A Low-Inference Indicator of Lesson Structure in Mathematics.

High school geometry students were presented lessons with either a high degree of structure or a low degree of structure. Structure was defined in terms of the frequency with which concepts were repeated from one sentence to the next. After the lessons, students were tested for comprehension of the material covered and then they rated the lessons in terms of perceived effectiveness. Students presented the high structure lesson achieved significantly higher and rated the lessons higher. These findings are discussed in relation to previous research on structure. (Author)
A Low-Inference Indicator of Lesson Structure in Mathematics

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According to Rosenshine (1971), the degree to which a teacher organizes lessons is a critical dimension of teacher effectiveness. Rosenshine noted, however, that nearly all of the research he reviewed on lesson organization focused on high-inference teacher behaviors. High-inference behaviors are open to subjectivity. Low-inference behaviors, on the other hand, can be observed and objectively quantified. This article focuses on a variable related to lesson organization, referred to as "structure", and its relation to student performance in high school geometry.

The structure of lessons has been studied by several researchers. Bruner (1962, 1964) referred to structure in terms of inducing students to recognize meaningful relationships among concepts. \textit{\textsuperscript{agne} (1970) studied structure as it relates to hierarchical organizations of knowledge. Ausubel (1960, 1963) related structure to the use of advance organizers that introduce subsuming concepts involving the content to be learned. Bellack, Kliebard, Hyman, and Smith (1966) studied structure as a series of pedagogical moves that affect subsequent teacher-student interactions. Cooney, Davis, and Henderson (1975) analyzed pedagogical moves in teaching concepts, generalizations, and skills in mathematics.}

In this study, lesson structure was examined in terms of definitions and findings reported by Anderson (1967, 1969, 1970, 1974), Anderson and Lee (1975), Browne and Anderson (1974), Mathis and Shrum (1977), and Trindade (1972). These researchers indicated that structure, as they defined it, significantly affected student achievement in science.
Using the same definition of lesson structure, Butterworth (1974) found that structure significantly affected attitudes and perceptions of science students, and Simmons (1977) concluded that structure influenced success in mastering psychomotor skills (such as using a compound microscope) of science students. Ferraro, Lee, and Anderson (1977) found that structure affects the learning of science students of various mental abilities. To date, little, if any, research of this nature has been conducted outside of the science classroom. Therefore, the purpose of this study was to examine the influence of the degree of structure on the achievement and perceptions of high school geometry students.

Method

Subjects

The 84 students were enrolled in high school geometry classes in a Richmond County (Georgia) public school. The students participated by virtue of their teachers’ willingness to release them from their regularly scheduled geometry classes for one hour. Of the 84 students, 48 were males and approximately 83 percent were Caucasian. Students were randomly assigned to be presented lessons involving one of two structure conditions, either high structure or low structure. Originally, 88 students were to participate, with 44 students randomly assigned to each group. However, four students were absent on the day of the experiment, leaving 43 in the high structure group and 41 in the low structure group.

Structure

Anderson (1969) studied the structure of verbal communication in terms of the way in which new concepts are introduced and in terms of the frequency
with which concepts are repeated from one sentence of communication to
the next. Anderson proposed the following equation to define $B_1$, the
degree of structure:

$$B_1 = \frac{2n_1}{n_0 + 2n_1},$$

where $n_1$ equals the number of concepts repeated in a pair of consecutive
sentences and $n_0$ equals the number of concepts in one or the other of a
contiguous pair of sentences that are not repeated from one to the other.
The mean of all values of $B_1$ in a lesson indicates the degree of structure
of the lesson. Table 1 presents corresponding excerpts from the high
structure lesson and the low structure lesson that were used in this study,
as well as the key concepts upon which the structure computations were based.
The excerpt of the low structure lesson is longer, because both excerpts
represent the portion of the lessons from the beginning to the presentation
of the first theorem. The key concepts are numbered in Table 1 to facilitate
computations. The numbers in parentheses by the key concepts represent the
number of sentences in the lessons that referred to particular concepts.
For example, the concept "circle" was referred to 22 times in the lessons.
The mean for all values of $B_1$ in the high structure lesson was .604.
The mean for the $B_1$ values in the low structure was .396. Anderson (1970)
suggested that a mean above .50 represents a high structure lesson.

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Procedure

Each of the two groups of students was presented a mimeographed three-
page lesson concerning three geometry theorems. The first theorem presented
states that "if two chords intersect in a circle, the product of the parts of
one chord is equal to the product of the parts of the other chord" (Taylor & Bartoo, 1962). The second theorem states that "if from a point outside a circle two secants are drawn, the product of one secant and its external segment is equal to the product of the other secant and its external segment" (Taylor & Bartoo, 1962). The third theorem states that "if from a point outside a circle a tangent and a secant are drawn to the circle, the tangent is the mean proportional between the whole secant and its external segment" (Taylor & Bartoo, 1962). None of the students had prior instruction concerning these theorems, although they all had been introduced to concepts such as secants, chords, and tangents. The lessons focused on applications of the theorems and did not present proofs of the theorems.

