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ABSTRACT

The concern of this paper is with techniques for improving the understandability of statistics textbooks for novices. Understandability is measured by tests of the reader's performance on creative transfer problems that require using text material in novel situations. The focus is primarily on the instructional objective of conceptual understanding. Techniques used for its achievement include: presentation of prerequisite information; use of familiar models and manipulatives; sequencing from familiar-to-formal; incorporating adjunct questions that encourage active processing; using worked out examples to teach strategies; and providing explicit training in representation and categorization of problems.
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What Does Educational Research Say About
How to Write and Select Statistics Textbooks?

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Abstract

This paper is concerned with techniques for increasing the under-
standability of statistics textbooks. The paper focuses on six major issues:
(1) presentation of prerequisite information, (2) use of familiar models and
manipulatives, (3) sequencing from familiar-to-formal, (4) incorporating
adjunct questions that encourage active processing, (5) using worked out
examples to teach strategies, and (6) providing explicit training in represen-
tation and categorization of problems. For each issue, an explanation, example,
and rationale is provided.

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Suppose you were going to write a textbook section to teach the concept of binomial probability, as shown in Table 1. Suppose you wanted to teach in such a way that the reader would be able to use the information in new situations, i.e. so that there would be "transfer of training." How should you design the textbook lesson to best achieve your goal of conceptual understanding of the concept of binomial probability?

The above example motivates the present paper. This paper is concerned with techniques for improving the understandability of statistics textbooks for novices. Understandability is measured by tests of the reader's performance on creative transfer problems that require using the material from the text in novel situations.

The goal of this paper is restricted in several ways. First, the design of a good statistics textbook involves decisions concerning both content (i.e. "what to teach") and instructional methods (i.e. "how to teach"). In light of the distinguished statisticians in attendance at this symposium, I feel it would be presumptuous of me to make recommendations concerning curriculum. Thus, I have decided to restrict this paper only to the latter issue--instructional design of statistics textbooks.

Second this paper is restricted in terms of the instructional objectives it addresses. A statistics textbook writer (or user) may have several different kinds of objectives in mind, including:

procedural skill--This objective involves teaching students how to use formulas and procedures in order to compute a correct numerical answer. The goal is follow a procedure and to "get the right answer."

conceptual understanding--This objective involves relating the computational procedure to other basic knowledge that the learner already possess.

The goal is to "understand what you are doing." A student who has

related the procedure to his own existing knowledge should be able to answer questions involving when to use the formula, recognize that some problems are impossible, determine whether some additional information is needed, be able to use the formula in novel transfer situations.

theoretical competence--This goal involves deriving the formula or procedure from more basic premises. Generally, algebraic representations or geometric representations are used.

Table 2 provides examples of problems related to each of these instructional goals.

The present paper will focus primarily on the second kind of instructional objective--conceptual understanding. Techniques commonly used to achieve the first objective involve: explicit presentation of the formula and procedure in steps such as a cookbook, providing drill and practice on how to compute the correct answer, evaluating progress using a mastery procedure. Since the procedures for enhancing computational skill are well known, they will not be dealt with further. Since the third objective is more frequently involved when the learner is not a novice, it will not be dealt with. Thus, the remainder of this paper deals with recommendations for how to address the goal of "conceptual understanding."

What can be done to increase the understandability of statistics textbooks? Below are listed six recommendations for either writing or selecting textbooks. These recommendations are relevant only when the instructional objective is "conceptual understanding." For each recommendation, a brief explanation, example, and rationale are given.

(1) Provide prerequisite information.

Explanation. Using standard task analysis techniques (e.g. see Greeno, 1976, Resnick, 1976), determine the concepts, ideas, and skills that are needed for understanding the new information. In order to make sure the learner possesses the prerequisite information, provide a short introductory "story" at the beginning of each chapter or section. The introductory story should explain the prerequisite information. (Another technique is to provide a short test of prerequisite knowledge at the beginning of each lesson, then refer readers the appropriate pages or appendices for instruction that is needed.)

