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ABSTRACT

This study involved the development and experimental tryout of 22 lessons designed to teach 23 primary grade children a set of specific procedures, focusing on the drawing of meaningful diagrams, for analyzing and solving arithmetic story problems. Major dependent variables studied were gain on (1) a 20-item test on stories of the eight types taught and (2) a transfer 20-item test. Gains on both variables were significant. Samples of student work from each class session provided the basis for a qualitative analysis of student progress. The study also investigated differences in pupil performance as these were related to differences in story type. (Author/MNS)

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Abstract

This study involved the development and experimental tryout of 22 lessons designed to teach primary grade children a set of specific procedures, focusing on the drawing of meaningful diagrams, for analyzing and solving arithmetic story problems. Major dependent variables studied were gain on (1) a test on stories of the eight types taught and (2) a transfer test. Gains on both variables were significant. Samples of student work from each class session provided the basis for a qualitative analysis of student progress. The study also investigated differences in pupil performance as these were related to differences in story type.

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Research conducted in a variety of content areas (e.g., Larkin, 1977; Simon & Simon, 1978; Heller & Greeno, 1979) suggests that effective problem solvers, at all age levels, develop some type of qualitative representation of a problem before they identify and carry out the mathematical operations necessary for solution. This appears to be their way of analyzing the problem situation so that they really understand it. Of course, teachers and mathematics educators have stressed the importance of developing some type of representation of the problem situation through such steps as drawing a diagram (Schoenfeld, 1980) or constructing a table (Yeshuran, 1979). In an attempt to follow up on the implications of these findings and suggestions, the present study explored the effect on primary grade children of formal instruction in the use of specific diagramatic models as an aid to understanding and solving arithmetic story problems.

The specific types of story problems used in this study involve simple addition and subtraction situations, and an important initial concern was the nature and content of the diagrams that were to be taught. In efforts to describe the essential content of addition and subtraction stories several investigators have analyzed the semantic structure of the stories and have developed categories of story types.

Heller and Greeno (1978), in a restructuring and relabeling of story categories used in earlier studies (e.g., Shores & Underhill, 1976), identified three major categories, combine, change, and compare. Carpenter and Moser (1979) have outlined a scheme that involves seven different categories. As an aspect of this type of analysis, Heller and Greeno (1978) have proposed specific structures of knowledge, or schemata, that pupils must possess if they are to be successful in solving the various simple addition and subtraction stories. Riley (1981) has used data from her own study as well as that from other investigators to support the idea that differences in pupils' abilities to solve these types of story problems can be explained in terms of the extent to which they possess the types of knowledge represented by the Heller and Greeno (1978) schemata. Using the results of these types of analyses, one of the present writers, in a study (Lindvall & Ibarra, 1980) involving the clinical observation of pupils attempting to build representations of stories through the use of manipulative materials (counting cubes), identified four major components to be included in a modeling of the essentials of a story; (1) set identity, (2) set numerosity, (3) operations on sets, and (4) identity of answer set. These, then, were used as guidelines in determining the components of the diagrams to be taught in the present study.

<u>Purpose of this study</u>. This study involved the development and experimental tryout of a sequence of 22 lessons designed to teach primary grade children a set of procedures for analyzing and solving arithmetic story problem. The procedures taught included the drawing of specific types of diagrams to model each of eight types of addition and subtraction stories and then using this model to identify the arithmetic

operation needed for problem solution. Purposes of the study were to determine (1) if primary grade children could master these procedures, (2) if mastery of the procedures resulted in improved performance in story problem solution, and (3) if pupils could transfer the procedure learned with respect to simple one-step problems to the solution of more complex problems (without any direct teaching for such transfer).

Lesson content and teaching procedures. The lessons used in this study were designed to help pupils to understand and to solve eight basic types of one-step addition and subtraction story problems. The eight types of problems, labeled both with the name used in many recent research studies and the name used in talking with students, are listed in Figure 1. Also shown is the basic format of the diagram taught in conjunction with each story type. Although our scheme of categorization does not represent an exact parallel to that outlined by Carpenter and Moser (1979), it is based largely on their analysis and may be thought of as our adaptation of their work as influenced by Heller and Greeno (1978). With each of these eight story types pupils were taught to solve for the unknown value when it might be in any of the possible positions in the story sequence. For example, with the Change Increase type of story they were taught to solve for an unknown starting amount, an unknown increase, or an unknown result from the increase.

