The Use of the Sequential Probability Ratio Test in Making Grade Classifications in Conjunction with Tailored Testing.

Missouri Univ., Columbia. Tailored Testing Research Lab.


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THE USE OF THE SEQUENTIAL PROBABILITY RATIO TEST
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IN CONJUNCTION WITH TAILORED TESTING

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THE USE OF THE SEQUENTIAL PROBABILITY RATIO TEST IN MAKING GRADE CLASSIFICATIONS IN CONJUNCTION WITH TAILORED TESTING

In many testing applications, the major use of the obtained score is to classify a person as being above or below some criterion score. Examples of such uses of test results include the screening of job applicants and the classification of students as masters and non-masters when using the mastery learning paradigm (Bloom, 1971). For such applications it is not necessarily required that the person's ability be accurately estimated, but only that the measurements be sufficiently precise that the examinees can be accurately classified.

When making such classifications, the accuracy of measurement required in making the decision is dependent upon how far from the cutting score the person is located. If the examinee is far above or below the cutting score, minimal accuracy will be required. If the examinee is close to the cutting score, high precision will be required. Since the accuracy of an ability estimate is dependent to a large extent on test length, it follows that shorter tests can be used if a person's ability were a substantial distance from the cutting score. Depending on the number of individuals who are far from the cutting score, the average length of test needed for classification might be substantially reduced over what is commonly used.

Based on this analysis, an optimal procedure for testing examinees for classification purposes would be to check the accuracy of classification after each item is administered. If the accuracy were sufficiently high, testing could stop. If the accuracy were not high enough, another item would be administered.

Exactly this type of procedure was developed by Wald (1947) to assist in quality control work during World War II. His procedure was designed to determine whether a batch of parts was acceptable based on whether it contained a sufficiently low number of defectives. The basic concept behind the procedure is to take an observation from the batch and determine the probability of the observation under the hypothesis of an acceptable or unacceptable batch. A ratio is formed by dividing the probability of the observation coming from an acceptable batch by the probability of it coming from an unacceptable batch. If the ratio is sufficiently large, the batch is considered acceptable and if it is sufficiently small, the batch is considered unacceptable. If the ratio is near 1.0, another observation is randomly selected. A new ratio is then formed using all of the previous observations. The process continues until a decision is reached. Because of the sequential nature of the process, it has been labeled the Sequential Probability Ratio Test (SPRT).

Since its development, the SPRT has been widely used for quality control work (Govindaraju, 1975). However, only recently has it appeared in the mental testing literature. Ferguson (1970) used the SPRT procedure to determine whether 75 students had mastered material in a hierarchically arranged set of instructional units. His procedure randomly generated items by computer using item forms and then administered the items using a computer terminal. He found a substantial reduction in testing time and in the number of items.
required to make a decision. The procedure was found to be in 99% agreement with the longer tests traditionally used to make the decisions.

No other studies were found that actually made real time decisions using the SPRT procedure. However, Epstein & Knerr (1978) did present the results of a real data simulation using Army proficiency testing response data. They found that only 33% as many items were needed for the SPRT based procedure without loss in decision accuracy. Sixti1 (1974), Kalish (1980), and Kingsbury and Weiss (1980) present the results of simulation studies showing that the SPRT procedures result in a substantial reduction in the number of items required to make decisions. Thus, all the research to date supports the contention that SPRT based procedures lead to increased testing efficiency.

Despite the promising results reported in the studies listed above, none of the procedures described take full advantage of the quality items in the item pool. That is, by randomly selecting items, the best items for making the classification decision may not be administered. A better procedure would be to select the items from the item pool that would be most informative for making the decision using a tailored testing paradigm. Reckase (1978) has shown that such a procedure could be used with the SPRT as long as local independence could be assumed. In a series of simulation studies (Reckase, 1980a, 1980b), he demonstrated that SPRT procedures will work with tailored testing. Further, a three-parameter logistic based procedure was found to give better results than a one-parameter logistic based procedure.

With the positive results obtained at this time it seems prudent to evaluate the quality of SPRT/tailored testing procedures for actual decisions. The purpose of this report is to present some results of the operation of the SPRT/tailored testing hybrid in the context of grade classification. Further, one-parameter and three-parameter logistic model based procedures will be compared on the basis of decision consistency. The overall criterion for success will be a comparison with traditional grading procedures.

