programs is directly related to how well the programs do in the budgetary process each year. Politically skillful administrators usually head agencies and programs which grow faster, and defend more successfully against cuts than less politically skilled directors of agencies and programs (Natchez and Bupp 1970).

If one looks at the share of the budget which various agencies and programs receive, clear patterns of trade-offs are evident. Figures 1 and 2 below provide obvious examples. Notice in Figure 1 that defense spending has not been able to maintain its slice of the budget, while payments to individuals have replaced the military's dominance in the budgetary arena. In fact, defense spending and personal entitlements almost sum to a constant each year over the entire period which has been plotted.

The trade-off between defense and domestic spending is an indirect form of political competition for budgetary resources. Figure 2 provides an example of a more direct form of budgetary interdependence. Here we are examining the functional spending category which groups all dollars allocated for energy, natural resources, and the environment. Figure 2 illustrates the share of these dollars which is acquired, over time, by pollution and energy programs, and by water and conservation programs. It is clear that pollution abatement and energy research have stored major victories, at the apparent expense of water projects and conservation programs. The pattern which emerges is hardly surprising, given the very high priority recently achieved by pollution and energy programs, and the controversy surrounding many of the recent water projects conducted by the Army Corps of Engineers.

The question, then, is how to model this competitive, interactive process. And, if a model can be developed, what are its deductive consequences?
A model of essentially unrestrained competition between two interacting agencies or programs may be written as:

\[ \Delta X_t = \left( p_x(L_X - X_t) - c_y Y_t \right) X_t \]

\[ \Delta Y_t = \left( p_y(L_Y - Y_t) - c_x X_t \right) Y_t \]

where the variables \( X_t \) and \( Y_t \) are proportions of some relevant budget total received by programs or agencies "X" and "Y." The parameters \( L_X \) and \( L_Y \) denote the maximal share of the total budget which the agency would receive in the absence of competition, assuming zero exogenous inputs. These upper limits \( L_X \) and \( L_Y \) are assumed to be constant for significant historical periods and are determined by the broad policy objectives extant during the era and by the general ability of the competitors to establish policy priorities for their agencies/programs, their administrator's political skill, and similar factors.

The parameters \( p_x \) and \( p_y \) denote the general acquisitiveness of \( X \) and \( Y \), respectively, as budgetary players. Descriptive adequacy imposes the constraint that \( 0 \leq p_x, p_y \leq 1 \). In general, the larger \( p_x \) or \( p_y \), the greater is that agency's aggressiveness in securing its optimal funding level, \( L_X \) or \( L_Y \).

Notice that as the model is written, the greater the agency's acquisitiveness, the more rapidly it will tend to approach some optimal share of the budget. This assumption has been suggested in at least two empirical studies (Sharkansky 1965, 1968). In this context, an agency or program is likely to have a greater success as a budgetary competitor if it is headed by persons experienced in bureaucratic infighting, who are highly motivated and...
articulate political entrepreneurs. Alternatively, a newly established agency or program with relatively unskilled or inexperienced leadership would tend to have a lower effectiveness as a budgetary player for some significant historical period. Finally, the parameter \( c_y \) denotes the rate at which \( Y \) competes against \( X \), and \( c_x \) denotes the rate of competition by \( X \) against \( Y \). These parameters provide a measure of how significantly each agency encroaches on the other's funds.

The logic of the model is straightforward. It asserts that in the absence of competition \( (c_y = c_x = 0) \), \( X_t \) and \( Y_t \) approach their upper limits \( (L_x \) and \( L_y) \) according to the logistic law:

\[
\Delta X_t = p_x(L_x - X_t)X_t
\]

\[
\Delta Y_t = p_y(L_y - Y_t)Y_t.
\]

Assuming, of course, that \( 0 < p_x < 1 \), Equations (3) and (4) produce the familiar S-curve typical of many diffusion processes (Rapoport 1963; Bartholomew 1967; Coleman 1964).

