Presented is a collection of reports on the status of competency-based mathematical education in the schools of 25 countries, and is the culmination of five years' work by the International Study Group on Minimal Competencies in Mathematics. Most reports summarize the trends and prevailing opinions about curricular implications of emphasis on minimal competencies, and a diverse spectrum of attitudes, approaches, concerns, and programs is revealed. Included is an historical perspective of prior investigations, a synthesis of the national reports, and a list of selected international references alphabetized by author. The goal of this manuscript is to answer some questions about the competency-based approaches of the following countries: Australia, Austria, Bangladesh, Belgium, Brazil, Canada (British Columbia and Ontario), Chile, England and Wales, Finland, Hong Kong, Ireland, Israel, Japan, Kenya, Luxembourg, Netherlands, New Zealand, Norway, Philippines, Scotland, Sweden, Thailand, U.S., and West Germany.
THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
Center for Science and Mathematics Education
The Ohio State University
Prepared for the
International Study Group on Minimal Competencies in Mathematics
in cooperation with the
Second International Mathematics Study
International Association for the Evaluation of Education Achievement (IEA)

AN INTERNATIONAL REVIEW
OF
MINIMAL COMPETENCY PROGRAMS IN MATHEMATICS
prepared by
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-1-
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Planned Publications


Applications of Mathematics. Editor to be chosen. March-April 1983.
Competency-based mathematical education (with the emphasis on "minimal" competencies) may be growing into the most significant educational trend of the 1980's. In one form or another, it is emerging in numerous countries around the world; and, as with educational trends of the past, this particular trend is gathering both supporters and critics. Many of its supporters consider it the ideal educational structure for emphasizing the essential, eliminating the extraneous, and therefore maximizing useful learning in mathematics. Its critics, on the other hand, are more skeptical. Many of them consider it an over-reaction to the problems associated with the "new math;" they believe that minimal competency programs can cause new problems of their own, and that they would be a change for the worse—not a change for the better.

Certainly curriculum planners in the 1980's will be seriously considering minimal competency programs to possibly cure whatever ills their own educational programs may be suffering from at the time. It is the hope of this author that AN INTERNATIONAL REVIEW OF MINIMAL COMPETENCY PROGRAMS IN MATHEMATICS will allow those curriculum planners to more easily learn from the experiences and insights of others.

It would indeed be unfortunate if school systems in every country had to individually experiment with the various aspects of minimal competency programs—unfortunate because experiments sometimes produce failures, and, in education, the victims of those program failures would be children.

In gathering and preparing the information for this document, there were many individuals who worked hard on the various tasks required. Some of those individuals are given credit in other portions of the text (for example, the contributors of national reports, many of whom did much more than they are specifically given credit for). Others include Jane Kulp, Mary Morrow, Robert Lindsay, Lynne Maloy, Albert Berg, Peggy Kasten, and James Schatzman.

Robert J. Riehs
Editor
A NOTE OF THANKS

It will be clear to everyone who reads this publication that a great many people were involved in preparing it. To each of the authors in the 25 countries represented in this report, we extend our thanks for contributing so willingly in order that a better understanding of the minimal competency trend can be attained.

Our deep appreciation is given to Robert J. Riehs. He undertook the task of editing this report with enthusiasm. His concern for having a large number of countries represented is evident. He attended to all the details of putting the publication together with dispatch. Despite the short timeline, he managed to find the time to correspond with authors, edit the manuscripts, obtain translations, arrange for typesetting of the section dividers, and even type the manuscript. We are indebted to him for giving of his time and talents so freely.

Kenneth J. Travers
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Preface</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Investigating the Minimal Competency Trend Around the World in 1981</td>
<td>13</td>
</tr>
<tr>
<td>II. NATIONAL REPORTS</td>
<td>15</td>
</tr>
<tr>
<td>Australia</td>
<td>17</td>
</tr>
<tr>
<td>Austria</td>
<td>24</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>29</td>
</tr>
<tr>
<td>Belgium</td>
<td>33</td>
</tr>
<tr>
<td>Brazil</td>
<td>39</td>
</tr>
<tr>
<td>Canada (British Columbia)</td>
<td>40</td>
</tr>
<tr>
<td>Canada (Ontario)</td>
<td>43</td>
</tr>
<tr>
<td>Chile</td>
<td>48</td>
</tr>
<tr>
<td>England and Wales</td>
<td>53</td>
</tr>
<tr>
<td>Finland</td>
<td>56</td>
</tr>
<tr>
<td>Federal Republic of Germany</td>
<td>60</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>64</td>
</tr>
<tr>
<td>Republic of Ireland</td>
<td>65</td>
</tr>
<tr>
<td>Israel</td>
<td>70</td>
</tr>
<tr>
<td>Japan</td>
<td>75</td>
</tr>
<tr>
<td>Kenya</td>
<td>77</td>
</tr>
<tr>
<td>Grand Duchy of Luxembourg</td>
<td>82</td>
</tr>
<tr>
<td>Netherlands</td>
<td>85</td>
</tr>
<tr>
<td>New Zealand</td>
<td>89</td>
</tr>
<tr>
<td>Norway</td>
<td>94</td>
</tr>
<tr>
<td>Phillippines</td>
<td>97</td>
</tr>
<tr>
<td>Scotland</td>
<td>101</td>
</tr>
<tr>
<td>Sweden</td>
<td>110</td>
</tr>
<tr>
<td>Thailand</td>
<td>118</td>
</tr>
<tr>
<td>United States of America</td>
<td>119</td>
</tr>
<tr>
<td>III. SYNTHESIS OF NATIONAL REPORTS</td>
<td>131</td>
</tr>
<tr>
<td>IV. SELECTED REFERENCES</td>
<td>149</td>
</tr>
<tr>
<td>APPENDICES</td>
<td>Page</td>
</tr>
<tr>
<td>---------------------</td>
<td>------</td>
</tr>
<tr>
<td>A. Programas de Destrezas Mínimas en Matemática en Chile</td>
<td>167</td>
</tr>
<tr>
<td>B. Rapport sur les Compétences Minimales en Mathématique en le Grand-Duché de Luxembourg</td>
<td>173</td>
</tr>
<tr>
<td>C. Minimal Competency Testing Programs in the United States</td>
<td>176</td>
</tr>
<tr>
<td>D. A Few Additional Words on the Learning Process by the Authors of the Chilean Report/ Algunas Palabras Acerca del Proceso de Aprendizaje Experimentado por los Autores</td>
<td>177</td>
</tr>
</tbody>
</table>
Part I

Introduction
"If today's school children were told to go forth and multiply, many of them wouldn't be able to do it." That, according to Thomas Hine, was the view of some critics of mathematics education in the United States in the early 1970's. Those critics feared a generation unable to balance its checkbooks or to decide whether two for 29 cents is cheaper than six for a dollar—in short, a population frightfully deficient in mathematical "survival" or "life" skills.

Hine's comments were primarily directed at deficiencies in the "new math," as implemented in many United States schools. However, comments similar to his have been made many times before, and in countries outside of North America. The last hundred years, especially, have been a time of turmoil in mathematics education, marked by massive adjustments in mathematics curricula and frequent criticism of instructional techniques. Rapid industrial development, and the growing impact of modern technology on daily life, have been making the learning of mathematics increasingly important in more and more countries. Also, a growing political willingness to educate the masses, rather than just the elite, has resulted in the tremendous expansion of public educational systems. With all of this exciting change has come the less exciting observation that many children, even after attending classes regularly, have nevertheless not learned much of what the schools were expected to teach.

It is not surprising, then, that parents, political leaders, and mathematical educators throughout the world continually search for ways to improve their local educational programs, with the hope of significantly increasing the level of useful learning. Unfortunately, while looking for programs that are better, they sometimes find and implement programs that are worse.

Undoubtedly, during the next decade, many of those parents, political

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leaders, and mathematical educators will consider experimenting with some form of competency-based curriculum in mathematics. As they do, they will be asking several important questions: "What have other countries done?" "Are there any successful programs after which we can pattern our own?" "Have any other countries made mistakes which we wish to carefully avoid?"

This document is an effort to gather the responses to those questions.

To whatever extent this effort succeeds, it is the culmination of five years' work by the International Study Group on Minimal Competencies in Mathematics, working together with other groups, in trying to identify successful approaches to the teaching of mathematics and to communicate them to others. While the early investigations of these groups have been somewhat hampered because of the infancy of the programs being investigated, they have, nevertheless, laid the groundwork for this study and, most helpfully, revealed what specific curricular issues should be most closely examined. Quite appropriately, then, before going into the details involved in formulating this particular investigation, a few pages have been devoted to some historical perspective—a description of some of the earlier investigations, by the chairman of the Study Group, Dr. James T. Fey.
In the United States, the focus on minimal standards of school achievement began building in the early 1970's. By the time of the first conference to plan the second IEA mathematics study in Urbana (1976), similar concerns were reported in several European countries. Consequently, at the Third International Congress on Mathematical Education (ICME III), in Karlsruhe, a study group formed to address this problem area.

The study group attracted 40 participants from 15 countries, but it quickly became clear that the only common factor in their interests was the phrase "minimal competencies." In Germany, the Deutsche Mathematiker Vereinigung was developing guidelines for mathematical preparation of students intending on university study. In Brazil, the focus was on standards for the vast number of students leaving school at the end of the primary grades. In the United States and Sweden, the most common focus was basic skills for all students leaving secondary school. These diverse foci seemed to share only the concern for setting minimal standards to which all students in some particular school group should reasonably be held. The Karlsruhe study group sub-divided to consider, in their different contexts, some common questions:

- For what purposes are "minimums" sought and what is the genesis of concern for finding such agreement on standards?
- How has concern for minimal competencies led to changes in school mathematics syllabi, teaching approaches, or testing?
- What implications of various minimal competence initiatives can be forecast?

Discussions of the Karlsruhe study group sessions were summarized in a report circulated to participants and published in the ARITHMETIC TEACHER (Fey, 1977). Since 1976, interest has, if anything, increased. An inquiry...

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2 This is a shortened version of a paper based on work of a symposium on curriculum change held at Bielefeld, FRG in January, 1980. Support for the conference came from the government of the Federal Republic of Germany, the Institut for Didactics in Mathematics, and the IEA.
addressed to ICME III participants elicited reports from Japan, Finland, Scotland, Germany, Austria, Brazil, Norway, and the United States. These reports and the information from Haus Ohrbeck Minimal Competencies Study Group members were combined to produce the following description and critical analysis.

Level, Purpose, and Character of Existing Minimal Competence Programs

Since the 1976 ICME III survey of minimal competence-related activities, the number and variety of countries engaged in some aspect of the issue have increased, and the purposes and character of programs have been more clearly defined.

I. University entrance. Nearly every country has a long tradition of setting standards, syllabi, and examinations for admission to higher education. Thus, in some sense, the minimal competence issue is not at all new on this level. However, the past twenty years have seen striking expansion of opportunities for admission to university study, and the question of appropriate minimal competence in mathematics has been addressed with new perspective. As mentioned above, the German DMV has developed general syllabus guidelines for students intending advanced study. In the United States, the Mathematical Association of America (MAA) has recently published similar guides and communicated their advice to secondary school teachers and counselors. Neither advisory report has any official force, and it is not clear that the suggestions have had major influence on secondary schools.

In Brazil, as in many developing countries, access to higher education is still sharply limited by resources. As a result, there remains competition by examination for available spots. Each university administers its own exams (in some countries, such as Thailand, there is a uniform national exam) and publishes syllabi for those exams.

Taken as a whole, the concerns for minimal competencies among applicants to higher education seem a diffuse and not strikingly influential force in mathematics education today. There are individual professors and institutions critical of incoming students' quality, but response to this concern has not taken any strong, organized, or widespread activist form.
2. **Secondary school leaving.** There are at least three distinct ways that mathematics educators are expressing concern about minimal competencies for those leaving secondary school. In the United States, the general public, and consequently political leaders, have expressed rising dissatisfaction with the apparent inability of secondary school graduates to function effectively in application of mathematical skills to what might be called "life survival" problem solving. There is doubt that many students can use their mathematics to make intelligent choices as consumers, to deal with government policies in areas such as taxation, or to perform rudimentary problem solving of the type commonly required in basic occupations or around the home. As a result, many of the states have initiated basic skills or minimal competence testing programs, with passage of a test being one prerequisite to secondary school graduation.

In Scotland, the focus of minimal competence program efforts has been not only on life skill survival competence, but also on adequate preparation for entering the world of work. The effort underway at present is to establish national certification standards for various student ability levels. Consultations with industry are planned or are in progress so that the aims of schools and the needs of employers can be mutually reconciled.

The third type of minimal competence concern directed at secondary school leaving standards is easier to define by describing the problem that motivates program activities than by clear characterization of the programs themselves. In many countries--Federal Republic of Germany, Holland, Norway, and Swaziland to mention only four--the past 10-15 years have seen substantial broadening of opportunities to continue education into the secondary and upper secondary levels. The diversified student body that results must be offered a more diverse program, and both teachers and the public have raised questions about appropriate performance expectations for each group.

3. **Minimal competence for progress through school grades.** In the early 1970's, the state of Michigan (USA) became concerned about minimal mathematical competencies in the various grades of school and, as a result, produced a detailed outline of specific objectives and tests to measure attainment of those objectives in the first six grades of school. The
objectives are as precise as "adds two-digit numbers with one regrouping" or "divides a three-digit number by a single-digit divisor." The tests measure achievement on an objective-by-objective basis, and, though the test results have no formal power to affect grade placement of students, there is a recommended passing criterion at each grade in each objective. This type of interest in minimal attainments for progress from grade to grade in school has been manifest in a number of other countries, though not with the same degree of specific testing.

The Netherlands, Norway, Sweden, and Finland have all reported recent efforts to specify the expectations at each level of schooling. Though the definition is not usually in terms of minimums, the phrase "basic material" is used to describe some sort of core mathematics syllabus which all students should meet.

4. Minimal competence programs in developing countries. It is not at all uncommon in developing countries for 4, 5, or 6 years of schooling to be the most attained by the vast majority of students. Thus, in such situations, the problem of defining minimal mathematical goals for school leavers is a problem for primary education. An international conference on developing mathematics in Third World Countries was held in Khartoum in 1978, and the question of goals for primary education was a major issue. The conference participants agreed on five broad objectives to be sought for the end of primary schools: (1) functional numeracy implying knowing how and when to perform arithmetic operations involving whole numbers and decimal fractions; (2) acquisition of techniques of representing and interpreting data; (3) indication of abilities in measurement, approximation, and estimation; (4) development of spatial concepts and the ability to represent them on maps and scale drawings; (5) acquisition of certain mental attitudes which enable the development of problem solving strategies. There is no indication yet of the influence of these proposals, but the goals do seem a fair statement of the aspirations of some 40 countries represented at the Khartoum meeting.

Impact of Minimal Competence Programs on Syllabi, Teaching, and Testing

In many countries currently interested in one or more aspects of minimal competence programs, it is too early to report the impact that such initiatives will have on the existing programs. Since implementation of
any syllabus, teaching, or testing system change should reasonably be preceded by thoughtful projection of the likely benefits and drawbacks of such change, the Haus Ohnbeck Study Group attempted to make a first approach to such an analysis. The conjectures fall into two broad categories.

First, it seems likely that the effort to define minimal competencies in various situations has the potential to greatly clarify school aims. Specification of minimum expectations has the potential to sharpen the focus of instruction, to clear away out-dated or unimportant topics, and to forge a clearer understanding between the schools and society about goals for school mathematics. There is promise that students will leave school with a much more precise description of their school-acquired abilities and that those abilities will be more closely matched to what the outside world will be looking for.

Second, the interest in minimal competence specification offers promise of helping to improve the effectiveness of instruction. With precise statement and measurement of goals for instruction, the teachers will be provided with potentially useful feedback on the needs of each student. Further, by studying in detail the abilities that students can reasonably attain at various levels of education, the curriculum designers will be provided with valuable information and an opportunity to plan syllabi and instructional methods that are carefully conceived to reflect latest knowledge about children's mathematical development.

While the potential benefits of minimal competence activities are attractive, it is not hard to imagine some less favorable results as well. Among the first concerns to be voiced have been worry that minimums will become maximums of school and student expectations, that curricula will focus on achievement of low level "basic" skills such as arithmetic computation, that testing will dominate and constrain creative teaching, or that instruction for short-term minimal competence objectives will be ineffective for development of long-range problem solving ability.

Minimal Competence as a Case Study in Curricular Stability and Change

In many ways, the most intriguing aspect of the current minimal competence movement is the way it reveals forces shaping education in
general and school mathematics in particular. Why did the desire for specification and measurement of minimal school objectives spring up nearly simultaneously in countries all over the world—countries with widely differing educational and socio-political traditions? Some explanations are common to all countries represented at the Ohrbeck Conference, but others are unique to the situations of particular countries, and, as such, they reveal the different influences of different systems.

1. **New math backlash.** The ambitions of "new math" curriculum developers were high. For many students or teachers the great expectations were not or could not be fulfilled, and it should not be surprising that debate ensued about which of the ambitious objectives were really important. From questions like "What good is all this set theory?" or "Why do I have to know the commutative property?" it is not too great a leap to concern for establishing the basic material that really is essential. From one of the most ambitious of the new European programs, Denmark has backed off to a much more modest plan, partially as a result of reconsidering just what is basic material. Similar experiences have led to revised USA mathematics courses that have a much more basic flavor than those of the 1960's.

2. **Economic forces and accountability.** For many countries, the decade of the 1960's was a period of great economic prosperity and optimism about growth. School expenditures grew because people believed in the importance of schooling (particularly in mathematics and the sciences) and because economic prosperity made such growth easy to support. In some countries, the 1970's have seen a substantially changed economic picture for society as a whole and for individuals. Inflation has driven school costs and taxes up, and, when pressed by demands for more funds, the public has challenged the schools to demonstrate that their programs are effective. This call for accountability of schools has in turn led to efforts to define school objectives more precisely and to measure the attainment of those goals. Only the reckless would set unrealistically ambitious objectives in such a test situation, so the interest in minimums seems a natural result of the pressure. Whether by coincidence or design, the resurgence of behaviorist learning theory and instructional approaches has provided a convenient tool for these accountabilists, lending the minimal competence movement a method and "scientific legitimacy."
3. **Social policy.** In many countries, the past 15 years have been devoted to a striking democratization of educational opportunity as social policy. For instance, in Swaziland, the period following recent independence has led to greatly increased access to secondary education. This has meant that the average ability and achievement of school graduates has declined (or has appeared in public view to decline). This, in turn, has provoked public calls to guarantee a bottom to the decline—to make sure that democratization of educational opportunity does not lead to complete loss of achievement standards. Again, the natural response is to define minimal acceptable levels of school mathematics performance and to initiate testing programs to see that the standards are met. While few countries have moved this far, the pressure was reported from Norway and Germany.

4. **Political forces.** In several countries, the minimal competence issue has been cause for involvement of education in partisan politics. As mentioned earlier, one of the most troublesome problems in any minimal competence testing program is setting the passing criterion. Since passing a competence test is often the required certificate for entry into a wide range of occupational and schooling opportunities, those who favor a very high criterion of competence are often seen as elitists. By controlling access to economic progress for individuals or groups within a society, the competence criterion acts as a conservative agent and, in several countries of Europe, high standards have become part of the political program for conservative parties. Conversely, those who seek to diminish social and economic class differences often argue for modest minimums. This is identified with social democrat type parties in the political process.

5. **Search for stability.** Underlying each of the preceding explanations for interest in minimal levels of mathematical achievement is a broad mood of insecurity in the schools and in the societies they serve. Representatives of widely diverse countries reported to the Hauß Ohrbeck conference that this pervasive sense of uncertainty about purposes and values of social institutions has encouraged people to seek a clear and familiar foundation for school programs. In most cases, this desire has been expressed in two phrases: "back to basics," "no experiments." Just as the vague public understanding of need for advanced technical education supported innovations in the 1960's, a similar non-specific concern for
stability is acting as a powerful force in the retreat from those innovations.

The implications of current minimal competence activity as a case study of curricular change are neither simple nor certain. What is most striking about the collection of factors driving the minimal competence movement is the contrast with influential forces during the 1960's. The "new math" movement was noteworthy because of the leadership assumed by specialists in the academic discipline of mathematics. Explanations for the failure of this movement to achieve anything near its ambitious goals have frequently emphasized underestimates by innovators of the conservatism inherent in education at the local school level. It might be that the net result of much minimal competence discussion will also be modest change in the actual school program. But, looking at the issue as it has developed so far, one cannot help but be impressed by the educational and social context within which it is only one program.
INVESTIGATING THE MINIMAL COMPETENCY TREND AROUND THE WORLD IN 1981
(COMMENTS ON THE FORMULATION OF THIS DOCUMENT)

In order to bring about a truly international sharing of information on emerging minimal competency programs around the world, mathematical educators on six continents were asked to contribute national reports for this document. Additionally, significant attempts were made to obtain a representative sampling of countries in various stages of economic, technological, and political development. A particular effort, finally, was made to include all of the countries participating in the Second International Mathematics Study.

Based on concerns raised in some of the previous investigations discussed by Dr. Fey, each contributor to this review was asked to deal in one way or another with the following questions:

- Have there been any recent efforts in your country to identify minimal levels of mathematical competence?
- What pressures (political, professional, societal, etc.) have been most influential in causing the imposition of new minimal standards?
- What efforts are being made to measure student achievement of the minimal mathematical competencies?
- What is being done for the "incompetent" student?
- Are the minimal competence goals or tests having any noticeable impact on course content or teaching styles in your country? If so, what?
- Have any special efforts been made in your country to assure that minimal competencies do not become educational goals--the maximum expectations of parents, students, and teachers?

All of the responses to these questions were included in Part II of this document, "National Reports."

Part III is a synthesis of the national reports, prepared with the very helpful suggestions of many of the contributors.

The list of selected references in Part IV is intended to provide a starting place for those who wish to obtain more information than this document provides. It may be particularly helpful in identifying available information on countries not represented in Part II.
Although based on a foundation of previous investigations, this particular study was not a long-term project. Except where specifically indicated otherwise, the national reports were prepared between November, 1980, and January, 1981. An endeavor was then made to prepare the material for distribution around the world as quickly as possible.

In the effort to get this review out so quickly, some countries that are normally supportive of the international sharing of information have been left out. This in no way reflects a lack of interest on the part of the omitted countries, but simply a difficulty on the part of the author in overcoming slow international mail delivery and in finding the right person in time to obtain a report. Those reports that were received in time to be included, of course, represent the latest available information on the educational programs in those countries.
Part II

National Reports
MINIMAL MATHEMATICAL COMPETENCIES IN AUSTRALIA

John Ellis
Education Department of South Australia

Schools in Australia are in the main administered by State Education Departments. The national (or federal) education scene is one of support funding for curriculum and research projects, and, although offering some monitoring services through the Australian Council for Educational Research (A.C.E.R.), does not specifically direct any assessment of minimal competencies. A.C.E.R. does administer, via sampling, choice of answer type items in mathematics, for both 10 year old and 14 year old cohorts, providing state and national norms on these items. Such assessment also assists in responses to I.E.A. surveys. Thus, any minimal competency assessment would currently be the province of each State Education Department. The absence of such assessment may reflect the disquiet of mathematics educators over any such setting of "minimum" standards and the possible abuse in both community reaction and lowering of mathematical goals.

Input to this Report from the various states indicates a majority are attacking the "minimal competency" area by trying to inculcate standards in the "core" curriculum of years 6-10 in formal schooling.

The Australian Association of Mathematics Teachers (A.A.M.T.) has recently been granted federal funds to complete the Australian Mathematics Education Project, which is aimed at improving the teaching of mathematics, particularly in the years 6-9, rather than setting standards for all, minimal or otherwise. Again, this reflects an attitude amongst key mathematics teachers in response to any outcry about student performance.

The following very brief reports from each state may expand the above general comments.

South Australia

Prior to 1977, separate Divisions of the Education Department administered Primary (K-7) and Secondary (8-12) education.

A report in 1977 made comments and recommendations about the K-12 Mathematics Curriculum, and work has proceeded since then on producing syllabus documents and, most importantly, teacher guidelines.
One of the key recommendations of the 1977 Report was that of identifying "Social Mathematics" as one of three end points in the curriculum. Social Mathematics, in syllabus content, contained the usual mathematical skills which are considered necessary for formal schooling, but also contained specific objectives on consumer awareness (e.g., insurances, handling money) which were considered desirable for every student to achieve before leaving school.

To some extent, Social Mathematics can be seen as a level of minimal competency. However, in defining this in the curriculum, there was no attempt made to suggest that a particular "level" occurs in the curriculum like a cut-off point. It is, in fact, most likely that all students will study mathematics further than Social Mathematics in pursuing the other two end points of the curriculum, one of which is an externally examined level of matriculation for tertiary institutions at the end of year 12 of schooling.

Traditionally, Australian schools have established subject courses in the discrete year levels from K to 12. Thus, a student in year n would be confronted by "year n mathematics." The 1977 Report very firmly commented that there is no such thing as "year n mathematics"—it is not desirable to so discretely associate a mathematics course with a student's age. This is a deliberate attempt to cater more for individual differences (although in class size groups rather than for each student, because of the sheer limitations of staffing). At the time of writing, there are many Primary Schools trying this new approach, but as yet there is not much experience of it in secondary schools.

In 1980, trial surveys were conducted in testing students on samples of items from Social Mathematics. From 1981, this testing will be carried out, using samples, for every year level 8 to 11. All schools may participate by conducting their own tests, and the data bank developed on the items is not to suggest pass or fail at items, but to provide feedback to teachers on student performance and thus monitor the performance over these years. The important service of this testing is to give teachers a clearer picture of skill retention over these years and thus a means of directly affecting the work in the classroom rather than making assumptions.
Victoria

At this time, there are no Education Department policy statements directly stating minimal competencies.

