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ABSTRACT

An information-processing task analysis of what kindergarten children might do if they are successful in representing and solving certain types of simple arithmetic story problems served to identify five key components. Data obtained from the interview testing of 66 kindergarten children served to suggest a major difference between successful and partially successful students in the knowledge or understanding that they employ. Successful students understand a set-joining or subset-removal operation both: (1) as a procedure to be carried out; and (2) as establishing specific relationships among the quantities involved while partially successful students have only the first type of understanding. (Author)

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INFORMATION PROCESSING CAPABILITIES USED BY KINDERGARTEN CHILDREN WHEN SOLVING SIMPLE ARITHMETIC STORY PROBLEMS

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There is considerable support for the assumption that persons who are successful in solving problems that ultimately require a mathematical solution generate some type of qualitative representation or model of the problem as an intermediate step between initial comprehension of the problem and the carrying out of the mathematical operations needed to arrive at the answer (Heller & Greeno, 1979; Larkin, 1977; Newell & Simon, 1972; Simon & Simon, 1978). It has also been shown (Lindvall & Ibarra, 1979) that when primary grade children are asked to solve simple arithmetic story problems, the successful problem solvers are capable of developing a physical model as an aid to solution. This would seem to suggest that acquiring this ability to develop appropriate models for story problems may be an important step in children's acquisition of mathematical competency. Of course, many kindergarten children display this desired modelling ability when they are faced with simple quantitative problems. They use their fingers, or other countable elements, to represent the sets of objects involved and carry out simple operations, such as joining sets or removing a subset, to derive answers. They display a type of understanding which can provide an essential basis for the continuing study of arithmetic and which should be an important goal of early instruction in this subject. The focus of the study reported in this paper is on the components of this type of understanding. Specifically, it is concerned with the types of knowledge that

students must have and must apply if they are to be able to solve various forms of story problems through a procedure of building a physical model of the story and using this model to arrive at the solution.

Rationale

A number of persons have analyzed the process that successful problem solvers employ in solving arithmetic word problems by outlining information processing models (Heller & Greeno, 1979; Riley, 1981). Heller (1980) has shown that the procedures used by primary grade children in solving simple "change" and "combine" type stories might be modelled by a computer program and has used this analysis to explain why certain types of stories are more difficult. Reasons for differences in problem difficulty have also been investigated by other researchers (Carpenter & Moser, 1979; Nesher, 1979).

In an investigation having a focus somewhat similar to the present study, Riley (1981) (See also Riley, Greeno, & Heller, in preparation) presented empirical data to support the hypothesis that primary grade children function at one of three levels with respect to their capability for processing the information presented in simple one-step story problems. The first level includes those children who can build sets of blocks to provide a representation of the sets described in a story and can operate on these externally available sets. However, children at this level cannot solve stories that require that they hold and manipulate information internally. They

appear to lack certain essential knowledge concerning the relationships that must exist among the sets involved in a set union or a subset removal operation. Under the Riley (1981) hypothesis, children at the second level have a limited capacity for internal representation and processing and, hence, can solve somewhat more complex stories than level one children. The third level includes those children who have relatively complete knowledge concerning essential relationships and can carry out internal processing that enables them to solve one-step story problems no matter at which point in the story sequence the unknown term is presented. Riley (1981) has developed a "model" which, for each of these levels, explains how attempted problem solution is carried out. Each model, in turn, is associated with the types of problems that it can solve successfully.

An earlier study by one of the writers (Lindvall & Ibarra, 1980) used a clinical interview procedure to describe what kindergarten children did when they processed the information provided in simple story problems and used counting cubes to develop physical representations of the stories and to arrive at solutions. That study represented an initial step in identifying the types of essential information that the successful problem solver had to abstract from the story in order to model it and solve it. The present study represents an effort to conduct a more formal and complete analysis of what children do in comprehending a story and in using a physical model to arrive at a solution. It also offers an explanation of why children have difficulty with certain types of stories. The paper first presents a flow chart that represents a type of information

processing task analysis (Resnick & Ford, 1976) of how children may process the information presented in the story in such a way as to enable them to develop a model of the story and to manipulate this model to obtain a solution. This flow chart serves as an aid in the identification of the specific decisions and actions that are involved in this problem solving process. The chart also helps identify the types of knowledge that the student must have in order to solve different types of stories.

