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ABSTRACT

This report illustrates procedures of item construction for addition and subtraction examples involving decimal fractions. Using a procedural network of skills required to solve such examples, an item characteristic matrix of skills analysis was developed to describe the characteristics of the content domain by projected student difficulties. Then 24-item tests were constructed for addition and subtraction with decimals. Pre- and posttest versions of each were administered to more than 100 junior high school students. After the pretest, each student was provided with some remediation geared to the particular "bugs" (persistent errors); then the posttest was given. A prediction of item dominance based on test characteristics is compared with item dominance determined from an item relational structure analysis of the data. (MNS)

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DECIMAL FRACTION ARITHMETIC: LOGICAL ERROR ANALYSIS AND ITS VALIDATION

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COMPUTERIZED ADAPTIVE TESTING AND MEASUREMENT — RESEARCH REPORT 82-1-NIE

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Abstract

This technical report illustrates a procedural network of skills needed to solve problems involving the addition and subtraction of decimal fractions. The network is used to define an item characteristic matrix of skills analysis which is then used as the basis for constructing error diagnostic tests. Several examples of student response patterns are used to illustrate the construction of a few selected items so that a test will have both content and construct validity.

The report includes lists of projected errors which are predicted from the nodes of the procedural network or from the content domain relative skills analysis. A prediction of item dominance based on test characteristics is compared with item dominance determined from an item relational structure (IRS) analysis of pre-and posttests administered to more than 100 junior high school students.

The report includes the procedural network, content domain relative skills analysis, and complete tests for both addition and subtraction.

Introduction

The assessment of student achievement and the diagnosis of student learning difficulties require the use of tests which have both content validity and construct validity. In order to determine uniquely student "bugs" (assuming there are such errors), test items must be selected to reflect the underlying cognitive process used by most students. Error diagnostic tests for whole number subtraction (Brown & Burton, 1978) and for signed-number arithmetic (Tatsuoka, Birenbaum, Tatsuoka, & Baillie 1980) have reported the successful diagnosis of hundreds of erroneous rules of computation.

This report illustrates procedures of item construction for addition and subtraction problems involving decimal fractions. First we describe a procedural network of skills that are required to solve decimal fraction arithmetic problems. Using that network, we develop an item characteristic matrix of skills analysis that describes the characteristics of the content domain (decimal arithmetic) by projected student difficulties. Both the network and the content domain relative skills analysis are used to describe the construction of a 24-item addition test and a 24-item subtraction test (Appendix I). The content domain relative skills analysis is used to predict item dominances for both addition and subtraction of decimals. Also, the analysis was used to predict relative problem difficulty among all items. Actual item dominances and relative item difficulties were determined using item relational structure analysis (Tatsuoka and Tatsuoka, 1981).

Pre- and posttest versions of each were administered to more than 100 junior high school students over a period of two weeks. After pretests were scored, each student was provided with some remediation geared to the particular "bugs" and to the student's prior knowledge of decimal fraction arithmetic (self reported). Posttests were then administered. Although we did not have control over several variables such as prior knowledge or method of prior instruction, we can use the results to suggest approaches to the construction of error-diagnostic tests and the remediation of student difficulties -- particularly those which can be implemented as part of computerized adaptive testing systems which can be used in the classroom. Our suggestions and concerns are based on the test results, item dominance predictors, and interviews with students who missed more than 15% of the items.

Student "Bugs" in Decimal Addition

The content domain of decimal fraction arithmetic is an appropriate one for investigating initial efforts to computerize diagnostic tests and remediation in that the number of student "bugs" is considerably more limited than either signed-number or fractions arithmetic. For this study we projected nine erroneous rules which are peculiar to decimal fraction addition and eleven for decimal fraction subtraction. (We did not include erroneous rules which also belong to signed-number arithmetic.) On the other hand, 88 erroneous rules were projected for fraction arithmetic (Klein, Birenbaum, Standiford, and Tatsuoka, 1981). The use of decimal fraction arithmetic is also more convenient in that there are few problem types for which application of a "bug" results in a correct answer.

As with other error diagnostic tests, the diagnosis of student difficulties and the assessment of student achievement requires more than a single raw score, such as percentage of correct items. It is possible for several students to have identical scores yet have different degrees of seriousness of error, such as persistence or resistance to remediation (Birenbaum & Tatsuoka, 1981; Klein, et al., 1981).

To illustrate, we have constructed answers to selected decimal addition and decimal subtraction problems using "bugs" consistently applied by junior high school students. On the addition "test" students 1 and 2 have the same percentage correct. Both have a single "bug." Student 3, who has a higher correct percentage, has two "bugs."

Insert Table 1 about here

Student 1 applies the "multiplication" rule to determine the number of decimal places in the sum. Problems are set up correctly but the decimal point in the sum is located by adding the number of decimal places in the two addends. The only correctly answered items are those involving whole numbers.

Student 2 applies the "right justification" rule in setting up the problem. The addends are written so that the digits are aligned along the right hand edge. The decimal is usually placed in the answer matching the addend with the greater number of digits to the right of

Table 1

Decimal Addition Test

Item	Student Answers		
	#1	#2	#3
.8 + .6 = 1.4	.14 X	1.4	.14 X
.23 + .5 = .73	.073 X	.28 X	.73
4 + 3.2 = 7.2	7.2	3.6 X	3.6 X
0.83 + 0.5 = 1.33	.133 X	.88 X	1.33
PERCENT CORRECT	25 %	25 %	50 %

X incorrect response

the decimal. The only correctly answered items are those whose addends have the same number of digits to the right of the decimal.

Student 3 has two "bugs." The first is the "carry rule" applied to problems whose sum of the tenth's place is greater than one. The "1" carry is written to the right of the decimal point only if the addends do not have an explicit whole number digit. Thus the student fails to answer .6 + .8 but correctly deals with 0.83 + 0.5. The second bug is the "whole number" error in which numbers with no explicit decimal points are aligned along the right edge.

<u>Insert Table 2 about here</u>

The subtraction test illustrates three different student "bugs" which result, in this case, in identical test scores. Student 1 equates "more digits" with "larger number" which is always written as the minuend. Also, the student right justifies the numbers. The decimal point in the difference reflects the placement in the student's "minuend."

Student 2 "brings down" numbers in the subtrahend if there are no corresponding digits in the minuend. This may be a carry over from addition thinking that there is "nothing to do." This will be correct when the minuend is longer (i.e., there are no corresponding digits in the subtrahend).

Student 3 borrows correctly except across the decimal point. In those problems the student merely adds ten to the tenth's place without adjusting the units digit.

<u>Problem Types</u>

Problem types were classified according to three attributes: number type, representation, and complexity. Number type was either whole number (W) or decimal fraction (D). Representations included implicit decimals (e.g., 3) or explicit decimal (e.g., 14.); leading zeros (e.g., 0.53) or trailing zeros (e.g., 6.30). Complexity included carrying or borrowing (especially "across the decimal point").

If we consider whole numbers (W) with implicit decimal points (W_I) and explicit decimal points (W_E) and decimal fractions (D) we can identify nine addition problem types: $W_E + W_E$, $W_I + W_I$, $W_E + W_I$, $W_I + W_E$, $W_E + D$, $D + W_E$, $W_I + D$, $D + W_I$, and $D + D$.