Example. For example, for the concept of binomial probability, the learner must have a good understanding of "combinatorial analysis" and "joint probability." To understand combinatorial analysis, a student must understand the concepts of trial, outcome, success, failure, sequence of outcomes; to understand joint probability, a student must understand the concept of probability of an event. Thus, an introductory story is needed that teaches this prerequisite information. An example is given in Table 3.

Rationale. In a series of experiments (Mayer, Stiehl & Greeno, 1975; Mayer, 1975a), subjects were taught basic statistical information such as how to use the formula for binomial probability. Some subjects were taught in a way that emphasized conceptual understanding while others were taught in a way that emphasized following the steps in the formula like a cookbook. All subjects were given pretests for prerequisite concepts such as intuitions about probability and combinations. Aptitude-treatment interactions (ATIs) were obtained in which subjects given conceptual instruction performed better if they possessed appropriate prerequisite ideas, but subjects given cookbook instruction performed just as well with high as with low levels of prerequisite ideas. Similarly, in other studies subjects were either given pretraining in prerequisite ideas or not.

The performance of subjects given conceptual instruction was aided by this pre-training but subjects given cookbook instruction were not as strongly influenced by pre-training. Thus, there is evidence that when conceptual understanding is the goal of instruction, students should possess the appropriate anchoring ideas prior to instruction. It must be noted that students who scored low in prerequisite knowledge actually learned more from a cookbook approach than from a conceptual approach, while students who scored high (or were given pretraining) in prerequisite knowledge learned more from a conceptual approach than a cookbook approach. An implication is that if a conceptual approach is going to be used in a textbook then it is important to present the specifically relevant prerequisite information.

(2) Begin with a concrete, familiar model.

Explanation. Begin by building on knowledge that the learner already is familiar with. Select a concrete situational example that is analogous to the formula or procedure. The text may even suggest a manipulative that can be built by the learner using simple materials.

Example. For example, in the binomial probability lesson, separate models may be needed for combinations and joint probability. For combinations, a concrete, familiar situation involves the problem of how to seat a certain number of guests at a dinner table. The number of available seats is analogous to the number of trials (N), and the number of guests is analogous to the number of successes (r). Asking, "how many ways can you seat 3 people at a table for 4" is analogous to asking, "how many ways can you have 3 successes out of 4 trials." Table 4a provides an example text.

Rationale. The use of concrete advance organizers and manipulatives in science and mathematics prose has been subjected to intensive study (see Mayer, 1979a, 1979b for reviews). In a typical study, subjects read a text passage on computer programming. Some subjects received a one page introduction that presented a concrete model of the computer, showing memory as an erasable score-board, input as a ticket window, output as a message note pad, and executive control as a shopping list. Subjects who were given the familiar model prior to reading the lesson performed better on solving transfer problems while those who were not given the model performed best on retention of the simple facts in the booklet. The results of series of studies is consistent with the idea that the model serves as a context to which new formal or technical information can be assimilated. The role of concrete models in improving transfer performance has long been acknowledged in mathematics education (Browell & Moser, 1949, Wertheimer, 1959).

(3) Sequence from the familiar to the formal.

Explanation. Do not provide a formal statement of the rule or the computational steps in the procedure until you have given a conceptual explanation using the concrete model. When the formula is given, it should follow the concrete model and be related to it. Thus, the lesson should move from the familiar, and concrete to the formula rather than the other way around.

Example. For example, in teaching the concept of combinatorial analysis, the table sitting model should come before the formula. The example in Table 4b shows how the formula is given as a summary or description after a conceptual explanation has been provided.