Method

Subjects

To analyze the performance of pupils on the types of tasks that were the focus of this study a total of 66 kindergarten students were given individual interview tests. This sample represented all kindergarten pupils in three classes in two different schools, a university-affiliated private school in a large urban setting and a suburban school serving a middle-class neighborhood.

Types of Story Problems Used

Eight forms of one-step addition and subtraction story problems were the focus of this study. Using the categorization scheme outlined by Heller (1980), these can be described as two forms of the "combine" type of story and six forms of the "change" type. The stories used are shown in Table 1, where an open sentence is associated with each type in order to show the point in the story sequence where the unknown value is introduced.

Insert Table 1 about here

In testing students on these problems, the tester, using an individual interview testing procedure, read the story line-by-line, pausing after each line to permit the student to respond. The student was told to "Use these blocks to show what this story says as I read each line." The student was expected to respond by doing such things as building sets of the appropriate size, joining sets, removing subsets, or taking other actions appropriate for representing what the story described and for obtaining an answer to the problem.

<u>The Information Processing Analysis</u>

As noted above, a first stage in the present study involved an effort to develop a detailed analysis of what kindergarten children do when they are successful in carrying out problem solving tasks of the specific type used in this study. This analysis involved an information processing task analysis of the type described by Resnick and Ford (1976) and was based on the writers' earlier experiences in observing kindergarten children carrying out the type of problem solving activities that were of concern (Lindvall & Ibarra, 1980). The information processing analysis took the form of a flow chart which shows the questions a child might raise in processing the information given in each line of the story and the action that would need to be taken on the basis of how each question is answered. The flow chart, shown in Figure 1, represents our hypotheses (derived from

Table 1

Story Problems Used in the Study, Categorized by Story Type

1. Combine: $a + b = \square$

Joe has 3 marbles.
Tom has 5 marbles.
How many marbles do they have altogether?

Mark has 6 apples.
Bob has 3 apples.
How many apples do they have altogether?

2. Combine: $a + \square = c$

Carol has 4 flowers.
Sue has some flowers.
Together Carol and Sue have 9 flowers.
How many flowers does Sue have?

Jenny has 3 books.
Amy has some books.
Together Jenny and Amy have 7 books.
How many books does Amy have?

3. Change: $a + b = \square$

John had 3 pencils.
He found 4 more pencils.
How many pencils did John have then?

Rich had 2 cookies.
He found 3 more cookies.
How many cookies did Rich have then?

4. Change: $a + \square = c$

Jill had 6 pennies.
She found some more pennies.
Then Jill had 8 pennies.
How many pennies did Jill find?

Elaine had 4 pieces of candy.
She found some more pieces of candy.
Then Elaine had 7 pieces of candy.
How many pieces of candy did Elaine find?

5. Change: $\square + b = c$

Jack had some baseballs.
He found 2 more baseballs.
Then he had 5 baseballs.
How many baseballs did Jack have to begin with?

Jim had some marbles.
He found 4 more marbles.
Then he had 6 marbles.
How many marbles did Jim have to begin with?

6. Change: $c - b = \square$

Mary had 9 dolls.
She lost 3 dolls.
How many dolls did Mary have then?

Linda had 8 flowers.
She lost 5 flowers.
How many flowers did Linda have then?

7. Change: $c - \square = a$

Bill had 6 toys.
He lost some toys.
Then he had 2 toys.
How many toys did Bill lose?

Gary had 5 baseball cards.
He lost some baseball cards.
Then he had 3 baseball cards.
How many baseball cards did Gary lose?

8. Change: $\square - b = a$

Jane had some buttons.
She lost 2 buttons.
Then she had 7 buttons.
How many buttons did Jane have to begin with?

