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ABSTRACT

This document focuses on a series of studies that viewed aspects of conditional reasoning. The investigations were concerned with the processes children use to acquire patterns of deductive inference and logical relationships pertaining to propositional reasoning. The first experiment, involving 96 children each in grades 2 and 5, investigated the ability of children to construct a formal representation of a Modus Tollens inference. A follow-up experiment and a pilot study dealing with acquisition of patterns of deductive inference are also discussed in the first of the document's three major sections. Section two focuses on effects of content on reasoning, with three experiments described. The last major section of the report looks at an experiment in geometry instruction that involved two phases, one devoted to teaching deductive reasoning processes and one concerning direct instruction in formal geometry. The document ends with concluding remarks reviewing implications of all the research noted. (MP)

Analysis of the development of propositional reasoning

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Final Report

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1. Introduction

To define the task domain for clarity, our focus is on logical reasoning as embodied in language comprehension and language use, on the processes and competencies involved in reasoning with linguistic material, and on the development of those processes. The more specific focus of the research reported here is on conditional reasoning. (A brief sidetrack: while the statement above specifies a focus of inquiry, it is not intended to suggest that verbal reasoning is entirely separate from reasoning about empirical phenomena; rather, it exercises caution in distinguishing the task domains for now. An attempt to characterize the linguistic and the Piagetian task domains respectively has been presented in Falmagne (1980), both in terms of a task analysis and in terms of the processes that may underlie each of these. A number of theoretical issues and empirical questions are related to this distinction, but clearly cannot be discussed here.

In general terms, the theoretical questions addressed by this project concern the process whereby children acquire patterns of deductive inference and logical relationships pertaining to propositional reasoning, and the resulting changes in their mental representation of the logical relationship embodied in linguistic material.

A preliminary clarification is necessary: clearly, even young children do reason correctly sometimes with simple rules of deductive inference, when the content of the problem is familiar to them or "concrete". This has been documented by Hill (1961), Ennis (1971), Kodroff and Roberge (1975), Kuhn (1977), Peel (1967), Roberge (1970, 1972), Roberge and Paulus (1971), and Taplin, Staudemayer and Taddonio (1970), among others, regarding some patterns of conditional reasoning and of quantifier inferences. So, the questions addressed here concern more specifically (a) the distinction between the various modes in which a deduction can be carried out, the functional characteristics of each medium, and the factors that govern the use of one or the other medium; (b) the process whereby a formal mode of representation becomes available for a given pattern of deductive inference, that is, the process whereby children become able to rely on formal rather than referential aspects of the statements when carrying out a deduction of a given logical form.

While the findings mentioned above testify to some (limited) logical competence in children, they do not elucidate the specific process underlying the subject's responses, and it is therefore not known whether children's reasoning was of a formal kind in the studies reported; and if not, it is not known whether it could have been of a formal kind. Thus the questions addressed in our work aim at a deeper analysis of the children's competence and of its implementation in the reasoning process than has been provided heretofore.

A somewhat related question has been addressed by Osherson and Markman (1974, 1975) who found that, when asked to evaluate statements about non-visible objects, children in grades 1-6 failed to recognize that tautological statements were true and contradictory statements false by virtue of their logical form and regardless of what the actual object was, thus treating these statements as empirical statements. These results are informative and relevant to our concerns in revealing that children have difficulty relying on logical form in that situation. However, the task used by Osherson and Markman implicates the child's meta-cognitive knowledge and "task orientation"

to a major extent[1] in addition to logical competence per se, and while their results are informative and relevant to our concerns, they do not preclude the possibility that the logical form of statements does guide the ordinary reasoning process without the child being aware of it, (in the same way as the syntactic form of sentences guides the comprehension process). Indirect support for the notion that children's reasoning sometimes relies on logical form has been provided by Osherson (1974, 1975) in a series of experiments with children and adolescents, aimed at testing a process model of deduction. "Indirect" because Osherson postulated a particular set of operations and because the results speak to the joint involvement of these operations rather than providing evidence about single operations. But the aspect of his results which is relevant here is the fact that a system of formal rules was able to predict a number of regularities in children's responses and furthermore that some of these predictions held across contents, thus indicating that perhaps formal patterns of inference were at play (Falmagne, 1977) instead of, or in addition to, content-specific modes of reasoning.

Returning to the issues mentioned previously, concerning the processes involved in deductive reasoning, these issues are addressed in the context of a theoretical framework discussed in Falmagne (1980) (enclosed in appendix).

It is assumed that, when a problem is encountered and a deduction has to be carried out, a representation of the problem has to be set up in working memory. Thus, a distinction is stressed here between the representation of knowledge in long term memory (including the repertoire of logical knowledge currently available to an individual), and the representation of a given problem in working memory in the course of reasoning, or functional representation.

The functional representation can be seen as a mapping between the input problem and the representational repertoire and structures available in LTM, which is then held in working memory for the purpose of reasoning. When this representation is formal, the mapping is between the verbal problem and the logical relationships available in LTM; this mapping relies both on surface cues in the sentences and on cues in the structural representation of the sentence. In that case, the logical form of the problem is abstracted -- or rather, the form it has in people's natural logic -- and represented as such for the purposes of reasoning. However, the functional representation can be couched in media other than formal as well. It may be of an imaginal kind, thus involving a more general knowledge of the world, or it may utilize a schematize medium, in which the information is concretized into prototypical exemplars of the situation described (see Johnson-Laird and Steedman, 1978, who use the term "analogical" for a representation of this kind in the case of quantified sentences). Thus, the representation in working memory has structural properties given in part by the content of LTM, and a modality determined in part by the content of the problem. The important point here is that a person may have a given pattern

1. Indeed, this was one main focus of the study, i.e., whether children know that some statements are analytic and more generally, that language is an object distinct from the world it represents.

of inference in his/her conceptual repertoire, yet use a different medium for representing a particular problem.

The mode used for representing a given problem is assumed to depend jointly upon the three following factors at least:

(i) the nature of the material, e.g., an imaginal representation is certainly more available for some sentences than for others, irrespective of their being part of a reasoning problem or not;

(ii) the form of the problem: if the statements are of a kind that the subject is able to recognize at some formal level, then he/she may encode these in a formal mode as outlined above. (So, a pattern recognition process is involved here, whereby the logical form of the problem is perhaps identified.) If not, some other modality is used and the problem is operated upon in that modality;

(iii) the salience of cues in the sentence (or the problem) that cue its logical form.

In terms of this framework, logical development has at least two facets (and a third one that space will not allow us to discuss here). It involves an enrichment of the logico-semantic network and in particular, of the patterns of deductive inference available. And it involves an increasing availability of the formal mode for representing natural language expressions in working memory. This is related to the enrichment of the logico-semantic network and to the increased range and generality of the cues that are used to identify the logical structure underlying natural language expressions. Thus, one aspect of logical development would consist in changes in the overall bias and in the distribution of availability of alternative modes of representation: as the formal repertoire of the child increases, as the range of surface cues pointing to each given logical relation expands, and as the cues increase in salience, a formal mode of representation and processing would become available for a wider range of inferential situations.

The studies we conducted in this project are motivated by two sets of questions that are closely interrelated within the framework above:

(1) What are the processes whereby this enrichment of the logical network occurs?

(2) How does the content of the problems being reasoned about affect the reasoning process at various levels of mastery of a given rule of inference and at various developmental levels?

The two sets of studies will be reported in that order, in Sections 2 and 3 respectively. In addition, an extension of the project and, specifically, of the notions above, to the teaching of geometry at the secondary level has been conducted on a pilot basis, and will be reported in Section 4.

2. Acquisition of patterns of deductive inference

2.1 Rationale and previous results

This first set of studies examines the assumption that the elements of

propositional reasoning can be acquired, at least in part, through a concept learning process: by being exposed in everyday life to various examples of a valid rule of inference, the child would abstract the structural concept of that rule. Thus it is assumed that children are capable of abstracting those regularities from their linguistic environment.

Two previous studies had examined this assumption with respect to a specific pattern of inference, Modus Tollens ("if p then q; not q; therefore not p", where p and q can be affirmative or negative statements, with supportive results. Children were given a number of different "word problems" in written form and were given feedback after their responses; the children underwent two sessions of that sort, which included 32 problems each, half of which were Modus Tollens in the experimental group. Whether they had abstracted the concept of a Modus Tollens inference, and the level of generality of that concept, was then assessed in a transfer session administered a few days later. The transfer session included several types of problems of the same logical form as the problems encountered during training, but which differed from the training problems in surface structure. The feature of the transfer task was to permit assessing whether the concept acquired was the relevant one, and to distinguish acquisition of a specific conept tied to the surface structure of the problem from acquisition of a more general concept based on the logical form of the argument. Thus in some transfer problems the conditional statement in the first premise was expressed with the consequent clause preceding the antecedent clause (e.g., "Mary goes to school if it is Tuesday"), in contrast to the phrasing of the training problems (e.g., "If it is Tuesday, then Mary goes to school") in order to differentiate acquisition of the relevant rule from an improvement due to strategies based on clause order. In other transfer problems, the connective "when-then" (logically equivalent to "if-then") was used, in order to distinguish concepts based on logical form from concepts restricted to the specific connective used during training. Similarly, in other problems the second premise (the "not q" clause) lexically negated the "q" clause in the first premise, rather than negating it expolicitly (e.g., "The book is red" is denied lexically by "The book is blue" and explicitly by "The book is not red"). FInally, problems similar in form to the training problems but containing nonsense content words were also included (e.g., "If Paul fibbles than he thabbles") in order to assess the level of abstraction of the concept acquired. All the problems used in the study were different from each other in content.

In these two previous studies, which included third, fourth and fifth graders, training had a significant effect on all variations of the Modus Tollens inference included in the transfer task, in particular, on the problems containing nonsense content. Thus, exposure to instances of a standard Modus Tollens inference led to learning of that structural concept and to generalization to logically equivalent but linguistically dissimilar problems. Of particular interest is the fact that training generalized to "nonsense" problems as well, thus indicating that the concept had been learned at a fairly abstract level. The two new studies completed during the first year of this project and described in the next two sections, extended those results and addressed more specific questions.

Aside from the studies just summarized, the question concerning children's ability to abstract patterns of deductive inference from linguistic input, has been almost unexplored so far, with the exception of a study by Hadar (Hadar, 1977; Hadar and Henkin, 1978; Hadar, 1978). The aims of the Hadar study were similar

to the aims of the studies just reported, and positive results were also obtained. However, the materials used apparently were biased to be congruent with factual states of affairs (e.g., "If this card is a heart, then it is red"), in contradistinction to our materials. Furthermore, one major difference is that the materials and procedures used in training in the Hadar experiment, were closer to providing direct instruction than did our procedure (since Hadar used pictorial aids and group discussions), thereby leaving open the questions of whether children can carry out those abstractions spontaneously, without direct instruction.

2.2. <u>Experiment CL4: "when-then" vs. "if-then" in second and fifth grades</u>

The aim of the first experiment conducted in this project was to assess whether the previous results would extend to a younger age group, and to compare the effectiveness of a training condition using the "if-then" connective (as was done in the previous studies), with that of a training condition in which conditional statements would be expressed in an laternative, more "concrete" fashion ("when...then"). It was hypothesized that experience with this more concrete connective may be more conducive to the abstraction of the deductive pattern and furthermore, that it may mediate more effectively the transfer to the other lexical formulations of that pattern and, in particular the transfer to "if-then" formulations. Such a result would be interesting because it seems reasonable to suppose that the meaning of conditional connectives may be built upon the meaning of more "concrete" connectives and gradually differentiated from it.

The procedure used in this experiment was similar to that of the previous studies, with minor modifications: children underwent four training sessions consisting of 16 "word problems" each, which were administered on four consecutive days whenever possible. As previously, problems were administered in written form and assembled in booklets, one problem per page, with the three response choices (<u>yes</u>, <u>no</u>, <u>can't tell</u>) typed below the problem, and with the correct answer being typed on the back of the page. In the training sessions half of the problems were of Modus Tollens form, for which the first premise was an If-Then sentence in one experimental group (IF group), and a When-Then sentence in the other group (WHEN group). The other problems included in the training sessions were miscellaneous kinds of inferences, as were all the problems given to a third group (Control group).

The Modus Tollens problems included problems in which both the antecedent and consequent of the conditional statement were in affirmative form, or AA problems (e.g. "If Mary goes to school, then she is walking down Hyde Street"; Mary is not walking down Hyde Street; Is she going to school?"); problems in which the antecedent was affirmative and the consequent negative, or AN problems (e.g. "If Jim is studying, then he is not sitting in the blue chair; Jim is sitting in the blue chair; Is he studying?"); and NA and NN problems.

96 children at each grade level (Second and Fifth) served as subjects, 32 in each group. An attempt was made to constitute groups that were comparable in terms of reading ability and math ability as expressed by the teachers' judgment of each child.

The transfer session was identical for all groups and included six types of Modus Tollens problems, to be described shortly, and it was designed to assess whether children had acquired a Modus Tollens concept based on logical form or, instead, a pattern tied to the surface structure of the training

problems. The transfer session, also, included indeterminate inferences. The six Modus Tollens types included in the transfer task consisted of problems in which: (a) the first premise was an if-then statement (If, Standard problems); (b) the first premise was an if-then statement in which the consequent clause preceded the "if" clause (e.g. "Mary is going down Hyde Street if she is going to school") (If, Reverse); (c) When, Standard problems; (d) When, Reverse problems; (e) problems in which nonsense words were substituted for content words, in if-then statements (Nonsense If); (f) Nonsense When problems. The Nonsense problems were only presented in Standard form (not Reversed).

Each Modus Tollens type was presented once in AA form, thus requiring a No answer, and once iether in NA or in NN form, thus requiring a Yes answer. The additional indeterminate inferences obviously required a Can't tell response. Each child received a total of 16 problems, and no feedback was provided.

The results of interest are those from the transfer task in the three groups and can be summarized as follows:

(1) In grade 5, training had a significant effect for both the IF and the WHEN group ($p < .01$).

(2) Interestingly, the resulting improvement was largest for the nonsense problems whose error rate in the experimental group became equivalent to the error rate on meaningful problems, whereas it was close to chance for the control group. The percentages of correct responses for each problem type and each group are shown in Figure 1.

Figure 1 about here

Since children had not been presented nonsense problems during training, this result strongly supports the notion that children had abstracted the structure of a Modus Tollens inference at an abstract level and were able to recognize it in problems ostensibly devoid of referential meaning. This result also indicates that more than improvement due to training occurred: though Modus Tollens was certainly not an entirely novel acquisition since children do use that pattern correctly in certain cases, the results concerning nonsense problems indicate that training resulted in at least a new level of abstraction in children's representation of that pattern. In terms of the framework outlined above concerning changes in mental representation in the course of logical development, the interpretation would be that training perhaps led children to construct a formal representation of a Modus Tollens inference.

(3) Another result congruent with this interpretation is that training primarily affected those problems involving a negation in either the p or the q clause (See Figure 2 for the percent correct responses in the three groups for problems in which both the p and q clause are affirmative, and problems in which one of these involves a negation).

Figure 2 about here

Since problems involving a negation are typically answered at a chance level in the control group and in the literature, this again suggests that training

resulted in a Modus Tollens concept more abstract than the procedure used previously. This result provides weaker support for this notion than the results on nonsense problems (since training did inlcude problems with negations) but is congruent with it.

(4) As can be verified from Figures 1 and 2, training involving the When-Then connective and training involving the If-Then connective were equally effective for each of the problem types and for either "affirmative" or "negative" problems.

(5) One more question relevant to the comparison of IF and WHEN training, is the extent to which training is connective-specific, that is, whether performance in the transfer task mostly improves for the specific connective used during training, or whether improvement in the IF group is equally distributed over if-then and when-then transfer problems. As can be seen from Figure 3, the latter is clearly the case (none of the differences in Figure 3 are significant except for those between the control values and the corresponding values in the other groups).

Figure 3 about here

Thus, training involving one given connective generalized to logically equivalent problems formulated with the other connective. These results indicate again that the learning process occurred at the level of the logical form of the problems rather than being tied to their surface form. (Incidentally, the pattern seen in Figure 3 also holds for each of the problem types separately, and in particular for nonsense problems which offer the most straightforward assessment of training effects.)

To summarize so far, the fifth grade results indicate that training resulted in improvement based on the lgoical form of a Modus Tollens inference rather than on surface features (since training uniformaly generalized to problems in which the antecedent and consequent clause were permuted, to problems with nonsense content, and to problems involving a different connective). Furthermore, training appeared to result in a new level of abstraction regarding the Modus Tollens inference, since transfer effects were largest for nonsense problems (which had never been presented during training) and for problems involving negation.

In contrast, no effect of training was obtained in the second grade group. Several reasons may account for this. The most interesting one from a theoretical point of view, is the notion that children of that age are in a transitional phase in the process of acquiring a new, written medium for conveying meaning, and that the novel medium may not be utilized in the same way as the oral medium during this transitional phase; the mental representation and mode of processing may rely on surface features of written language to a greater extent than it does in the oral medium, in which meaning is the primary mode of organization. If so, our training procedure based on logical form could not be effective under written presentation until children acquire a deeper level of processing of that particular medium. This tentative account is consonant with more general notions concerning the acquisition of literacy and its congitive consequences.

