A Note on the Factor Structure of Some Piagetian Tasks.

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NOTE

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ABSTRACT

Some evidence supports the hypothesis that formal operational reasoning ability (at least that measured by Piagetian tasks) is a unified process. The purpose of this research was to determine: (1) if conservation tasks, such as conservation of number, liquid amount, weight and volume, are unifactor; and (2) if conservation tasks form a scale of early concrete to early formal reasoning ability, would analysis yield two principal components - a concrete component and a formal component. Ten Piagetian tasks were administered to 96 seventh-grade urban science students. The factor structure of the tasks was determined using principal components analysis. The analysis did reveal two components. One component was identified as an early concrete operational component. The other component was identified as an early formal operational component. The results were consistent with Piaget's theoretical discussions about what the tasks measure and represent a factorial validation of the tasks. This indicates that the tasks can be used in a meaningful manner to measure concrete and formal operational thought.

(Author/DC)
A NOTE ON THE FACTOR STRUCTURE OF SOME PIAGETIAN TASKS

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Introduction

A number of recent studies have attempted to analyze the factor structure of a variety of Piagetian tasks. The general intent of these studies was to identify the number of underlying cognitive parameters or abilities which determine success on the tasks, and in so doing, gain some insight into the validity of Piaget's insistence on viewing concrete and formal thought as unified stages of cognition. For example, Bart administered four formal operational tasks (the pendulum, equilibrium in the balance, motion on an inclined plane, and the shadows task) to a group of adolescents. He hypothesized, in accordance with Piagetian theory, that the four tasks would have a unifactor structure. This hypothesis was confirmed. Studies by Lovell and Butterworth, Lovell and Shields, and Lawson and Renner also found a single intellectual ability to underlie success on formal tasks involving proportionality, combinatorial reasoning and controlling variables.

Thus there is some evidence that supports the hypothesis that formal operational reasoning ability (at least that measured by Piagetian tasks) is a unified process. Is it also the case that the classical conservation tasks (such as conservation of number, liquid amount, weight and volume) are unifactor? This seems a reasonable hypothesis in that these tasks all appear to require similar reasoning processes (i.e., in the face of transformations the child must reason that each quantity under consideration remains invariant). Although the logic behind each task appears the same, certain conservations occur earlier in development than others. Elkind, for example, demonstrated that conservation
of substance invariably precedes conservation of weight which invariably precedes conservation of volume. In view of this, an alternative hypothesis regarding the factor structure of these tasks could be advanced. The conservation tasks are not unifactor. Rather, since they purport to measure either early or middle concrete operational thought, or early formal thought depending upon the quantity involved, the factor structure of these tasks may consist of as many as three components - a component for early concrete thought, middle concrete thought, and a component for early formal thought. To test these hypotheses, eight conservation tasks, which, according to the literature, require early or middle concrete thought or early formal thought for successful completion, were administered to a sample of 96 seventh grade science students. In addition to the conservation tasks, two formal tasks (separation of variables and equilibrium in the balance) known to load on a single factor were included in the battery of tasks. This was done to provide a formal component to serve as a reference point for the conservation tasks. The addition of the formal tasks allows for a third hypothesized factor pattern. If the conservation tasks do form a scale of early concrete to early formal reasoning ability, then the analysis may yield two principal components - a concrete component and a formal component. The conservation tasks which are solved early in the stage of concrete operations should load on one component and the formal tasks should load on the other component. The conservation tasks which are solved late in the concrete stage or early in the formal stage should load moderately on both components - presumably because they measure some concrete thought and some formal thought.
Method

Subjects. --Ninety-six seventh grade students (ages 11.7 years to 14.4 years; mean age = 12.6 years) were randomly selected from science classes of a predominantly black and Spanish-American urban junior high school located in a large midwestern city.

The Tasks. --The following tasks were administered in individual interviews. The conservation tasks were administered in reverse order followed by the separation of variables and equilibrium in the balance tasks. Only brief descriptions of the tasks and materials used are included since each task has been employed by previous investigators.

**Number.** Two rows of plastic chips were placed on the table. Each row contained six plastic chips. One row was shortened by pushing the chips together while the other row was lengthened by spreading the chips apart.

