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ABSTRACT

Efforts have been underway since 1973 to determine if there are elements of Navajo culture that potentially impact upon a Navajo student's attainment of mathematics concepts. Extensive reading and discussion with Navajo students and educational leaders on the Navajo reservation show that basic concepts and objectives of Navajo philosophy (a quest for unity, harmony, order, and beauty) appear to be entirely consistent with views of the greatest contributors to the development of mathematics. However, there are no words for "multiply" and "divide" in Navajo, and no agreed upon word for "if," and difficulties arise when Navajo students handle situations calling for use of those functions or syllogistic reasoning. While Anglo folklore has many references to numbers (e.g., the three little pigs), Navajo folklore is more personal. Sentence word order (nouns in particular), the role of imaging in problem solving, clan characteristics, and cultural views of geometry pose additional hazards. Navajo culture has traditionally been transmitted orally, and it is possible that the students' facility for memorization has not been fully appreciated. The teacher should have visual classroom models of certain basic mathematical entities, and should make every effort to close the gap between the hypothesis form of teaching and the real world of the Navajo. (BRR)

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THE NAVAJO CULTURE AND THE LEARNING OF MATHEMATICS

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The Navajo Culture and the Learning of Mathematics

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Introduction. It is the intent of this report to describe some observations that have been made during an investigation of those elements of the Navajo culture that potentially impact either positively or negatively upon a Navajo student's attainment of mathematics concepts. It is not anticipated that the contents of the report will form the rationale for hypotheses leading to controlled experiments. Most of the observations to be made are of subtle cultural variations which, when understood by a mathematics teacher, may lead to an enhancement of his understanding of his Navajo students as individuals and thereby to an improvement of the classroom atmosphere for teaching and learning.

Basic Cultural Climate. I wish to state at the outset that as the result of extensive reading and discussion with my Navajo students, both former and present, and also as a result of talks with educational leaders at centers of education on the Navajo reservation, the basic concepts and objectives of the philosophy underlying the Navajo culture do not appear to be incompatible with, but appear to be entirely consistent with, those views of the greatest contributors to the development of mathematics. For example, assuming that the philosophical views of the Navajo singers are representative of those of the Navajo people as a whole, Reichard concluded: "All chanters share the same assumptions...a common belief in the universal order, assurance that man has or may obtain power to fit into the world securely and smoothly and faith in their power to correct error when it becomes necessary to reduce the friction generated by ignorance of the universal machine."[1]

Through both a study of the literature and in discussions with Navajo acquaintances I find this quest for unity, harmony, order and beauty to be a powerful influence upon which their philosophy for daily living is based. Descarte, working to unify the concepts of algebra and geometry, or Kepler, searching for a mathematical model which would be in harmony with observations of the movement of the planets, or Einstein striving for a unified field theory which would eliminate apparent inconsistencies between the action of objects moving at everyday speeds and those moving at velocities approaching the speed of light, would have felt entirely comfortable with the view of the chanter as described by Reichard.

These observations, along with the fact that numerous Navajo students can be identified who have proven themselves to be exceptional mathematics students, would tend to indicate that any differences between the Navajo and Anglo cultures which would impact upon the learning of mathematics would be incidental and specific rather than fundamental and pervasive. Nevertheless, if it is possible for the mathematics education of Navajos to be improved through the increased awareness of the cultural background of his student, then no stone should be left unturned in an attempt to achieve this end. On the basis of the information to be partially described in this report, it has become easier for me to establish a friendly, personal relationship with Navajo students in my mathematics classes. If better personal relations can be established then communication is facilitated and the way is opened for improved teaching and learning.

<u>Geographical Setting of the Study</u>. Funds made available through a grant from the National Institute of Education enabled me to do background reading, conduct on-campus interviews and to talk to leaders at centers of education throughout the Navajo reservation. The investigation was essentially an ex-

tension of research begun in 1973-74 and funded then and in intervening years by Northern Arizona University.

