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ABSTRACT

The two main objectives of the present study were to get a better understanding of sixth-graders' solution processes with respect to arithmetic word problems and to investigate the possibility of improving children's problem-solving ability through instruction. As background for the study, a hypothetical model of the problem-solving process was developed based on previous research. In the first part of the investigation, quantitative and qualitative data on the forms of problem-solving behavior were collected in two classes. In this ascertaining study, important shortcomings in children's solution strategies were discovered, and it was hypothesized that these shortcomings could be overcome by instruction. Therefore a teaching experiment was undertaken during a two-week period involving teaching the experimental class a solution-strategy for word problems in which estimating the outcome of a problem was a central concern. Estimating a problem's outcome systematically before working out the solution was expected to be an effective heuristic strategy that induces pupils to analyze the problem on the one hand and to anticipate the solution on the other. At the end of the teaching program, a posttest was administered to the experimental and the control groups. The results of this experiment are discussed. (Author/MP)

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ESTIMATING THE OUTCOME OF A TASK AS A HEURISTIC STRATEGY IN ARITHMETIC PROBLEM SOLVING : A TEACHING EXPERIMENT WITH SIXTH-GRADERS

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Abstract

The two main objectives of the present study were to get a better understanding of sixth-graders' solution processes with respect to arithmetic word problems and to investigate the possibility of improving children's problem-solving ability through instruction. As background for the study, a hypothetical model of the problem-solving process was developed based on previous research. In the first part of the investigation, quantitative and qualitative data on the forms of problem-solving behavior were collected in two classes. In this ascertaining study, important shortcomings in children's solution strategies were discovered, and it was hypothesized that these shortcomings could be overcome by instruction. Therefore a teaching experiment was undertaken during a two-week period involving teaching the experimental class a solution-strategy for word problems in which estimating the outcome of a problem was a central concern.
We thought that estimating the outcome of a problem systematically before working out the solution would be an effective heuristic strategy that induces pupils to analyze the problem on the one hand and to anticipate the solution on the other. At the end of the teaching program, a posttest was administered to the experimental and the control groups. The results of this experiment are discussed.

1. Background and framework of the study

The present study is concerned with sixth-graders' solution processes in simple and more complex arithmetic word problems. This investigation is part of a more comprehensive research project where we try to make a contribution to a theory of learning in solving arithmetic problems among elementary school children. This research project is delineated by the following basic principles, that are in line with recent developments in instructional psychology (see e.g. De Corte, Carpay & Span, Note 1; Glaser, 1981)

(1) Trying to integrate two kinds of research that have been often contrasted in the past : basic research oriented toward theory-construction and applied research having an accent on practical relevance. As a consequence, our studies have explicitly a knowledge-domain orientation; they take place in settings that are ecologically valid; and they have as their ultimate goal contributing to the improvement of instructional design and practice.

(2) A central characteristic is the process-orientation toward actions : in studying children's learning we are not satisfied with observing external behavior but instead try to analyze the mental actions and cognitive structures that underly performance.

(3) Methodologically, a variety of data-gathering techniques and research designs are applied. The background idea of this approach is that one needs a diversity of information to get a complete understanding of children's cognitive processes. We mention here especially that the teaching experiment is considered to be a valid research strategy to test hypotheses on problem-volving processes.

The view of problem solving behind the present study can be summarized as follows (De Corte & Verschaffel, Note 2). Children's ability to solve arithmetic problems largely depends on the degree to which they have an appropriate orientation basis that enables them to approach intelligently and systematically unfamiliar tasks for which they do not immediately have a ready-made solution procedure. Possessing such an appropriate orientation basis involves children's being equipped with two complementary kinds of actions : (1) actions that consist of being able to use and apply relevant conceptual knowledge of subject-matter content, such as concept, principles, etc.; and (2) thinking procedures for handling, analyzing, and transforming problem situations so that they make contact with specific subject-matter content.

2. A hypothetical model of the problem-solving process

Different researchers have suggested more or less explicitly a division of the problem-solving process into stages. We mention Polya's (1945) problem-solving rules, Resnick & Glaser's (1976) hypothetical model relating to invention problems which is in line with the current information-processing approach to cognition, and Van Parreren's (1974-1975) action-oriented approach which is based on work by Selz, and Duncker on the one hand, and on recent research in Russian instructional psychology on the other (see also De Corte & Borremans, 1980). Starting from these conceptions, we have developed a model of the optimal solution process with respect to arithmetic tasks in which different stages are distinguished. We explicitly state arithmetic tasks and not arithmetic problems because a certain task does not necessarily constitute a problem for all pupils. By a problem we mean, then, a task for which a pupil does not have an answer or a ready-made solution strategy immediately available. Tasks for which a pupil has automatically a solution method will further be callled familiar or routine tasks. The following word problem can constitute a problem for some sixth-graders, and a routine tasks for others : "Ann's score on an arithmetic test is 60 out of 75; Joe who goes to another school, scored 48 out of 60. Who achieved the better result ?" The model of the problem-solving process represented in Figure 1 applies to both groups of pupils.

Insert Figure 1 here

Identification of the task

Taking into account the distinction between a problem and a familiar or routine task, the presentation of an arithmetic task can evoke four different identification actions in pupils.

(1) The task is immediately recognized as a routine one. In this case the pupil has a ready-made solution method or routine procedure immediately at his disposal.
(2) The task is, after a short while, recognized as a routine one.
(3) The task is immediately identified as a problem. In this case, the pupil realizes immediately that he has no available ready-made solution procedure.
(4) The task is, after a while, detected as a problem. According to Resnick & Glaser (1976, p. 211), problem detection can arise in different ways : either no solution procedure relevant to the task is found; a tentative solution procedure was found, but the necessary conditions for running it are not met; or the inappropriate procedure is applied, but the action is not succesful.