Table 2

Decimal Subtraction Test

Item	Student Answers		
	#1	#2	#3
.4 - .236 = .164	.232 X	.236 X	.164
4.05 - .68 = 3.37	3.37	3.37	4.37 X
4.2 - .367 = 3.833	.325 X	3.967 X	4.833 X
6.23 - 4 = 2.23	2.23	2.23	2.23
PERCENT CORRECT	50 %	50 %	50 %

X incorrect response

Since addition is commutative, three pairs are identical: $W_E + W_I$ and $W_I + W_E$; $W_E + D$ and $D + W_E$; and, $W_I + D$ and $D + W_I$.

In addition to the combinations of number types, we considered representation and complexity. In an effort to keep the decimal tests to a reasonable length, we limited our initial test items to problem types $W_I + D$ (or $D + W_I$) and $D + D$ but included various combinations of representation and complexity. This limited domain allowed us to test combinations of critical attributes of problem types as well as attributes related to procedure (see next section.)

Nine different types of subtraction were also identified: $W_E - W_E$, $W_E - W_I$, $W_I - W_I$, $W_I - W_E$, $W_E - D$, $D - W_E$, $W_I - D$, $D - W_I$, $D - D$. Since subtraction is not cummutative, all problem types are different. As in the addition test, problems involving W_E were not included on the test administered during this stage of the investigation. Also, problem type $W_I - W_I$ was not included as representative of a decimal test. Finally, care was taken to avoid problems which would yield a negative answer.

The Procedural Network

There are several different methods for adding or subtracting decimals. The procedural network is a representation of different algorithms whch can be used to solve different types of problems. Figures 1 and 2 are procedural networks for addition and subtraction respectively.

Insert Figure 1 about here

Insert Figure 2 about here

Five sided cells indicate that the particular step in a procedure which is explained in another chart. The number of the chart is included in the cell label.

Using the procedural networks, we can classify the student errors as either "set up" or "decimal placement" errors. Set up errors occur when the student rewrites a horizontal addition (subtraction) problem as a vertical addition (subtraction) problem. Errors such as "right-justification," "inside out justification," and "whole number errors"

