By participating in these class activities, students will acquire mathematical skills and at the same time learn about aging. Topics related to aging are often quantitative, and therefore, subject to mathematical analysis and procedures. The activities, which contain teacher suggestions and all student handouts, deal with seven topics. In the first activity, "Graying of America," students extract data from a graph of U.S. population growth to solve problems which illustrate changes in the age structure of society as they and their parents grow older. "Life Expectancy" is the topic of the second activity. Students use actuarial data to compute the probability of surviving a given number of years beyond a certain age. In the third activity, "Compound Interest and Population Growth," students gain insight into population growth by working through a problem involving compound interest and then applying the same mathematical concepts to a problem involving birthrates and deathrates. The fourth activity deals with "Changing Age Structure and Implications." Using bar graphs and mathematical skills, students estimate by age groups the proportion of people in society who are economically productive compared to those who are primarily consumers of goods and services. The last three activities deal with social security. Students compute the amount a typical worker can expect to pay into social security and the amount he or she can expect to receive during retirement. They also write a computer program to estimate 1978 revenues and expenditures. (Author/RM)
MATH ACTIVITIES FOR TEACHING ABOUT AGING

ROBERTA E. DOYLE
ANTHONY A. PAPPAS

Mathematics Department
Acton-Boxborough Regional High School

HARA ANN BOUGANIM, EDITOR
EQUITY ASSOCIATES

1982

"TEACHING AND LEARNING ABOUT AGING PROJECT"

We encourage you to duplicate this material freely for non-profit educational purposes.

SUPPORTED BY A GRANT TO THE ACTON-BOXBOURGH REGIONAL SCHOOL DISTRICT UNDEP THE ELEMENTARY AND SECONDARY EDUCATION ACT, TITLE IV-C.

TEACHING AND LEARNING ABOUT AGING

McCarthy-Towne School, Acton, MA 01720 (617) 263-8773
INTRODUCTION

The teaching of mathematics is generally aimed at helping students understand mathematical concepts and procedures and developing skills they can use to solve problems. Because the emphasis is on process, the specific content of problems is usually incidental. Students can be taught concepts and skills relating to fractions whether the problem calls for dividing a cake into servings, dividing a dollar into coins, or establishing voting districts according to population. To put it another way, once the skills for using fractions have been mastered they can be used by a caterer, a cashier, a politician or anyone else who has the need to divide anything into equivalent parts. Since the content of mathematical problems is secondary to process, any subject which is quantitative in nature can serve as a focus for mathematics instruction.

Topics related to aging, are often quantitative, and therefore subject to mathematical analysis and procedures. Such matters as life expectancy, changing age distribution of the society and financing of pension programs are just a few of the many age-related topics that are mathematical in nature. These are among the topics dealt with in the seven math activities of this unit.

The activities are not intended to comprise a single unit. Each activity may be used independently. The activities are designed for use in the context of different courses with students of varying levels of mathematical understanding and skill. Any teacher considering using this material should review the table below. The teacher should also closely examine the lessons to determine which activities are most suitable for the specific course and types of students involved. Together the activities involve a wide variety of concepts and skills, and they deal with a variety of issues related to aging.

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>CONTENT RELATED TO AGING</th>
<th>MATH SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. GRAYING OF AMERICA</td>
<td>Changing age structure, impact on society</td>
<td>Interpret graph, use calculator for percentage problems</td>
</tr>
<tr>
<td>2. LIFE EXPECTANCY</td>
<td>Changing life expectancy at different ages</td>
<td>Use actuarial data, compute mean and standard deviation on normal distribution curves</td>
</tr>
<tr>
<td>ACTIVITY</td>
<td>CONTENT RELATED TO AGING</td>
<td>MATH SKILLS</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>3. COMPOUND INTEREST AND POPULATION GROWTH</td>
<td>Application of concepts learned in computing, compound interest rate, and birth- and death rates</td>
<td>Compute interest and birth- and death rates using log tables, pocket calculators or a computer</td>
</tr>
<tr>
<td>4. CHANGING AGE STRUCTURE AND IMPLICATIONS</td>
<td>Effects of changing age structure on the economy and Social Security System</td>
<td>Use bar graphs, compute ratios</td>
</tr>
<tr>
<td>5. SOCIAL SECURITY AND YOU</td>
<td>Social Security System and what a worker pays in and gets</td>
<td>Compute addition, subtraction, multiplication and division</td>
</tr>
<tr>
<td>6. ESTIMATING SOCIAL SECURITY EXPENDITURES</td>
<td>Social Security System and compute rising Social Security expenditures</td>
<td>Interpret graph, compute advanced multiplication</td>
</tr>
<tr>
<td>7. CAN WE AFFORD SOCIAL SECURITY?</td>
<td>Difficulties faced by Social Security because of demographic and economic factors</td>
<td>Write computer program to estimate 1978 revenues and expenditures</td>
</tr>
</tbody>
</table>

Teaching these lessons involves discussing the issues involved, not simply learning the mathematical procedures for analyzing the problems. Thus students can not only increase their math skills, but see the relevance of these skills to real life issues.
ACTIVITY 1: GRAYING OF AMERICA

Overview:

Working from a graph of United States population growth, students extract the necessary data to solve problems which illustrate changes in the age structure of society as they and their parents grow older. Discussion of the problems emphasizes the impact of the changing age structure on society now and in the future.

Objectives:

At the conclusion of this activity students will be better able to:

1. Interpret a graph and extract data needed to solve mathematical problems.
2. Solve mathematical problems involving advanced skills of addition, subtraction, multiplication and division.
3. Use a calculator to solve percentage problems.
4. Describe specific changes taking place in the age structure of society.
5. Identify specific ways in which changing age structure may influence society during their lifetimes.

Materials needed:

For each student:


b. Handout #1-2: "Where Have All the Babies Gone?"

c. Handout #1-3: "Population Growth Assignment"

d. Calculators

For the teacher:

a. Transparency copy of Handout #1-1

b. Overhead projector and screen
Advance preparation:

Make copies of Handouts #1-1, 1-2 and 1-3 for each student.

Make transparency copy of Handout #1-1.

Set up overhead transparency machine and screen (both Day 1 and 2).

Estimated teaching time:

2 class periods

Guidelines, day 1:

1. Distribute copies of Handout #1-1, "U.S. Population Over and Under Age Sixty-five...", to all students.

2. Project transparency copy of Handout #1-1.

3. To check students' understanding of the graph, ask individual students to answer the following questions:

   a. What was the total population of the U.S. in 1880, 1930 and 1975?

      Approximately 55 million; 127 million; 213 million.

   b. How much is the under sixty-five population expected to grow between 1970 and 2000?

      From approximately 185 million to 238 million, a growth of about 53 million.

   c. How much is the over sixty-five population expected to grow between 2000 and 2030?

      From approximately 30 million to 45 million, a growth of about 15 million, in spite of general decline of total population in the same period.

4. In order to be sure that all students know how to read the graph, take time to explain, or have students explain, how each answer to the above questions was obtained.
5. Ask students what will happen to the average age of the U.S. population between 1880 and 2030.

As the percentage of older people increases, the average age of the population will increase.

6. Distribute copies of Handout #1-2, "Where Have All the Babies Gone?" Students should read the handout and discuss briefly the question at the end.

7. Distribute copies of Handout #1-3, "Population Growth Assignment," and have students individually solve the problems.

8. Collect the completed assignments.

Guidelines, day 2:

9. Project the transparency copy of Handout #1-1.

10. Demonstrate, or ask individual students to demonstrate, how to solve the problems on Handout #1-3, taking time to answer questions for students who don't understand the process.

Answers:

a. Total approximate population of the U.S.