Curriculum guidelines incorporate that students be exposed to, and master where possible, a core of mathematical content; but no specific assessment is carried out. Thus, the only impact on teaching styles comes directly to teachers from the continuing public debates on "standards."

There have been some attempts to measure essential skills, but these were not warmly received by teachers, and nothing has been continued. Evidence suggests that many groups of primary schools liaise with secondary schools, on a district basis, to clarify the transition period and mathematical skills involved in that transition period. Such liaison is school-initiated, and it is difficult to gauge success, and it certainly does not involve specific statements of minimal competencies.

Queensland

Primary mathematics. A survey of teachers' opinions about the Program in Mathematics is being undertaken by the Research Branch of the Department's Planning and Services Division. Data will be gathered on the degree of importance attached to various aspects of "content" in primary mathematics.

Transition, year 7 (primary) to year 8 (secondary). A series of workshops has been prepared by officers of the Curriculum Branch of the Department's Planning and Services Division to guide secondary schools and their feeder primary schools in identifying basic/common areas of mathematics regarded as important in both sectors.

Some Regions have prepared summaries of expected outcomes in mathematics for the end of year 7/beginning of year 8. These include statements of "minimal competencies in mathematics" which students may reasonably be expected to have developed prior to entering secondary school.

Transition, year 10 to technical and further education. A study conducted by the Curriculum and Evaluation Section of the Department's Technical and Further Education Division has identified the mathematical (and science) skills required by students before entering apprenticeship
courses conducted at colleges of TAFE. The report of this study (March, 1980) has been used as a basis for discussions between Technical and Further Education Department and the Secondary Division of Education Department in order to determine "reasonable expectations of year 10 students in the Advanced, Ordinary, and General Mathematics."

Transition, years 10, 11, and 12 to employment. Supported by Commonwealth funds, the Department has stimulated the development and implementation of "transition education" (pre-employment) programs in secondary schools. To assist schools in this regard, the Curriculum Branch of the Department's Planning and Services Division has prepared guidelines for the identification of relevant "competencies" in mathematics.

"Core" mathematics. A small working party of teachers and officers of the Curriculum Branch has recently completed a statement of core/common areas of study in mathematics at the year 8 and year 9 levels. Topics and expected student outcomes have been stated for number theory, algebra, mensuration, geometry, and graphing/tables.

Western Australia
The Education Department has no statement of minimal competencies for any age level and no testing programs in this area.

Some future monitoring of standards in numeracy will occur at year 6 and year 9 levels through the formation of mathematics item banks incorporating numeracy items.

Flowing on from this project, the Department has undertaken some statewide testing of numeracy standards—in 1980, testing about 4% of the year groups. Future testing may occur annually, but the items will not all be repeated. The initiative for such testing is Departmental and aimed at heading off demands for such action by other outside groups.

Tasmania
During 1980, all subject committees of the Schools Board of Tasmania were asked to address themselves to the task of defining more clearly those skills and competencies which should be exhibited by students given awards at the various levels in the School Certificate (grade 10) assessments.
These are not minimal standards as such, but are intended to be a guide to users of the School Certificate as to what should be a reasonable expectation from students presenting themselves with awards at the various levels. They could be variously described as job preparation skills or basic skills for those who begin jobs which require them to undergo further training or as basic skills only for those who proceed through the school system.

This request from the Board has been brought about by a general dissatisfaction amongst users of the School Certificate about interpretations which can be placed on the various awards. Many employer groups are finding that awards in mathematics, at the lower levels in particular, are not good predictors of success in trade apprenticeship calculations (however unrealistic and unreasonable these may be) and are trying to bring pressure to bear for a separate arithmetic award.

Whatever measures are introduced in this state, it is likely that any measures of achievement will be determined using a scheme similar to our moderation scheme. Broadly speaking, the general criteria to be used in testing will be decided by a state moderation committee; these will be transmitted to the schools, which will then apply them in their own testing programs, and the results will be moderated at meetings arranged among schools within particular geographical regions.

The State is providing an increasing number of trained "special" teachers who are attached to schools to help teachers cope with the problems caused by the inclusion of "special" students in their classes. In addition, most schools have some sort of an intervention program to identify students with weaknesses and to work with them to resolve these weaknesses. The effectiveness of these programs is, to a large extent, dependent on the availability of suitable staff—suitable in the sense of being interested in this sort of intervention and having the requisite background knowledge. Some do very good work; others may serve to make the problem worse.

Individual schools have embarked on their own "calculations" courses to develop and maintain competence in the number area. In some of these cases, up to one-half of the time allocated to mathematics is spent in this
area. In schools where this is not being done, the time allocated to calculations would be less.

There was a strong movement to introduce a subject "Calculations" into the list of those included on the School Certificate. The Mathematics Subject Committee recommended against it because of the dangers inferred in the question. The Committee did, however, recommend that attempts be made to improve the correlation between awards in mathematics and competence in arithmetic.

**Northern Territory**

During 1980, the Northern Territory Department of Education set up Subject Area Committees in seven major subject areas, including mathematics. These Subject Area Committees have recently defined Core Curricula for Primary Schools, and these curricula will be implemented during 1981.

The Core Curriculum in each subject consists of those skills and understandings deemed essential for students to master and those educational experiences they must have had before leaving school. The junior secondary core curricula are to be developed by the S.A.C.'s during 1981.

In mathematics, the Core Curriculum has been defined in three phases, the first phase being Early Childhood (stages T-3). The second phase involves Middle Primary (stages 4-5), and the third phase involves Upper Primary (stages 6-7). Within each phase, the core has been identified in space, number, and measurement strands. The emphasis in mathematics has been on the application of computational skills and the development of problem solving techniques.

The move towards Core Curricula in the Northern Territory has involved political, professional, and societal pressures. Moves within the Department several years ago, initially at a Principals' conference, led to the establishment of a Task Force to report on curriculum, assessment, and accreditation. The Task Force Report, which set out the parameters of Core Curriculum, obtained Ministerial approval in early 1980 and is now the basis of policy in these areas. The professional and political pressures were supported by societal views which arose from concerns by some sectors of
the community who believed there was a need to establish what they regard as minimal standards across the educational system.

Subject Area Committees including Mathematics have established Subcommittees on Assessment and Accreditation. These Subcommittees will assist in constructing tests for use by secondary schools. The aim of these tests, in the lower secondary school, is to ensure a common standard throughout the Territory. In general, primary schools will be responsible for their own assessment. The Mathematics S.A.C. and the Mathematics Unit have developed tests for stages 3-6 primary level. It is intended that schools may use these tests to assist in the placement of students, in the development of remediation programs, and as measures of competence. The results of these tests, however, will not be used to make comparisons between schools.

The work of the S.A.C's is coordinated by the Northern Territory Curriculum Advisory Committee. This Committee is planning a Certificate of Competence which will be issued in year 10.
Didactics and pedagogy are, to some extent, dependent on the structure of the school system and also on the structure of the societal system. They therefore have to take these structures into account as external boundary conditions. So, for understanding the state of affairs in Austria with respect to the discussion on minimal competencies, it seems to be helpful to first give some characteristic features of our school system.

The first four grades (6-10 years old) are covered by the comprehensive elementary school. After grade 4, a choice already has to be made, because the lower secondary level in Austria does not have a comprehensive school. Either the student chooses the lower part of the Gymnasium, or he chooses the Hauptschule (with, in general, a lower level of teaching leading up to the eighth grade). After the Hauptschule, there is one more obligatory year of education, after which one can leave the school system (so 9 years in school is the minimum requirement). After the eighth grade (either at the Hauptschule or at the Gymnasium), there may follow either 3 years or 5 years in a vocational school or 4 years in the upper part of the Gymnasium (which is divided into many branches). Graduation from the higher vocational schools (graduation after the 13th grade) or from the Gymnasium (graduation after the 12th grade) gives the license for enrolling at any Austrian university. Approximately two-thirds of those graduating from the Gymnasium will enroll at a university. For the vocational schools, this percentage is much less.

Referring specifically to mathematics, the elementary school, the Hauptschule, and partly the lower part of the Gymnasium all have to provide education in "life skills." In the Gymnasium, skills basic for reaching the higher grades are also covered. But this does not mean that mathematics teaching in these schools is delimited by any minimal competency program. The upper part of the Gymnasium mainly serves as preparation for university studies, and at the vocational schools there is a slight emphasis on applications of mathematics (but by no means to a sufficient extent).
In brief, one can say that the mathematical training at the elementary level (and partly at the lower secondary level) focuses on the mathematical knowledge and abilities necessary for everyday life in our western, developed society (with some additional topics, motivated by inner-mathematical and systematic considerations). At the upper secondary level, preparation for university education and "mathematics for its own sake" are the structuring guidelines—much less, the teaching of "job preparation skills."

The main point, and the point of greatest influence on the issues under discussion in this volume, is the fact that the Austrian school system is a highly centralized and centrally organized bureaucratic system. Despite Austria being a federal state, all competencies regarding curricula and school legislation are held by the federal government and the competent ministry. The schools are mainly financed by the federal government; even the costs for personnel of most private schools are reimbursed (at least this holds for the secondary schools). The individual school has very little autonomy and almost no influence on the form and content of syllabi. The syllabi are designed centrally by ministerially authorized commissions, and they have the status of law to which every teacher must adhere. They describe, in more or less detail, the aims and goals of the various types of schools, for the different subjects; they also contain didactical hints which, in principle, should be observed by the teacher. To this normalizing and unifying effect of general syllabi is added the fact that only licensed textbooks are permitted to be used in the schools. It is mainly by these textbooks that a certain standard and a well-defined level of mathematics teaching is prescribed. Empirical studies (Universität Klagenfurt, 1979) show that the vast majority of teachers (at the secondary level) use the licensed textbook to a great extent for organizing and structuring their teaching; the exercises, especially, are taken out of these books.

Knowing this general situation as a background, it will come as no surprise that there is no public discussion on minimal competencies in mathematics at all school levels. The syllabi, by law, define a minimum program, which is interpreted by the licensed textbooks. This program, with only some exceptions, is carried through by many teachers so that
teaching of mathematics reaches a quite high level with respect to the topics covered (not necessarily with respect to the abilities and qualifications attained by the students). So, in some sense, there is no necessity for minimal competency programs in schools, since, by our syllabi in general, it is granted that a certain standard is achieved by most students.

Apparently, this standard suffices for what is expected by the later employers of the students, since there are no essential critics of the mathematical education. Empirical studies carried out at the University in Klagenfurt (1981) show, for example, that more than 90% of the students graduating from the Gymnasium and from upper secondary vocational schools judge their mathematical education to be good or at least to be sufficient for their needs. A hint in the same direction is that only a minority of these students have to undergo further education in mathematics at their working place.

There is, therefore, no pressure whatsoever for the imposition of minimal standards. One could mention complaints (by the parents) that the students are overworked at school. However, I guess minimal competency discussions are usually motivated by the feeling that student achievement has fallen below a certain minimal standard. But this minimal standard (with respect to topics covered!) is defined by the syllabi in Austria.

The preceding paragraphs might give too good an impression of the situation in Austria; a closer look might therefore be necessary. This has been done in an empirical study at the Department of Mathematics in Klagenfurt (Dörfler & Peschek, 1980). This study shows that the achievement in mathematics at the upper secondary level predominantly lies in the ability to perform some well-explained algorithm. That is, mathematical routines are acquired, but much less common are deeper understanding, problem-solving abilities, and other higher (in a taxonomical sense) abilities. This is explained by a "hidden minimal standard" which is established by the (final) examinations. It is mainly for these that students are educated and students are willing to learn. Without great difficulty, it should be possible to write down the "average exam" which, by a not-outspoken convention, defines a minimal competencies catalogue. So, we have a great many traditional norms regulating the teaching of mathematics in our schools.
In giving a complete survey, it should be mentioned that there is a ministerial commission which dedicates some of its work to research into the following question. The syllabi (of the upper secondary level) being given, what are the competencies (not necessarily minimal ones) which possibly can be achieved by teaching and learning the prescribed topics? In other words, what is the best we can make out of given contents of teaching by using appropriate methods of teaching? Finally, there has been an empirical study (Universität Klagenfurt, 1981) on determining the expectations of the employers (in both private and public sectors) concerning mathematical abilities of graduates from the Gymnasium and from higher vocational schools.

A final general remark—when discussing minimal competencies, much more emphasis should be placed on abilities which do not consist of knowing something (a fact, a method, an algorithm). Such minimal competencies of factual knowledge are not very reliable and are subject to rapid change. One could instead also ask for minimal cognitive competencies, which guarantee intelligent and creative behavior in many different contexts. What are, in this respect, minimal standards to be achieved by mathematical training, and how can such achievements be tested? In other words, what is needed besides the discussion of minimal competencies relative to certain needs (there are no absolute minimal competencies!) is a discussion about the possible taxonomical cognitive structure of (minimal) competencies in mathematics and about mathematics. It should also become much clearer what has to be and can be the scale against which to measure minimality of competencies. Who defines minimality—the mathematicians, the didacticians, the employers, the politicians? One should be more sensitive to this problem! Also, definitions of minimal standards tend to result in inflexibility and rigidity of the teaching system.

In closing, I wish to thank Prof. Dr. H. Bürgner, University of Vienna, for valuable discussions on the subject matter of this paper.

References


MINIMAL MATHEMATICAL COMPETENCIES IN BANGLADESH

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Bangladesh came into being in 1971. In all developing countries like Bangladesh, the impetus for reform in the contents of mathematics education comes from international movements which disseminate new points of view through conferences and publications. Local professionals, who have the opportunities to familiarize themselves with the new trends and events abroad, are prime factors for the establishment of the same in their own country. In curriculum changes at the secondary level, some resistance comes from the secondary teachers: however, this gradually fades away as the teachers recognize their weaknesses, and the controversy is generally not protracted. Resistance from students is almost nil, because they know that the changes have been made under pressure and advice from university teachers, and because they will be going on to higher study at the university (as it is not easy to get a job after secondary education). Most of the parents never received secondary education as such, so changes in curriculum have little effect on them.

The importance of mathematics in the field of education has been fully recognized in Bangladesh. Mathematics is compulsory for all disciplines up to Secondary School Certificate level (grade I to grade X) and has been made compulsory recently for all higher secondary (grades XI and XII) science students. In Bangladesh, the same syllabus is followed in all institutions. We feel that to be appropriate a curriculum must be within the comprehension of pupils. Efforts to teach too much can end in practically nothing being learned. A balance between the capacity of the children and the demands which society will make on them has to be maintained. A syllabus that is appropriate has great value in building up a balanced personality.

Determining a uniform syllabus for the whole country was a very difficult task. There was the danger that the uniform standard might be determined by the lowest. This would lower the general standard. Equally so, there was the danger that uniformity might be brought about by accepting the highest norm. This could be desirable for some, but it
might create insuperable difficulties for others. Both of these
alternatives would be harmful, and they would not be conducive to the
evolution of a balanced, progressive pattern. In determining the
suitability of a syllabus that is neither too heavy nor too light, the
curriculum committee of Bangladesh kept the needs of the average child in
mind. In advanced countries, the syllabuses are highly flexible, so that
children of the same age have different programs—one for brilliant,
another for average, and still another for retarded pupils. Such a system,
however, needs not only teachers of higher quality, but also teachers in
adequate numbers (besides, of course, the necessary classrooms). The
present resources of our country do not permit the implementation of such
a system. Therefore, we have to content ourselves with framing the
mathematics syllabus for the average child, taking due care that it
contains certain portions which only brilliant children can absorb. When
the condition of the country in respect to provision of teachers, classrooms,
and equipment permits, separate syllabuses for different categories of
children may be prepared and introduced simultaneously. However, this is
an expression of hope. There is hardly any chance of such a program being
possible in the future. We hope that intelligent teachers will
particularly bear in mind the needs of brilliant children and take suitable
steps to see that such children make rapid progress and full use of their
talents. If suitable collections of books are available in classrooms and
school libraries, and if the teachers pay due attention to guiding children
in voluntary reading and writing, a separate syllabus for brilliant
children will not be missed.

Our primary level begins at an average age of 5 years. A 5 year old
boy cannot do more than what is within the limits of his maturation and
learning capacity. He therefore requires a program elastic enough to allow
him to exercise initiative and learn to adapt himself to changing
conditions. In grades I-V, we have introduced "elementary mathematics."
It deals, besides number, with simple spatial problems (e.g., concepts of
size, shape, position, direction, and areas of plane figures), which the
term "arithmetic" does not ordinarily include. The term "elementary
mathematics" has therefore been used, but it should not be interpreted to
include algebra or formal geometry. We expect that the pupil must gain
skill and understanding in the fundamental processes of addition,
subtraction, multiplication, and division of whole numbers, easy fractions, and decimals (as are necessary in everyday life). We also expect minimal competency in problems with money, weight, and measure at this level.

In preparing our mathematics curriculum, we attached importance to those "basic skills" needed to go on to the next educational levels. Our syllabus in mathematics from grade VI to grade X has been prepared keeping this objective in view. We feel that mathematics is required for two purposes—first, for use in the everyday life of the average citizen ("survival skills"), and secondly, as a tool for the scientist, engineer, or technician ("job preparation skills"). To meet these needs, two separate syllabuses have been framed. One consists of "general mathematics," which covers all that is likely to be required by a normal citizen in his routine daily tasks. This has been made compulsory for grade VI to grade X. The second syllabus, which we call "elective mathematics," contains the basic mathematics needed by pupils who wish to specialize in science and technology or who wish to study the subject because they have the necessary attitude and liking. This syllabus is therefore elective at the secondary level. The course content of general mathematics is a modest one, appealing to actual experience rather than to abstract principles. Algebra, arithmetic, Euclid (Parts I-IV), graphs, and a little bit of trigonometry and coordinate geometry are the main topics. The so-called "solid geometry" course has been omitted.

Mathematics has now been made compulsory for all science students at the higher secondary level. The syllabuses have been prepared to meet the entrance requirements for studying engineering, medicine, or mathematics at the University; the minimal competencies are determined by the requirements of these different disciplines. Present conditions have not permitted us to provide different syllabuses for each of these disciplines. We have had to develop a single, comprehensive one. Consequently, we have had to be satisfied with the basics. As in all developing and developed countries, there is murmuring that our higher secondary students do not have sufficient preparation for University Mathematics. However, in the present circumstances, with limited resources, we cannot think of an alternative.
We have public examinations at the end of grade V and also at the end of grade X. These are organized by the Board of Intermediate and Secondary Education. Nearly 50% of the questions are on basic concepts, so that average students can get passing marks. The passing mark is 40% at present. This is the only means by which we currently measure the students' achievement of minimal competencies. Some schools arrange tutorial classes for incompetent students, and the students themselves also arrange private tutoring on their own. The latter practice is particularly prevalent.

In countries like ours, "change in curriculum" and "change in textbooks" are synonymous. The curriculum is framed by the National Curriculum Committee, and textbooks are produced by the Bangladesh School Textbook Board (an autonomous organization). A single textbook is followed in all schools of Bangladesh. The authors, therefore, have some flexibility in translating the curriculum. This gives a check from going too far in the contents.

The mathematics curricula at the university level are framed by the respective universities. They differ in subjects as well as in contents.

For the last two years, there has been talk about "back to basics"—which got its impetus at the 2nd National Conference of the Bangladesh Mathematics Association. But in Bangladesh, this movement cannot gain ground, as the contents of our mathematics curriculum already stress basic skills. Our slogan is "FORWARD TO BASICS."
MINIMAL MATHEMATICAL COMPETENCIES IN BELGIUM

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Introduction

The problem of identifying minimal competencies is a relatively new one in the Belgian school system, as it is in most European countries. Two main reasons may explain this phenomenon:

- In Belgium, the concept of "curriculum" is a very recent concept. A large majority of school or ministry administrators and of teachers (probably more than 90%) ignores this concept. The practice of these educators is therefore mainly guided by school programs or by existing textbooks. The programs have been so far considered as minimal standards to be attained by the students. Students able to pass exams based on the existing school programs are admitted into the next higher grade level and are oriented towards different tracks. In fact, contents included in school programs for academic subjects may be considered as basic skills. There is, moreover, very little reference to mathematical life skills or to mathematical job preparation skills in the school programs.

- The concept of educational objectives is also a relatively recent one in Belgium (especially in the French part). However, for more than ten years, much effort has been devoted to studies attempting to identify educational objectives in both (French and Flemish) Belgian school systems.

The movement towards identifying minimal competencies is therefore exogenous to the school system itself. In most cases, it is the result of the influence of American studies which have been disseminated throughout Belgium by University Institutes of Education. The movement is not really institutionalized, and therefore no general framework exists coordinating efforts made by different people or by different institutions.

Primary Education

1. Educational objectives. Much work has been done to define objectives for grades 1 to 6.
In the French-speaking part of Belgium, lists of educational objectives have been elaborated by educational researchers and disseminated by the Ministry of Education and the University of Liège. Efforts have been made too for surveying student and school achievement by using these lists. Scores of "opportunity to learn" and of "educational importance" have been obtained from random samples of mathematics teachers. An item bank is being developed and will be available in the near future. These studies are not really "minimal competency studies," but this material will nevertheless be available for further studies in this field.

In the Flemish part of the country, much effort is actually devoted to the "re-definition" of the general objectives of the Primary Education for providing equal opportunities for everyone. Within these new educational objectives, problems of individualization are important, and hence, problems of minimal and differential objectives. For mathematics, the Laboratory of Educational Sciences (Gent) has been invited as consultant by the Curriculum Committee of Basic Education. Extensive lists of operational objectives have been elaborated (grades 1 to 6) according to the taxonomy of De Block. Minimal and differential objectives have been mentioned.

2. Teaching sequences. Some research has been done in French Belgium, at grade one, for constructing teaching sequences in mathematics. These sequences include:

- educational objectives
- evaluation instruments
- teaching strategies.

They intend to lead a majority of children to the mastery of high-priority objectives. The methodology is as follows:

- selection of terminal objectives and analysis of mediating objectives
- systematic control of the achievement of these objectives through formative tests
- remedial procedures (for non-mastery students) and enrichment activities (for mastery students).

This methodology could avoid the eventuality that minimal competencies become educational goals and maximal expectations for parents, students, or teachers.
Transition Between Primary Education and Secondary Education

Many educational administrators (at the Ministry level) are concerned with the gulf which exists between primary school and secondary school education.

In the Flemish part of Belgium, "minimal competencies" for the elementary school are reviewed in the first year of secondary education. This focus is explicitly mentioned in the curriculum. Because of the introduction in 1978 of the so-called "new math" within the curriculum of primary education, the problem of the transition in this field will be reviewed in the near future.

For the French part of Belgium, a primary school inspector (specializing in the field of mathematics) is in charge of this problem. Through a cooperative research project with primary school and secondary school teachers, he is attempting to establish a common set of educational objectives for both levels. These objectives, which are considered as basic skills, should be mastered by the end of primary education. It is, moreover, expected that secondary school teachers could use these objectives as minimal competencies for constructing school programs.

In another research activity funded by the Ministry of Education (French part of Belgium), educational researchers at the University of Liège and school teachers of a local community are attempting to reduce school failure at the primary school level. In this project, they are trying to identify "basic skills" for grades 3 to 6. They will undoubtedly be confronted very soon by the acute problem of transition from primary to secondary education.

Secondary Education

1. Academic programs. "Academic programs" refers to those tracks which are designed for preparing students for further studies (Higher Technical School, University, Teacher's College).

In mathematics, the school programs intend to provide the best preparation for these students who will be math-oriented students (i.e., for students who apply to Institutes of Technology or to Graduate Schools of Mathematics and Physics). The whole program from grade 7 to grade 12 is intended to meet the needs of those students who have to pass exams at the
end of grade 12 in order to enter the Institutes of Technology. In one sense, we can consider that the contents of the program correspond to the basic skills which are required for these students.

For all other students (non-math-oriented), the school programs are developed by deleting contents from the math-oriented programs. There is, therefore, no real adjustment of the programs to the needs of these students.

In both parts of Belgium, some efforts are made by Universities (University of Mons, University of Gent, University of Leuven) or by Inspectors of the State and Catholic schools to define mathematical objectives. Unfortunately, these efforts are not coordinated enough, and much energy is often spent for few significant results. Moreover, this operationalization of the educational objectives remains very formal. The main purpose is to establish a hierarchy among objectives without changing anything in the existing school programs. In some cases, there are attempts to survey achievement; in other cases, however, there is only an identification of the objectives and no evaluation.

In the Flemish part of the country, new curricula had to be constructed to keep pace with the introduction of the Renewed Secondary Education (which is in line with the basic trend in the post-war development of school systems in Western Europe towards the so-called comprehensive schools).

At the University of Gent, tremendous effort has been made to introduce, on a theoretical base, the concept of "curriculum" and of "minimal competencies" (defined as the "core curriculum" which applies for all the pupils of a grade level, in every school track). Within this area, research projects were set up by the Ministry of Education within the Laboratory of Educational Sciences (Gent). Behavioral objectives for mathematics (first year Renewed Secondary Education) were listed according to the taxonomical model of De Block, and research was carried out to reduce the gap between intended and implemented curricula. Research data was provided to the Curriculum Committee of the State Education to enable them to list minimal and differential objectives. So far, however, a fundamental scientific approach to curriculum innovation has not been
possible to a sufficient degree. The lack of infrastructure for curriculum development and implementation is becoming very obvious.