Approximately 130,000 Navajos live on the reservation which spans an area of almost 250,000 square miles and contains portions of three states. The area is rich in minerals, including one-fourth of the coal found in the United States. Young Navajos trained in the technical fields are sorely needed to assist in the intelligent development of these natural resources in a manner consistent with the objectives of the Navajo nation.

Language and Research in Mathematics Education. A potential researcher in the area of the relation of culture to mathematics learning should be aware of the "Whorfian hypothesis". Benjamin Lee Whorf was a linguist who made extensive studies of the language of the Indians of the southwest in the 1920's. He assumed and asserted that the characteristics of an individual's native language impacts upon the manner in which he approaches a problem solving situation. If this hypothesis is valid the mathematics educator has a right to delve into a language for clues. Not all linguists accept the Whorfian hypothesis, however, claiming that any difference in mathematical performance could be attributed to individual motivation and social factors. I shall proceed on the assumption that the Whorfian hypothesis is valid, citing two authorities in addition to my own observations.

Benjamin Whorf wrote of the Hopi language, "Hopi, with its preference for verbs, as contrasted to our liking for nouns, perpetually turn our propositions about things into propositions about events..."[2]

Kluckholm and Leighton state in regard to the Navajo language, "The structure is too different. The lanaguage of The People represents an importantly different mode of thinking and must be regarded as such."[3]

Examples of Language Related Difficulties. It was during a discussion of my research with Dr. Paul Rozier, a linguist residing on the reservation, that I received a clue to a language difference that may influence the learning of mathematics by Navajo students. He mentioned that there is no word for "multiply" in the Navajo language. At the time, I made a note of this fact and considered it to be an interesting bit of information. However, in a later discussion with Dr. Ward of the Northern Arizona University Physics Department, the interesting, though isolated, note concerning the lack of a Navajo word for "multiply" began to acquire new significance.

Dr. Ward was teaching an elementary physics course in which many Navajo students were enrolled. The course was based largely upon the concepts of Jean Piaget. One test problem was to determine the volume and surface area of a two-inch cube. The answers were 8 cubic inches and 24 square inches respectively. The following problem was to assume that the cube was divided by three cuts into one inch cubes and again to find the total volume and surface area. If the students were conservers, the answer of eight cubic inches for the volume was immediate, but the new total surface area required some thought. Dr. Ward reported that one young Navajo student, who was a good student, solved the problem of the area by drawing an expanded view of one of the one-inch cubes and noting correctly that its surface area was six square inches. He then repeated this expanded drawing seven more times and counted the one-inch squares (1 through 48) arriving at the correct answer of 48 square inches. The point is that even though he was a good student, he did not think to multiply the area of one of the one-inch cubes by the number of cubes or if he did consider it; he did not have enough confidence in the operation to rest his answer upon it.

Following is another example brought to my attention by Dr. Ward: One Indian girl, again a good student, was to solve a problem, part of the solution of which involved the division of 13 by 3. At the point where the division was to be accomplished, the girl (who knew how to divide) subtracted 3 from 13 getting 10. She next subtracted 3 from 10 getting 7. The process was continued until she had recorded that 13 contained four threes with one left over.

The latter example suggested that the Navajo language also lacked a word for "divide" and this suspicion was quickly verified by a dictionary. In each of the foregoing examples, the failure to carry out a formal multiplication or division occurred within the course of solving a more complicated problem. In these instances, the concepts of multiplication or division were not readily available when needed. Possibly the students' confidence in the formal processes was not adequate to make them useful tools as a result of an inadequate concept of those operations. Such an inadequately developed concept may have been due to the fact that in the language first learned by the student, a name for the concept had been absent. This possibility will require more investigation.

I shall now describe what is possibly another problem caused by an imperfectly developed concept of division. A former student, now working in the Engineering department at a large coal company reported that the engineers in that office had noted that the Navajo assistants had trouble with the trigonometric concepts of sine and cosine. The engineers thus had hypothesized that their difficulty was due to the absence in the Navajo language of a word for sine or cosine. However, upon a little reflection, it is noted that the words "sine" and "cosine" mean no more to the Anglo student than the Navajo student when first introduced in trigonometry. The real difficulty for the

Navajo student may arise because the names "sine" and "cosine" refer to ratios or divisions which may have been inadequately formed concepts in the assistants' first language.