   2005: 262 million
   2030: 260 million

b. Estimated population over sixty-five

   2005: 30 million
   2030: 45 million

c. Percentage of population over sixty-five

   2005: \( \frac{30}{262} = 11.1\% \)

   2030: \( \frac{45}{260} = 17.1\% \)
II. Ask the students: How might the "Graying of America" affect the future of society in terms of each of the following areas? (It is not necessary to try to identify all the implications, but simply to help students see that there will be many implications that might affect them.)

a. Employment

Unless people work longer, there will be fewer workers to supply society's needs. This could mean a decrease in unemployment and higher wages because of a scarcity of workers. However, this could also increase inflation.

b. Retirement and pensions

With more retired people, the total outlay for pensions and other retirement benefits will increase while there will be fewer workers paying into retirement systems. This could put a strain on Social Security and other pension programs.

c. Advertising and marketing

Industry and business will have to shift toward production of goods and services to meet the wants and needs of older people and reduce production of goods for the young. Specific examples, such as baby foods, children's clothing, toys or motorcycles will help students to visualize the kinds of adjustments that might have to be made.

d. Government

Politicians will probably respond to a growing constituency of older people by expanding programs and services to older people. With relatively fewer children of school age, the proportion of tax money devoted to public education and other services for youth may decline as programs for older people expand.
Handout #1-2: Where Have All the Babies Gone?

When George Washington was President in the late 1700s the average person in the United States was sixteen years old. There were several reasons for such a young population. Most immigrants to America tended to be younger people. Life expectancy was much lower than it is today, and families generally were larger.

Since the eighteenth century, the trend has been toward fewer births and longer life expectancy. Thus by the middle 1970s, the average age of Americans had climbed to about thirty, and it is expected to continue to climb in the decades ahead. Through the nineteenth and twentieth centuries the birthrate has been in constant decline except for one important break in the pattern. In the years following World War II (the late 1940s and 1950s) large families once again became popular and the birthrate climbed sharply. This “baby boom” produced a bulge in the age structure of society which is still affecting us and will continue to do so for many years to come.

Yesterday’s “baby boom” has now become a bulge in the proportion of young adults and middle-aged people. Soon after the turn of the century the “baby boom” will become a “senior boom” as the babies of the 1940s and 1950s become the elders of tomorrow. Currently the birthrate is again declining
and life expectancy continues to climb as more medical advances are made. As a society, and as individuals, we are becoming older as we move toward the twenty-first century.

Question for consideration:

How does the graph showing "U.S. Population Over and Under Age Sixty-five" reflect what you have read above?
Handout #1-3: Population Growth Assignment

Using the graph, Handout #1-1, "U.S. Population Over and Under Age Sixty-five," solve the following problems:

1. Estimate the total population of the U.S. for the years 2005 and 2030.
2. Estimate the population over age sixty-five for the years 2005 and 2030.
3. Calculate the percent of population over sixty-five for the years 2005 and 2030.
ACTIVITY 2: LIFE EXPECTANCY

Overview:

Students use actuarial data to compute the probability of surviving a given number of years beyond a certain age. They compute the mean and standard deviations of the distribution of a life expectancy curve. Then they use these deviations to compute the probability that each member of a married couple will survive a given number of years beyond their seventieth birthday. The activity provides experience in working with statistical data, a normal distribution curve, mean and standard deviations. The activity also illustrates that women in general live longer than men and that this imbalance of the sexes poses certain problems for older people and for society.

Objectives:

At the conclusion of this activity students will be better able to:

1. Solve problems involving statistical data, distribution curves and statistical deviations.
2. Explain how analysis of statistical data of life expectancy can be used in planning for the future.
3. Explain how life expectancy for subgroups changes with age.
4. Describe social problems that exist now or may develop in the future as a result of the differences in life expectancy for males and females.

Materials needed:

For each student:

a. Handout #2-1: "Expectation of Life at Various Ages in the United States (1974 data)"

b. Handout #2-2: "Life Expectancy Worksheet"

c. Normal distribution table
Advance preparation:

Make copies of Handouts #2-1 and #2-2 for each student.

Estimated teaching time:

2 class periods

Guidelines, day 1:

1. Write the following terms on the chalkboard and define them for students.
   a. **Life span**: the average length of life for a given species (for example, ninety days for house flies, twelve to fourteen years for dogs, seventy years for elephants).

   b. **Longevity**: actual length of an individual life (term implies long life).

   c. **Life expectancy**: the average age to which a particular category of people can be expected to live. For example, newborn males in 1974 had a life expectancy of 68.2 years while newborn females had a life expectancy of 75.9 years. Life expectancy can also be stated as the number of remaining years a person may be expected to live. A forty-year-old man may be expected to live 32.2 more years. A forty-year-old woman may be expected to live 38.5 more years.

2. Explain that, in a sense, life expectancy increases as people grow older. For example, seventy-year-old men in 1974 had already lived beyond the average life expectancy for males of their generation, yet having survived that long they could expect to live an average of 10.7 more years. Similarly, seventy-year-old women in 1974 could expect to live an average of 13.9 more years.

3. Ask students to identify other ways of categorizing people besides male and female for which it might be possible to establish different statistical averages for life expectancy. Factors might include:

   a. **Race or ethnicity**: Many racial and ethnic minorities have shorter life expectancy than the American population as a whole, including Blacks, Chicanos and Native Americans.
b. occupation: Farmers generally live longer than factory workers.

c. health habits: Smokers, people who overeat and heavy drinkers all have lower average life expectancies.

d. year of birth: Life expectancy for people born in 1900 was only about forty-seven years, but the average baby born today can be expected to live more than seventy years.

Ask how statistical information about life expectancy for various subgroups might be useful and to whom.

Life insurance companies need this information to set premiums and anticipate future payments of benefits. This information is also needed by private industries for planning pension programs and by government in planning for social security and services for older persons.

5. Distribute copies of Handout #2-1: "Expectation of Life at Various Ages in the United States (1974 data)," and Handout #2-2: "Life Expectancy Worksheet."

6. Check to see that all students understand the table on Handout #2-1 and that they understand the problems they are to solve on the worksheet.

(Note that this assignment assumes that students have the necessary skills for working out solutions to the problems. If not, further instruction is needed.)

7. Have students work through the problem individually. If done in class, provide help as needed, and collect the completed assignments. If done as homework, set a date for assignments to be turned in.

Guidelines: day 2:

8. After assignments have been collected, demonstrate the following solutions on the chalkboard taking time to answer questions and explain the processes.
Solutions to worksheet problems:

1a. Seventy-year-old man's expectancy distribution.

\[ \text{Use table to compute mean } \mu = 70 + 10.7 = 80.7 \text{ years} \]

To compute standard deviation:

1. Draw normal curve (as above).

2. Mark off B area of .049 at far left to indicate one tail of the distribution, from age 70 to 71. From normal probability table, Z for B area of .049 is 1.55.

3. Label mean 80.7.

4. Using Normal Distribution Table Z values

\[ 0.5 - 0.049 = 0.451 \]

Look up Z value for .451 = 1.655.

This gives the number of standard deviations from age 71 to 80.7 (mean).

\[ 1.55\sigma = 10.7 - 1 \text{ or } 9.7 \]

\[ \sigma = 5.86 \text{ years} \]
1a. Seventy-year-old woman's expectancy distribution.

\[ Z = \frac{83.9 - 70}{13.9} = 1.675 \]

\[ Z 	ext{ for } 1.675 = 0.453 \]

\[ 0.5 - 0.453 = 0.047 \]

B area = 0.047

Mean: \[ \mu = 70 + 13.9 = 83.9 \text{ years} \]

Standard deviation:

(1) "Draw normal curve (as above)."

(2) Mark off B area of .047 at far left to indicate distribution from 70 to 71.

(3) Label mean 83.9.