In solving all types of stories pupils were taught to follow a general procedure, or strategy, consisting of the following four steps:

1. Read (or listen to the reading of) the story.

2. Draw a diagram to represent the sets and the operations or relationships described.

3. Write a number sentence for the story.

4. Solve the number sentence.

The right hand column of Figure 1 provides an example of the basic diagram used with each story type. Of course, adaptations were made in a diagram on the basis of which set was unknown. Also, with most story types the pupils were taught to use numerals to represent set size as well as to use an appropriate number of dots. Each of the eight types of diagrams can be considered as representing a specific strategy for analyzing and solving the given story type.

Examples of the steps used in instruction are provided in the portions of two lesson outlines presented in Figure 2. The initial modeling of the story provided in the instructional example was carried out by using chips to represent the elements in the sets and loops of yarn to encircle the sets. This was demonstrated by the teacher and carried out by each student individually. This was followed by an explanation of the diagram and by having the student draw the appropriate diagram for the example. The teacher also explained additional examples as this was deemed necessary. The total time devoted to this type of group instruction was approximately 10 minutes for each lesson. This was followed by a study period during which pupils worked independently on ten practice problems, using the paper and pencil diagrams to solve each story.

Method

The sample of students used in this investigation consisted of 23 children from the primary grade division of a university laboratory school operating under a program of individualized instruction. After being pretested both on their ability to solve the types of stories that were to be the focus of instruction and on stories to be used in testing transfer, the students were provided with 22 instructional sessions, each of about 40 minutes in length, and presented on 22 different days. This instruction focused on the types of stories described in the preceding section. At the completion of instruction the students were given a 20 item posttest and a 20 item transfer test. The transfer test consisted of seven two-step stories involving successive application of the same story operation (e.g., combine-combine), seven two-step stories involving successive application of two different story operations (e.g., combine-change increase), three one-step stories involving length units, and three one-step stories comparable in form to those taught but involving two-digit numbers. The purpose in testing transfer was to determine the extent to which pupils could use the procedures taught for use with one-step problems to create their own diagrams for more complex stories.

Results

The means and standard deviations on the twenty-item pretest and posttest, for all three performance measures, are presented in Table 1. It can be seen that the instruction resulted in a significant improvement on all three criterion tasks, finding the answer, drawing the diagram, and writing the appropriate number sentence. The absolute

size of the gain was somewhat restricted because of the relatively high mean pretest score, the latter being due largely to the fact that four students had perfect scores on the pretest. (These students were retained in the study because of an interest in studying their gains on the transfer test.) It should be noted here, too, that the pretest on modeling involved having students respond to the instructions "Would you explain this story using these blocks, or these chips, or by making a drawing." That is, since these students had not received any prior instruction on using diagrams, it was considered appropriate to get a general measure of their modeling ability by permitting them to select the materials they wished to use. However, on the posttest they were required to use paper and pencil diagrams, of the type emphasized in instruction, in developing their models.

Of at least equal interest to quantitative data obtained with respect to what pupils learned is the qualitative information gained by examining children's solution procedures, both on the tests and on the daily lesson sheets. From this, it was obvious that the majority of the students learned to follow the general procedure taught and to use diagrams that aided understanding of the story. Although students were taught a very specific type of diagram for each type of story, they were told that the basic requirement was that a story diagram represent the essential elements of the story. As a result, they were given credit for producing a correct model if the diagram showed the proper number and identification for any sets described in the story and if it depicted any relationships or operations in a way that served to identify the answer.

As can be seen in Table 2, the mean gain on the transfer test was slightly larger than the mean gain for getting the correct answer on the one-step story achievement test (as reported in Table 1). Perhaps of more interest, here, was information concerning procedures used by the students as this was obtained by examining their tests to note the diagrams used. This revealed that in solving the transfer stories on the posttest the pupils used diagrams that were their own adaptations and extensions of those they had been taught to use with the one-step stories.

In many cases the arithmetic operation appeared to be too complex for them to represent (for example, they had not been instructed on how to do two-step problems by carrying out two successive operations) but they were, nevertheless, able to analyze the story through the use of a meaningful diagram and to apply counting procedures to arrive at the answer. That is, they exhibited a real understanding of the story.

The transfer test items were analyzed further by investigating differences among the different types of items involved. Results from this analysis are summarized in Tables 3 through 6. For the most part these results are in line with what might have been predicted. A comparison of posttest performance on the first four items listed in Table 3 with that on the last four items shows that two-step stories in which the answer is produced by applying the two described operations are easier than two-step stories in which the story describes an operation on an unknown quantity which produces a known result. This, of course, parallels results found with one-step problems (Carpenter & Moser, 1979; Lindvall & Tamburino, 1981; Riley, 1981). Also, paralleling results from these earlier studies of one-step problems are

the posttest results summarized in Table 4 which indicate that stories involving the compare or equalize operations (the last four stories listed) are more difficult than stories which only involve combine and change. In general, the data in Tables 3 through 6 indicate that the greatest improvement from pretest to posttest was shown on those stories where the proportion passing on the pretest was the lowest.