Langauge and the Hypothetical in Mathematics Education. Dr. Carroll, a linguist at Northern Arizona University, made me aware that in the Navajo language there is no word for the concept "if". When one considers that the "if" concept is basic to any field, the logical foundations of which are deductive or hypothetical patterns of thought, the absence of a word for "if" in the native language can present potential difficulties in certain problem solving situations. Furthermore, there is some evidence that hypothetical thinking is not readily accepted by some Native American students. Dr. Carroll gave the following example of responses obtained when attempting to teach syllogistic reasoning to Indian students. The problem was: "All bears that live within 1000 miles of the North Pole are white and bear B lives 700 miles from the North Pole. What color is bear B?" One answer was "brown". When Dr. Carroll inquired as to why this answer was given, the student replied, "When I went to the zoo in Phoenix last summer I saw a bear and it was brown". The student had not accepted the hypothetical situation upon which the problem was based.

Professor Bill Leap of the American University cited an example of the same mode of thought at a meeting at Navajo Community College. The problem given to his Navajo students was:

If your grandfather drove his truck forty miles per hour for three hours how far would he travel? The response was: "Grandfather doesn't have a truck".

Sister Michael, a teacher of mathematics at Winslow, Arizona, noted that her students have difficulty in fantasizing.

Research at Northern Arizona University. The lack of a Navajo word for "if" may also be part of the cause for the poor performance of Navajo students at Northern Arizona Unviersity on a test of their grasp of the concepts of conservation as defined by Jean Piaget. In that study conducted by the author in 1973-74 [15], a set of six demonstrations were developed, each designed to test the student's understanding of a different conservation concept. The concepts were:

1. Conservation of substance, shape varied.
2. Conservation of weight, shape varied.
3. Conservation of displacement, shape varied.
4. Conservation of displacement, weight varied.
5. Conservation of area, arrangement varied.
6. Extinction of conservation of weight.

Of possible significance is the format of the demonstrations. At the start of each demonstration the students were presented with a particular physical situation; e.g., two balls of clay of equal size and weight, two metal cylinders of equal size and shape but of different metal, etc. A transformation was then made on one of the physical objects and the students were then asked what the result would be "if" the other object were treated in the same way. The students would then check an answer and write out their explanation for their answer. The demonstrations were presented to 284 students ranging from freshmen to seniors.

At the time the research was conducted, I had no hypothesis whatsoever concerring our Indian students and their development with respect to conservation concepts. However, a cursory examination of the first few sets of completed answer sheets provided a rationale for the null hypothesis that Indian students have no more difficulty answering questions concerning conservation tasks than Anglo students.

In order to test the proposed hypothesis, two classes consisting entirely of Indian students, mostly Navajos, were identified and those students were presented with the same set of demonstrations and answered the same set of questions. The percentages of incorrect responses were computed and compared with those of the general student body tested earlier. That comparison is exhibited in Table 1.

Table 1. Comparison of Incorrect Responses to Conservation Demonstrations

Task No.	Proportion of Incorrect Responses Indian	General
1	.33	.03
2	.64	.17
3	.67	.25
4	.33	.16
5	.20	.06
6	.94	.74

For each conservation concept, the hypothesis that Indian students make no more incorrect responses on those six conservation tests than the general students was rejected at the .01 level of confidence.

Worthy of note is an observation made when studying the answer sheets of the Indian students. Those who had responded correctly had written explanations that were entirely as correct, rational and lucid as any of those in the general student population. In subsequent interviews with those students, they appeared to be amazed that not everyone had answered all of the questions as easily as they. Although there were differences in their thoughts concerning the physical transformation of the demonstrations, those differences apparently had never been detected by students who were friends on a social basis. This would tend to indicate, as suggested earlier in the report, that these differences are superficial and could possibly be remedied by teacher understanding and appropriate modifications of methods of teaching basic mathematics concepts rather than by drastic revisions in the curriculum or in teaching techniques.