For competitive processes, \( c_x \) and \( c_y \) are assumed to be between zero and unity, and hence the larger either agency becomes, the greater its competitive impact on the other. There is nothing in the structure of the process which limits how much the agencies can influence each other. In this sense, then, the competition may be characterized as "pure" or "unrestrained."

Equilibria for the system are obtained as always, by setting \( \Delta X_t = \Delta Y_t = 0: \)

\[
0 = [p_x(L_x - X^*) - c_yX^*]X^*
\]

\[
0 = [p_y(L_y - Y^*) - c_xX^*]Y^*.
\]

There are in fact four simultaneous solutions for Equations (5) and (6). Clearly, (5) is always true if \( X^* = 0 \) and (6) is true if \( Y^* = 0 \). Hence, one equilibrium point is

\[
(X^*, Y^*) = (0, 0).
\]

If we use \( X^* = 0 \) from (5) and substitute into (6), we obtain

\[
0 = [p_y(L_y - Y^*) - c_x0]Y^*.
\]

The right-hand side of (8) will then equal zero if \( Y^* = L_y \). A second equilibrium point, thus, is

\[
(X^*, Y^*) = (0, L_y).
\]

Similarly, if we use \( Y^* = 0 \) from Equation (6) and substitute into (5), we obtain

\[
0 = [p_x(L_x - X^*) - c_y0]X^*.
\]

The right-hand side of (10) goes to zero if \( X^* = L_x \). Thus a third equilibrium point is

\[
(X^*, Y^*) = (L_x, 0).
\]

A fourth equilibrium point occurs when both \( X^* \) and \( Y^* \) are nonzero. We may find this point as follows. First, divide (5) by \( X^* \) and (6) by \( Y^* \) to obtain

\[
0 = p_x(L_x - X^*) - c_yX^*
\]

\[
0 = p_y(L_y - Y^*) - c_xX^*.
\]

Equations (12) and (13) may thus be rewritten as

\[
p_xX^* + c_yY^* = p_xL_x
\]

\[
c_xX^* + p_yY^* = p_yL_y.
\]

The simultaneous solution of (14) and (15) for \( X^* \) and \( Y^* \) thus produces a final equilibrium point:

\[
(X^*, Y^*) = \left\{ \frac{p_y(p_xL_x - c_yL_y)}{p_y-p_x} \right\} \left\{ \frac{p_xL_y - c_xL_x}{p_y-p_x} \right\}.
\]
Question 4: If the competitive process had the structure
\[ \Delta X_t = aX_t - bY_t \]
\[ \Delta Y_t = cY_t - dX_t \]
what would its equilibrium point(s) be?

To summarize, our four equilibria are:

(17) \[ (X^*, Y^*) = (0,0) \]
(18) \[ (X^*, Y^*) = (0, L_y) \]
(19) \[ (X^*, Y^*) = (L_x, 0) \]
(20) \[ (X^*, Y^*) = \left( \frac{p_x(p_y - c_y)(L_x - L_y)(p_x - p_y)(L_y - L_x)}{p_y p_x c_y}, \frac{p_y(p_x - c_x)(p_y - c_y)(L_x - L_y)}{p_y p_x c_y} \right) \]

Net change ceases, then, under four conditions:
1. When both agencies are eliminated; (2) when agency X is eliminated and agency Y obtains its upper limit, L_y;
3. When agency Y is eliminated and X achieves its optimal level, L_x; and (4) when both achieve some competitive level between zero and their upper limits. Given that three of the four possibilities end in elimination of one or both agencies, it is clear that unrestrained budgetary competition is quite Darwinian. In addition, this unlimited competition—with its rather extreme consequences—is unrealistic in the context of contemporary federal budgeting. Agencies and programs in the real world are seldom totally eliminated. At worst they tend to move to some minimal level of funding which they then maintain year after year.

This model might well describe budgetary competition in political settings which are not highly bureaucratized. It may describe, for example, programmatic conflict in newly initiated agencies where priorities are not well established and bureaucratic inertia has not yet mounted.