(4) Using Normal Distribution Table Z values

\[ Z \text{ for } 0.453 = 1.675 \]

\[ 1.675 \sigma = 13.9 - 1 = 12.9 \]

\[ \frac{12.9}{1.675} = 7.701 \text{ years} \]

2a. For a seventy-five-year-old man:

\[ Z = \frac{80.7 - 75}{6.26} = 1.33 \text{ or } Z = 1.33, \text{ this } Z \text{ corresponds to a B area of } 0.9037. \]

So probability a man is alive at seventy-five = 0.9037

For a seventy-five-year-old woman:

\[ Z = \frac{83.9 - 75}{7.82} = 1.14 \text{ or } Z = 1.14, \text{ this } Z \text{ corresponds to a B area of } 0.8707. \]

So probability woman is alive at seventy-five = 0.8707

Probability that both are alive at seventy-five is (.9037) (.8707) = .787

2-5
2b. Probability that woman is alive and husband is dead when woman is seventy-five = (.8729)(1-.8186) = (.8729)(.1814) = .158

9. Ask students what practical implications they see for individuals and for society in the solutions to these abstract problems.

Students should recognize the extreme imbalance of females to males in the population of elderly people. Many older widows have traditionally depended heavily on their husbands for financial support. The current movement toward equal rights for women and the entry of more women into full-time careers may change the pattern in years to come. Until now, however, the paid work force has been male dominated, and Social Security and retirement benefits have been linked to the husband’s earnings rather than the wife’s. It is not uncommon for a woman to find herself without any regular financial support after her husband’s death except, perhaps, for a small monthly Social Security benefit check. The hypothetical problem assigned to students (Handout #2-2, Question 2) assumes that the husband and wife were the same age when they married. Students may suggest that the problem of widowhood could be alleviated if women married younger men. This hypothetical way out may seem workable in the classroom, but in fact our society operates in the opposite direction. On the average, men marry women who are about three years younger. Since women outlive men by an average of seven years, this means that women who marry and stay married to the same man can expect to be widows for an average of ten years. Statistically, three out of four such wives become widows.
Handout #2-1: Expectation of Life at Various Ages in the United States
(1974 data)*

From the National Center for Health Statistics:

<table>
<thead>
<tr>
<th>Age</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>68.2</td>
<td>75.9</td>
</tr>
<tr>
<td>20</td>
<td>50.4</td>
<td>57.5</td>
</tr>
<tr>
<td>40</td>
<td>32.2</td>
<td>38.5</td>
</tr>
<tr>
<td>45</td>
<td>27.9</td>
<td>33.9</td>
</tr>
<tr>
<td>50</td>
<td>23.8</td>
<td>29.5</td>
</tr>
<tr>
<td>55</td>
<td>19.9</td>
<td>25.3</td>
</tr>
<tr>
<td>60</td>
<td>16.5</td>
<td>21.3</td>
</tr>
<tr>
<td>65</td>
<td>13.4</td>
<td>17.5</td>
</tr>
<tr>
<td>70</td>
<td>10.7</td>
<td>13.9</td>
</tr>
</tbody>
</table>

Definition: Expectation is the average number of years of life remaining.

Information from the John Hancock Insurance Company:

1. A seventy-year-old man has a 0.951 probability of living at least one additional year.

2. A seventy-year-old woman has a 0.953 probability of living at least one additional year.
Handout #2-2: Life Expectancy Worksheet

1. Using the information provided in the table on Handout #6-1, compute the mean (μ) and standard deviations (σ) for the expectancy distributions for (a) seventy-year-old men and (b) seventy-year-old women. Assume that life expectancy is normally distributed.

2. a. Calculate the probability that a seventy-year-old man and his seventy-year-old wife will both celebrate their seventy-fifth birthdays.

b. Calculate the probability that the woman will be a widow when she is seventy-five.
ACTIVITY 3: COMPOUND INTEREST AND POPULATION GROWTH

Overview:

Students gain insight into population growth by working through a problem involving compound interest and then applying the same mathematical concepts to a problem involving birthrates and deathrates. First the students analyze a table of savings showing the annual growth over five years of compounding interest at a given rate, using pocket calculators or logarithm tables. They then construct a similar table for estimating population growth and rate of population growth for a given number of years.

Objectives:

At the conclusion of this activity students will be better able to:

1. Solve complex problems involving compounding growth.
2. Transfer advanced mathematical concepts and skills from one type of problem to another.
3. Explain how knowledge of birthrates and deathrates can be used to predict future population.

Materials needed:

For each student:

a. Handout #3-1: "Computing Compound Interest"
b. Calculators for logarithm tables

For the teacher:

a. Transparency #3-1, "Computing Compound Interest"
b. Transparency #3-2, "U.S. Population: Two- vs. Three-Child Family"
c. Transparency #3-3, "World Birth and Death Rates (Estimated)"
d. Overhead projector and screen
Advance preparation:
Prepare copies of Transparencies #3-1, #3-2 and #3-3.
Make copies of Handout #3-1 for each student.
Set up overhead projector and screen for days one and two.

Estimated teaching time:
2 class periods

Guidelines, day 1:
1. Project Transparency #3-1 and distribute copies of Handout #3-1 to all students.
2. Review the computations in Part A of Handout #3-1 if using calculators, or Part B if using logarithm tables. (If using a digital computer, a program would have to be developed.)
3. Ask students to identify the three essential factors that must be known in order to solve this type of problem involving compound growth. Write the factors on the chalkboard. The factors involved are:
   a. The original amount (in this case the original amount of principal)
   b. The rate at which growth occurs (in this case compound interest rate)
   c. The number of compounding periods
4. Project Transparency #3-2, "U.S. Population: Two- vs. Three-Child Family." Ask whether the same three factors can be applied to a problem of projecting population growth.

   Students should be able to recognize that the graph illustrates the same factors:
   a. The original amount (population in 1870),
   b. The rate at which growth occurs (illustrated by the curves for two- and three-child families), and
   c. The number of compounding periods (in this case the decades from 1870 to 2070).
5. Ask whether it is possible from this graph (an exponential curve obtained at constant growth rate) to obtain the actual growth rates for two- and three-child families.

The actual growth rate can be obtained from this graph, since both changes in the mortality and immigration rates were taken into account.

6. Ask students to estimate what the population will be when they reach age forty, then age sixty-five, according to two-child family and three-child family rates of growth. Students should locate these points on the graph and report back to the class.

Students should recognize that future population will depend heavily on the size of families. An average increase of one child per family will drastically change the population of the United States within their own lifetimes. It might be noted that in recent years the birthrate has been declining and that the United States has been moving closer to the two-child family rate, but there is no assurance that this will continue.

7. Hold an open discussion on the difference it might make to the United States (and to the students personally) if we become a nation of 300 million, 500 million, or 700 million people in the twenty-first century. Ask how constant analysis of growth rates and population projections might help the country to make long-range plans for the future. Show how the mathematical skills and principles involved in this lesson have practical uses for society.

Guidelines, day 2:

8. Project Transparency #3-3, "World Birth and Death Rates (Estimated)." Ask students how the growth rate of a population could be determined. Students should recognize that the rate of growth, or "natural increase," can be determined by measuring the difference between the birthrate and deathrate.

9. Demonstrate (or have a student demonstrate) on the chalkboard how to determine the rate of growth in 1975 for developed countries.

Solution is: \[
\frac{17.0 - 9.2}{1000} = .0078 \text{ or } .78\%
\]
10. Present the following problem to the class and have each student work on it individually.

Problem: Assume the population of the United States was 200 million in 1975 (actually, it was slightly higher) and that the growth rate would continue indefinitely at .78% (the rate of growth for developed countries in 1975). Compute the population in the year 2000.

Solution using compound interest formula:

\[ A = 200,000,000(1 + .0078)^{25} \]

\[ A = 2 \times 10^8(1 + .0078)^{25} \]

\[ \log A = \log (2 \times 10^8) + 25 \log 1.0078 \]

\[ \log A = 8.3010 + 25(0.0034) \]

\[ \log A = 8.3010 + .0850 \]

\[ \log A = 8.3860 \]

\[ A = 2.4322 \times 10^8 \]

Variations on the problem might be to graph the population at five- or ten-year intervals or to plot the same curve for different growth rates.