Problem analysis

In situations (1) and (2) described above, the pupil can proceed to employ the method which he or she knows will lead to the solution of the task. On the contrary, in situations (3) and (4) the pupil is confronted with a real problem. This means that he realizes that he does not have a ready-made solution method available, but, at the same time, he knows that the task probably contains the cues and the data which are necessary to obtain a solution. This state gives rise to problem analysis guided by questions such as the following : What are the data; What should I look for; How can I find this; Have I met a similar task before for which I found the solution ?

In the literature, this problem analysis, in which heuristic procedures play a great part, is often considered to be the core activity of the problem-solving process (Van Parreren, 1974-1975; Frijda & Elshout, 1976). These heuristic procedures are search strategies that make it possible to approach a problem in an intelligent and systematic way; although they do not guarantee finding a solution, they increase the probability of finding it. The ultimate objective of problem analysis is to transform the initial problem to the point where it has reached the form of a routine task.

Solving the routine task

Very often the problem transformation results in a statement of the task that is familiar to the pupil but still requires the application of subject-matter content (such as a concept, a principle, a formula, or an algorithm) to obtain the solution. This can be illustrated by means of the word problem mentioned above. The problem could be transformed into the following familiar task : 60/75 is greater than, equal to or smaller than 48/60. The solution can then easily be found by means of the procedure for reducing fractions to the same common denominator or for converting fractions to decimals.

Carrying out verification actions

Carrying out verification actions is the stage which finishes, possibly only provisionally, the optimal problem-solving process for those pupils, who recognized the task, either immediately or after a short while, as a familiar one as well as for those for whom it was a problem. In a previous study (De Corte & Verschaffel, Note 2), we have stated that mastering verification actions implies the following : (1) the pupil knows which successive steps he has to take in order to verify the answer to a task; (2) the pupil can justify these actions in terms of meaningfulness and efficiency; and (3) the pupil applies the verification actions spontaneously as a final step in his solution process to a problem.

arithmetic problems is strongly determined by the degree to which they have an appropriate orientation basis which enables them to approach new and unfamiliar tasks for which they lack a ready-made solution procedure intelligently and systematically. As we have stated before, such a complete orientation basis involves pupils being equipped with two complementary types of actions : (1) actions that consist of being able to use and apply relevant conceptual knowledge of subject-matter content such as concept, principles, etc.; and (2) thinking procedures for analyzing and transforming a problem to the point where it has reached a form that is familiar and makes contact with specific subject-matter content.

With respect to pupils of the sixth-grade, the highest class of the elementary school in Belgium, it is often established that they are not very successful in solving more or less complex word problems. It was hypothesized in our study that this is mainly due to a lack of the second kind of actions mentioned above - namely, thinking procedures. Therefore, to improve sixth-graders' problem-solving ability they should acquire the attitude and the skills to analyze and represent the relations between the data of the problem before starting to perform computations. Besides techniques for problem analysis, verification actions from another component of the equipment of an efficient problem solver. We thought, then, that systematically estimating the outcome of a word problem before working out the solution would be an effective heuristic strategy that leads pupils to analyze the problem on the one hand and to anticipate the solution on the other . The analysis of the problem provides the problem solver with an appropriate orientation toward the solution process, while the anticipated solution provides him with a means for verifying his final outcome.

In this study we have defined the concept estimating as follows : estimating is trying to get the approximate solution to an arithmetic task - a familiar one as well as a problem - by passing roughly and in an abbreviated way through the solution process. We have tried to analyze this estimating activity in terms of its main characteristics. Therefore, an analysis of a number of textbooks designed for the last two years of the elementary school was undertaken, and a series of twenty arithmetic tasks was administered to one pupil of the sixth-grade. Figure 2 gives a schematic overview of the outcome of our analysis.

Insert Figure 2 here

A first cluster of characteristics derives from the fact that the solution to the task is anticipated by estimating. In other words, the solution is already known, albeit only approximatly, before the "real" solution process is carried out (1). As a consequence of this anticipation of the solution, the pupil will be more goal-oriented during his problem-solving activity (3). At the same time, his resources for verification will increase (4). Indeed, the estimated outcome provides him with a useful criterion to which he can compare his final answer during the verification stage. It is expected that the pupil's chances to obtain the correct solution will be enhanced by these considerations (13).

Estimating the outcome of a task also implies an orientation toward the solution process (2). From this aspect a second cluster of characteristics is derived. Estimating the outcome starts the pupil's thinking about the task (5) : "Does the task confront me with a problem or is it familiar to me ?" After the identification of the task either as a routine one (6) or as a problem (7), the orienting process differs further in each case.

To estimate the solution to a problem, it is necessary to undertake some degree of problem analysis (8). It is precisely this activity which constitutes the heuristic value of the estimating strategy and which, at the same time, represents its importance for learning how to solve problems. That a pupil has gone through a kind of problem analysis in view of getting the approximate answer to the problem will influence the proper solution process in the sense that the problem space is reduced (9). Such a reduction in the search space implies better insight into the nature of the problem (10) It will now be obvious that the ultimate objective of teaching the estimating strategy rests in providing pupils with a method that enables them to approach a problem more thoughtfully and less routinely.

Orientation also plays a role when one estimates the outcome of a familiar task. Although, the process is less complex than with a problem, it is nevertheless, similar. Estimating leads in this case to a reduction in the search space as well (11). Consider e.g. the following routine task : "Divide 129 by 39 until there is no remainder". (Solution : 3.307692308). Pupils who estimate correctly know that the solution lies between 3 and 4. As a consequence, they will pay more attention to the size of the numbers and to the relations between the data and the unknown. In other words, the task becomes, so to speak, more transparent (12).

Altogether these characteristics enhance the probability of obtaining the correct solution (13) so that the estimating strategy can correctly be considered to be a heuristic procedure. Finally, we would like to emphasize that, in reality, both clusters of characterics are closely connected (14).