The qualitative behavior of the model is most easily studied in the phase-space. That is, rather than the usual strategy of plotting \( X_t \) across time and \( Y_t \) across time, we will study \( Y_t \) as a function of \( X_t \) (or vice versa).

In fact, what we are doing is projecting the three-dimensional graph of \( X_t, Y_t, \) and \( t \) onto the \( X_t, Y_t \)-plane. Such a projection is often referred to as a "phase-portrait."

The principal question we hope to answer, then, is: Under what substantive conditions is each of the equilibria reached? What is the likelihood that one or both agencies may be eliminated, or that both agencies will survive over time?

If we set Equations (1) and (2) to zero, we obtain two zero-isoclines for \( \Delta X_t \) and \( \Delta Y_t \). For \( \Delta X_t = 0 \), these zero-isoclines are:

(21) \[ X_t = 0 \]
(22) \[ Y_t = \frac{-p_y X_t}{c_y} + \frac{p_x L_x}{c_y} \]

At any point along these two lines, we are guaranteed by definition that \( \Delta X_t = 0 \). And for \( \Delta Y_t = 0 \), the zero-isoclines are:

(23) \[ Y_t = 0 \]
(24) \[ Y_t = \frac{-c_x}{p_y} X_t + L_y \]

These lines are very useful in determining the qualitative behavior of the model. On the lines, by definition, \( \Delta X_t \) and \( \Delta Y_t \) are zero. But what happens to \( \Delta X_t \) and \( \Delta Y_t \) if the point \((X_t, Y_t)\) moves off the zero-isoclines?

It is easy to see that when \( X_t \) is to the right of the zero-isocline, \( \Delta X_t \) is less than zero. For example, if we assume that \( X_t = a \), then \((X_t, Y_t)\) is on the \( \Delta X = 0 \) isocline if

(25) \[ Y_t = \frac{-p_x a}{c_y} + \frac{p_x L_x}{c_y} = \frac{p_x L_x - p_x a}{c_y} \]

Verify that \( \Delta X_t = 0 \) by substituting (25) into (1)

(26) \[ \Delta X_t = p_x (L_x a - c_y p_x a) \]
(27) \[ = p_x L_x a - p_x a^2 - p_x L_x a + p_x a^2 \]
(28) \[ \Delta X_t = 0. \]
But if we increase $X_t$ by amount $c$ and make the same substitution into (23) we obtain

$$ \Delta X_t = \left[ pX_t - \left( \frac{p}{c} + \frac{1}{c} \right) X_t \right] (t+\theta) $$

$$ \Delta Y_t = \left[ pY_t - \left( \frac{p}{c} + \frac{1}{c} \right) Y_t \right] (t+\theta) $$

(31) 

If we decrease $X_t$ by $\theta$, assuming we were originally at $X_t = 0$,

(32) 

Thus, at any point to the right of the $\Delta X_t = 0$ isocline in the phase space, $\Delta X_t$ is negative to the left, $\Delta X_t$ is positive. Similarly, above the $\Delta Y_t = 0$ zero-isocline, $\Delta Y_t$ is positive. The student should convince him/herself of this assertion. In effect, then, the zero-isoclines here "pull" the point $(X_t, Y_t)$ toward themselves, with $Y_t$ able to move only up and down, $X_t$ only left and right. These simultaneous "pulls" combined determine where $(X_t, Y_t)$ moves at time $t+\theta$.

Question 2: Show that above the $\Delta Y_t$ isocline, $\Delta Y_t < 0$ and below it $Y_t > 0$.

If we know where a point $(X_t, Y_t)$ occurs in the phase space with respect to the $\Delta X_t = 0$ and $\Delta Y_t = 0$ isoclines, we can readily determine what its general trajectory will be. For example, if at time $t$ the point $(X_t, Y_t)$ is above the $\Delta Y_t = 0$ isocline and to the left of the $\Delta X_t = 0$ isocline, $(X_t, Y_t)$ will move to the right and down at time $t+\theta$. This is illustrated in Figure 3 below.