11. Collect papers when completed, and demonstrate the solution to the problem on the chalkboard, answering any questions students have about the process.
Handout #3-1: Computing Compound Interest

1. Using calculators: table showing principal on an initial investment of $1000 when this investment is compounded at 5% per year for 5 years.

\[ A = P + PRT \]

- \( A \) = Amount
- \( P \) = Principal
- \( Prt \) = Interest

<table>
<thead>
<tr>
<th>YEAR</th>
<th>PRINCIPAL</th>
<th>INTEREST</th>
<th>AMOUNT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>1000 x .05 x 1 = 50</td>
<td>1050</td>
</tr>
<tr>
<td>2</td>
<td>1050</td>
<td>1050 x .05 x 1 = 52.5</td>
<td>1102.5</td>
</tr>
<tr>
<td>3</td>
<td>1102.5</td>
<td>1102.5 x .05 x 1 = 55.13</td>
<td>1157.63</td>
</tr>
<tr>
<td>4</td>
<td>1157.63</td>
<td>1157.63 x .05 x 1 = 57.88</td>
<td>1215.51</td>
</tr>
<tr>
<td>5</td>
<td>1215.51</td>
<td>1215.51 x .05 x 1 = 60.76</td>
<td>1276.27</td>
</tr>
</tbody>
</table>

2. Using log tables: principal after 5 years computed using the compound interest formula.

\[ A = P(1 + r)^t \]

\[ A = 1000(1 + .05)^5 \]

\[ \ln A = \ln 1000 + 5(\ln 1.05) \]

\[ 5 \ln A = 6.9077 + 5(.04879) \]

\[ 5 \ln A = 7.1565 \]

\[ A = 1276.2 \]
COMPUTING COMPOUND INTEREST

A. Using calculator:

<table>
<thead>
<tr>
<th>YEAR</th>
<th>PRINCIPAL</th>
<th>INTEREST</th>
<th>AMOUNT (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>1000 × .05 × 1 = 50</td>
<td>1050</td>
</tr>
<tr>
<td>2</td>
<td>1050</td>
<td>1050 × .05 × 1 = 52.5</td>
<td>1102.50</td>
</tr>
<tr>
<td>3</td>
<td>1102.50</td>
<td>1102.50 × .05 × 1 = 55.13</td>
<td>1157.63</td>
</tr>
<tr>
<td>4</td>
<td>1157.63</td>
<td>1157.63 × .05 × 1 = 57.88</td>
<td>1215.51</td>
</tr>
<tr>
<td>5</td>
<td>1215.51</td>
<td>1215.51 × .05 × 1 = 60.76</td>
<td>1276.27</td>
</tr>
</tbody>
</table>

B. Using log tables:

\[ A = P(1 + r)^t \]

\[ A = 1000(1 + .05)^5 \]

\[ \ln A = \ln 1000 + 5(\ln 1.05) \]

\[ \ln A = 6.9077 + 5(.04879) \]

\[ \ln A = 7.1565 \]

\[ A = 1276.2 \]
U.S. population passed the 100 million mark in 1915 and reached 200 million in 1968. At the two-child rate, population in 2015 will reach 300 million. At the three-child rate population will reach 300 million in this century and 400 million in 2013. (Projections assume small future reduction in mortality and assume future immigration at present levels.)
WORLD BIRTH AND DEATH RATES
(ESTIMATED)

Developed Countries,
1775 to 1975

Rate of Natural Increase = Birth Rate - Death Rate

Developing Countries,
1775 to 1975

Population Reference Bureau, Inc., 1754 N Street, N.W., Washington, D.C. 20036

Transparency 3-3

29
ACTIVITY 4: CHANGING AGE STRUCTURE AND IMPLICATIONS

Overview

Using bar graphs and mathematical skills involving ratios, students estimate by age groups the proportion of people in society who are economically productive compared to those who are primarily consumers of goods and services. Using projections of the future age distribution of the population, students see how the changing age structure may affect the economy and, more specifically, the Social Security program.

Objectives

At the conclusion of this activity students will be better able to:

1. Interpret bar graphs.
2. Compute ratios.
3. Describe expected changes in the age structure of society and their implications for the future.
4. Explain how the changing age structure of society may affect the future of Social Security.

Materials needed

For each student:

a. Handout #4-1: "Age Structure of the U.S. Population in the Twentieth Century"

b. Handout #4-2; "Assignment on Age Distribution"

c. Calculators

For the teacher:

a. Overhead projector and screen

b. Transparency copy of Handout #4-1
Advance preparation:

Set up overhead projector and screen.
Make copies of Handouts #4-1 and #4-2 for each student.
Make transparency copy of Handout #4-1.

Estimated teaching time:

2 class periods

Guidelines, day 1:

1. Distribute copies of Handout #4-1, "Age Structure of U.S. Population in Twentieth Century."

2. Project the transparency copy of Handout #4-1.

3. Make sure that everyone knows how to read the graph, then discuss the following questions.
   
   a. What major change(s) do you see in the age structure of the United States population between the years 1900 and 2000?
      
      The key change is that the proportion of younger people is declining as the proportion of older people increases. They should also notice the "baby boom" of the 1950's and its effect on the age distribution for 1975 and 2000. They may also notice the significant rise in the ratio of older women to older men.

   b. What is happening in the twentieth century that might account for these changes in the age structure of society?
      
      Life expectancy is increasing, largely due to medical advances that enable people to live longer. Since more children survive to live a long life, the number and proportion of people in the higher age groups increase. At the same time, the birthrate has generally been declining except for the "baby boom" period around the 1950's. As these "baby boom" children mature and age, they greatly increase the number and proportion of older people. Important medical advances have also greatly decreased the mortality rate of women in childbirth, and females now have a much higher life expectancy than males.
c. What trend in the birthrate is noticeable after the "baby boom" of the 1950s?

The number and proportion of children is in constant decline in the graphs for 1970, 1975, and 2000. Many demographers (population experts) expect that we will reach a state of zero population growth (ZPG) by the year 2015. If so, the birthrate and deathrate will be in balance and population will remain stable, but with a considerably older population than we have now. The average age will be approximately forty, as compared to an average age of thirty in the 1970's. Approximately 20 percent of the population will be over the age of sixty-five, compared to about 10 percent in 1975.

4. Ask students at what age they think people usually become mainly producers instead of mainly consumers of goods and services.

Answers may vary, but students will probably recognize that most children and adolescents are primarily consumers depending on adults to produce goods and services for them. They may think of people typically beginning their working careers in their upper teens or as young adults.

5. Ask students at what age they think most adults cease to be producers and become mainly consumers.

Answers will vary, but probably most students will think of people retiring during their sixties.

6. Ask students if consumers are a "burden" to society and whether there are other ways people may contribute to society besides the production of goods and services.

Students should recognize that we are all producers and consumers throughout our lives, that there would be no production without consumption. Increased consumption of goods and services means more job opportunities and ordinarily a decline in unemployment. They should also recognize that people can make important non-economic contributions to society, whether they are "gainfully employed" in the work force or not.
7. Explain that, while there is no way to arbitrarily set a precise age at which the majority of people will begin to be primarily producers or cease their working years, demographers usually consider persons aged fifteen to sixty-four as the "economically productive" segment of the population. They regard people under fifteen or over sixty-five as primarily consumers.

8. Distribute Handout #4-2: "Assignment on Age Distribution," and ask each student to complete the assignment individually. (If done in class instead of as homework, help students as needed without doing it for them.)

Guidelines, day 2:

9. Project the transparency of Handout #4-1.

10. Collect assignments and demonstrate (or ask individual students to demonstrate) how to answer the questions in Handout #4-2. Take time to answer their questions and check that all students understand the process. The answers are:

1. Producers in 1975 = 65.5%
2. Consumers in 1975 = 34.5%
3. Producers in 2000 = 62.7%
4. Consumers in 2000 = 37.9%
5. Ratio of consumers to producers in 1975 = 53 to 100 (approximately 1 to 2)
6. Ratio of consumers to producers in 2000 = 60 to 100 (approximately 3 to 5)

11. Discuss the questions for consideration on Handout #4-2.

1. How might the change in the ratio of consumers to producers affect the production and distribution of goods and services in the future?