Based on these notions a follow-up study was designed with the aim of examining concept learning of Modus Tollens with oral rather than written presentation. Thus, the procedure was modified so that children were tested individually, and iether the experimenter or the child read the problem aloud, depending on the child's preference. (This flexible procedure was adopted in an attempt to minimize the memory requirements of the task in this exploratory test of the viability of the assumption above. Presumably, the procedure remained in keeping with the rationale of the study.)

2.3. Follow-up experiment CL5: Training in second grade (When-then vs. If-then with oral presentation

The same general materials and design was used in this study as those of the main study, but subjects were tested individually as described below.

Each child met individually with the experimenter for three sessions spaced evenly over the ocurse of a week. Each session lasted from 15 to 30 minutes. An attempt was made to control for reading ability in the assignment of subjects to groups by the experimenter's informal rating of subjects' oral reading level and subjects' report of marks received the previous school year.

In the first session, the general nature of thestudy was described (see CL4 instructions) and the child was given several linear syllogisms as practice in order to familiarize him/her with the possible categories of answer (yes, no, can't tell). If the child seemed to understand these quickly, only three practice problems were given; otherwise, six problems were administered. The rest of Session 1 consisted of the administration of the first two "training" booklets. During Session 2 the next 2 "training" booklets were administered, and during Session 3 the transfer booklet was administered. Mode of presentation varied, depending on the child's preference: most children preferred reading silently while the experimenter read aloud, but some preferred to read aloud themselves and one child chose to read silently to himself.

As in the previous studies, the two training sessions included 16 problems each, half of which were Modus Tollens in the experimental group; children were asked to answer each problem and were given feedback (but no direct instruction) as to the correctness of each response. The training problems were phrased using the "if-then" connective in one experimental group and the "when-then" connective (logically equivalent to "if-then")in the other group. As in the previous experiment, a control group only received problems unrelated to the Modus Tollens inference during training. The third session consisted of a transfer task in which no feedback was given, and the materials included a variety of problems logically equivalent to the standard Modus Tollens problems but varying from those in surface form (using a different connective, i.e. "when-then" problems vs. "if-then" problems; permuting the antecedent and consequent clauses, or "Reverse" problems; using nonsense content words, or "Nonsense" problems; and "Reverse Nonsense" problems).

The transfer booklets used in this follow-up differed in only one respect from those used in the primary study: in addition to the 16 original transfer problems, the follow-up included one Reverse Nonsense IF and one Reverse Nonsense WHEN problem. Thus, each transfer booklet contained 18 problems.

At the end of the transfer session each child was asked:

(a) How they figured out the answers;

(b) Whether or not they used "images" in solving the problems;

(c) How they handled the nonsense problems;

(d) To attempt to solve abstract problems (e.g., If A, then B.
Not B.
(A?);

(e) To make up a problem like the ones they had been administered.

The results of main interest are as follows:

(1) There is a clear improvement in the group who received Modus Tollens training with the If-then connective (IF Group, 78% correct in transfer task vs. 57% in control group, $p < .05$). Furthermore, interestingly, this improvement was as marked for nonsense problems (72% correct vs. 45% in control group) as for theother transfer problems, even those of the same surface form as the problems encountered in training. Thus, here again, there is tentative evidence that the children abstracted the concept at a fairly abstract level. The relevant results may be seen in Figure 4.

Figure 4 about here

This result provides tentative support for the assumption that the absence of training effects in grade 2 for the previous experiment may be due, at least in part, to the specifics of the reading process at that transitional level. In this respect it is interesting to note that the grade 2 performance after IF training in the prsent experiment is comparable to the results of grade 5 in the previous study (Figure 1).

(2) In contrast, no training effects were obtained in the WHEN group, as can be seen in Figure 4. This result will be discussed shortly.

(3) Coming back to the results of the IF group, the improvement obtained both for those transfer problems using the same (if-then) connective as was used in training, and the transfer problems using the when-then connective (See Table 1 for results not including the Reverse Nonsense problems, and Figure 5 for the same results including those problems). This result is similar to the grade 5 results in the main study, and indicates that learning was not connective-specific.

Table 1, about here

Figure 5, about here

Taken together with the result concerning nonsense problems, this result suggests again that a pattern based on logical form was learned.

This result is particularly interesting given the fact that children in this age group typically do not master Modus Tollens inferences even with familiar content (e.g. in this experiment the proportion correct for meaningful problems in the control group was 63%). Thus this result is presumably closer to illustrating a genuine acquisition effect than the results from fifth graders discussed above for the main study.

(4) Turning now to the comparison of learning effects for affirmative and negative problems, Table 2 reveals that the largest improvement in the IF group occurs for affirmative problems (83% vs. 58% correct), although there is marked improvement on negative problems as well (73% vs. 55% correct). The Affirmative-Negative and the training main effects are significant $p < .05$; the interaction is not. In comparison with the corresponding results for grade 5 in Experiment CL4 (Figure 2), it is interesting to note that the present subjects started from a much lower percent correct on affirmative problems than did the fifth graders and that this difference disappears after training.

Table 2 about here

(5) Finally, it is of interest to examine individual learning data, with the aim of identifying individual patterns of acquisition as opposed to group improvement. This analysis is presented in Table 3, which displays the percent correct for each subject on the first training block of Modus Tollens problems, on the last training block, and on the transfer task. These data indicate that eight among the ten subjects in the IF group benefitted from training with performance in the transfer task exceeding 75% correct (whereas there is a much higher degree of variation in the WHEN group).

Table 3 about here

Furthermore, strikingly, the initial performance for all subjects except two, and for the group as a whole, was at the chance level, which suggests again that these data reflect the acquisition of a Modus Tollens pattern of inference as opposed to mere improvement under practice.

(6) A result mentioned previously, in Point 2, requires discussion, namely the fact that, contrary to what was hypothesized and in contrast with the results from the "IF" group just discussed, the group trained with "WHEN" does not exhibit any training effect in the transfer task, even on problems including the "when-then" connective (62% correct vs. 60% for controls). This result is superficially puzzling and potentially of great theoretical interest. The initial hypothesis was that "when-then", being more concrete and more familiar to children of that age, might mediate learning and transfer more effectively. The results indicate exactly the opposite. The possible, highly interesting (though speculative) account of this result is that "when-then" may be an intrinsically content-bound connective, whose semantic representation does

not contain those formal features, cutting across contents, that are required to sustain a general concept. Interesting evidence in support of this hypothesis can be found in Table 3 and in Figure 6. The WHEN group displays the same initial level of performance as the IF group in the first training block (thus indicating that both groups were equivalent initially), and the same level of performance as the IF group in the last training block (thus indicating that specific training was equally effective); but the WHEN group fails to transfer to problems differing in surface form (though logically equivalent), as indicated by its performance on the transfer session.

Figure 6 about here

Although the difference between the two groups in this respect (difference between the last training block and the transfer task) is not statistically significant, the fact that the trend depicted in Figure 6 is replicated in most of the individual data adds to its validity. This pattern of results clearly supports the hypothesis that the representation of the when-then connective may be intrinsically content-bound and therefore ineffective in mediating a general concept.

Another possible account for the absence of training effects in the WHEN group is that, in children's language, "when-then" statements presuppose the truth of the proposition expressed in the first clause (p) so that the Modus Tollens inference, which results in negating that clause, is not natural for that connective. This is supported by a number of spontaneous reactions from the younger subjects, who when the (not p) conclusion was given, exclaimed "but you just said that (p)!". However, on that account, one would predict the Modus Tollens inferences using "when-then" to be more difficult than those using "if-then", generally; judging from the control group's data, this is not the case (see Table 1).

(7) In addition to these responses, at the end of the transfer session, children were asked to make up problems "like the ones they had had before", in the hope of assessing the child's logical construal of the training problems. The problems made up by each subject are presented in Appendix 1, in relation to their performance (number correct) on the transfer task. One interesting aspect of those data is that, while 3 of the 8 subjects in the WHEN group who were able to invent problems did so using If, none of the 9 subjects in the IF group used WHEN. Exposure was the same in both cases (i.e., same number of "If" and "When" on the transfer task). So, If does seem either more salient, or more transferable across contents, which is consonant with speculations discussed previously.

2.4. Discussion

To summarize, the two experiments reported here, as well as those completed previously, have provided support for the assumption that children are cognitively equipped for abstracting patterns of deductive inference through a concept learning process of the kind described above.

Several points need to be stressed here, to clarify the status of this statement. First, although the results of these studies indicate that a logical

rule of inference can be abstracted through a concept learning process, they do not indicate that this is how children acquire such rules in natural circumstances. Rather, they demonstrate that children have the requisite ability for this to be possible, that is, the ability to abstract a structural concept of this kind from linguistic input, without direct instruction.

Second, in some of the cases discussed above and most probably at the fifth grade level, children presumably did have a limited "concept" of a Modus Tollens inference initially, limited perhaps in the sense of being content-bound, perhaps in the sense of applying to affirmative sentences only and perhaps in other ways. (The term "concept" is used between quotes above because distinguishing a logical concept from a relatively general but content-bound procedure, is an interesting and complex issue, both conceptually and empirically). Thus, in these cases, the appropriate interpretation of our results is that training led to a new level of abstraction regarding that pattern of inference (or, more specifically, in terms of the framework proposed earlier, that a formal representation became available through exposure to a range of exemplars of the pattern), rather than leading to an entirely novel acquisition of a previously non-existent inference. However, this is most likely not the case in the study with second graders reported in the previous section. The initial performance of the control group, as well as other results in the literature, seem to indicate that children in this age range do not master Modus Tollens inferences, so that this experiment comes closer to demonstrating genuine acquisition than did the previous studies.

Next, if the feasibility of an acquisition process of the sort hypothesized here, is demonstrated by this line of studies, one will require data indicating that the natural inguistic environment does indeed provide the input and feedback that render such an acquisition mechanism possible. That is, naturalistic observations and analyses of naturally occurring verbal and nonverbal interactions of the relevant kind, are needed before the results of the present studies (which presumably indicate that the child is cognitively equipped for this acquisition mechanism) can be extrapolated into a tentative statement about logical development in "real life".

A final comment on the status of the notions discussed here concerns their intended scope with respect to a comprehensive account of logical development. It is not suggested here that logical development can be accounted for entirely by a concept learning process of the sort just outlined, nor is it implied that the overall theoretical account of this development must be of an empiricist kind. On the latter point, we should leave open the possibility that natural logic is constrained by fundamental properties of the mind, and on the former we should acknowledge that (a) what can be acquired at a given age (or stage) is presumably subject to structural constraints imposed by the level of cognitive functioning of the child, and (b) in addition to "direct" learning and abstraction, autonomous rational constructions and regulations presumably occur (for example, it is unlikely that a child who has acquired three of the four basic patterns for conditional inference, is dependent upon environmental input for acquisition of the fourth pattern). In fact, analysis of feedback mechanisms between concurrent acquisitions would be extremely interesting.

2.5. Pilot study on CI 5: Concept learning of indeterminate inferences

The next step, in this line of study, is to investigate whether novel patterns can be acquired through the same process as discussed above. Two obvious candidates are the indeterminate inferences associated with the conditional, which are known from previous research to be essentially absent from children's deductions. Thus the next step contemplated in this set will examine whether indeterminate patterns of inferences can be acquired through a concept learning process of the sort described here. In addition to its import for the "acquisiiton" question discussed here, it is of intrinsic interest to analyze the sources of difficulty of indeterminacy. The difficulty of indeterminate inferences even for adults is well known, but it is an open question whether their difficulty stems from fundamental cognitive factors, or from the scarcity of their instances in everyday situations. Should positive results be obtained for these patterns in a concept learning paradigm such as the one used here, this would lend support to the latter hypothesis.

Thus, the aims of this experiment are (1) to assess whether the unfamiliar indeterminate inference patterns can be learned, as was the Modus Tollens inference, via exposure to examples together with feedback as to the correct conclusion; and (2) to gain a better understanding of the sources of the apparent difficulty reasoners have with the indeterminate conditional inferences.

The present pilot research follows the same general design used in Experiment 1. In training sessions, children answered problems presented in written form, and they were given immediate feedback as to the correct answer for each problem. The task was administered to children individually in order to collect interview data regarding, for example, changes in the children's approaches to the problems and their use of feedback as to the correct answers. Since the indeterminate inferences are known to be difficult for adolescents, as well as for children, the pilot work included children ranging from grade 3 to grade 7. In this pilot study, approximately five children participated in each training condition.

Children were trained with <u>either</u> the AC inference <u>or</u> the DA inference. In order to balance the "can't tell" answers for the indeterminate problems, an equal number of "yes" and "no" items were included in the training task; among these "filler" problems were sevearl MP and MT problems, as well as nonconditional propositional reasoning problems (linear syllogisms, spatial inference problems, and classical syllogisms). The same set of contents appeared in the AC training problems and the DA training problems; and the set of filler items was identical in each case. Approximately 22 problems were presented in each training session. Each subject received from one to three training sessions (administered on different days); and each child received the transfer task either immediately following the last training trial, or after an interval of one day following the last training trial. As was the case in Experiment 1, no feedback was provided in the transfer task, and the surface form of the transfer task problems was varied (i.e., the transfer task included items with negaties in the first premise and items constructed of nonsense material, as well as standard conditional problems). Of particular interest, in this experiment, the transfer task included all four inference forms; thus, it is possible to examine how training on one indeterminate inference may affect performance with the other indeterminate inference

for which no training was given.

Very generally, the results of Experiment 2 are as follows:

(1) Some children clearly improved in their performance with the indeterminate inference on which they were trained (as was assessed during training and with the transfer task).

(2) Typically, after receiving feedback, children initially overgeneralized the "can't tell" response, and, in particular seemed to have difficulty discriminating between problems both of which include a negative (i.e., between MT and DA) as well as between problems which did not include a negative (i.e., between MP and AC). Thus, in the training tasks, children's performance on determinate problems frequently deteriorated, if only temporarily, as children began to use the "can't tell" response more frequently.

(3) In the course of the training trials, most subjects gradually differentiated between the determinate and indeterminate conditional inferences and, in some cases, were able to verbalize a general interpretation of the conditional meaning. Most of the subjects' performance on the transfer task was suprior to their initial performance in the training sessions.

(4) Of the children who learned one indeterminate inference, performance with the other indeterminate inference on the transfer task was highly variable between subjects, and was somewhat variable within-subject (i.e., across the examples of that inference which appeared in the transfer task). There is some evidence that effective transfer to the inference form that was <u>not</u> trained may be correlated with the speed of learning the original indeterminate inference in training.

(5) In addition to the variables just discussed, another training variable was also examined. In describing the results from the previous experiments, the importance of the logical connective ("if" versus "when") has been emphasized. The present pilot research attempts to explore the possible effects of more radically varying the language expressing the conditional relationship. While necessity and indeterminacy are difficult notions in the context of the explicit propositional reasoning discussed so far, they appear to have a more natural expression at the lexical level via terms such as "must", "can", "always", and "has to". It was hypothesized that training with indeterminate inferences couched in these terms would be effectie (and, especially, would be <u>more</u> effective) in mediating later recognition of indeterminacy in conditional inferences phrased in the standard manner. For this pilot work in lexical reasoning with conditionals, the subjects, the design of the study, and the procedure are identical to the experiment just described. Children received training with indeterminate conditional syllogisms (either AC or DA) which included modal terms in conjunction with the standard "if-then" connective. A typical example is as follows:

> If Jack is at home, then Jill <u>has to be</u> at home.
> Jack is not at home.
> Is Jill at home?

Other children received training with problems in which the conditional premise was expressed by means of modal terms together with relational terms other than the "if-then" connective. An example of a typical problem is as follows:

Jill has to be at home before Jack can be at home.
Jack is not at home.
Is Jill at home?

At the present time, the pilot data are too limited to permit assessing the relative effectiveness of lexical training versus standard training with the indeterminate problem types. In all six training conditions (Standard Conditional AC, Standard Conditional, DA; Conditional Modal AC, Conditional Modal DA; Modal AC, Modal DA), performance with the indeterminate inference form was improved, and, to at least a limited extent, training on one indeterminate inference form transferred to the other indeterminate inference form.

3. Effects of content on reasoning

3.1 General description of aims

Within the framework outlined earlier, the second line of research was aimed at providing insight into the various modalities in which logical problems can be represented and into the interplay between those representational moes and the way in which the deductive process operates; developmental trends in that regard were also examined. The specific focus was upon the role of imagery in deductive reasoning.

A dual motivation underlies this line of research. (i) Although the role of imagery in cognition has elicited much interest (and debate) recently, it has been primarily investigated in relatively simple memory or language comprehension tasks. One aim of the research to be reported, was to examine the interplay of imaginal and linguistic processes in a deductive reasoning task in which information from more than one sentence must be integrated. Especially, the analysis focused upon the way in which images are related to the logical form of the deductive inference at hand and upon developmental trends in that respect. (ii) Conversely, research on deductive processes in children has mostly restricted itself to processes of a linguistic kind, despite the considerable interest that several theoretical approaches to cognitive development have with respect to imaginal processes in thinking.