Rationale. In a series of experiments (Mayer & Greeho, 1972; Mayer, 1974, 1975a), subjects read a lesson on binomial probability. For some subjects the lesson began with a statement of the formula and a list of computational steps as

in a cookbook, with conceptual information presented after computation with the formula had been emphasized. For other subjects the lesson began with a description of the concrete, familiar concepts and built up to the formula, only stating the formula after it had been explained in English. Both booklets presented the same basic information and examples; only the sequencing and emphasis differed. Subjects given the formal-to-familiar sequencing performed best on solving problems like those given in the lesson--i.e. on fast and accurate application of the formula. Subjects given the familiar-to-formal sequencing performed best on solving creative transfer questions and on recognizing unanswerable problems. Thus, there was a consistently obtained pattern, in which the familiar-to-formal sequence improved transfer performance while the formal-to-familiar sequence improved retention.

(4) Maintain a balance between computational and conceptual questions

Explanation. Students look for cues in the text for what to pay attention to. If the text emphasizes the formula, and gives many practice problems that involve computing an answer using the formula, the student will assume that the goal of instruction is computational proficiency. If the text emphasizes questions on conceptual understanding as well as strictly computational questions, the learner will direct his/her attention to more portions of the text. Thus, the text should include a variety of adjunct questions, rather than emphasize only computation questions.

Example. Table 5 provides some example questions that may be inserted in the text for binomial probability, or given at the start of the text as a sort of "instructional objective." Some questions focus on computation and some focus on verbatim definitions, and some focus on relating the formula to familiar knowledge.

7

Rationale. In a series of studies (Mayer, 1975b), we asked students to read an 8-lesson sequence on probability theory and combinatorial analysis. After each of the first six lessons, subjects received questions that asked for computing a numerical answer using the formula (computation), giving a verbatim definition from the text (definition), or relating to a concrete model that was given in the text (model). Although subjects received only one kind of question for each of the first six lessons, they were given all three types of questions for the material in lessons 7 and 8. The students who expected computation questions or definition questions performed well on what they expected but not on other types of questions, the subjects who expected model questions performed well on all types of questions. The same result was obtained when the questions were given before each lesson as an instructional objective. Apparently, adjunct questions and behavioral objectives may serve to direct the learner's attention--either to focus on how to use the formula or to conceptual information that explains the underlying ideas.

(5) Provide many detailed worked-out examples of good solution strategies

Explanation. Once the text has presented the prerequisite information, the conceptual model, and the formula, it is time to provide practice in how to solve problems. One way to allow the learner to develop a sense of elegance and style in solving statistical problems is to provide many worked out examples. The examples should be provided in sufficient detail for the student to see every step and what each step means. Thus, the worked out examples should show the strategy that a good statistician goes through, including commentary, rather than emphasizing the final answer. Worked out examples give the reader a chance to see the relationship between problems and solution strategies.

Example. Table 6 provides some worked out examples for the binomial probability lesson. Some problems show how to use the formula efficiently, others show

how to answer other kinds of questions. Students should try a problem, and then compare his answer to that of the "expert."

Rationale. Recent research (Larkin, McDermott, Simon & Simon, 1980) suggests that there are major differences between the strategies that experts and novices use to solve physics problems. In comparing expert and novice problem solving, Simon (1980) has suggested that experts are more effective because they have been exposed to many problems. Thus, one factor in developing expert solving problems is to see lots of worked out problems. A second factor that enhances expert-like problem solving is learning of general problem solving skills. For example, Schoenfeld (1979) was able to teach heuristics for solving mathematics problems to students by explicitly stating the strategy and giving examples. Similarly, in a classic study Bloom & Broder (1950) improved performance on problem solving tests by having novices observe the process that experts go through in solving problems. Thus, there is some evidence that practice in seeing examples worked out by experts can be useful in learning expert problem solving.

(6) Give explicit training in representation and categorization of problems

Explanation. Many of the difficulties that students experience in solving story problems of all kinds, is an inability to know what to do. In spite of computational proficiency, students are often "stuck" when asked to solve story problems. First, explicit instruction may be given for categorization of problems, the distinguishing conditions for the use of each formula (or test) should be clearly stated, and readers should be given practice in "pattern matching" between problems and appropriate statistical tests. Second, explicit instruction may be given for how to represent problems; subjects need training in how to find the key variables in a story problem.