At the University of Leuven, curriculum development for Renewed Secondary Education in Catholic Schools used a "social sciences project" as a model. The influence of the social sciences project is most obvious in the rather uniform schema of the curricula. They all contain the following components: a global description of entering behaviors, objectives of the subject, content specifications, suggestions as to teacher models, indications related to evaluation, suggestions and sources for further study and information. In the mathematics curricula of the Renewed Secondary Education, a differentiation has been made between basic contents (imperative for all pupils in a certain school year), and differential objectives—which enable them to make differentiation according to

- the available time
- the intellectual level of the pupils.

2. Technical and vocational education. An extensive project funded jointly by the European Communities and the Belgian Ministries of Education attempts to build a system of certification by credits for students following technical or vocational tracks.

Within the French part of Belgium (UNICAP Project), minimal competencies have been identified by a group of mathematics teachers under the supervision of a mathematics inspector. An educational researcher is acting as a consultant to this group.

In a first step, the project has analyzed the whole set of mathematical objectives pursued by the school programs. A hierarchical model (tree-diagram) has been used.

In a second step, subsets of these objectives have been built which are relevant for different grade levels. These subsets are also organized according to different fields of mathematical education (arithmetic, algebra,...). It is worthwhile to note that three levels of minimal competencies have been defined:

- life skills
- basic skills
- job preparation skills.
Life skills (which are supposed to be mastered at grade 6—the end of primary education) are considered as the most important ones. It is expected that remedial procedures should be provided during grades 7 and 8 for those students who do not show mastery on these skills.

Within the IEA Second Mathematics Study, documents of the IEA International Curriculum Symposium (Osnabrück, January, 1980) are actually used in the Flemish part of Belgium, for focusing on problems of minimal competencies with regard to life and basic skills. Within this area, the "comparative" dimension of these international studies can be of utmost importance for further curriculum Research and Development within national centers.

Conclusion

There is a slow but real movement towards a more sophisticated identification of educational objectives and towards the definition of minimal competencies in the field of mathematics. We can guess that this domain of research will become more and more important in the near future.

So far, this movement is mainly restrained by the lack of information of the educational community and by the prevailing conception of the school programs in Belgium. It is restrained also by the lack of institutionalization.

It is worthwhile to note that the movement principally concerns the highest grades in primary education and the lowest grades in secondary education. This fact is probably tied to the problems encountered in adjusting primary and secondary education; it is probably tied too to the existence of remedial teaching programs in all Belgian Junior High Schools.
MINIMAL MATHEMATICAL COMPETENCIES IN BRAZIL

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In Brazil, there are no significant efforts to identify minimal levels of mathematical competence.

Many individual teachers in my country are very interested in this subject; but, as of January, 1981, we have had no formal meeting to study it. At the Fifth Inter-American Conference on Mathematical Education, that took place in Campinas, São Paulo, nothing about minimal levels of mathematical competence was included.

Here, in Bahia, we have a lay group very interested in establishing "basic skills" and in devising teaching strategies that ensure student achievement of those skills. (For example, we have asked ourselves what geometric skills are essential for mathematical education in the Junior High School. Specifically, what kind of geometry? Euclidean geometry? Affine geometry? Both?) Our group is trying to do something for this, but our work is not official, and the success of our efforts is very limited.
Mathematics curriculum in British Columbia is prescribed centrally, whereas implementation and student evaluation is a joint responsibility at provincial, district, and school levels. In the final analysis, however, decisions related to determination of the competence levels of students rest with classroom teachers. In order to assist them in this decision-making role, direction and support services are provided at both district and provincial levels.

Identification of Basic Skills

To identify the skills and knowledge which are generally accepted as fundamental or basic to all students, the Ministry of Education published a document in 1977 titled "Guide to the Core Curriculum." This document lists the learning outcomes in mathematics, as well as other discipline areas, which are considered essential and should be learned at a mastery level by students. The aim of the core curriculum is to provide all students with the opportunity to develop their full potential as individuals and as members of society. With this goal in mind, these objectives encompass the basic, life, and job skills necessary for this purpose. Specific learning outcomes which are identified in this document are referenced to courses, but they do not attempt to be course-descriptive; rather, they are derived from different grade levels and subject areas.

Evaluation of Student Performance

A province-wide assessment of mathematics is scheduled every four years. Results are used to monitor student performance on these essential skills, as well as on others which reflect overall objectives of the mathematics program. In order to define satisfactory levels of performance on student tests, and then to judge the acceptability of the actual provincial results in light of these standards, the Learning Assessment Branch convenes provincial review panels (consisting of educators and
informed members of the public from throughout the province). Student performance on individual items, and objectives tested by those items, is judged on a scale of five, ranging from showing weakness to showing strength. In light of student performance on tests, as well as findings from questionnaires, a series of recommendations aimed at each educational decision-making level is arrived at.

District data, in addition to provincial results, are forwarded to each school district during the next stage of interpretation. A similar procedure is followed in which local performance ratings and recommendations are arrived at. District reports are then forwarded to the Learning Assessment Branch.

To provide follow-up, regional meetings between district representatives and the Learning Assessment Branch are held annually. Recommendations and subsequent actions are reviewed at that time.

Aids to Evaluation

To assist teachers in evaluation of students relative to their achievement in specific courses, the Learning Assessment Branch of the Ministry of Education has developed a series of achievement tests. The content of each test contains topic and cognitive behavior level weightings, which reflect expected student learning outcomes, as determined by a committee of experienced teachers and described in tables of specifications. Test items are referenced to curriculum objectives and include a range of difficulty levels. Each test is accompanied by a manual which contains provincial norms and instructions for administration and scoring, as well as instruction on the interpretation of results.

Interpretation of results includes judgements which are made to identify standards of performance. These standards relate to letter grade equivalents which include the critical "pass-fail" cut-off. In this manner, teachers are provided with direction on what is considered minimally satisfactory performance on each specific test. To determine cut-off scores, a representative panel of teachers assigns expected "p" values at each student performance category for every item. After discussion and review of these assignments, cut-off scores are arrived at through summing the "p" values. Details of this procedure are outlined in
a publication of the Learning Assessment Branch titled "Construction and Use of Classroom Tests." Teachers are cautioned, however, that these performance standards are relative to the specific test at hand and should not be the sole determiner of a student's standing.

Position of the Ministry

At present, the Ministry does not define minimal levels of mathematical competence in terms of an external benchmark which must be attained on a specific test. It does, however, identify basic skills for which a mastery level of performance is expected; and it provides instruments which measure student achievement within a mathematics course, relative to a set of performance standards. Provision of these resources serves as a valuable aid to assist teachers in their assessment of student performance.

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Division of Public Instruction, MATHEMATICS CURRICULUM GUIDE YEARS ONE TO TWELVE, Richmond: Ministry of Education, 1978.


There are no formal minimal competency testing programs in operation in, or planned for, Ontario as of late 1980. In fact, there are no province-wide every-pupil testing programs in any subject at any grade level, nor has there been any such testing program since the 1967 demise of the provincial examinations for grade thirteen students. There has been much concern in the province about the apparent loss of accountability which resulted from the termination of provincial examinations, and there has been continuing pressure to introduce minimal competency tests or to reinstate some type of monitoring system. The fact that the Secondary/Postsecondary Interface Study conducted in 1976 showed no loss in student performance in mathematics following 1968 may account in part for the seemingly unique status of Ontario on the issue of province-wide testing (Russell, 1977).

In spite of Ontario's decision to abandon province-wide every-pupil testing, the province is truly a part of the same world that has called for attention to the problems of the poorly educated student. Indeed, there has been political pressure, professional pressure, and societal pressure on this point and on the more general point that standards seem to be falling. By 1976, these forces combined to persuade the Minister of Education to establish a Task Force to identify the kind of province-wide testing program that Ontario should adopt. After 18 months of study, the task group reported to the Government and to the public that most of the testing programs they were able to observe in other countries were not really solutions to Ontario's problems. In fact, such testing programs were not uniformly well received by teachers and politicians in the jurisdictions where they were in place. The resulting report (1977) recommended an approach to provincial testing which would employ anonymous sampling strategies, similar to those found in the National Assessment of Educational Progress (in the United States) and in some item banking operations. Furthermore, the report recommended against every-pupil surveys, which could lead to comparisons among teachers and schools. As a
consequence of this report and other factors, both anonymity and sampling have been built into the province's blueprint for meeting some of the accountability demands of society. The new plan is called the Ontario Assessment Instrument Pool (OAIP), and it was initiated in 1978 with the development phase, which focused on creating assessment instruments or test items directly related to the government-produced curriculum guidelines. The first instruments are now openly available to classroom teachers for their own use in their own classrooms. Also, the provincial government and municipal governments are free to use the instruments in sample surveys which ensure that all individuals involved remain anonymous.

It is true that many of the items or instruments in the OAIP for mathematics are indistinguishable from items on minimal competency tests. The differences arise in what is done with the student responses to the items. In the case of minimal competency tests, the responses of individual students which are correct are aggregated for each student, and the resulting student score is seen to be either above or below the defined cut-off level. In the case of the Ontario Pool, the student responses which are correct are aggregated for each item and then converted to a "p" value (or a portion of students trying an item who get it right). Thus, Ontario is adding up the columns (items), rather than across the rows (students), in the student response matrix. The Ontario data therefore focuses on program monitoring, rather than on people monitoring. The Ontario scheme is quite different from most currently popular accountability schemes, but it is really not all that novel when it is considered feature by feature.

The Ontario plan, as it is presently conceived, does not specify passing standards for individuals or for groups. It does, however, result in the provision of useful data—namely, the proportion of students in a given grade who are taught a particular topic, and the proportion of successful students (in both tutored and untutored groups). What is to be done as a consequence of low proportions of successful students is a programming problem which is to be resolved, if possible, by the person(s) responsible for the program. The definition of what constitutes a "low" proportion is also the responsibility of these same persons, who should know what the program purposes are in terms of both participation rates and
success rates of students. Programming changes which are introduced as a result of the above type of data should lead to general increases in the product of education, but they do not focus specifically on problems related to individuals.

The task of identifying individual students who need more than the above mentioned general programming changes remains that of the classroom teacher. Furthermore, the responsibility for providing such attention is also in the hands of the classroom teacher. The task of identifying teachers who need special attention is beyond the scope of the OAIP purposes, and that therefore remains the responsibility of a supervisor (who must use other data to determine the nature and extent of any problem). Hence, the Ontario scheme does not provide the kind of individual student and teacher accountability which seem to be important features of most minimal competency testing programs.

The OAIP operates across a number of subject areas, and the rationale is the same for all. However, in the mathematics domain, there are some special ingredients in Ontario. The OAIP mathematics pool was developed by the same OISE professors who built a computer assisted instruction (CAI) remedial program, in parallel with the OAIP. They followed the same procedures which they had used successfully in the development of a remedial CAI mathematics program for freshman college students. Their program was needed because Ontario's freshman college students have had varying amounts of instruction in secondary school mathematics; however, in some college courses, a rather complete set of mathematics skills is required as a prerequisite (Gershman & Sakamoto, 1980).

The potential steering effects of testing programs such as OAIP and minimal competency tests is an issue which has caused some concern among Ontario educators, who wish to preserve both breadth and diversity in programs. The Province is populated densely along the shores of the Great Lakes and sparsely in the north. Diversity in interests and needs is accepted as a fact of life here, and this is reflected in the official curriculum guidelines through options which can be selected into the school program or omitted, depending on the needs and wishes of the communities concerned. It is an important requirement, then, of any testing program in Ontario, that this officially endorsed diversity be preserved, if not
fostered. The claim that province-wide tests tend to collapse the school programs around a narrow band of objectives has been voiced many times in Ontario, and the evidence available to support the claim seems especially convincing to educators. The arguments focus on public or widely known comparisons of teachers or schools based on test results of students; and the impact such comparisons have on teachers' allocation of time to topics known to be (or presumed to be) on the test. The OAIP plan is to avoid the use of student performance data in such comparisons and to ensure that individuals surveyed in data-gathering activities are not identifiable. The anonymity policy thus removes the pressure to inflate scores, and, although it reduces the number of functions which can be performed with the data, it nevertheless preserves the freedoms the various diverse groups are expected to enjoy as they select program topics on the basis of local community needs rather than on the more general needs of the larger provincial community.

The goals of the Ontario mathematics programs are set out in official guidelines which apply to the entire Province. The pool items and instruments which reflect the guideline goals cover more content than any one teacher can cover, and hence the guideline requirement that teachers have some freedom of choice among the goals expressed is almost automatically fulfilled. The goals, and the pool, have a steering effect, but that effect seems to be nothing more than what was intended by the authors of the guidelines. As pool data becomes available from the provincial surveys, there will be new opportunities for Ontario educators to debate what is in our programs and what is out, as well as how well the students learn what is in.

It would be misleading to suggest that the current Ontario plan is a solution to the minimal competency problem. It may be a solution to some of the continuing problems related to minimal competency. Hopefully, it will become a solution in time to the most serious programming problems. As such, it will lead to further deliberations about what it is that students really need to learn, how many of them need to learn which things, and what are the best ways of improving the proportions of successful students in critical topic domains.
References


What has been done with regard to the determination of minimal mathematical competencies in Chile? Which programs utilize this approach? What impact has it had on the teaching of mathematics? This report summarizes the results of a brief investigation of this topic; it indicates which programs these were; it characterizes the actual nature of these programs and proposes some actions which can be recommended in the light of the information obtained.

The educational system in Chile consists of eight years of basic education (educación obligatoria) and four years of secondary education (educación media). It can be qualified as being centralized, in the sense that it poses national plans and programs and that the coordination is in the charge of the Minister of Education. This characteristic is important in examining the impact of an innovation. At the present time, the system is experiencing a profound transformation in administrative technique, which affects, specifically, its centralized character. The mentioned reform tends to leave in the hands of municipal power the coordination of the educational subsystems. The reform implies, in the same way, the decentralization of the curricular decisions and of educational evaluation. It is in this system, and within this context, that you have to read the following conclusions about minimal competency programs in mathematics.

Minimal Competency Programs in Chile

As a consequence of consultation with educational leaders and with the centers for mathematical education in Chile, it is possible to conclude:

1. In the last few years, there has been a small group of experiences tending to define minimal mathematical competencies; the consultation produced a total of four programs of this nature (see the accompanying chart).

Translated from the Spanish by Robert J. Riehs and Diego Bigas. The original version appears in Appendix A.
Minimal Competency Programs in Chile

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<td>Fidel Oteíza Patricio Montero G. Pischedda Pierina Zanocco and others</td>
<td>Teodoro Jarufe M. A. Luque Pierre Maret Julio Crellana and others</td>
<td>Irigoin Valdivieso Ponce Avilés Cepeda</td>
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*Correspondence to the authors mentioned above can be sent to the addresses listed in Appendix A.
2. These experiences had a limited scope, their impact being more technical than massive. So, for example, they have influenced the programs for teacher education and the technology for the production of educational material.

3. The experiences identified through these consultations can be qualified as cautious; as for the techniques utilized, it would be in the determination of necessary skills and also in the production or validation of the average instructions for which they provided a basis.

4. The four experiences identified did not limit themselves to determining minimal standards. They actually provided a basis for teaching, supported by instructional material. (In one of the courses of study, there was generated a complete modular system for the basic education of adults.)

5. The initiative was principally from investigators; only in one case did the initiative have its origin in the system of formal education.

6. The two efforts with the greatest span were employed for the education of adults, with the specific instruction depending on their individual capabilities.

7. In the two programs designed for children, the fundamental criteria used for their selection were in the area of basic skills; in the two programs for adults, job-preparation skills were emphasized.

8. The educational levels in which the difficulty was realized corresponded to elementary education (grades 1-8).

9. The resulting instructional programs utilized criterion-referenced evaluation, and their administration was consistent with the principles of learning for mastery.

Other Educational Programs

The process of analyzing the experiments referred to, and the personal participation of the authors in the development of some of them, have resulted in the omission of some educational programs, which might profitably be synthesized in this section.

The determination of necessary skills should be re-examined. Their value is unquestionable, but the instruments and the procedures should be
revised. It follows that:

- The procedures utilized should take into account the community and all of the aspects of life which can determine the necessity of various skills.
- The processes for determining necessary skills should be permanent and should therefore accompany any curricular development and play a part in any program implementation.

The experience showed the value of mathematics, applied as an instrument of community development; as such, it should be offered as it is needed, and it should be more accessible to settlers and rural folk. In this sense, it is possible to re-examine the social value of the basic learning, for the purpose of putting it into the service of both personal and community development.

The programs analyzed show the incalculable value of the educational format that they affirm:

- Self-directed learning,
- Formative evaluation frequently during the learning process,
- Use of remedial alternatives or the provision for attaining mastery of what is learned, and
- The existence of material for valid teaching that will provide the pre-conditions for learning.

Finally, it is important to point out that the mathematical programs analyzed show the importance of formulating a system of learning which is a suitable component of an over-all system of economic and social development.

**Recommended Policies**

In the light of the previous analysis, it is suggested that:

1. The results of these efforts should be evaluated.
2. The results of these experiences should be communicated to others, and the results of such experiences should be exchanged with other nations. (The results of this paper were gathered in 1980 at a seminar in Chile.)
3. Greater coverage should be given to the existing programs, both in
terms of the diffusion of techniques and in terms of applying them.

4. Certain options should be pushed--those tending to create a consciousness of the necessity to have these programs (with or without minimum standards).

5. The political environment should be adapted to the transformation that is being experienced in the Chilean educational system.

6. These programs should be coordinated with programs for social and community development.
MINIMAL MATHEMATICAL COMPETENCIES IN ENGLAND AND WALES

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Strictly speaking, there are no minimal competency programs in England and Wales. Although there have been and are pressures in that direction, these should be interpreted in the context of a decentralized system of education, administered by more than a hundred local education authorities (LEA's). In this system, control of the curriculum rests largely with the LEA's and the schools themselves, and examinations are in the hands of 8 autonomous bodies for the General Certificate of Education and 13 such bodies for the Certificate of Secondary Education. There are no prescribed minimum standards for grade levels, and indeed there is a strong body of opinion in England and Wales which would question their value. However, a government committee of inquiry (the Cockroft Committee) is looking at the teaching of mathematics in primary and secondary schools with particular regard to the mathematics required in further and higher education, in employment, and in adult life generally. This committee is due to report in 1981.

There are pressures to establish "basic skills," "life skills," and "job-preparation skills." These pressures are rooted in a widespread belief that standards have dropped or that skills in mathematics have become less relevant to the needs of the outside world—particularly as a consequence of the introduction of "New Math" in the 1960's (although one should add that similar complaints have been heard for at least the last hundred years). The Royal Society's Working Party into school mathematics in relation to craft and technician apprentices in the engineering industry (1976), for example, noted a mismatch between the needs of industry and many of the CSE and GCE 0 level syllabuses. Also, the study entitled "Standards of Numeracy and Literacy in Wales," published by the Confederation of British Industries in 1977, was mainly an attack on standards achieved in schools. However, though some of the work done in this area attempted to define the mathematics thought to be needed for particular purposes, informed opinion tends to see the problem as how best
to improve current standards of performance, and to see this as a rather more complex matter than listing things which pupils (or teachers) should achieve.

In an attempt to improve on the present state of knowledge in this field, the Assessment of Performance Unit (APU) was set up in 1975, within the Department of Education and Science, to provide information about the general levels of performance of children and young people at school and how these change over the years. The APU has since carried out three surveys of the performance in mathematics of 11 year olds and 15 year olds. It should be stressed, however, that the APU is concerned with what children can do rather than what they ought to be able to do, and that far from creating levels of minimal competency the APU is at pains to minimize the "backwash" effect of its work on the schools. Of more direct relevance to the subject of minimal competencies is the work of the Shell Centre for Mathematical Education, Nottingham University, which has sponsored research and conferences on the mathematical needs of school leavers entering employment. The Shell Centre has also developed the SLAPONS (School Leaver's Attainment Profile of Numerical Skills) test for school leavers entering industry—a test which is already in a usable form. As its name suggests, this is a profile test rather than a pass/fail test; therefore, the question of establishing passing criteria does not arise.

As responsibility for education lies with more than 100 local education authorities, each is likely to tackle the problem of low competence in its own way. Although some form of testing is used in over 80% of the LEA's, very few of them at present use tests systematically to diagnose learning difficulties in mathematics and to allocate resources to schools for remedial action—nor indeed is there any clear evidence that administrative action of this kind would produce the desired results.

With so little done in the field of minimal competence goals or tests in England and Wales, it would be fair to assume that the impact on course content or teaching styles is negligible. Other external factors, notably public examinations, have had a far greater effect on the teaching of mathematics. In the view of some observers (including Her Majesty's Inspectorate), public examinations, while having certain beneficial effects
in setting standards, carry attendant risks (notably reinforcing tendencies towards a narrowly didactical approach with emphasis on the repetitive practice of isolated skills divorced from application and illustration). Although the GCE O level and CSE examinations are, between them, designed for the top 60% of the ability range at 16+ (and therefore cover much more than minimal competency), one result of raising the school leaving age from 15 to 16 has been that perhaps 80% (and, in some areas, more) of the pupils of this age are on courses leading to a CSE or GCE mathematics examination.

For most people at 16, the main educational goals are the passing of public examinations; so, it could be said that, generally, the standards set are considerably higher than minimal competency. The work of the lowest 10-30% of the ability range (depending on the school) is more at a "minimal competency" level; but, in a decentralized system, the programs and goals are decided by individual schools and are likely to vary greatly.
MINIMAL MATHEMATICAL COMPETENCIES IN FINLAND

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Prompted by the general concern about raising the level of school performance, and influenced by the discussion on minimal competencies, extensive work has recently been done in Finland by the National Board of Schools to define the "basic material" in the mathematics, foreign languages, and native language programs in the comprehensive school attended by all children from age 7 to age 16. This work is in response to an urgent practical demand from teachers of all levels. The involvement of the teachers is illustrated by the fact that more than half of the members of the committee which drew up the guide containing the basic material in mathematics were classroom teachers. A draft of the guide was also sent to several organizations in Finland, including the National Union of Teachers of Mathematics, and was revised in accordance with their comments.

The work was based on the following principles:
- Certain aims that give a basis for further education are set for the whole age group.
- The basic material leads to the attainment of these aims and is taught to the entire age group.
- Every effort is made to lead as many pupils as possible to a specified level of achievement with respect to the basic material.

Why Basic Material?

The main practical significance of the work described is to furnish teachers with a guide for planning and emphasizing appropriate areas of their teaching. The need for this type of guide arises partly from the fact that the modernized mathematics program of 1970 has proved too vague and has even rise to the use of widely differing textbook materials. Selecting topics to be emphasized from this wealth of material has proved

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2This is an updated version of a paper prepared in August, 1977, and submitted to the International Study Group on Minimal Competencies in Mathematics.
especially difficult for the non-specialized teachers of grades 1-6, teaching pupils aged 7-13. Specialized teachers of grades 7-9, teaching pupils aged 13-16, have also complained about problems caused by the new program and textbooks. The work of secondary school teachers of grades 10-12 will also be more clearcut when they can rely on practically all of the pupils having a solid common core in the basic material.

Evaluation

No national testing program to control the achievement of the basic aims has so far been planned. It is felt that organizing such a testing program would be a task of great responsibility and cost; and it could have harmful side effects, such as directing teachers to concentrate too heavily on teaching only the basic material which, after all, is at a relatively low level of achievement. The shortcomings of testing programs were also pointed out in the NACOME report. The task of evaluating the attainment of the aims thus rests primarily on the individual teachers.  

Scope and Sequence

The basic material has been grouped under several general headings. For grades 1-6, these headings are number concept, addition, subtraction, multiplication, division, functions, equations and inequalities, geometry, applications, and statistics. For grades 7-9, the headings are number concept, algebra, functions, equations and inequalities, geometry, applied mathematics, and statistics and probability.

Our term "basic material" is somewhat more encompassing than what is usually understood by "minimal competencies." The basic material should be teachable to all students (practically) and should suffice as a basis for study at the high school (gymnasium) level and in the trade schools.

The material is organized for teachers in two ways. First, there are lists of topics to be mastered by all of the pupils at the end of the

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Quite recent and tentative results of one testing program, intended to measure the degree to which the basic material has been mastered, seem to indicate that, at present, about 60% of the students master it to a satisfactory degree. When considering such results, however, one must bear in mind that only the very newest textbooks have been written taking the basic material guide into consideration.
second, fourth, sixth, and ninth grades respectively. This information is followed by more detailed methodological advice, as well as suggestions for additional material for pupils who master the basic material in less time than others. It might be pointed out that most textbooks contain an abundance of such additional material.

The guide also contains a sequence chart that enables the teacher to see at a glance what has been taught, as well as what will be taught later.

**Underlying Aims**

The basic aims and basic material are not explicitly deduced from general cognitive aims. There has been some discussion as to whether the basic aims should be stated in behavioral terms of the form "The student should be able to..." Lists of skills of this type tend to be quite forbidding to the average teacher-reader, however, so the method of presenting the basic aims by means of the material associated with them was chosen.

As to general mathematical aims, the importance of fostering abilities such as concept formation, deduction, understanding of types of ordering, estimation, problem solving, and creativity is stressed. In connection with these abilities, such mathematical skills as calculation with numbers and algebraic symbols, handling equations, basic geometry, interpreting and graphing functions, etc. are evidently quite central.

In drawing up the basic material, use was made of results of the work on harmonizing the mathematics programs in Sweden, Norway, Denmark, Iceland, and Finland. This work has been sponsored by the Northern Secretariat for Cultural Cooperation. Extensive use was also made of Finnish investigations and practical experience.

**Future Plans**

At this stage, the basic material is, to a large extent, based on practical experience instead of authoritative and detailed analysis of the aims of mathematical education. A thorough theoretical analysis of

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4 In 1979, it was decided that some minor revisions in the contents of the basic material should be made and that the lists of topics should be given for all grades from 1 through 9.
this work is felt to be of great importance, and follow-up research concerning the value and use of the basic material will serve as empirical data for such an analysis.