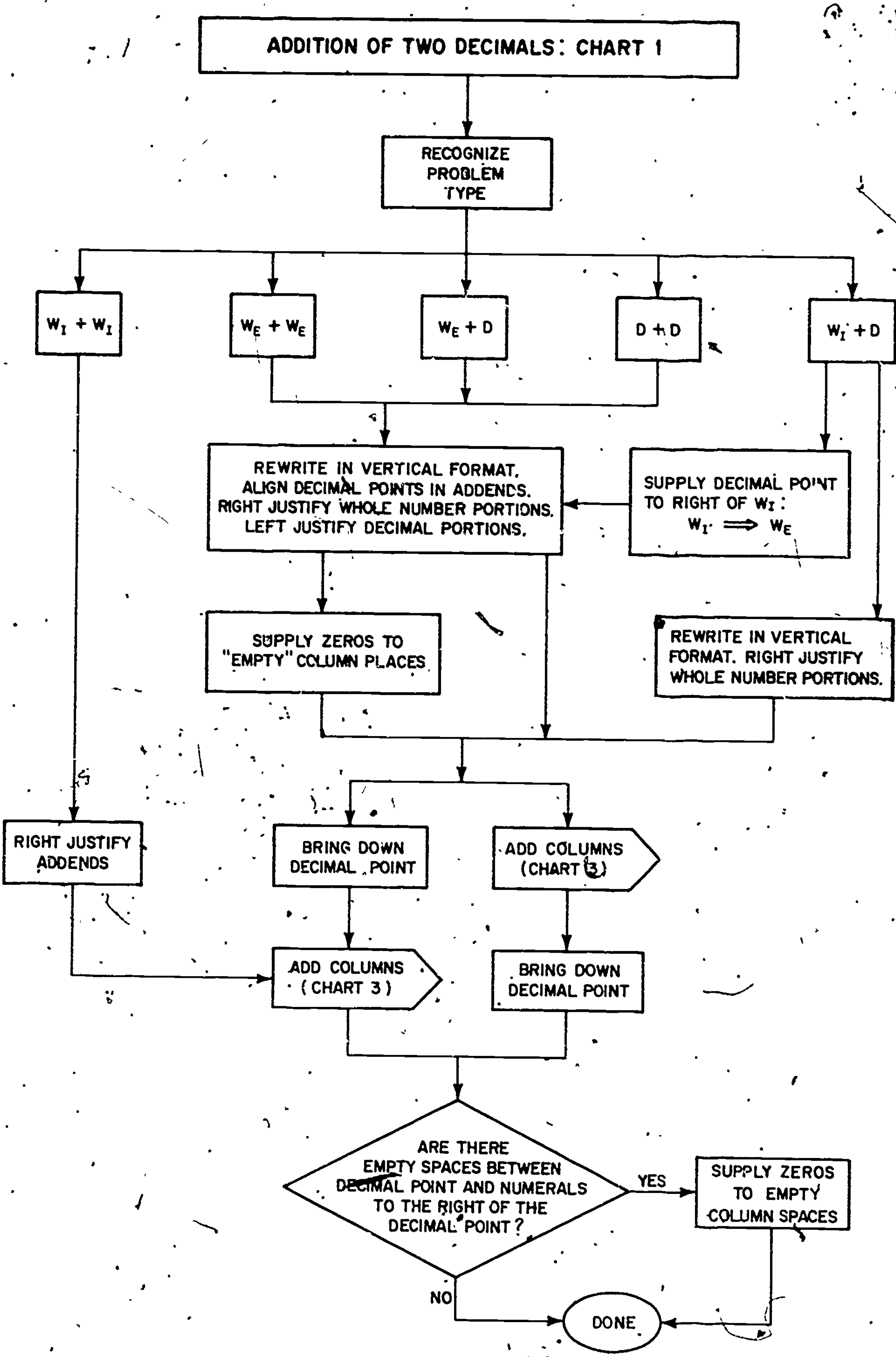

FIGURE 1: A Procedural Network for Adding Two Decimal Fractions

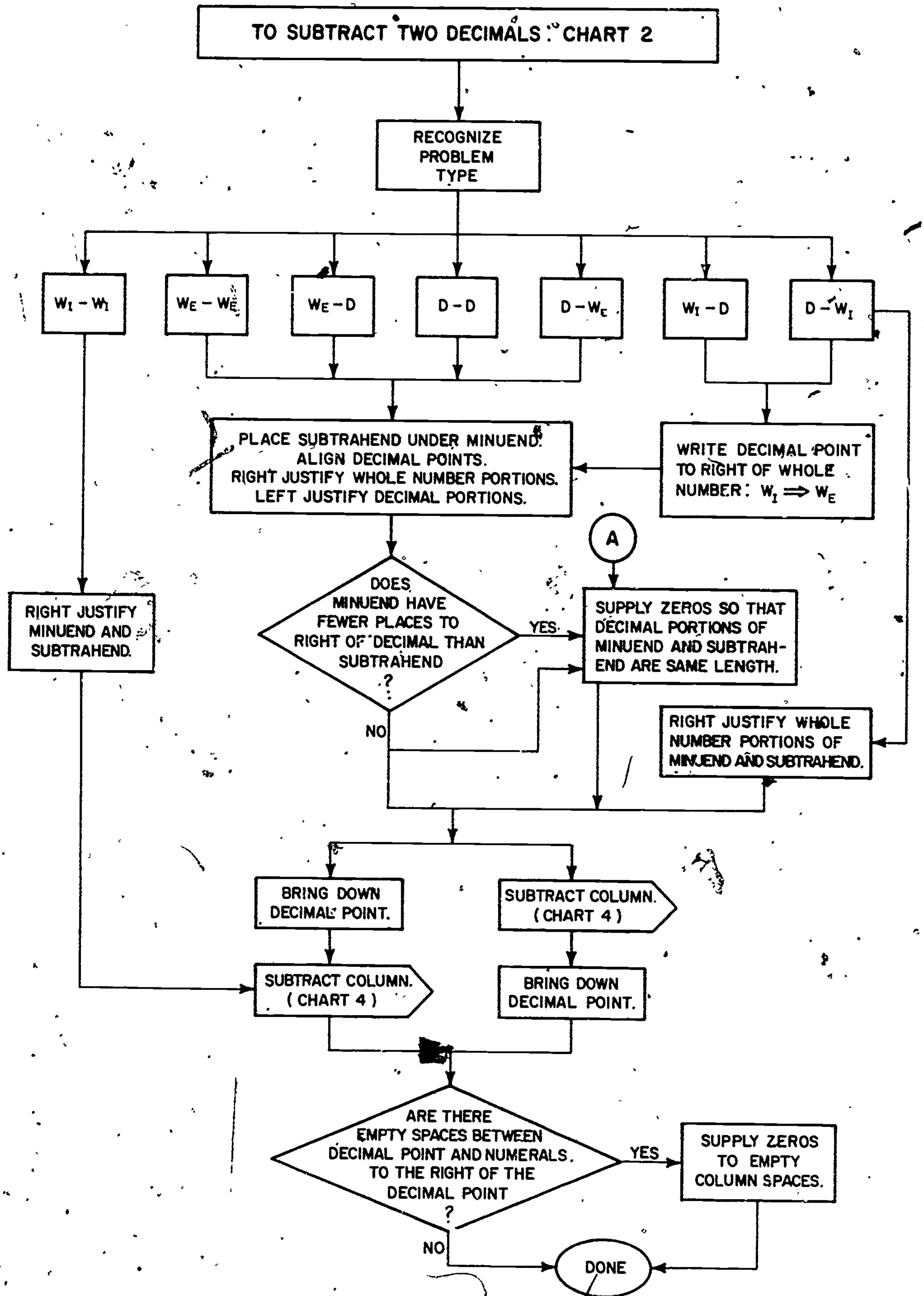

FIGURE 2 : A Procedural Network for Subtracting Two Decimal Fractions.

fall into this category. Decimal placement errors occur in placing the decimal in the sum. Errors such as "carry errors," or "multiplication rule" belong to this category.

One of the purposes of this study was to investigate the relationship between problem type attributes and the incidence of certain procedural errors. For example, do leading zeros in addends diminish the incidence of "carry errors?"

Levels of Competency

Computation with decimals can be represented in a skills hierarchy for adding (subtracting) decimals. Using a skills hierarchy, several levels of competency needed to solve addition and subtraction problems can be identified.

Level 0. The student understands the relationship between whole numbers and decimal fractions. The general concept of decimals is understood.

Level I. The student is able to add (subtract) numbers with explicit decimals with the same number of digits to right of the decimal point.

Level II. The student is able to add (subtract) numbers with explicit decimals with different numbers of digits to right of decimal point. Failure at this level may be due to incorrect alignment of addends due to different lengths.

Level III. The student is able to add (subtract) numbers with implicit decimals. Failure at this level may be due to incorrect alignment of addends due to unwritten decimals.

Level IV. The student is able to add (subtract) numbers involving carrying (borrowing) to (from) the whole number portion of the addend (minuend).

Level V. The student is able to add (subtract) involving several problem types.

Level 0 mastery indicates conceptual understanding of decimals. Levels I - V can be achieved by adhering to a "recipe" or procedure for solving the particular problem. It is likely that students can "master" levels I -V without a clear understanding of level 0. Our tests were designed to test mastery of the procedural skills (levels I - V). Mastery of level 0 was to be determined through interviews with individual students.

Procedures of Item Construction

A common error in adding two fractions is a "carry error" in which the student does not carry a sum "across the decimal point." For example, for the item .8 + .6 the student might write .14 as the answer. There are two possible explanations for this error: the "carry error" or the placing the decimal point according to the rule for multiplication. To determine which rule is being used, a second item such as .82 + .73, is needed. If the student answers .155 (carry error) or .0155 (multiplication rule) the rule is diagnosed. It may be, however, the student applies a particular rule only in certain problems. For example, the student may not use the "carry error" if there is a non-zero whole number digit in either addend.

One way to approach item construction to help identify possible contexts in which errors always (or never) occur is to construct a table indicating item characteristic by relative level of skill required to correctly answer the item. Such a table for addition is shown in Table 3.

Insert Table 3 about here

Problem types can be identified from a content domain relative skills matrix. characteristic table. Particular items can be constructed to match the combinations selected. For example, an item such as .63 + 0.4 would reflect the following combinations:

1.	Problem form	Horizontal
2.	Number of digits right of d.p.	Different
3.	Decimal point	Explicit
4.	Carry across zero	Yes
5.	Leading zero in addend	Yes (one)
6.	Embedded zeros in answer	Yes
7.	Number of decimal places	Different

It should be noted that not all item characteristic selections are independent. For example "implicit decimal" indicates one addend is a whole number. As a result, "number of digits to right of dp" must be different, and "number of decimal places in answer" must be equal to sum of decimal places in the addends. This is not to say, however, that

Table 3: Characteristics of the Content Domain by Projected Difficulties in Decimal Fraction Addition Problems

ITEM CHARACTERISTIC	RELATIVE SKILL LEVEL REQUIRED		
	EASY	MODERATE	DIFFICULT
Problem Form			
Format	Vertical		Horizontal
Number digits right of d.p. in addends	Equal (and equal addend lengths)	Different (and unequal addend lengths)	Different (and equal addend lengths)
Decimal notation	Explicit		Implicit
Complexity			
Carry across decimal point	No		Yes
Leading zeros	Yes		No
Answer Form			
Embedded Zeros	None (few)		Some (many)
Number of decimal places in answer c/w sum of places in addends	Equal		Different

student bugs related to these categories are not possible. To illustrate, the item 7 + .4 might be interpreted as:

	.4	.4	.4	.4
	.7	7	7	.7
	1.1	1.1	.11	.11
possible errors	whole number	mult. rule whole number right just.	carry right just. whole number	carry mult. rule whole number

Finally, to reduce the number of problems to test likely student bugs, we did not construct items for two categories: problem form "vertical" and whole number "explicit decimal."