Example. As an example, let's consider how to teach about categorization and representation of binomial probability problems. Table 7a shows some possible training on how to categorize. The distinguishing characteristics of binomial, probability, joint probability, and combinations problems are given. The reader is given pattern matching practice in when each formula is relevant (and not relevant). Table 7b shows some possible training on how to represent a story: the learner must find the key variables in the problem and fit the values into an equation.

Rationale. Recent research by Hayes and his colleagues (Hinsley, Hayes & Simon, 1977; Hayes, Waterman & Robinson, 1977; Robinson & Hayes, 1978) suggests that students are easily able to read a story problem and classify it by type. Many apparent difficulties occur because students have classified a problem as one type when it is really another. Recently, Mayer (1982) has developed a taxonomy of over 100 different basic types of algebra story problems. Apparently, subjects use certain cues from the problem to determine which "formula" is relevant. In another line of studies, Bobrow (1968) developed a computer program that translates story problems into equations. More recently, Mary Johnson and I have explicitly taught the steps that the program uses to human subjects, explicit training increased the translation performance of low ability subjects but not high ability subjects. Thus, there is evidence that students may benefit from training in how to categorize problems (i.e. how to determine which formula is relevant) and how to translate problems (i.e. how to find the values for the key variables).

The six suggestions given above are not fool-proof guidelines based on solid research support. Rather, they are "state of the art" proposals that require further testing and refinement. Certainly, the experience of the textbook author (or selector) must be used in conjunction with any set of recommendations.

However, I hope that research will continue in the area of textbook design, and that these recommendations will stimulate further improvements in statistics textbooks that are used in our nation's classrooms.

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Table 1

The Formula for Binomial Probability

$$P(r/N) = \left(\frac{N!}{r!(N-r)!} \right) \times (P^r) \times (1-P)^{N-r}$$

where $P(r/N)$ stands for binomial probability, N stands for number of trials, r stands for number of successes, and p stands for probability of success.

Table 2

Questions for Three Kinds of Instructional Objectives

Procedural Skill

If $N = 4$, $r = 3$, $p = .2$, use the formula to find $P(r/N)$.

Conceptual Understanding

Suppose that two people out of every nine in a certain town like John Wayne movies. If a sample is taken, what is the probability that two people in the sample like John Wayne movies?

(Not answerable)

Is there a difference between the probability that two dice rolled at once both come up 6 and the probability that one die rolled twice comes up 6 both times?

Theoretical Competence

Derive the formula for number of combinations, $C = \frac{N!}{r!(N-r)!}$.

Table 3

Introduction to Some Prerequisite Concepts

Two important concepts are trials and outcomes. A trial is something we do. The outcome of a trial is just what happens on the trial. Usually there are several possible outcomes of a trial.

For example, imagine rolling a die. Rolling the die is a trial. The possible outcomes are the numbers 1, 2, 3, 4, 5, and 6. The number that comes up when we roll the die is called the outcome of the trial.

Another important concept is success. We define a success as one or more of the possible outcomes. Then if one of those outcomes occurs, we have a success.

In rolling a die we could decide to define success as rolling a 5 or a 6. Then if 5 or 6 comes up, a success has occurred.

We can define success in different ways. A success might be rolling an even number. Then if the outcome is 2, 4, or 6 we have a success. Or a success might be not 4. Then success outcomes are 1, 2, 3, 4, 5, and 6.

The next concept to learn is the probability of success. The probability of success is the proportion of trials on which a success would occur if there were a large number of trials. For example, if a success is an outcome of 5 or 6 in rolling a die, then the probability of success is $1/3$. This is so because we expect a 5 or 6 to come up on about one-third of the times the die is rolled.

In other words, the probability of a success is the number of success outcomes divided by the total number of outcomes (including success outcomes) if all the outcomes have an equal chance. If a success is 5 or 6 and all the possible outcomes have an equal chance, then

probability of success is $2/6$ or $1/3$ because there are two possible outcomes with six outcomes in all.