This part of the work is, to a large extent, exploratory in that, aside from some general predictions that were put to an initial test, the hope is that the data may suggest some specific aspects of the interplay of imaginal and linguistic processes in deductive reasoning, and some questions to be followed up. So, the story below is tentative and in flux. It consists of three experiments and of some intensive pilot work that was initiated after completion of the first two experiments with the aim of probing into some questions they brought up. One of the studies, conducted during the first year of the project varied the imagery value of conditional problems as well as the temporal relation (simultaneous vs. temporally ordered) between the "if" event and the "then" event (Experiment R3). An other study (Experiment R1), conducted previously, had also varied temporal relation in a partly similar way, as well as the type of clauses included in the problems (clauses referring to events or actions - e.g., "If Mary is walking..." - versus clauses stating attributes of objects or persons - e.g., "If the book is red..."). Only Experiment R3,

conducted within this project, will be reported here, although the pilot work to be descrived in a subsequent section is aimed at clarifying the results from both studies.

Subsequent to the completion of Experiment R3, a pilot study (Experiment R4) was conducted with the aim of exploring children's imagery for complex linguistic material, including simple and complex sentences and conditional inference problems.

Finally, a third experiment (Experiment R5) followed up on the previous results regarding the effects of content upon reasoning and on the pilot data regarding children's imagery for sentences and conditional problems. The purpose of Experiment R5 was to more systematically investigate the role of imagery in children's reasoning with conditional inferences with a particular focus on whether imagery appears to play a functional role in reasoning, or whether it appears to be epiphenomenal.

3.2 Experiment R3: Effect of imagery and temporal structure on reasoning with conditional inferences

This study, conducted with third and sixth graders, examined conditional reasoning in problems differeing in imagery value, as well as in the temporal relation between the events referred to by the "if" and "then" clause (simultaneous versus temporally ordered). These variables are important in the context of the question addressed, because problems with high imagery value can conceivably be solved in some cases, by forming a compound image of the statements of the problem and applying processes appropriate for that representational mode in order to answer the problem. With problems low in imagery, more formal cues may have to be used (although the problems low in imagery involved meaningful familiar statements as well). Furthermore, since the particular characteristic of imaginal representations is to afford simultanoues access to all its components, it was speculated that the imagery effects would be amplified for sentences referring to simultaneous events. The four conditional inferences were examined (Modus Tollens, Modus Ponens, and the two indeterminate inferences).

Four sets of problems were constructed by varying the imagery value of the clauses and the temporal relation between the events. Thus a problem with the conditional premise "If the girl left school alone, then she is walking the two dogs now" would be an Ordered problem. A problem such as "If the girl is watching television now, then she is sitting in a blue chair ...etc..," would be a Simultaneous problem. Imagery value of the clauses was assessed by collecting imagery ratings from another group of subjects of the same age range (from second to sixth grade). Clauses were read to them, and they were asked to give ratings from 0 to 5 to each clause, depending on whether a mental picture of what was said occurred naturally and how clear the picture was. Each child rated wither 24 or (if he/she was willing to go on) 48 clauses. Only affirmative clauses were presented for rating. After completion of this phase, clauses with average rating in the highest and lowest third of the distribution, with the least variable ratings and whose rating by younger and older children was comparable, were used to construct the problems for the reasoning task.

The reasoning task was administered to 72 children at each grade level (third and sixth). Each grade level included four groups of 18 subjects, receiving respectively one of the Imagery (High vs. Low) X Temporal structure

(Simultaneous vs. Ordered) conditions.

The entire set of materials included six "contents" for each condition, a "content" being defined by the first premise of a problem (e.g., "If Mary goes to school, then she is walking down May Street"). Each content was cast in six different problem types: Modus Ponens, Modus Tollens, Denying the Antecedent, Affirming the Consequent, and Modus Ponens and Modus Tollens with a negation in the first premise (e.g., "If Mary is going to school, then she is not walking down May Street; Mary is going to school; Is she walking down May Street?").

Each child received three blocks of six problems with all six problem types being included in each block, and the same "content" being paired with a different logical type across the three blocks.

The focus of the analysis was on patterns of interaction of the content variables with the logical form of the problems. Roughly, the assumption is that problems whose strcuture is within the repertoire of the subject's logical knowledge are more likely to be encoded and processed with reliance on formal cues. Therefore, the variability in performance due to content should decrease for problems whose form is within the child's competence. In a similar study with adults, it was found that neither imagery value nor temporal structure had a significant main effect on performance, but that both variables significantly interacted with the logical form of the problem, both in terms of the magnitude of their effect and in terms of its direction. Specifically, for example, errors increased for Modus Tollens and decreased for the indeterminate inferences, in the High Imagery Condition. The importance of this result is to indicate that the locus of the imagery effect was in the deductive process, not language comprehension per se. (Furthermore, the imagery effects were strikingly amplified in combination with "simultaneous" material, thus supporting the notion that a compound representation of the events described by both clauses was formed.)

In the present study, the focus was again on these interactions, and also on the interactions of those factors with age. As in the adult study, no significant main effect of imagery or temporal structure was obtained, but both variabels interacted significantly with the logical form of the problems, suggesting that different modes of processing and different types of representation may be involved for the various patterns of inference. Table 4 presents the pattern of significance in the ANOVA conducted with Age, Imagery, and Temporal structure as Between factors, and Logical Type as Within factor. The ANOVA included Modus Ponens, Modus Tollens, and Modus Ponens with a negative only; the error rate to indeterminate inferences was at a ceiling value for all conditions, and it was judged that including those problems in the ANOVA would yield unrevealing results regarding the interactions of interest. However, obviously, the fact that the high error rate to indeterminate problems was insensitive to content variations, needs to be accounted for: this is indeed one focus of the subsequent pilot work.

Table 4 about here

The arrows in Table 4 point at the results of main interest: Neither Imagery nor Temporal structure yield significant main effect, but both interact significantly with logical type. This is consistent with the initial theoretical rationale in general terms.

Looking at these interactions in more specific terms reveals some interesting regularities and some puzzling results. Figures 7, 8, and 9 display the proportion of errors to Modus Ponens (Figure 7), Modus Tollens (Figure 8) and Modus Ponens with negation (Figure 9) for the four content conditions in Grade 6. The Grade 3 results are qualitatively similar. The results for Affirming the Consequent and Denying the Antecedent haven't been included; the error rate is uniformly close to 1.0, in keeping with other findings in the literature.

Figures 7, 8, and 9 about here

One regularity is that, as was the case for adults, Modus Tollens was negatively affected by imagery, i.e., the error rate for that pattern was higher for high imagery material. Two accounts are possible (and complementary) for this finding:

(i) High imagery material may be conducive to imagining alternatives to what is being negated by the premise, and therefore to asserting that the conclusion is indeterminate. This should improve performance with indeterminate inferences (as was found, indeed, in the adult study) but impair performance to Modus Tollens, which requires a determinate answer.

(ii) Low imagery content may be conducive to dealing with Modus Tollens at a more formal level, which should improve performance if that pattern of inference is in the subject's repertoire and if content-bound procedures are sometimes misleading as suggested in (i).

On the other hand, the increase in the errors to Modus Ponens as well in the High Imagery condition does not fit the adult data and is presently being examined for possible interpretations.

3.3 Experiment R4: Pilot work on imagery to simple clauses, two-clause sentences, and problems

While conducting the study just described, it became clear that the workings of mental imagery for this kind of material required closer scrutiny, and we engaged in an interesting excursion into those questions, in intensive pilot work in which we asked children to evaluate various kinds of sentences in terms of the clarity of the image they elicit, and to describe those images. Some of the questions addressed concern the relationship between images corresponding to negative sentences and the "affirmative" image; the relationship between these images; the imagery value of problems of different logical form but otherwise similar content; age-related differences in these respects, etc.

The aim of this pilot work was to promote an intuitive understanding of the ways in which imagery appears to be used to represent this complex linguistic material. Of course, we are not overlooking either the issues related to mental imagery as a theoretical construct, or the ambiguities of introspective reports, but rather sidestepping those issues at the moment for hypothesis-generating purposes. The aim is to use the suggestions provided by this imagery work to reanalyze the results from the reasoning studies (the study just summarized and the previously ocmpleted study varying the type of clauses used in the problems -- clauses referring to attributes of objects or persons, or to states of affairs versus clauses referring to events or actions), and to initiate relevant follow-ups.

Twenty children ranging from grade 2 through 7 have been tested so far. The subjects were interviewed individually. Each child ~~was told~~ that the experimenter was interested in what happens when people listen to sentences. Several examples of high and low imagery sentences were presented with a discussion that "sometimes a picture of what the sentence says will pop into your head. Other times, for other sentences, you will understand them but no picture will pop up". The experimenter explained that she would be reading sentences to the child and that, for each sentence, the child was to indicate whether a picture "automatically" came up or not. The child was further instructed as to gradations of clarity among various images; the experimenter discussed with the child that a mental picture might appear to be "very clear", "pretty clear", or "fuzzy" (not very clear at all).

The child was cautioned to report having a picture for a sentence only if it was a picture of the whole sentence and not just of part of the sentence. Further, the subject was instructed not to force pictures to come to mind for the sentences but, rather, to report them only if they "automatically popped into his/her head".

The materials included simple clauses (e.g., "The boy is at the park"); negated clauses ("The girl is not opening the window"); conditional sentences ("If the boy is at the park, then he is wearing shows"); "when" sentences ("When the girl is watching TV then she is combing her hair"); and problems similar to those of the reasoning study, that is, the four conditional inferences. An effort was made to present lists that would constitute a relatively balanced design across children so that the comparisons of interest could be made, but priority was put on on-line flexibility and responsiveness to potentially interesting features of the responses of each subject.

Qunatitative and qualitative analyses of the results were conducted. Some aspects of these results can be summarized as follows:

(1) Compared to imagery for simple declarative sentences, the imagery reported for negative sentences decreases. This can be seen in Figure 10 which shows the distribution of ratings to affirmative and negative sentences: More "no picture" responses and fewer "very clear" responses are obtained for negatives.

Figure 10 about here

One comment is in order. In a literal sense, negative sentences do not afford an imaginal representation and if subjects were using images as a literal representation of meaning (or as a literal code for meaning), one would expect imagery for negatives to be nonexistent. However, this is not what children appear to do. First the images described are often far from literal representations of the sentences: they often are elaborations, concretizations (beyond the degree of concretization that is inherent in any imaginal representation of a meaningful statement), associative images, etc... In the case of negatives (e.g., "The girl is not opening the window"), the images reported frequently describe a specific alternative ("I saw a girl sitting") or, most interestingly, seem to express a presupposition of the sentence ("I see the girl looking through the window"). Qualitative analyses of these responses, especially to negative sentences, are extremely interesting. The present comment is simply aimed at pointing out that, given this variety of devices effectively used for representing negation, the fact that the vividness of imagery to negative sentences is nevertheless lower than for affirmative sentences, is non-trivial psychologically.

(2) Younger children report less imagery than older children, both to affirmative and negative sentences. The relevant data can be seen in Table 5: the proportion of "No picture" responses decreases from the second and third grade group to grade 4 and 5, to grade 6 and 7. Concurrently, the proportion of "very clear" responses sharply increases.

Table 5 about here

This seems counterintuitive at face value. However, imagery reports are ambiguous, and several possibilities need to be kept in mind and eventually tested:

(i) the younger children's responses may reflect a general tendency not to attach certainty to any report on internal processes;

(ii) young children's conceptions of what constitutes an image ("a picture in the head") may be more stringent, if the most vivid imagery they sometimes experience is more vivid than that of older children or adults (as one would intuitively tend to believe based on extensions of Paivio's and Bruner's ideas among others). In other words, the data of Table 5 may reflect a criterion shift. This could be tested, at least in principle, by examining the correlation between imagery reports and other indices of the occurrence of imagery (e.g., selective visual interferences data of the kind used by Brooks).

(3) Another question of interest is whether the effect of negation on imagery varies with age. Judging from Table 5, it appears that, at the level of imagery reports (see discussion above), the effect is similar in the youngest and oldest age group: the vertical difference between underlined values is the same as the vertical difference between circled values. On the other hand, the effect of negation seems smaller in grade 4 and 5. This U-shaped phenomenon is potentially interesting, but larger subject samples and more tightly controlled materials need to be studied before any interpretation is warranted.

(4) In keeping with the variables manipulated in the reasoning studies, one focus of the analysis is on comparing imagery to conditional sentences in which the "if" and "then" clause refer to simultaneous events or states of affairs (e.g., "If the girl is wathicng TV now, then she is also combing her hair")

These cases were compared in terms of the imagery ratings (presumably reflecting the vividness of the images), and in terms of qualitative aspects of the images described. In the latter respect, four tentative categories were defined: compound images in which the two events are integrated into a unitized representation ("I see a girl watching TV and combing her hair"); sequential images in which both events seem to be represented separately and temporally tagged (e.g., "I see a girl watching TV, and then I see her sitting and combing her hair"); descriptions in which only the event referred to by the first clause is included; and descriptions that only include the event corresponding to the second clause. Figure 11 presents the results of those two analyses for "simultaneous" and "ordered" sentences respectively.

Figure 11 about here

As is evident in the top part of Figure 11, the imagery reported for "simultaneous" sentences seems to be higher than for "ordered" sentences (i.e., fewer "No picture" responses and more "very clear" responses). This is congruent with the initial intuition that led us to introduce the temporal factor in the reasoning study reported above: it was assumed that the "simultaneous" condition would amplify imagery effects.

Looking at the qualitative properties of the images described (bottom part of Figure 11), it seems that the "simultaneous" condition leads to more frequent compound images, which again supports the initial expectation underlying the reasoning study, (Experiment R3).

The "ordered" condition leads to more frequent reports of "sequential" images which is not surprising yet indicates that the temporal order is preserved in some of the imaginal representations at least. These results are of interest in the context of the question to be discussed next.

(5) As was stated, one focus of this pilot work was on attempting to relate the imagery corresponding to entire problems, to the imagery corresponding to their more elementary constituents. Thus, having taken notice of the results described in (4), it is of interest to examine the imagery reports to problems that involve "simultaneous" vs. "ordered" material. It can be seen in Figure 12 that the greater effectiveness of "simultaneous" material in eliciting images seems to hold up in the context of problems as well.

Figure 12 about here

(6) Comparing the values in Figure 11 (top) and in Figure 12 leads to an interesting observation: there seems to be more imagery elicited in the context of problems than for the conditional sentences alone. This observation, as the previous ones, is provisional because as was mentioned initially the specific materials used could not be counterbalanced or controlled rigorously across conditions (if a condition is defined as simple clause, conditional sentence, problem, simultaneous, ordered, etc...). However, the difference mentioned here seems substantial and likely to hold up under more strictly controlled conditions. This difference is extremely interesting in the context of the theoretical notions discussed above: to the extent that the representation of a given content

in the context of a deductive inference appears to differ from the representation of that content when presented in isolation, there are empirical grounds for positing a level of representation functional for interential processing, and there is hope that the present kind of comparison may be used for assessing variations in the functional representation across logical form or level of competence.

(7) Finally, concerning the comparison between the four problem types in terms of imagery value, no obvious difference seems to obtain: the histograms (in the vein of Figure 12) are very similar across problem types. This result is only tentative, however, because the imagery value of the basic contents (simple clauses) of which the problems are composed, is not strictly controlled for across problem types, in this pilot phase.

(8) Turning to a comparison of problem types in terms of the types of images reported, it can be seen in Table 6 that a higher percentage of compound images obtains for the two indeterminate inferences (AC and DA) than for the two determinate inferences (MP and MT).

Table 6 about here

Again, this comparison needs to be replicated with controlled materials, but the difference seems substantial. If it holds up, this difference may provide informative suggestions regarding the processing of indeterminate inferences. It must be realized that indeterminacy vs. determinacy is the only factor that distinguishes the two top inferences (MP and MT) in Table 6 from the two bottom inferences. Surface features of the argument are unconfounded by definitions:

(i) neither MP nor AC contain a negation; MT and DA both do;

(ii) the "direction of processing" in MP goes from left to right, so to speak, and similarly for DA (i.e., the second premise concerns the antecedent and the conclusion concerns the consequent); the "direction of processing" in MT goes from right to left, and similarly for AC (i.e., the second premise concerns the consequent and the conlcusion concerns the antecedent). Thus the two groups of inferences (determinate vs. indeterminate) naturally form a balanced design, so that any difference in imagery between those two groups would have to be related to the indeterminacy factor.

Of course, two caveats apply: first, reports of imagery are ambiguous evidence for its occurrence; second, occurrence of imagery does not imply that it had a functional role in the reasoning process. While these caveats can be bracketed in this pilot hypothesis-generating phase of the project, they will have to be addressed constructively in subsequent work. In the next study to be reported, we focus on this question more closely.