This table can be used to predict item "difficulty" which can be tested experimentally. Table 4 shows the analysis for the first 12 addition problems (the second 12 are parallel items).

Insert Table 4 about here

Based on this analysis, items with characteristics matching "easy" should, in fact, be answered correctly more often than items with characteristics matching "difficult." One would predict the following dominances:

1,2 < 7,8,9,11
4 < 7,11
9 < 8
10 < 5,6,8,12

If there is an interaction between certain characteristics, such as "carry" and "leading zeros" then one would predict the following dominances:

1,2 < 9,8
4 < 7,11
9 < 8
10 < 5,6,8,12

If one were to give equal weights to the independent characteristics "digits", "notation", and "embed", and weigh "carry" when "leading zeros" would influence its application, one would predict the following relative item difficulties for addition:

1,2,3,4	easiest
10	easy
6,7,9,11	difficult
5,8,12	most difficult

A similar technique is used for subtraction problems. The item

Table 4

Judged Rating for SKILL x ITEM CHARACTERISTIC Interaction
Addition Test

	Item Characteristic					
Item	Digits rt. of d.p. (note 1,2)	Decimal notation (note 3)	Carry across d.p. (note 4)	Lead Zero	Embed answer zero (note 5)	Answer decimal places (note 6)
.24 + .61	E	E	E	D	E	D
.36 + .45	E	E	E	D	E	D
14.3 + 2.4	E	E	E	-	E	D
0.53 + 0.81	E	E	D	E	E	D
4.32 + 3	M	D	E	-	E	E
.047 + 0.31	H	E	E	M	E	D
.54 + .83	E	E	D	D	E	D
.032 + .06	M	E	E	D	D	D
.035 + .062	E	E	E	D	D	D
3.24 + 6.5	M	E	E	-	E	D
.6 + .8	E	E	D	D	E	D
6 + 4.13	M	D	E	-	E	E

note:
1 → right justification error
2 → inside-outside justification
3 → whole number
4 → carry across zero
5 → missing zero
6 → multiplication rule

characteristic x skill table shown in table 5. The table showing the analysis for the first 12 (of 24) subtraction problems is presented in table 6.

Insert Table 5 about here

Insert Table 6 about here

Using the analysis in table 6, one would predict the following dominances:

$$5 < 2,12$$
$$11 < 4$$
$$6,8,10 < 7 < 3 < 9$$
$$1 < 4,9$$
$$12 < 9$$

If one were to give equal weights to the independent characteristics "digits", "notation", "multiple borrows", "borrow across zero", and "fractional part of subtrahend greater than fractional part of minuend," one would predict the following relative item difficulties in subtraction:

5	easiest
6,8,10,2	easy
1,7,11,12	neutral
3,9	difficult
4	most difficult

Test Results

The pre- and posttests for addition and subtraction were analyzed to determine item dominance using techniques discussed in detail in Tatsuoka and Tatsuoka (1981). The dominance relationships among all items on the addition pre- and posttests are illustrated in figures 3 and 4.

Insert Figure 3 about here

Insert Figure 4 about here

These figures indicate that problems with little inherent

Table 5: Characteristics of the Content Domain by Projected Difficulties in Decimal Subtraction Problems

ITEM CHARACTERISTIC	RELATIVE SKILL LEVEL REQUIRED: EASY	MODERATE	DIFFICULT
Problem Form			
Format	Vertical		Horizontal
Number digits right of d.p. $\#(M)$ c/w $\#(S)$	$\#(M) = \#(S)$	$\#(M) > \#(S)$	$\#(M) < \#(S)$
Decimal notation	Explicit		Implicit
Complexity			
Borrow across decimal point	No (no borrows)	(few borrows)	Yes (many borrows)
Trailing zeros	Yes (and $\#(M) \geq \#(S)$)	Yes (and $\#(M) < \#(S)$)	No (and $\#(M) < \#(S)$)
Answer Form			
Embedded Zeros	None (few)		Some (many)
Number of decimal places in answer c/w sum of places in Min. and Subt.	Equal		Different

Table 6

Judged Rating for SKILL x ITEM CHARACTERISTIC Interaction: Subtraction Test

Item Characteristic

Item	Digits rt. of d.p. (nt. 1,2)	Decimal notation (note 3)	Borrow across d.p. (note 4)	Borrow over 2 digits	Digits in Subt. less than number in Min. (notes 5,6)
14.25-3.4	D	E	D	E	E
1.4-.8	E	E	D	E	E
6.4-3.235	D	E	E	D	D
6-3.8	D	D	D	E	D
.Ø35-.Ø14	E	E	E	E	E
.456-Ø.31	D	E	E	E	E
6.35-3.248	D	E	E	E	D
.576-.23	D	E	E	E	E
2Ø.ØØ4-Ø.Ø3	D	E	D	D	E
.631-.31ØØ	D	E	E	E	E
8.7-5	D	D	E	E	E
4.ØØ2-.Ø81	E	E	D	D	E

note: 1 → right justification
2 → inside-outside justification
3 → whole number placement
4 → borrow across zero
5 → minuend subtrahend exchange
6 → recopy subtrahend digits