### The SPRT Procedure

The SPRT procedure has been described in detail elsewhere (Wald, 1947; Epstein & Knerr, 1978; Reckase, 1980a) so only a brief description will be given here. The basic equations will be presented along with the procedures for describing the characteristics of the decision making process.

As described above, the basic philosophy behind the SPRT procedure is to determine the probability of the observed responses for two alternative hypotheses and then form the ratio of the probabilities. A large ratio favors one of the hypotheses and a small ratio favors the other. For example, if \( H_1 \) is the hypothesis that the ability \( \theta \) for a person is equal to \( \theta_1 \), and \( H_2 \) is the hypothesis that the ability equals \( \theta_2 \), the probability of the obtained responses, \( x_1, x_2, \ldots, x_n \), given these hypotheses would be:

\[
p(x_1, x_2, \ldots, x_n | 1) = \prod_{i=1}^{n} p(x_i | \theta_1) \quad (1)
\]

and

\[
p(x_1, x_2, \ldots, x_n | 2) = \prod_{i=1}^{n} p(x_i | \theta_2) \quad (2)
\]
under the local independence assumption of latent trait theory. The values of \( P(x_2) \) would be computed using the appropriate latent trait model assuming known item parameters from a previous item calibration. Assuming \( H_2 \), the probability ratio would then be formed as

\[
P(x_1, x_2, \ldots, x_n | H_1) \quad \text{and} \quad \frac{P(x_1, x_2, \ldots, x_n | H_1)}{P(x_1, x_2, \ldots, x_n | H_2)}
\]

(3)

If this ratio were sufficiently large \( H_2 \) would be rejected, and if the ratio were sufficiently small \( H_1 \) would be rejected. The determination of what constitutes large and small depends upon the error rates that are considered acceptable.

Suppose \( \alpha \) is the probability of accepting \( H_1 \) when \( H_2 \) is really true and \( \beta \) is the probability of accepting \( H_2 \) when \( H_1 \) is really true. Wald (1947) has shown that a good approximation to the decision points needed for the probability ratio (Equation 3) can be obtained by the following two expressions:

Upper decision point = \( A = \frac{1-\beta}{\alpha} \)

(4)

and

Lower decision point = \( B = \frac{\beta}{1-\alpha} \)

(5)

Thus, if Equation 3 gives a result larger than \( A \), \( H_1 \) should be accepted with an error rate of approximately \( \alpha \), and if the expression yields a value less than \( B \), \( H_2 \) should be accepted with an error rate of approximately \( \beta \).

The procedure described above assumes that a decision is to be made between two simple hypotheses: \( H_1: \theta = \theta_1 \) or \( H_2: \theta = \theta_2 \). Wald (1947) has generalized this procedure to making decisions concerning complex hypotheses such as \( H_2: \theta < \theta_0 \) and \( H_1: \theta > \theta_0 \). This is a much more useful set of hypotheses because it matches the decision process used in making classification, above or below a criterion score.

In order to test a complex hypothesis using the SPRT, an indifference region must first be specified around the cutting score, \( \theta_0 \), for the decision. The indifference region is the area around the cutting score in which either classification is considered equally good. For example, if \( \theta_1 \) is the cutting score for making the decision, persons sufficiently close to \( \theta_1 \) could be classified either high or low without appreciable loss. Sufficiently close is defined here as being between \( \theta_1 \) and \( \theta_2 \) when \( \theta_1 > \theta_0 > \theta_2 \). If a person were outside the region from \( \theta_1 \) to \( \theta_2 \) and were misclassified, the error would be considered serious.

The use of the SPRT to test complex hypotheses works the same as for the simple hypotheses except that the limits of the indifference region are used in Equation 3 to form the probability ratio instead of the hypothesized true values. The upper and lower decision points for the test are determined in exactly the same way as before (Equations 4 and 5). However, now the operation of the SPRT is controlled not only by the \( \alpha \) and \( \beta \) error rates, but also by the width of the indifference region. The higher the error rates and the wider the indifference region, the fewer the items that need to be administered.
The quality of operation of the SPRT procedure is usually judged on the basis of two mathematical functions called the operating characteristic (OC) function and the average sample number (ASN) function. The OC function is defined as

\[ OC(\theta) = P(\text{classified below } \theta | \theta). \]

This function should have values close to 1.0 for \( \theta < \theta_c \) and values close to 0.0 for \( \theta > \theta_c \). To the extent that this function drops quickly from a value near 1.0 to near 0.0 in the indifference region, the SPRT procedure is working well.