3.4. Experiment R5: Imagery in conditional reasoning

The pilot work described above was aimed at promoting an intuitive understanding of the workings of mental imagery for representing complex linguistic material. Further, it was an attempt to explore how imagery may be used in solving conditional

inference problems.

In the present study, we examined more systematically the imagery reports to the four conditional inferences, with carefully controlled materials with a focus on the interplay between linguistic and imaginal processes, and particularly with a focus on whether imagery apperas to have a functional role in the deductive process, or whether it appears to be epiphenomenal.

Sixteen third graders and sixteen sixth graders (volunteers) solved conditional inferences of the following kind:

If Mark is working, then he is writing.
Mark is not writing.
Is Mark working?

The problems were constructed from materials previously rated as high or low in imagery value by another gorup of 20 subjects in the same age range. Four types of conditional inference were administered, two of which permitted no definite conclusion (indeterminate inference). Each inference type was presented once with high imagery material and once with low imagery material to each subject and the pairing of specific content (e.g. Mark writing...) with inference type was counterbalanced between subjects that is, across subjects each inference type appeared with each content with equal frequency.

After answering each problem, the subject was asked whether an image occurred to him/her as he/she solved the problem, and how clear that image was (on a 3-point scale). In addition, subjects were asked to describe their images in each case, and a content analysis of those descriptions was conducted subsequently.

The data analysis focuses on the frequency and vividness of imagery reported for the four inference types; the qualitative properties of the iamges reported, in terms of the information encoded therein; the relation between those two variabels and the correctness of the responses to problems; and age-related trends in those respects. What follows is a preliminary description of those complex, qualitative data.

(1) A first question concerns whether the frequency and/or clarity of the images reported varies according to the logical type of the problem. The structure of the first premise (If. ., then...) is the same for each of the four problem types; and the content of the first premise was identical for the four problem types across the group of subjects. Thus, any general differences among the problem types in the frequency of images reported, the clarity of images reported, or the type of images reported, must be attributed to the deductive process, that is, to reasoning at a later stage of processing than the encoding ofthe first premise.

It can be seen in Table 7 that, in terms of frequency of images reported, both age groups report fewest images for the DA problems.

Table 7 about here

One factor that might be hypothesized to account for this difference is that the DA problem type includes a negation in the second premise (and the results of a previous study indicated that sentences with negation yield lower imagery). However, the negation alone cannot account for the low frequency of imagery reports, since the MT inference also contains a negation in the second premise, and yet does receive a relatively high frequency of imagery reports in both grades. Similarly, the fact that DA problems permit no conclusion might be considered in accounting for the low frequency of iamgery reported for those problems. However, the AC inference is also indeterminate, and yet does receive a high frequency of imagery reports.

Tentatively, this finding may be due to the fact that in DA problems it is the antecedent which is negated. A result to be discussed below (see Point 2 and Table 10) indicates that, when images are incomplete, it is the antecedent (IF) clause which tends to be reported. Thus, negating that clause could be expected to destroy imagery more effectively, which would account for the present result.

Turning now to the clarity of images reported for the various problem types, Table 8 (bottom) indicates that the problems with negation, and especially DA, receive relatively fewer "very clear" reports, among the images reported. This result is consonant with the corresponding result concerning frequency of images.

Table 8 about here

Other differences potentially of interest between the problem types can be observed, but will not be discussed here.

(2) Up to this point, we have considered the frequency of imagery reports to the four problem types, and the clarity of those images. Some of the questions and speculations mentioned in relation to those data will be addressed by examining the children's descriptions of images for problems.

In order to examine general trends in the imagery descriptions, the descriptions were categorized according to sevearl general features. It was not possible to establish precisely which parts of the problem the child represented in his/her image. For example, in the following problem, if the child reported, "I see a boy writing on the blackboard", it is not clear whether the image stems from the consequent (in the first premise) or the second premise -- since the event imaged occurs in both the first premise and the second premise.

If Mark is working on his math homework, then
he is writing on the blackboard.

Mark is writing on the blackboard.

Is Mark working on his math homework?

Thus, the images reported are analyzed only in terms of the antecedent and consequent events, although these events are also involved in the second premise and conclusion.

The images reported can be described primarily by three general classes:

1. Image of a single clause, either the antecedent or the consequent event (an affirmative or negative image).

Example: "Him with chalk, writing on the blackboard."

Example: "I saw him sitting in a room -- a blackboard there, but he wasn't writing."

2. Image of the antecedent and consequent events integrated into a single image -- a "Compound Image".

Example: "Him writing on the blackboard -- writing about math."

3. Image of the antecedent and consequent events ordered in time -- a "sequential image".

Example: "He was working on the project and then he did the blackboard writing."

Several aspects of these results are of interest. First, is worth noting that, when an image is reported it is most often a compound image integrating the antecedent and consequent clauses into a simple scene, as if it were an image corresponding to the first, conditional premise. Table 9 presents the frequency and proportion of compound images for each of the four problem types. I can be seen that this frequency is higher for "affirmative" problems.

Table 9 about here

Table 10 presents the frequency of image reports describing a single clause, that is, images for either the antecedent or the consequent event. For both grades, but especially for the third grade, there are more frequent reports of images for the antecedent; and this holds across problem types.

Table 10 about here

(3) Turning now to age-related differences in the frequency, clarity and type of image reported, it can be seen in Table 7 that the sixth graders reported a higher frequency of images than did third graders (61% vs. 56%). This result replicates pilot findings that older children report more imagery for sentences than younger children (see discussion, page 20). Additionally, it is interesting to note that sixth graders report more images for the determinate inferences (MP and MT), the inference forms that appear to be most familiar to older children and adults (as we will see later), in comparison with the indeterminate inferences (AC and DA), which may not be mastered even in adulthood. This difference does not hold for third graders. This result will be discussed below in relation to error data.

Concerning the clarity of images reported for the various problem types, a return to Table 8 indicates that the third and sixth graders report similar total

proportions of "Very clear", "Pretty clear" and "Fuzzy" images. This suggests that the children used consistent criteria for assessing the clarity of their images. This result appears to conflict with the results from the previous pilot study (see Table 5) according to which older children reported relatively more very clear images than younger children. However, this previous result concerned isolated sentences, whereas the present result concerns imagery to problems. The implications of this discrepancy (if it is reliable) need to be understood.

Finally, it can be seen in Table 9 that the older children report compound images more often than the third grade group, which might indicate either a more complete encoding of the information, or (as will be argued below) a more effective use of images.

(4) Thus far, we have discussed children's imagery for the four problem types. A further question concerns the specific role played by imagery in the reasoning process. First, it is interesting to examine the relationship between the materials as categorized previously (High or Low imagery), and the actual imagery reports for the various problems. The relevant data are presented in Table 11. It can be noted that, when looking separately at problems whose component clauses had been categorized previously as high versus low in imagery, these two types of material remain rated as high and low, respectively, when combined with determinate inferences. In contrast, the distinction vanishes for the indeterminate inferences; indeterminate inferences constructed from high and low imagery material yield nearly the same amount of imagery. This pattern holds for both age groups as well as for the two age groups combined.

A possible explanation for this result, is that the logical processing factors involved iwth indeterminate inferences, overrode the contribution of the material per se to the formation of images. In contrast, with determinate inferences this did not occur. More specifically, our speculation is that there may be two kinds of images involved. Some images may be evoked directly by the initial statement, in the same way as they would be evoked by that sentence in any context. These "literal images" would be influenced by particular material and would not have a functional role in deduction. In other cases the image reported would be an image constructed in the course of the deductive process, as part of the deductive process itself. These can be called "constructed images". In line with this speculation, for Modus Ponens and Modus Tollens, problems with High imagery material remain high in imagery (and low imagery problems remain low) because most of the imagery to these problems is literal imagery. It has to do with the encoding of the premise, but not with the deductive process. This occurs for determinate problems because they are known at a relatively abstract level, so that the reasoning process primarily operates with reliance on logical form rather than utilizing the content. Imagery may play only an auxilliary role.

In contrast, no formal pattern of reasoning is available for the indeterminate inferences; reasoning does not primarily rely on logical form. Therefore most images here may be constructed during deduction and their occurance has little to do with whether the material was previously rated high or low in imagery. It is suggested then, that images to indeterminate problems may be functional, whereas images to determinate problems may play only an auxilliary role in deduction.

Table 11 about here

(5) A second result supports this speculation. Figures 13 and 14 present data on the relationship between the occurence of imagery and wether a "yes" or "no" response is given to a problem ("can't tell" responses were not included for sake of clarity; they behave in the same manner as "no" responses). It is seen that for determinate inferences there is little relationship between whether a respone is yest or no, and the occurance of an image. However, with indeterminate inferences, children tend to say "yes" when they report an image and "no" when they do not report an image. Again this indicates that when a logical pattern is available, imagery does not play a functional role in determining respones. When a pattern is not available, as for indeterminate inferences, the response is largely determined by whether an image was constructed or not.

Figures 13 and 14 about here

(6) The final question to be addressed, concerns developmental trends in the use of imagery. First, in Table 12 we will consider the accuracy of responses to problems independently of imagery reported. Note that, overall, sixth graders perform better than third graders (47% correct vs. 35% correct). Sixth graders usually respond correctly to the determinate problems (MP and MT), whereas third graders are not generally very accurate in responding to the determinate problems. It is interesting to note that (as pointed out in (3) sixth graders report more frequent imagery than do third graders on determinate problems specifically, which are also those problems on which they improve most compared to third graders. This result suggests that sixth graders may use auxilliary imagery more effectively.

Table 12 about here

(7) In support of this speculation, it is interesting to examine data concerning the correctness of the responses as a function of whether an image was reported or not for that problem. These data are presented in Table 13. It can be seen that, in general, the sixth grader's performance on determinate problems tends to be better whenan image is reported, in contrast to the third graders' results which show a trend in the opposite direction. Again, these results suggest that sixth graders may use imagery more effectively.

Table 13 about here

In sum, these results offer tentative support for the notion that imgery can serve either a functional role or an auxilliary role in the deductive process. Imagery appears to be functional primarily for those problems for which a logical pattern of inference is not available. Additionally, results indicate that older children utilize auxilliary imagery more effectively, perhaps because -- speculatively -- it would be guided by an increased logical competence.

4. The teaching of Geometry: content and deductive inference

4.1 Background

The mathematic curriculum in our elementary and secondary schools, despite almost two decades of curricular reform, is still under pressure to "do something" about geometry. A critical problem for the mathematic's teacher is determining what the subject is all about. There are many geometries and many basic facts and ideas inherent in these geometries. But there are a number of consistencies to be noted in the objectives of and approaches to the teaching of geometry both in this country and abroad. One of these consistencies is the use of geometry as a vehicle for the teaching of the deductive method and for an introduction to imaginative thinking through the solution of "original problems".

A common approach to the teaching of geometry engages the student in learning a fixed theorem following a prescribed order for deriving solutions. Typically absent is attention directed towards developing an understanding of what constitutes a proof and the usual rules of logical reasoning used in mathematics. Students are engaged in direct geometry tasks with problem content and logical processes experienced simultaneously.

The extension of the present research to the natural circumstances of the secondary classroom is to provide training cycles consisting of two phases of instruction geared to the degree of complexity of the logical demands embodied in a given geometry task. The first phase is considered preparatory to the second which involves direct instruction in formal geometry.

4.2 General procedure

In each training cycle the preparatory phase (I) consists of training in selected patterns of deductive inference that are exemplified in the theorems to be dealt with in the isntructional phase (II). Phase I involves a four stage sequence, beginning with: (a) exercises in a specific logical form (e.g., Modus Ponens, Modus Tollens, etc.) represented linguistically followed by parallel exercises expressed; (b) in the language of geometry; (c) in geometric symbolization; and (d) in symbolization of formal logic. The instructional phase (II) explicates a geometry task in terms of logical forms considered in Phase I. (See Appendix 2 for a more detailed description of the materials and procedure.)

The preparatory phase was planned for a period of about three weeks, with special emphasis on stages 3 and 4. It was hypothesized that during these stages, the student would gain necessary exposure to, and experience with the form and symbolization of the relevant patterns of deductive inference. The instructional phase would vary in time according to the geometry task at hand.

4.3 Experiment G1: Deductive reasoning and teaching of geometry

A pilot study conducted for 2½ weeks at a privaet secondary school in Worcester, Massachusetts (Spring, 1980) yielded encouraging results. Twenty high school sophomores selected randomly from two classes of college level geometry were randomly assigned to four groups of five students. Using teaching method (Logical Training vs. Traditional) and level of task difficulty as independent variables with achievement on performance tests as the dependent

variable, a (2x2) ANOVA indicated that (i) students instructed under the Logical Training method performed significantly better ($p < .05$) than did students under the traditional approach and (ii) there was a significant interaction effect ($p < .01$), i.e., the Logical Training method was particularly useful when the task was difficult (i.e., logically more complex).

Additional analyses using Student's t and comparing only the groups who were given difficult tasks revealed that few steps were required on the average for successful completion of a task when the Logical Training method was used, although the difference did not reach statistical significance.

Finally, the students given the Logical Training instruction demonstrated a better understanding of the relevant technical vocabulary than did the students taught through the traditional approach.

Experiment GI investigating this question systematically is now in progress. This classroom study covers the conventional tenth grade geometry curriculum and is conducted in a private secondary school in Worcester, Massachusetts. Two groups of students have been randomly assigned to experimental and comparison groups. In the course of the instructional sequence, the experimental group is undergoing the two training cycels described above. The control group only receives conventional instruction in the same content topics, without prior logical instruction.

At this time, Cycle I (covering, as content topics, Order on a number line; Absolute Value; The Distance Postulate; The Ruler Postulate, and the Ruler Replacement Postulate; Betweeness; Line Postulate; Plane Postulate; Convex Sets; Plane and Space Separation Postulates; Angles and Definitions; Angle Addition Postulate; Angle Construction Postulate, and, Supplement Postulate) has been implemented. The preparatory logical training for that cyle, implemented at the beginning of the school year dealt with the logical connectives $\wedge$ (conjunction) and $\vee$ (disjunction - both inclusive and exclusive), negation and DeMorgan's Laws. Necessary definitions for subsequent stages of the Logical training were given informally, to appeal to intuition and experience. Truth tables for each form were constructed at the end of this phase.

The Logical training phase of Cycle 2 has also been completed. The content topics to be covered in that cyle include: Equivalence Relations; Vertical Theorems; Notions of Congruence; Congruence Postulates; Isoscales Triangles, etc.; Quadrilaterals; Mediums; Bisectors; Theorems in the form of hypothesis and conclusion, and Coverses. The logical training for this cycle focused on conditional statements ($p \to q$) and included converses, inverses, contrapositives, biconditionals, and logically equivalent forms of $p \to q$ (e.g., $\sim p \vee q$; $\sim p \sim\vee q$). Once again, truth tables were constructed at the end of the training phase.

A third cycle, extending to Polygonal regions, similarity and proportionality, and circles and spheres, will complete the curriculum.

Data is being collected presently but no evaluation will occur until all three cycles have been implemented. Differential outcomes will be assessed by specially designed measures of the student's ability to use technical vocabulary and apply deductive reasoning to a series of original problems. Those measures, focusing on specific aspects of the deductive steps carried out by students in proofs, have been worked out in some detail at the present time, but will not be described here for the sake of brevity, and will have to be refined as indicated by

instructional experience.

5. Conclusion

The theoretical notions underlying the work reported, are an attempt to integrate the view that logical reasoning relies, at some level, on a formal system of rules, with the recognition that a variety of representational media are available to represent and process verbally given information. It has been suggested thatlogical development consists both in an increased availability of the formal mode for an increasingly wide range of situations, and in elaboration o the logico-semantic network; it has also been argued that the increased availability of the formal mode involves not only an enrichment of the formal rules themselves but also an elaboration of the pattern recognition device through which the logical structure of a problem or sentence is abstracted from its natural language expression; and one possible mechanism of acquisition of patterns of deductive inference has been discussed. However, the qualification stressed earlier with regard to this acquisition process may be recalled, namely that it should be seen as operating within the context of more general cognitive and developmental constraints and should not be mistaken as reflecting an empiricist epistemology.

The work completed so far has provided support for the "acquisition" notions; has begun to explore some interactions between content and form that are of relevance to the theoretical questions addressed; and has explored some relevant aspects of the workings of mental imagery for complex meaningful material. The projected work will examine acquisition of indeterminate inferences; will attempt to replicate the main "imagery" results under controlled conditions in small-scale studies; and will use these results to focus again on the content-form interactions of interest.

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Footnotes

1. Thanks go to Catherine Clement for assitance in preparation of this report and to Bobbi Karman for typing.

Table 1

CL5, Grade 2

Percent correct in each group for If-Then and When-Then Modus Tollens problems [a].

	If-Then Problems	When-Then Problems	All MT Problems
IF Group (n = 10)	.82	.75	.78
WHEN Group (n = 10)	.63	.62	.63
Control (n = 10)	.53	.60	.57

[a] 60 observations per cell. Reverse Nonsense problems are not included for purpose of comparison with Experiment CL4.