Follow-up Research. Although the proportions of incorrect responses to the conservation tasks was not out of line with results of conservation research with students at other universities, they did seem rather high. It was thought that perhaps the poor proportion of correct responses results from the students' never having actually tested the concepts on a physical level. Follow-up research was conducted during the subsequent year to test the hypothesis "If students have conducted tests of the conservation concepts under question on physical materials with mechanical apparatus they will thereafter have a firmer grasp of those concepts".[16]

Fifty sets of balance scales were constructed. The individual members of classes of undergraduates were provided with a balance scale and a quantity of clay. They performed experiments where they each tested personally the conservation concepts under consideration and together with the teacher they identified, stated explicitly, and agreed upon each conservation concept they had confirmed; e.g., "If two quantities weigh the same and are changed only in shape then they will still weigh the same". Thus, the principal investigator knew the students had physical experience with the conservation concepts to be tested.

After a period of two weeks the original six demonstrations were presented to those classes. The results showed no improvement over results obtained from students who had not had this experience. The hypothesis was rejected and it was concluded that the source of difficulty with conservation concepts was rooted somewhere in their earlier childhood environment.

Implication of Conservation Concepts. The implications of a poorly developed concept of conservation for the learning of mathematics are not difficult to understand. We may write on the board an equation like

$$ax^2+bx+c = 0.$$

We ask the students to believe that the two sides of the equation are equal when they do not even look equal. The two balls of clay were weighed before the eyes of the students and they could see that the balls of clay were equal and they themselves decided when they were equal with respect to weight. Nevertheless, they had difficulty believing that the clay was still equal in weight when one ball was transformed in shape (or some other transformation was performed). Now assume that we make a series of transformations on the equation mentioned above such as

$$x^2+\frac{b}{a}x+\frac{c}{a} = 0$$

$$x^2+\frac{b}{a}x \qquad = -\frac{c}{a}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2} = \frac{b^2-4ac}{4a^2}$$

We ask the students to believe that equality is conserved through each transformation by some set of abstract logical principles and that the student should accept the final conclusion

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

It is almost ludicrous to expect the students to believe that an equality which is no more than a mathematical assertion in the beginning is conserved through a series of algebraic manipulations based upon successive applications of abstract logical principles when they do not even have a firm grasp of the conservation of equality on a physical level when a single physical transformation is applied. Consequently, instead of our students learning mathematics (conclusions based upon the deductive process), they tend to learn to accept those conclusions and believe those ideas they perceive as being pleasing to the professor.

If the foregoing indictment of our methods of teaching the scientific method is valid to any degree, the situation is even worse for any group of students for whom the concept of conservation may constitute a special problem.

Folklore and Mathematics Education. The difference between the emphasis of the folklore of the Navajo and the Anglo students is of potential relevance to the investigation. The folklore of the Anglo provides many opportunities for the student, in his early years, to contemplate numbers and their meanings. There was Snow White and the seven dwarfs, three wise men, three bears, three little pigs, twelve disciples, the dimensions of Noah's Ark and Soloman's temple and the unlucky number thirteen.

On the other hand, the folklore of the Navajo appears to be more personal in nature with the emphasis often being placed upon the characteristics of certain animals such as the coyote, badger, or deer. Very few number concepts are encountered in the book, Coyote Stories of the Navaho People. [4]

The conjecture of the preceding paragraphs concerning the folklore of the Navajo loses some of whatever validity it possessed upon encountering some of the Navajo attitudes toward numbers that parallel those of the Anglo culture.

One finds the uses of the number four occurring frequently in Navajo rituals. Examples contrasting the ritualistic use of numbers by Anglos and Navajos follows:

1. Anglo stars have five points.[5]
   Navajo stars have four points.
2. The Anglo starts a race with '1,2,3,go!' The Navajo starts with '1,2,3,4,go!'[6]
3. For the Anglo, the third time is the charm. For the Navajo, success usually comes on the fourth attempt.[7]

In Navajo religion, a distinction is also made between even and odd numbers. The rule is that blessing and divinity are represented by even numbers, evil and harm by odd.[8]

The probable reason for groups of five in the sandpainting of the Bead Chant is that the major figures are predatory animals.[9]

The degree to which these uses of numbers in rituals becomes part of a Navajo child's folklore is a question which could form the basis for interesting research.