The reactions of classroom teachers after four years of having the basic material available have been quite favorable. It seems, however, that in order to have a really wide impact on teaching practice, mathematics textbooks should be edited anew, so that the basic material is clearly brought out in the text.

It is felt that a central aim of teaching mathematics in school is to assure a sufficient level of mathematical literacy for all students, without early discrimination between those who have barely mastered minimal competencies necessary for survival and those eligible for secondary education. That should also be kept in mind when discussing minimal competencies. The approach described above sets forth one way to attack the problem of minimal competencies, from the point of view of curriculum development and teaching practice.
Minimale Mathematische Kompetenzen in der Bundesrepublik Deutschland (FRG)

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Efforts

Disappointment over the inefficiency of the "new math" and low numerical, algebraic, and geometric competencies of many pupils in FRG put in motion the discussion about educational goals and mathematical competencies. This discussion has not focused on "minimal" mathematical competencies, but on "useful," "necessary," "worthy" competencies. A mixture of basic skills, life skills, and job preparation skills under the heading of "mathematische Allgemeinbildung" (this is a non-translatable German term) is being established. Let us call this pursued set of goals, contents, and educational background "mathematical standard competencies."

There are efforts to define two levels of mathematical standard competencies:

- Level 1 for all pupils leaving school after 10th grade (at the age of 16): arithmetic, ratios, algebra, functions, and geometry.
- Level 2 for all pupils leaving school after 13th grade with Abitur/university entrance (at the age of 19/20): Level 1 + calculus.

For both levels, there is also a trend to add statistical and algorithmical contents.

Pressures

Nearly all groups of the society influence the discussion about mathematical standard competencies. Policy makers are divided into two groups:

a) The first group is interested mainly in unifying the essentials of level 1 for all types of schools in the German school-system; and, within the set of standard competencies, they emphasize life skills and job preparation skills.

b) The second group tries to raise various Levels 1 and especially keep Level 2 on a high standard. This group is focusing on the aspects of "Allgemeinbildung."
All educational decisions in the FRG are compromises between these two views, a fact which results from the federal structure of Western Germany.

A couple of years ago, university teachers of mathematics and mathematicians in industry published a catalogue of competencies for Level 2. This catalogue was called too big, not realistic, by others in university and in industry. Reality in schools often differs from the ideas of policy makers, university teachers, and industry managers. The efficiency of current mathematical teaching and learning is lower than the standard levels mentioned above. Therefore, many teachers wish to reduce and to simplify contents and goals.

After having left school, the members of the non-professional society consider mathematics as a very, very difficult topic—rather useless for their own daily life, but, nevertheless, necessary for their own children.

Most teachers of didactics follow the international trend and wish to establish standard mathematical competencies as a set of skills, focused on problem-solving and on applications of mathematics to the real world. Parallel to this, more and more teachers of didactics try to find out conditions for better cognitional development of pupils in mathematics. They compare results of empirical research on learning and teaching mathematics with goals and contents found in the standard levels.

To summarize the efficiency of pressure groups, we can point out a trend to make those standard levels more realistic, more minimal, and more life-worth. However, this trend has not yet been very successful.

Testing

Level 1 competencies are tested by vocational organizations, when pupils change from school to industrial jobs. Level 2 competencies are tested by schools, not by universities. Only some universities make tests (e.g., for students of medicine).

External tests are primarily intelligence tests. They do not test coherent mathematical competencies. Testing in school is curricular testing of topics current in actual teaching.

Testing of schools is not emphasized, except for the comparison of school-systems—not for the development of realistic standards.
The FRG does not participate in the current IEA study. On the whole, testing efforts in this country are not yet efficient and are not yet designed to evaluate minimal competencies (nor have these competencies been developed). But the results of external school testing have shown the gap between the possibilities of school mathematics and the goals of outside-school-groups.

Remediation

Level 1. All comprehensive schools have established remediation programs for incompetent students, but only a few other schools have done so. For about 50% of the members of remediation groups, the work is successful. The others remain on a low level. An increase in remediation programs is avoided because of the high costs, the expenditure of time and energy, and the prejudices of many teachers.

Level 2. Some universities have established mathematical pre-courses for students of mathematics, science, medicine, and economics.

Impact

There is an increasing feeling of teachers and society for the gap between expectations and reality. Information coming out from the teaching and learning of mathematics implies that, as a matter of fact, the goals in reach are much lower than they "should" be. This feeling might be the source of a coming impact of "basic thinking" in all mathematical curricula in the near future. So far, however, in the FRG at least, this feeling has not yet taken the definite shape of a catalogue of mathematical competencies.

References


MINIMAL MATHEMATICAL COMPETENCIES IN HONG KONG

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In Hong Kong, there is not and has never been to my knowledge a minimal competency program or movement in mathematics.

It would be difficult to see what support such a movement would achieve here. Mathematics education here is primarily concerned with the achievement of excellence! There is little evidence that there has been any fall in "national" standards of achievement that would have prompted a concern for minimal standards. There have been regular surveys over the past twelve years which bear this out, despite high immigration and 100% enrollment at the first level.

In fact, one of the reasons for our joining the IEA Second Mathematics Study was to bring home to our teachers the absurdity of maintaining the extremely high standards that we do. This is achieved in part at the expense of other subjects in the curriculum. It may be attributable in some measure to the difficult bi-lingual situations of the medium of instruction which lends to Mathematics a greater value as a language free from ambiguity.
MINIMAL MATHEMATICAL COMPETENCIES IN IRELAND

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Interest in minimal mathematical competence has been growing in Ireland throughout the last decade. While, at present, there is no formal national program dealing specially with minimal standards, it is possible that the next few years will see either the development of such programs or the emergence of a "minimal competence" component in the ordinary national examinations. In this paper, the present situation and the possible developments are explained as follows. First, a brief description is given of the Irish examination system, and its effect on the current view of minimal competence is considered. The discussion is then broadened to include the curricular reforms of the last twenty years, and, in particular, to indicate teachers' growing involvement in them. Next, various projects or movements which have relevance to minimal competence are described; and the paper concludes with some brief observations on possible future trends.

The present situation cannot be understood without considering the examination system. School examinations in Ireland are centrally controlled; courses are prescribed, and the main examinations are conducted by the Government's Department of Education. There is no official test at the primary level, the former Primary Certificate having been abolished—-one of the many reforms of the 1960's. At the second level, however, there are three main examinations: the Day Vocational Certificate (taken after two or three years of the junior cycle), the Intermediate Certificate (which follows a junior cycle course of three or four years' duration), and the Leaving Certificate (taken at the end of a two-year senior cycle). Some 80% of junior cycle students sit for the Intermediate Certificate, and almost all of them take one of the two mathematics courses ("Lower" and "Higher") that are offered. The Day Vocational Certificate, or "Group Certificate" (as it is usually called), is taken by smaller numbers, and typically by weaker students; some use it as a terminal examination, while others go on to take the Intermediate
Certificate the following year. More than 50% of the entire age group proceed to the Leaving Certificate, and, of these, over 90% sit for one of the two examinations ("Ordinary" and "Higher") in mathematics. Thus, altogether, it can be seen that around 90% of the second-level school population follow an examination course in mathematics, and sit for a national examination at the end of it.

As a result, minimal competence at the second level has tended to be associated with performance in national examinations. Minimal standards can therefore be identified at two levels: at the end of the junior cycle (when students are aged 15+), and at the end of the senior cycle (when students are aged 17+). The respective standards are a pass in the Intermediate Certificate or the Group Certificate, in the first case, and a satisfactory grade in the Leaving Certificate, in the second. (The Intermediate Certificate program issued by the Department of Education is set out in a way that suggests what should be covered at 13+ and at 14+, but there are no national tests or agreed standards at these levels.) However, such a definition of minimal competencies leads to problems. For example, a pass or satisfactory grade does not define uniquely the skills that a candidate has mastered. The examination papers offer considerable choice; from the published results, the candidate who obtains his marks on questions demanding "life" or "survival" skills cannot be distinguished from the one who gathers the same total mark from questions on abstract and academic topics. While that particular problem could be overcome (by a suitable restructuring of the examination papers, for example, or by a different presentation of results), other difficulties would remain. Some of them are indicated below.

Thus, the present situation is not satisfactory. In order to appreciate the attempts that are being made to improve it, a closer look must be taken at the history of curriculum reform, and, in particular, at teacher involvement in it, over the last twenty years.

As mentioned above, the school courses are prescribed by the Department of Education. However, second-level courses are sanctioned only after consultation with syllabus committees, on which the various teaching and managerial organizations are represented. The committees themselves,
like so much else, were started in the 1960's. Before that time, syllabuses had remained unchanged for years; now, it is accepted that courses are periodically reviewed. The first new courses, introduced between 1964 and 1966, seem to have been welcomed by many teachers. However, with the passage of time and the further modernization of courses, the mood changed, and became predominantly one of disenchantment. Some of the reasons for this are outside the scope of the present paper, but one is entirely relevant: teachers felt that the courses were too abstract for the weaker students. This seems to have been an important factor in prompting teachers (other than the few on syllabus committees) to play a more active role in processes that can lead to curriculum change. The involvement is still on a fairly small scale, for the tradition of central control of syllabuses is strong; but the trend is regarded as exciting nonetheless.

It would be wrong to suggest that second-level teachers are the only people concerned about student achievement. The "back to the basics" movement reached Ireland—or maybe grew up spontaneously—in the mid-1970's; there is evidence of interest among employers, third-level teachers, and perhaps also parents. However, the school teachers and their third-level colleagues have been the most active. Their overt concern has been mainly with courses for the weaker students, rather than with minimal competence as such. Attempts to formulate alternative, less abstract courses began as early as 1973, but did not immediately lead anywhere. In 1977, there was another attempt. Like the earlier ones, it began in the Irish Mathematics Teachers' Association; but developments elsewhere, notably in the Royal Irish Academy's National Sub-Commission for Mathematical Instruction, lent impetus to the endeavors and widened their scope. After considerable discussion among the members, the Association drew up a whole battery of syllabuses (for both Intermediate and Leaving Certificate) and presented them in a draft document to the Department of Education. This document specifies a "core" of material, mastery learning of which is expected; thus, although the term "minimal competence" is not used, a minimal standard is in fact defined. It involves both "mathematical" and "survival" skills. However, the Association is still facing the problem caused by the dual
role of the suggested Lower Intermediate and Ordinary Leaving Certificate courses, which would provide the essential core for the weaker students, but would also have to cater for those of medium ability. (The more able students, naturally, would take the Highest courses at both levels.) The danger that minimal standards could become general standards has been appreciated.

Other groups have taken different approaches. For example, a project run by the Co. Tipperary (North Riding) Vocational Education Committee and the Thomond College Centre for the Advancement of Mathematical Education in Technology (CAMET [Ireland]) is considering an alternative Leaving Certificate course which again involves the ideas of "core" material and mastery learning, though in this case there is a specific emphasis on preparation for employment; and the City of Dublin Vocational Education Committee has concentrated on a "remedial" type of course for junior cycle students. It is too early to judge the success of these projects.

Finally, little has been said so far about minimal competencies at the primary level. In fact, some research has been done at this level (notably by the Educational Research Centre at Drumcondra, Dublin) using criterion-referenced tests, and aspects of the research have been carried on to the early post-primary level. However, the emphasis has been on achievement of the objectives of the standard courses, rather than specifically on minimal competence.

Attempts are now being made to link the work of some of the groups. More generally, there are moves to start a national debate on mathematics education. The social climate is sympathetic to such discussion, and the information is needed for the next revision of the national examination courses, which is due shortly. The IEA study may help by providing relevant data. Thus, the potential exists for a meaningful discussion of minimal competence, in the context of mathematics education in general. The interest is there; preliminary work on different aspects is being provided by the various projects; and the impending review of school mathematics courses offers an immediate incentive. Perhaps, therefore, the next few years will see the emergence of a formal policy and program on minimal competencies as part of mathematics education in Ireland.
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One of the most salient characteristics of the Israeli educational system is the high proportion of school children whose parents immigrated to the country. At the beginning of the 1970's, about 88% of the elementary school children were born to immigrant parents whose mother tongue was different from that used in the schools. Moreover, 68% of the immigrant parents came from Asian or African countries, and only a few of them had completed elementary school, let alone a secondary or tertiary education. From the early 1950's, educators have been deeply concerned about the failure of a large proportion of immigrant children to acquire basic skills in reading, writing, and arithmetic. Systematic testing, performed at the terminal grade of the elementary school during two decades (the 1950's and the 1960's), provided massive evidence that a very high proportion of immigrant children—mostly from Asian and African countries—had failed to attain satisfactory achievement levels in reading and in arithmetic.

Educational psychologists attributed the failure to the environmental deprivation of these learners, rather than to their lack of ability; accordingly, the educational system launched a variety of improvement programs. Findings about the failure to master basic skills led educators to specify a reduced set of skills and concepts which should be mastered by children of certain immigrant groups.

In 1962, the Pedagogical Secretariat of the Ministry of Education and Culture (the body responsible for school curricula) published a series of booklets, each one devoted to a single subject taught in schools, which specified the restricted list of skills and concept units drawn from the general school curriculum, and recommended that it serve as the minimum learning requirements for those students who are unable to cope with the complexity of the regular curriculum. Within each subject, detailed specifications were given for the minimum learning requirements at each grade level (from grade 1 up to grade 8). One booklet specified the Restricted Program (in Israel, it is called the "Parenthesis Program") in Mathematics.
The Parenthesis Program was based on considerations rooted in educational psychology, rather than in the analysis of social needs or functional significance. A committee of educational psychologists, school supervisors, and teachers determined the scope of the Parenthesis Program on the basis of their judgement of what this special population of learners could master during a given period of time. Social and functional demands were considered only in determining the priority among skills and concept units of the regular curriculum, but they were subordinated to the considerations concerning the estimated limitations with respect to ability.

The Parenthesis Program booklets were published as a series of suggestions which have never been approved by the decision-making bodies of the Ministry of Education and Culture. They were criticized for two reasons: Teachers disliked it for its stigmatizing effect; and educational psychologists ridiculed its naivete, claiming that the reduction of the scope of the skills and concept units to be mastered is not likely to improve the achievement level of the "deprived children." Rather, they should learn skills and concepts which are not included in the regular school program, because "these children" enter school with certain intellectual deficits stemming from the lack of educational stimulation at home. Nevertheless, the Parenthesis Program has had an impact on what has actually been taught in schools and on the kinds of demands teachers have made upon "deprived" groups of school children.

Textbooks as a Source for Determining Minimum Requirements

Since 1962, the Israeli school authorities have not issued any formal statements or regulations concerning Minimum Achievement Requirements. Nevertheless, given the fact that Israel has a central educational system, and the school program for the whole country is set by the Ministry of Education and Culture, the textbooks (especially those prepared for the deprived or disadvantaged children) do set the standards for achievement at each grade level for the low achieving group of learners. Textbook writers have used various approaches to Minimum-Requirement features of the mathematics program. For example, the core elements of the program in regular textbooks prepared for the whole school population are marked, and
the teachers are instructed to focus on those portions of the program for disadvantaged groups of learners. The principle of this approach does not differ from that in the Parenthesis Program outlined in 1962. Another approach is the preparation of parallel textbooks containing only the basic skills and concept units required for disadvantaged groups. These textbooks also contain supplementary explanations of some prerequisite knowledge, and they present a carefully graded series of exercises related to the core elements of the program.

 Schools are instructed to ensure that all learners master these core elements of the program. Accordingly, they are requested to test the learners and to identify those who fail to master the core elements of the program. A special supplementary budget is allocated to schools for setting up small remedial or corrective teaching classes and small group or individual tutoring activities for those who need such treatment.

An extensive network of improvement programs has been established in the schools, as well as in the Community Learning Centers, which has helped to increase the number of those who have mastered skills and concept units (in mathematics, as well as in other basic subjects) emphasized in textbooks for deprived groups of learners.

While improvement programs and corrective teaching activities are more common in classes with a large proportion of disadvantaged children, schools are requested to employ such corrective measures for all children who fail to master basic skills and core concept units, without regard to their socio-cultural background.

Basic Skills and Basic Concepts

In 1979, the Pedagogical Secretariat of the Ministry of Education instructed the Curriculum Development Units operating within the country to prepare a list of basic skills and concepts, which should be emphasized in each subject and at each elementary school grade level. Whenever the curriculum contains alternative units, the basic concepts should be specified separately for each alternative. The learner is requested to master those basic concepts which are treated in the units and which are selected by the teacher to be taught in his class. This request reflects a new approach in determining what should be learned by the low-achieving
groups in the school. This new conception of specifying minimum achievement requirements is based on the belief that the existing curriculum adequately represents social needs. However, since the quantity of concepts treated in the curricula on the one hand, and the level of their complexity on the other hand, correspond to the level of ability of the average or of above average learner, it is necessary to prepare a restricted list of basic concepts and also to specify the most basic parameters of these concepts (which then should constitute the core elements of the program to be mastered by all learners). The approach of anchoring the list of the basic concepts in the existing curricula, rather than in surveying societal or occupational needs, reflects the educational philosophy that the preparation of the learner for his role in the society, as an adult, can be done through a variety of curriculum content sets. It is difficult, if not impossible, to specify a list of concepts, the mastering of which is absolutely necessary for a person to function in our society, beyond a very limited set of basic concepts. This by no means can be considered the basis for a worthwhile educational program, even for the lowest level of learners. This conception applies as much to mathematics as to other subjects taught in schools. Therefore, according to this conception, Minimum Requirements should be determined for each curriculum unit. In elementary mathematics, one may include such topics as isometric transformations, visual representation (graphs), numeration systems, etc., and a series of basic concepts should be specified for each of these alternative topics. These concepts should be mastered by those who studied a given curriculum unit, but they should not constitute a Minimum Achievement Requirement for the entire student population.

Minimum Achievement Requirements Defined in Terms of Distribution of Scores

Israeli educators tend to define Minimum Achievement Requirement in terms of setting standards for distribution of scores in a class or in a well defined subgroup of the population. Teachers are expected to ensure that the lower limit of the score distribution be above a certain specified achievement level. At the same time, they are expected to ensure that the class average surpass the minimum limit, and that at least a small proportion of the learners get scores which can be considered at a level of excellence. Knowledge accumulated in this country about the
distribution of cognitive entry behaviors of learners in various socio-
economic groups makes it possible to set different standards for the
distribution of achievement scores, in keeping with the specific socio-
economic composition of a particular class. While the mastery of the
required basic concepts may be considered a positive achievement for
certain classes, it would not be viewed as such for other classes.
In Japan, one of the social problems for the last several years has been that the curricula at primary, secondary, and high school levels might be a burden to many pupils. Many students have been forced to drop out of school. Because of this situation, the Ministry of Education has formed a committee to re-examine those curricula from various angles. After much discussion, the committee submitted the report titled "The Leisured Education." Following the report, the Ministry formed committees for each subject ("Committee of Japanese," "Committee of Mathematics," etc.). The members of the "Committee of Mathematics" were school teachers from each level and mathematicians. Those committees re-examined the existing curricula and submitted a revised program for the high school level (ages 16-18) in the early part of 1979. The revised program will be put in force from 1982.

The Contents of the New High School Mathematics Program in Japan

Mathematics I. (Compulsory subject, 4 lessons per week.)
I. Numbers and algebraic expressions.
II. Equations and inequalities.
III. Functions.
IV. Figures (graphs) and trigonometric ratios.

Mathematics II. (Optional subject, 3 lessons per week.)
I. Probability and statistics.
II. Vectors.
III. Differentiation and integration.
IV. Arithmetic and geometric sequences.
V. Exponential, logarithmic, and trigonometric functions.
VI. Electronic computers and flow charts.

This report was prepared and submitted to the International Study Group on Minimal Competencies in Mathematics in August, 1980.
Algebra and geometry. (Optional subject, 3 lessons per week.)

I. Quadratic curves.
II. Vectors in a plane.
III. Matrices.
IV. Figures in space.

Pre-calculus. (Optional subject, 3 lessons per week.)

I. Sequences of numbers.
II. Exponential, logarithmic, and trigonometric functions.
III. Changes of values of functions (differentiation and integration).

Differentiation and integration. (Optional subject, 3 lessons per week.)

I. Limits.
II. Differentiation and its applications.
III. Integration and its applications.

Probability and statistics. (Optional subject, 3 lessons per week.)

I. Arrangement of data.
II. Number of events (permutations and combinations).
III. Probability.
IV. Probability distribution.
V. Statistical inference.

Minimal Competence

The "minimal competence" in mathematics for an applicant to enter a university is Math I, because most university entrance examinations contain Math I. Math I has been the subject of common entrance examination by the National Entrance Examination Board since 1978. (That is applied for 87 national universities and 33 public universities.) Applicants for science or engineering must take the examination with Math I plus additional topics contained in the 2nd and 3rd year courses.

The most important changes in the new program, compared with the present program, are a cutting down in the contents of Math I (which is a compulsory subject) and a decrease in its lessons per week from 6 to 4.
MINTMAL MATHEMATICAL COMPETENCIES IN KENYA

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The last ten years mark a period of unprecedented change in the Mathematics Curriculum in Kenya. The change started with the so-called "New Mathematics" program that was made compulsory for all primary school children in 1970. After a few years of experimentation with this program, there was a public outcry about the failures of the "New Mathematics." It was generally claimed by parents that children could not carry out basic arithmetical operations and that the mathematics they were learning was irrelevant to their day-to-day needs. To the politicians and policy makers, this problem seemed to occupy a low priority as far as primary education was concerned, particularly in the early 1970's. What seemed to matter then was to expand the primary school system so that as many children as possible could obtain basic education. Primary Education was made free, and, as a result, the enrollment rates rose rapidly to 85 per cent by 1980. On the surface, it seemed that Kenya had achieved one of its stated goals in education.

In addition to the complaints about incompetencies in mathematics, parents castigated the primary education in that it was producing primary school learners who were illiterate. This seemed to undermine the Government's commitment to provide basic education to all her youth. Three other related problems surfaced. It was noted that:

- Less than 50 per cent of those who started school completed the course; the rest dropped out during the first three years of primary school.
- The primary school education was terminal for the majority of those going through the education system.
- Those who complete primary education do so at a relatively young age of between eleven and twelve years.

These and several other factors led the government to consider more seriously the question of basic education. In terms of the quality of education, the pertinent question was: "What should be the working
definition of basic education in the Kenyan context?"

The National Committee on Educational Policies and Objectives

As a result of the concerns outlined above, the Kenya Government set up a National Committee on Education to examine the existing policies and objectives in education and to make recommendations for curriculum change for the 1980's. In its deliberations, the committee paid attention to the fact that more than 90 per cent of Kenyan youth live in the rural areas. They therefore defined guidelines for curriculum change with this fact in mind.

Mathematics Education

One of the major questions that has been considered while redefining objectives for primary mathematics in Kenya has been: "What kind of mathematics does the Kenyan child need in order not to be socially deprived?" This question had to be answered for all categories of children discussed in the above paragraphs.

The first step was to replace foreign and irrelevant material with local and immediately meaningful curriculum. Secondly, it was recognized that the future success of mathematics education depends not on the number of units of mathematical knowledge a child has or is able to recite on request, but on how the mathematics he has learned has prepared him to live effectively. Considering this, the list of minimal mathematics competencies in Kenya is very short. The child who has had basic education in Kenya should:

1. Recognize size of numbers up to one hundred (e.g., a number of people in a meeting).
2. Count up to 100.
3. Read and write numbers up to 20 (for the urban child to know what 2 means when he sees this number written on a bus).
4. Handle money in a fairly complex way to avoid being robbed through short-changing.
5. Be able to tell time—although one can always ask; and in some situations other measures of time like the position of the sun, are not precise.
6. Be able to measure (distance, weight, etc.).
That is all that I would teach to a child for whom primary school education at whatever level is terminal. It is not very much, but it is what all students attending primary school, compulsory or non-compulsory, must master and be confident that they can handle before leaving school.

The first thing these children should be taught, and I mean taught, should be the basic operations of addition, subtraction, multiplication, and division of whole numbers. Where electronic gadgets like calculators or other means are available, these must be used with confidence and efficiency.

The second objective that I will want the child going through primary education to achieve is confidence. Some of these children will certainly want to study mathematics beyond primary school, and some of them will be working during their adult lives in situations demanding some level of mathematical sophistication. They therefore need both knowledge and confidence. There are a number of "confidences" that are needed:

a. Confidence with numbers. Too often one comes across a 12-year-old child who believes that although 8 and 7 make 15 today, they could be something else tomorrow. Such a child needs to trust numbers as orderly and well-behaved. One way to achieve this is to give a lot of work in number patterns that focus on the pattern and do not require computation except of a very simple sort. There are many examples that can be illustrated on the 100 board, the 100 strip etc. Number relationships is another big confidence area: number pairs that add up to 12, say; number pairs that multiply to make 24, say—all developed from practical situations.

b. Confidence in abstracting from concrete. The ability to think in abstract terms is probably one of the major causes of the inability to cope with mathematics, so that it is unlikely that much can be done—certainly in the early stages—to help with this. On the other hand, the "modeling" of concrete situations in symbols is something that can be done, provided that the concrete situation is not then removed and only the abstract symbols remain. It must be a continued process of concrete to symbols and then back to concrete so that they run side by side.
Structural apparatus, such as the Cuisenaire rods or the use of the cowries on the bao game, are all ideal for the number situation; but the older primary school children, at least in Kenya, tend to regard them as "kids' stuff." This is a challenge to the teacher.

c. **Confidence in problem solving.** This is perhaps the greatest contribution mathematics can make to any person who has gone through a formal school. This is not to suggest by any means that problem solving skills cannot be acquired from other disciplines. In analyzing the kind of mathematics that all students going through primary school should master, special attention should be paid to this area. The child should be encouraged to ask helpful questions. He should be taught to have patience while solving problems, and he should certainly be made aware that the world does not have neat answers to all problems. Furthermore, while success in problem solving is expected, there will be times of failure, and we must not be discouraged.