**Solid Amount.** Two balls of clay were presented S. One ball was transformed into a "hotdog" shape.

**Liquid Amount.** Two identical beakers (100-ml) were filled with equal amounts of water. The water from one beaker was then poured into a 50-ml pyrex graduated cylinder.

**Length.** Three wooden sticks were placed end to end. Parallel to these sticks a similar row of wooden sticks was constructed. After S agreed that both rows were the same length, E removed one stick from the end of one of the rows and placed it on the opposite end of the row.

**Area.** Two pieces of green paper of equal areas and twelve small wooden
blocks were used. Six blocks were placed on each piece of paper; one set of blocks was grouped together, the other set was spread apart.

Weight. Two balls of clay were presented S. One ball was then transformed into a "pancake" shape.

Volume-clay. The two balls of clay from the previous task and two beakers (400-ml) which contained equal amounts of water were used. S was asked about the relative level of water displacement by the clay pieces.

Volume-metal cylinders. Two metal cylinders of equal volume but different weight (18-g and 55-g) were handed S. The equal height and thickness of the metal cylinders were pointed out. After the light cylinder was placed into one test tube, S was asked to predict the height the water will rise when the heavy cylinder is placed into the other test tube.

Separation of variables. This task tested S's ability to identify and control variables, e.g.: given six flexible rods of varying length, diameter, shape, and material and hanging weights, S was asked to demonstrate proof of the effect of each variable on the amount of bending of the rods. This demonstration required understanding of the concept "all other things being equal."

Equilibrium in the balance Using a balance beam and hanging weights, this task tested S's ability to balance various combinations of weights at various locations along the beam, e.g.: given a 10 unit weight 5 units of length from the fulcrum, S was asked to predict the proper location of a 5 unit weight to achieve a balance. Successful completion of this task implied understanding of inverse proportion.

Scoring. Two points were awarded for successful completion of the conservation
tasks. One point was awarded for a correct conservation response and one point was awarded for a correct explanation, e.g.: They are the same because you did not add anything or take anything away. It is the same because you could pour the water back into the glass to the same level. Or it's the same because it is shorter but it's also wider. A total of three points each was possible on the bending rods and balance beam tasks. Subject responses were categorized and points awarded as follows:

IIA. Early concrete operational - 0 points
IIB. Fully concrete operational - 1 point
IIIA. Early formal operational - 2 points
IIB. Fully formal operational - 3 points*

Results

The following percentages of students demonstrated conservation responses and satisfactory explanations on the eight conservation tasks: number 95.8%, substance 95.8%, liquid amount 97.9%, length 72.9%, area 71.8%, weight 47.9%, volume-clay 3.1%, volume-cylinders 10.4%. On the separation of variables task 69.7% of the subjects were classified at the early concrete operational level, 26.0% at the fully concrete operational level, and 4.1% at the early formal operational level. No subjects were classified at the fully formal operational level. On the equilibrium in the balance task the percentages were: 79.1% early

* For a description of scoring criteria see Lawson, Nordland and DeVito, 1974.
concrete operational, 17.7% fully concrete operational, 3.1% early formal operational and 0.0% fully formal operational.

To determine the factor structure of the ten tasks the scores were subjected to a principal components analysis. The analysis was conducted using program FACTOR. This program extracts only components with eigenvalues greater than one and uses the varimax method of axes rotation. The results of the analysis appear in Table 1. Only two components were isolated which accounted for 55.5% of the total variance of the sample scores. The conservation of number, solid amount and liquid amount tasks loaded heavily on the second component (.73 to .86). The conservation of length, area, and weight tasks loaded moderately on both components while the remaining tasks (volume-clay, volume-cylinders, separation, and equilibrium) loaded substantially on the first component (.54 to .86). Figure 1 shows the same data in graphical form.
### TABLE 1