Memory in Mathematics Education. The non-literate heritage of the Navajo is also of possible importance in relation to the question of mathematics education. An Anglo freshman student beginning his studies at a university may feel overwhelmed by it all but he knows that higher education is part of his heritage even if the members of his immediate family did not attend a college. This knowledge provides him with a source of confidence and support.

By contrast, the Navajo freshman does not have such knowledge to comfort him. The Navajo student may have had a famous singer for a grandfather, a fact of which he can be justly proud. The singers provided a very important humanitarian service by memorizing long, intricate rituals and legends thereby preserving a rich culture. However, they did not write research reports and study the delta-epsilon definition of limit.

Gladys Reichard wrote of the mental feat of Navajo ladies who weave rugs without reference to a pattern.

> "The weaver must keep the composition of the entire rug surface in her mind, but she must see it as a huge succession of stripes only one weft strand wide. It matters not how ideal her general conception may be, if she cannot see it in terms of the narrowest stripe, meaning a row, of properly placed wefts, it will fail of execution." [10]

I have recently heard my colleagues who are mathematicians complain that their mathematics memory is not as good as they wish, and that their research would progress more smoothly without this problem. The Indian student has the advantage of a heritage where a highly developed memory was of paramount importance if accurate transmission of the culture were to occur. It is possible that the Indian student's facility for memorization has not been fully appreciated in the mathematics classroom.

<u>Twelve Commandments for Teachers of Traditional Navajo Students</u>. I am indebted to mathematics teachers at Monument Valley High School in Kayenta, Arizona who were kind enough to prepare the following set of guidelines for mathematics teachers to abide by in teaching their traditional Navajo students.

A. Do <u>not</u> "expect your students" to acquire certain skills.

B. Do <u>not</u> determine that your students "need to do something".

C. Take your time explaining short cuts in detail.

D. Allow students to talk openly in Navajo about math concepts.

E. Be able to count to ten in Navajo.

F. Do <u>not</u> single out one person for something they did not finish, but mention three or four people who did not finish the same thing.

G. Find time to get to know your students.

H. Encourage traditional students with their success daily.

I. Compliment traditional students for the good work that they do.

J. Have a simple grading system based on achievement, not a curved system based on competition.

K. Type all tests.

L. Be courteous to and show consideration for your traditional students at all times. (Give more attention to your traditional students than to those who are not traditional in your classroom setting.)

Signed by Tommie Yazzie
Assistant Principal
Monument Valley High School
Kayenta, Arizona

Discussion of Language and Mathematics Education. Critics of my investigation say that I have overemphasized the importance of language as a determinant of thought processes in mathematics education. One opposing view is that any identifiable differences in performance in mathematics between cultural groups is due solely to variations in motivation and social outlook.

The study often cited in support of the opposing view is that of Bernstein[11] who identified two styles of language used in England. The elaborated mode was used by children of the professional classes while the restricted mode was used by children of the working classes. Educational programs based upon these differences proved to be unsuccessful. I recognize the study and its implications but do not believe that the differences between the elaborated mode and the restricted mode in England is closely analogous to the differences between the Navajo language and scientific English used in mathematics. I shall now cite another example indicating the manner in which the structure of a native language can impact upon the interpretation of a concept expressed in a second language.

David Witherspoon, in his book Art and Language of the Navajos[12] discusses the importance of the order of the nouns in a sentence in Navajo. The rule is that the first noun mentioned holds the dominant position. As an example, he considered the two sentences:

1. The girl drank the water.
2. The water was drunk by the girl.

To the native English speaking person, the two sentences appear to convey essentially the same information. Witherspoon's wife, a Navajo, said that the second sentence was ridiculous because it is impossible for water to drink the girl.

Informal Research in Language. In an attempt to test the impact of the order of the nouns, I showed the same two sentences to five groups of Navajo university students. Each group consisted of two to four Navajo students who happened to be in my office.