The ASN function is defined as the average number of observations needed to make a decision as a function of \( \theta \). This function is typically peaked, with high values near the cutting score and decreasing values with increased distance from the cutting score. Both the OC function and the ASN function are dependent on the size of the error rates and the width of the indifference region. A narrow indifference region and/or low error rates result in a steep OC function and require a large number of observations for decisions. High error rates and/or a wide indifference region flatten the OC function and reduce the number of observations required. Thus, the price paid for high precision is a greater number of observations. More detailed information concerning the OC and ASN functions can be found in Wald (1947), Reckase (1980a), or Epstein and Knerr (1978).

Tailored Testing Procedure

Tailored testing procedures are defined by their methods of item selection and ability estimation. The procedure used in this study selects items to maximize the value of the information function (Birnbaum, 1968) at the previous ability estimate. Ability was estimated using an empirical maximum likelihood approach. The procedure is described in detail by McKinley & Reckase (1980), so it will not be described again here. The above tailored testing procedure was used with both the one-parameter logistic (1PL) and the three-parameter logistic (3PL) models in the study reported here.

Tailored Testing/SPRT Hybrid

The procedure used to administer the test items in this study used components of both tailored testing methodology and the SPRT. Items to be administered in the process of the computerized test were selected using the maximum information criterion (Birnbaum, 1968; McKinley & Reckase, 1980). After the response to each item was obtained, the value of the probability ratio (Equation 3) was computed and a decision was made to classify high, classify low, or to administer another item. If another item were to be administered, a maximum likelihood ability estimate was obtained and a new item was selected to maximize the information function at that ability estimate and administered to the examinee. The process continued until a classification decision had been made or until 20 items had been administered. After 20 items, ratios above 1.0 resulted in a high classification, and ratios below 1.0 resulted in low classification.
Research Design

The purpose of the research reported here was to compare 1PL and 3PL based procedures for making classification decisions using the SPRT. Since the true classifications were unknown, a consistency of classification design was used as a criterion for evaluation. To facilitate the comparison of decision consistency a test-retest design was used in which tailored tests based on both the 1PL and 3PL models were administered to the same individuals in two sessions one week apart. In the first session the 1PL and 3PL tailored tests were administered as described above without a break in between. From the student's point of view, only one test was administered. In the second session, the same procedure was followed, only the order of presentation of the 1PL and 3PL procedures was reversed to counterbalance fatigue effects. The initial order of presentation of the 1PL and 3PL procedures was randomly assigned to the students.

Within the tailored tests, three grade placement decisions were made using the SPRT procedure. Based on the test information, students were placed above or below the A/B grade cutoff, the B/C grade cutoff, and the C/D grade cutoff. Thus, if a student were classified below the A/B cutoff, and above the B/C cutoff, a grade of B would be assigned. The grade cutoffs for the study were set to be consistent with those used on the traditional test using the test characteristic curve.

Before the cutoffs could be set, the traditional test first had to be linked to the tailored testing item pool. This was done so that the cutoffs determined from the traditional test would be on the same scale as the tailored test ability estimates. The linking was performed using the major axis method for the 1PL model, and the maximum likelihood method for the 3PL model. See Reckase (1979a) for a more detailed description of these procedures.

The traditional test used as a basis for the grade cutoffs was a 50 item multiple choice test over the area of classroom evaluation procedures. The test and the population of students who took part in the study were from an introductory course on educational measurement techniques. The grade classification region for the traditional test in terms of raw scores were: 42-50, A; 33-41, B; 29-32, C; and 28 and below, D. Based on these score ranges, the A/B cutoff was set at 41½, the B/C cutoff at 32½, and the C/D cutoff at 28½. The 1PL ability scale cutoffs corresponding to the raw score cutoffs were A/B, 2.24; B/C, .95; and C/D, .46. The cutoffs on the 3PL ability scale were: A/B, .78; B/C, -.85; and C/D, -.39. These values were determined by finding the points in the latent trait scales that were equivalent to the raw score points.