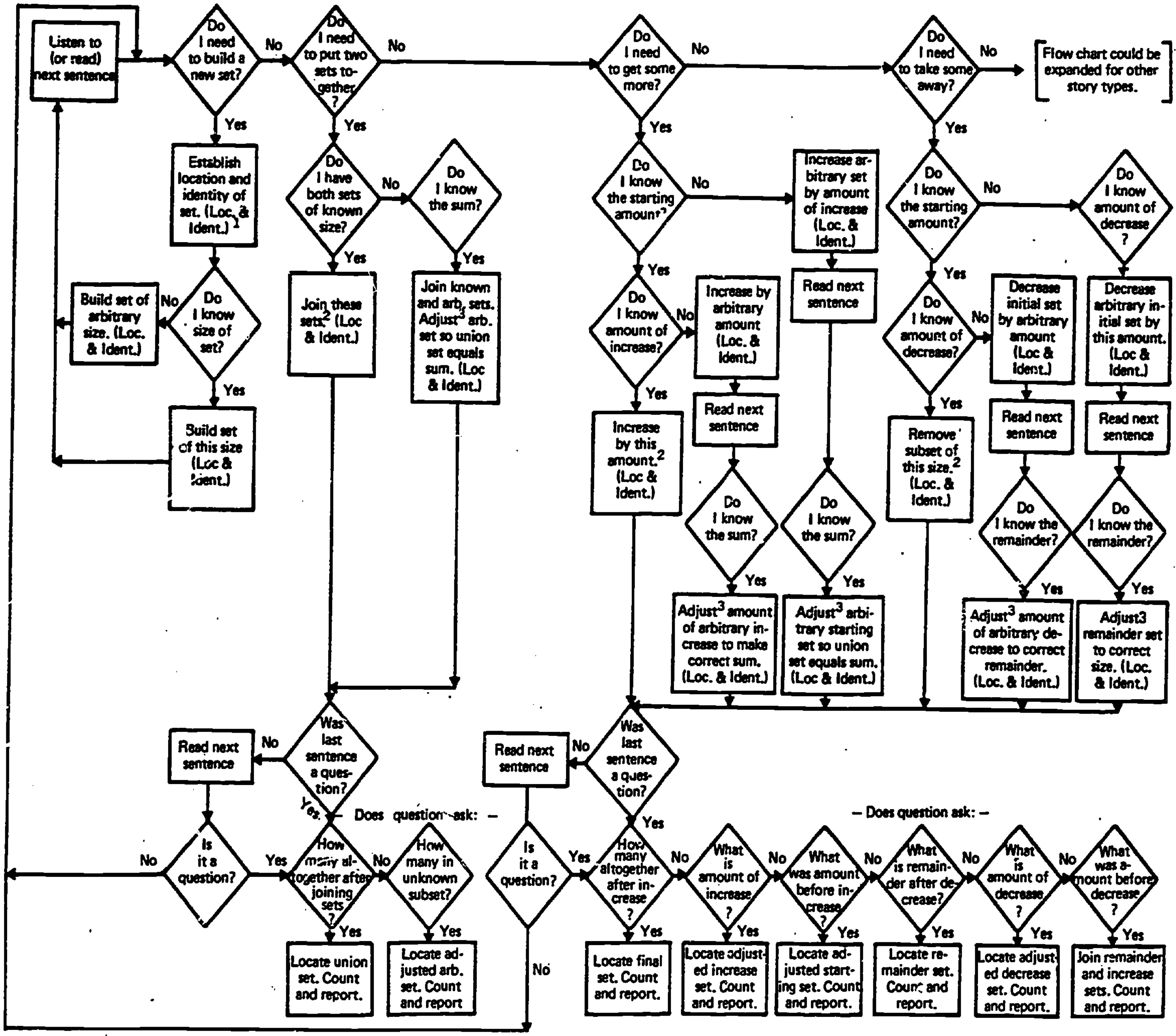
Maria had some pencils.
She lost 5 pencils.
Then she had 3 pencils.
How many pencils did Maria have to begin with?

rather extensive observations of a large sample of children) as to how successful problem solvers process the information, how they decide what to do, how they distinguish among different types of stories, and how they manipulate the counting cubes to arrive at the correct answer. For purposes of our research, this analysis served to: (1) clarify the specific actions that a pupil must take to represent and solve each story; (2) suggest the type of knowledge that the pupil must have in order to solve each problem type; and (3) indicate why certain types of problems are more difficult than others.