Discussion

If, as a number of studies have shown, effective adult problem solvers develop some type of quantitative representation of a problem before determining the mathematical procedures appropriate for solution, it would seem to be important to investigate the effect of efforts to teach this skill as an aspect of instruction in problem solving. In a limited exploration of this type of instruction, the results from the present study suggest (1) that primary grade children can master this modeling ability, (2) that having this ability increases a student's performance in problem solution, and (3) that this ability has some transfer value.

It is the feeling of the writers, as a result of this experience, that teaching pupils to use paper and pencil diagrams to develop models of stories provides pupils with an efficient and effective means for analyzing and comprehending a story problem. However, the diagrams taught should not be considered as a new type of algorithm which, when the proper one has been selected, the student can use in a rather rote fashion merely by plugging in the appropriate numbers and carrying out the operation suggested by what is unknown in the diagram. Obviously, it is useful and probably necessary to start by teaching some rather

specific diagram forms and components as was done in the present effort. However, the real goal is to get the pupil to really analyze the problem and to develop a diagram that is a valid representation of the situation and that is meaningful to the pupil. In our work with children we frequently found pupils using a diagram that was a variation on the one taught for that particular story type or of the form taught for a different story type. However, quite often, a questionning of the student would serve to convince us that the diagram was indeed a valid and meaningful representation of the problem. Also, as we discussed earlier, pupil performance on the transfer test indicated that pupils displayed considerable creative ability in sketching diagrams that were meaningful for more complex stories. Instruction of the specific type presented in the present study should probably be thought of as a useful way for introducing pupils to the technique of developing diagrammatic representations of stories and providing them with an orientation to the essential components of such diagrams. The ultimate goal of such instruction, however, must be to help the student become proficient in developing his or her own diagrams.

Quite obviously, the present study has only been an exploratory investigation of this type of instruction. However, it appears to have some implications for further development efforts and for research. Development work might include attempts to refine the instructional procedures and materials so that they could be used by a broader sample of classroom teachers and students. Such trials, with the feedback provided by participants, could then be used as a basis for determining the practicality of the procedures and materials for general classroom use and could provide suggestions for improvement. Research conducted

in conjunction with such tryout efforts might also provide a basis for determining if the "correlational" relationship between the use of a qualitative representation of a problem and success in problem solution, as revealed in descriptive studies of differences between experts and novices (Simon & Simon, 1978; Chi, Feltovich & Glaser, 1980) is actually a causal relationship.

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Table 1

Means and Standard Deviations for Pretest and Posttest on Each of the Three Performance Measures (N=23)

Performance Measure	Means (S.D.) Pretest	Posttest	Gain	t for signif. of gain
Answer	14.00 (3.91)	17.61 (3.03)	3.61 (3.22)	5.38*
Model	14.52 (3.27)	17.04 (3.17)	2.52 (3.03)	3.99*
Number Sent.	13.17 (4.41)	17.17 (3.60)	4.00 (3.48)	5.52*

*$p < .001$

Table 2

Means and Standard Deviations for Pretest and Posttest on Giving the Correct Answer on Transfer Test (N=23)

Pretest Mean	Pretest S.D.	Posttest Mean	Posttest S.D.	Mean Gain	t for signif. of gain
11.65	4.10	16.26	2.65	4.61	5.73*

*$p < .001$

Table 3

Proportion of Students Giving Correct Answer on Pretest and Posttest Transfer Items Involving Two Applications of the Same Operation

Problem Type	Measure		
	Pretest	Posttest	Gain
Combine, Combine Total Unknown	.96	.96	.00
Combine, Combine Subset Unknown	.60	1.00	.40
Change Inc., Change Inc., Final Amount Unknown	.91	1.00	.09
Change Dec., Change Dec., Final Amount Unknown	.77	.82	.05
Combine, Combine Subset Unknown	.55	.86	.31
Change Inc., Change Inc., Subset Unknown	.69	.82	.13
Change Dec., Change Dec., Subset Unknown	.50	.73	.23
Compare, Compare Set Unknown	.46	.69	.23

Table 4

Proportion of Students Giving Correct Answer on Pretest and Posttest Transfer Items Involving Successive Application of Two Different Operations

Problem Type	Measure		
	Pretest	Posttest	Gain
Change Dec., Change Inc.	.41	.82	.41
Combine, Change Dec.	.55	.87	.32
Change Dec., Compare	.09	.46	.37
Change Inc., Compare	.64	.64	.00
Change Inc., Equalize	.36	.78	.42
Change Dec., Equalize	.36	.60	.24