Along with the cutting points, an indifference region and the α and β error rates were needed to totally specify the SPRT procedure. A reasonable indifference region for the test was thought to be one standard error of measurement on either side of the cutting point. Based on the traditional test reliability of .60 for the sample of students used in the study, the standard error of measurement in 1PL and 3PL ability units was .45. Thus, the indifference regions were set at A/B, 2.69 to 1.79; B/C, 1.40 to .50; and C/D, .91 to .01 for the 1PL procedure and A/B, 2.23 to 1.33; B/C, -1.30 to -.40; and C/D, -1.84 to -.94 for the 3PL procedure. The differences in indifference regions for the two procedures were due to differences in the way the origins of the ability scales were defined.
Since it was considered a more serious error to classify someone high incorrectly than low incorrectly, \( \alpha \) was set at .02 and \( \beta \) was set at .10. Using Equations 4 and 5, the decision points for the SPRT were computed to be \( A=45 \) and \( B=.102 \). This resulted in a classification in the higher grade category if Equation 3 resulted in a value greater than 45, in the lower grade category if the value was below .102, and continued testing if the result was between 45 and .102. The same \( A \) and \( B \) values were used for both the 1PL and 3PL procedures.

The sample used in this study consisted of 88 student volunteers from an undergraduate introductory measurement course. Of the 88 students, 21 were male and 67 female. The group consisted of 19 juniors, 67 seniors, and 2 graduate students. The tailored tests were administered the week following a classroom test over the same content. The examinees were told that the tailored test score would be substituted for the classroom test score if they performed better on the tailored test, and that they would receive extra credit points for completing the requirements of the study.

Analyses

The major analysis performed in this study was the comparison of the grade classifications over the test-retest period. This analysis was to show which procedure (1PL or 3PL) gave more consistent grade classification over the one week time period. Since the grade scale yields mainly categorical results, a phi coefficient derived from the chi-square contingency table was used for this analysis. The same analysis was also performed to determine which procedure made grade classifications that were more similar to those obtained from a traditional classroom test.

Along with the above analyses, the distributions of grades for the two procedures were determined and compared. The number of items required for a decision were also tabulated for each procedure and the mean number of items required were compared using a two-way ANOVA. Session and procedure were the independent variables in this analysis, with repeated measures over both session and procedure.

Results

The direct result of the tailored testing procedure in this study is the classification of students into grade categories using the SPRT paradigm. The results of this grade classification for the 1PL and 3PL tailored testing procedure, and the traditional classroom test are shown in Table 1. This table presents the frequency distribution of the grades for each procedure and each testing session. The means and standard deviations are also presented to summarize the distributions even though the data are only ordinal.

From these results, a tendency can be seen for the 1PL procedure to grade slightly easier than the 3PL procedure. The traditional test assigned the highest average grade of all the procedures. This can probably be explained by the fact that the classroom test was the test studied for and it was taken first. The standard deviations of grades for the 1PL and 3PL procedures were about the same, with a slight increase in the second testing session. The traditional test had the smallest standard deviation of all of the procedures.
Table 1
Grade Distributions for the 1PL and 3PL Tailored Tests
and the Traditional Classroom Test

<table>
<thead>
<tr>
<th>Session</th>
<th>Grade</th>
<th>1PL</th>
<th>3PL</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A(4)</td>
<td>13</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>B(3)</td>
<td>60 $\bar{x}=2.78$</td>
<td>58 $\bar{x}=2.59$</td>
<td>78 $\bar{x}=2.91$</td>
</tr>
<tr>
<td></td>
<td>C(2)</td>
<td>20 s.d.=.75</td>
<td>26 s.d.=.75</td>
<td>10 s.d.=.56</td>
</tr>
<tr>
<td></td>
<td>D(1)</td>
<td>7</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>A(4)</td>
<td>18</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B(3)</td>
<td>54 $\bar{x}=2.78$</td>
<td>50 $\bar{x}=2.65$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C(2)</td>
<td>17 s.d.=.88</td>
<td>27 s.d.=.83</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D(1)</td>
<td>11</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Note: The values presented in the table are percentages of 88 cases.