Insert Figure 1 about here

As the flow chart shows, when the child hears a story sentence, he or she must first decide whether this means that a new set must be built or whether some operation must be carried out with sets previously built. When a set is built, it must be given its own location (to keep it separate from other sets or from the common store) and its own identity (e.g., "Tom's pieces of candy"). Of course, the set must also be of the correct size. It will be noted that our analysis assumes that whenever a set of unknown size is described (e.g., "John had some apples."), the child builds a set of some arbitrary size. This is done because in our experience, using the instructions that we employed (i.e., "Use these blocks to show what each line means.") and reading the story on a line-by-line basis, essentially all our students did build this type of arbitrary set.

When a story line does not indicate that a new set should be



1 Each set generation or modification step should include attention to the location and identity (Loc. & Ident.) of the set.
2 In this type of story the problem operation produces the correct answer set number.
3 In this type of story an adjustment, based on relationships associated with this operation, must be made to produce correct answer set number.

Figure 1. Information Processing Task Analysis for Children's Modeling of Combine and Change Stories.

built, the pupil must decide if some specific action should be carried out. With the eight story forms used in this study, it was decided that one of three major operations might be described in a story: (1) putting two sets together (combine); (2) increasing the size of an existing set (change increase); or (3) decreasing the size of an existing set (change decrease). When the described operation for the given story is identified, the student will proceed to carry it out. If the sets involved in the described operation are of known size (i.e., if the story gives the size of the two sets in a "combine" problem or if it gives the size of the initial set and either the amount of increase or decrease in a "change" problem), carrying out the operation described in the story will produce the answer to the problem. With such stories the student only needs to know how to carry out the operation in order to obtain the correct answer. However, if one of the sets involved in the operation is the unknown set (a set of "arbitrary size" in the student's model of the story), carrying out the operation will not automatically produce the answer. Here an adjustment must be made in the size of the arbitrary set in order to maintain the relationships that are a part of the given set operation (e.g., In a story giving the size of one subset and the union set, the size of the unknown, or arbitrary-size, set must be adjusted so that the sum of the two subsets equals the number in the given union set.) To solve such stories, a student must not only know how to carry out a described operation but must also know what relationships must be maintained among the sets involved in the operation. Only if the student knows and applies such knowledge will

he or she arrive at the correct number in the answer set.

Of course, the final bit of information processing that the successful problem solver must carry out is to respond to the story question by identifying the specific set that represents the answer. (It will be noted that at certain decision points the flow chart makes provision for only a "yes" response. A "no" response at such points would mean that the story had two unknown terms and, hence, would have no unique solution. Such "no" responses were deleted from the chart merely to reduce overcrowding in the chart format.)

Procedure

On the basis of the foregoing analysis, five basic components of the types of stories used here were identified as being those aspects of the story and its solution that the student must identify and represent correctly if the problem is to be solved. Each of these aspects was then used as a criterion task which a student was judged as passing or failing as he or she built a representation of the story and carried out the steps necessary to solve it. These five aspects of story solution are the following.

1. The initial identity of each given set. The pupil must establish the identity of sets such as "Tom's marbles," "the apples that Sue and Joe have together," "the pieces of candy that Mary had in the beginning," etc. Such sets will be represented by a set of blocks, or fingers, or other countable elements, but this representation must be identified with the specific set described in the story.

2. The initial number in the set. The number must be represented correctly.

3. The problem operation. The action or relationship described in the story must be translated into an operation on the sets built to model the story.

4. The answer set identity. The problem question must be translated so that it refers to a specific set involved in the "problem operation."

5. The answer set number. Obviously, if the problem is to be solved correctly, the number in the answer set must be correct. This means that operations and any adjustments necessary to meet problem conditions must be carried out correctly. Specifically, in those stories where the child has initially represented a set of unknown size by building a set of arbitrary size the child must adjust the size of that set so that it is compatible with the relationships associated with the given set operation.

The individual interview testing procedure used in this study permitted the performance of each child to be judged as correct or incorrect with respect to representing each of these five components as the child used sets of counting cubes to model each of the different types of story problems.

Hypotheses and Methods of Analysis

Since an initial analysis of the data for this study supported the findings of the earlier research (Lindvall & Ibarra, 1980) that kindergarten children have minimum difficulty in the physical

representation of the initial identity and correct number in the sets described in a story, the present study focused on what students did in: (1) carrying out the operation; (2) identifying the answer set; and (3) making any necessary adjustments to obtain the correct answer set number.