Table 2

CL5, Grade 2

Percent correct in each group, for affirmative and negative Modus Tollens problems [a]

	Affirmative	Negative	All Problems
IF Group (n = 10)	.83	.73	.78
WHEN Group (n = 10)	.68	.57	.63
Control (n = 10)	.58	.55	.57

[a] 60 observations per cell. Reverse Nonsense problems are excluded from totals for purposes of comparison with CL4.

Table 3

CL5, Grade 2

Individual learning data: Percent correct by each subject on Modus Tollens problems in the first training block; in the last training black; and in the transfer task

	IF Group				WHEN Group		
	First Block (n=8)	Last Block (n=8)	Transfer Task (n=12)		First Block (n=8)	Last Block (n=8)	Transfer Task (n=12)
S_1	.00	.63	.75	S_{11}	.38	.50	.42
S_2	.38	.75	.83	S_{12}	.50	.75	.75
S_3	.38	.75	.83	S_{13}	.63	.63	.50
S_4	.88	.88	1.00	S_{14}	.25	.63	.17
S_5	.38	.63	.92	S_{15}	.38	.88	.75
S_6	.50	.75	.83	S_{16}	.38	1.00	.83
S_7	.88	.88	.83	S_{17}	.38	.38	.75
S_8	.63	.63	.83	S_{18}	.88	.75	.67
S_9	.38	1.00	.33	S_{19}	.63	.75	.92
S_{10}	.38	.63	.67	S_{20}	.25	.75	.33
All IF S's	.48	.75	.78	All WHEN S's	.47	.70	.53

Table 4

Experiment R3. Pattern of significance from the ANOVA including Modus Ponens, Modus Tollens and Modus Ponens with negation as Logical Types

G: Grade (third vs sixth)

IM: Imagery (High vs Low)

TS: Temporal Structure (Simultaneous vs Ordered)

LT: Logical Type (MP, MT, MP_{an})

	P
G	.01
IM	NS ←
TS	NS ←
G x IM	NS
G x TS	NS
IM x TS	NS
LT	.01
G x LT	NS
IM x LT	.01 ←
TS x LT	.01 ←
G x IM x LT	NS
A x TS x LT	NS
IM x TS x LT	NS

Table 5

Imagery ratings to affirmative and negative sentences as a function of age

Affirmatives

Grade	No Picture	Fuzzy	Pretty Clear	Very Clear
2-3	.29	.13	.29	.26
4-5	.20	.13	.22	.33
6-7	.08	.08	.32	.53

Negatives

Grade	No Picture	Fuzzy	Pretty Clear	Very Clear
2-3	.59	.18	.20	.04
4-5	.34	.09	.11	.34
6-7	.34	.11	.23	.26

Table 6

Types of images reported for the four problem types:

Percentage of images of each type, among the images reported

	Compound	Sequential	First Clause	Second Clause
Modus Ponens	30	20	20	30
Modus Tollens	31	25	25	19
AC (indeterminate)	64	4	23	9
DA (indeterminate)	52	23	16	10

Table 7: Proportion of images reported for the four problem types

	3rd. Grade	6th Grade	3rd. & 6th Grades
MP	.50 (16/32)	.69 (22/32)	.59 (38/64)
MT	.59 (19/32)	.69 (22/32)	.64 (41/64)
AC	.66 (21/32)	.63 (20/32)	.64 (41/64)
DA	.50 (16/32)	.44 (14/32)	.47 (30/64)
TOTAL	.56 (72/128)	.61 (78/128)	.59 (150/256)

Table 8. Proportion of "Very Clear", "Pretty Clear", and "Fuzzy" imagery ratings relative to total images reported

3rd. Grade		VC/Total Pic.	PC/Total Pic.	F/Total Pic.
	MP	.19 (3/16)	.56 (9/16)	.25 (4/16)
	MT	.16 (3/19)	.47 (9/19)	.37 (7/19)
	AC	.33 (7/21)	.38 (8/21)	.29 (6/21)
	DA	.25 (4/16)	.44 (7/16)	.31 (5/16)
	TOTAL	.24 (17/72)	.46 (33/72)	.31 (22/72)

6th Grade		VC/Total Pic.	PC/Total Pic.	F/Total Pic.
	MP	.32 (7/22)	.50 (11/22)	.18 (4/22)
	MT	.22 (5/22)	.55 (12/22)	.23 (5/22)
	AC	.35 (7/20)	.40 (8/20)	.25 (5/20)
	DA	.07 (1/14)	.72 (10/14)	.21 (3/14)
	TOTAL	.26 (20/78)	.53 (41/78)	.22 (17/78)

3rd. & 6th Grades		VC/total Pic.	PC/Total Pic.	F/Total Pic.
	MP	.26 (10/38)	.53 (20/38)	.21 (8/38)
	MT	.20 (8/41)	51 (21/41)	.29 (12/41)
	AC	.34 (14/41)	.39 (16/41)	.27 (11/41)
	DA	.17 (5/30)	.57 (17/30)	.27 (8/30)
	TOTAL	.25 (37/150)	.49 (74/150)	.26 (39/150)

Table 9: Proportion of compound images among the images reported

	Grade 3	Grade 6	Total
MP	.31	.64	.50
MT	.21	.32	.25
AC	.43	.40	.41
DA	.31	.36	.33
TOTAL	.31	.44	.34

Table 10: Proportion of images describing the antecedent only or the consequent only, among the images reported

	3rd. Grade			6th Grade		
	Anteced.	Conseq.	Total # Images Reported	Anteced.	Conseq.	Total # Images Reported
MP	.38	.19	16	.09	.14	22
MT	.37	.05	19	.27	.14	22
AC	.29	.10	21	.25	.20	20
DA	.31	.19	16	.21	.16	14
TOTAL	.33	.11	72	.21	.13	78

Table 11: Proportion of images reported for the various problem types, respectively with High Imagery material (Hl) and Low Imagery material. (Ll)

	Grade 3		Grade 6		All Subjects	
	Hi	Li	Hi	Li	Hi	Li
MP	.69	.31	.94	.44	.81	.37
MT	.75	.44	.88	.50	.81	.47
AC	.56	.75	.69	.56	.62	.65
DA	.56	.44	.50	.38	.53	.41

Table 12: Proportion of correct answers for the four problem types

	3rd. Grade	6th Grade	All S's
MP (yes)	.69	.94	.81
MT (no)	.41	.78	.59
AC (ct)	.09	.06	.08
DA (ct)	.22	.09	.16
TOTAL	.35	.47	.41

Table 13: Proportion of correct answers for the four problem types as a function of whether an image is reported ("Picture" report) or not ("No Picture" report)

	3rd. Grade		6th Grade		All Ss.	
	Pic.	No Pic.	Pic.	No Pic.	Pic.	No Pic.
MP (yes)	.69	.69	1.00	.80	.84	.74
MT (no)	.37	.46	.82	.70	.59	.58
AC (ct)	.05	.18	.05	.08	.05	.13
DA (ct)	.20	.25	.00	.17	.09	.21
TOTAL	.32	.40	.47	.43	.39	.42

Fig 1.