The results of the consistency of classification analysis are presented in Table 2 along with a comparison with the grades assigned by the traditional classroom exam over the same course content and the final grade in the course. As can be seen from this table, the consistency of the 3PL/SPRT procedure was substantially higher than the 1PL/SPRT procedure ($\phi = .938$ vs. .662; $t = 5.19$, $p < .01$).

Table 2
Phi Coefficients Showing the Consistency of Grade Classifications and the Relationship With Traditional Grading Practices

<table>
<thead>
<tr>
<th>Test</th>
<th>1PL-1</th>
<th>1PL-2</th>
<th>3PL-1</th>
<th>3PL-2</th>
<th>Course Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1PL-1</td>
<td>.662</td>
<td>.340</td>
<td>.489</td>
<td>.486</td>
<td>.679</td>
</tr>
<tr>
<td>1PL-2</td>
<td>.448</td>
<td>.645</td>
<td>.495</td>
<td>.467</td>
<td>.710</td>
</tr>
<tr>
<td>3PL-1</td>
<td>.938</td>
<td>.376</td>
<td></td>
<td></td>
<td>.461</td>
</tr>
<tr>
<td>3PL-2</td>
<td>.490</td>
<td></td>
<td></td>
<td></td>
<td>.649</td>
</tr>
</tbody>
</table>

Note: All phi coefficients are based on 88 cases.
The relationship between the tailored testing results and the traditional grading schemes show a more confusing pattern. The 1PL procedure had a correlation of around .5 with the exam grades and about .7 with the final grades. This was unexpected because the course exam was on the same material as the tailored test, while the final grade was based on a composite of three exams over different content areas. The correlations of the 3PL procedure with the course grade gave a similar pattern of results, but the grades assigned by the first 3PL session had lower phi coefficients. The results from the second testing were about the same magnitude as the 1PL results.

The data on the mean number of test items required to make the grade classifications are presented in Table 3. Since the tailored testing procedures were terminated if a grade decision was not made at or before 20 items, the table also gives the percent of cases making classifications in 20 items or less. As can be seen from this table, the 1PL procedure seldom was able to make classification decisions in 20 items or less, while about half the time the 3PL procedure could. Overall, the 3PL procedure required significantly fewer items to make a decision than the 1PL procedure (t=13.41 vs. 18.14). Significantly fewer items were also required for the second testing session. The ANOVA on the number of items required for classification is given in Table 4. The low number of items required for a grade classification is even more dramatic when compared to the 50 items used to make the grade classifications with the traditional test.

Table 3
Average Number of Items Required To Make Grade Classifications by Procedure and Session

<table>
<thead>
<tr>
<th>Session</th>
<th>1PL</th>
<th>2</th>
<th>3PL</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent using 20 items or less</td>
<td>5.7</td>
<td>6.6</td>
<td>50.0</td>
<td>53.4</td>
</tr>
<tr>
<td>\bar{x} for cases</td>
<td>12.0</td>
<td>14.5</td>
<td>9.0</td>
<td>11.8</td>
</tr>
<tr>
<td>\bar{x} for all cases (N=88)</td>
<td>13.6</td>
<td>17.6</td>
<td>13.9</td>
<td>14.3</td>
</tr>
<tr>
<td>s.d. for all cases</td>
<td>2.8</td>
<td>4.0</td>
<td>4.9</td>
<td>5.0</td>
</tr>
</tbody>
</table>

The data on the mean number of test items required to make the grade classifications are presented in Table 3. Since the tailored testing procedures were terminated if a grade decision was not made at or before 20 items, the table also gives the percent of cases making classifications in 20 items or less. As can be seen from this table, the 1PL procedure seldom was able to make classification decisions in 20 items or less, while about half the time the 3PL procedure could. Overall, the 3PL procedure required significantly fewer items to make a decision than the 1PL procedure (t=13.41 vs. 18.14). Significantly fewer items were also required for the second testing session. The ANOVA on the number of items required for classification is given in Table 4. The low number of items required for a grade classification is even more dramatic when compared to the 50 items used to make the grade classifications with the traditional test.
Table 4
ANOVA Results on Number of Items Administered With Model and Session as Independent Variables and Repeated Measures on P-Ah Variable