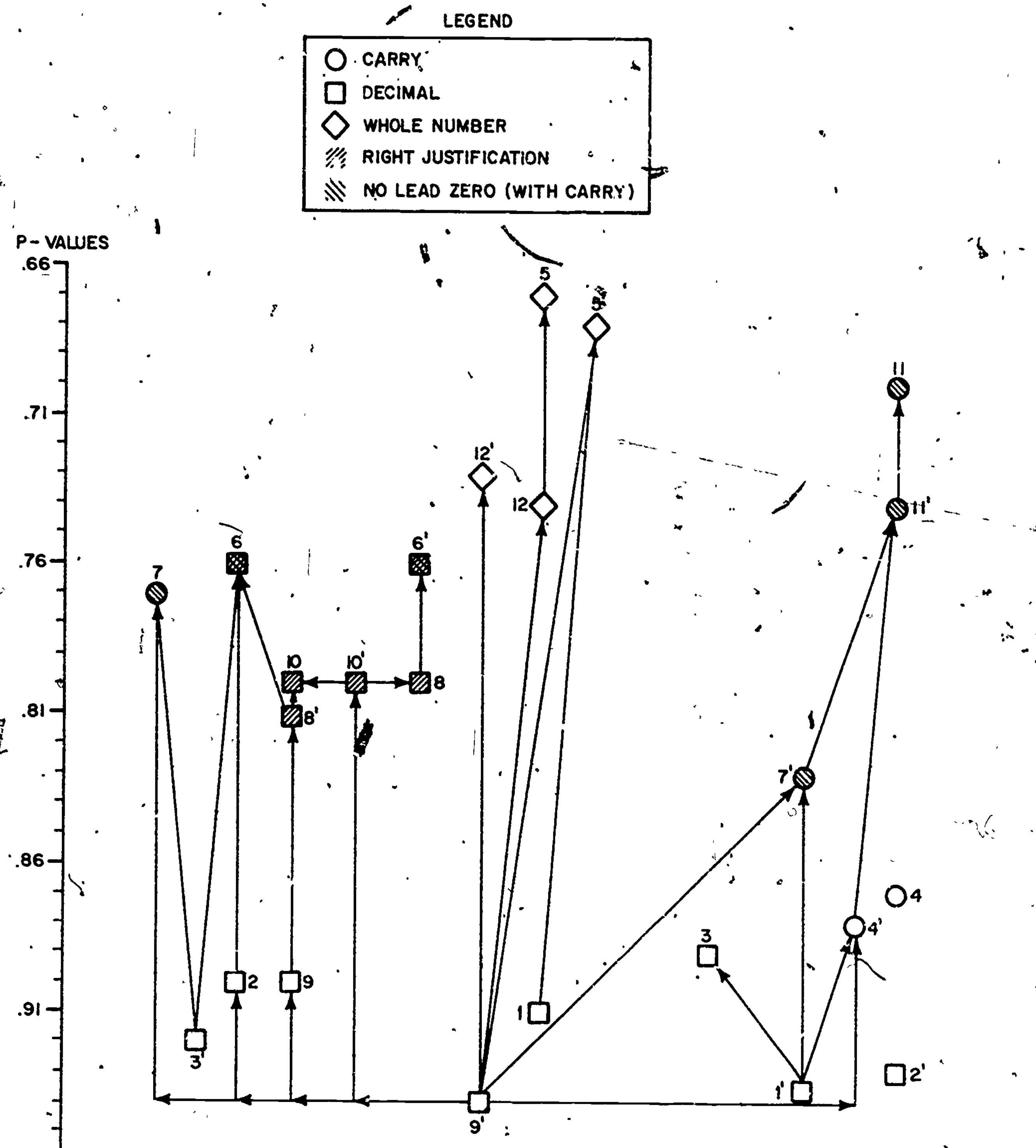

FIGURE 3: A Directed Graph of Addition Pre-Test

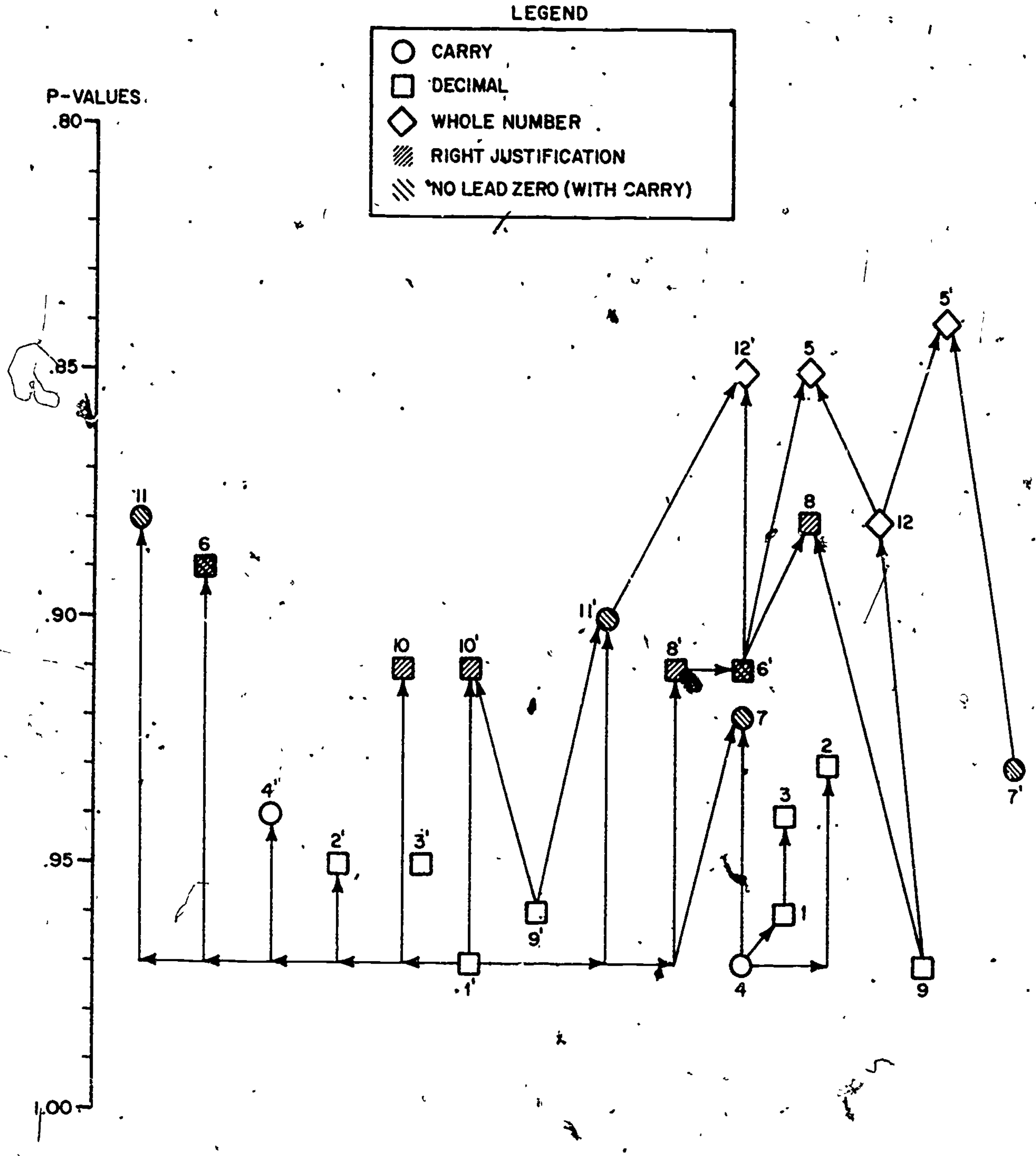

FIGURE 4: A Directed Graph of Addition Post-Test

difficulty associated with "set-up" (e.g, same number of digits to right of decimal point) are easier for students while problems involving multiple difficulties are harder. This is indicated by the presence of the former items at the beginning of chains and the location of the latter type at the end of the chains.

The directed graphs are constructed from chains (sequences) of items that demonstrate a dominance relationship. Specifically, the sequences for the addition pretest include:

$5 > 12 > 9'$
$5' > 9'$ (or 1)
$11 > 11' > 7' > 9'$
$\qquad > 4' > 9'$ (or $1'$)
$12' > 9'$
$6 > 8' > 9 > 9'$
$\quad > 2 > 9'$
$\quad > 3$
$6' > 8' > 10' > 9'$
$7 > 9'$ (or $3'$)
$7' > 9'$ (or $1'$)
$10 > 10' > 9'$
$\quad > 8 > 9 > 9'$
$4' > 9'$ (or $1'$)
$3 > 1'$
$9 > 9'$
$2 > 9'$

The relationships of particular interest are those involving items for which predictions of dominance were made based on the content domain relative skill table (page 13). Support for *a priori* predictions were found for several of the items:

$2 > 9'$	(but not $1,2 > 9$)
$4 > 11,11'$	(but not $4 > 7$)
$9' > 8,8'$	
$9 > 8$	
$10,10' > 8' > 6'$	(but not $10 > 5,12$)

The relationships appear to be stronger for the items in the second half of the test (items are parallel to those in the first half). This may indicate that students tried several different strategies initially and only applied a strategy consistently after several items had been answered.