There will be fewer workers to produce and more people (especially older people) to consume goods and services. This could mean less unemployment and higher wages as the supply of workers shrinks. A relative scarcity of workers might also mean higher wages and, in turn, higher prices for goods. Less emphasis might be placed on production of goods and services for youth and more emphasis
on meeting the needs of older people. Length of the work day, typical age at retirement, government revenues and expenditures are other areas that might be affected by the changing age structure.

2. What special problems might the change in the age structure present for the Social Security system?

Social Security is set up on a "pay-as-you-go" basis: in any given year revenues should be sufficient to cover expenditures. If the proportion of workers paying into the system declines and the proportion of retired persons receiving benefits increases, the Social Security system could be in financial trouble. Social Security taxes would have to be increased substantially unless Social Security was supported in some other way. In most other modern industrial nations all people beyond a certain age are entitled to a retirement pension from the government, and general revenues are used to support these pensions. The United States program of Social Security is unique and relatively conservative in several respects. In the United States, Social Security benefits are not provided to all retired workers. The benefits are proportionately smaller, since they are meant only as a supplementary income, not as a full pension. Also, general tax revenues are not used to support the program. In many European countries, it is not uncommon for people to retire with a government pension equal to, or even greater than, the amount earned in their working years.
Age Structure of the U.S. Population in the 20th Century

Handout #4-1:

Handout #4-2: Assignment on Age Distribution

Use the graph, "Age Structure of the U.S. Population in the Twentieth Century," and your calculator to solve the following problems. Consider people ages fifteen to sixty-four as primarily producers. Consider people ages zero to fourteen and over sixty-five as primarily consumers. Write your answers in the spaces provided.

1. What percentage of the population was made up of primarily producers in 1975?
2. What percentage of the population was made up of primarily consumers in 1975?
3. What percentage of the population will be primarily producers in 2000?
4. What percentage of the population will be primarily consumers in 2000?
5. What was the ratio of consumers to producers in 1975?
6. What will be the ratio of consumers to producers in 2000?

Consider the following questions and be prepared to discuss them in class.

1. How might the change in the ratio of consumers to producers affect the production and distribution of goods and services in the future?
2. What special problems might the change in the age structure present for the Social Security System?
ACTIVITY 5: SOCIAL SECURITY AND YOU

Overview:

Through a filmstrip and discussion, students learn about the Social Security System—its history and background, how it functions, and whom it serves. They employ a variety of math skills to compute the amount a typical worker can expect to pay into Social Security during his working years and the total benefits the worker and his wife can expect to receive during retirement.

Objectives

At the conclusion of this activity students will be better able to:

1. Solve mathematical problems involving advanced skills of addition, subtraction, multiplication and division, through use of a calculator.

2. Describe the Social Security System in terms of its background and how it works.

3. Explain how much a worker can expect to pay in to Social Security and how much the worker can expect to receive in benefits.

Materials needed:

For each student:

a. Handout #5-1: "Discussion Guide for 'Social Security System'"


c. Calculators
For the teacher:


b. Filmstrip projector and screen

c. Cassette tape recorder

Advance preparation:

Make sufficient copies of Handouts #5-1 and #5-2 for each student. Obtain, preview and set up the filmstrips and cassettes for day 1.

Estimated teaching time:

3 class periods

Guidelines, day 1:

1. Distribute copies of Handout #5-1, "Discussion Guide for Social Security System." Explain that these questions will be discussed after viewing the filmstrips.

2. Project the filmstrips with cassette tapes (27 min.).

3. Discuss the following questions from the study guide with the class. Add information as necessary.

1. Until the 1930s who was considered responsible for the economic welfare of the individual? What happened to change this attitude?

Following the tradition of "rugged individualism," it was generally believed that people through their own initiative, hard work and thrift should be able to save enough money to meet all threats to their economic security. People did not expect the government to assume responsibility for individual welfare. By the 1930s the depression had deprived millions of workers of their jobs. With about one out of four workers unemployed, there was only one place to turn—the government. Welfare or relief programs were set up to provide immediate aid to the poor, but Social Security was established as a long-range program for the future.

5-2
2. When was Social Security established, and how does it work?

Social Security was established as a government insurance program in 1935. Originally it was intended to provide a supplementary income to retired workers, and retirement benefits are still a major part of the program. A special payroll tax (Federal Insurance Contributions Act, or FICA) pays for benefits. A percentage of each covered worker's pay is withheld and put in a trust fund or reserve. Employers are required to match the "contributions" paid by their workers. At age sixty-five retired workers begin to receive monthly benefits for the rest of their lives. In the event of a worker's or retiree's death, benefits are paid to the worker's spouse and dependent children. Workers may choose to retire at age sixty-two and receive lower monthly benefits. Monthly benefits may also be paid earlier if a worker becomes disabled. Since the first Social Security benefit check was received in 1940, Social Security has paid out about one trillion dollars in benefits to hundreds of millions of retired, widowed, orphaned, sick and disabled people.

3. How can individuals know what Social Security costs them and what they can expect in benefits?

An account of Social Security payments is maintained for each individual, so that when the worker retires it will be possible to determine how much money the worker will receive. The monthly benefits are based partly on average earnings over the highest five years paid in. The worker's pay check indicates how much is withheld, and a worker may keep personal records of these withholdings, keeping in mind that the employer's contribution is supposed to be equal to what is withheld from pay. At any time a worker may request information from the Social Security Administration about the status of his or her account. There are over 1300 regional Social Security offices throughout the country that can provide up-to-date information on benefits.

4. How secure is the Social Security System financially? What has happened in recent decades to affect Social Security finances?
When Congress set up the program in 1935 it was ensured that current revenues from FICA and interest on the trust fund would cover the cost of benefits. The reserves would be used only to meet special needs created by a recession or depression. By 1960 it was planned that the reserves would be equivalent to two years’ benefits payments. However, by the 1970s it became necessary to start using some of the reserves to meet obligations, and it was estimated that the program would be in debt by the 1990s. Several factors account for the strain on Social Security. At first only commercial and industrial workers were covered, but today over 90 percent of all workers are included. Thus, many are receiving benefits who have not contributed during their entire work years. Increased life expectancy has more workers living to retirement and receiving benefits over a longer period of time. Rapid inflation has caused monthly benefit payments to increase, and by law benefits now increase with each rise in the cost of living index. New programs added to Social Security have sharply increased expenditures. Total annual benefit payments have also climbed due to the rising number of workers choosing to retire at age sixty-five instead of sixty-five. In an effort to avert bankruptcy, Congress has set up a schedule for increasing FICA taxes over the next several years. This is expected to keep Social Security paying for itself for some years to come, but it also means that younger generations will be paying heavily to support a growing older population of beneficiaries.

**Guidelines, day 2:**


5. Review the handout, explaining the assignment. Students should complete it in class. Provide help as needed. Students can use the back of the handout to show how they solved the problem.