Students were presented mimeographed lessons because this allowed precise control over the degree of structure the lessons contained and also eliminated extraneous variables that are present in "live" lesson presentations.

The high structure lesson was begun by reviewing the concepts circles and chords, and then the first theorem (involving chords that intersect in a circle) was presented. An example of an application of the first theorem then was presented. The concepts secants and external segments of secants were then reviewed, and then the second theorem (involving secants) was presented, followed by an application. Finally, the concept tangents was reviewed, and then the third theorem (involving secants and tangents) was presented, followed by an application.

The low structure lesson was begun by reviewing the concepts circles,
chords, secants, external segments of secants, and tangents. Each of the three theorems was then presented and then an application of each theorem was presented. Care was taken to construct the low structure lesson so that the logic of communication was not violated. Concepts that had not been introduced previously in the lesson were not referred to in such a way that semantic nonsense would result. That is, statements without logical introductions or with misplaced reference were not presented. As can be seen by examining the lesson excerpts in Table 1, the high and low structure lessons covered exactly the same content. The only difference in the lessons was the order in which the content was presented. Both lessons contained 23 statements and 10 corresponding figures.

Students were given ample time (up to 30 minutes) to read the mimeographed handouts. Immediately after they finished reading the lessons, student comprehension was determined by administering a 10-item test that focused on using the three theorems previously mentioned to solve for lengths of given line segments. Therefore, to solve each problem, students first were required to select the appropriate theorem and then to perform correct computations. The split-half reliability of the test was .90.

Immediately after the students completed the test, they were administered a four-item lesson evaluation (see Table 2). Similar items were reported by Butterworth (1974) to reflect the degree of lesson structure presented to college biology students.

Results

A t-test was performed on the student achievement scores as well as
on the lesson evaluation scores. The results of the analysis of both sets of scores are presented in Table 3. The achievement scores of the students presented the high structure lesson were superior to the achievement scores of the students presented the low structure lesson, $t(82) = 2.52, p < .05$. The lesson evaluation scores were obtained by totaling the scores for the four lesson evaluation items. The lesson evaluation scores of the high structure group were superior to those of the low structure group, $t(82) = 2.84, p < .01$.

Treating each of the four lesson evaluation items as dependent variables, $t$ tests indicated that, although students generally gave higher ratings to the high structure lesson, these ratings were not significantly higher for Items 1, 2, and 3. However, the difference was significantly higher for Item 4, $t(82) = 2.70, p < .01$.

Discussion

The results of this study support the research conducted in science classes in that the degree of structure significantly affected student achievement scores. This study provides initial evidence that structure affects achievement in mathematics as well as in science, although more research is needed on the effects of various levels of structure on mathematics achievement. The results of this study also indicate that mathematics textbooks should be examined in terms of their degrees of structure.

It should be noted that students did not differentiate significantly between the high structure lesson and the low structure lesson on Item 1 ("In this lesson the same idea was repeated when moving from one statement..."
to the next"), Item 2 ("Each idea presented was closely related to the preceding idea"), and Item 3 ("The lesson was well organized"). Yet these three items characterize the nature of structure as it has been defined.

Students did differentiate between the high structure lesson and the low structure lesson on Item 4 ("I understood the material presented in the lesson"). Thus, it appears that students felt uncomfortable with the low structure lesson, but did not know exactly why.

This study differs from the studies conducted in science in that students were presented their lessons in written form. Therefore, further research should be conducted on the effect of structure on mathematics achievement when material is presented verbally. When lessons are presented verbally, there is usually interaction between the teacher and the students. Therefore, student discourse should be included in the analysis of structure. Anderson (1969) explained the process for conducting such an analysis. The process is not difficult to understand, but it is tedious in that it involves tape-recording the lesson and then transcribing it and breaking it into units of discourse. Of course, students affect the degree of lesson structure according to the degree with which they make "on task" remarks.