4. Overall research design

Within the theoretical framework described above, we designed and carried out an investigation consisting of an ascertaining and a teaching experiment (Kalmykova, 1970, p. 128; see also De Corte & Verschaffel, Note 2).

In an ascertaining experiment, one determines how learning takes place and which outcomes are achieved under given, already formed, conditions of learning; there is no question of systematic instruction to improve the learning process. In the present investigation, the objective of the ascertaining experiment was to determine how well and how sixth-graders solve simple and more complex word problems. We were, then, not only interested in pupils' performances but also in the processes and actions underlying those performances.

In a teaching experiment, one tries to design favorable conditions for learning based on hypotheses concerning the optimal course of the teaching-learning process. Such hypotheses are developed on the basis of systematic observation and theoretical reflection on the data collected. Starting from those hypotheses, a teaching device expected to produce high learning outcomes is constructed. Through implementation of the teaching device in a well-controlled situation, it is possible to test the underlying hypotheses concerning the optimal course of the learning process. In the present investigation, we examined whether teaching pupils a problem-solving strategy leads to better performance, and we also tried to interpret possible effects in terms of underlying actions and processes.

The overall-design of the study corresponds to Campbell & Stanley's (1963) pretest-posttest design with control group. The ascertaining part of the investigation constitutes the pretest stage. The ability to solve arithmetic word problems was established in an experimental class (N=20) and in a control class (N=21). On the basis of the findings of this ascertaining experiment on the one hand, and the ideas explained above on estimating as a heuristic strategy on the other, a teaching experiment was designed in which we developed an experimental teaching program. The objective of the program was to teach children a strategy for solving word problems in which estimating the outcome before working out the solution occupies a central place.

One of the researchers implemented the program in the experimental class during a two-week period, and, at the end of the program, a posttest was administered to the experimental and the control groups. Due to certain unforeseen circumstances, it was not possible to administer a retention test.

5. The ascertaining study

5.1. Method

As stated before, the objective of the ascertaining study was to determine how well (performance data) and how (process data) sixth-graders solve simple and more complex word problems. A specially designed test was administered in the experimental and the control class. The test consisted of ten items : one numerical task (multiply .523 by 289.25) and nine word problems. Four of the nine word problems were of the more simple type : for example, "Maria got 180 fr. (francs) to go to the bakery; she received 11.25 fr. in change; how much did she have to pay ?" The other five word problems were more complex : for example, "Five workers got their wages after ten days; altogether they received 50,000 fr.; under te same conditions, how much would the total amount of the wages of six workers after eight days be ?"

In the control class all the items of the entire test were administered at the same time. In the experimental class, eight items were administered first, and the pupils were asked to write all the arithmetic operations carried out on their answer sheets. To get even more information about the processes and actions underlying performance, the remaining two tasks, one simple and one complex word problem, were presented on a separate sheet, and the pupils were asked to solve the problems in the same way as the preceding ones, followed by a short written report describing solution processes : "What have you done and thought to solve the problem ?" Finally, after the full session, three problems, a simple one and two complex ones, were administered individually to three children; they were asked to think aloud while solving the items and their comments were recorded on audiotape.

An extensive quantitative and qualitative analysis was performed on all those data paying, thereby, special attention to pupils' errors.

5.2. Results

Table 1 gives the average results of the experimental and control groups on the pretest.

Insert Table 1 here

First of all, Table 1 shows that, on the average, these sixth-graders commit a great many errors, namely 53 % in the experimental class and 42 % in the control group. This finding is in line with the more or less systematic data reported by educational practitioners to which we referred before (see section 3). On the other hand, there is a notable significant difference between the two groups [t (2.0876) = 2.1304 (two-tail test), $p < .05$]. We mention that the variability of both groups differs also significantly [experimental class : 10; control class : 19; F (2.94) = 3.4152, $p < .01$]. In spite of the significant difference between the two groups on the pretest, we have, nevertheless, continued the study with those classes. When comparing the results on the posttest it will, of course, be necessary to take the difference in initial level between the groups into account.

A second finding in Table 1 which is also in line with evidence from educational practice is that simple problems are solved better than complex word problems, as attested by the result that, in both groups, the ratio approaches 2 : 1. This can be interpreted in terms of the difference between a routine task and a real problem. The simple tasks will more frequently be identified as familiar tasks, while the more complex word problems will more frequently constitute a real problem for sixth-graders. Because the important difference between the solution process of a routine task and a real problem lies in the stage of problem analysis, we hypothesize that pupils do not sufficiently apply methods for problem analysis to the more complex word problems. The qualitative data reported below support this interpretation.

In a further analysis, two categories of errors were distinguished : thinking errors and technical errors (De Corte & Verschaffel, Note 2). Thinking errors are due to choosing and carrying out an incorrect arithmetic operation during the problem-solving process. Technical errors, on the contrary, are due to mistakes in the execution of an arithmetic operation. In the present study, pupils' thinking errors primarily came about as a result of the following narrowly connected factors : lack of insight into the task, an inappropriate approach to the task, and insufficient methods for problem analysis or failure to apply them. As an illustration, we mention a frequently occurring thinking error in relation to the problem about the workers, given above as an example of the test items. The error consists of merely taking into account either the number of workers or the number of days. The distribution of all the observed errors in the experimental class over the two categories is as follows : 78 % thinking errors and 22 % technical errors. This result justifies the conclusion that, for those

sixth-graders, the difficulties with respect to word problems are set primarily in the thinking phase of the solution process. At the same time, this finding supports the hypothesis put forward above concerning pupils' weakness in the matter of problem analysis.

The written reports of the experimental class with respect to two word problems and the thinking-aloud protocols of three pupils provided additional qualitative data concerning the cognitive processes which occur during problem solving. Although they were generally rather short, the written reports contained a great deal of relevant information. The audio-tapes of the individual sessions did not reveal much new data; nevertheless, they were useful in confirmation of the findings extracted on the basis of the other material.