6. Collect the completed assignments.
Guidelines, day 3:

7. Demonstrate (or ask a student to demonstrate) on the chalkboard how to complete the assignment done in class on day 2. Respond to any questions students may have about the mathematical process. The solutions are as follows:

Problem 1: The amount paid in 1987 was $3,000 and increased by $200 each year for the next forty-one years. Thus his contribution in the year 2029 will be $11,200.00.

\[ 3,000 + (200)(41) = 11,200 \]

Problem 2: His lowest contribution in the year 1987 was $3,000. His highest contribution in the year 2029 was $11,200. Thus his average annual contribution was $7,100.

\[ \frac{3,000 + 11,200}{2} = 7,100 \]

Problem 3: His total contribution between 1987 and 2029 will be forty-two times his average annual contribution, or $298,200.

\[ 42 \times 7,100 = 298,200 \]

Problem 4: Single share maximum benefits over fourteen years will amount to $1,328,426.40. Calculators are needed for proof.

\[
\begin{align*}
\text{Year 1:} & \quad 68,364 \\
\text{Year 2:} & \quad 68,364 + (.05)(68,364) = 71,782.20 \\
\text{Year 3:} & \quad 71,782.2 + (.05)(71,782.2) = 75,371.31 \\
\text{Year 4:} & \quad 75,371.31 + (.05)(75,371.31) = 79,119.87 \\
\text{Year 5:} & \quad 79,119.87 + (.05)(79,119.87) = 83,096.86 \\
\text{Year 6:} & \quad 83,096.86 + (.05)(83,096.86) = 87,251.70 \\
\text{Year 7:} & \quad 87,251.70 + (.05)(87,251.70) = 91,614.28 \\
\text{Year 8:} & \quad 91,614.28 + (.05)(91,614.28) = 96,194.99 \\
\text{Year 9:} & \quad 96,194.99 + (.05)(96,194.99) = 101,004.73 \\
\text{Year 10:} & \quad 101,004.73 + (.05)(101,004.73) = 106,054.97 \\
\text{Year 11:} & \quad 106,054.97 + (.05)(106,054.97) = 111,357.22 \\
\text{Year 12:} & \quad 111,357.22 + (.05)(111,357.22) = 116,925.61 \\
\text{Year 13:} & \quad 116,925.61 + (.05)(116,925.61) = 122,041.00 \\
\text{Year 14:} & \quad 122,041.00 + (.05)(122,041.00) = 127,004.20 \\
\end{align*}
\]

Single Share Total: $1,328,426.40
Problem 5: As a couple they receive one and one half times total maximum benefits for fourteen years, and after his death his widow receives single share benefits for four years.

\[(1 \frac{1}{2})(1,328,426.40) + \text{(single share benefits years 15-18)} = 1,992,639.60\]

- **Year 15:** \[123,064.2 + (0.05)(123,064.2) = 129,217.41\]
- **Year 16:** \[129,217.41 + (0.05)(129,217.41) = 135,678.28\]
- **Year 17:** \[135,678.28 + (0.05)(135,678.28) = 142,462.19\]
- **Year 18:** \[142,462.19 + (0.05)(142,462.19) = 149,585.30\]

**Total:** \[\$566,943.18\]

\[1,992,639.60 + 566,943.18 = \$2,549,582.60\]

8. Discuss the following questions:

**Question 1:** Is Social Security a good investment for the typical worker?

The solution shows that a "typical worker" contributing for forty-two years beginning in 1987 could expect to pay \$298,200 into the system. Given average life expectancy, he and his wife would receive total benefits amounting to \$2,549,582.60, or approximately eight and one half times the amount paid in.

**Question 2:** Should workers be required by the government to pay for Social Security if they don't want to invest their earnings in this way?

The answer is a matter of opinion, and responses will vary. If students don't bring it up, point out that under a voluntary system, many people might not save for retirement at all. In that case society might be burdened with large numbers of elderly people who have no income or savings, and society would have to rely on families, public assistance, or welfare. Remind students also that employers have to contribute to the system an amount equal to the contributions of employees, making the real total FICA contributions to Social Security double those computed in problems 1-3 for the worker.
Question 3: What would happen to Social Security if the system were made voluntary?

If the system were made voluntary it would probably soon be deep in debt and go bankrupt. The "pay-as-you-go" principle means that, at any given time, there is sufficient money coming in from workers to pay benefits to those who are retired. If the number of people paying were substantially reduced, the Social Security System would not be able to meet its obligations to those already entitled to benefits. This would be especially true in a time of rising life expectancy and inflation when the number of beneficiaries and amount of benefits are rising.
Handout #5-1: Discussion Guide for "Social Security System"

Familiarize yourself with the following questions. Keep them in mind as you view the filmstrips and listen to the recordings. The questions will be discussed following the filmstrips.

1. Until the 1930s, who was considered responsible for the economic welfare of the individual? What happened to change this attitude?

2. When was the Social Security System established, and how does it work?

3. How can individuals know what Social Security costs them and what they can expect in benefits?

4. How secure is the Social Security System financially? What has happened in recent decades to affect Social Security finances?
Many workers who must pay Social Security taxes wonder whether the system really pays off. Will they ever receive as much from Social Security as they paid in FICA withholding taxes week after week, year after year, throughout their working careers? Of course since no one can predict how long workers will live or whether they might become disabled, there is no way to answer the question for a specific individual. However, it is possible to make some realistic assumptions about Social Security costs and benefits based on "typical" situations.

Let's take the situation of a married man who, starting in the year 1987 when he is twenty-three years old, works at jobs covered by Social Security until his retirement at age sixty-five in the year 2029. He earns a fairly high income, paying the maximum contribution in FICA taxes. His wife does not work at a job covered by Social Security and pays no FICA contribution. Assuming that he and his wife have a normal life expectancy, will Social Security pay off for them?

Using the following data, solve the problems listed below.
Data needed:

1. He pays FICA taxes for 42 years.
2. His total FICA payments in the first year (1987) are $3,000.
3. His FICA payments increase by $200 annually over the next 41 years until the year 2029.
4. He retires in the year 2029 and lives for 14 years beyond retirement.
5. His wife lives 4 more years after he dies (or 18 years after his retirement).
6. The maximum retirement benefit in the first year of his retirement (2029) is $68,364.
7. After the first year, benefits increase 5% annually.
8. As a married couple, he and his wife are entitled to 1-1/2 times the maximum benefit. After his death, his widow receives a single benefit share.

Problems:

1. How much will he contribute to Social Security in the year 2029? (This will be the greatest yearly contribution he will make.)
2. Using the least and greatest contribution made, find the average annual contribution he will make during his working career.
3. Using the average contribution, compute the total contributions paid between 1987 and 2029.
4. What are the total maximum benefits (single share) Social Security will pay to him during his retirement? (Use a calculator.)
5. What are the total benefits that will be paid to him and his wife? (Note that as a couple they will receive 1-1/2 times the maximum benefit, but after his death his widow will receive a single benefit share.)
ACTIVITY 6: ESTIMATING SOCIAL SECURITY EXPENDITURES

Overview:

Students view and discuss a filmstrip explaining what Social Security is and how the system operates. They then estimate from a graph the size of the "over sixty-five" population in the United States in 1978, and combine this information with other data to compute the Social Security expenditures for that year. Finally, the students discuss why Social Security expenditures are on the rise and are likely to continue to increase.

Objectives:

At the conclusion of this activity students will be able to:

1. Explain what Social Security is and how the system operates.
2. Estimate from a graph the size of the "over sixty-five" population in a given year.
3. Use data to estimate total expenditures for Social Security benefits in a given year.
4. Give reasons why Social Security expenditures are rapidly increasing.

Materials needed:

For each student:

a. Handout #6-1: "Discussion Guide for Social Security System"

b. Handout #6-2: "Population of the United States Over and Under Sixty-five Years Old"

c. Handout #6-3: "Assignment on Social Security Expenditures"
For the teacher:


b. Filmstrip projector and screen

c. Cassette tape recorder

d. Transparency copy of Handout #6-2

e. Overhead projector

Advance preparation:

Make copies of Handouts #6-1, #6-2 and #6-3.

Obtain filmstrips and cassettes. Preview them and set up for day 1.

Prepare transparency copy of Handout #6-3. Set up overhead projector for days 1 and 2.

Estimated teaching time:

2 class periods

Guidelines, day 1:


2. Project the filmstrips with cassette tapes.

3. Discuss the following questions from the study guide with the class.

   1. Until the 1930s who was considered responsible for the economic welfare of the individual?

      Following the tradition of "rugged individualism," it was generally believed that people through their own initiative, hard work and thrift should be able to save enough money to meet all threats to their economic security. People did not expect the government to assume
responsibility for individual welfare. By the 1930s the depression had deprived millions of workers of their jobs. With about one out of four workers unemployed, there was only one place to turn—the government. Welfare or relief programs were set up to provide immediate aid to the poor, but Social Security was established as a long-range program for the future.