In an outline, the minimal mathematical competencies in Kenya include:

1. **Acquisition of basic understanding of number and numeration.**
   a. Count.
   b. Recognize, read, and write numbers, including fractions and decimals.
   c. Determine the order of two or more numbers by comparison.
   d. Develop the decimal numeration system through the idea of grouping of objects.

2. **Development of ability to perform the basic operations.**
   a. Develop proficiency in adding, subtracting, multiplying, and dividing.
   b. Use addition and multiplication facts with ease.
   c. Understand and make use of the interrelationship between:
      i. Addition and subtraction.
      ii. Addition and multiplication.
      iii. Multiplication and division.
      iv. Subtraction and division.
   d. Identify and use the relevant operation in everyday life.
3. Development of skills in measurement (i.e., acquire knowledge of various units available for measuring length, capacity, weight, time, and money).

4. (Introduction to) Development of spatial concepts and ability to use them.
   a. Identify from the environment objects having regular and irregular shapes.
   b. Categorize objects of regular shapes.
   c. Acquire knowledge of common properties about shape and size of objects in their environment.

5. (Introduction to) Acquisition of the techniques of collecting, representing, and interpreting data.

6. Development of desirable attitudes; making good use of leisure time, and utilization of senses appropriately through:
   a. Discovering and making patterns, magic squares, and other puzzles.
   b. Playing mathematical, and other, games.

   a. Identify and understand the problem.
   b. Analyze and choose the appropriate line of action.

Evaluation

Materials for the planned mathematics curriculum will be introduced in Kenya Primary Schools in 1981. Evaluation has been planned. Through this evaluation, it will be possible to measure student achievement and to recommend any remedial measures that will be necessary. In the past, the only evaluation that was available was the national examination (the certificate of Primary Examination) that is taken at the end of seven-year primary education. This has tended to be more of a selection tool, and not an evaluation instrument for mathematical competencies.
Preschool Instruction

The duration of preschool instruction is two years. The children are admitted from the age of four. At five years of age, this instruction is obligatory. There is no official curriculum. The teachers prepare the children, through activities at the pre-math level, for the elementary program.

Elementary Instruction

The duration of elementary instruction is six years. The children are enrolled at the age of 6. The program is identical in all of the schools, both private (of which there is only a small number) and public. A fixed plan of study, established by ministerial ordinance, specifies the course content. It is understood that, in the classes at the weakest level, the teacher has the right to adapt the program to the actual capabilities of the students—making sure, however, to cover the essential parts with them. Given that the children have to learn two foreign languages (German and French) and also that in the classes there is a large proportion of children of immigrants (which prompts the study of a third, native language), the teachers try to do their best to take into consideration the demands of the program. Experiences in the classes of the mentally handicapped children have shown that the utilization of non-verbal instructions, tildes (accent marks), multi-colored arrows, or micro-computers will be able to help the disadvantaged children to surmount their handicaps.

Post-Elementary Instruction

The post-elementary instruction consists of three parts:

- Secondary instruction, lasting seven years, prepares for university studies.

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6 Translated from the French by Ellen J. Tamburri and Robert J. Riehs. The original version appears in Appendix B.
Secondary technical instruction prepares the students for their average and low-level careers in administration and also for the professional life eventually, with future craftsmen following the appropriate vocational courses. (This instruction is only in the experimental stage; it is expected to replace the average high school, professional, and trade school instruction.) It is anticipated that this level of instruction will have a duration of 7 years.

Finally, the complementary instruction, lasting three years (the end of scholastic obligation).

1. **Secondary instruction.** The students must take an admissions test, identical in all schools throughout the country. After the 7th grade, the students have to choose between classical (Latin) and modern studies. Within each of the two courses of study, the student can choose (after the 9th grade) one of the following 6 tracks:

   - languages
   - mathematics
   - science
   - economics
   - art
   - music.

For each of these tracks, some specialized programs are provided. With the exception of the languages track, the mathematics instruction is approved by an official test (one for each track) at the end of the last year (13th grade). In certain schools with large student populations, the final exams are constructed through common agreement among the heads of the different classes.

2. **Secondary technical instruction.** The students must take an admissions test, but of the weakest level. In fact, a student who has failed in a branch of secondary instruction is admitted to the technical course if his grade multiplied by 1.5 is sufficient. We have said that the technical instruction is still in the experimental stage. More specifically, the program is in a three-year observation cycle. In each class, and by technical branch, the students are grouped by levels. Level "A" corresponds approximately to those of the "secondary instruction"
ability level, Level "D" roughly to those of "complementary instruction" ability level. By level, the programs are the same for all children, with some possible adjustments. Some courses of support organized by the school permit the passage from one level to the next. In the current observation cycle, the mathematics course takes 4 to 5 hours a week, out of a total of 30 hours.

3. Complementary instruction. This instruction takes in children who have not passed their admissions test for the other two orders of post-elementary instruction, and also some who have not taken the admissions test for varying reasons. There is again an official, prescribed program, but the teachers have a lot of liberty in the choice of material that they want to teach. Very often, it deals with repetitions of previous material from elementary school.

Special Instruction

In the country different classes function which take in mentally handicapped children or those with personality problems. However, we have at our disposal a resource center. Theoretically, the official program of elementary school is prescribed, but one bears in mind to a large extent the capabilities of the students. It has been shown with some experimental classes that the educational resources implemented make it possible, especially in math, with small classes, to bring the students up to a reasonable level, at least from the point of view of observable attainments.
MINIMAL MATHEMATICAL COMPETENCIES IN THE NETHERLANDS

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Introduction

In the Netherlands, there has been a tremendous increase in the social and economic demand for education, as a result of industrialization and of technological, social, and scientific advances.

Within a period of about twenty years, we have witnessed a change from secondary education for a minority to secondary education for a large majority, if not all.

The main objective of present-day educational policy is to prepare young people for active and critical participation in the society of tomorrow. In order to fulfill that purpose, education should serve to equip every individual to develop his cognitive, creative, and social faculties and capacities to their full potential.

Only recently, there has been a demand for minimal competence standards in mathematics. The discussions cover two topics:
- What are the basic skills in mathematics, needed for everyone?
- What level of skill and understanding should be demanded from students stopping mathematical study before they leave secondary school?

To understand the discussions of these topics, a short description of the Dutch system of education is needed.

Structure of Education in The Netherlands

Primary education has a duration of six years. In order to be eligible for admission to a primary school, a child must have reached the age of six before 1 October of the year in question. The mathematics program is merely arithmetic.

Secondary education includes the following types of schools:
- Pre-university schools offer a 6-year course in preparation for study at a university. The student has to pass a final examination in 7 subjects, for example Mathematics I (algebra, trigonometry, calculus, and probability) and Mathematics II (linear algebra and
The Current Structure of Education in The Netherlands*

**Primary Education (age 6-12)**
- LBO (4 years)
- MAVO (4 years)
- HAVO (5 years)
- VWO (6 years)

**Secondary Education**
- MBO (2-4 years)
- HBO (2-4 years)

**Post Secondary Education**
- University Education (5-6 years)

*Meaning of acronyms:

HBO = Hoger beroepsonderwijs [higher vocational education]
MBO = Middelbaar beroepsonderwijs [intermediate vocational education]
LBO = Lager beroepsonderwijs [elementary vocational education]
VWO = Voorbereidend wetenschappelijk onderwijs [pre-university education]
HAVO = Hoger algemeen voortgezet onderwijs [higher general secondary education]
MAVO = Middelbaar algemeen voorgezet onderwijs [intermediate general secondary education]
vector geometry. Mathematics is obliged during grades 1-4.

- General secondary schools at two levels.

HAVO: a general education in preparation for study at institutes of higher education, other than universities. The student has to pass a final examination in 6 subjects, for example mathematics (algebra, calculus, probability, and geometry). The first three years is the same for all students and includes mathematics.

MAVO: a general education with a four-year course. A MAVO certificate is a qualification for admission to grade 4 at a HAVO school and grade 1 at MBO schools. The final examination comprises 6 subjects. Mathematics is at a lower level than HAVO. Mathematics is required during 3 years.

- Elementary vocational education (LBO). It has been found that a large number of students in LBO schools do not form a homogeneous group. There are major differences in aptitude and capacities. That is why, since 1975, every student can do his 6 exam subjects on different levels (A, B, and C). The mathematics program is comparable with the MAVO program.

Minimal Competence in Basic Mathematical Skills

As the society is changing with regard to the growth of technology, the influence of computers and calculators, and the scientific advances, the discussions about the basic skills in mathematics education have increased. Is it not necessary to teach in the primary school the ability to handle pocket-calculators in addition to arithmetic?

The scientific council for government policy is asked to answer this and other questions and give its advice to the government.

Minimal Competence in Secondary Education

On the pre-university level (VWO), every student has to take a four-year course in mathematics. If he doesn't choose mathematics as an exam subject, he has to pass the fourth grade tests. The standards for these tests are being set by his own teacher. There is a rough syllabus determined by the government, but the standard differs from school to school. That is why there is a demand for an official determination of minimal standards.
The same problem occurs in the third grade of higher general secondary education (HAVO).

In elementary vocational education (LBO), the problem is to advise the student on which level (A, B, or C) he must do his final examination in mathematics. The teachers ask for standardized tests to distinguish the levels; but who can make these tests?

In The Netherlands, many questions concerning minimal competencies are raised. The discussions have just started. We hope that in the near future some answers can be found.

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MINIMAL MATHEMATICAL COMPETENCIES IN NEW ZEALAND

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About three or four years ago, when the Second IEA Mathematics Study was collecting information from participating countries, one of the requests was for a report on minimal competencies. At that time, there was a good deal of public discussion about standards of literacy and numeracy, and I outlined what I thought would be a likely course of events with respect to minimal competencies in New Zealand. Excerpts from that paper follow.

Minimal Mathematical Competencies in New Zealand (1977)

In New Zealand, criticism of weaknesses in basic mathematical skills in school-leavers is not new; but, in recent years, the introduction of new mathematics programs has provided a convenient peg for critics to hang their claims on. Those critics concerned about computational skills have had some justification for their criticism. The adoption of much wider goals for primary school mathematics, and for less able students in secondary schools, has inevitably meant that less time has been available for the practice of elementary skills. This factor has been exacerbated by teacher adjustment problems to new mathematical material and to new teaching methods in mathematics. The increasing demand for school-leavers with some mathematical ability, coupled with a fairly rapid shift of emphasis from an agriculture-based economy to industrialization, has sharpened criticism, as students of much lower intelligence than was formerly the case enter industry and commerce.

There is some indication that now that teachers have had time to adjust to the new programs, students are receiving a better balanced mathematics education. Increasingly, too, more account is being taken of individual differences in the classroom and more suitable resources being used for less able students.

Until recently, little serious attention has been paid to the idea of minimal competencies. The recently formed National Consultative Committee on Mathematics, comprised of representatives from schools, universities,
technical institutes, the Department of Education, and the general public, has turned its attention to this field and will shortly formulate a policy for minimal mathematical competencies for New Zealand Schools. The following is, therefore, not official policy and is based on preliminary discussion only. It is likely that minimal competence will be defined at the age for the end of compulsory schooling (i.e., 15 years) and will be for all students. The Committee on Secondary Education commented (1976) "There are major faults in our education system when students can enter and leave secondary schools as poorly equipped as some of them do," and recommended that "secondary schools ensure that every student becomes as competent as possible in basic language and number skills." If this recommendation is to be effectively implemented, there is clearly a need to identify the desired basic skills and to design instruments to determine whether students are reaching the required level of performance in them.

Minimal competencies in mathematics in New Zealand are likely to be drawn from:

1. **Computational skills.** The hand-held calculator is having a considerable impact on computation, but it is essential that students be able to perform relatively simple computations in addition, subtraction, multiplication, and division and to develop algorithms. Students will also need practice with calculators to enable them to determine when it is appropriate to use the calculator. Some appreciation of the concept of a ratio and facility with percentages and simple fractions is also needed.

2. **Application of mathematics to physical situations.** It is important that students be able to apply their mathematical skills to physical situations. There is no value in the student being able to subtract proficiently if he/she has no knowledge when confronted by a practical problem that subtraction is the operation required.

3. **Approximation and estimation.** Students should know some simple techniques for estimating quantity, length, distance, weight, and so on, and be able to carry out approximate, rapid calculations by first rounding off numbers. Students need to be able to use approximation to determine whether or not an answer is reasonable.

4. **Organization and interpretation of numerical data.** Much of the information presented to people as consumers, as citizens and voters, and
as workers is in the form of numbers and graphs. Students should be able to set up simple tables, charts, and graphs and be able to interpret them and draw valid conclusions from them.

5. **Measurement.** In many practical situations in which mathematics is used by the citizen or worker, measurement is involved. At a minimum, students should know how to measure length, distance, weight, area, volume, temperature, and angles, and know the units of measure for these.

6. **Qualitative understanding of and drawing inferences from functions and rates of change.** A general understanding of how one quantity can "depend" on another, together with a qualitative grasp of rates of change, are basic to making reasoned predictions and recognizing trends in many everyday situations.

7. **Spatial relationships.** Minimal geometrical competencies should include the following:
   a. An appreciation of the relationship between points, lines, and planes and some experience of tessellation and space-filling.
   b. Line and rotational symmetry and the properties of squares, rectangles, isosceles and equilateral triangles.

8. **Problem solving.** All students should acquire the skills needed to devise a procedure for solving a simple practical problem.

**Minimal Mathematical Competencies in New Zealand (1981)**

That early report on minimal competencies (of which the above is only a portion) was presented to the National Consultative Committee on Mathematics, a fairly representative group which advises the Department of Education on developments in mathematics education. There was considerable interest and discussion, but insufficient agreement to ensure consequent action.

Among the reasons for this were the following:

1. Several members of the group were in favor of some form of competency checks, but each had a different view of what constituted minimal competencies, of which levels testing should be administered at, and of who should have responsibility for the testing.
2. All members of the group were aware of the dangers of minimal competency testing (as outlined in the Report of the Study Group on Minimal Competencies in Mathematics, 1980) and several argued strongly that advantages would be greatly outweighed by disadvantages of the introduction of the notion of minimal competencies.

Since then, the debate about "standards" has, by and large, "fizzled out."

Almost all students in New Zealand enter for an external examination (in several subjects) at the end of their 5th form (grade 10) year. The examination is norm-referenced, and movements which approximate the minimal competency notion are related to pressures induced into the system by this (School Certificate) examination. In most parts of the country, local certificate courses in mathematics have been developed by mathematics teachers from regional groupings of schools to cater for the mathematically less able 20-30% of the grade 10 population. These courses include material which would be classified as minimal competencies, but most aim to go well beyond this. Recently, a spokesman for the New Zealand Employers' Federation has been publicly advocating what is essentially a criterion-referenced system for the award of School Certificate and receiving significant support.

This may have been encouraged by a trial scheme in one district for the award of School Certificate mathematics based on a series of criterion-referenced levels of attainment with testing for mastery.

While the minimal competencies movement has (at least so far) passed us by, the debate (such as it was in New Zealand) appears to have sensitized teachers to the need to place more emphasis on basic skills than they had been doing over the last few years. I think that provided the sensitizing process is translated into action in the classrooms, we can well do without the added stimulus of a formal testing program.

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MINIMAL MATHEMATICAL COMPETENCIES IN NORWAY

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The curricula in Norwegian schools today are generally frames, in which the teachers and students, in principle, to some degree, are free to choose. The tendency today is to define a core curriculum and, besides, propose some optimal items.

Situation

In Norway, we have a 9-year compulsory comprehensive school. It is divided into primary school (7-13 years old) and lower secondary school (13-16 years old).

Upper secondary education consists of one- or two-year basic courses, and one-year additional courses. It encompasses a general study course (normally 3 years) and vocational courses.

The curricula are set by national councils. Formerly, the curricula were strictly defined lists of items. According to the principles of our school reforms of the 1960's and 1970's, the national curricula should now define the aims and topics of a school subject in more general terms.

We have formal marks in lower and upper secondary schools, but not in primary school. In compulsory school, there are national tests in some subjects, among them mathematics. They are intended to have an advisory function, and schools are free to use or not use them.

At the end of compulsory school, and in the general course in the upper secondary school, some of the pupils are drawn out each year for a common national examination paper in mathematics (and in some other subjects). All students are marked by their subject teachers.

As there are no formal minimal standards, there are likewise no established passing criteria. Students are, in principle, marked according to a statistical normal distribution.

The intention is to have no strict ability grouping, but to have
mixed-ability groups. Milder forms of ability grouping are however practiced.

In principle, the upper secondary school is open to all youth, and teaching starts at the individual level each student has achieved at the completion of compulsory school. Due to a shortage of space, however, the marks from compulsory school will, in most parts of the country, limit the free choice of upper secondary education.

Results from upper secondary education are used to select students for further studies.

Tendencies

The 1974 national plan for compulsory school is a frame plan, but stricter definition of central themes and skills was announced to be coming in some subjects. Such guidelines are now prepared in the national council, but they have not yet been put into practice. The council warns that it is only pointing out central themes, and not defining a minimum standard.

In the upper secondary school, there is a similar tendency toward defining a core curriculum. It has been done for the basic course, and it may also come in other courses.

In compulsory school, the pressure to define a core curriculum or minimal standard comes mainly from non-socialistic political parties; and, in the upper secondary school, it also comes from professional organizations.

In theory, as mentioned above, all of our curricula are frames and don't set minimal standards. In practice, marks and examinations define more or less common standards in the lower and upper secondary schools. This conflict between the principle of frame curriculum and national examination papers is one reason why many teachers want directives which narrow the frames.

The professional tendency to want a core curriculum and a setting of minimal standards may also be a reaction to the "comprehensiveness" of students, and to the fact that schools have to adapt to a growing proportion of each age group. The alleged falling standards are also a concern of many established parents and politicians.
There is a strong professional fear of defining the curriculum too strictly. It may define essentials of the subject in too narrow terms, and thus exclude broader aims (such as mathematical awareness, problem-solving, and the use of mathematics in unstructured settings). It may also exclude students in an unwanted manner.

Even if we have no formal minimal competency goals, many teachers feel that such goals and programs are defined by curricula and examinations. An impact of too extensive curricula (when they are construed as minimal competency lists, and not as guiding frames) is that teaching styles tend to be deductive rather than inductive. However, in lower secondary education, some local curricula have been established which stress, for instance, practical uses of mathematics and project work more than in usual teaching styles.
Colleges and universities in the Philippines have often commented about the widely disparate mathematical knowledges and skills of entering freshmen. While differences in mental abilities and motivations of students can partly account for this disparity, also to blame, according to a number of mathematics educators, could be the teachers' poor perception of the relative importance of different learning-outcomes. This is evidenced in the overemphasis placed on some topics at the expense of others and in the faulty budgetting of time. It is important that mathematics teachers be clear about what mathematics can and should contribute to an individual. Hence, a project was undertaken to come up with an accepted list of minimal competencies for the average Philippine high school student. It was felt that the making of such a list would be a positive way to focus attention on the essential.

It was also felt that the list could be useful in many other ways—e.g., for competency-based teaching, for testing programs, for inservice training, and for curriculum development. In remote places, where textbooks or teaching guides are hard to come by (and such places do exist in the Philippines), the list could be the skeleton of a syllabus.

The study was an outgrowth of an exchange of ideas among a number of agencies. It was conceived by an ad hoc Committee for Mathematics under the National Capital Region, Ministry of Education and Culture, and was undertaken by the Science Education Center of the University of the Philippines, with the cooperation of the Mathematics Teachers Association of the Philippines.

Procedure

In a nutshell, the procedure decided upon was to make a comprehensive list of competencies, then to ask certain knowledgeable people to pass judgement on each item on the list using the following ratings:
A. Required of all students to the point of mastery, and therefore regarded as a basic skill.

B. Required of all students, but not necessarily to the point of mastery.

C. Optional.

D. Discard or move to a different year level.

In deciding what competencies to include in the list, one consideration was that each must satisfy either a social need or a subject matter need. The mathematics learned in school should be useful in various life-roles— as a student, as a consumer, and later on as a parent and a professional/vocational worker. This made necessary an expanded notion of mathematical skill beyond mere computational ability. Educators nowadays are agreed that computational ability by itself will contribute little to enabling one to meet the expectations of society. The satisfaction of a subject matter need was in recognition of the idea that the learning and development of mathematical competencies is a continuing process that extends through the different levels of schooling, and that in the elementary and secondary levels the learning is sequential in nature and cumulative in effect. That is, prior learning makes subsequent learning more understandable and easier.

One other consideration is that the prepared list should have the average high school student in mind; it should not be taken to be the complete mathematics curriculum that applies to every student.

Reactions to the list were secured from mathematics supervisors, mathematics department heads, and qualified high school mathematics teachers from the different regions—approximately 160 respondents in all.

Results So Far

The project was undertaken quite recently (mid 1979), so the results are not yet definitive. While the study did come up with a list of competencies that were judged as basic, the project team feels that the judgements were overambitious—there were TOO MANY skills that were considered minimal or basic. The partial list below gives the cognitive skills expected to be mastered during the tenth year of schooling (in the Philippine educational system, the fourth year of high school):
Computation skills. Give examples of functional relationships in tabular, equation, graphical, or verbal form which are linear functions; power functions; quadratic functions; exponential functions.

Give examples of functional relationships in tabular, equation, graphical, or verbal form which are direct variations; direct-square variations; inverse variations.

Define: - intercepts of a graph - parabola
- slope of a line - vertex of a parabola
- constant of variation - axis of symmetry
- domain of a function - sequence
- range of a function

Manipulative skills. Manipulate numbers in power-of-ten or scientific notation—i.e., obtain equivalent expressions, perform operations.

Given a linear equation, a line graph, or two points of a line graph, determine slope, y-intercept.

Given a line graph or two points on a line graph, determine the equation.

Given the equation of a linear or quadratic function, draw the graph.

Given a set of values of the variables in a variation, determine the constant of variation.

Application skills. Interpret time-rate graphs.

Looking Ahead

At this point, the project team has yet to make a decision as to the direction towards which efforts at identifying minimal competencies will go. Since the resulting list is a collective view of representative supervisors, department heads, and teachers of mathematics, should it be proposed for nationwide use as a new basis for mathematics learning and teaching? Or, should there be a revalidation of the list from other groups before further work is undertaken?

One happy outcome of the project so far is the heightened concern of mathematics educators for mathematics as a body of competencies, rather than merely as a body of facts, and the strengthening of continued lines of communication and cooperation among different groups.
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MINIMAL MATHEMATICAL COMPETENCIES IN SCOTLAND

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Introduction

In the wake of changes which have taken place in organization and syllabi in secondary schools since the 1960's, a great interest has developed in the competence of the students who are the products of these changes.

Criticisms are often heard from employers, social agencies, etc., particularly in regard to the basic skills of numeracy and literacy. These complaints of "falling standards" have been expressed at regular intervals for many years, but they have recently become more widespread and vociferous.

For example, the U.K. Prime Minister in late 1976 called for "a national debate on educational standards," since he had been concerned "to hear complaints from industry that new recruits from the schools do not have the basic skills to do the job that is required." He added "there is concern about the standards of numeracy of school leavers. Is there not

7 The opinions expressed in this article are those of the author alone and should not be regarded as necessarily representing the view of the Scottish Council for Research in Education. For further information on the national curriculum in Scotland, contact:

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a case for a professional review of the mathematics needed by industry at different levels?"

In a similar vein, a recent report (1977) of the CBI (Confederation of British Industry) in Wales included a survey of the selection test scores obtained by all of the young people who had applied for craft-apprenticeships with a large industrial firm over an eight year period (1966-1974). The survey showed a noticeable drop in levels of attainment. In 1966, 30% of applicants had an arithmetical competence too low, in terms of the criteria set by the employers, to justify further consideration. By 1974, this figure had reached 86%.

While the employers recognized that the sector of the school-leaving population from which applicants were drawn had changed due to better opportunities for sixth form and further education, nevertheless they did not accept that this was the sole reason for the decline in standards, and believed that given more and perhaps better teaching, higher levels of attainment could be achieved. I quote from their comments:

"It is apparent that in addition to the inability to manipulate numbers at an elementary level, there is a general lack of understanding of the basic arithmetical functions—to the point of not knowing when to add, subtract, multiply or divide.

"In the case of the addition of fractions, a common practice was to add together all the numerators and then add together all the denominators, e.g.

\[ \frac{3}{4} + \frac{3}{8} + \frac{1}{4} = \frac{7}{16} \]

"When multiplying simple fractions it was common practice to cross-multiply, e.g.

\[ \frac{3}{4} \times \frac{2}{9} = \frac{8}{27} \]

"Place value and the use of the decimal point were little understood and often ignored altogether.

"The most serious shortcomings are to be found in the understanding of percentages, decimals, square roots and simple equations."

The employers did admit, however, that only those of the lowest ability failed to add, divide, subtract, and multiply correctly, provided they made a correct decision as to the required operation.

Similar comments are made frequently, all relating to the extent to
which today's students are equipped to deal with the mathematical tasks required to function effectively in an increasingly technological society.

The Educational Viewpoint

Two recent reports in Scotland, commissioned by the Secretary of State, have addressed the problem of competence in basic skills. One report (the Munn Report) was concerned with the desirable curriculum for the 14-16 year old (16 being the age at which the student may terminate secondary education if he wishes). The second report (the Dunning Report) was concerned with the assessment of school education at age 16.