The zero-isoclines given by Equations (22) and (24) yield six distinct geometries in the phase-space. There are three possibilities which occur when the slope of the $\Delta X_t = 0$ isocline is greater than the slope of the $\Delta Y_t = 0$ isocline. And three possibilities exist where the opposite relationship between the slopes is true. All six are illustrated below, along with the qualitative behavior of $(X_t, Y_t)$ which occurs with each geometry.

Inspection of Figure 4 reveals several features of the system's dynamics. Notice first that the $(0,0)$ equilibrium point is never stable: under no circumstance will competition between agencies $X$ and $Y$ end in the elimination of both.

The $(0,L_y)$ equilibrium is always obtained in III and $V$ and sometimes in I (depending on initial conditions). Notice that in both III and $V$ the $\Delta Y_t = 0$ isocline intersects the $Y$-axis above the $Y$-intersection of the $\Delta X_t = 0$ isocline. That is,

(33) 

$$ L_y = \frac{pX_t}{cY} $$

Figure 3. Zero-isoclines for $\Delta X_t$ and $\Delta Y_t$, with resulting motion of $(X_t, Y_t)$ illustrated.
Figure 4. Possible geometries and resulting qualitative behaviors of the zero-isoclines for the model (equilibria are circled).

Slope for $\Delta X_t = 0$ Isocline > Slope for $\Delta Y = 0$ Isocline:
\[ \left( \frac{p_x}{c_x} \right) > \left( -\frac{c_y}{p_y} \right) \]

or
\[ (36) \quad c_x L_x < p_y L_y \]

In fact, III and V are the only geometries for which both inequalities hold. We may deduce, then, that the $(0, L_y)$ equilibrium point is stable throughout the unit phase-space if two conditions hold:
\[ (37) \quad p_x L_x < c_y L_y \]
\[ (38) \quad c_x L_x < p_y L_y \]

By a similar comparison of the geometries of zero-isoclines, the $(L_x, 0)$ equilibrium may be seen to be stable if
\[ (39) \quad p_x L_x > c_y L_y \]
\[ (40) \quad c_x L_x > p_y L_y \]

The remaining equilibrium is stable if
\[ (41) \quad p_x L_x > c_y L_y \]
\[ (42) \quad p_y L_y < c_x L_x \]
\[ (43) \quad p_y p_x < c_y c_x \]

but is unstable if the inequality in (43) is reversed. We now have a complete analysis of the dynamics of the system within the $0 < X_t, Y_t < 1$ state space.

These conditions are easily given substantive interpretation. Recall that $p_x$ is interpreted as the aggressiveness of agency $X$, and $L_x$ is essentially a measure of the maximal position of $X$ with respect to all other relevant budgetary players. The term $(p_x L_x)$ may thus be conceptualized as the net political acquisitiveness of $X$ as a budgetary player. In simple terms, $(p_x L_x)$ measures the agency's budgetary clout. Since the parameter $c_x$ is an expression of $X$'s impact on $Y$, the term $c_x L_x$ may be interpreted as the net competitive impact of $X$ on the
budgetary success of Y. Similar interpretations hold for the terms \( p_y L_y \) and \( c_y L_y \).

Given these interpretations, Inequalities (37) through (43) suggest the following substantive conclusions:

1. Agency X eliminates Y if: the net acquisitiveness of X exceeds the competitive impact of agency Y \( (p_x L_x > c_y L_y) \), and if the net acquisitiveness of Y is insufficient to defend adequately against competition by X \( (c_x L_x > p_y L_y) \).

Typical phase-portraits are exhibited in Figures 5 and 6 below.

2. Agency Y eliminates X if: the above inequalities are reversed. That is, the acquisitiveness of Y overcomes the competitive impact of X, while X lacks sufficient clout to defend adequately against encroachment by Y. Phase-portraits resulting from this set of political conditions are exhibited in Figure 7 and 8.