In the context of this study, structure is related to the repetition of key concepts from one statement to the next. Excessive use of repetitive patterns without introduction of new concepts could produce habituation and reduce student attention. Therefore, caution should be exercised in planning lessons with extremely high degrees of structure. Bearing this caution in mind, research by Lamb, Davis, Leflore, Hall, Griffin, and Holmes (1979) has shown that teachers can be trained to present lessons with higher degrees
of structure. At present, most supervisors or trainers of mathematics teachers use high-inference indicators to evaluate the degree of organization (structure) of lessons. Teacher education programs should consider using low-inference indicators, such as structure as defined in this study, since structure appears to be a powerful variable related to student achievement.
"References"


Simmons, E. S. The effects of kinetic structure on knowledge about and performance of a psychomotor skill: Teaching students to use the compound microscope. *Science Education*, 1977, 61, 7-28.


### Table 1

**Excerpts from High and Low Structure Lessons**

<table>
<thead>
<tr>
<th>Key Concepts</th>
<th>Key Concepts</th>
<th>B1</th>
</tr>
</thead>
</table>

#### High Structure Lesson

Today you will be introduced to three theorems dealing with circles, chords, secants, and tangents.

From our previous discussion, you will remember that a circle is the set of all points in a plane that are a given distance from a given point in the plane. The following diagram represents circle 0. (Figure is shown.)

You will also remember that a chord is a segment whose endpoints lie on a circle. Both segments AB and DE are chords of circle 0. (Figure is shown.)

Chords AB and DE in the figure intersect in circle 0 at point O. (Figure is shown.)

Theorem 1. If two chords intersect in a circle, the product of the parts of one chord is equal to the product of the parts of the other chord.

#### Low Structure Lesson

Today you will be introduced to three theorems dealing with circles, chords, secants, and tangents.
Low Structure Lesson (continued)

<table>
<thead>
<tr>
<th>Key Concepts</th>
<th>$B_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 5, 6</td>
<td>$\frac{2}{5+2} = .286$</td>
</tr>
<tr>
<td>2, 7, 1</td>
<td>$\frac{2}{4+2} = .333$</td>
</tr>
<tr>
<td>3, 1, 10, 2</td>
<td>$\frac{4}{3+4} = .571$</td>
</tr>
<tr>
<td>4, 1, 10, 7, 11, 6, 8, 5</td>
<td>$\frac{4}{8+4} = .333$</td>
</tr>
<tr>
<td>2, 8, 1, 5</td>
<td>$\frac{6}{6+6} = .500$</td>
</tr>
<tr>
<td>7, 2, 1, 10, 3</td>
<td>$\frac{4}{5+4} = .444$</td>
</tr>
<tr>
<td>7, 4, 1</td>
<td>$\frac{4}{4+4} = .500$</td>
</tr>
<tr>
<td>2, 8, 1, 9</td>
<td>$\frac{2}{5+2} = .286$</td>
</tr>
</tbody>
</table>

Note. Key concepts are numbered to facilitate reference. Numbers in parentheses by key concepts represent number of sentences in lessons that referred to particular concepts.
### Table 2
Lesson Evaluation

<table>
<thead>
<tr>
<th>Item</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>In this lesson the same idea was repeated when moving from one statement to the next.</td>
<td></td>
</tr>
<tr>
<td>Each idea presented was closely related to the preceding idea.</td>
<td></td>
</tr>
<tr>
<td>The lesson was well organized.</td>
<td></td>
</tr>
<tr>
<td>I understood the material presented in the lesson.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Definite no</th>
<th>No</th>
<th>Yes</th>
<th>Definite yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Note.** The numbers indicate the values given to the responses for scoring purposes.
Table 3
Results of Analysis of Scores

<table>
<thead>
<tr>
<th>Dependent variables</th>
<th>High structure (N=43)</th>
<th>Low structure (N=41)</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cummings</td>
<td>M = 8.67, SD = 1.93</td>
<td>M = 7.24, SD = 3.15</td>
<td>2.52*</td>
</tr>
<tr>
<td>Lesson evaluation Item 1</td>
<td>M = 2.77, SD = 0.61</td>
<td>M = 2.56, SD = 0.78</td>
<td>1.38</td>
</tr>
<tr>
<td>Lesson evaluation Item 2</td>
<td>M = 3.19, SD = 0.45</td>
<td>M = 3.05, SD = 0.22</td>
<td>1.79</td>
</tr>
<tr>
<td>Lesson evaluation Item 3</td>
<td>M = 3.28, SD = 0.45</td>
<td>M = 3.07, SD = 0.79</td>
<td>1.51</td>
</tr>
<tr>
<td>Lesson evaluation Item 4</td>
<td>M = 3.42, SD = 0.59</td>
<td>M = 3.02, SD = 0.75</td>
<td>2.70**</td>
</tr>
<tr>
<td>Total lesson evaluation scores</td>
<td>M = 12.67, SD = 1.41</td>
<td>M = 11.71, SD = 1.68</td>
<td>2.84**</td>
</tr>
</tbody>
</table>

*P < .05
**P < .01