Exp. CL4, Grade 5

Percent correct in each group, to

Standard, Reverse and Nonsense problems

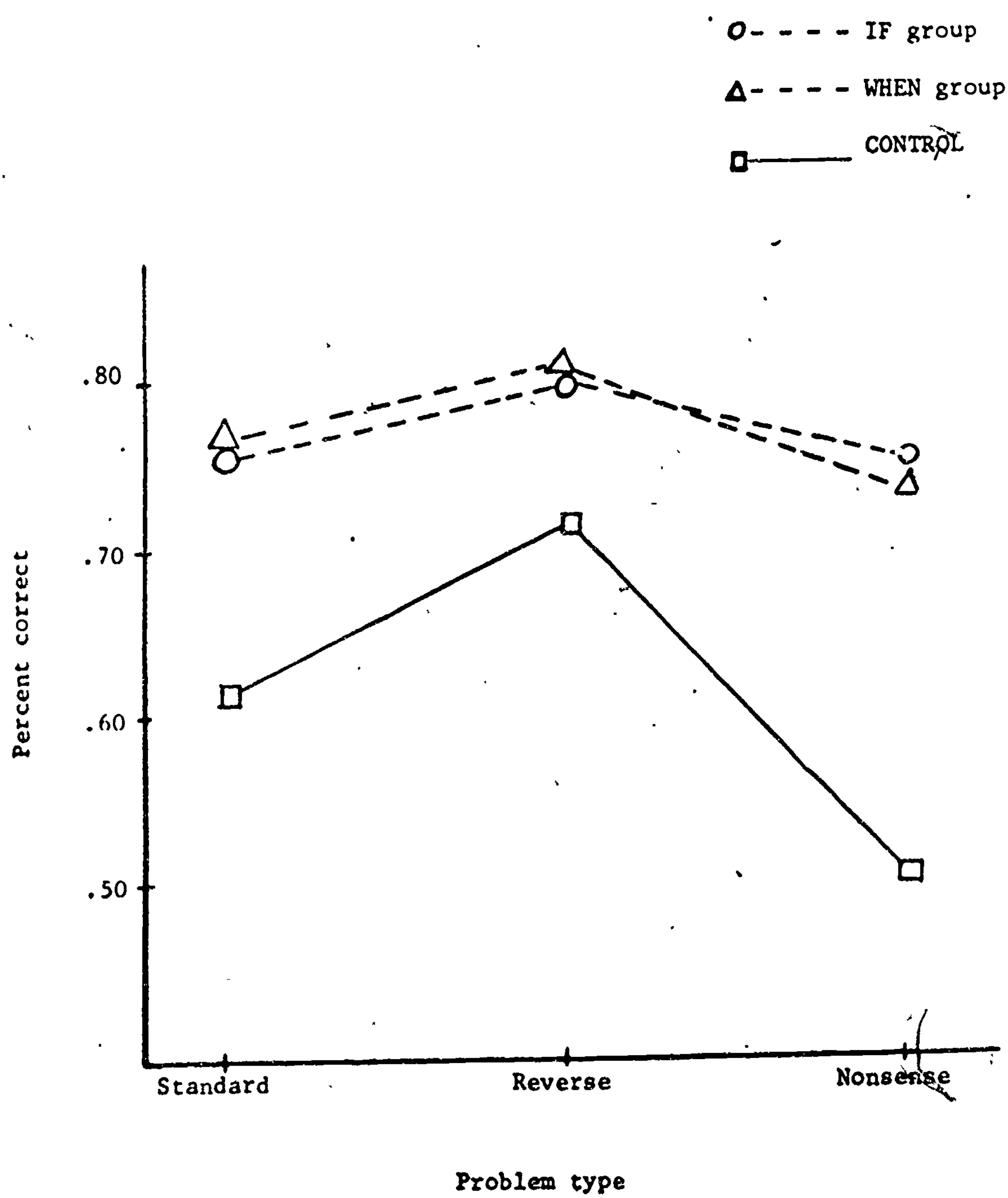


Fig. 2

Exp. CL4, Grade 5

Percent correct in each group

AA problems versus NA and NN

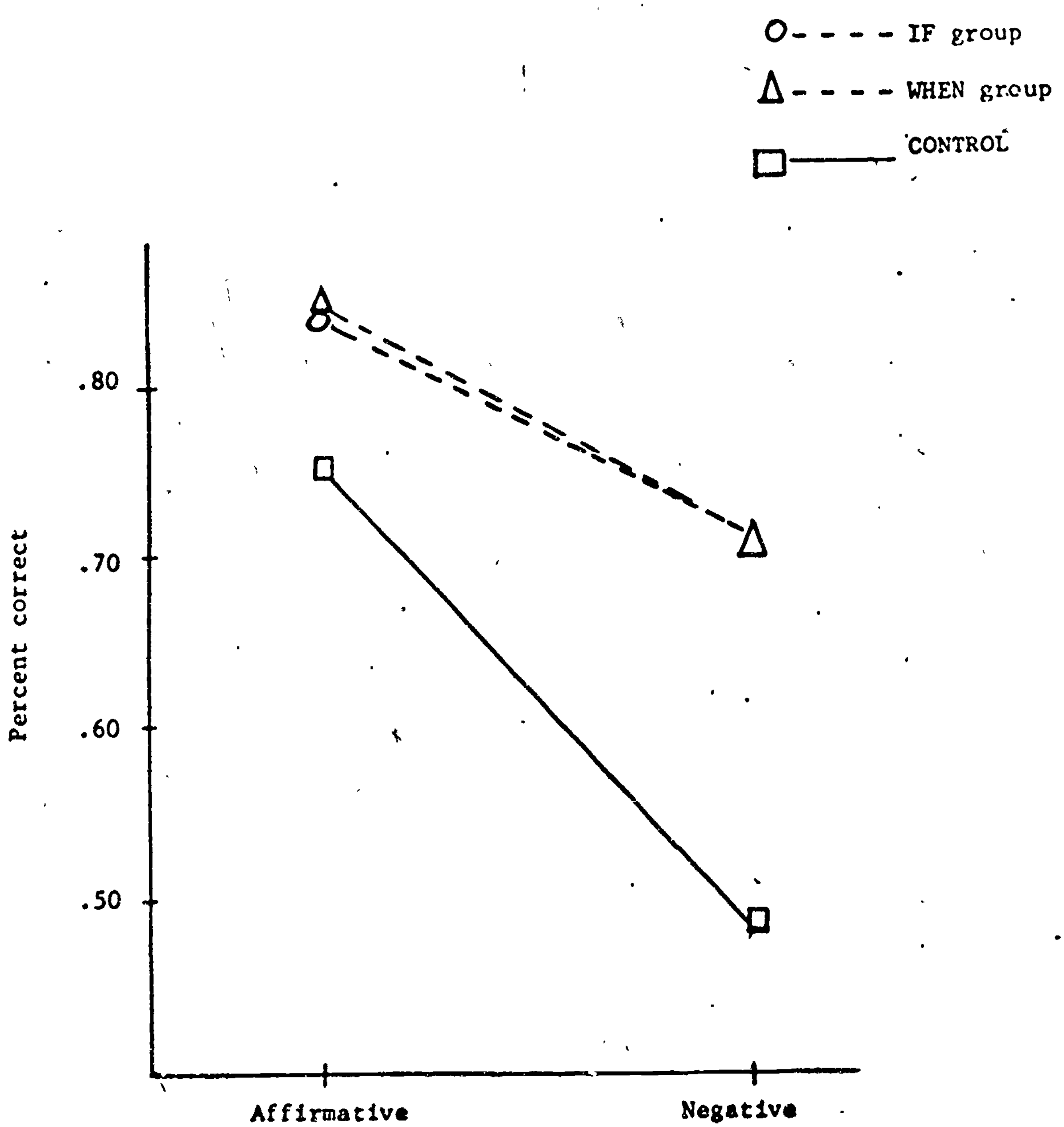


Fig. 3

Exp. CL4, Grade 5

Percent correct in each group for

transfer problems including

"if-then" or "when-then" respectively

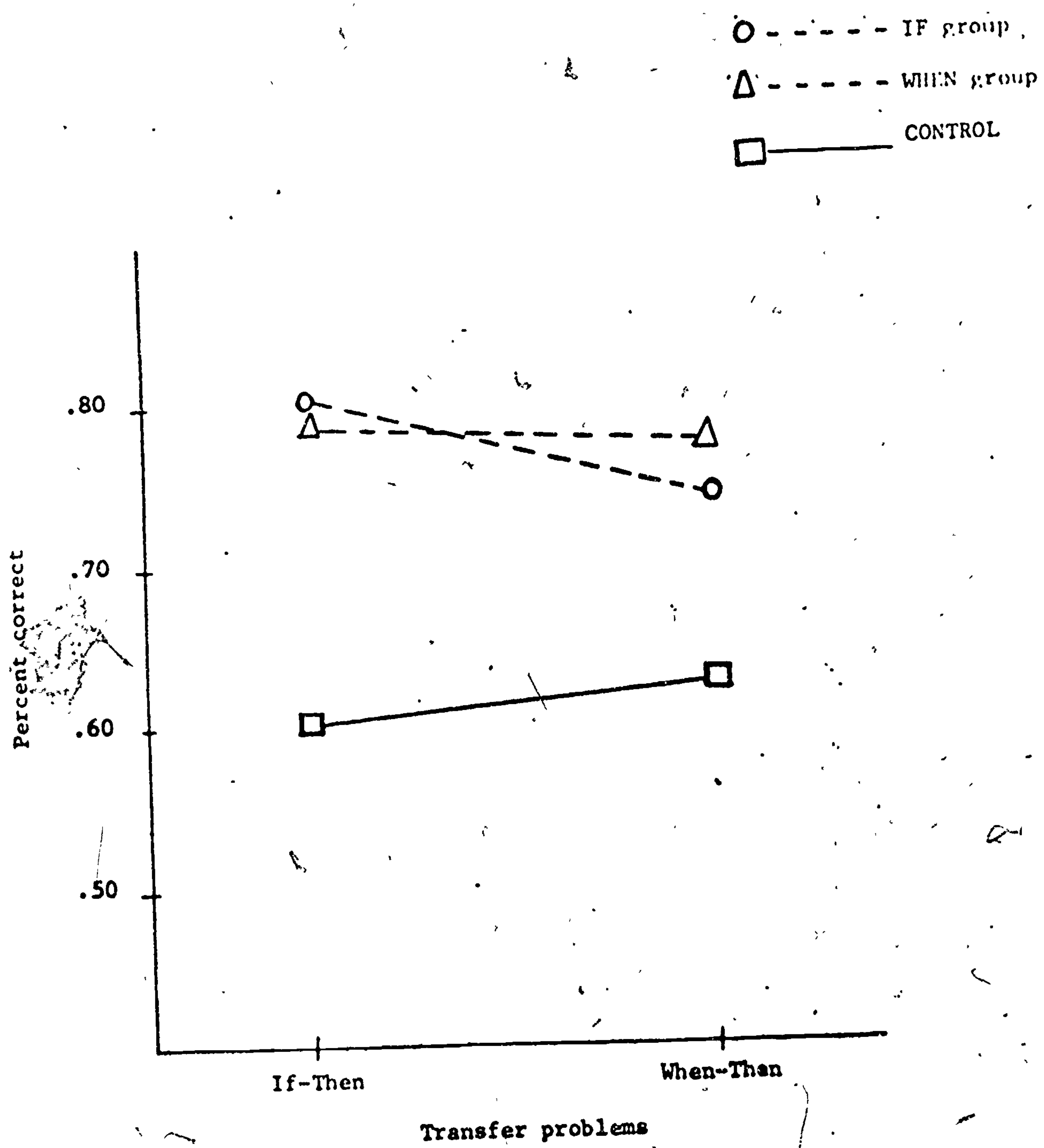


Figure 4

CL5, Grade 2

Percent correct in each group, to the various types of Modus Tollens problems

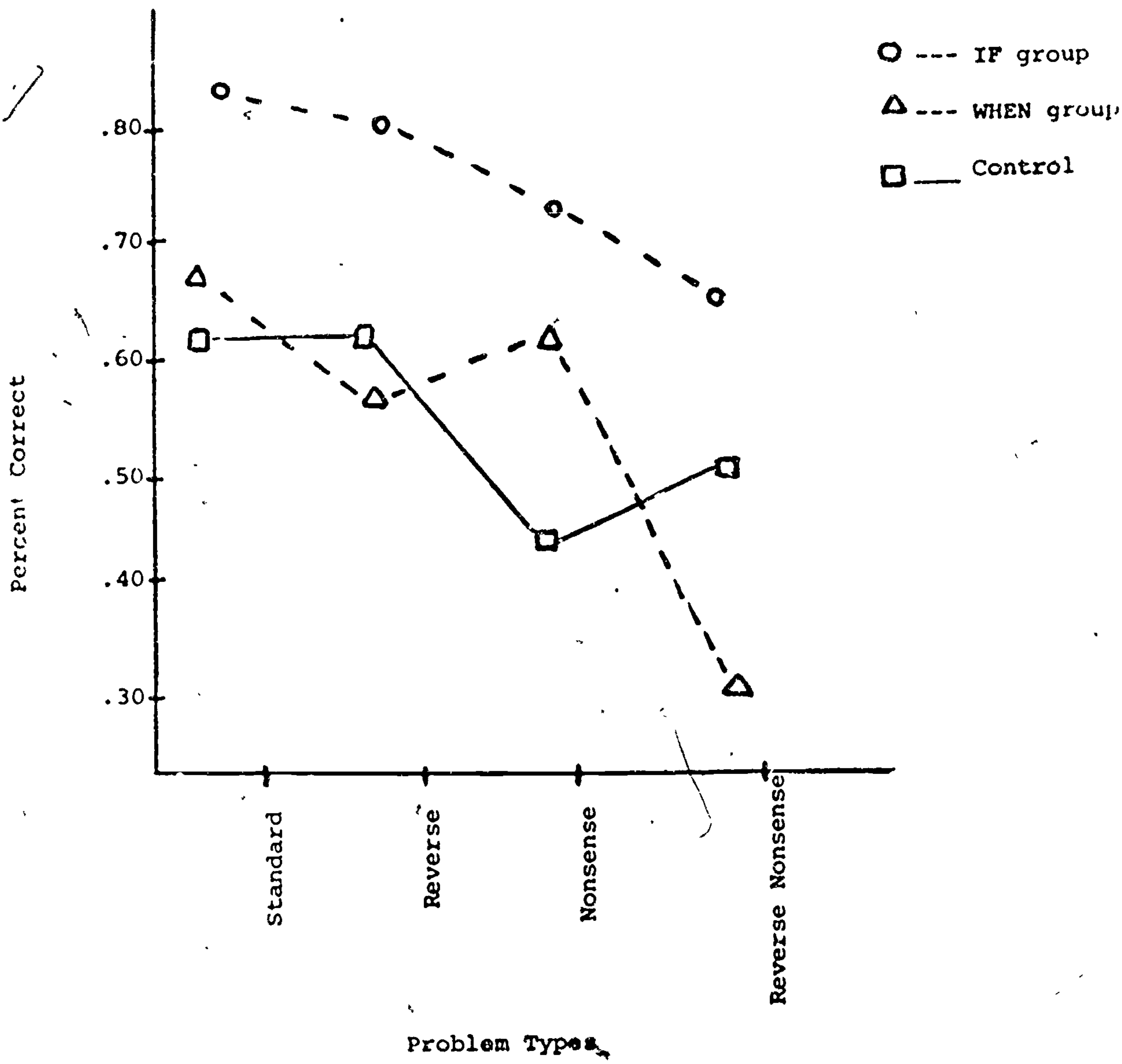


Figure 5
CL5, Grade 2

Percent correct in each group for transfer problems including the "if-then" vs. the "when-then" connective

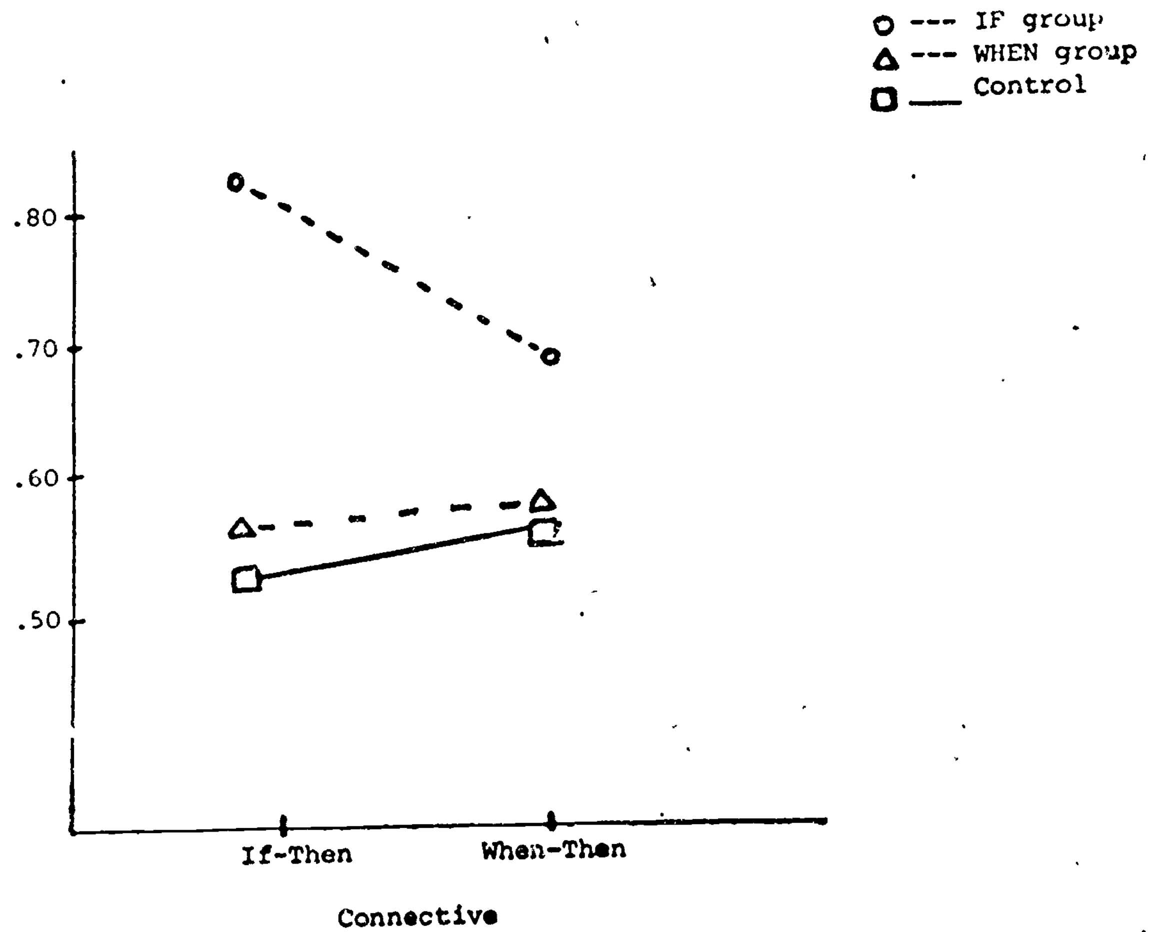


Figure 6
CL5, Grade 2

Percent correct in the first training block, the last training block, and on the transfer task for the IF and the WHEN groups