The prediction of relative item difficulty was supported by the statistical analysis:

prediction		actual
1,2,3,4	(easiest)	1,2,3, ,9
10	(easy)	4
6,7,9,11	(difficult)	6,7,8,10
5,8,12	(most difficult)	5,11,12

The only item badly located by the prediction was item 9 which was easier than predicted. This may indicate that the category "embedded zero" may not be a common difficulty and, as a result, may not need equal weight with other categories in predicting item difficulty.

The posttest prediction and actual results are similar:

prediction		actual
1,2,3,4	(easiest)	1,2,3,4,9
10	(easy)	7
6,7,,9,11	(difficult)	6,8,10
5,8,12	(most difficult)	5,11,12

The directed graph for the second set of items on the subtraction pre test is shown in Figure 5. The sequences of items which were used to generate this graph include:

7′ > 3′ > 10′ > 5′
  > 6′ > 8′
9′ > 6′ > 8′
  > 5′
12′ > 5′
4′ > 11′
  > 1′

where the prime notation indicates the item is from the second half of the test and tells its item parallel.

Insert Figure 5 about here

The chains of particular interest are those involving items in the *a priori* predictions (see page 16). There is support for most of the predictions. Specifically,

5<12	(but not 5<2)
11<4	
6,8,10<7<3	(but not 7<3<9)
6,8<9	
1<4	(but not 1<9)
---	(and not 12<9)

Most of the predictions which were not supported involved item 9 (20.004 - 0.03) which was predicted to be relatively difficult, but was, in fact, only moderately difficult.

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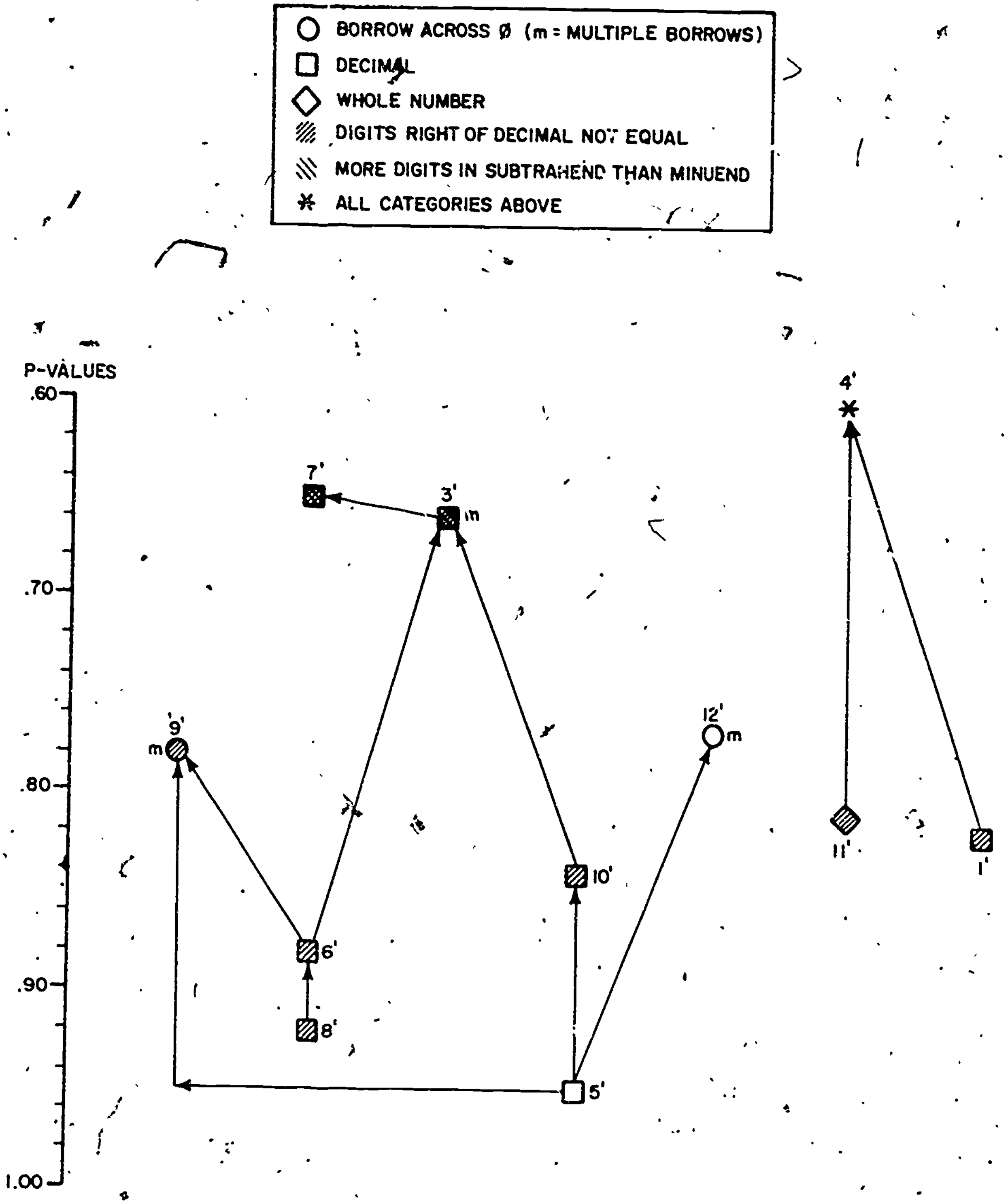

FIGURE 5 : A Directed Graph of Subtraction Pre-Test
(second set of items: 13 - 24)

prediction		actual
5	(easiest)	5
2,6,8,10	(easy)	2,6,8,10,11
1,7,11,12	(moderate)	1,9,12
3,9	(difficult)	3,7
4	(most difficult)	4

Posttest results reflect these same relative item difficulty ratings.

Summary

This report describes the application of item relational structure (IRS) analysis (a graph theoretic approach to Order analysis) to the construction of error diagnostic tests (Tatsuoka and Tatsuoka, 1981b) which can be used to describe specific misconceptions involved in addition and subtraction of decimal fractions. The IRS analysis was compared with predictions of item dominance and relative item difficulty which were based on a content-domain relative skills analysis. Projected student difficulties, identified with item characteristics pertinent to decimal fraction arithmetic such as "leading zeros in addends" or "implicit decimals in whole number subtrahends," suggested several item dominances. For example, students who

could answer	should be able to answer
.032 + .06	.035 + .062
.54 + .83	0.53 + 0.81
.6 + .8	0.53 + 0.81
.047 + 0.31	.035 + .062

The IRS analysis supports our item construction procedures in that it confirms the relative skills analysis of several important item charactersits including: digits to right of decimal point, decimal notation, carry across zero interacting with leading zeros (addition), number of digits to right of decimal point in subtrahend compared with number in minuend (subtraction). The IRS analysis did, however, indicate one characteristic that was not as important as predicted: zeros embedded in sum (difference) between decimal point and first non-zero digit to right of decimal point. There were also some questions raised about some item characteristic interactions and the construction of parallel items.

There are two very critical questions which relate directly to this research. First, there remains an important question of how to use specific description of errors in designing effective and efficient remedial instruction. Second, to be practical, we must be able to design our psychometric models so that they are easily and correctly used and interpreted by classroom instructors.