The Munn Committee (of which I was a member) recommended that a basic core of subject including mathematics be provided for all students in secondary education. The emphasis on basic skills was mainly in regard to the societal needs of the nation. The report recognized the importance of ensuring that students become skilled in quantitative thinking and statistical reasoning so as to enable them to cope with a society which depends increasingly on statistics, computers, etc.

The Munn Report restricted itself to broad general statements of goals, rather than to producing the specific detailed elements of syllabi which would achieve these goals. It shelved the issue of deciding what the basic social needs were, and left it to others to carry out this task, suggesting that mathematics teachers, in conjunction with industrialists and others, should identify these needs.

The report also pointed out that there was a need to identify those students who were having specific difficulties in mathematics in particular, so that appropriate remedial action could be taken as early as possible in secondary schooling.

While the report supported the need to maintain standards in respect to the basic skills of numeracy and literacy and agreed that there was a need for more emphasis on basic skills than currently existed, nevertheless it did not favor creating a separate test of basic skills for 16-year-olds, since almost certainly this would lead to bad instructional practices designed to fit the test rather than the desired educational objectives.

The Munn Committee stated strongly their opinion that assessment
practices in education must be geared to the objectives of the curriculum. The national assessment system must not be allowed to control the curriculum.

Moreover, the Committee felt strongly that such tests would serve little educational purpose at age 16, since failing the test would simply record inadequacy at a point in the student's career when the school could do nothing to remedy his/her deficiencies. Hence the report suggested that any test of basic skills should be designed for use at around age 14, the aim being to identify failure to reach adequate standards at that point in time. Such identification could lead to a remedial program being put into action with a view to improving national standards in basic skills.

The report also considered the problem of differentiation within subject areas and recommended that there should be up to three different but overlapping syllabi. The Dunning Report made a similar recommendation in regard to assessment practices.

In mathematics, for example, there would be three such courses:
- a credit course for the top 20 to 25% of an age group,
- a general course to be taken by approximately the next 70% of the age group, and
- a foundation course to be taken by the remaining 10 to 20% of the age group.

At age 16, there would be three corresponding national examinations in mathematics; and entrants could obtain passes at either a credit level, general level, or foundation level.

Both reports asked for substantial research expenditures to be made available to identify the problems of implementation and to enable feasibility studies to be carried out to assist in the production of course guidelines and syllabi for the various levels and subjects.

These particular recommendations have been accepted by the Government; and much development work is now in progress looking at the problems of implementing the proposals, and the feasibility, in particular, of constructing foundation level courses and foundation level assessments. It is hoped to start the first foundation level examinations in the mid 1980's.
The Need for Consensus

One of the main problems is how we (society) arrive at a consensus as to the required mathematical tasks. The expression "mathematical competence" has many meanings according to the context in which it is used. It may, for many teachers and educationists, imply a command of the skills necessary for survival as a citizen in society. For employers, it is generally the skills required to function adequately in a particular business or skilled trade. In terms of the "accountability of the public education system," it may mean demands for minimal levels of performance at particular grades of schooling.

Even assuming agreement could be reached on the broad meaning of the term, there is no clear consensus at the present time as to which specific basic skills should be possessed by the "competent" student.

An example of a list of survival skills for citizenship postulated by the "Institute of Mathematics" in the U.K. is given below.

1. The addition and subtraction of whole numbers and decimals of up to two decimal digits; the multiplication and division of whole numbers less than 100; the multiplication and division of decimals by whole numbers less than 10; ratios of integers less than 10, their conversion to and from percentages and decimals.
2. The common units of measurement; money; volumes and areas of circles, squares, rectangles, cubes, and right-angled blocks.
3. An understanding of the size of numbers; the use of approximations with one significant figure; averages within the preceding limitations and an idea of statistical spread; statistical presentation; interpretation of simple graphs and diagrams.

The first group requires the acquisition of simple mechanical skills, and some might argue that the standard was set too low (e.g., in multiplication and division). The second requires mechanical skills combined with the knowledge of simple formulae (e.g., for the area of a circle). The third group is much more open-ended. For example, an understanding of simple graphs is advocated as a basic tool for late-twentieth-century adult life, to prevent people from being misled by advertisements or political propaganda.
The Institute suggested that the possession of these minimum skills should be monitored by test sets of items, the passing standard being set at 95% and to be such that able children would be able to pass the test at around nine, while average children would pass at about thirteen.

Some evidence from employers as to the basic mathematical topics required for entry to a wide variety of industries is exemplified in a survey of employers' needs carried out by Sheffield Region Centre for Science and Technology.

The survey was carried out between November, 1976, and March, 1977, in an 8% sample of all firms in South Yorkshire and North East Derbyshire. 186 firms were contacted, of which 113 had taken on 1427 school leavers during 1976. Each firm was visited and asked
  - about its selection procedures, as far as mathematics is concerned,
  - to consider the relative importance of 28 mathematics topics for each young employee.

Eight mathematics topics were found to be so important in all types of firm and employment that they could be regarded as an essential "core" of mathematics education for boys and girls. They were:
  - Four arithmetic operations with whole numbers
  - Mental arithmetic
  - Length
  - Metrication (transfer from imperial to metric units and converse)
  - Decimals
  - Weight
  - Use of tables (reading a value from any two sets of information, e.g. bus and train timetables, tire pressures, etc.)
  - Area

Apart from these topics, other topics regarded as important varied according to the sex of the young person and the type of job.

Relatively unimportant topics included:
  - Use of slide rule
  - Simple trigonometry
  - Simple algebra
Statistics
- Technical drawings and their interpretation
- Simple circle work
- Simple triangles.

While the "Use of slide rule" rating may be no surprise, the low importance of "simple triangles," "simple algebra," and "statistics" may be of interest to mathematics teachers considering their own syllabi.

Estimation of errors, use of a calculator, and interpretation of graphs also rated low for boys.

For girls, many mathematics topics were rated unimportant. Only percentages, statistics, estimations of errors in sets of figures, money, and calculators needed to be added to the core as "important," particularly so for those in commercial and sales departments of firms.

For craft apprentices, additional important topics included simple angles and directions, speed, simple fractions, and tolerances. The use of calculators was regarded as relatively unimportant.

These findings obviously only relate to the industrial area covered by the survey, and it is obviously essential to carry out similar surveys in other parts of the country and different types of industries in order to obtain a more general consensus. Indeed, the Association of British Chambers of Commerce has recently published a report "Education and Employment" (September, 1979) asking the Government to set standards of basic numeracy for school leavers and recommending that the standards required be determined by "market research" among employers.

The basic core of items specified in this survey are all met by young people while still at Primary School (up to age 12).

While there is some overlap in content between the two statements listed, there is also considerable disagreement.

Clearly, there exist fairly wide differences in conception and philosophy among, for example, teachers and employers; and there is an obvious need for the development of a greater understanding among both groups, concerning the extent of and the reasons underlying these differences in philosophy.
The Need for Empirical Data

In order to be able to make meaningful judgements about the standards to be expected of young people passing through the educational system, it is essential to have empirical evidence available as to the current levels of performance in schools.

Since 1974, the writer has been involved in a number of surveys of mathematics and arithmetic achievement both at regional and national level in Scotland at a variety of age levels (8, 10, 11, 14, 16, and 18). This work has involved the preparation of test materials designed to provide sub-scores or item scores relating to specific aspects of content or performance, rather than to overall total scores.

Data have been and will be analyzed both by content area and by level of ability to produce profiles of performance in basic skills at various ability levels (e.g., 90th, 75th, 50th, 25th, and 10th percentile levels), which have direct relevance for the teacher in the classroom.

The 16-year-old data will provide quantitative data on school leavers on a national basis. The 14-year-old data will enable us to identify the percentage of 14-year-olds currently failing to reach particular levels of performance. Both sets of data will provide information on which schools could base "remedial" programs with a view to eliminating or at least reducing such deficiencies over the last two years of compulsory schooling.

The secondary school data will also be collated with similar work in the primary schools at ages 8 and 11 to provide a picture of mathematical competence across the whole range of schooling.

By relating the current work to evidence available from earlier survey data, it will be possible to examine "standards of performance" over time, and to provide additional evidence for the continuing debate on whether or not educational standards are falling. It should be noted that the recent work of the Council in this respect has shown little evidence of any real falling off in the standards of performance of young people in the educational system.

Other relevant work in progress in Scotland at the present time includes, for example:

1. The development of item banks in mathematics, based on detailed
specifications of content area and behavioral objectives, at both primary and secondary school levels.

2. The five-year project "Education for the Industrial Society" (launched by the Consultative Committee on the Curriculum and the Scottish Committee for Schools/Industry Liaison in 1977) which is investigating the relationships between education and industry. A series of study groups, including one on Numeracy, have been set up to examine the contributions made by particular curriculum areas to the topic under investigation.

Much of the work described in this paper is still in process of completion and therefore detailed results and information are not yet available. However, when they are, the data will provide educationists with an opportunity to make realistic appraisals of what is within the competence of young students at the present time. The data will also provide a basis for protagonists to make value judgements as to whether or not existing standards are satisfactory, and should assist in clarifying some of the main issues of the continuing debate on appropriate basic skills.

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MINIMAL MATHEMATICAL COMPETENCIES IN SWEDEN

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In Sweden, the efforts to give all pupils minimal mathematical competencies have not been directed towards constructing testing programs or identifying minimal levels of mathematical competencies. The direction has instead been a movement towards limiting the teaching matter and towards attempting to make the teaching for low performers more concrete and experimental.

In the early seventies, most teachers in Sweden agreed that the new comprehensive school curriculum, Lgr 69, in its syllabus for mathematics, had a course content that was too big, and that the teaching of several items was characterized by difficult terminology and abstract arguments. To help the various schools and teachers solve these problems, the Board (SÖ, 1973) published a manual entitled "Basic Skills in Mathematics," specifying a central nucleus of knowledge and skills which practically all pupils passing through comprehensive school should have acquired by the end of each level (grade 3, grade 6, and grade 9). This manual restricted the goals for the lowest (15%) performers in mathematics and also indicated different ways to give these pupils better basic skills. The manual is not to be regarded as an official minimum course.

In spite of great exertions from further training consultants and other people, the manual did not have the impact on course content and teaching styles that was intended. On the other hand, it had some influence on the textbooks in mathematics, and special books with basic skills for low performers were developed.

Research by Wiggo Kilborn and Bengt Johansson (The PUMP project) has been of great importance to the debate on minimal mathematical competencies in Sweden. By means of tests, classroom interviews, and observations, the two investigators gave clear evidence of defective basic skills among a great number of pupils in Swedish schools. They also evinced that the construction of textbooks in mathematics was not very logical and had many deficiencies. As a result of this, most textbooks for the junior and intermediate levels of the comprehensive school were reconstructed.
During all of the 1970's, the possibilities to give auxiliary teaching extended. In 1976, the Commission on Working Conditions in Swedish Schools (SIA) handed over to the government its main report. In the report, the focus is on the individual pupil, especially on the situation of low performers and slow learners. The report resulted in the government's decision to set aside a general financial resource to be used for urgent local needs as defined by boards or school conferences. It also resulted in a decision to review the curriculum.

In the proposition of a new curriculum for the comprehensive school (Prop 1978/79:180), the demand of the politicians that school shall give all pupils a foundation of basic skills (especially in the Swedish language and mathematics) is very clear. Measures for this are above all:

- Increased financial resources for training in basic skills from the school year 1980/81.
- A commission to the Board to perform diagnostic tasks and, as soon as possible, place these at the disposal of the schools.
- The emphasis on a foundation of basic skills in the core curriculum of 1980 (Lgr 80) shall leave its mark on school work from the school year 1980/81.
- The new curriculum (Lgr 80), in its entirety, is to be used from the school year 1982/83. The syllabus in mathematics lays special stress on problem solving and a foundation in basic skills.

A tentative version of diagnostic tasks was published at the beginning of 1981. The intention with the tasks is to give teachers an instrument to identify pupils with difficulties in mathematical basic skills, as well as to point out probable reasons for these difficulties and to give indications of ways to overcome the problems. The diagnostic tasks can also be used as one basis for the distribution of the general financial resource.

The syllabus in the new curriculum (Lgr 80) points out ways to give all pupils a foundation in basic skills. It is constructed with central items (which are necessary for all pupils to master) and items to be desired for as many pupils as possible.
The following is an extract from the syllabus of mathematics in the curriculum of 1980 for the Swedish comprehensive compulsory school:

Goals

The teaching of mathematics must take as its starting point the experiences and needs of the pupils and must prepare them for the role of adult citizens. Primarily, therefore, the pupils must acquire a good capacity for solving the type of mathematical problems commonly occurring in everyday life. This means that the pupils must derive the following from the instruction they receive:

- a firm command of numerical calculation, with and without aids,
- proficiency in mental arithmetic and rough estimates,
- a knowledge above all of percentage calculation, practical geometry, units and unit changes, and descriptive statistics.

Mathematics teaching must be sufficiently concrete for every pupil to be able to comprehend the concepts and understand the use of mathematics in practical situations. Teaching must be arranged in such a way that the pupils discover the necessity of being able to use mathematics and can experience the satisfaction of being able to apply the skills they have learned. Mathematics teaching must utilize the pupils' curiosity and imagination and develop their logical thinking. In this way, mathematics will become a tool for the understanding of reality and a source of benefit and gratification.

Main Teaching Items

The structure of mathematics is such that a new teaching item is generally based on a prior knowledge of other elements. This must be carefully borne in mind when planning the individual pupil's instruction. A pupil must not start a new teaching item without a sufficient grounding in preceding items. Guidelines in this respect are available in the following division of the subject content of the main teaching items.

Junior level. Teaching items which all pupils are to study and acquire a basic knowledge of and proficiency in during the junior level grades.

Junior and intermediate levels. Items with which most pupils should become acquainted during the junior level grades and which all pupils are
to study and acquire a basic knowledge of and proficiency in during the intermediate level grades:

Intermediate and senior levels. Items with which most pupils should become acquainted during the intermediate level grades and which all pupils are to study and acquire a basic knowledge of and proficiency in during the senior level grades.

Senior level. Items with which the pupils should become acquainted during the senior level grades. These items are particularly important to pupils intending to go on to lines of upper secondary school or comparable forms of subsequent education involving a great deal of mathematics.

Problem Solving

The fundamental goal of mathematics is for all pupils to acquire a good capacity for solving the mathematical problems they encounter at home and in the community at large. In order to be able to solve such problems, one usually has to meet the following requirements:

- One must understand the problem and have a method of solving it.
- One must be able to cope with the necessary numerical calculations.
- One must be able to analyze and evaluate the result and draw conclusions from it.

All these stages in the solution of problems must be covered by teaching. Teaching must include practice in discussing and adopting standpoints concerning both the nature of the problem and the plausibility of the solution, and it must not be confined to the one-sided practice of predetermined calculations. Talking mathematics is an important part of teaching.

Problem solving must be included in all main teaching items. Generous scope must be given to practical problems of everyday life.

Problems ought primarily to be selected from the pupils' experience and interests and from the immediate environment, but they should also be capable of shedding light on social and global problems. Calculations must be closely adapted to the skills of each individual pupil.

Basic Arithmetic

Arithmetic teaching must start with and be rooted in everyday problems
and situations of concrete relevance to the pupils. In order to be able to solve mathematical problems of an everyday nature, the pupils need, among other things, to have attained a good standard of proficiency in various methods of calculation. Revision and individual diagnosis are particularly essential in arithmetic, because the various modes of calculation are interdependent. The practice of skills must continue individually until each pupil has mastered the subject matter concerned.

Junior level. The concept of numbers is built up by means of collaborative exercises and comparisons of numbers. Natural numbers up to 1000 are dealt with in connection with everyday problems leading to addition and subtraction. The concepts of multiplication and division are raised, but the treatment of algorithms should be left until the pupils have acquired a firm command of addition and subtraction. The latter in turn demand firmly grounded knowledge of addition and subtraction tables up to 18. The multiplication table is now learned with one factor not exceeding five.

Junior and intermediate levels. Natural numbers are expanded to 10,000 and decimal numbers, primarily to two decimal places, are dealt with in connection with arithmetic. The multiplication and division tables are practiced, because they constitute important prior knowledge both for mental arithmetic and rough calculation and also for the learning of the corresponding algorithms. Previously learned algorithms are revised, and multiplication is learned algorithmically with one factor limited to units. Practical applications of the algorithms thus learned are practiced, mainly in the form of mental arithmetic and rough estimates applied to relevant everyday problems.

Intermediate and senior levels. Arithmetic primarily comprises natural numbers up to a million and decimal figures with up to three decimal places. Algorithms learned previously are revised, and the multiplication algorithm is expanded to include two factors running into several figures. The aim is for pupils to acquire confidence in dividing by at least a single-digit denominator. Mental arithmetic and rough calculation are practiced on everyday problems with reference to the algorithms which have been learned. Generous scope is given to practical applications of the four modes of calculation, with and without aids, for
example in connection with wages, costs, unit prices, and suchlike.

Senior level. Revision and practical implementation of the various modes of calculation, using calculation aids where appropriate. Mental arithmetic and rough calculation.

Real Numbers

Most people have occasion above all to use natural numbers and decimals. Negative numbers occur in practical life to a limited extent, and in the junior grades they are only dealt with in response to pupils' questions and with reference to other teaching subjects. The same applies to squares, square roots, and numbers to the power of ten somewhat higher up in the elementary school grades. Numbers such as $\sqrt{2}$ and $\frac{3}{7}$ can, for example, be rounded off to decimals with the aid of a pocket calculator. It should also be noted that subtractions of negative numbers, like multiplications of two negative numbers, hardly ever occur in practical situations.

Percentages

The concept of percentages occurs in a number of practical situations (e.g., economic deliberations concerning such matters as prices, discounts, interest, installments and loans and, in the teaching of social and natural sciences, with reference to population growth, statistical data, and the concentration of chemical solutions). The pupils should solve problems connected with these and similar situations. In doing so, they should acquire a good standard of proficiency in performing calculations based on percentages and should learn to employ such calculations as a basis for their decisions in everyday situations.

This analytical aspect of problem solving must be given at least the same amount of scope as the calculations, which can often be performed using calculating aids.

Measurement and Units

Units form an important part of the data employed in most problem solving situations in the home, at work, and during leisure. Often one needs to be able to choose the right unit for a calculation and to have mastered the commonest changes of units. In practical situations, the unit
is seldom given and must instead be worked out in connection with measurement. Great attention must therefore be paid in teaching to measurement using different units and instruments and to the coordination of this work with the teaching of other subjects (above all home economics, handicraft, and science subjects).

**Junior level.** Length, mass, and volume are measured using the units and instruments most commonly occurring in the home. Telling the time is practiced, together with specifications of time in hours and minutes. The use and denominations of the commonest notes and coins are dealt with in connection with practical situations.

**Junior and intermediate levels.** Units of time are expanded from seconds to years. Writing the date is also dealt with. Simple changes of units and mental arithmetic are practiced in connection with problem solving with reference to cooking, the calculation of traveling times, prices, etc.

**Intermediate and senior levels.** The commonest prefixes are dealt with, as are accuracy of measurement and measuring techniques. The solution of everyday problems, for example in connection with geometry, speed, local times, trade and currency, to consolidate the commonest units.

**Senior level.** Calculation of error in connection with measurement and rounding off into whole numbers. Units and changes of units in technical and scientific contexts.

**Geometry.**

Geometry teaching must help pupils to organize their comprehension of their surroundings. Teaching should start with the home and the immediate environment and tie in with other subjects, especially geography, pictorial studies, and handicraft. Teaching must also give pupils the capacity to interpret and apply geometric formulas and models.

The capacity for thinking in geometrical models is closely connected with the pupils' development. Most junior and intermediate level pupils have a very limited ability to comprehend and make theoretical use of such concepts as area and volume. The same applies to formal, formula-oriented geometry teaching at the senior level.
Geometry teaching ought therefore to be concrete and practically oriented, especially at junior and intermediate levels. Furthermore, the arithmetic used in geometrical calculations should be adapted in such a way that the basic geometrical ideas will not be obscured by calculation difficulties.

Algebra and the Theory of Functions

This element is of minor importance in everyday life, but all pupils should have some acquaintance with the material. Careful individualization, based on pupils' preferences and capacities, is called for.

Descriptive Statistics and Theory of Probability

A great deal of the information we receive about the community around us is expressed in the form of tables, charts, and probabilities. Instruction is aimed primarily at teaching the pupils to interpret and evaluate this information. It should be integrated with instruction in general subjects and based on real data from the local government, society, and the world at large. Work on this item is particularly suitable for group exercises, and is recommended for projects.

Data Theory

All pupils should be informed about the use of computers in society and the rapid developments taking place in this sector. It is particularly important for pupils to realize that the computer is a technical aid controlled by man.

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In Thailand, no programs in minimal competencies have yet been set up. In fact, the idea has not even been contemplated.
MINIMAL MATHEMATICAL COMPETENCIES IN THE UNITED STATES

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The minimum competency movement could be the major school reform of the twentieth century; it has been called that. More cautious observers deem it "a potential tool for fundamental school reform." Critics call it the "great American educational fad of the 1970's." But whatever it turns out to be, the movement has already brought education into the headlines more than any topic since the passage of the Elementary and Secondary Education Act of 1965. ESEA focused attention on disadvantaged pupils.

Similarly, the competency movement is an attempt to give all students a chance to succeed in school and life. The students most affected are disadvantaged youth and those unmotivated to learn. Legislators and policy makers, in responding to public concern about education, have taken action or are studying how to guarantee that students will emerge from schools equipped with minimum proficiency in basic skills. At this writing, thirty-nine states have established some form of minimal competency testing. All fifty states have experienced legislative or state education department activity (study, planning, discussing, drafting, implementation) in the area of setting standards for schools or students. Federal minimal competency legislation has been proposed, but administration officials have opposed it on the grounds that education is essentially a state and local function. Passage appears unlikely. However, Congress has included a provision in the renewed Elementary and Secondary Education Act that enables the Department of Education to award grants to local or state school systems which desire to develop educational proficiency standards.

Some of the actions emanating from minimal competency legislation require that students be able to read, write, and compute (generally at the eighth grade level) before receiving a high school diploma. Others require that students become proficient in solving everyday problems that adults face. To illustrate: instead of asking students to add and subtract columns of figures correctly, students would be asked to balance a checkbook— an everyday "survival" or "life" skill.
The rationale behind legislation on required competencies seems to be this: requiring states or local school districts to set minimum standards of performance and to test standard proficiency in meeting those standards will result in more competent students.

The push to minimal competency testing began in the early 1970's, when a rash of studies appeared reporting that American students were performing considerably below previous levels. Employers began to complain that a great number of job applicants lacked reading, writing, and mathematical skills for entry level positions. Citizens charged that schools were failing in their task and needed to be held accountable. In 1976, George Gallup found that 65% of the public supported the idea of required competencies as prerequisites for high school graduation. Other polls have found the percentage to be as high as 80%. A poll of school board presidents, by the National School Boards Association, found that 76% believed that every student should be required to pass a minimum competency test as a prerequisite for graduation. Public criticism has been fueled by well-publicized law suits in which ill-prepared students have sought (so far unsuccessfully) redress from the schools. The Peter Doe case (San Francisco, 1972) was the first malpractice case against a school district ever pursued in this country. Doe charged that his fifth grade reading level at high school graduation was below the competency level necessary for holding a job. Although Doe lost his case, some educators view minimal competency testing as a fail-safe mechanism to prevent such suits in the future. Others see minimal competency testing as providing the necessary documentation to substantiate claims of educational malpractice. Some legal experts believe that Doe would have won his case if minimal competency testing had been in operation during his school years. Public interest in minimal competency testing has been supplemented by research on competencies, behavioral objectives, and the measurement of educational outcomes (all of which have become common in public education). This combination of forces has made minimal competency testing politically feasible.

Oregon, the first state to become involved in the competency movement, offers an example of fundamental school reform, attempted through the use of a competency-based system. The State Board of Education requires all
schools to set goals for elementary and secondary education. Failure to do so could result in the loss of state aid. The intent is to ensure that schools will make available to students the opportunity to learn to function effectively in six life roles. They are: individual, learner, producer, citizen, consumer, and family member. The Oregon Board requires all districts to assess how well students can read, write, and compute. It also asks them to devise policies in new graduation requirements (started in 1978). Districts must identify the minimal competencies needed for a diploma.

Other states, such as Florida and New York, have introduced new tests which help determine whether a student graduates with a diploma or with a certificate of attendance. Florida introduced the functional literacy test in October, 1977. A passing score of 70% is required both in communication and in mathematics. In the first administration of the test, one-third to one-half of the students in several districts failed mathematics. Officials estimated that five to seven percent of the students would still be unable to pass it after being given three chances.

Students in New York must make a minimum score of 65% on five new basic competency tests administered state-wide in reading, mathematics, practical sciences and health, civics, citizenship, and writing skills. Otherwise, they cannot receive a diploma (started in 1980).

Other states are taking different routes to guarantee minimal performance in basic skills. "There is almost universal acceptance of the need for the concept, but only minimal agreement on how best to attack the problem," says Chris Pipho, Associate Director of the Department of Research and Information for the Education Commission of the States. He maintains it would take daily bulletins to keep up with the frenzied activity characterizing the movement.

At the local level, the Denver Public Schools developed and started to use minimal competency tests in 1960, to assure that students are proficient in reading, language, spelling, and mathematics. Only about one and one-half percent of the students in the district have failed to receive a diploma because they cannot pass the tests. By comparison, the failure rate for Denver students who have taken the tests only once or
twice is twenty to thirty-five percent. Students have eight chances to pass.