The most important result of the analysis of the qualitative data is that sixth-graders, in fact, employ systematic problem analysis rather rarely when they are confronted with a new and unfamiliar task. On the contrary, it seems quite customary for them to start performing computations almost immediately after they have read a task. Also when the latter is not the case and the learners are really confronted with a problem, this situation does not give rise, for the most part, to an attempt to analyze and understand the problem. They rather try to get external cues - for example, by asking questions - concerning the computations that have to be performed. All this shows that the pupils either do not consider problem analysis to be an essential stage in the process of problem solving or neither have available nor master methods for problem analysis. It is quite understandable that, in these circumstances, the learners look for external cues. As an illustration, we mention the following passage from a written report : "At first I didn't know how to start, but, as soon as the hint came, I could start; I just multiplied."

Another finding is the almost complete lack of verification actions performed by pupils. They do not even try to test if their answer is plausible or possible or to check it roughly by paying attention to the size of the numbers. An example of the first point is the frequently occurring answer to a simple word problem. "With his savings, Peter can buy 11.444 cookies." To illustrate the second point, we first mention the task : "Jan runs around a ring which is 379.25 m long. Snoopy, Jan's dog, runs with him, but quits after one and a half rounds. How many meters dit Snoopy ran ?" In a written report we read : "I thought that one has to compute 379.25 x 1.5; so Snoopy has run 5688.75 m."

We can conclude that the results of the ascertaining study confirm the facts and the theoretical considerations discussed in section 3. (1) We have indeed established that sixth-graders are not very successful in solving a series of more or less complex word problems. (2) Our findings support the hypothesis that this is mainly due to the fact that those pupils do not possess the attitude and/or the skills to analyze and represent the relations existing within the data of a new and unfamiliar task before proceeding to execute computations. The main objective of the teaching experiment lies in the further elaboration and verification of this hypothesis.

6. The teaching experiment

In view of the teaching experiment, the preceding hypothesis was transformed as follows : if we can foster pupils' thinking skills by equipping them with appropriate and useful methods for problem analysis, their ability to solve word problems will improve. Based on the psychological analysis of estimating the outcome of a task discussed in section 3, this hypothesis was specified as follows : if we can teach the learners a solution procedure for word problems in which estimating as a heuristic strategy is of prime importance, their ability to solve such tasks will increase.

As said before, the teaching experiment consists of implementing an experimental teaching program in the experimental class and, afterwards, administering a posttest in both the experimental and the control groups. We will successively discuss both points; this discussion will be followed by a review of the results obtained.

6.1. The experimental teaching program

The main objective of the experimental program was to teach children to estimate the solution to a word problem systematically before starting to perform computations. It was expected that this would (1) lead them to problem analysis and, in so doing, point them appropriately toward the solution process and (2), through the anticipation of the solution, provide them with a means to verify the result obtained. To guarantee that the heuristic estimation strategy will function as efficiently as possible in the pupils' problem-solving behavior, we thought that it would be useful to teach them the strategy as part of a more comprehensive systematic solution procedure for approaching an arithmetic word problem. By solution procedure, we mean here the whole sequence of successive actions that the learner should perform to reach the solution to a task.

In the present case we developed a five point strategy consisting of the following sequence of actions :

(1) Read the task;

(2) Estimate the solution and represent the result of the estimation grafically on the number scale;

(3) Solve the task;

(4) Verify the solution - that is, carry out the proof and compare the result obtained with the estimated solution;

(5) Note the solution.

It will be obvious that the estimation strategy can only perform its heuristic function if the pupils master the subject-matter knowledge and skills necessary to estimate the outcome of a task - for example, rounding off, mental arithmetic with round numbers, being able to apply the properties of numbers and of arithmetic operations, insight into the number system.

The experimental teaching program was implemented during a two-week period and proceeded as follows.

During two lesson periods, the subject-matter knowledge and skills related to estimating the outcome of a task was reviewed and practised. At the end, a specially designed test was administered, the results of which showing that the learners mastered the required knowledge and skill sufficiently.

Thereafter, the proper learning phase, in which the solution procedure described above was taught, began; it covered ten lesson periods and was implemented according to a completely preplanned schedule in which four substages can be distinguished. In the first stage, the pupils were given a series of word problems of the same type as the tasks in the pretest. The second substage consisted of a classroom discussion on the pupils' solutions, thereby paying special attention to the causes of the errors. The discussion led to the following conclusions : (1) we have to read the task more attentively; (2) we can avoid a lot of errors by estimating the outcome beforehand; (3) we should first look closely at the task before starting to perform computations; (4) we should ask ourselves if our final solution is plausible and really possible. In accordance with those conclusions, the five point procedure was systematically introduced in the next substage. Each step of the procedure was associated with difficulties and errors that were put forward during the classroom discussion. The five point strategy was also translated in terms of an action schedule which was presented to the learners in the

last substage as a resource, importing to them the attitude of estimating the outcome of a task (see Figure 3). The schedule was practised throughout three lesson periods. To be able to ascertain whether the pupils applied the procedure correctly, they were asked, for each problem, to note the code numbers of the different steps together with the corresponding action on their answer sheets.

Insert Figure 3 here

Originally we had planned one more substage in the learning phase, in which the pupils would be given a series of problems to solve without the support of the action schedule. During the implementation, it seemed necessary to spend more time on practising with the schedule. Because of the limited time available for the study, the phase of practising without the schedule was eliminated. As a consequence, we allowed the pupils of the experimental class to use the action schedule during the posttest.

While the experimental teaching program was implemented in the experimental group, the control class was taught according to the normally prescribed arithmetic program. To this end, the teacher presented tasks which were similar to those discussed in the experimental class though treated in the usual way - namely, without instructing the pupils systematically in the heuristic estimation strategy.