**PRINCIPAL COMPONENTS ANALYSIS, SEVENTH GRADE URBAN SCIENCE CLASSES, N=96, VARIANCE MAXIMIZING ROTATION**

<table>
<thead>
<tr>
<th>Task</th>
<th>First Component</th>
<th></th>
<th>Second Component</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loading, 37.2</td>
<td>Percent of Variance</td>
<td>Loading, 18.3</td>
<td>Percent of Variance</td>
</tr>
<tr>
<td>1. Conservation of Number</td>
<td>.06</td>
<td>.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Conservation of Solid Amount</td>
<td>.08</td>
<td>.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Conservation of Liquid Amount</td>
<td>.01</td>
<td>.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Conservation of Length</td>
<td>.50</td>
<td>.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Conservation of Area</td>
<td>.57</td>
<td>.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Conservation of Weight</td>
<td>.76</td>
<td>.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Conservation of Volume-Clay</td>
<td>.70</td>
<td>-.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Conservation of Volume-Cylinders</td>
<td>.54</td>
<td>.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Separation of Variables</td>
<td>.86</td>
<td>.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Equilibrium in the Balance</td>
<td>.70</td>
<td>.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1

Factor Structure of Piagetian Tasks, Seventh Grade Urban Science Classes, N = 96

1. Conservation of Number
2. Conservation of Solid Amount
3. Conservation of Liquid Amount
4. Conservation of Length
5. Conservation of Area
6. Conservation of Weight
7. Conservation of Volume Using Clay
8. Conservation of Volume Using Cylinders
9. Separation of Variables
10. Equilibrium in the Balance
Discussion

The results of the principal components analysis support hypothesis number three. Since the conservation of number, solid amount and liquid amount tasks loaded on the second component, this component was identified as representing early concrete operational reasoning ability. These three quantities are normally conserved at ages 5-6. In this sample of students, approximately 96% of the subjects demonstrated conservation of these quantities. Since both formal tasks (separation of variables and equilibrium in the balance) loaded on the second component it was identified as representing early formal reasoning ability (formal - IIIA). It was not considered to represent fully formal reasoning in that none of the 96 subjects demonstrated fully formal (formal - IIIB) responses on the two tasks. In fact the label "early formal" may even be somewhat artificial since only seven of the 192 responses on the combined tasks were classified at the formal - IIIA level. This component, rather than representing the presence of formal thought, may simply indicate the lack of success on these tasks. However, in other samples, subjects did demonstrate success on these tasks and the tasks were identified as "formal," therefore the label "early formal" may still be appropriate. The fact that only two components were extracted is consistent with Piaget's position that these tasks measure basically two types of reasoning - namely concrete and formal operational thought. However, only 55.4% of the total variance was accounted for by these two components. This percentage is not as large as might be expected and suggests that psychological parameters not identified determine to
some extent success on the tasks.

The conservation of length, area, and weight tasks loaded moderately on both components. This indicates that these tasks measure something intermediate between early concrete thought and early formal thought. This is consistent with the empirical finding that these conservations occur later than number, substance, and liquid amount conservation and that they represent more advanced reasoning than the earlier conservations. The fact that the conservation of volume using clay and conservation of volume using metal cylinders tasks loaded substantially on the formal component indicates that these tasks measure early formal reasoning. This result is once again consistent with Piaget's analysis of these tasks as indicators of beginning formal operational thought.

What then do these results suggest? Piaget has developed a stage model of intellectual development. In this model he has identified stages and substages he calls early concrete operational-IIA, concrete operational-IIIB, early formal operational-III A, and fully formal operational-III B. Also he has developed task... and argued on theoretical grounds about what those tasks measure. On the other hand, this statistical treatment was entirely objective in its analysis of the tasks. No psychological theory played a role in the results. With these points in mind, the fact that the mathematical results and predictions based on the theory were so similar supports Piaget's distinction between concrete and formal thinking and these tasks' ability to measure these thinking abilities. The results, however, should not be construed to be supportive of the theory in general or the idea that the formal operations constitute
a unified stage of development. This latter notion is certainly open to debate and is a question in need of continued investigation. Nevertheless these results do represent a factorial validation of these specific tasks and support their use as meaningful measures of concrete and formal thinking ability.
Synopsis

Ten Piagetian tasks were administered to 96 seventh grade urban science students. The factor structure of the tasks was determined using principal components analysis. The analysis revealed two components. One component was identified as an early concrete operational component. The other component was identified as an early formal operational component. The results were consistent with Piaget's theoretical discussions about what the tasks measure and represent a factorial validation of the tasks. This indicates that the tasks can be used in a meaningful manner to measure concrete and formal operational thought.
References