I first wrote on the board "The girl drank the water". The students read the sentence and exhibited a "so what" attitude. Next I wrote "The water was drunk by the girl". In each case the students laughed at the second sentence. When asked to explain what was funny about the sentence, they said that the second sentence implied that the water drank the girl or some similar absurdity.

These students knew how to read English and they knew that they person writing the sentences knew no Navajo, yet the structure of their native language had a notable impact upon their interpretation of the sentence.

It is difficult to believe that these students would not consider initially ridiculous a problem that is introduced with the sentence, "A house was built by

John in 20 days." At least two levels of absurdity can be identified; e.g. a house making a decision and a house not yet built making a decision.

I asked the students if the wording used in the mathematics texts caused them trouble and they said that it did but they were unable to show me specific examples at the time. The degree to which conflicts between a first language and a second language in which a problem is phrased affects the process of solving the problem requires further investigation.

The Role of Imaging in Problem Solving. In discussions with colleagues who do not accept the Whorfian hypothesis, I have asked the question, "If a person does not interpret and solve problems in his native language then how does he think about them?" One answer given was that the process of imaging is utilized where imaging is a process of visualizing a desired result. Although imaging may play a beneficial role in the problem solving process, I maintain that it may be used with quite negative results. Consider the problem:

> How much water must be added to 20 pounds of an 80% acid solution to get a 10% acid solution? Let x represent the number of gallons of water to be added.

The student who proposes the equation

$$.80(20)+x = .10(20+x)$$

as a model for the problem is using imaging. He has written an equation corresponding to his image of how the equation for problems of this type should appear. The problems at the beginning of a set of mixture problems usually give rise to equations with two terms on the left and one on the right. The student wrote an equation that was consistent with his image of how the equation ought to appear on paper. Here imaging was used as a basis for the construction of an equation the solution of which yields an answer which is not only incorrect but absurd; i.e., $-15\frac{5}{9}$ gallons.

It would appear that if a statement of a problem is made in a language in which it is awkward or difficult to state the pertinent relations the problem solving process is rendered more difficult. If the mathematics teacher of Navajo students is sensitive to those elements of the Navajo language that have a bearing upon the problem solving techniques to be taught, he can avoid the use of examples and phrases which result in conflict in the student's thought processes and he can be ready to help with more understanding when students encounter difficulty with problems from various printed sources.

Laboratory Techniques. The concept upon which correct equations for mixture problems can be based is, "If everything in a large container came from two small containers then the number of pounds of acid in the large container is equal to the number of pounds of acid in the first small container plus the number of pounds of acid in the second small container". Students may assert a belief in this principle but if they are really basing equations upon this image of how a suitable equation should appear they stand in danger of obtaining ridiculous results and they will derive little satisfaction from their work.

Before solving problems concerning mixtures of chemicals, the teacher can demonstrate the proper underlying concept by placing available items such as chalk, paper clips, etc. in two small coffee cans and then emptying them noisily into a large coffee can. The principal derived from the demonstration should be discussed until it is thoroughly understood by all of the class before problems dealing with mixtures are attempted. This is one example of the use of laboratory techniques being used to help the student understand the relation between the hypotheses of a problem and the equation to serve as a model for the problem. A correct equation will yield a correct answer to the problem.

Introduction of a Word for "number". Of further possible relevance to mathematics educators is the fact that in the original Navajo language there is no word corresponding to the English word "number". Although the Navajo numbering system was developed to 999, the lack of a word for the concept of number may indicate that the abstract concept of number was not a conceptual reality to the Navajos prior to 1900. A word that is sometimes used today is nóomba which is clearly a cognate borrowed from English.

Lest the Anglo become too smug over the fact that his language has a name for the concept "number", let him note the English "number" is a cognate borrowed from the latin "numerus". When in the course of the development of the English language the English speaking people realized the need for a word for the abstraction "number" they borrowed a word from an existing language. The Navajo people have done the same thing by incorporating the word nóomba into their language.