Basically, this study involved an investigation of the hypothesis that children need two types of knowledge concerning the set operations if they are to be able to solve various types of simple one-step addition and subtraction story problems. These two types of knowledge are:

1. Knowing the operation as a procedure to be carried out in order to produce an answer.

2. Knowing the various specific relationships that must exist among the quantities involved when an operation is carried out.

The extent to which children differ in their possession of one or both of these types of knowledge was investigated in this study by noting differences in patterns of pupil performance (1) on different types of problems, and (2) on different components of the same type of problem.

As discussed previously, our information processing task analysis indicated that certain types of stories can be solved solely by applying knowledge of an operation as a procedure to be carried out. On the assumption that this type of knowledge is simpler and acquired more readily than knowledge of relationships, this study hypothesized that problems requiring only this simpler knowledge will be solved by a greater proportion of students than will stories requiring a

knowledge of relationships. This hypothesis can be stated more specifically as follows:

Hypothesis 1. With stories in which the correct answer set is produced by applying the procedure described in the story to sets of known size (e.g., joining two known sets or removing a known subset from a known superset) the proportion of students getting the "answer set number" correct will be greater than it will be with stories where the story procedure must be applied to a set of unknown size and then adjustments must be made to get the correct answer set number.

As indicated previously, our method of data collection, among other things, provided information on whether or not the student: (1) carried out the story operation correctly (i.e., joining sets, removing a subset, increasing the size of a given set, decreasing the size of a given set); and (2) identified the correct set as representing the desired answer. With these additional types of information it was possible to shed some light on the basic hypothesis of this study by investigating the following four additional hypotheses:

Hypothesis 2. With stories in which the correct answer set is produced by applying the procedure described in the story to sets of known size, the proportion correct in carrying out the "problem operation" will be approximately equal to the proportion getting the "answer set number" correct.

Hypothesis 3. With stories in which the correct answer set is not directly produced by applying the procedure described in the story to sets of known size, the proportion correct in carrying out the

"problem operation" will be much larger than the proportion getting the "answer set number" correct.

Hypothesis 4. With stories in which the correct answer set is produced by applying the procedure described in the story to sets of known size, the proportion correct in establishing "answer set identity" will be approximately equal to the proportion getting the "answer set number" correct.

Hypothesis 5. With stories in which the correct answer set is not directly produced by applying the procedure described in the story to sets of known size, the proportion correct in establishing "answer set identity" will be much larger than the proportion getting the "answer set number" correct.

To investigate the hypotheses of this study the performance data of children were summarized in terms of proportion of children passing each of the five component steps in story representation and solution for each of eight different types of addition and subtraction stories. A comparison of the appropriate proportions was then made to investigate each of the five hypotheses.

Results

The major results from the study are summarized in Table 2 where the proportion of students performing correctly on each of the five components involved in abstracting the meaning of the story and solving it for each of the eight problem types is shown. It can be seen that these students had little difficulty in establishing the initial identity of the sets in the story and in representing set size

correctly. Other results can be examined in terms of the five hypotheses posed for the study.

Insert Table 2 about here

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1. Differences associated with two categories of problems, categorized on basis of type of knowledge needed to solve story. The data in Table 2 show that the proportions of students (.83, .89, .91) arriving at the correct "answer set number" for problems 1, 3, and 6 were definitely higher than the proportions for the other stories (.18, .08, .32, .64, and .12). These data support the hypothesis that more children can solve stories where a direct application of the described set operation produces the answer than can solve stories where this is not the case. That is, more students have a knowledge of these set operations as procedures to be carried out than have a knowledge of them in terms of the relationships that must exist among the quantities involved.