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1966.55</td>
<td>1</td>
<td>1966.55</td>
<td>96.55</td>
<td>.00</td>
</tr>
<tr>
<td>Session</td>
<td>94.10</td>
<td>1</td>
<td>94.10</td>
<td>6.59</td>
<td>.01</td>
</tr>
<tr>
<td>Model x Session</td>
<td>.56</td>
<td>1</td>
<td>.56</td>
<td>.03</td>
<td>.85</td>
</tr>
<tr>
<td>Error (model)</td>
<td>1771.95</td>
<td>87</td>
<td>20.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error (session)</td>
<td>1242.40</td>
<td>87</td>
<td>14.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error (interaction)</td>
<td>1397.94</td>
<td>87</td>
<td>16.07</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discussion

The major thesis of this paper is that the number of items required to make a decision concerning the classification of individuals above or below a cutting score can be substantially reduced from the number traditionally used. This can be done because abilities far removed from the cutting score need not be measured as precisely as those who are near the cutting score. In order to implement a testing procedure that can modify the length of the test as a function of the examinee's ability, a tailored testing procedure based on maximum information item selection and maximum likelihood ability estimation (McKinley and Reckase, 1980) was combined with Wald's (1947) Sequential Probability Ratio Test.

Common wisdom in test theory indicates that in order to accurately classify individuals into two groups, the items should be selected to be most informative at the cutting score (Lord & Novick, 1968). This could be done in this situation by selecting items with maximum information at the cutting score and using the usual SPRT procedure. However, in this case three cutting scores were present (A/B, B/C, C/D) so the usual tailored testing item selection procedure of choosing items to give maximum information at the most recent ability estimate was used.

Beyond demonstrating the economics of the tailored testing/SPRT hybrid over traditional testing, the purpose of this paper was to compare tailored tests based on the 1P model with tailored tests based on the 3PL model. The results showed that the 3PL procedure is clearly more consistent than the 1PL procedure, but that the relationship to the grades based on the classroom tests was about the same or a little worse for the 3PL procedure. This may be explained by the fact that the 1PL model tends to give ability estimates that are the sum of the components in a test while the 3PL based tests tend to give ability estimates that are more pure measures of the first principal component of a test (coe
Reckase, 1979, for a more thorough discussion). The larger correlations with the final grades than with the exam grades is probably due to the higher reliability of the final composite based on the sum of three exams. The generally low correlations with the course grades were probably due to the low reliability of the course exams (.60) and differences in method variance.

The test length analysis resulted in several interesting findings. First, the 1PL based procedure had great difficulty in classifying students into grade categories with less than 20 items. The three parameter procedure could make the classification with less than 20 items about half the time. On the average, the 3PL procedure required about 5 items less for classification than the 1PL procedure. This shorter test length with higher consistency of classification is probably a result of the advantage obtained by using the item discrimination parameter in item selection. Since the 1PL procedure assumes that all items are of equal discriminating power, only the nearness of the item difficulty parameter to the most recent ability estimate affects item selection. In selecting items using maximum information with the 3PL procedure, discrimination, guessing, and difficulty parameters contribute to selection. This results in the administration of higher quality items overall. The fewer test items required in the second session may be due to greater familiarity with the testing system resulting in fewer mistakes in using the terminals. McKinley & Reckase (1980) give more details concerning the characteristics of the items actually administered in this study.

Summary and Conclusions

The purpose of this paper has been to compare two tailored testing based decision making procedures using the Sequential Probability Ratio Test. The procedures were based on the one-parameter logistic model and the three-parameter logistic model. The procedures were also compared to traditional paper and pencil test based grades.

The results of the study showed that the 3PL based tailored test/SPRT procedure had higher decision consistency and required fewer test items than the 1PL based procedure. The tailored testing/SPRT procedure also required substantially fewer items than the traditional classroom test (X=13.4 vs. 50). These results indicate that a substantial increase in efficiency can be obtained through the use of tailored testing/SPRT procedures, but that the grades assigned may not be the same as those given using a traditional method. Of the two procedures used in this study, the 3PL based method was superior to the 1PL method in decision consistency and number of items required. Both procedures had about the same correlations with the traditional grades.
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