Learning Partners: Reading and Mathematics.

Although many of the word recognition skills needed to read narrative material are applicable to the language of mathematics, other specialized skills are also required to read mathematics materials. Among the factors contributing to students' problems in reading mathematics are the students' interest and motivational level, their readiness for reading mathematics, the reading level in narrative material, and the students' understanding of the purpose for reading mathematics. To help students associate the spoken words with the written word, teachers can point to words written on the chalkboard as they are spoken. In addition, an understanding of mathematical concepts can be developed through the use of a mathematics dictionary. Teachers should also be sensitive to the roots, prefixes, and suffixes as new words are introduced. In order to work successfully with symbols, students could be asked to interchange symbols, words, and pictorial representations. When reading graphic materials, students should be told to first survey the graphic to determine what it is about, the manner in which it is organized, and the information that it supplies. For the comprehension of mathematical sentences, students could be asked to write the sentences in as many different forms as possible. Finally, to prepare for reading word problems, they must be able to state the problem situation correctly in their own words. (HOD)
Teaching reading in the content areas, that is, teaching students to read social studies, home economics, or mathematics materials, has a two-fold purpose. One is to help students improve reading skills; the other is to help students understand the assignment and therefore learn more about the subject being taught.

Mathematics teachers, as well as teachers of other content areas, have become increasingly sensitive to students' deficiencies in reading and study skills. Before an attempt is made to remediate this problem, we must challenge the traditional practice of treating reading as a separate subject primarily taught at the elementary level. Reading is a process necessary to all subjects at all levels and must be integrated with course content.

Since reading skills are part of the language and intellectual development that continues from childhood through adulthood, there is crucial need to teach reading skills in content areas. Although many reading skills are taught in elementary grades, such skills as vocabulary development, comprehension, and study skills must be taught throughout the student's school career. Some students at secondary level may still need help with word recognition skills.

The reading of mathematics involves abstractions, specialized symbolism, and technical terminology. Although many of the word recognition skills learned to read narrative material are applicable to the language of mathematics, other specialized skills are also required to read mathematics material. Thus, students' effective reading of narrative does not imply they will be equally as effective
in reading mathematics. Research indicates that mathematics material is the most difficult content area material to read with more concepts per word, per sentence, and per paragraph than any other area.

Since effective reading in mathematics is closely associated with the development of mathematics skills and concepts, the mathematics teacher must accept the major responsibility for teaching students to read mathematics. The reading teacher cannot be expected to teach students to read, interpret, translate, and comprehend specialized words and symbols used in mathematics. Meaningful interpretations of many mathematical words and symbols require considerable content knowledge on the part of the student before proper meaning can be attained.

There are no magic techniques that will enable all students to learn the language of mathematics. Mathematics teachers should be aware of some of the factors that make the language of mathematics difficult to read in order to provide help to students. Among the factors contributing to a student's problems in reading mathematics are: the student's interest and motivational level, readiness for reading mathematics, reading level in narrative material, and understanding of the purpose for reading mathematics. The structure of the language of mathematics makes it difficult for many students to read. Differences exist, for example, between the language of mathematics and standard English prose. Such differences include:

- English uses a standard set of 26 symbols. Mathematics uses these symbols plus many nonalphabetic mathematics symbols.
- The sentence structure in mathematics is different from that of standard English.
- In mathematics, the student does not always read from left to right and down the page.
If teachers would provide necessary information to students in reading the language of mathematics, student comprehension would improve and student success in the development of concepts and computational and problem solving skills would increase. When students develop skills in the reading of mathematics, much of the frustration and anxiety about the subject is eliminated and mathematics becomes less threatening.

**SPECIALIZED VOCABULARY AND SYMBOLS**

The reading of mathematics requires the understanding of a specialized vocabulary and of abstract symbols. Ideas and concepts in mathematics are sometimes difficult, if not impossible, to be adequately communicated in narrative language; therefore, abstract concepts are often explained by using symbols. Specialized words and phrases such as prism and least common multiple are used to express broad, complex ideas. These words often cannot be defined simply. They are sometimes new or unfamiliar words, or familiar words that take on different or more technical meanings, such as prime and base.

Although the language of mathematics is often abstract in nature, understanding of this language can be enhanced through the use of concrete objects. If students are allowed to work initially with concrete experiences, the mathematical abstractions can become meaningful.

**Reading Mathematical Words**

Reading mathematical words involves:

- developing word meanings by using definitions and examining context clues, prefixes, suffixes, and root words
- interpreting words with special meanings.
Teachers sometimes use words orally before students are ready to associate those words with their meanings. To help students associate the spoken words with the written word, teachers can point to words written on the chalkboard or on charts as they are spoken.