Adding numbers vs. Adding Objects. Dr. Hal Hamlow, Chairman of the Department of Mathematics at Navajo Community College, has informed me that teachers of Navajo grade school students observed that their students often added

$$10 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 13.$$

Here the students were adding objects and not numbers. Presumably, the concept of the addition operation as well as the concept of fractions was faulty.

Research in the Hypothetical. The 1973 study provided the rationale for an informative study which was conducted by Ruth Palmer, a graduate student at Northern Arizona University.[17] Her study was based upon the suggestion made in the 1973 report that the lack of a word for "if" in the Navajo language was an indication that Navajos did not regularly utilize the hypothetical "If---then---" mode of thought.

The subjects of Palmer's study were forty Navajo students in attendance at Aztec High School at Aztec, New Mexico. The primary technique employed was to ask the students to paraphrase a given "If---then---" statement without using the word "if". Two hypotheses were tested in the Palmer study. They were:

1. Navajo students would paraphrase distinctly differently than native English speaking students.
2. The more proficient the Navajo student was in English, the fewer would be the difficulties with the word "if".

Both hypotheses were supported by the results of the study. The Navajo students utilized extensively the temporal (when) in paraphrasing as compared with non-Navajos who made use of "and" and "or".

The subjects were identified by Palmer as being poor, fair, or excellent with respect to their mastery of English and her analysis of their answers indicated that as a Navajo's mastery of English increased, his use of the temporal "when" in paraphrasing "If---then---" sentences decreased.

With respect to mathematics education, the concepts of "and" and "or" are more directly related logically and mathematically than the concept "when". Further research will possibly clarify this issue.

Palmer also found that Navajos do have words they use for "if". There is, however, no standard word for the concept. When a group of students were together and they were from different parts of the reservation, there was always a great deal of controversy over exactly what word meant "if".

The word "joh" meant "if" in the area of Sweetwater, Arizona but in the area of Chaco Canyon, New Mexico, it meant "well" as in "Well, I think I'll go to sleep". The word "shii" was used for "if" in Two Grey Hills, Arizona and in Young and Morgan [14] "shii" means "probably". In Tuba City, Arizona, no word for "if" was found to be in common use and when asked to paraphrase "If the sun shines, the rain will dry up", those Navajos resorted to the temporal "The rain drys up when the sun comes out".

Plamer identified the present stage of the "if" concept in the Navajo language as "neologian". Repeatedly in the history of the Navajo people, they have proven themselves to be extremely adaptable and resilient and if a familiarity with the hypothetical "If---then---" mode of thought is deemed to be essential to an acceptable understanding of modern science and technology, then a word for the "if" concept will be introduced into the language. It appears that this process is underway at present.

The dictionary The Navajo Language by Young and Morgan [13] published in 1967 does not list a Navajo word for "if". However, Young and Morgan in The Navajo Language published in 1980 list for "if"

> --dą́ą́', ládą́ą́'. #If not, doo (verb +)--góó. #If (not = unless), t'ą́ą́doo + perfective mode + -góogo. (If it does not rain my corn will all dry up, t'ą́ądoo nahóółtą́ągóogo shin aadą́ą́' 'ałtso dadoogą́ął.

The implications for the mathematics educator are evident. He should be keenly aware of those situations where he or a textbook is making statements in the "If---then---" form and then make every effort to help his Navajo students derive as nearly as possible the precise meaning and the mathematical implications of those statements.

Clan Characteristics and Mathematics Education. Lola Wilson, a Navajo student at Northern Arizona University came to my office and explained how she thought that still another facet of the Navajo culture may have implications for mathematics educators. She described the importance of the clans in her culture. A Navajo not only feels strong emotional ties to his immediate family, but also to a larger related group, the clan. The various clans are, furthermore, characterized by a particular trait. Lola's clan is the Edgewater clan and the characteristic trait of that clan is slowness. To the non-Indian American, slowness usually possesses a negative connotation. One does not become a professional athlete on the basis of being slow. Lola, however, explained that this slowness of her clan was more akin to the deliberateness associated with a quest for perfection. This is one of the positive aspects of slowness.