6.2. The posttest

When the experimental teaching program was terminated, the posttest was administered to the pupils of the experimental and control groups. The test consisted of two parts : the ten items of the pretest (part 1) and ten new items (part 2), similar in nature to the pretest tasks. The structure of each part of the posttest corresponded also to the pretest - namely, one numerical task, four simple word problems, and five more complex word problems.

The pupils of the experimental class were again asked to write a short description of the solution process employed in the same two items as in the pretest.

6.3. Results

Table 2 gives an overview of the average results of the two groups on the pretest and the posttest. In addition to the data in the table, we mention that, on the posttest, there are no significant differences between the experimental class and the control class. We reiterate here that the pretest scores were significantly better in the control group than in the experimental class.

Insert Table 2 here

Table 2 shows that the result of the experimental group on the posttest is significantly better than on the pretest. This is the case for the total test [t (2.878) = 6.3631, $p < .01$] as well as for part 1 [t (2.878) = 4.3557, $p < .01$] and part 2 [t (2.878) = 7.5678, $p < .01$]. The result of the control group on part 1 is on the same level as on the pretest [t (2.861) = .6454, $p > .05$], while there is a significant increase on part 2 [t (2.861) = 3.22, $p < .01$]. This last finding means that part 2 of the posttest was probably easier than part 1; the difference between parts 1 and 2 in the experimental class points in the same direction. The result of the control group on the total test does not differ significantly from the pretest score [t (2.861) = 1.3905, $p > .05$]. From all these data, we can conclude that the findings support the hypothesis which was the starting point of this teaching experiment : when we teach pupils a solution procedure for word problems in which estimating as a heuristic strategy is of prime importance, their ability to solve such problems will increase.

The direct comparison of the experimental and control classes is thwarted by the significant difference between the initial levels of both groups (see 5.2.). Nevertheless, the comparison leads to findings that are convergent with the preceding conclusion. As we mentioned above, the significant difference between the experimental and control groups established on the pretest in favor of the control class, no longer occurs on part 1 of the posttest; there is even an obvious tendency in the opposite direction. Indeed, the score of the experimental class is here 10 % higher than in the control class, and this difference is almost significant at the .05 level [t (2.03) = 1.9106, $p > .05$]. The score of the experimental group on part 2 of the posttest is also higher than the result of the control group; the same is, then, true for the total test. However, none

of these differences is significant [part 2 : t (2.03) = 1.0439, $p > .05$; total test : t (2.03) = 1.6755, $p > .05$].

It is also interesting to examine whether the established progress of the experimental group relates to the simple word problems as well as to the complex ones. The results are presented in Table 3.

Insert Table 3 here

Table 3 shows that the score of the experimental pupils on the simple word problems in the posttest (part 1) is 21 % higher than on the pretest; this difference is significant at the .01 level. For the complex word problems the difference is 15 %, which is just below the .05 level of significance. In the control class pupils performance on the posttest for the two types of problems lies at the same level as on the pretest for the simple word problems, there is even a slight decline which is, however, not significant. In other words, in contrast with the control group, the experimental class makes considerable progress on both types of word problems. This progress is most strongly marked and significant in the case of the simple word problems; for the complex word problems, there is an obvious tendency in the expected direction though it is not significant. The result would undoubtedly have been better if, during the experimental teaching program, we had been able to devote more time to the systematic instruction of the solution procedure, especially to the heuristic estimation strategy. Indeed, during the implementation of the program, we had the impression that, for a thorough mastery of this heuristic strategy, considerably more training would be necessary, and, as we mentioned before, there was insufficient time to implement the program entirely as it had been conceived (see 6.1).

A further analysis of the difference between the scores on the pretest and on part 1 of the posttest revealed also that the pupils of average ability profit most from the experimental program but that the children of low ability also make good progress. On the other hand, the learners of high ability show almost no improvement. It is obvious that one should not attach much importance to these differential results, because they are based on very small groups of pupils.

We attempted to collect qualitative data concerning the processes and actions underlying children's problem-solving performance on the posttest by means of an analysis of their answer protocols and of the written reports. However, we have not been very successful in this regard. The

written reports concerning the two posttest items yielded particularly few data in contrast to the pretest. This is most likely the case because the pupils worked very intensively to solve the posttest problems using the learned solution procedure; in these circumstances, the question asking for a descriptive report afterwards may have been an overburdening of those sixth-graders. A consequence is, however, that we have much less data available on the qualitative aspects of the problem-solving processes than was the case with the pretest. For this reason, the interpretation of the quantitave results in process terms becomes rather difficult. Taking this into account, we can, nevertheless, provide the following information.

The results discussed show that the children of the experimental group, who have acquired a solution strategy during the experimental teaching program, achieve better results in solving arithmetic word problems. Undoubtedly this has to do with the fact that, by applying and following the solution procedure, they are more appropriately oriented toward solving the tasks. The data collected do, however, not allow us to decide whether or not the quantitative improvement in achievement is due to the acquisition of the heuristic estimation strategy as such. It is true that this heuristic strategy was central to the experimental program; however, further research is needed to examine if this is the determining factor in the solution procedure. Meanwhile, in view of such research, we wish to state the hypothesis that the acquisition by the learners of the heuristic estimation strategy leads to a qualitative improvement in their problem-solving activity with respect to word problems, through which a quantitative increase in achievement occurs. This hypothesis is not only based on the central position of the estimation-strategy in the solution procedure taught to the children but also on certain data that emerge from an analysis of the scores and the answer protocols from the posttest. Indeed, a number of observations indicate that, through the experimental program, the pupils had learned consider the problems more carefully. As an illustration we give the following example. On the pretest, the problem concerning the wages of the workers was solved correctly by 59 % of the experimental group; as mentioned before, we observed a lot of thinking errors due to the fact that the pupils only took into account either the number of workers or the number of days (see 5.2). On the posttest, however, the percentage of correct answers increased to 90 %, and such thinking errors no longer occured. This shows that, in the posttest, the pupils perform some degree problem analysis, and it is our view that this is induced by the application of the estimation strategy.