3. Agencies X and Y survive: if both are strong enough to defend adequately against encroachment by the other and if the effects of competition are not so great as to destabilize the agencies' interaction \( (p_x p_y < c_x c_y) \). Figure 10 provides a typical phase-portrait when the agencies manage mutual coexistence. When competition becomes sufficiently intense, however, it tends to destabilize even this situation and the result is the elimination of one agency or the other. As Figure 9 illustrates, initial conditions determine the ultimate outcome, with the relatively stronger agency at time \( t = 0 \) finally prevailing.

It is apparent from these phase-portraits of the model that the variety of histories may be generated for the competing agencies. While all of the time series which are possible cannot be represented here, it is instructive to present several of the more common interaction patterns.

Predicted by the model. Figure 11 exhibits some of the typical deterministic histories which are generated by the model.

![Figure 5](image5.png)

![Figure 6](image6.png)
Question 3: In Figure 7, one trajectory in the phase-space is marked "Q." Draw the approximate time-paths for $X_t$ and $Y_t$ across time for this phase-portrait.

Figure 7.

$P_x = 0.3$
$P_y = 0.4$
$C_y = 0.9$
$C_x = 0.8$
$L_x = 0.2$
$L_y = 0.5$

Figure 8.

$P_x = 0.3$
$P_y = 0.4$
$C_y = 0.9$
$C_x = 0.8$
$L_x = 0.2$
$L_y = 0.5$

Figures 7 and 8. Y eliminates $X$ over time. For both $P_x < C_y$ and $C_x < P_y$, but in Figure 7 $P_y > C_x$, while the reverse is true in Figure 8.

Figures 9 and 10. Destabilizing competition and mutual coexistence. For both $P_x > C_x$ and $P_y > C_y$, but the effects of competition are much more severe in Figure 9, where $P_y > P_x$. For Figure 10 $P_y > C_x$. 

$P_x = 0.5$
$P_y = 0.5$
$C_y = 0.3$
$C_x = 0.3$
$L_x = 0.3$
$L_y = 0.3$
The incremental theory of budgeting derives from a well-established literature on human behavior in complex organizations. It should be stressed that in this analysis no quarrel has been taken with the view that administrative decision makers necessarily work under bureaucratic constraints.

The criticisms of budgetary incrementalism suggested here come not from these ideas, but from the way in which they have been applied to the study of budgetary politics. The appropriations process has tended to lose one of its most interesting qualities, the fact that it is political. Budgeting becomes little else than a compartmentalized series of bureaucratic routines. Competition and conflict are overlooked. Political interaction and fiscal interdependencies are ignored. And the shape of the budget becomes dependent on little else except the passage of time. In fact the incremental model bears little resemblance to the tactical maneuvers and political stratagems qualitatively described in the "classic" budgetary studies of Fenno or Wildavsky.

The model presented in this module is an attempt to express the dynamics of the appropriations process in terms which are more consistent with a view of politics as inherently conflictive and necessarily interdependent. We have focused on the interactions of two competing programs, agencies or departments. It is worth emphasizing that while the analysis does not depend critically on the presence of only two competitors, increasing the number of players is not without consequence. From a technical perspective, increasing the cardinality of states in a system, particularly if the system is nonlinear, can make it quite difficult to obtain a global stability analysis of the system. Usually a local stability analysis must suffice, and even that can easily become quite intractable. Substantively, increasing the number of direct competitors...
will produce an increasingly rich political fabric, where
diverse dynamics may be exhibited.

In the sense of La Porte (1975) or Brunner and
Brewer (1971), increasing the number of competitors (and
hence the level of interdependence in the process)
naturally increases the probability of generating unantici-
pated consequences by only small modifications of the
parameters of the process or small perturbations of its
states.

It should be stressed, finally, that competitive
processes can result in incremental as well as nonincremental
outcomes. In the models presented here, as the competitors
approach an equilibrium each becomes able to maintain a
constant proportion of the total budget. The time-path
which results is descriptively incremental, since each is
able to maintain its "fair share" over its existing "base"
each year. The process which is generating these histories,
however, is not an incremental decisional strategy. Rather,
it is a conflictive, interactive process in which the
competitors have moved over time to fiscal positions which
they are able to defend each year. Even if one observes a
descriptively incremental time series, it is not possible
to deduce that an incremental process generated the observed
history of appropriations.