The mathematics educator of Navajos must determine whether the slowness which is an admirable trait for members of the Edgewater clan becomes negative and detrimental to a member of that clan who is enrolled in his mathematics courses. If the course content and the pace at which various topics are taken up are covered is determined by a tightly packed syllabus with a rigid time schedule, then a member of the Edgewater clan, enrolled in that course, could feel frustration at the revelation that what he had always considered an admirable characteristic, slowness, had become and was now treated as a hindrance as he strives to learn mathematics concepts.

The preceding observation concerning the traits of clans does not imply that the mathematics teacher should be ready to adopt a drastic revision of either his syllabus or his goals. The implication is that the teacher should learn as much about the background of his students as possible. He cannot expect to learn all that he should know through research, but if his students feel that their teacher is culturally sensitive to their problems, they will feel less hesitant to share their concerns with him. Being aware of the conflict over a particular clan's concept of slowness may cause him to modify his requirements for particular students or at least modify his attitude of of the role of the concepts of fastness and slowness in mathematics education and thereby possibly learn to utilize the positive aspects of slowness or any other trait that may be emphasized by those clans represented by his students.

Conclusions and Recommendations. Though the study to date cannot be considered complete, tentative implications for mathematics educators can be drawn. An area of difficulty appears to be hypothetical thought and abstraction based upon hypotheses. If this be true, the mathematics teacher must attempt to close the gap between the hypotheses of his teaching and the real world of the Navajo by every means available to him. Teachers can take courage from the fact that in recent years professional mathematics educators have been much concerned with the laboratory method and a vast literature on the subject has resulted from which the teacher can draw ideas.

Each algebra teacher should have a plywood model of the identities $(a+b)^2$ and $(a+b+c)^2$. He should also have at hand a three dimensional mode of $(a+b)^3$.

Navajo students with whom I have discussed these models in my laboratory have expressed delight upon learning that there is a physical basis for the algebraic expansion of $(a+b)^2$ and $(a+b)^3$ and they state that such "demonstrations" if used earlier would have given them a different attitude towards mathematics.

The utilization of laboratory techniques is consistent with the Navajo philosophy of education. The Navajo verb for "to teach" means "to show".

Culture and Geometry. I shall conclude this chapter with a comment on the relation between the heritage of our Indian students and the teaching of geometry in the secondary schools. The subject matter content of the secondary geometry course is totally dominated by the straight edge straight line and mechanical constructions as dictated by Fuclid in his Elements in 300 B.C. Prior to the U.S. Government decision in 1880 that Indian children should attend schools, the geometry of the straight line had played little or no role whatsoever in the cultures of the Indian tribes of North America. The heavens, the hills, the stream, and even the animals of his world showed no evidence of or need for the straight line or the straight edge. The hunter was well aware that a straight arrow shaft flew truer than a crooked one. Yet all he could strive for was to find the most suitable branch or reed available to him in nature and then to decrease its degree of crookedness as much as possible by whatever tools he could devise. Straight lines were not much known and the need for them was negligible.

A hasty comparison of an early diagram showing the territories occupied by the various Indian tribes with boundaries determined by mountain ranges or streams with a map of the states of the western United States dominated by straight lines and square corners provides almost shocking evidence of the imposition of the concept of the straight line upon a people whose cultural heritage shows no need for them.

In support of the forgoing idea, I present a poem written by an Indian entitled "Roundness of Life".

The light of the sun is my life;
The living breeze is my life;
The turning of day and night is my life;
My life is never ending
For everything is a part of
The roundness of the seasons.
The sun is round---
The moon is round---
The world is round.

And my life follows the same path.
The eyes of my grandson, the bear, are round;
The eyes of my grandson, the chipmunk, are round:
The face of my grandson, the cougar, is round;
And like them, my eyes and face are round.
Because life was made that way---
To be round---never ending.

---From Butterfly of Hope

I have yet to find a poem written to such an aberration of the natural order as a straight line laid down by the surveyor's transit. Perhaps an awareness of this difference in the cultural background of students may influence some geometry teacher to be somewhat more sympathetic in his approach.

The conclusions suggested in this report are intended to be tentative and further attempts at verification and corroboration will be made as the work continues.

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