7. Summary and discussion

The results of the ascertaining study have confirmed a frequently heard complaint in educational practice : that sixth-graders are not very successful in solving arithmetic word problems. A further analysis showed that the bulk of the incorrect solutions is due to thinking errors. It appeared from the qualitative data concerning pupils' problem-solving processes that they very often start performing computations almost immediately after they have read a task, instead of first trying to analyze and understand the problem, and also that they almost never verify their results.

The teaching experiment was an attempt to contribute to the verification of the following hypothesis : fostering pupils' thinking skills by equipping them with appropriate methods for problem analysis will lead to an improvement in their ability to solve arithmetic word problems. To stimulate skills in problem analysis among the experimental learners, they were taught a solution procedure, the core of which consisted of the use of estimating as a heuristic strategy. The quantitative results of the posttest support the hypothesis; indeed, a significant increase in achievement was observed in the experimental class. Because the sample involved in the present study was small, we are very well aware of the limited scope of our findings, and, therefore, we guard against the hasty generalization of the conclusions.

During the teaching experiment we were not successful in collecting the appropriate data on the children's cognitive processes and actions required to be able to give a more precise qualitative interpretation of the observed increase in achievement. Yet, we seem to have sufficient indications to retain the following hypothesis for further study : it is especially the central aspect of the proposed solution strategy for word problems - namely, the heuristic estimation strategy - which induces a qualitative improvement in pupils' problem-solving activity and which, therefore, is the determining factor of the increase in performance. The qualitative improvement has to do with the fact that estimating the solution of a task beforehand leads the learners to analyze the problem and provides them with a means for verifying their solutions.

In further research with respect to this hypothesis, special attention should be paid to the collection of qualitative data on pupils' problem-solving processes before, during, and after the experimental teaching program. It will certainly be necessary to collect better process information during the posttest than was possible in the present study. It

is also our opinion, however, that, in view of theory-construction about learning to solve problems in elementary school children, it is extremely desirable as well to conduct, in addition to classroom teaching experiments, clinical teaching experiments in which the experimental teaching program is implemented with an individual learner or with small groups of children. Such small-scale teaching experiments in which learning is guided and stimulated almost individually are essential for theory-building because they make it possible to observe and record the effects of all sorts of interventions on the course of the learning process with a high degree of precision. This methodological proposal meets Resnick's (Note 3) comments during the 1980 AERA-meeting in Boston on a previous classroom teaching experiment (De Corte & Verschaffel, Note 2).

Otherwise, the rather insufficient data collected from the posttest is not the only weak aspect of the present study. We have already pointed out that the experimental teaching program could not be implemented as it was planned, because there was not enough time available to train the pupils thoroughly in the application of the solution procedure, especially the heuristic estimation strategy. This should also be taken into account in future research. On the other hand, in spite of the shortcomings of the teaching program, there was still an increase in achievement within the experimental group - a hopeful result with respect to the possibilities of improving children's problem-solving abilities. By providing them with procedures for analyzing problem situations, we can probably equip them with an appropriate orientation basis to approach problems. The present investigation points toward the possible usefulness of the estimating procedure in this regard.

The latter considerations sound optimistic in view of the potential optimization of instructional design and practice. However, such optimism is only justified when research results like those reported above actually give rise to thorough reflection on current teaching practice with regard to arithmetic problem solving. In any case, our findings seem to support those who are convinced that the causes of the many complaints about the poor results of instruction on word problems should, for the most part, be sought within the teaching practices now in use.

Reference Notes

1. De Corte, E., Carpay, J.A.M. & Span, P., Between East and West : instructional psychology in Western Europe as a possible integrating force. Introduction. Paper presented at the Annual Meeting of the American Educational Research Association, Boston, April 1980.

2. De Corte, E. & Verschaffel, L., Children's solution processes in elementary arithmetic problems : analysis and improvement. Paper presented at the Annual Meeting of the American Educational Research Association, Boston, April 1980. (ERIC Document Reproduction Service ED 187552.)

3. Resnick, L.B., Discussion of the papers presented at a symposium entitled "Between East and West : instructional psychology in Western Europe as a possible integrating force". Annual Meeting of the American Educational Research Association, Boston, April 1980. (Cassette Program from the AERA 1980 Annual Meeting : R - 23.02. Instructional psychology in Western Europe.)

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Table 1. Average results (in %) on the pretest in the experimental and the control groups.

	Experimental class (N=20)	Control class (N=21)
Simple word problems (including the numerical task)	60 %	74 %
Complex word problems	34 %	41 %
Total pretest	47 %	58 %

Table 2. Average results (in %) on the pretest and the posttest in the experimental and the control groups (1)

	Experimental class (N=20)	Control class (N=21)
Pretest	47 %	58 %
Posttest,part 1	65 % **	55 % n.s.
Posttest,part 2	76 % **	70 % **
Posttest,total	71 % **	62 % n.s.

(1) For each of the posttest results, an indication is given whether there is a significant difference with the pretest score :

** : significant at the .01 level (t-test)

n.s. : not significant.

Table 3. Average results (in %) on the pretest and the posttest, part 1 for the simple and the complex problems in the experimental and the control groups.