2. Relationship between correct performance in carrying out the operation and arriving at the correct answer for problems where carrying out the operation produces the answer. The data in Table 2 show that with stories 1, 3, and 6 (those in which carrying out the described operation produces the answer), the proportions of students correct on "problem operation" (.83, .91, .94) are essentially equal to the paired proportions correct on "answer set number" (.83, .89, 91, respectively). These results merely say that, with this type of problem, if you can carry out the operation described in the story,

Table 2

Proportion of Pupils Displaying Correct Performance on Each of Five Major Components in Story Representation and Solution for the Eight Story Problems

Problem Type	Essential Components of Problem to be Abstracted from Story: Initial Identity of Set	Initial Number in Set	Problem Operation	Components of Answer to be Gained from Model: Answer Set Identity	Answer Set Number
1. Combine (a + b = $\square$)	.85	.83	.83	.82	.83
2. Combine (a + $\square$ = c)	1.00	1.00	.52	.44	.18
3. Change (a + b = $\square$)	.95	.95	.91	.91	.89
4. Change (a + $\square$ = c)	1.00	1.00	.41	.12	.08
5. Change ($\square$ + b = c)	1.00	1.00	.62	.52	.32
6. Change (c - a = $\square$)	1.00	1.00	.94	.92	.91
7. Change (c - $\square$ = b)	1.00	1.00	.76	.89	.64
8. Change ($\square$ - a = b)	.98	.98	.50	.50	.12

you can arrive at the correct answer.

3. Relationship between correct performance in carrying out the operation and arriving at the correct answer for problems where carrying out the operation will not produce the answer directly. In examining the same type of data as in 2, above, but looking at stories 2, 4, 5, 7, and 8 (those in which carrying out the described operation will not produce the answer directly) it can be seen that the proportions of students correct on "problem operation" (.52, .41, .62, .76, and .50, respectively) are uniformly much larger than the paired proportions correct on "answer set number" (.18, .08, .32, .64, and .12, respectively). That is, with these types of problems, being able to carry out the operation described does not necessarily mean that you will be able to solve the problem. It may be noted, here, that problem type 7 produced proportions (.76 and .64) that are somewhat out of line with those produced by the other problem types. The explanation for this would appear to be that this type of story can be solved by a minor adjustment in carrying out the procedure. It does not require as full an understanding of relationships as do Problems 1, 4, 5, and 8. With stories of the type represented by Problem 7, the child builds the given initial set and removes an arbitrary size set in response to the phrase "...lost some." This step produces two subsets, one representing the number lost, the other representing the number left. When the child next hears that the person in the story had "2 left" (for example), all that is necessary is that enough blocks be moved from one set to the other so that there are exactly 2 in the set representing the number left. Although this adjustment can

be considered as being made on the basis of an understanding of essential relationships established in the subset removal operation, a student might also think of it as merely correcting a temporary mistake in carrying out the procedure.

4. Relationship between establishing the correct answer set identity and getting the answer set number correct for problems where carrying out the operation produces the answer. The data in Table 1 show that with Stories 1, 3, and 6 the proportions establishing the correct answer set identity (.82, .91, .92, respectively) are essentially equal to the proportion correct on the answer set number (.83, 89, 91, respectively) This, again, indicates that wit these types of stories, since the story operation produces the correct answer, if the student is able to identify the correct set as representing the answer, the number in that set will be the answer.

5. Relationship beteen establishing the correct answer set identity and getting the answer set number correct for problems where carrying out the operation will not produce the answer directly. The data for Stories 2, 4, 5, 7, and 8 in Table 1 show that the proportion of students establishing the answer set identity correctly is much larger than the proportion getting the answer set number correct. This suggests that identifying the correct answer set is not sufficient for answering these stories correctly. If adjustments are not made to establish the correct quantitative relationships among the sets, the answer set number will be incorrect.

Discussion

The results of this study provide additional support for the finding of previous studies (Buckingham & MacLatchy, 1930; Ibarra & Lindvall, 1979) that kindergarten children can solve certain types of addition and subtraction story problems before they have any formal introduction to arithmetic and can give evidence of a real understanding of the solution process. That is, these children are able to develop physical models of stories that are of a type which should provide a meaningful basis for the later application of simple mathematical models to the solution of such stories. If, as has been indicated by the work of several researchers (Larkin, 1977; Lindvall & Ibarra, 1979; Simon & Simon, 1978), the development of such models is a procedure typically employed by effective problem solvers at all age levels, then this ability demonstrated by kindergarten pupils should be further developed, on a continuing basis, as children move up through the elementary grade levels. This, in turn, suggests the need for research on how children can best be taught to develop effective qualitative models for the many types of problems they will encounter as they progress in their study of mathematics.