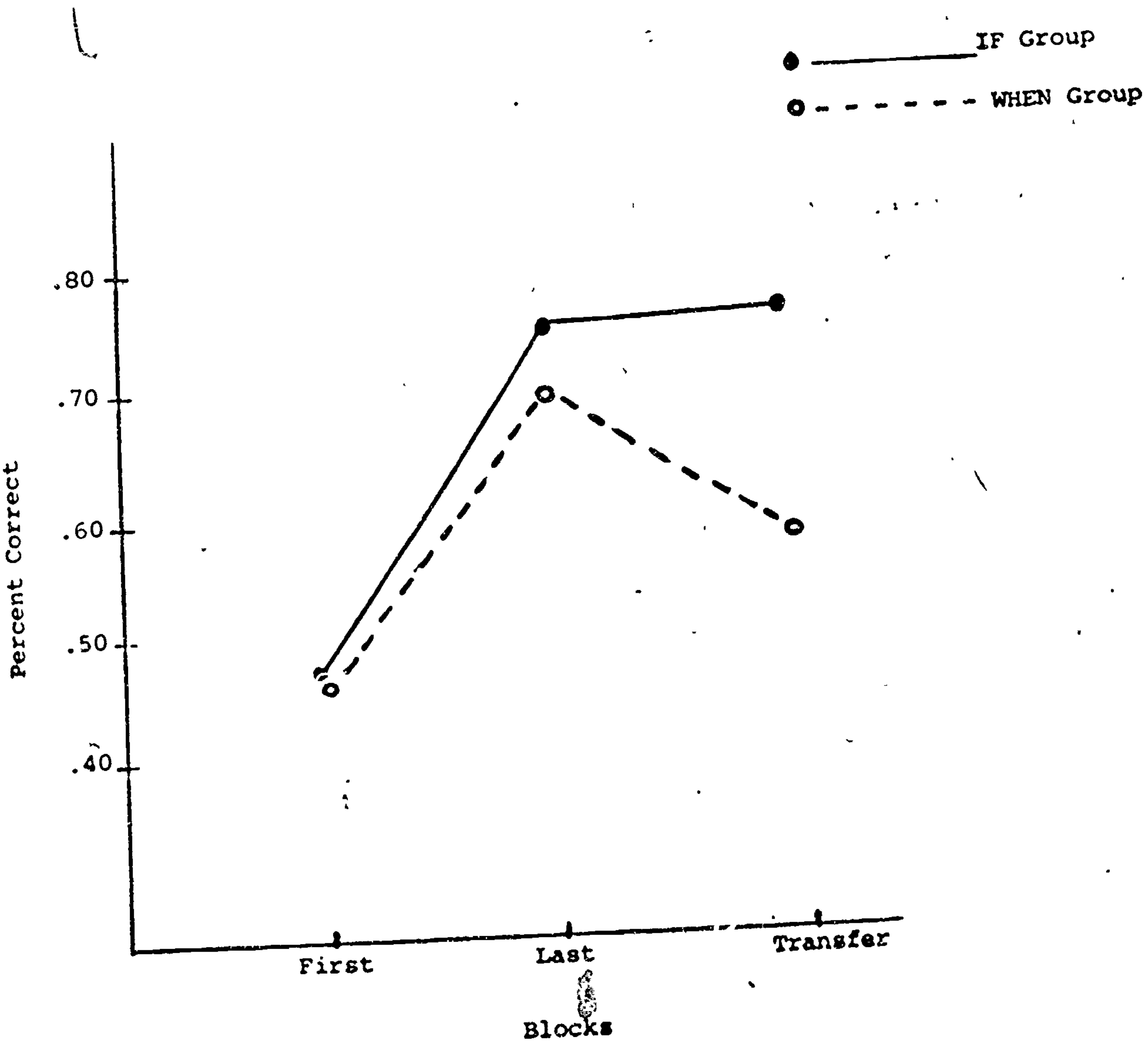


Fig. 7

Grade 6: Percent errors to Modus Ponens in the various conditions

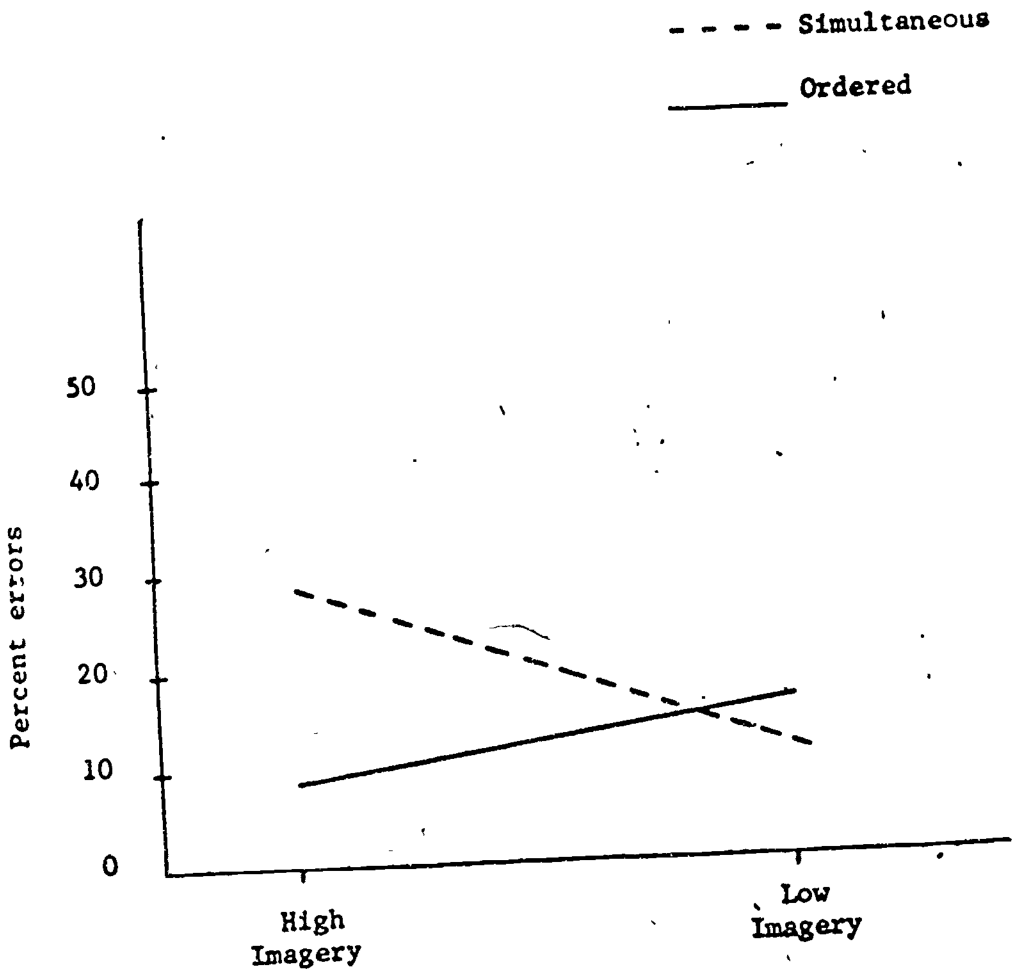


Fig. 8

Grade 6: Percent errors to Modus Tollens in the various conditions

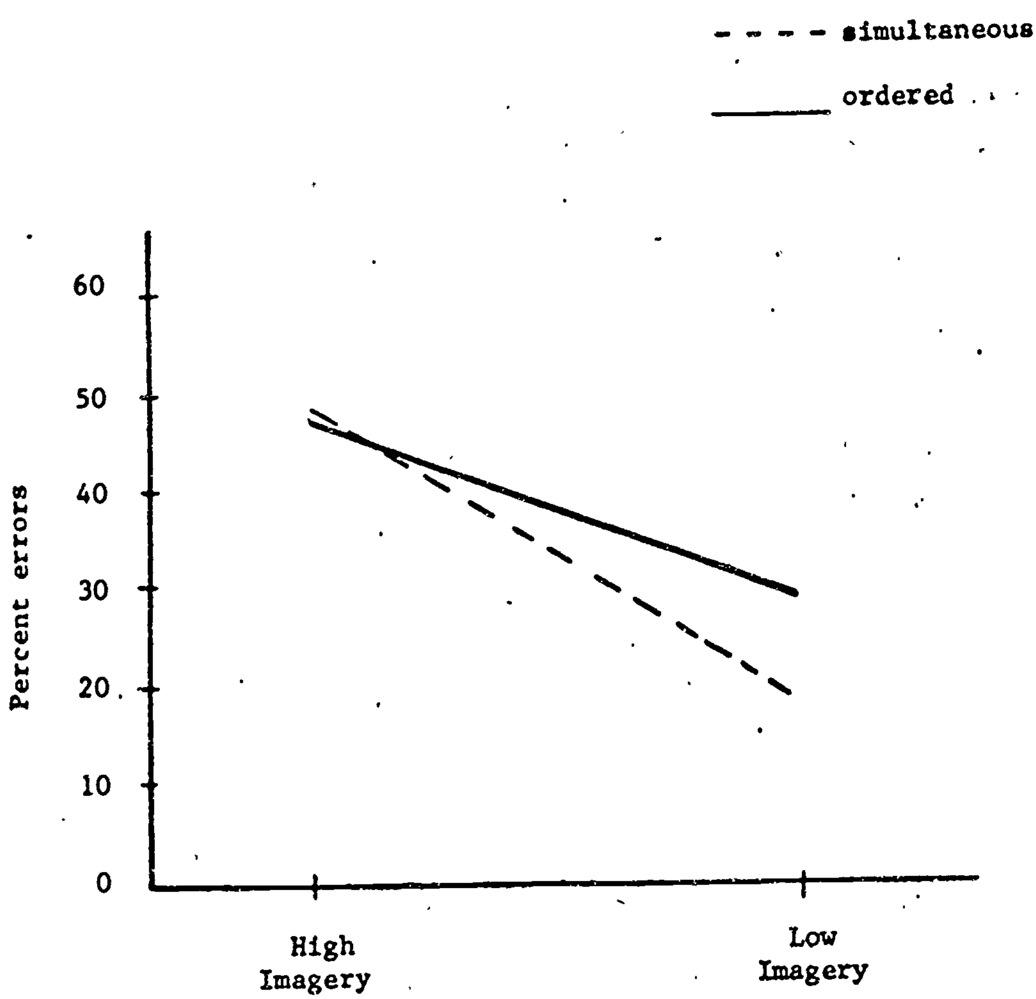


Fig. 9

Grade 6: Percent errors to Modus Ponens (MP_{an}) with negation in the various conditions

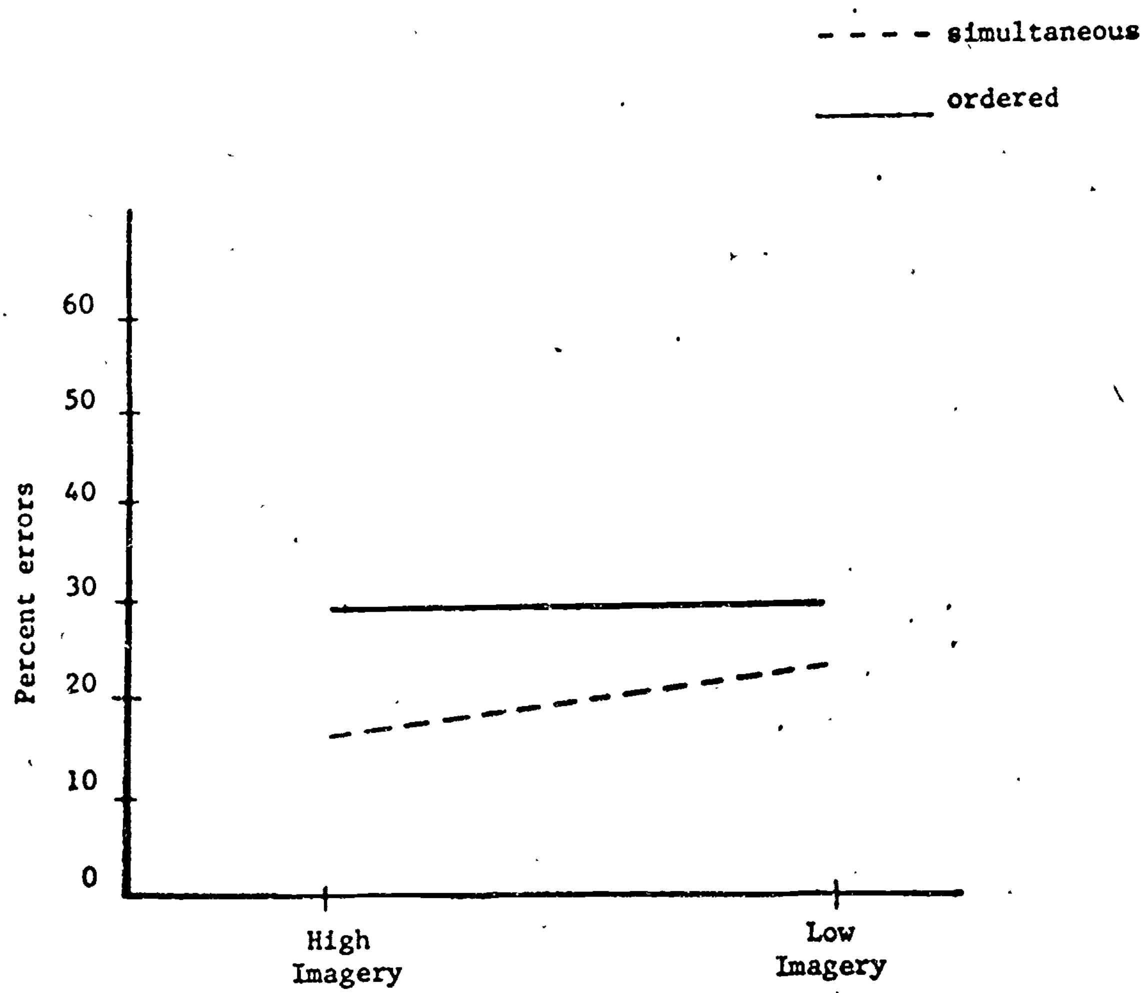


Fig. 10

Negation

Distribution of imagery ratings for

affirmative and negative simple sentences

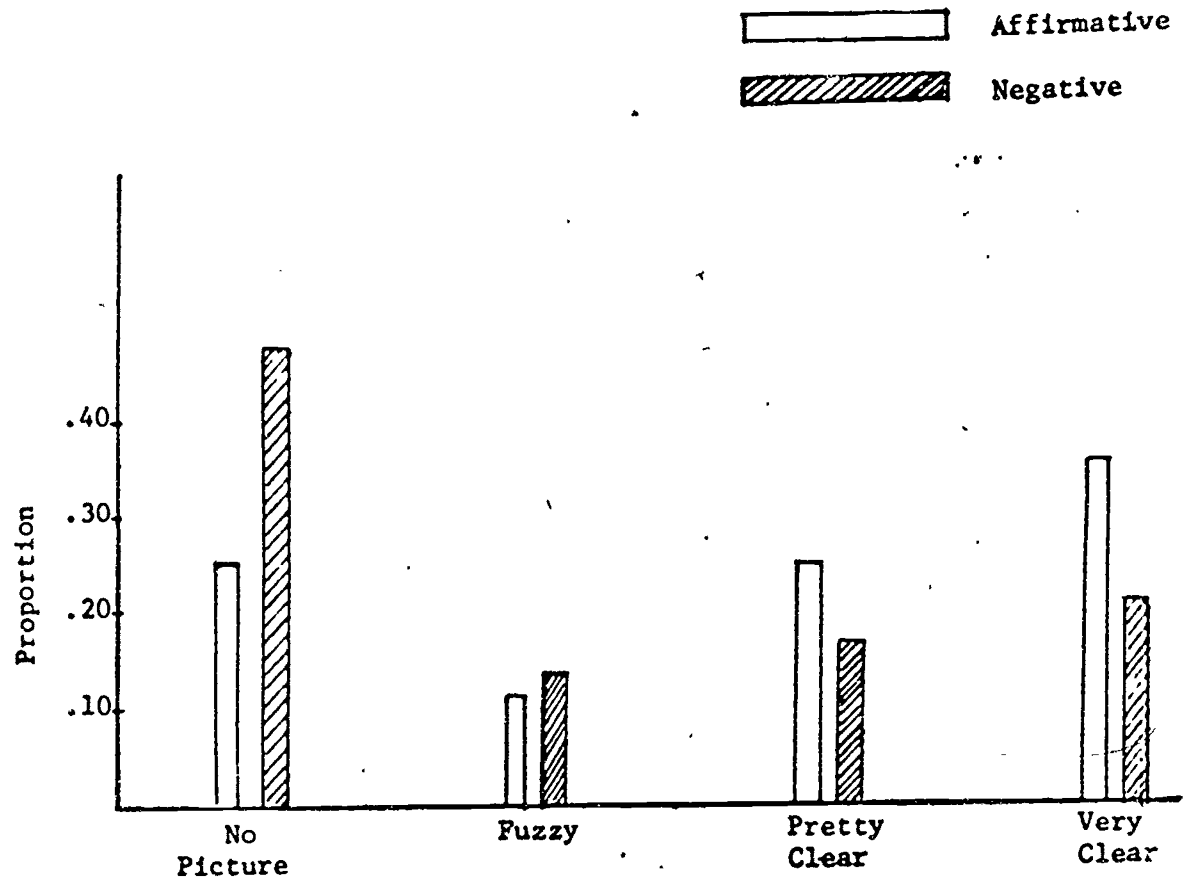
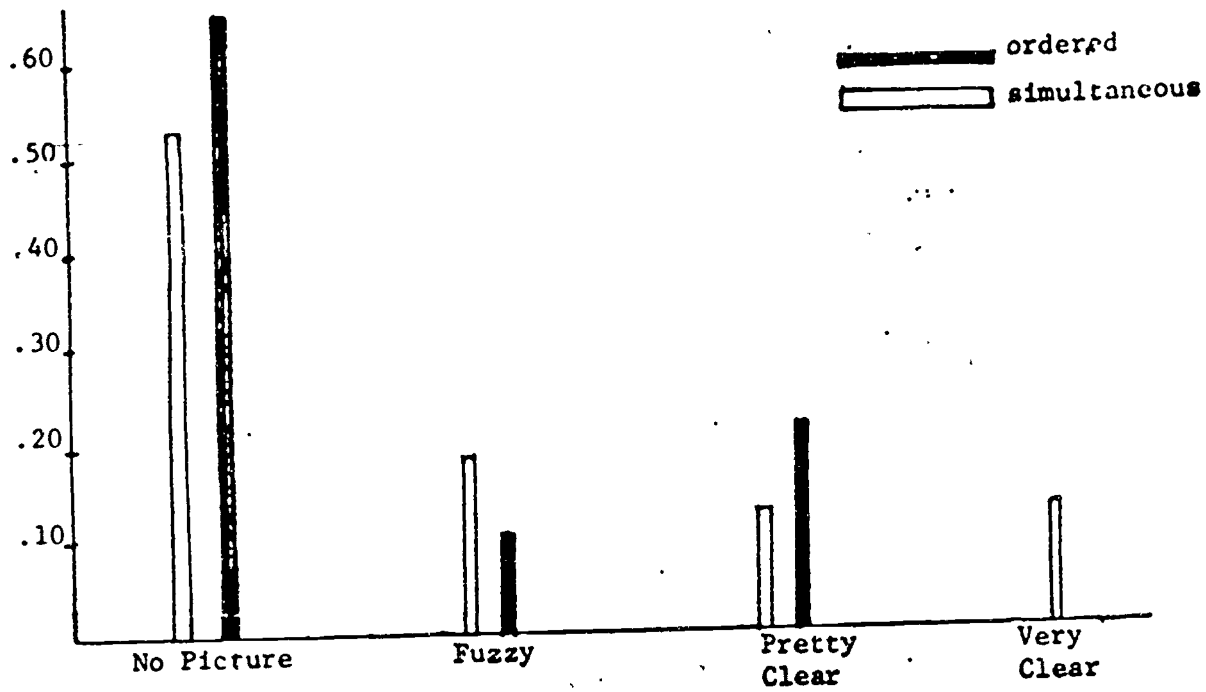


Fig. 11

Conditional sentences" "simultaneous" vs. "ordered"

1) Distribution of imagery ratings



2) Types of images reported

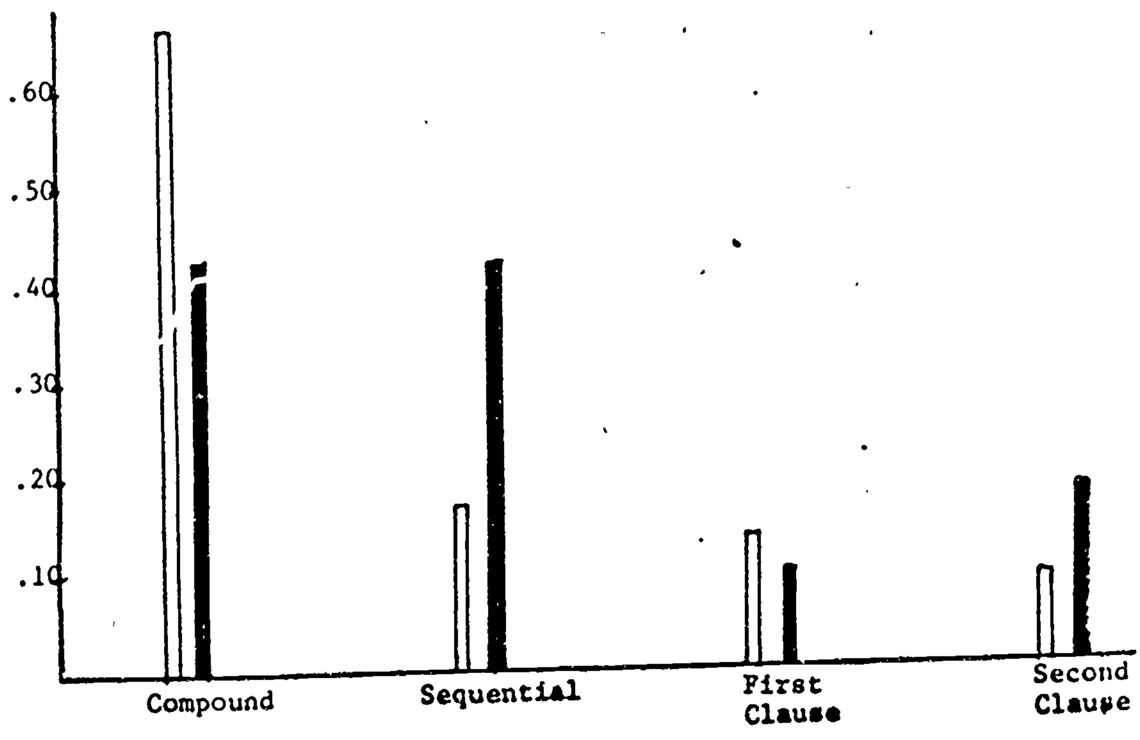
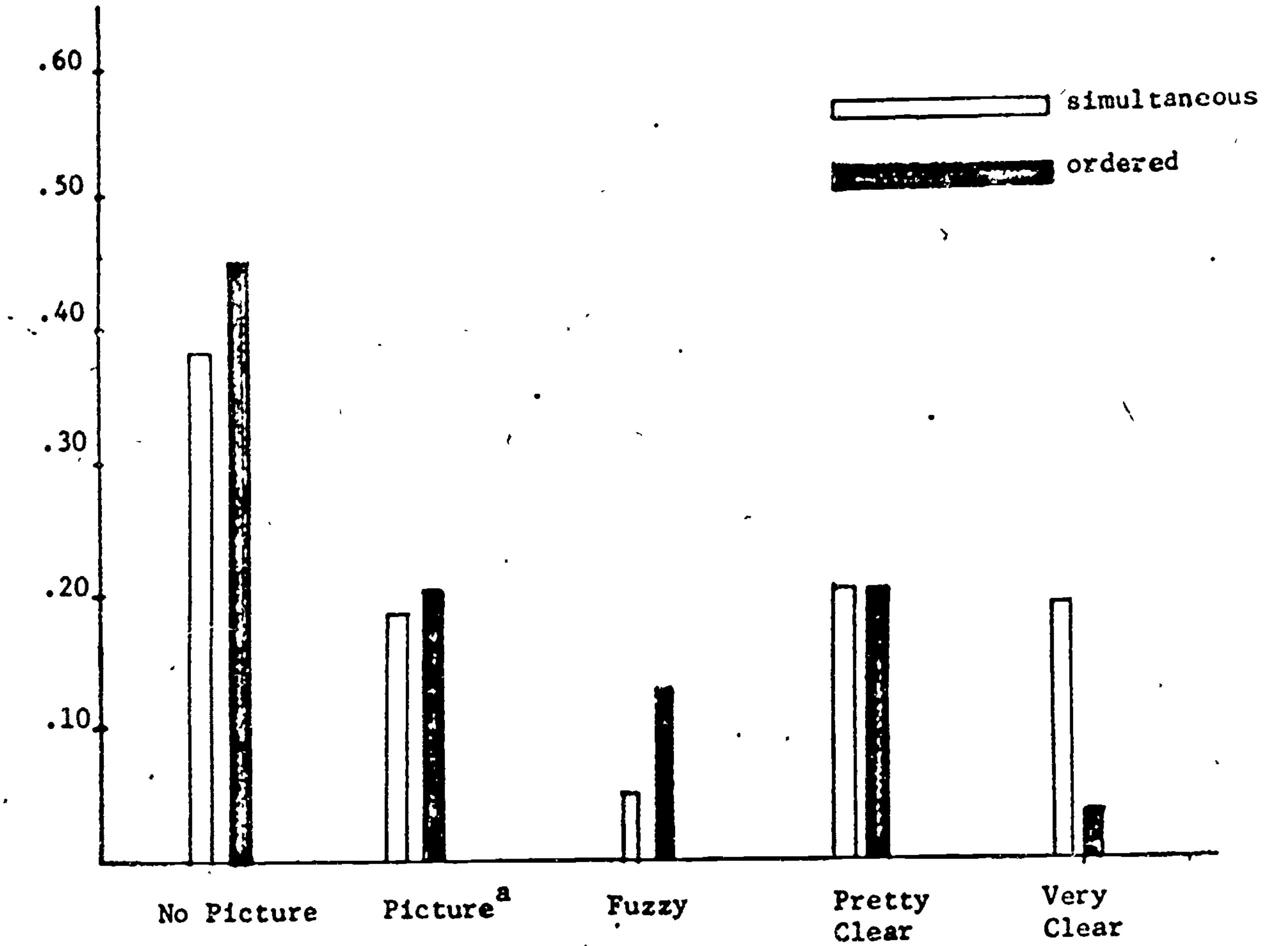