**Using Definitions**

The meanings of words in the mathematics vocabulary are generally precise, with clearly delineated definitions. It is important, however, that students learning the vocabulary of mathematics do more than merely memorize and vocalize word meanings. To grasp the material's meaning, they must develop ideas and concepts represented by the word.

A basic understanding of the concepts involved is required for a student to completely understand a definition. Furthermore, many terms require varying levels of understanding. For example, one could define a square as "a quadrilateral with equal sides and equal angles". However, a student may be able to quote this definition without having a grasp of the meaning. The student may not understand the construction of the square, the relationship of the diagonals, or the relationship of sides and area. All of these factors build on the student's complete understanding of the properties of the square.

Many activities can be devised for developing and assessing students' understanding of mathematical words and concepts. Students should be encouraged to utilize a mathematics dictionary, which should be part of every school library. If students are using a standard dictionary, the teacher must emphasize the importance of context: the definition must make sense in the sentence in which the student found the word.
Using Prefixes, Suffixes, and Root Words

The meaning of certain mathematical words can be determined by observing the meaning of prefixes, suffixes, and root words. For example, the words add, adding, addend, additive, and addition share a common root. Students will know many of these words, suffixes, and root words, and they can be guided into adapting them for mathematics.

Teachers can direct students to become more aware of the roots, prefixes, and suffixes as new words are introduced. The activity that follows can improve student awareness of word meanings:

Directions: Write down as many words as you can think of that begin with the prefix at the top of the column.

<table>
<thead>
<tr>
<th>bi-</th>
<th>tri-</th>
<th>sub-</th>
<th>poly-</th>
<th>ex-</th>
<th>bi-</th>
<th>milli-</th>
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Interpreting Words with Special Meanings

Many words used in a mathematical context take on a different or specialized meaning. A few examples of words with specialized meanings include solution, plane, volume, power, root, and group. As previously mentioned, words in mathematics generally have precise definitions. However, there are words, such as degree, square and base that have multiple meanings in mathematics.
As an activity involving words which have specialized meanings, have students write two sentences for each word, one using its mathematical meaning and one using its common meaning: "Joan covered her legs with a blanket" and "The legs of an isosceles triangle have equal measures."

READING MATHEMATICAL SYMBOLS

A symbol in mathematics may be a character, abbreviation, or mark which represents an idea, concept, expression, quantity, or operation. Many mathematical symbols, such as the "less than" sign or the "equal" sign are entirely different from customary reading symbols. Others, however, such as the "x" to indicate multiplication, or the "-" to indicate subtraction, are standard reading symbols that have specific meanings in mathematics.

In order to work successfully with symbols, students need to be able to:

- associate a word or phrase with a symbol
- express an idea in objects, pictures, words, and symbols
- be aware that there may be more than one meaning for some of the symbols.

A sample activity could ask students to change between symbols, words, and pictorial representations:
Directions: Fill in the missing symbol, word(s), or picture to complete the chart.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Word</th>
<th>Picture</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>one-fourth</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>greater than</td>
<td><img src="image2" alt="Diagram" /></td>
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<td></td>
<td>equals</td>
<td><img src="image3" alt="Diagram" /></td>
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<td></td>
<td>angle</td>
<td><img src="image4" alt="Diagram" /></td>
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<td>3</td>
<td></td>
<td><img src="image5" alt="Diagram" /></td>
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**DIRECTIONALITY OF MATHEMATICAL SYMBOLS**

Directionality and spatial relationships often cause difficulty in the reading of mathematics. For example, elementary teachers may have students who find confusion between 65 and 56. Algebra teachers may have students who confuse $5x$ and $x^5$. In each case, the meaning of the 5 has changed according to its position. Furthermore, there is not necessarily a one-to-one relationship between the mathematical symbols and the spoken word. For example, $x^5$ can be read as "the fifth power of x."

Because of the nature of our place value system, one skims a number in a back-and-forth motion to read it. To read the number 748, for example, one must note that there are three digits, indicating a number in the hundreds.
One then reads the first digit and follows it with a "hundred." One then observes the 4 in the tens place, indicating 40. Then the 8 is observed in the ones column to be added to the 40. Reading 748 as seven hundred forty-eight, there, is a more complex activity for students than we often realize. Pronouncing a phrase such as \((x+5)^3\) as the cube of the sum of \(x\) and 5 requires a similar back-and-forth motion which is sometimes difficult for older students.

The following examples illustrate some of the directions and eye movements involved in reading mathematics material:

The Smith's spend more money on transportation than on what other expenses?