2. When was Social Security established, and how does it work?

Social Security was established as a government insurance program in 1935. Originally it was intended to provide a supplementary income to retired workers, and retirement benefits are still a major part of the program. A special payroll tax (Federal Insurance Contributions Act, or FICA) pays for benefits. A percentage of each covered worker's pay is withheld and put in a trust fund or reserve. Employers are required to match the contributions paid by their workers. At age sixty-five retired workers begin to receive monthly benefits for the rest of their lives. In the event of a worker's or retiree's death, benefits are paid to the worker's spouse and dependent children. Workers may choose to retire at age sixty-two and receive lower monthly benefits. Monthly benefits may also be paid earlier if a worker becomes disabled. Since the first Social Security benefit check was received in 1940, Social Security has paid out about one trillion dollars in benefits to hundreds of millions of retired, widowed, orphaned, sick, and disabled people.

3. How can individuals know what Social Security costs them and what they can expect in benefits?

An account of Social Security payments is maintained for each individual, so that when the worker retires it will be possible to determine how much money the worker will receive. The monthly benefits are based partly on average earnings over the highest five years paid in. The worker's pay check indicates how much is withheld, and a worker may keep personal records of these withholdings, keeping in mind that the employer's contribution is supposed to be equal to what is withheld from pay. At any time a worker may request information from the Social Security Administration about the status of his or her account. There are over 1300 regional Social Security offices throughout the country that can provide up-to-date information on benefits.
How secure is the Social Security System financially? What has happened in recent decades to affect Social Security finances?

When Congress set up the program in 1935 it was assured that current revenues from FICA and interest on the trust fund would cover the cost of benefits. The reserves would be used only to meet special needs created by a recession or depression. By 1960 it was planned that the reserves would be equivalent to two years' benefits payments. However, by the 1970s it became necessary to start using some of the reserves to meet obligations, and it was estimated that the program would be in debt by the 1990s. Several factors account for the strain on Social Security. At first only commercial and industrial workers were covered, but today over 90 percent of all workers are included. Thus, many are receiving benefits who have not contributed during their entire work years. Increased life expectancy has meant more workers living to retirement and receiving benefits over a longer period of time. Rapid inflation has caused monthly benefit payments to increase, and by law benefits now increase with each rise in the cost of living index. New programs added to Social Security have sharply increased expenditures. Total annual benefit payments have also climbed due to the rising number of workers choosing to retire at age sixty-two instead of sixty-five. In an effort to avert bankruptcy Congress has set up a schedule for increasing FICA taxes over the next several years. This is expected to keep Social Security paying for itself for some years to come, but it also means that younger generations will be paying heavily to support a growing older population of beneficiaries.


5. Project transparency copy of Handout #6-2, and check that all students know how to read it.

6. Explain the assignment on Handout #6-3. (If the assignment is done in class instead of as homework, help individual students as needed.)
Guidelines, day 2:

7. Collect the assignments.


9. Demonstrate (or ask the students to demonstrate) how to solve the problem. Steps to solution are:
   a. Population over sixty-five in 1978 was approximately 26 million (26 x 10⁶). Note that the figure 26 million is only approximate since it is derived by sight from a graph curve. Students may come up with a slightly different number which would make some difference in the outcome. The solution should be accepted if the correct process is used and the answers are reasonably close to those given below.
   
   b. Number eligible for retirement benefits = .95 x 26 x 10⁶ = 24.7 x 10⁶
   
   c. Monthly Social Security outlay for retirement benefits = 24.7 x 10⁶ x $148 = $3,655,600,000
   
   d. Social Security outlay for 1978 = 12 x 3,655,600,000 = $43,999,999,992 or $43.9 billion

10. Take time to answer questions and be sure that all students understand how to solve the problem.

11. Briefly discuss the following questions:

   1. According to the graph, what can be expected to happen to Social Security expenditures in the future?

   Since the graph projects a constant increase in the over sixty-five population to the year 2000, it can be expected that expenditures will continue to rise annually.

   2. What will happen to Social Security expenditures if inflation continues?

   Since retirement benefits are increased with every rise in the cost of living index, it follows that overall expenditures will continue to rise as long as inflation continues.
Handout #6-1: Discussion Guide for "Social Security System"

Familiarize yourself with the following questions. Keep them in mind as you view the filmstrips and listen to the recordings. The questions will be discussed following the filmstrips.

1. Until the 1930s, who was considered responsible for the economic welfare of the individual? What happened to change this attitude?

2. When was the Social Security System established, and how does it work?

3. How can individuals know what Social Security costs them and what they can expect in benefits?

4. How secure is the Social Security System financially? What has happened in recent decades to affect Social Security finances?
Handout #6-2: Population of the United States Over and Under Sixty-five Years Old

Handout #6-3: Assignment on Social Security Expenditures for 1978

Using Handout #6-1, the graph "Population of the United States Over and Under Sixty-five Years Old," compute the total expenditure of Social Security for retirement benefits in 1978. Use the following assumptions:

1. Consider only retirement benefits for people over sixty-five.

2. Assume that 95% of the over sixty-five population receives retirement benefits.

3. Assume that the average retirement benefit is $148 per month.

Show the steps to your solution as well as your final answer below.
ACTIVITY 7: CAN WE AFFORD SOCIAL SECURITY?

Overview:

Students read and discuss a short selection describing the increasingly difficult problem of financing the Social Security system. They next write a computer program to estimate Social Security revenues and expenditures for the year 1975. In the process they gain appreciation for the value of computer programming, as opposed to "paper and pencil" methods, in dealing with complex mathematical problems. They learn how computer programs can help in administrative planning, and they gain insight into the problems faced by Social Security in meeting its obligations.

Objectives:

At the conclusion of this activity students will be able to:

1. Use given data to write a computer program estimating annual revenues and expenditures.

2. Cite advantages for computer programs and ways in which they can help administrators plan.

3. Explain the difficulties faced by Social Security in meeting financial obligations.

Materials needed:

For each student:

a. Handout #7-1: "Social Security--The Budget Crunch"

b. Handout #7-2: "Income Distribution for U.S. Workers (1975)"

c. Handout #7-3: "Programming Assignment"

For the teacher:

a. Transparency copy of Handout #7-2

b. Overhead projector and screen

c. Computer
Advance preparation:

Make copies of Handouts #7-1, #7-2 and #7-3 for each student.

Make transparency of Handout #7-2.

Set up overhead projector for days 1 and 2.

Estimated teaching time:

2 class periods

Guidelines, day 1:

1. Distribute copies of Handout #7-1, "Social Security--The Budget Crunch." Ask students to read the selection carefully.

2. Review their comprehension by getting verbal responses to the "check-up questions" at the end. Answers:

   1. Why was Social Security established?

      The system was set up to provide financial assistance to retired workers. Note that it was meant to provide only supplemental income, not a pension sufficient to cover all of a retired worker's financial needs.

   2. How is the system financed?

      Workers covered by Social Security pay a percentage of their wages into the system, and their "contribution" is matched by their employers. Although called a contribution, it is in effect a payroll tax which employers and employees are required to pay. Moneys received are placed in trust funds which draw interest which adds to Social Security reserves.

   3. How and when does someone benefit from the program?

      At age sixty-five retired workers are entitled to receive monthly benefits as long as they live. They may opt to retire at age sixty-two and receive lower benefits. If a worker or retiree dies, benefits are paid to the surviving spouse and dependent children.
4. What are the main factors that make it increasingly difficult to finance Social Security?

Several factors are involved, including the rising percentage of retired workers compared to workers paying into the system, inflation-indexed rising benefits paid to retirees and new programs which increase administrative costs.

5. Why do some people worry about the future of Social Security?

Continued inflation and growing life expectancy mean increased demands on Social Security reserves in the future, while a declining birthrate means fewer workers to provide revenue to the system.