The domain of decimal addition and subtraction is an attractive one to begin the development of classroom based error-diagnostic utilities since it is a managable size for initial efforts. It also offers an opportunity to examine effective remediation directed at the conceptual level and remediation directed at the procedural level.

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Appendix I

Addition and Subtraction Test

DECIMAL ADDITION TEST (PRE and POST)

1) .24 + .61 = .85	1') .42 + .16 = .58
2) .36 + .45 = .81	2') .15 + .26 = .41
3) 14.3 + 2.4 = 16.7	3') 12.4 + 3.2 = 15.6
4) 0.53 + 0.81 = 1.34	4') 0.64 + 0.72 = 1.36
5) 4.32 + 3 = 7.32	5') 5.41 + 2 = 7.41
6) .047 + 0.31 = .357	6') .036 + 0.51 = .546
7) .54 + .83 = 1.37	7') .45 + .92 = 1.37
8) .032 + .06 = .092	8') .041 + .05 = .091
9) .035 + .062 = .097	9') .016 + .023 = .039
10) 3.24 + 6.5 = 9.74	10') 2.34 + 7.6 = 9.94
11) .6 + .8 = 1.4	11') .7 + .9 = 1.6
12) 6 + 4.13 = 10.13	12') 5 + 3.24 = 8.24

DECIMAL SUBTRACTION TEST (PRE and POST)

1)	14.25 - 3.4 = 10.85	1')	23.36 - 2.4 = 20.96
2)	1.4 - .8 = .6	2')	1.6 - .9 = .7
3)	6.4 - 3.235 = 3.165	3')	5.7 - 2.528 = 3.172
4)	6 - 3.8 = 2.2	4')	4 - 2.6 = 1.4
5)	.035 - .014 = .021	5')	.057 - .023 = .034
6)	.456 - 0.31 = .146	6')	.832 - 0.71 = .122
7)	6.35 - 3.248 = 3.102	7')	5.63 - 2.527 = 3.103
8)	.576 - .23 = .346	8')	.659 - .34 = .319
9)	20.004 - 0.03 = 19.974	9')	30.005 - 0.04 = 29.965
10)	.631 - .3100 = .321	10')	.742 - .4200 = .322
11)	8.7 - 5 = 3.7	11')	9.6 - 5 = 4.6
12)	4.002 - .081 = 3.921	12')	5.003 - .072 = 4.931

Appendix II

Projected Errors for Addition and

Subtraction of Decimals

Typical Errors in Addition of Decimals

1. Right Justification: The student justifies the numbers along the right hand side and does not align the decimal points. For example, 3.24 + 6.5 becomes:

```
 3.2 4
   6.5
 -----
 3 8 9   (without decimal point)
```

The decimal point may be placed corresponding to either addend or, in rare cases, in both positions. Occasionally the student will omit the decimal point. Students may also use the "multiplication rule" for placing the decimal point.

2. Left Justification: Similar to right justification with the difference in aligning the numbers along the left hand side. For example, 3.24 + 6.5 becomes:

```
 3.2 4
 .6 5
 -----
 9 7 4
```

Placement of the decimal point is described in 1 (above).

3. Inside-out justification: Students may use right justification for the whole number portion and right justification for the decimal portion. For example, 3.24 + 6.5 becomes:

```
 3.2 4
 6.  5
 -----
 9.2 9
```

The decimal point in the answer is usually correctly aligned (relative to the whole number).

4. Whole number error: If the whole number does not have an explicit decimal point, students might write the decimal point to the left of the whole number. For example, 3 + 6.58 becomes:

```
 6.5 8
  .3
 -----
 6.8 8
```

Occasionally the whole number is right justified with the decimal portion of the other addend (see #1). In this case, the decimal point of the answer usually corresponds to the explicit decimal point of the addend. For example, 3. + 6.58 becomes:

```
 5.5 8
     3
 -----
 6.6 1
```

5. The carry error: If the sum of the tenths digits involves a carry to the ones position, the answer is written as hundreds instead.

```
 . 6        . 4 5
 . 8        . 9 2
 ---        -----
 .14        .13 7
```

This error usually occurs only if there are no digits (zero or non-zero) written to the left of either decimal point.

```
 correct      correct      incorrect
  0 . 6        0 . 6          . 6
  0 . 8          . 8          . 8
  -----        -----          ---
  1 . 4        1 . 4          .14
```

6. The lost zero: zeros written between the decimal point and the first non-zero digit to the right of the decimal point are omitted.

```
 .035
 .027
 ----
 . 62
```

7. Multiplication rule: The student places the decimal point in the answer by following the rule for multiplication. That is, the student adds the number of decimal places in the addends. For example, .35 + .86 becomes:

```
   .35
   .86
 -----
 .0121
```

8. Multiple decimal points: Student will include more than one decimal point. This usually occurs when the student does not align the problem correctly and has more than one column with an explicit decimal point.

9. Missing decimal point: Student omits a necessary decimal point.

Typical Errors in Subtraction of Decimals

1. Right Justification: Similiar to right justification in addition.
2. Left Justification: Similar to left justification in addition.
3. Inside-out justification: Similiar to additon.
4. Whole number error: Similiar to addition.
5. The borrow error: Typical difficulty encountered in whole number subtraction. Another difficulty is encountered in decimal subtraction

in that some students occasionally have difficulty borrowing across the decimal point often neglecting to adjust the whole number portion of the decimal.

6. The lost zero: Similiar to addition.

7. Multiplication rule: Similiar to addition.

8. Longer number error: The student assumes that the longer number is the larger and, as a result, it is written as the minuend. This often results in serious borrowing problems unless the student also right justifies the numbers.

```
6.35 - 3.2                 6.35 - 3.248
    6.3 5                      3.2 4 8
    3.2                          6.3 5
    3.1 5                      2 6 1 3
                          (without decimal point)
```

9. Recopying some digits from subtrahend: If the minuend has fewer digits to the right of the decimal than the subtrahend, some students will merely recopy some of the subtrahend digits without subtracting.

```
  6.4
- 3.235
  3.235
```

This may be a carry over from addition in that the student may think that there is "nothing to do."

10. Multiple decimal points: Student will include more than one decimal point. This usually occurs when the student does not align the problem correctly and has more than one column with an explicit decimal point.