On the other hand, a failure occurs whenever success does not occur. For example, if we say a success is either of the outcomes 5 or 6 in rolling a die, then the outcomes 1, 2, 3, or 4 could be a failure. The probability of failure is: Probability of success subtracted from one. If success is rolling 5 or 6, probability of success is $1/3$ and probability of failure is $2/3$.

The next concept to learn is a sequence. A sequence is what happens when we conduct several trials, one after the other. Suppose each trial is rolling a die and we define a success in some way. If we roll the die five times we might obtain the sequence (failure, failure, success, failure, success), or more simply (F, F, S, F, S).

Any sequence has a probability. The probability of a sequence is the product of the probabilities of the individual events. For example, if probability of success is $1/3$, then the sequence (success, failure, success) has probability $(1/3) \times (2/3) \times (1/3) = 2/27$. The sequence (F, F, S, S, F) has probability $(2/3) \times (2/3) \times (1/3) \times (1/3) \times (2/3) = 8/243$.

We let the number of trials in a sequence be symbolized by the letter N and the number of successes in those trials is called R . The number of failures is $N-R$.

N = number of trials

R = number of successes

$N-R$ = number of failures

For example, in the sequence (S, F, S) there are three trials and two successes. Therefore, $N=3$, $R=2$, and $N-R=1$.

Table 4a

A Concrete Model for Combinations

Suppose you wanted to seat 3 elves at a table set for 5 people.
How many ways can you seat them (assuming that all elves are identical)?

You can think of this problem as beginning with 5 spaces.



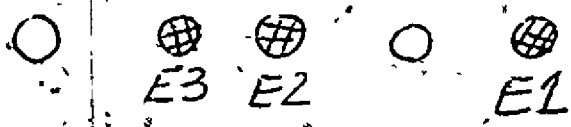
One elf can go in each space. The first elf that is seated has his choice of five different spaces. Let's say he is seated in seat 5.



No matter which seat is chosen for the first elf, there are four possible seats left for you to put the second elf. Let's say you pick the third space.



And no matter which spaces you choose for the first two elves, there are three spaces left to put the last elf. Let's say you picked the second seat.



The sequence of elves at the table is:

$$x E_3 E_2 x E_1$$

This yields one possible sequence of 3 elves in 5 seats. Notice that there are 5 possible first choices and 4 possible second choices and 3 possible third choices, yielding $5 \times 4 \times 3$ or 60 orderings.



Table 4a (cont.)

But wait a minute--since all elves are identical, there could be several ways to generate each sequence. For example, $xEExE$ could be generated as $xE_1E_2E_3$ or $E_1E_2E_3$ or $xE_1E_3xE_2$ or $xE_3E_1xE_2$, etc. There are $3 \times 2 \times 1$ ways to arrange three elves in three seats, so whichever three seats are selected there will be 6 duplicates. To compensate for all these duplicates, you must divide 60 orderings by the number of duplicates of each sequence--thus, the number of combinations is $60/6 = 10$.

Table 4b

Relating the Model to the Formula

Let's say that the number spaces at the table is N and the number of elves is R . Thus, in the above example $N = 5$ and $R = 3$. To find the number of orderings we multiplied $N(N-1)(N-2)$ or $5 \times 4 \times 3$. The general formula for this is:

$$\text{Orderings} = \frac{N!}{(N-R)!}$$

Remember that $!$ means "factorial". $5! = 5 \times 4 \times 3 \times 2 \times 1$ and $(5-3)! = 2! = 2 \times 1$ so $5!/2! = 5 \times 4 \times 3$.

We must also divide by the number of duplicates. This corresponds to the number of ways 3 elves can sit in 3 places or $3 \times 2 \times 1$. The general formula for number of duplicates is $R!$. Thus the formula for number of combinations is:

$$\text{Combinations} = \frac{N!}{(N-R)! (R!)}$$

Thus, for the elves problem $N = 5$ and $R = 3$ so combinations = $\frac{5!}{2!3!}$
or $5 \times 4 \times 3 / 3 \times 2 \times 1$ or 10 .