Other districts have established new policies which do not allow students to be promoted to a higher grade until necessary skills or competencies are mastered. Both California and Florida allow students to leave school early by taking a form of proficiency test. Students who pass receive the equivalent of a high school diploma and are permitted to by-pass other graduation requirements. The flurry of activity at the state and local level was the reason for legislatures to propose that national standards be set by minimal competencies. These proposals received little support from educators and federal officials, despite the insistence of one legislator that "parents want schools to get back to the basics, and this is one way to do it."

Administrators responding to minimal competency testing view the movement in various ways. Some call it "another swing of the pendulum." Others welcome the movement and consider it an impetus for examination, renewal, and reform. But to many administrators, the competency movement represents overreaction to reports of declining test scores by legislators, parents, and the public.

The demand that schools prove that students are competent before they graduate more often than not takes the form of a pencil and paper minimal competency test. While, on the surface, requiring a test for graduation appears simple, the potential is great that it can turn into an educational or legal quagmire. Students denied a diploma may sue, charging that the graduation test is culturally biased or that they had not been given sufficient preparation for this new requirement; or the test content could be so elementary that it creates political problems, as is the case in New York, where the proposed test was termed "laughable and deceptive." Competency tests can be hazardous if testing content does not reflect the curriculum of the school district. In this instance, students are penalized for something they could not control, and schools are judged on areas they did not purport to teach.

All of these cautions were discussed in a report from the National Academy of Education to HEW Secretary, Joseph Califano, Jr. Released at a National Conference on Achievement Testing and Basic Skills sponsored by
HEW in the spring of 1978, the report warned that "any setting of state-wide minimum competency standards for awarding the high school diploma--however understandable the public clamor which has produced the current movement and expectation--is basically unworkable, exceeds the present measurement arts of the teaching profession, and will create more social problems than it can conceivably solve." The report recommended, instead, that schools use diagnostic testing in the early grades and that more time be spent in the classroom on basic skills.

That report and the conference's strong decision that the federal government not get into the area of a national testing program, even a voluntary one, probably will be looked on as the first brakes on the competency testing fervor. But although it may be slowed, competency testing is an accountability technique that will remain, in some form, a part of public education in the immediate future.

The currently popular slogan "back to basics" has become a rallying cry of many who perceive a need for certain changes in education. The result is a trend that has gained considerable momentum and has initiated demands for programs and evaluations which emphasize narrowly defined skills.

Mathematics educators find themselves under considerable pressure from members of boards of education, legislators, and citizens' groups who are demanding instructional programs which will guarantee acquisition of computational skills. Leaders in mathematics education have experienced a need for clarifying what are the basic skills needed by students who hope to participate successfully in adult society. The narrow definition of basic skills which equates mathematical competencies with computational ability has evolved as a result of several forces:

- Declining scores on standardized achievement tests and college entrance exams.
- Reactions to the results of the National Assessment of Educational Progress.
- Rising costs of education and increasing demands for accountability.
- Shifting emphasis in mathematics education from curriculum content to instructional methods and alternatives.
- Increased awareness of the need to provide remedial and compensatory programs.
- The wide-spread publicity given to each of the above by the media.

This wide-spread publicity, in particular, generated a call for action from governmental agencies, educational organizations, and community groups. In response to these calls, the National Institute of Education adopted the area of basic skills as a major priority. This resulted in a Conference on Basic Mathematical Skills and Learning, held in Euclid, Ohio, in October, 1975.

The National Council of Supervisors of Mathematics (NCSM), during its 1976 annual meeting in Atlanta, Georgia, met in a special session to discuss the Euclid Conference report. More than one hundred members participating in that session expressed the need for a unified position on basic mathematical skills—a position which would enable them to provide more effective leadership in their respective school systems, to give adequate rationale and direction in their task of implementing basic mathematics programs, and to appropriately expand the definition of basic skills. Hence, by an overwhelming majority, they mandated the NCSM to establish a task force to formulate a position on basic mathematical skills. In the summer of 1976, that task force was convened. The NCSM Position Paper on Basic Mathematical Skills was produced as a result of its efforts.

There are many reasons why basic skills must include more than computation. The present technological society requires daily use of such skills as estimating, problem solving, interpreting data, organizing data, measuring, predicting, and applying mathematics to everyday situations. The changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators all demand a redefining of the priorities for basic mathematical skills. In recognition of the inadequacy of computation alone, NCSM went on record as providing both a general list of basic mathematical skills and a clarification of the need for such an expanded definition of basic skills.

Any list of basic skills must include computation. However, the role
of computational skills in mathematics must be seen in the light of the contributions they make to one's ability to use mathematics in everyday living. In isolation, computational skills contribute little to one's ability to participate in mainstream society. Combined effectively with other skill areas, they provide the learner with the basic mathematical ability needed by adults.

Basic Skill Areas

The NCSM views basic mathematical skills as falling under ten vital areas. The ten skill areas are interrelated, and many overlap with each other and with other disciplines. All are basic to pupils' development of the ability to reason effectively in varied situations.

This list is presented with the conviction that mathematics education must not emphasize computational skills to the neglect of other critical areas of mathematics. The ten components of basic mathematical skills are listed below, but the order of their listing should not be interpreted as indicating either a priority of importance or a sequence for teaching and learning. Furthermore, as society changes, our ideas about which skills are basic also change. For example, today our students should learn to measure in both customary and metric systems; but, in the future, the significance of the customary system will be mostly historical. There will also be increasing emphasis on when and how to use hand-held calculators and other electronic devices in mathematics.

Problem solving. Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students should also be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions, and they must be willing to subject these conclusions to scrutiny.
Applying mathematics to everyday situations. The use of mathematics is interrelated with all computational activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in light of the initial situation.

Alertness to the reasonableness of results. Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential.

Estimation and approximation. Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand.

Appropriate computational skills. Students should gain facility with addition, subtraction, multiplication, and division with whole numbers and decimals. Today, it must be recognized that long, complicated computations will usually be done with a calculator. Knowledge of single-digit number facts is essential, and mental arithmetic is a valuable skill. Moreover, there are everyday situations which demand recognition of, and simple computation with, common fractions.

Because consumers continually deal with many situations that involve percentage, the ability to recognize and use percents should be developed and maintained.

Geometry. Students should learn the geometric concepts they will need to function effectively in the 3-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties which relate to measurement and problem-solving skills. They must also be able to recognize similarities and differences among objects.

Measurement. As a minimum skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles
and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems, using the appropriate tools.

Tables, charts, and graphs. Students should know how to read and draw conclusions from simple tables, maps, charts, and graphs. They should be able to condense numerical information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Using mathematics to predict. Students should learn how elementary notions of probability are used to determine the likelihood of future events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election forecasts.

Computer literacy. It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching/learning, financial transactions, and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations.

Even though some school boards and state legislatures are starting to mandate mastery of minimum essential skills in reading and mathematics as a requirement for high school graduation, certain caveats are in order. In the process, they should consider the potential pitfalls of doing this without an appropriate definition of "basic skills." If the mathematics requirements are set inordinately high, then a significant number of students may not be able to graduate. On the other hand, if the mathematics requirements are set too low, and mathematical skills are too narrowly defined, the result could be a sterile mathematics program concentrating exclusively on learning of low level mathematical skills. The NCSM Position Paper neither recommends nor condemns minimal competencies for high school graduation. However, the ten components of basic skills in the Position Paper can serve as guidelines for state and
local school systems that are considering the establishment of minimum essential graduation requirements.

Developing Basic Skills

One individual difference among students is still the way of learning. In offering opportunities to learn the basic skills, the objectives must be provided to meet these varying learning styles. The present "back to basics" movement, on the other hand, may lead to an emphasis on drill and practice as a way to learn.

Certainly drill and practice is a viable option, but it is only one of many possible ways to bring about learning and to create interest and motivation in students. Learning centers, contracts, tutorial sessions, individual and small group projects, games, simulations, and community based activities are some of the other options that can provide opportunities to learn basic skills. Furthermore, to help students fully understand basic mathematical concepts, teachers should utilize the full range of activities and materials available, including objects the students can actually handle.

The learning of basic mathematical skills is a continuing process which extends through all of the years a student is in school. In particular, a tendency to emphasize computation while neglecting the other nine skill areas at the elementary level must be avoided.

Conclusion

Competency activity in the states and local school districts has been described as a constantly changing landscape. Under such conditions, it is impossible to know for certain what its long-term effects on education will be. Since competency-based programs have emerged for diverse reasons and with differing approaches, it is likely that the movement will have some positive effects and some negative effects. Because it has been such a short time since minimal competency testing first emerged, and with implementation details scheduled all the way up to 1985, it may be years before any reasonable assessment of impact can be made. This fact, coupled with uneven implementation and/or financing, court cases, and other confounding variables, may well prohibit any final conclusions about minimal competency testing.
Any assessment of the impact of such testing programs must be tempered with the knowledge that new fads and terms tend to rise quickly in education, only to be short-lived. It must be noted, however, that even if minimal competency testing turns out to be a fad, the political forces that gave rise to that fad may require attention for quite some time. Moreover, because minimal competency testing has attained the status of law in several states, this movement may have more impact on education than most fads.

In the final analysis, it is quite likely that the implementation of minimal competency programs in mathematics will have both positive and negative effects. Thus far, neither the worst fears of critics nor the highest hopes of advocates have been realized.
Part III

Synthesis of National Reports
While the minimal competency "movement" has had far-reaching effects in many countries around the world, minimal competency "programs" are few and far between. If a minimal competency program is characterized by
- the identification of skills to be achieved by all students in a particular class, with
- the satisfactory ("minimal") level of mastery clearly identified for each skill, and
- testing to tell if a student has achieved the "minimal" level,
then four countries report large-scale minimal competency programs.

Minimal Competency Programs Reported

Some small, experimental, competency-based programs for mathematical education have been reported (e.g., in individual districts in Chile and in New Zealand). However, only 4 of the 25 countries report any large-scale minimal competency programs, either already implemented or planned for the near future.

Belgium. Especially in the French-speaking part, as described by G. Henry, "Lists of educational objectives (for grades 1-6) have been elaborated by educational researchers and disseminated by the Ministry of Education.... Efforts have been made too for surveying student and school achievement by using these lists." However, this approach is new, is not really institutionalized, and may, in practice, turn out to be very different from what is generally thought of as a "minimal competency program."

Israel. In 1962, according to A. Lewy, the Ministry of Education initiated a minimal competency program for low achievers (primarily children of immigrants), called the "Parenthesis Program." However, the program never received formal approval, and it was effectively dropped because of the poor response by both teachers and educational psychologists.

Kenya. Vital mathematical skills (particularly for rural, non-math-oriented students) have been identified, and, according to S. Eshiwani, "Materials for the planned mathematics curriculum will be introduced in
Kenya Primary Schools in 1981. Evaluation has been planned...to measure student achievement and to recommend any remedial measures that will be necessary." Since it is not yet in full operation, it is, of course, difficult to determine if this curriculum will actually be a minimal competency program.

United States. Unlike Belgium and Kenya (where their curriculum changes for the 1980's may or may not turn out to be minimal competency programs), in some states of the United States, curriculum changes of the 1970's were definitely minimal competency programs. In fact, according to A. Tobin, "Thirty-nine states have established some form of minimal competency testing." On the other hand, those states which have gone so far as to make passing test scores a requirement for receiving a diploma have been facing legal challenges (e.g., Debra P. v. Turlington, Florida, 1979). These have been initiated mostly by disgruntled parents, who thought that a minimal competency program would result in their children achieving minimal standards, not in their being prevented from graduating by the imposition of those standards. The rising enthusiasm over minimal competency testing in the United States is clearly leveling off.

Problems Associated with Minimal Competency Programs

In countries other than the four mentioned above, curricular changes have had emphases other than testing for the achievement of "minimal" standards. In some cases, the national curriculum planners just haven't given much thought to minimal competencies in mathematics; this is the situation described by C. Sutabutr in Thailand. In other countries, the curriculum developers have been deliberately avoiding minimal competency programs for a variety of reasons; that is the situation described by T. Fidje in Norway.

Listed below are some of the more commonly expressed difficulties associated with minimal competency endeavors. Some are concerns expressed in those countries which have, thus far, avoided minimal competency programs; others are problems actually experienced in countries which have implemented such programs.

1. Minimal competency programs do not necessarily result in increased
learning. A. Levy points out that one of the criticisms of the "Parenthesis Program" in Israel was that "the reduction of the scope of the skills and concept units to be mastered is not likely to improve the achievement level of the 'deprived children.'"

2. Minimal competency programs do take time and effort to develop and implement—time and effort which could otherwise be devoted to improving the quality of instruction. The concern of teachers in Ireland, for example, "has been mainly with courses for the weaker students, rather than with minimal competence as such." In Sweden, as outlined by L. Skoogh, "The efforts...have not been directed towards constructing testing programs or identifying minimal levels of mathematical competencies. The direction has instead been a movement towards limiting the teaching matter and towards attempting to make the teaching for low performers more concrete and experimental."

3. Minimal competencies are difficult to identify. Certainly the necessary mathematical competencies vary significantly from country to country. However, even within each country, "minimal" competencies may vary, depending on sex, geographical location, and job expectations. The significance of such differences was emphasized in a recent survey of employers in Scotland by the Sheffield Region Centre for Science and Technology (1978). "One of the main problems," to use the words of G. Pollock, "is how we (society) arrive at a consensus as to the required mathematical tasks."

4. "Definitions of minimal standards tend to result in inflexibility and rigidity of the teaching system." Those words of W. Dürfler (Austria) are particularly critical in this age of rapid technological development. Flexibility and adaptation are essential. As A. Tobin writes,

"As society changes, our ideas about which skills are basic also change. For example, today our students should learn to measure in both customary and metric systems; but, in the future, the significance of the customary system will be mostly historical. There will also be increasing emphasis on when and how to use hand-held calculators and other electronic devices in mathematics."

5. Theoretically "minimal" standards tend to collapse under the strain of implementation. In other words, to reduce the number of "failures" to a politically acceptable number, the standards may be
lowered to the point of uselessness. Such was the case in New York (U.S.A.), "where the proposed test was termed 'laughable and deceptive.'"

6. In nearly every country where some form of minimal competency movement is growing, there is a serious concern that the increasing emphasis on "minimal" competencies may result in those minimums becoming educational goals—the maximum expectations of parents, students, and even teachers. J. Ellis talks about this concern as it exists in Australia:

"Schools in Australia are in the main administered by State Education Departments. Thus, any minimal competency assessment would currently be the province of each State Education Department. The absence of such assessment may reflect the disquiet of mathematics educators over any such setting of 'minimum' standards and the possible abuse in both community reaction and lowering of mathematical goals."

7. Focusing on easily behavioralized items may result in the omission of "necessary" mathematical learning. In Norway, for example,

"There is a strong professional fear of defining the curriculum too strictly. It may define essentials of the subject in too narrow terms, and thus exclude broader aims (such as mathematical awareness, problem solving, and the use of mathematics in unstructured settings)."

**Essential Features of Effective Minimal Competency Programs**

If a country, a district, a school, or an individual teacher wishes to implement a "minimal competency program," then they must not limit their preparation to:

- a long list of "minimal competencies,

  carefully written as behavioral objectives,

  and accompanied by an extensive array of criterion-referenced evaluation instruments.

While the above list may identify the essential characteristics of a minimal competency program, they are not the essential features of a minimal competency program that works.

From the 25 national reports submitted for this review, it appears that, to whatever extent a minimal competency program can be a successful educational tool, it can only be successful if:

1. It includes instructional materials, thus providing a basis for teaching, not just evaluating. In one of the minimal competency programs...
in Chile, F. Oteiza and P. Zanocco report the generation of "a complete modular system for the basic education of adults."

2. It includes remediation. Identifying the "incompetencies" of students does not make those students more competent. G. Pollock reports on the recommendations of the Munn Committee (1977) in Scotland:

"The Committee felt strongly that such tests would serve little educational purpose at age 16, since failing the test would simply record inadequacy at a point in the student's career when the school could do nothing to remedy his/her deficiencies. Hence the report suggested that any test of basic skills should be designed for use at around age 14, the aim being to identify failure to reach adequate standards at that point in time. Such identification could lead to a remedial program being put into action with a view to improving national standards in basic skills."

Unfortunately, B. Andelfinger (FRG) adds on this point that increases in remediation programs are sometimes "avoided because of the high costs, the expenditure of time and energy, and the prejudices of many teachers."

3. It allows for individual differences (thus perhaps contradicting the generally accepted interpretation of minimal competency programs).

A. Lewy reports from Israel,

"Knowledge accumulated in this country about the distribution of cognitive entry behaviors of learners in various socio-economic groups makes it possible to set different standards for the distribution of achievement scores, in keeping with the specific socio-economic composition of a particular class. While the mastery of the required basic concepts may be considered a positive achievement for certain classes, it would not be viewed as such for other classes.

In Scotland, the Munn Report (1977) recommended "three different but overlapping syllabi," with "three corresponding national examinations in mathematics."

4. It avoids curricular rigidity. F. Oteiza and P. Zanocco (Chile) suggest that the "processes" for identifying necessary skills (rather than competencies themselves) be made a permanent part of the educational system and that they "play a part in any program implementation."

5. It avoids the formality of behavioral objectives. J. Paasonen reports from Finland,

"There has been some discussion as to whether the basic aims should be stated in behavioral terms of the form 'The student
should be able to....' Lists of skills of this type tend to be quite forbidding to the average teacher-reader, however, so the method of presenting the basic aims by means of the material associated with them was chosen."

Emerging Curricula and the Minimal Competency "Movement"

During the last decade, even in countries which have avoided fullfledged minimal competency programs, there has nevertheless been an apparent movement away from the very abstract "math for math's sake," and a movement toward minimal or "necessary" skills. In other words, there has emerged an increasing emphasis on mathematical topics which are demonstrably useful to the students. These topics have been grouped into three categories:

- Basic skills. The "back to basics" trend has been felt in several countries, including Finland. Recent curriculum modification there, as described by J. Paasonen, was based on the principles that, "Certain aims that give a basis for further education are set up for the whole age group," and that "the basic material leads to the attainment of these aims and is taught to the entire age group." At a slightly different level, the same trend is described by Y. Ozawa in the trimming down of the Japanese high school math program to the skills "necessary" for university study.

- Survival or life skills. The National Committee on Education in Kenya, for example, recently made recommendations for curriculum change in the 1980's. Their recommendations were based on the recognition that the future success of mathematics education depends not on the number of units of mathematical knowledge a child has or is able to recite on request, but on how the mathematics he has learned has prepared him to live effectively.

- Job-preparation skills. In England, as pointed out by D. Dawson, the Shell Centre for Mathematical Education has been doing extensive research on the mathematical needs of school leavers entering employment. The Education for the Industrial Society Project has been performing similar investigations in Scotland. These efforts are typical of investigations being made in many countries, including the United States.
There is a tremendous overlap among the categories listed above, since the main emphasis is the same—giving the student what he/she needs. In several countries, quite explicit attention has been given to the importance of all three categories of necessary skills. In Bangladesh, for example, as so clearly described by S. Sharfuddin,

"In preparing our mathematics curriculum, we attached importance to those 'basic skills' needed to go on to the next educational levels... We feel that mathematics is required for two purposes—first, for use in the every day life of the average citizen ('survival skills'), and secondly, as a tool for the scientist, engineer, or technician ('job preparation skills')."

In Germany, a single term—"mathematische Allgemeinbildung"—is used to include all three.

**Impact of the Minimal Competency Movement**

This emerging emphasis on "minimal" or "necessary" mathematical skills has had an impact on curricular change on six continents. Some of the more significant aspects of that impact follow:

1. It is fostering an awareness among instructors of the necessity for review of more basic skills or competencies. The situation in New Zealand is described by R. Garden:

   "While the minimal competencies movement has (at least so far) passed us by, the debate (such as it was in New Zealand) appears to have sensitized teachers to the need to place more emphasis on basic skills than they had been doing over the last few years. I think that provided the sensitizing process is translated into action in the classroom, we can well do without the added stimulus of a formal testing program."

Similar minimal competency thinking (or interest in necessary skills) is showing up among individual mathematics instructors in other countries (e.g., Australia, Brazil, Kenya, and the United States, just to mention a few).

2. There is a trend toward reducing the teaching matter for less able students. In Ireland, for example, E. Oldham reports that attempts are being made "to formulate alternative, less abstract courses." In some other countries, of course, the efforts are less ambitious and are illustrated by the example of Belgium:

   "In mathematics, the school programs intend to provide the best preparation for the...math-oriented students.... For all other students, the school programs are developed by deleting contents"
from the math-oriented programs. There is, therefore, no real adjustment of the programs to the needs of these students."

3. "There is an increasing feeling of teachers and society for the gap between expectations and reality." B. Andelfinger's words apply not only to the Federal Republic of Germany, but also to Finland (where follow-up testing to a basic skills program indicated that only 60% of the students had mastered the material to a "satisfactory" extent), to Bangladesh (where the passing mark on public mathematics examinations has been set at 40%), and, in the United States, to Florida (where one-third to one-half of the students in several districts failed the "minimum competency" test in mathematics).

4. Minimal competency investigations are supporting an expanded concept of necessary skills. Quite unexpectedly, it seems, inquiries concerning necessary mathematical skills are not only identifying those skills which are being taught but don't have to be; they are also revealing those topics which are not being emphasized but should be. J. Fonacier and L. Pascua report their surprise (and the surprise of the project team) over the wide range of skills considered essential by a survey in the Philippines:

"The project was undertaken quite recently (mid 1979), so the results are not yet definitive. While the study did come up with a list of competencies that were judged as basic,...there were TOO MANY skills that were considered minimal or basic."

A similar study, reported by B. Andelfinger in Germany, produced a catalogue of competencies that "was called too big." In the United States, the National Council of Supervisors of Mathematics published a position paper on basic mathematical skills, in which they identified ten vital areas:

- Problem solving
- Applying mathematics to everyday situations
- Alertness to the reasonableness of results
- Estimation and approximation
- Appropriate computational skills
- Geometry
- Measurement
- Tables, charts, and graphs
- Using mathematics to predict
- Computer literacy
Finally, from Brazil, M. Dantas reports the efforts of individual teachers to determine what geometrical skills are essential, and to perhaps bring the study of geometry back into the junior high school curriculum in Brazil.

Summary of Trends Observed

Needless to say, the minimal competency movement does not exist in isolation. It functions in conjunction with other trends, many of which may be of interest to curriculum planners.

1. Necessity of mathematics. Mathematics education (and education in general) is recognized as becoming increasingly necessary for the average citizen. This is, of course, a result of industrialization and (to use the words of H. Schuring) a vast array of "technological, social, and scientific advances," including "the influence of computers and calculators."

2. More are being educated. In Finland, as J. Paasonen reports, "It is felt that a central aim of teaching mathematics in school is to assure a sufficient level of mathematical literacy for all students, without early discrimination between those who have barely mastered minimal competencies necessary for survival and those eligible for secondary education."

In Kenya, in the early 1970's, a tremendous effort was made "to expand the primary school system so that as many children as possible could obtain basic education." As G. Eshiwani reports, "Primary Education was made free, and, as a result, the enrollment rates rose rapidly to 85 per cent by 1980."

It is perhaps worth mentioning that the responsibility to provide appropriate education for as many students as possible includes those with mental or emotional problems. This is particularly apparent in the program in Luxembourg, as reported by Robert Dieschbourg. Finally, and more generally, as H. Schuring reports from The Netherlands, "Within a period of about twenty years, we have witnessed a change from secondary education for a minority to secondary education for a large majority, if not all."

3. Poor math Skills. There is world-wide distress over poor development of math skills. A majority of the countries submitting reports for this review indicate more than mild displeasure with the level of achievement in mathematics among primary and secondary students.
4. *New math* is being rejected. While not being given all of the blame, the "new math" is nevertheless considered a major factor in a decrease in "useful learning." The displeasure over the effects of new math is expressed most strongly in the reports from England, Finland, Germany, Ireland, Kenya, New Zealand, Sweden, and the United States. From England, for example, D. Dawson reports that,

"There are pressures to establish 'basic skills,' 'life skills,' and 'job-preparation skills.' These pressures are rooted in a widespread belief that standards have dropped or that skills in mathematics have become less relevant to the needs of the outside world—particularly as a consequence of the introduction of the 'new math' in the 1960's (although one should add that similar complaints have been heard for at least the last hundred years)."

From Germany, B. Andelfinger reports that,

"Disappointment over the inefficiency of the 'new math' and low numerical, algebraic, and geometric competencies of many pupils in FRG put in motion the discussion about educational goals and mathematical competencies. This discussion has not focused on 'minimal' mathematical competencies, but on 'useful,' 'necessary,' 'worthy' competencies. A mixture of basic skills, life skills, and job preparation skills under the heading of 'mathematische Allgemeinbildung' is being established."

R. Garden adds that,

"In New Zealand, criticism of weaknesses in basic mathematical skills in school-leavers is not new; but, in recent years, the introduction of new mathematics programs has provided a convenient peg for critics to hang their claims on. Those critics concerned about computational skills have had some justification for their criticism. The adoption of much wider goals for primary school mathematics, and for less able students in secondary schools, has inevitably meant that less time has been available for the practice of elementary skills."

5. Core curricula. While there has not been an overwhelming enthusiasm over "minimal" competencies, there has been a great deal of interest in constructing a "Core Curriculum." From Canada (British Columbia), for example, A. Taylor reports,

"To identify the skills and knowledge which are generally accepted as fundamental or basic to all students, the Ministry of Education published a document in 1977 titled 'Guide to the Core Curriculum.'"