On the other hand, there are many examples of budget-
ing outcomes which do not appear to exhibit an incremental
pattern of change. This investigation, therefore, has
assumed that the allocation of resource is a conflictive
process whose central feature is interdependence. The
resulting model is far from being a complete picture of
budgeting, but it does at least provide a dynamic structure
by which some clearly nonincremental outcomes may be
partially explained.

4. ANSWERS TO QUESTIONS

1. The system is linear, therefore it can have only one equilibrium
point \((x^*, y^*)\)

\[
0 = a x^* - b y^*
\]

\[
0 = c y^* - d x^*
\]

Simultaneous solution of (1) and (2) yields \((x^*, y^*) = (0, 0)\).

Note that in this competitive process, net change ceases only
when both players are eliminated altogether.

2. The zero-isocline for

\[
\Delta y_t = [p_x (L - y_t) - c_x x_t] y_t
\]

is, again,

\[
y_t = \frac{a x_t}{p_y} + L
\]

Thus when

\[
y_t = \alpha, \quad x_t = \frac{p_x L - p_y \alpha}{c_x}
\]

on the zero-isocline. If we increment \(y_t\) by \(\theta\), and retain our
\(x_t\) value,

\[
\Delta y_t = \frac{p_x [L - (\theta + \alpha)] - c_x (p_y L - p_y \alpha)}{c_x} (\theta + \alpha)
\]

\[
= (p_x L - p_y \alpha - p_y \theta + p_y L + p_y \alpha) (\theta + \alpha)
\]

Thus, above the \(\Delta y_t = 0\) isocline, \(\Delta y_t < 0\). Similarly, decrement \(y_t\) by \(\theta\) produces \(\Delta y_t = +p_y \theta\); thus \(\Delta y_t > 0\) for \(y_t\) below the
\(\Delta y_t = 0\) isocline.

3. Notice in Figure 7 that \(x_t\) constantly decays toward zero, while
\(y_t\) begins at about 0.1, decreases slightly, then increases to 0.5.
The time-paths would approximately be:
5. BIBLIOGRAPHY


STUDENT FORM 1

Request for Help

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name.

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Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

- Corrected errors in materials? List corrections here:

- Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

- Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature

Please use reverse if necessary.
STUDENT FORM 2

Unit Questionnaire

Name ________________________________  Unit No. _____________  Date _____________

Institution ____________________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?
   ( ) Not enough detail to understand the unit
   ( ) Unit would have been clearer with more detail
   ( ) Appropriate amount of detail
   ( ) Unit was occasionally too detailed, but this was not distracting
   ( ) Too much detail; I was often distracted

2. How helpful were the problem answers?
   ( ) Sample solutions were too brief; I could not do the intermediate steps
   ( ) Sufficient information was given to solve the problems
   ( ) Sample solutions were too detailed; I didn’t need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for
   example, instructor, friends, or other books) in order to understand the unit?
   ( ) A Lot  ( ) Somewhat  ( ) A Little  ( ) Not at all

4. How long was this unit in comparison to the amount of time you generally spend
   on a lesson (lecture and homework assignment) in a typical math or science course?
   ( ) Much Longer  ( ) Somewhat Longer  ( ) About the Same  ( ) Somewhat Shorter
   ( ) Much Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as
   many as apply.)
   ( ) Prerequisites
   ( ) Statement of skills and concepts (objectives)
   ( ) Paragraph headings
   ( ) Examples
   ( ) Special Assistance Supplement (if present)
   ( ) Other, please explain

6. Were any of the following parts of the unit particularly helpful? (Check as many
   as apply.)
   ( ) Prerequisites
   ( ) Statement of skills and concepts (objectives)
   ( ) Examples
   ( ) Problems
   ( ) Paragraph Headings
   ( ) Table of Contents
   ( ) Special Assistance Supplement (if present)
   ( ) Other, please explain

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of
this sheet if you need more space.)