		Pretest	Posttest, part 1
Experim. class	Simple problems	60 %	81 %
	Complex problems	34 %	49 %
Control class	Simple problems	74 %	68 %
	Complex problems	41 %	42 %

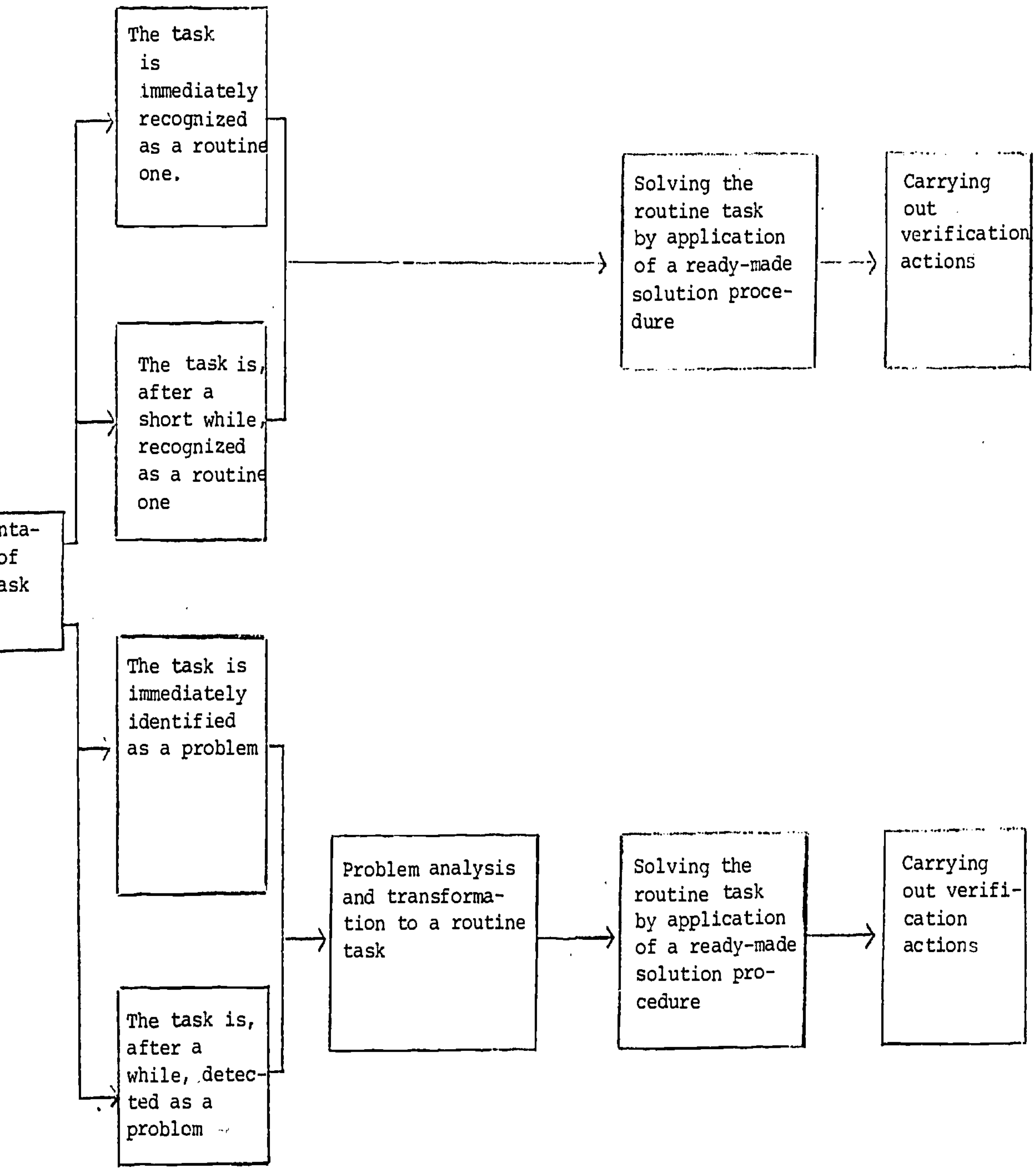

Figure 1. A model of the solution process with respect to arithmetic tasks