Of course, the present study also indicates the need for being aware of the limits to what children can fully comprehend at any given point in their careers and of the exact nature of such limits. The majority of the students displayed a definite limitation in their knowledge of the relevant set operations. That is, their lack of understanding of the necessary relationships associated with a set operation prevented them from achieving a successful modelling and

solving of certain types of stories. The data obtained in this study may be interpreted as indicating that the relative difficulty of one-step addition and substraction story problems can be explained largely on the basis of their falling into one of two categories: (1) stories in which the solution is produced merely by carrying out the operation described in the story, or (2) stories in which obtaining the correct solution requires that certain adjustments must be made after the operation described in the story has been carried out. This categorization differs slightly from that proposed by Riley (1981) who presents results supporting the need for three categories, or "models," related to problem difficulty and to level of ability at which a child is capable of functioning. A careful examination of the procedures used in the two studies may help to explain the differences in the results observed and in consequent difference in number of explanatory categories needed. As stated earlier, in our story when students heard a line in a story referring to a set with "some," they proceeded to build a set of arbitrary size whereas in the Riley study the students did nothing. This means that the children in our study had something in "external" representation that provided the identity of the unknown set. This meant that whether the "some" set was mentioned first or second in the story, the student was faced with the same task. At that point, he or she had to make an adjustment in the set of arbitrary size in order to satisfy the necessary relationships between it and the other two known sets. One way of interpreting this is to suggest that the procedure used by the students in the present study had the effect of consolidating the second and third levels

described by Riley. With the exception of this slight qualification, it can be said that the results of our study provide substantial support for the findings and conclusions of the Riley study.

It should be added here that we do not interpret our results as implying that if children are to be taught to use blocks or other manipulatives to model a story that they should be encouraged to use a set of arbitrary size to represent a set that has "some." Our observations of children indicate that it is very easy for children to forget that a set they have built is of arbitrary size and proceed to use it as being a set of that particular size. As a teaching strategy it would appear to be more effective to teach pupils to use something like a blank sheet of paper (or an empty loop of string, etc.) as a "place-holder" for an unknown set. That is, an arbitrary-size set (or any other place-holder) has value only in reminding the student that "here is a set that is an essential element in analyzing and solving the story, and its size has yet to be determined."

Some Implications for Instruction

One way of describing the performance of the kindergarten children observed in this study is to suggest that they have command of a rather specific strategy for solving story problems of the type used here. This strategy can be summarized in terms of three steps: (1) represent the sets described in the story by using available countable elements (e.g., blocks, fingers), (2) carry out the actions described in the story, and (3) obtain the answer by counting the number in the answer set. This is the only capability available for

the average student. However, the able student also has available a knowledge of the relationships associated with a given set operation (e.g., knowing that the size of an unknown subset can be found by removing a set of size equal to the known subset from the known union set) and can apply this knowledge to solve some of the more complex stories.

This view of the results of the present study emphasizes the need for problem solvers to possess specific types of knowledge if they are to be able to solve specific story forms. One type of knowledge is knowledge of procedures or strategies, the other is conceptual knowledge or knowledge of relationships. An essential part of the teaching task then is the identification of the knowledge necessary for modelling and solving each given story type and then the actual teaching of this knowledge and of its application.

To the extent that the results obtained in the present study can be considered as somewhat descriptive of the types of knowledge and understanding that children possess at the time that they are being formally introduced to the operations of addition and subtraction and to the writing of number sentences, they have certain implications concerning readiness for this instruction. Specifically, they suggest that essentially all students are ready to comprehend addition and subtraction sentences as models of <u>operations</u>. However, most children are probably not ready to comprehend "open sentences," that is, to interpret a number sentence as an <u>equation</u>. If students are to be ready to comprehend the latter, they probably need to take part in learning activities that will help them to understand the

relationships associated with set union and subset removal operations.

Finally, it should be emphasized that our use of kindergarten children in investigating what is involved in solving the various specific story types included in our study should not be taken as implying that we feel that kindergarten children should be taught to solve all such stories. The purpose of this study was to describe the present capabilities of kindergarten children in order to clarify the types of arithmetic concepts that they are ready to comprehend and also to identify those component capabilities that students, of any age, must possess before they are ready to study somewhat more advanced concepts. The question of when such component capabilities can be taught most effectively and efficiently is quite a separate consideration.

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