11. Missing decimal point: Student omits a necessary decimal point.

Appendix III

Procedural Network Charts

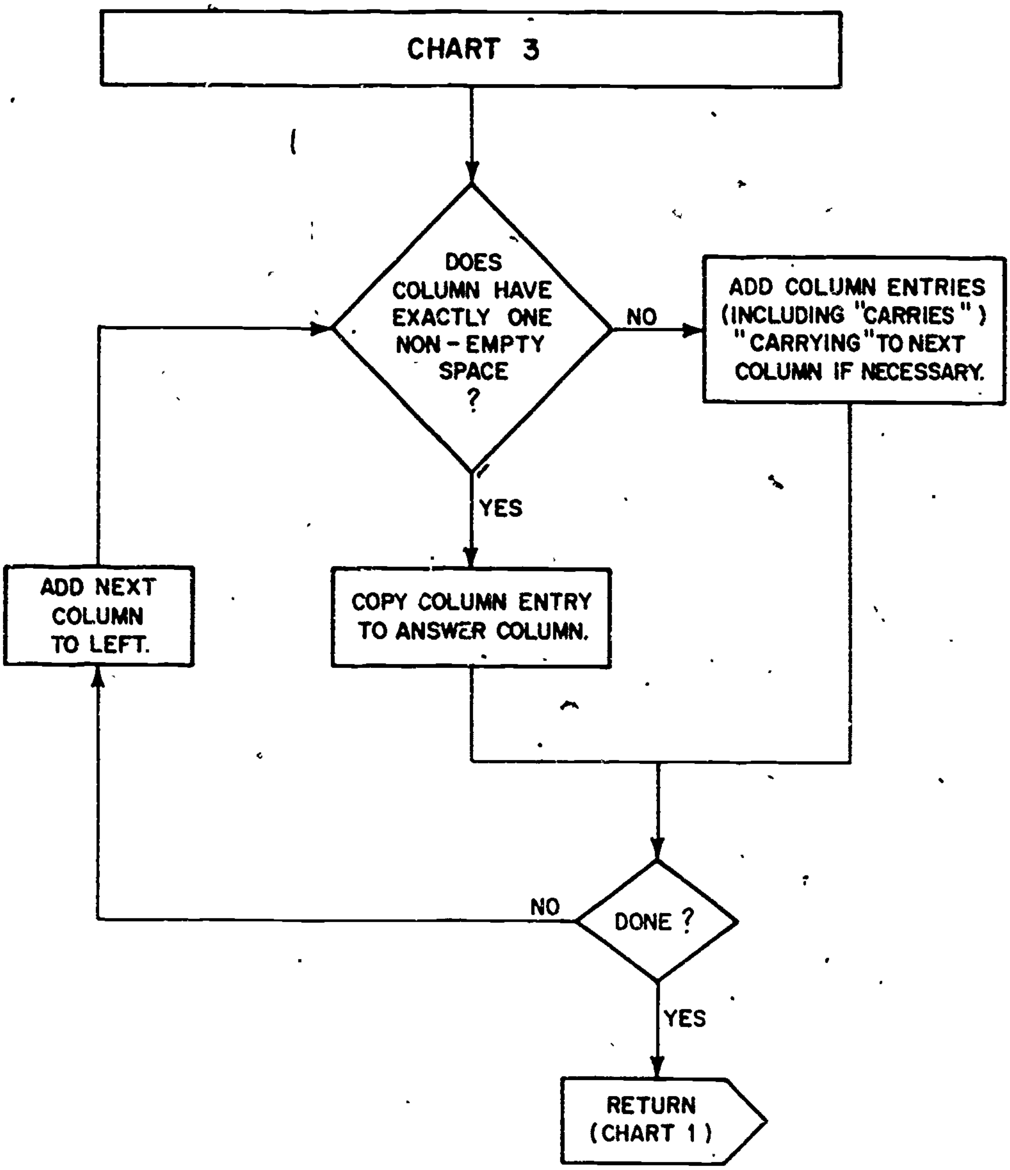

FIGURE 6 : A Flowchart to Add Columns of Numbers

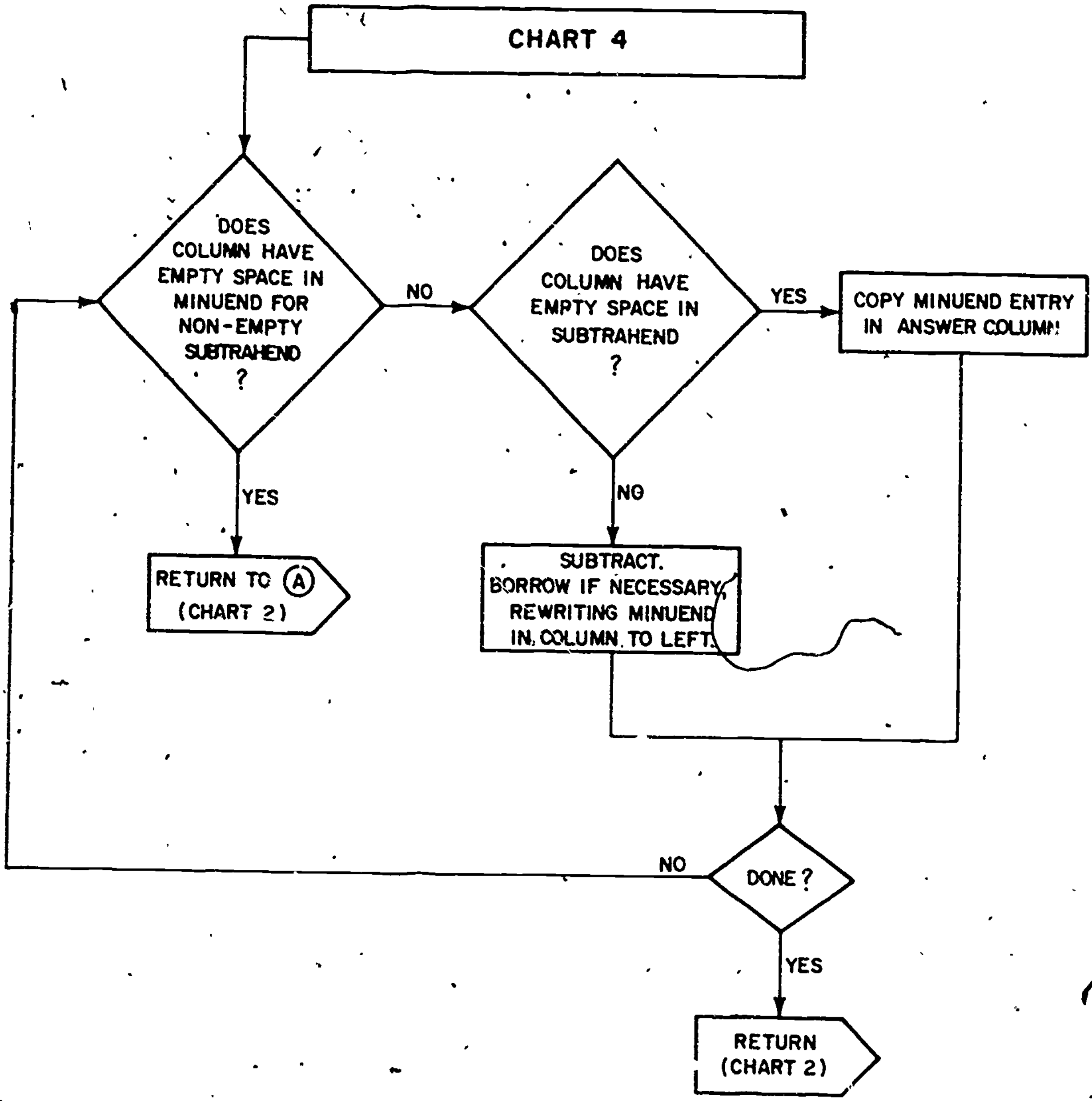

FIGURE 7: A Flowchart to Subtract Columns of Numbers