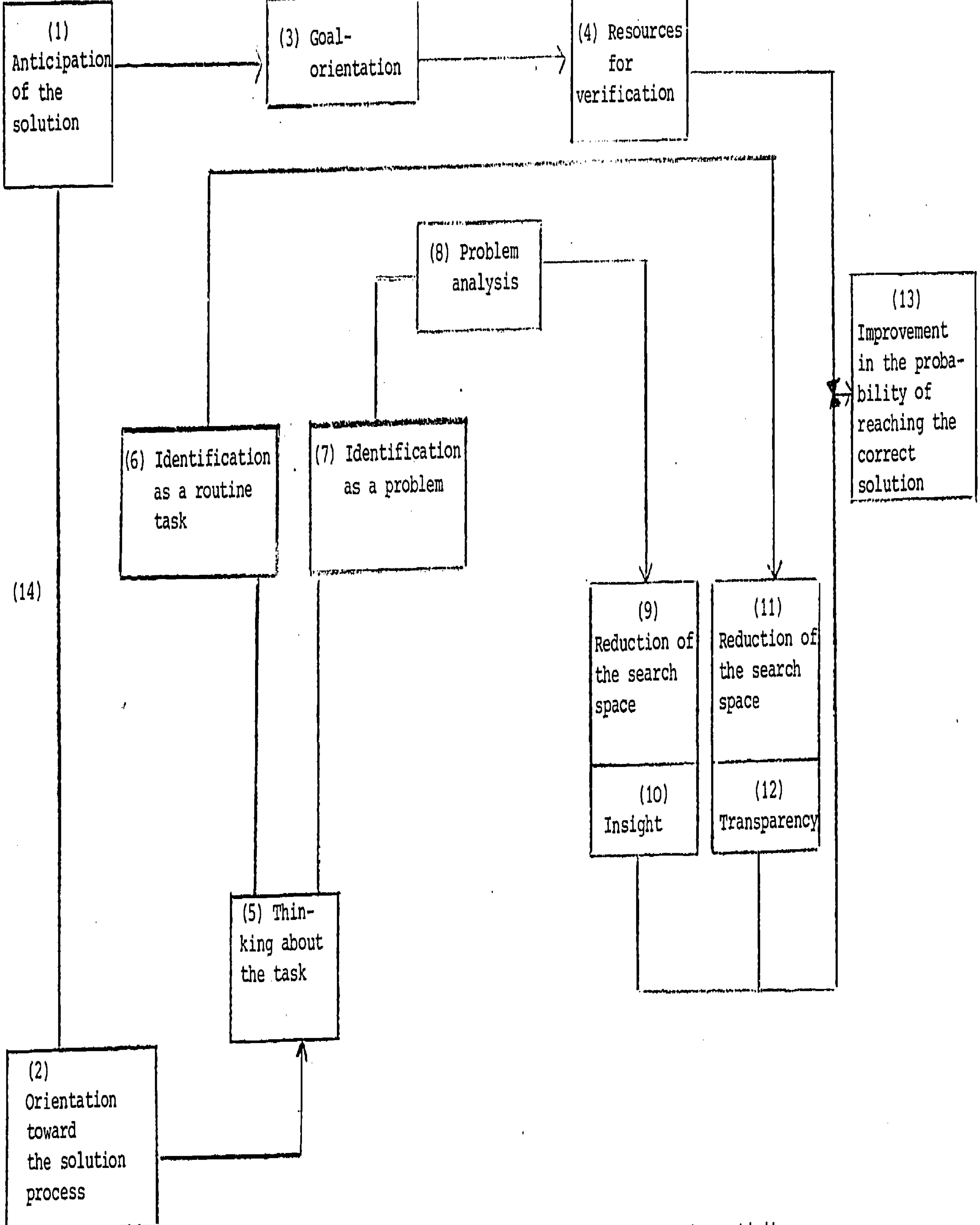

Figure 2. Psychological analysis of the estimating activity

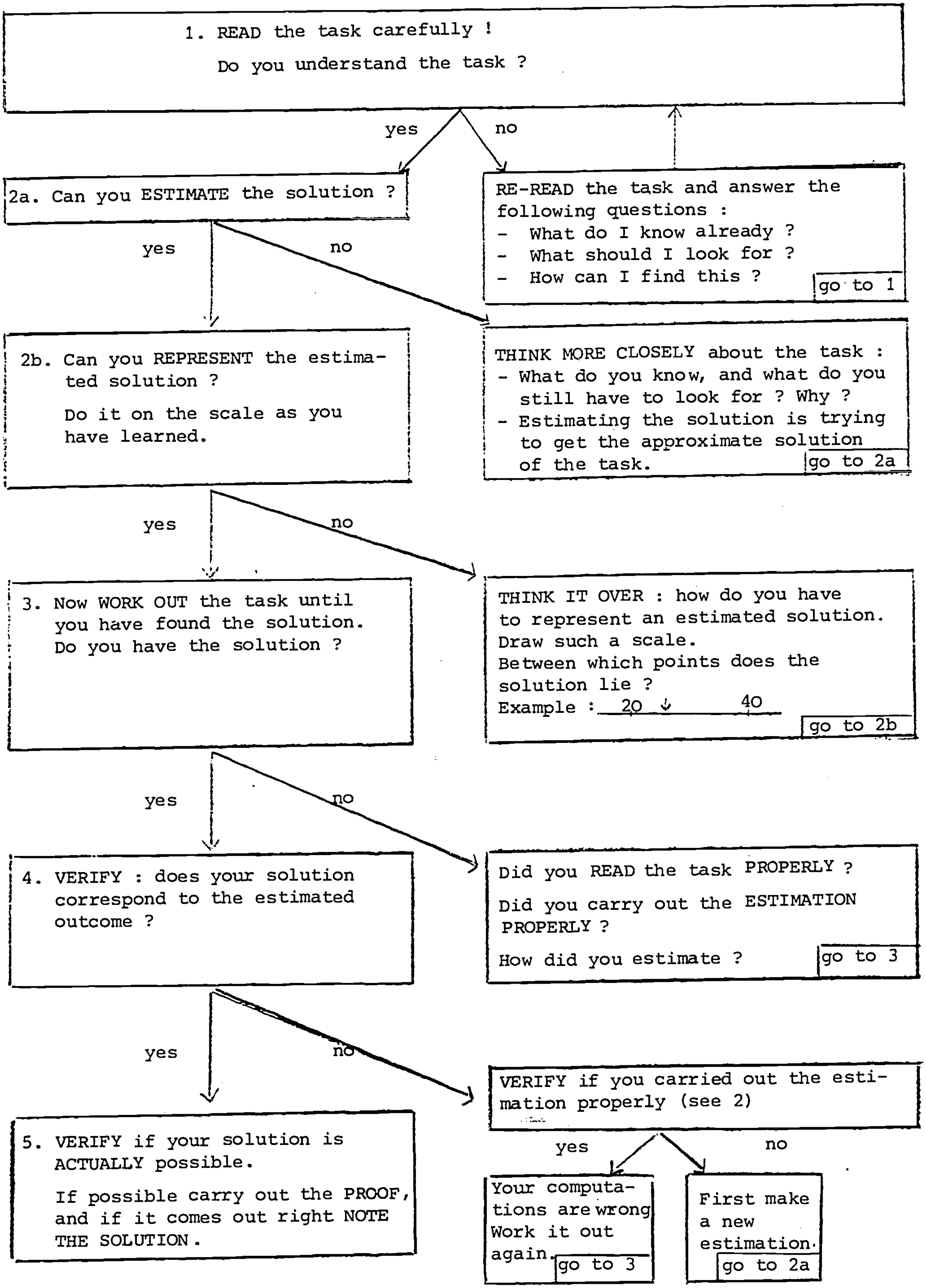

Figure 3. Action schedule for applying the solution procedure during the learning phase