Table 5

Proportion of Students Giving Correct Answer on Pretest and Posttest Transfer Items Involving One Operation But With Length Measures

Problem Type	Measure		
	Pretest	Posttest	Gain
Combine, (Mile Units)	1.00	1.00	.00
Change Dec. (Feet)	.78	.96	.18
Compare (Years)	.78	.82	.04

Table 6

Proportion of Students Giving Correct Answer on Pretest and Posttest Transfer Items Involving One Operation But With Two-digit Quantities

Problem Type	Measure		
	Pretest	Posttest	Gain
Combine, Subset Unknown	.60	.87	.27
Change Inc.	.50	.87	.37
Change Dec.	.46	.73	.27

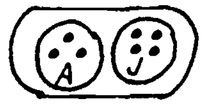
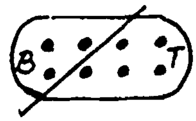
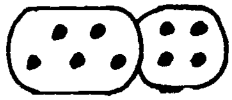
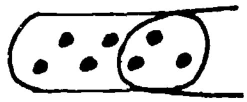
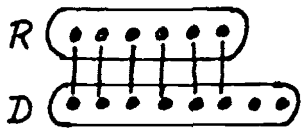
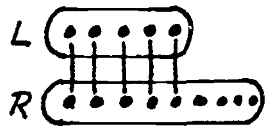
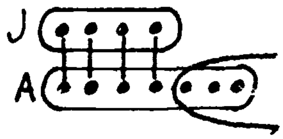
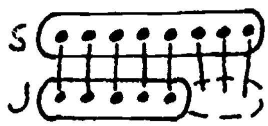
Problem Type	Story Example	Diagram Taught
1) Combine (Putting Sets Together)	Ann had 3 apples. Jill had 4 apples. How many apples did they have altogether?	
2) Separate (Taking Set Apart)	Together Bob and Tony had 8 toy cars. 3 of these were Bob's. How many did Tony have?	
3) Change Increase (Getting More Things)	Sue had 5 pencils. She got 4 more pencils. How many did she have then?	
4) Change Decrease (Losing Some Things)	May had 7 cookies. She then ate 3 of them. How many did she have left?	
5) Compare - More (How Many More)	Rick had 6 kites. Dan had 8 kites. How many more kites did Dan have than Rick?	
6) Compare - Less (How Many Less)	Len had 5 books. Rita had 9 books. How many less books did Len have than Rita?	
7) Equalize - Take Away (Making Same Size - Take Away)	Jim had 4 cookies. Al had 7 cookies. How many cookies would Al have to eat to have as many as Jim?	
8) Equalize - Add On (Making Same Size - Add On)	Sally had 8 rings. Jan had 5 rings. How many more would Jan have to get to have as many as Sally?	

Figure 1. Names and Examples of the Eight Problem Types Used in This Study Together With Basic Diagram Taught for Each Story.

Lesson 4: Losing Some Things (LST)

Lesson objective: Given a written story problem of the "losing some things" type, the student will be able to draw the appropriate story diagram and write and solve the correct number sentence.

Story example: "Jane had 6 pencils. She lost 2 of these pencils. How many did she have then?"

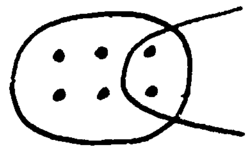
1. Show with blocks.
2. Write number sentence (on board)
3. Draw diagram for story. Emphasize each component:

   --the set "Jane had": number, identity

   --the "take away" loop

   --the set remaining

3. (cont.)



$6-2=4$

4. Write number sentence. Solve.
5. Check answer.

Lesson 15: Losing Some Things - "Some" (LST-S)

Lesson objective: Given a written story problem of the LST-S type, the pupil will be able to draw the appropriate story diagram and write and solve the correct number sentence.

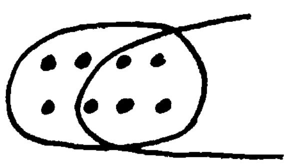
Story example: "Bob had 8 pencils. He lost some of his pencils. He then had 3 left. How many pencils did Bob lose?"

1. Show with blocks.
2. Draw story diagram (dots)

   --start set

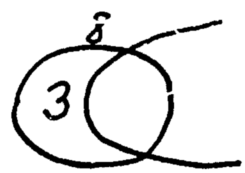
   --the "take away" loop

   --number remaining



$8-3=5$

3. Draw story diagram (numerals)



$8-3=5$

   --place number in start set above loop

   --draw "take away" loop

   --place number in "remaining set" space

4. Write number sentence. Solve.
5. Check answer.

Figure 2. Examples of Basic Components of Two of the Lessons Presented in the Twenty-Two Lesson Sequence.