The formula is just a short hand way to tell about number of first, second and third choices and the number of duplicates.

Table 5

Examples of Three Kinds of Adjunct Questions

Computation

$N = 4, R = 2$, Find $C(N,R)$

Definition

Write the formula for number of combinations.

In one sentence, write the definition of number of combinations.

Model

Suppose I pick five cards from a normal 52-card deck, with replacement, and exactly three of the cards I picked are red. Draw a picture to relate this problem to a situation in which a certain number of people will be seated at a table set for larger number of people. How many first choices are there? How many second choices? How many third choices?

Table 6.

A Worked Out Example

Tom's Problem

$N = 3, R = 5, p = 1/2$

What Tom Said

1. First, I'll try to think of a real situation that corresponds to this problem. Let's say I am flipping coins.
2. I'll say p refers to the probability of "heads" coming up on a flip.
3. Then, N refers to the number of times I flip the coin, which is 3.
4. And R refers to the number of times I get a heads to come up on the coin. That happens 5 times.
5. Hey, wait a minute. That cannot be right. How can the number of successes (5) be more than the number of trials (3). This is a trick problem!

Table 7a

Portion of a Section on Training Categorization Skill

Definitions

Binomial Probability is the probability that you obtain R successes out of N trials. The problem must give a value for p (the probability of success), a value for R (the number of successes), and a value for N (the number of trials). The problem must ask for a probability--namely the probability of this general outcome.

Joint Probability is the probability that you obtain a specific sequence of R successes in N trials. The problem must give you a value for p (the probability of success), and a specific sequence of successes and failures. The problem must ask for a probability--namely the probability of this one specific outcome.

Combinations is the number of different ways you can arrange R identical successes in N trials. The problem must give you a value for N (number of trials) and for R (the number of successes). The problem must ask you for a number--namely how many different combinations there are when you don't want to consider duplicates.

Problems

For each problem, fill the space with a, B (for binomial probability), J (for joint probability), C (for combinations) or X (for none).

_____ A bag has red and blue chips with twice as many reds as blues. What is the probability of picking 4 reds in a row and then one blue?

_____ A die is rolled six times. What is the probability of evens coming up twice as many times as odds?

_____ When a die is rolled four times, what is the probability of the sequence: success, success, failure, failure?

_____ One-tenth of the peanuts in a barrel are rotten. If you take five peanuts, what is the probability that the first four are good and the fifth one is rotten?

_____ If a fair die is rolled 4 times and the number of successes equals half the number of trials, how many different sequences could be generated?

Answers

J, B, X, J, C

Table 7b

Portion of a Section on Training Translation Skill

Definitions

The main variables are:

N = number of trials
 R = number of successes
 p = probability of success

Problems

For each problem list the values for N , R and p that are given.

A coin is flipped six times, giving a sequence of heads and tails.
 How many different sequences contain two heads and four tails?

$N = \underline{\quad}$, $R = \underline{\quad}$, $p = \underline{\quad}$

If a die is rolled 100 times and success is defined as 1 or 2, does the number of possible sequences with 47 successes equal the number of possible sequences with 47 failures?

$N = \underline{\quad}$, $R = \underline{\quad}$, $p = \underline{\quad}$

Define a success as rolling a 1 or 2 on a die. If the die is rolled five times, what is the probability that there are successes on exactly two of the trials?

$N = \underline{\quad}$, $R = \underline{\quad}$, $p = \underline{\quad}$

A die is rolled six times. What is the probability of evens coming up twice as many times as odds?

$N = \underline{\quad}$, $R = \underline{\quad}$, $p = \underline{\quad}$

Answers

$N = 6$, $R = 2$, $p = 1/2$

$N = 100$, $R = 47$, $p = 1/3$

$N = 5$, $R = 2$, $p = 1/3$

$N = 6$, $R = 4$, $p = 1/2$