Fig. 12

Imagery reported for problems with "simultaneous" vs. "ordered" material

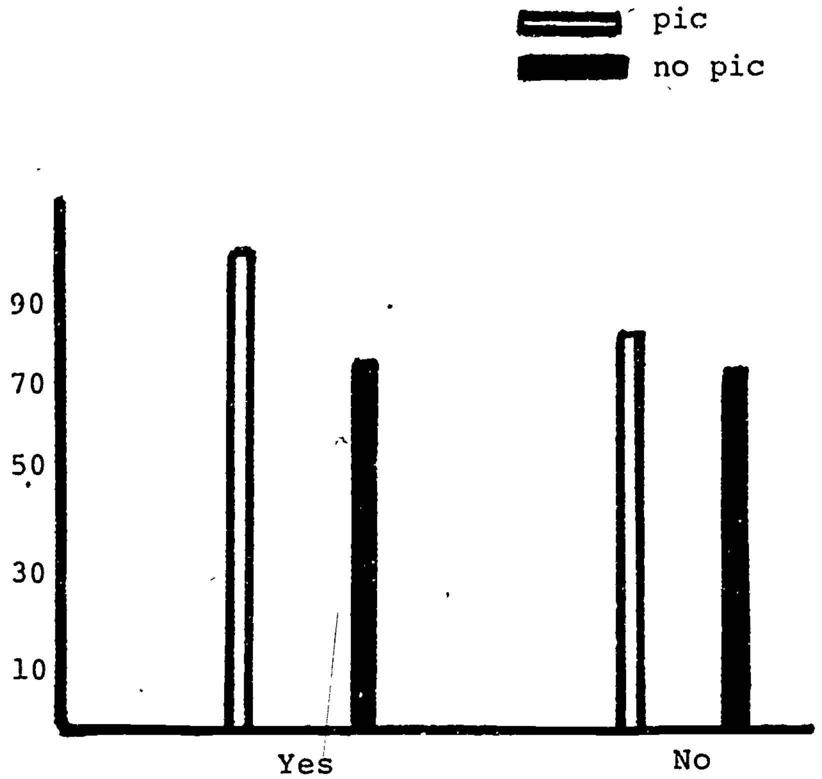


[a] Cases in which the child reported having a picture but couldn't rate it in terms of its clarity.

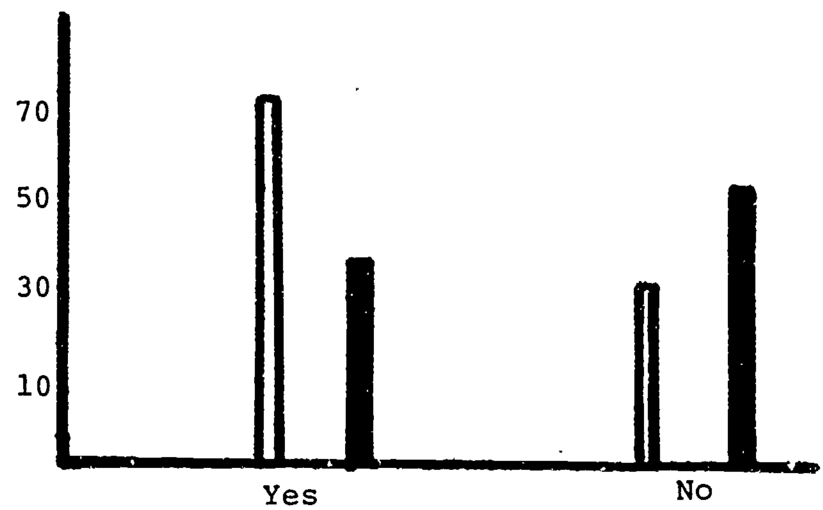
Fig. 13

Grade 6

Proportion of Yes and No as a function of whether an image was reported or not



(a) When yes or no is the correct response



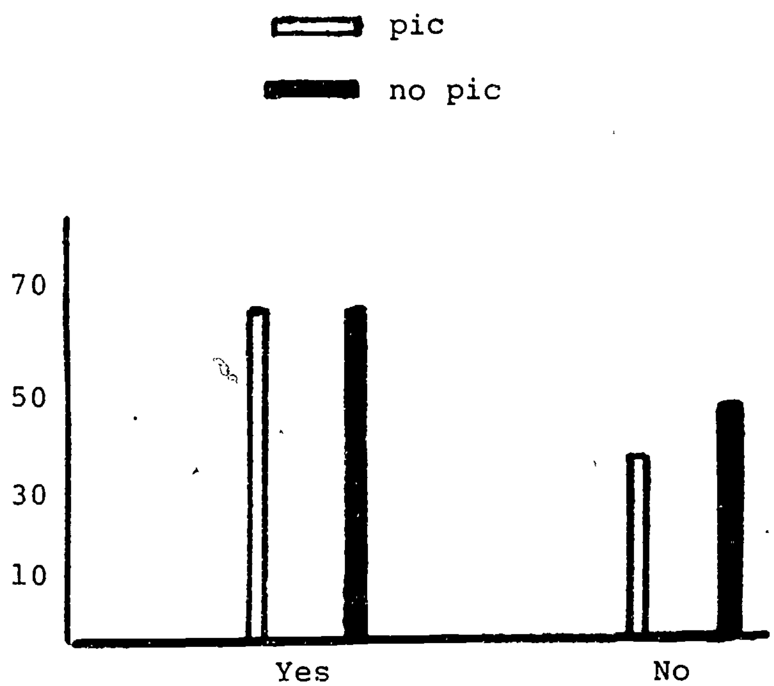
(b) Error to an indeterminate inference

Fig. 14

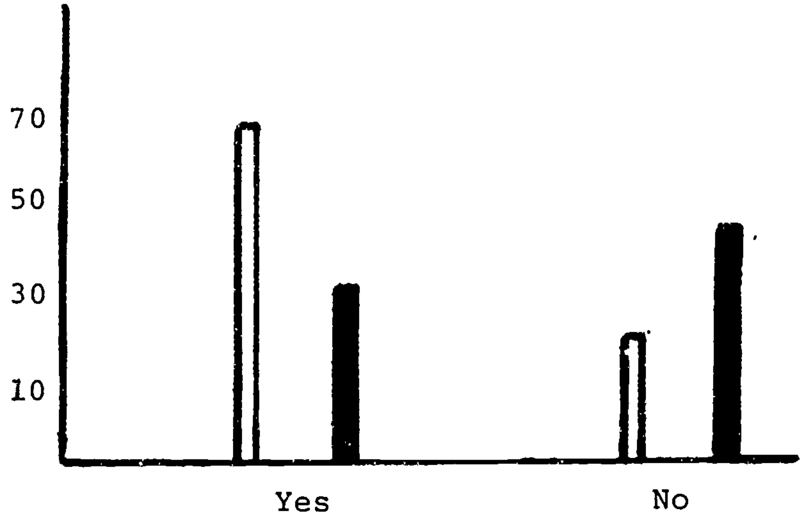
Grade 3

Proportion of Yes and No as a function of whether an image was reported or not

(a) When yes or no is the correct response



(b) When it is an erroneous response to an indeterminate inference



Appendix 1

Problems Made Up By Subjects

IF GROUP	Number correct Modus Tollens transfer problems (Max. = 14)
Subj. No.4	14
Subj. No. 5	12
Subj. No. 8	12
Subj. No. 1	11
Subj. No. 2	11
Subj. No. 3	11
Subj. No. 6	11
Subj. No. 7	11
Subj. No. 10	9

Subj. No.4 — 14

If Poirer is doing his work, then he is not talking to the teacher.
Poirer _is_ talking to the teacher.
Is Poirer doing his work? No.

Subj. No. 5 — 12

If Tara is playing catch, then she is wearing sneakers.
Tara is not playing catch.
Is Tara wearing sneakers? No.

Subj. No. 8 — 12

Mary and Joan are playing with the red dolls at school.
They are not at school.
Are they playing with the red dolls? No.

Subj. No. 1 — 11

Kevin and Bob are the oldest in their families.
Kevin is 13.
How old is Bob? Can't tell.

Subj. No. 2 — 11

If it is 7:30, then Cathy is getting up.
It is not 7:30.
Is Cathy getting up? No.

Subj. No. 3 — 11

If Abby was a monster and you were a person, what would happen? Yes.

Subj. No. 6 — 11

No problem.

Subj. No. 7 — 11

Joe is foodling.
Is Joe Thabbling? Can't tell.

Subj. No. 10 — 9

If John is in the car and the car door is open.
The car door is not open.
Is John in the car? Yes.

(Appendix 1)

IF GROUP

Number correct Modus Tollens transfer problems (Max. = 14)

5

Subj. No. 9

If Joe is watching T.V., then he is eating cookies.
Joe is not eating cookies.
Is Joe watching T.V.? No.

WHEN GROUP

12

Subj. No. 19

If the car is on the side, then it is near the house.
The car is not near the house.
Is the car on the side? No.

11

Subj. No. 16

When Marc is singing, then he is not playing the piano.
Marc is playing the piano.
Is Marc singing? No.

10

Subj. No. 12

Pam's cat is brown in the summer.
Pam's dog is in the yard.
Is the dog brown? Can't tell.

10

Subj. No. 17

John is riding a bike. When John is riding a bike then he is watching television.
John is not watching television.
Is John riding a bike? Yes.

9

Subj. No. 15

No problem.

8

Subj. No. 18

When Miss Mesale is reading, then she is not correcting the papers.
Miss Mesale is correcting the papers.
Is she reading? No.

7

Subj. No. 11

When Kelly is playing kickball she is not eating ice cream.
Kelly is eating ice cream.
Is Kelly playing kickball? No (because you can't eat when playing or you'll choke)

6

Subj. No. 13

If Donna is playing beachball, then she is not watching T.V.
Donna is watching T.V.
Is Donna playing beachball? No.

WHEN GROUP

Number correct Modus Tollens transfer problems (Max. = 14)

Subj. No. 20 — 5

If Kathleen is painting a picture at the
If she is not painting a picture at the beach, she is swimming.
Is she swimming? Yes.

Subj. No. 14 — 3

No problem.

CONTROL

Subj. No. 23 — 14

No problem.

Subj. No. 22 — 10

If Chantal does not want to ride her bike, she rides her skateboard.
Chantal is not riding her bike.
Is Chantal riding her skateboard? Yes.

Subj. No. 26 — 10

No problem.

Subj. No. 24 — 8

John is taller than Mary.
John is taller than Sue.
Is Mary taller than Sue? Can't tell.

Subj. No. 25 — 8

John goes in the car.
The car is blue.
Does John go in the car? Can't tell.

Subj. No. 30 — 8

No problem.

Subj. No. 27 — 7

The bricks are in a pile.
The bricks are not in a pile.
Are the bricks in a pile? No.

Subj. No. 28 — 6

Terri does not ride his bike when he's going to his friends.
Terri is at his friend's.
Is Terri riding his bike? No.

(Appendix 1)

CONTROL

Number correct Modus Tollens transfer problems (Max. = 14)

Subj. No. 29. 5

No problem.

Subj. No. 21 2

Paul is in the schoolyard playing ball.
But he is not playing ball.
Is he in the schoolyard? **Yes.**

Appendix 2.

Teaching of Geometry: Exemplar Training Cycles

Modus Ponens

Preparatory Phase

1. Linguistic

Example (Modus Ponens): If it rains, then I will not go to work.

It is raining.

Therefore, I will not go to work.

2. Language of Geometry

If two sides of a triangle are congruent, then the triangle is isosceles.

In a given triangle, two sides are congruent.

Therefore, the triangle is isosceles.

Remark: At this point the student should be made aware that the above form can more usefully be written as:

If $\triangle$ ABC has two sides that are $\cong$ (this means congruent), then the triangle is isosceles.

In $\triangle$ ABC, side AB $\cong$ side BC.

Therefore $\triangle$ ABC is isosceles.

Writing theorems (etc.) in this manner will serve as a natural transition to the next stage, symbolization.

3. Symbolization

Given: $\triangle$ ABC with AB $\cong$ BC.
Conclusion: $\triangle$ ABC is isosceles.

4. Generalization

If p then q. or even more symbolically $p \longrightarrow q$

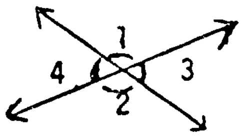
$\underline{p}$ $\underline{p}$

Therefore q $\therefore q$

Instructional Phase

Note: Necessary definitions and concepts are examined prior to the study of any theorem requiring them. Many of the concepts are explored in the preparatory phase, prior to the formal course work. Clearly an understanding of a theorem requires such an exposure to the terms from which it is formed.

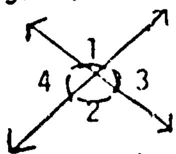
Example: A pair of vertical angles is formed by two intersecting lines such as angle 1 and angle 2;



Angles 3 and 4 are also vertical.

Theorem: Vertical angles are congruent.

Proof: Symbolization: if $\angle 3$ and $\angle 4$ are vertical angles, then $\angle 3 \cong \angle 4$.



1. If two angles form a linear pair (i.e. then "fit" together to form a line) then the sum of their measures is 180° (by definition of linear pair).
2. Angle 1 and angle 3 form a linear pair.
3. Therefore, measure $\angle 1 + \angle 3 = 180°$.

Note the modus ponens

4. Similar steps using modus ponens yield measure $\angle 1$ + measure $\angle 4 = 180°$.

Modus ponens

5. Measure $\angle 1$ + measure $\angle 3$ = measure $\angle 1$ + measure $\angle 4$.

(By the transitive property since they are both equal to 180°)

6. If equal amounts are substracted from equal amounts, then the remaining amounts will be equal (subtraction property of equality).
7. Subtract measure $\angle 1$ from both sides of the equation in step 5.
8. Therefore, measure $\angle 3$ = measure $\angle 4$.

Modus ponens

9. Conclusion $\angle 3 \cong \angle 4$ (two angles with equal measures are congruent).

Training Cycle: Other Logical Forms

Modus Tollens: If it is nighttime, then it is dark outside.

It is light outside (i.e. it is not dark outside).

Therefore, it is not nighttime.

Generalization: $p \longrightarrow q$

not q

Remark

not p $= \sim p$

$\therefore$ not p

Law of the Excluded Middle

It is either raining here or it is not raining here.

Generalization: p or $\sim p$

Hypothetical Syllogism

If it is raining then I will not go to work.

If I do not go to work, then I will not get paid.

Therefore, if it is raining, then I will not get paid.

Generalization:

$p \longrightarrow q$

$q \longrightarrow r$

$\therefore p \rightarrow r$

Statements and Contrapositives

If the fruit is an apple then it is red.

If the fruit is not red, then it is not an apple.

Generalization: $p \longrightarrow q$

$\sim q \longrightarrow \sim p$

Converses

If the fruit is red, then it is an apple.

$q \longrightarrow p$

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"The Development of Logical Competence: A Psycholinguistic Perspective,"
by Rachel Joffe Falmagne, Developmental Models of Thinking, New York:
Academic Press*, 1980, pp. 171-197.

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