What is the temperature reading on this thermometer?
Is there a candle for each birthday cake?

Find $\square \times \square$ in this table.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>1</td>
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<td>2</td>
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<td>6</td>
<td>8</td>
<td>10</td>
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<td>9</td>
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<td>5</td>
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<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>
And so ... We read -- and reread -- mathematics, left-to-right, sometimes, but other times we read around, or diagonally, or following arrows, or vertically.

Interpreting Graphic Materials

Mathematical ideas are often compacted through the use of graphic materials, such as tables, graphs, or charts. The reading and interpreting of these materials play an important part in the reading of mathematics. Students should begin developing skills in reading graphs at a simple level and progress gradually to more complex situations.

When reading graphic materials, the student should first survey to determine what the graphic is about, the manner in which it is organized, and the information that is supplied. Following this overview, the graphic can be studied in more detail for the specific information required.

READING MATHEMATICAL SENTENCES

At an early stage of their mathematical development, students must learn to read and understand mathematical sentences. The reading of these sentences involves recognizing and pronouncing words and symbols and attaching meaning to the total sentence. A mathematical sentence can be modeled in the following way: symbols that name things, followed by a symbol that names a relationship, followed by symbols that name things. In the sentence $9x + 3 = 4x + 2$, for example, $9x + 3$ and $4x + 2$ are symbols naming things, and $=$ is a symbol that names a relationship. Students must be able to interpret all the symbols involved and then put all the symbols together to attach meaning to the entire sentence.

Note: When we write $1 + 3 = \underline{}$ and do not explain it, we sometimes give the idea that this is the only order in which this sentence can be written.
Many times when students see $4 = 1 + 3$, they are confused about the order in which it should be read. This causes them to think that this is an incorrect form. One activity to help young students understand the flexibility in the sentence order is to ask them to write mathematical sentences in as many different forms as possible:

$$398 = 300 + 90 + 8$$
$$398 = (3 \times 100) + (9 \times 10) + (8 \times 1)$$
three hundred ninety-eight equals 300 + 90 + 8
$$398 = \text{three hundred plus ninety plus eight}$$

**READING AND SOLVING WORD PROBLEMS**

Many students experience difficulty when they are asked to solve word problems, although they are often able to solve the problems without difficulty when someone performs the translation for them. In order to experience success with solving word problems, students need to:

+ read the words in problems
+ analyze the information in word problems
+ translate and use signal words
+ identify sufficient, insufficient, and extraneous information in a word problem
+ employ some strategy (someone else's or their own) that will help them solve word problems.

**Preparing for Reading Word Problems**

In reading, the word "readiness" is used to refer to a student's preparedness to understand beginning reading instruction or a reading activity at any level of instruction. Readiness for reading must be present in all reading activities from kindergarten through adult education. Likewise, in mathematics, readiness to perform a given task is needed. If students cannot adequately read and understand a word problem, they must be given help to do so. A mathematics
teacher cannot afford to ignore the reading levels of the students nor of the material to be read. Every attempt should be made to facilitate the learning process.

Another consideration of readiness is ascertaining that students can perform the necessary arithmetic operations required to solve the problem. Even if the problem is analyzed, unless students can perform the required operations, the problem cannot be solved. Word problems should be assigned in small quantities! Effective teachers will keep this in mind when making assignments! Teachers should also attempt to arrange problems according to difficulty. Early success tends to motivate students' progress.

A good sample activity would be to have students form small groups, make up problems of their own, state them in their own words, and assign them to their classmates.

Analyzing Word Problems

Students need to be able to state meanings, relationships, and needed operations in order to solve a word problem. They must be able to state the problem situation correctly in their own words and to determine what is sought. They should be able to follow ideas in a sequential manner. In an effort to build skills in analyzing word problems, the teacher can use an activity in which students are required to respond to a list of questions about a problem situation. If students find it hard to organize their thoughts, it is the responsibility of the teacher to determine whether the material is too hard or whether students need help with organization.
As with general reading, there are no "cure-all" techniques which will allow us to easily teach all students to read the language of mathematics. Oftentimes, the battle is half won if we, as teachers, agree that a problem does, in fact, exist in reading mathematics materials and are aware of some of the factors which make the language of mathematics difficult to read. As teachers we need to devote much more effort to teaching students to read the language of mathematics. With improved comprehension, the student enjoys much more success in concept development, computational skills, and problem solving. When students develop the skills needed to read mathematics effectively, mathematics becomes less threatening. And much of the anxiety and frustration, for both the student and teacher, associated with learning mathematics may be eliminated.
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