3. Distribute Handout #7-2, "Income Distribution for U.S. Workers (1975)."

4. Project the transparency copy of Handout #7-2 and ask members of the class to orally interpret the graph. Answer students' questions until you are sure that all students understand how to draw from the graph the data needed for the assignment.

5. Distribute Handout #7-3, "Programming Assignment." Review the directions, indicating that the assignment is to be completed and handed in. (Set a completion date that is reasonable in terms of student abilities and schedule for computer use.)

Guidelines, day 2:

6. Collect completed programming assignments from the students.

7. Again project the transparency copy of Handout #7-2 and work through the assignment (or have some students demonstrate how to work through the assignment).

8. Respond to students' questions about the assignment, and help any students who were unable to complete the program.

9. Conduct a follow-up discussion on the following questions:

a. Would it be possible to estimate the revenue and expenditures of Social Security for a given year without a computer program?
b. What advantages would a computer program have for administrative planning?

The computer program assures speed and accuracy in dealing with complex mathematical problems which must be solved if administrators are to make intelligent decisions and plans. Once the computer has been programmed, input parameters may be changed as needed without reworking the entire program.

c. Do you think Social Security revenues in 1975 were sufficient to cover expenditures?

Responses will vary since the problem has not actually been solved.

10. Program the computer based on programs prepared by the students and compute the results. The solution to be expected from the computer is:

Total Payroll Contribution (FICA) Withholdings

\[
= .0585 \times 10^6 (15.1 \times 1500 + 18 \times 5000 + 11 \times 8500 + 12.3 \times 12050 + 25.2 \times 14100)
\]

\[
= 58500 (22850 + 90000 + 93500 + 148215 + 355320)
\]

\[
= 58500 (709685)
\]

\[
= 4.1516 \times 10^{10} = \$41.5 \text{ billion}
\]

However, since only 90 percent of workers contribute to Social Security, total FICA withholdings = .9 x 41.5 = $37.35 billion. Since employer matches FICA withholdings, income to Social Security = 2($37.35 billion) = $74.7 billion.

Amount paid by Social Security for retirement benefits:

\[
= (.95)(16 \times 10^6)(148)(12)
\]

\[
= \$43867.2 \times 10^6 = \$43.8672 \text{ billion}
\]

11. Ask students whether the estimated Social Security revenues for 1975 were sufficient to cover expenditures for retirement benefits.
The solution should show revenues of close to $75 billion, which is far more than anticipated expenditures of less than $44 billion for retirement benefits.

12. Ask students what other factors might account for a budget crunch when revenues appear to be so far in excess of needs. Among the factors that might be indicated are:

a. Administrative costs: Because Social Security deals with millions of people and billions of dollars, it requires an army of bureaucrats to administer the program. Offices are scattered throughout the country. Payroll for Social Security employees, buildings and maintenance, utilities, equipment and other expenses amount to billions of dollars in administrative costs.

b. Survivors' benefits: In addition to retirees, there are millions of spouses and dependent children who receive benefits after a worker dies.

13. Ask students what other factors might influence the financial picture for Social Security. Among the factors that might be mentioned are:

a. Changes in rates of employment: If unemployment rises, Social Security revenues will decrease since fewer payroll contributions will be received.

b. Inflation: Under present law benefits are automatically increased with each rise in the cost of living index. However, payroll contributions increase only when Congress decides to increase them.

c. As life expectancy increases, more workers live to retirement age and receive benefits for a longer period of time. A declining birthrate means proportionately fewer workers will be contributing to the system to finance retirement for a growing number of retirees.

14. Ask students whether it would be possible to constantly adjust projected revenues and expenditures for such things as unemployment, inflation or changing age structure of the population. Make the point that once a computer has been programmed to compute revenues and expenditures, new input parameters can be entered rather easily to adjust the outputs without starting from scratch.
15. In concluding the activity, discuss whether Social Security represents an economic burden to society. Responses will vary, but it is important that students understand that Social Security is not welfare. While it may be burdensome to pay retirement benefits, workers have paid for these benefits throughout their working lives and are entitled to the benefits they receive.
Social Security was established in 1935 to provide a regular income to retired workers to supplement whatever assets they were able to accumulate during their working years. At first the program only provided coverage for workers in business and industry. Over the years the program has been expanded until now over 90 percent of all workers in the United States are included.

Social Security is a self-supporting government insurance program financed by a special payroll tax (or mandatory "contribution") paid equally by employer and employee. For example, a worker in 1975 paid 5.85% of the first $14,100 earned into Social Security, and the worker's employer paid an equal amount. A personal account is maintained for each worker to keep track of amounts paid in. Payroll contributions provide the main revenue to finance the program, but some income is derived from interest on assets in Social Security trust funds.

Beginning at age sixty-five, a retired worker is entitled to receive monthly benefits until death based partly on the amount the worker earned and paid into the system. If a covered worker or retiree dies, benefits are paid to the surviving spouse and dependent children. A worker may choose to retire at age sixty-two and accept lower monthly benefits. The main expenditures from Social Security are in the form of these "Old Age and Survivors Benefits."
In recent years it has been more and more difficult to finance the program for a variety of reasons. One reason is that medical advances have increased life expectancy while birthrates have declined. Thus both the numbers of people over sixty-five and their proportion in the total population has been growing annually. The result is that proportionately fewer workers are paying into the system to support a growing percentage of retirees. Because of inflation, Congress has repeatedly increased the amount of monthly benefits. Now benefits are automatically increased each time there is a rise in the cost of living index.

Another complicating factor is that Congress has added new programs and administrative costs to Social Security. One major program is Medicare, a system of medical insurance which provides benefits to the elderly. Two others are financed directly by the government, but are managed by the Social Security Administration. Medicaid provides medical assistance to needy elderly who are not otherwise covered by Social Security. Managing these programs adds to administrative costs.

The big problem for Social Security now and in the future is one of "cash flow." Will enough money come in each year to enable Social Security to continue to meet its obligations to retired workers and their dependents? Congress has faced this challenge by increasing both the contribution which workers and employers must pay and the salary base from which it is withheld. Under current law, withholdings in 1985 will increase to 7.05% on the first $38,100 of income. While this "buys time" for Social Security, it
does not represent a long-range solution. Some workers wonder whether, after years of contributing to the system, the retirement money will be there when they need it and whether they will get a fair return for their investment in Social Security.

Check-up Questions:

1. Why was Social Security established?

2. How is the system financed?

3. How and when does someone benefit from the program?

4. What are the main factors that make it increasingly difficult to finance Social Security?

5. Why do some people worry about the future of Social Security?
Handout #7-3: Programming Assignment

Write a program to estimate the amount collected by the Social Security system in 1975.

Use the following assumptions, which are quite realistic:

a. Approximately 90 percent of the workers in each wage class were contributors to Social Security. You may assume that all workers in a wage class earn the average income for that wage class. For example: 15.1 million workers earn $1500 a year (see Handout #7-2).

b. In 1975 the Social Security tax (FICA) deducted from a worker's wages was 5.85 percent of the first $14,100 earned.

c. Employers contribute amounts to Social Security that match the contributions of each employee.

In addition, have your program compute the amount paid out by the Social Security system for Old Age Assistance in 1975.

Assume that 95 percent of the over sixty-five year old population (of 26 million) collected an average of $148 per month.
Handout #7-2: Income Distribution for U.S. Workers (1975)

<table>
<thead>
<tr>
<th>Income (Thousands)</th>
<th>Workers (Millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 3</td>
<td>15.1</td>
</tr>
<tr>
<td>3 - 7</td>
<td>18.0</td>
</tr>
<tr>
<td>7 - 10</td>
<td>11.0</td>
</tr>
<tr>
<td>10 - 14.1</td>
<td>12.3</td>
</tr>
<tr>
<td>Over 14.1</td>
<td>25.2</td>
</tr>
</tbody>
</table>

Number of People (in millions)

Income (in thousands of dollars)