From the other side of the Atlantic ocean, T. Fidje reports a similar development:
"The curricula in Norwegian schools today are generally frames, in which the teachers and students...to some degree, are free to choose. The tendency today is to define a core curriculum and, besides, propose some optimal items."

6. Textbook influence. There is an increasing awareness of the difficulty in achieving fundamental change in the educational system. In fact, it appears that there are some places where textbooks have a greater impact on course content and teaching methods than the "prescribed curriculum" does. "In Belgium," for example, as reported by Mrs. Brusselmans and Dr. Henry,

"The concept of 'curriculum' is a very recent concept. A large majority of school or ministry administrators and of teachers (probably more than 90%) ignores this concept. The practice of these educators is therefore mainly guided by school programs or by existing textbooks."

Of course, some countries actively recognize the importance of textbook content in relationship to curricular change. As S. Sharfuddin points out,

"In countries like ours, 'change in curriculum' and 'change in textbooks' are synonymous. The curriculum is framed by the National Curriculum Committee, and textbooks are produced by the Bangladesh School Textbook Board. A single textbook is followed in all schools of Bangladesh."

7. Primary/secondary transition. In an age of rapid curricular change, careful communication between primary and secondary teachers is increasingly being recognized as a prerequisite for effective mathematical education. As Mrs. Brusselmans reports,

"Many educational administrators are concerned with the gulf which exists between primary-school and secondary school education. In the Flemish part of Belgium, 'minimal competencies' for the elementary school are reviewed in the first year of secondary education. This focus is explicitly mentioned in the curriculum. Because of the introduction in 1978 of the so-called 'new math' within the curriculum of primary education, the problem of the transition in this field will be reviewed in the near future."

Similar concerns are expressed by J. Paasonen (Finland) and J. Ellis (Australia).

8. Nation-wide testing. Nation-wide testing, of some sort, exists in almost half of the countries contributing to this review; however, their methods and purposes vary tremendously

9. International sharing of information. International studies can have a significant impact on educational trends. The significance of the
international sharing of information was particularly identified in the reports from Belgium, Bangladesh, Brazil, Germany, Hong Kong, and New Zealand. To cite just one example, S. Sharfuddin reports,

"In all developing countries like Bangladesh, the impetus for reform in the contents of mathematical education comes from international movements which disseminate new points of view through conferences and publications."

Implications for Future Activity

In most of the countries contributing reports to this review, great concern over the quality of mathematics education is being shown. More so than ever before (perhaps because of the growing importance of mathematics in our increasingly technological society), parents, political leaders, and mathematical educators throughout the world are actively searching for a way to significantly improve their local educational programs—to significantly increase the level of useful mathematical learning.

More time devoted to mathematics instruction. The solution is almost too obvious to mention. However, since millions of people on six continents have been avoiding the obvious solution for the last several years, it may indeed be worth mentioning. MORE TIME NEEDS TO BE SPENT ON MATHEMATICS IN BOTH PRIMARY AND SECONDARY SCHOOLS, since there is more essential mathematics to be learned than there was 100 years (or even 20 years) ago. More time in the mathematics classroom and more effort on the part of the student are required, if today's students are to learn the mathematical skills "necessary" for both survival and employment in tomorrow's technological society.

It is interesting to note that the two countries reporting the least dissatisfaction with the level of mathematics achievement are also the two countries which have already followed, to some extent at least, the above suggestion. M. Brimer reports that, although "achieved in part at the expense of other subjects in the curriculum," in Hong Kong,

"There is little evidence that there has been any fall in national standards of achievement that would have prompted a concern for minimal standards. There have been regular surveys over the past twelve years which bear this out, despite high immigration and 100% enrollment at the first level."

Also, W. Dörfler reports from Austria that, although parents complain that
students are overworked at school, the results are significant:

"...There are no essential critics of the mathematical education. Empirical studies carried out at the University of Klagenfurt (1981) show, for example, that more than 90% of the students graduating from the Gymnasium and from upper secondary vocational schools judge their mathematical education to be good or at least to be sufficient for their needs. A hint in the same direction is that only a minority of these students have to undergo further education in mathematics at their working place."

With mathematics education no longer limited to the elite, to the college-bound, or to the math-oriented student, it could reasonably be expected that average or below-average students would need more time and effort to master the same amount of content (much less more content). This is the feeling expressed by industrial firms in Wales, as reported by G. Pollock:

"The employers recognized that the sector of the school-leaving population from which applicants were drawn had changed due to better opportunities for sixth form and further education... They believed that given more and perhaps better teaching, higher levels of attainment could be achieved."

It is, of course, particularly important to learn from the Japanese experience, that if the goal is to increase the level of math learning for all students, then simply raising the standards (increasing content and making students work harder), without increasing the time, can create a worse problem. As Y. Ozawa explains,

"In Japan, one of the social problems for the last several years has been that the curricula at primary, secondary, and high school levels might be a burden to many pupils. Many students have been forced to drop out of school."

What else can be done? In addition to spending more time on mathematics education, steps can be taken to make better use of that time. Some of these suggestions, by the way, can be implemented immediately (within hours of reading this document) by individual mathematics instructors.

1. Let the "new math" die a RAPID death. In an era when there are many more skills that need to be developed in the mathematics classroom, adding many extra ones simply makes a bad situation worse.
2. Within a framework of rapid curricular change, better communication between primary and secondary teachers must take place. One method of encouraging such communication and coordination is reported from Queensland, Australia:

"A series of workshops has been prepared...to guide secondary schools and their feeder primary schools in identifying basic/common areas of mathematics regarded as important in both sectors."

3. The authors of the Chilean report suggest that educational programs should be coordinated with programs for social and community development."

4. Large-scale testing programs should be approached cautiously, whether or not they are part of a minimal competency program. The 1977 Task Force report from Toronto, Ontario, suggests that anonymous sampling strategies be considered. It suggests further, as reported by H. Russell, that every-pupil surveys, "which could lead to comparisons among teachers and schools," be avoided. Concern over potential side effects of large-scale testing exists in several countries. From England, for example, D. Dawson reports,

"Public examinations, while having certain beneficial effects in setting standards, carry attendant risks (notably reinforcing tendencies towards a narrowly didactical approach with emphasis on the repetitive practice of isolated skills divorced from application and illustration)."

5. However much attention is paid to testing or setting minimal standards, substantially more effort should be devoted to improving the actual teaching/learning process. Others might follow the example being set in Australia:

"The Australian Association of Mathematics Teachers has recently been granted federal funds to complete the Australian Mathematics Education Project, which is aimed at improving the teaching of mathematics...rather than setting standards for all, minimal or otherwise."

6. Curriculum developers and individual teachers must gear instructional content and material to student abilities. As S. Sharfuddin points out,

"In Bangladesh...we feel that to be appropriate, a curriculum must be within the comprehension of pupils. Efforts to teach too much can end in practically nothing being learned."
The recent change in the Japanese high school curriculum is an example of how this suggestion can be implemented.

7. Whenever possible (and this will depend greatly on the country and the school's financial situation), utilize electronic calculators, not only to develop familiarity with the instruments, but also to cut down calculation time in upper level courses. L. Skoogh provides us with an appropriate example from the Swedish mathematics syllabus:

"Geometry teaching ought therefore to be concrete and practically oriented, especially at junior and intermediate levels. Furthermore, the arithmetic used in geometrical calculations should be adapted in such a way that the basic geometrical ideas will not be obscured by calculation difficulties."

By using a calculator in a geometry course, the work would progress more quickly, and the student would more clearly see the "flow" of logic involved in solving the various problems.

8. For curriculum developers, for textbook writers, and for individual teachers in the classroom, a careful balance must be maintained between reviewing (or learning) "basic" computational skills and fostering other more general abilities, such as those identified by J. Paasonen: "concept formation, deduction, understanding of types of ordering, estimation, problem solving, and creativity." As A. Tobin writes in the U. S. report,

"Any list of basic skills must include computation. However, the role of computational skills in mathematics must be seen in the light of the contributions they make to one's ability to use mathematics in everyday living. In isolation, computational skills contribute little to one's ability to participate in mainstream society. Combined effectively with other skill areas, they provide the learner with the basic mathematical ability needed by adults."

9. The international sharing of information concerning emerging curricula should be continued in the future. Long after the minimal competency movement has passed by, curriculum developers around the world will still be able to learn from the experiences of others.

Robert J. Riehs
Part IV

Selected References
Approximately 150 references are included in the list which follows. An attempt was made to select the items which are most informative, most recently published, and most readily available. (Well, two out of three isn't bad.) To facilitate further research into particular programs, there is a national notation following each item:

- Australia - AUS
- Austria - ÖST
- Belgium - BEL
- Brazil - BRA
- Canada - CAN
- Chile - CH
- England - ENG
- Finland - FIN
- France - FR
- Federal Republic of Germany - FRG
- Hong Kong - HK
- Hungary - HUN
- India - IND
- Republic of Ireland - IRE
- Israel - ISR
- Italy - IT
- Japan - JAP
- Kenya - KEN
- Grand Duchy of Luxembourg - LUX
- The Netherlands - NET
- New Zealand - NZ
- Nigeria - NIC
- Norway - NOR
- Philippines - PHL
- South Africa - SA
- Scotland - SCO
- Swaziland - SWA
- Sweden - SWE
- Switzerland - SWI
- United States of America - USA

In most cases, the national notations listed above indicate the country of publication. However, in a few special cases, to aid researchers, the notation is used to indicate the country of focus in the publication itself. In such special cases, the place of publication is clearly indicated in the annotation.
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Part V

Appendices
APPENDIX A: PROGRAMAS DE DESTREZAS MÍNIMAS EN MATEMÁTICA EN CHILE

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¿Qué se ha hecho en cuanto a la determinación de destrezas matemáticas mínimas en Chile?, ¿Cuáles son los programas que utilizan este enfoque?, ¿Qué impacto han tenido en la enseñanza de la matemática?. El presente informe resume las conclusiones de una breve exploración del tema, indica cuáles fueron estos programas, caracteriza la situación actual de estos programas y propone algunas acciones que serían recomendables a la luz de la información recabada.

El sistema educativo chileno comprende ocho años de educación obligatoria (basic education) y cuatro de educación media (secondary education). Puede ser calificado de centralizado, en el sentido que posee planes y programas nacionales y que la coordinación está a cargo del Ministerio de Educación. Esta característica es importante al analizar el impacto de una innovación en el sistema, en cuanto a iniciativa, del organismo central de coordinación. En la actualidad el sistema experimenta una profunda transformación administrativo técnica que afecta, específicamente, su carácter centralizado. La mencionada reforma tiende a dejar en manos del poder local (municipal) la coordinación de los subsistemas de educación. La reforma implica, así mismo la descentralización de las decisiones curriculares y de evaluación de los aprendizajes. Es en este sistema y en relación con este contexto que hay que leer las conclusiones acerca de programas de competencias matemáticas mínimas, que a continuación se enuncian.

Programas de Destrezas Mínimas en Chile

Como resultado de una consulta realizada a través de Líderes educacionales y de los centros en los que se imparte educación matemática en Chile, se puede concluir:

1. En los últimos años han existido un conjunto reducido de experiencias tendientes a definir destrezas matemáticas mínimos; la consulta arrojó un total de cuatro programas de esta naturaleza (ver Cuadro N°1).
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<th>Autores</th>
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<th>Fechas</th>
<th>Nivel Educativo</th>
<th>Población Estudiantil</th>
<th>Criterio enfatizado</th>
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CUADRO Nº1 (continuación).
2. Estas experiencias tuvieron un alcance limitado logrando un impacto más técnico que masivo. Así por ejemplo, han influido en los programas de formación de profesores o en la tecnología para la producción de material de enseñanza.

3. Las experiencias detectadas se pueden calificar de cuidadosas, en cuanto a las técnicas utilizadas sea en la determinación de necesidades, sea en la producción o en la validación de los medios instruccionales a que dieron origen.

4. Las cuatro experiencias detectadas no se limitaron a determinar estándares mínimos, sino que dieron origen a programas de enseñanza apoyados con material de instrucción, en uno de los cursos se generó un sistema modular completo para la educación básica de adultos.

5. La iniciativa fue, principalmente, de investigadores; sólo en un caso la iniciativa tuvo su origen en el sistema de educación formal propiamente tal.

6. Los dos esfuerzos de mayor envergadura estuvieron destinados a la educación de adultos en el área de capacitación.

7. Los programas destinados a niños (2) utilizaron fundamentalmente criterio de selección la noción de destrezas básicas (basic skills); los dos programas para adultos enfatizaron los criterios relacionados con job-preparation skills.

8. Los niveles educacionales en que se realizó el trabajo corresponde a la educación elemental (educación general básica 1º - 8º).

9. Los programas instruccionales resultantes, utilizaron evaluación referida a criterios y su administración fue consecuente con los principios del aprendizaje para el dominio.

Algunas Enseñanzas

El proceso de análisis de las referidas experiencias y la participación personal de los autores en el desarrollo de alguna de ellas, han dejado alguna enseñanza que podría ser de utilidad sintetizar en esta página.

La determinación de necesidades debe ser reconceptualizado. Su valor
es indiscutible pero la instrumentación y los procedimientos deben ser revisados. Se impone a) la utilización de procedimientos participativos en los que se haya junto la comunidad y la totalidad de los aspectos en los que una necesidad puede incidir y b) los procesos de determinación de necesidades deben ser permanentes y acompañar, así, al proceso de desarrollo curricular y de implantación de un programa.

La experiencia mostró el valor de la matemática aplicada como instrumento de desarrollo comunitario, como tal debe ser ofrecida tal y como se la necesita y es esta para que sea más accesible a pobladores y campesinos. En este sentido es posible conceptualizar el valor social del aprendizaje básico, con el fin de ponerlo al servicio del desarrollo personal y comunitario.

Los programas analizados muestran el valor inapreciable de forma educativa que aseguren a) aprendizaje autodirigido, b) evaluación formativa frecuente de los aprendizajes, c) uso de alternativas remediales o de provisión para el logro del dominio de lo aprendido y d) la existencia de material de enseñanza válido que permita que existan las condiciones antes señaladas.

Por último es importante señalar que los programas de matemática analizados muestran la importancia de disponer de sistema de aprendizaje que sean componentes propias de sistemas de desarrollo económico y social.

Políticas Recomendables
A la luz del análisis anterior, se sugiere:
1. Evaluar los resultados de estas aplicaciones.
2. Difundir los resultados de las presentes experiencias e intercambiar resultados con otras naciones.
3. Dar mayor cobertura a las experiencias existentes a) en términos de difusión técnica, b) en términos de aplicación.
4. Impulsar algunas medidas tendientes a crear conciencia en torno a la necesidad de contrar con programas y/o estudios mínimos. (Los resultados de la carta que dio origen a este informe, se realizó en Chile durante el año 80 un Seminario al respecto.)
5. Adecuar esta política a la transformación que experimenta el sistema educacional chileno.
6. Relacionar estos programas con programas de desarrollo social o comunitarios.
APPENDIX B: RAPPORT SUR LES COMPÉTENCES MINIMALES EN MATHEMATIQUE EN LE GRAND-DUCÉ DE LUXEMBOURG

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Walferdange

Enseignement Préscolaire
La durée de l'enseignement préscolaire est de deux ans. Les enfants y sont admis dès l'âge de 4 ans. À 5 ans cet enseignement est obligatoire. Il n'y a pas de programme officiel. Les institutrices préparent par des activités au niveau du précalcul les élèves au programme du primaire.

Enseignement Primaire
La durée de l'enseignement primaire est de six ans. Les enfants y sont scolarisés dès l'âge de 6 ans. Le programme est identique pour toutes les écoles tant privées (en petit nombre) que publiques. Un plan d'études fixé par arrêté ministériel en règle les détails. Il est prévu que dans des classes à niveau plus faible l'instituteur a le droit d'adapter le programme aux possibilités réelles des enfants en essayant cependant d'en couvrir les parties essentielles. Compte tenu d'une part que les enfants ont à apprendre deux langues étrangères (allemand et français) d'autre part que dans les classes il y a une forte proportion d'enfants d'immigrants (d'on l'étude d'une 3ème langue, maternelle il est vrai) les instituteurs s'efforcent de leur mieux à tenir compte des exigences du programme. Des expériences dans des classes d'enfants mentalement handicapés ont montré que l'utilisation de langages non verbaux tels flèches multicolores au minicomputer de Papy pourraient aider les enfants faibles à surmonter leur handicap.

Enseignement Postprimaire
L'enseignement postprimaire comprend 3 volets : l'enseignement secondaire proprement dit d'une durée de sept ans prépare aux études universitaires ; l'enseignement secondaire technique prépare les enfants aux carrières moyennes et inférieures dans l'administration ainsi qu'à la vie professionnelle éventuellement artisanales suivant les sections. (Cet enseignement n'est qu'en stade expérimental ; il est appelé à remplacer les collèges d'enseignement moyen, les écoles professionnelles et artisanales.) Il est prévu qu'il aura une durée de sept ans. Enfin,
l'enseignement primaire complémentaire d'une durée de trois ans (fin de l'obligation scolaire).

1. **Enseignement secondaire.** Les élèves doivent passer un examen d'admission identique pour tous les établissements du pays. Après la 7\textsuperscript{me} d'orientation les élèves ont à choisir entre l'enseignement classique (Latin) et l'enseignement moderne. Dans chacun des 2 ordres d'enseignement, l'élève peut se décider après les 9\textsuperscript{me} année pour l'une des six sections suivantes: section langues, section math, section sciences, section économie, section arts, section musique. Pour chacune des sections des programmes détaillés sont prescrits. Sauf en section langues, l'enseignement mathématique est sanctionné par un examen officiel, identique par section, à la fin de l'année terminale. Dans certains établissements à forte population, les compositions finales de fin d'année sont établis d'un commun accord par les titulaires des différentes classes.

2. **Enseignement secondaire technique.** Les élèves doivent passer un examen d'admission mais de niveau plus faible. En fait, un enfant ayant échoué dans une branche pour l'enseignement secondaire est admis au technique si se note multipliée par 1.5 est suffisante. Nous avons dit que l'enseignement technique était encore au stade expérimental; les expériences en sont encore au cycle d'observation d'une durée de trois ans. Dans chaque classe et par branche les élèves sont groupés par niveaux.

Le niveau A correspond approximativement à celui du secondaire, le niveau D à celui de l'enseignement complémentaire (voir plus loin). Par niveaux, les programmes sont les mêmes pour tous les enfants avec des ajustements possibles. Des cours d'appui organisés par l'établissement permettent le passage d'un niveau à un autre.

Dans le cycle d'observation le cours de math prend 5 à 4 h par semaine sur un total de 30 h.

3. **L'enseignement complémentaire.** Cet enseignement recueille les enfants n'ayant pas réussi leur examen d'admission pour les deux autres ordres d'enseignement post primaire ou n'ayant pas présenté d'examen d'admission pour raisons diverses. Il y a de nouveau un programme officiel prescrit mais les enseignements ont une large liberté dans le choix des matières qu'ils veulent enseigner. Bien souvent il s'agit de répétitions de matières vues au primaire.
Enseignement Special

Dans le pays fonctionnent différentes classes qui recueillent les enfants mentalement handicapés ou caractériels. De plus, nous disposons d'un centre de lagopédie. Théoriquement, le programme officiel du primaire est prescrit mais on tient compte largement des possibilités des enfants.

Il a été possible de montrer avec des classes expérimentales que les moyens pédagogiques mis en œuvre permettent surtout en mathématique avec des classes à effectif faible, à amener les enfants à un niveau honnête en point de vue connaissances.
APPENDIX C: MINIMAL COMPETENCY TESTING PROGRAMS IN THE UNITED STATES

Arizona   1972 1975  *  1976
Arkansas  1979  *  1982
California 1976  *  1980  local
Colorado  1975  *  1979  local
Connecticut 1977  *  1979
Delaware  1976  *  1978 1975 1977 1979 1985 (When tests are developed)
Florida  1975  *  1978 1975 1977 (Proficiency endorsement on diploma)
Georgia  1976  *  1982 (To be determined)
Idaho    1977  *  1978 1975 1977 (Use determined by local districts)
Illinois 1978  *  1979 1977 (Use determined by local districts)
Kentucky 1978 1977  *  1979 1976 (High school graduation removed from program by 1978 legislation)
Louisiana 1976  *  1985 (Debra P. v. Turlington, 1979, has delayed implementation)
Maryland 1977  *  1985 (Use determined by local districts)
Massachusetts 1977  *  1982, 1985 (Proficiency endorsement on diploma)
Michigan 1974  *  1978 1977 (For comparative purposes)
Missouri 1976  *  1978
Nebraska 1975  *  1979 1978 1977 (For local use; a competency-based high school diploma was studied but rejected)
New Jersey 1976  *  1979
New Mexico ***  *  1978
New York  1978  *  1979
North Carolina ****  *  1980
Oklahoma  ****  *  1978
Oregon   1972/76  *  1978
Rhode Island 1978  *  1978
South Carolina 1978  *  1978
Tennessee 1977  *  1978
Texas     ****
Utah     1977  *  1980
Virginia 1978  *  1978
Washington 1976  *  1978
Wyoming  1977  *  1978

This information is provided to facilitate further research into minimal competency testing. It was synthesized from information gathered by the Education Commission of the States, and it reflects state-wide activity as of February 1, 1980.
APPENDIX D: A FEW ADDITIONAL WORDS ON THE LEARNING PROCESS BY THE AUTHORS OF THE CHILEAN REPORT/ALGUNAS PALABRAS ACERCA DEL PROCESO DE APRENDIZAJE EXPERIMENTADO POR LOS AUTORES

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The participation the authors have had in the development of minimal competency programs has led them to reconsider some concepts and strategies commonly used in educational and curriculum planning. Within the limited space available, these lessons might be summarized in the following way:

1. Concepts and procedures involved in educational planning and curriculum development need to be reconsidered when they are being transferred to developing countries (our experience relates to three Latin American countries). Models and techniques used in developed societies cannot be transferred in a linear or simple fashion to be utilized, for example, in rural or deprived areas of a country like ours. Using the words which Torsten Husen utilized to criticize the universal schooling model, we could say:

"...it raises profound..."

1These observations by the authors of the Chilean report, prepared in January, 1981, were received too late to be included in Part II (National Reports).
questions about the adequacy of the Western European and North American model of formal schooling that has been exported wholesale to the developing countries where it may not be suitable."

The main problems we found were:
- the inability of existing models to cope with qualitative social characteristics, as well as with quantitative information, and
- their inadequacy in relating technical and political issues.

While our thinking is still quite tentative, let us consider the things we are trying to implement and the kinds of questions we are facing through the common notion of needs assessment.

First, in the implementation of new programs, the authors are using alternative methods that may allow a participative process in which needs may emerge. However, time, priorities, and resources (to name a few) are viewed differently by rural or marginal communities, as compared with the way these ideas are used by the so-called "educated" technician. The question arises: How can an investigative process for

...surgen preguntas de fondo al considerar el uso del modelo formal de educación desarrollo en Europa occidental y los Estados Unidos de Norte América, en países en desarrollo. Puede que allí no sea aplicable."

En el uso de los modelos conocidos para el desarrollo del currículum, en programas de destrezas mínimas en zonas de extrema pobreza, encontramos dos dificultades principales: la ineficacia de estos modelos para manejar apropiadamente información cualitativa y su inoperancia para relacionar cuestiones de índole político-administrativo.

Nuestro pensamiento, en torno a este problema, es sólo tentativo. Para ejemplificarlo, consideramos lo que estamos tratando de implementar, y las preguntas que surgen, al utilizar las ideas corrientes acerca de determinación de necesidades.

Primeros, en el desarrollo de nuevos programas, los autores están utilizando estrategias que permitan un proceso participativo, del cual se espera emerjan las necesidades de una comunidad. Pero sucede que conceptos, como tiempo, prioridades, recursos, para nombrar sólo algunos, son comprendidos en forma diferente por las comunidades rurales y los...
identifying needs account for the differences between the aims and ways of thinking of community members and researchers, and yet lead to valid outcomes?

Second, the process of identifying needs should continue throughout the program development and implementation. A good reason for this is that practice frequently teaches the planner more than all of the previous studies. Moreover, experience shows that the program itself changes the actual conditions, making previous information largely irrelevant. Again a question arises: How can newly identified needs or a better understanding of reality be incorporated into a curriculum development process that is, by itself, expensive, technically hard to alter, and tends to generate rigid and expensive educational products?

2. Experiences with adult education have shown the value of applied mathematics as an instrument for community development. In this perspective, mathematical contents and/or objectives should be selected according to their potential to bring about an awareness...
among poor adults, concerning the kinds and sources of their problems,
- help communities in their organization,
- help individuals to cope with an increasingly complex civilization,
- prepare communities and individuals to understand financial and administrative problems, and
- help adults of marginal ability to comprehend social relations and commercial transactions as forces with which they have to cope for survival.

It is an interesting challenge for the mathematical community and for educators to find the mathematical structures or programs which would most appropriately clarify these issues for the illiterate or for the recently incorporated marginal population.

los contenidos o los objetivos matemáticos debieran ser seleccionados de acuerdo con su potencialidad para
- esclarecer, al adulto, la clase de problemas que debe enfrentar y las causas de dichos problemas,
- reforzar la organización comunitaria,
- ayudar al individuo a comprender una civilización de complejidad creciente,
- preparar al individuo y a las organizaciones comunitarias para el manejo de cuestionar financieras y administrativas y
- ayudar al adulto marginalo a comprender relaciones sociales y laborales en términos de fuerza con la que poder luchar para su sobrevivencia.

Es un desafío interesante para la comunidad de matemáticos y de educadores el esclarecer cuáles son las estructuras matemáticas que mejor podrían esclarecer estos aspectos al analítabo o al recientemente alfabetizado.