This paper develops an approach to estimating the effect of government employment and training programs on measured unemployment. The theoretical aspects of the method draw heavily on earlier work on labor market flow equilibrium (see note). Previous estimates of the direct or statistical impact of government programs on the unemployment rate have not been based on a general equilibrium model and no attempt was made in these studies to constrain the labor market to be in equilibrium. This paper describes an empirical procedure based on markov flow equilibrium and then implements it with data on youth employment and training programs. Models with both constant and variable transition probabilities are analyzed. The results are contrasted with results of the alternative methodology used in previous studies. (Author)
This research was supported under contract No. J-9-M-8-0035 under grant No. 51-11-77-04 from the Department of Labor. The opinions expressed in this paper are the author's own and do not necessarily reflect those of the Department of Labor or the Urban Institute and its sponsors.

IMPACT OF EMPLOYMENT AND TRAINING PROGRAMS ON MEASURED UNEMPLOYMENT

by

Richard S. Toikka

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WASHINGTON, D.C.
ABSTRACT

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This paper develops an approach to estimating the effect of government employment and training programs on measured unemployment. The theoretical aspects of the method draw heavily on earlier work on labor market flow equilibrium. Previous estimates of the direct or statistical impact of government programs on the unemployment rate have not been based on a general equilibrium model, and no attempt was made in these studies to constrain the labor market to be in equilibrium. This paper describes an empirical procedure based on a Markov flow equilibrium and then implements it with data on youth employment and training programs. Models with both constant and variable transition probabilities are analyzed. The results are contrasted with results of the alternative methodology used in previous studies.
This paper develops an approach to estimating the effect of government employment and training programs on measured unemployment. The theoretical aspects of the method draw heavily on earlier work on labor market flow equilibrium by Holt (1969), and the application of Markov processes to model labor market behavior by Toikka (1976). Previous estimates of the direct or statistical impact of government programs on the unemployment rate have been made by Cohen (1959), Small (1972), and Killingsworth and Killingsworth (1978). The approach taken in this paper differs from those earlier efforts in that it is based on a labor market flow equilibrium which is modeled by a Markov process.

In section I, a model of labor market equilibrium is presented. In section II, it is shown how a government program may be introduced into the labor market model and its impact on equilibrium unemployment assessed. In section III, a special case of the general model is analyzed in which the equilibrium unemployment rate of non-participants in the program is independent of the size of the program. In section IV, the methods of the earlier studies are compared with the method of this study. In section V, the assumption of constant labor market transition probabilities is relaxed and the impact of allowing hiring and labor force participation rates to respond to program induced tightening of the labor is estimated.
I. LABOR MARKET EQUILIBRIUM

The three state labor market model used by Toikka (1976) and others to describe transitions among the states of employment, unemployment, and out of the labor force will be adapted to incorporate a government program or set of programs. The program(s) will be introduced as a fourth labor market state. In the official unemployment statistics, persons in the program will be counted as in one of the other three labor market states (see Killingsworth and Killingsworth, 1978). However, in modelling the impact of a program, it is important that participants be identified as being in the program because flows between the program state and the other states play an important role in determining the statistical impact of programs on unemployment.

At any point in time, the population \( T \) is divided into four sub-populations: \( G, W, U, \) and \( N \), the number of persons in the program, employment, unemployment, and out of the labor force, respectively. Define a \( 4 \times 4 \) Markov matrix \( P_t = \{p_{ijt}\} \) with elements \( p_{ij} \) denoting the probability of moving from state \( i \) to state \( j \) in period \( t \). Define a \( 4 \times 1 \) state vector \( \pi_t = \{\pi_{it}\} \) with elements \( \pi_{it} \) denoting the expected fraction of persons in state \( i \) at time \( t \) after the Markov process has operated for a long period of time. The dynamics of the process can be described by the transition equation:

\[
\pi_t = \pi_{t-1} P_t
\]

If \( P \) is stationary (\( P_t = P \) for all \( t \)), the process approaches a stationary state defined by:

\[
\pi = \pi P
\]
Equation (2) may be used to describe a labor market in "flow equilibrium." In flow equilibrium, there is no tendency for any of the states to change in size. Such a model will be used to estimate the impact of a program which has been in existence for some time and has a constant enrollment with the number of new enrollees equalling the number of terminations.

To model a program's impact within such a framework, it is necessary to relate the program's characteristics to the transition probabilities in the Markov matrix. To understand how the program affects the labor market equilibrium, it is instructive to begin with a labor market in equilibrium without the program. The equilibrium condition is given by (2)

where \( \pi = (\pi_E \pi_U \pi_N 0) \)

\[
P = \begin{pmatrix}
P_{EE} & P_{EU} & P_{EN} & 0 \\
P_{UE} & P_{UU} & P_{UN} & 0 \\
P_{NE} & P_{NU} & P_{NN} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} = \begin{pmatrix}
P^* & 1 & \emptyset \\
\emptyset & 1 & 0 \\
\emptyset & 0 & 0
\end{pmatrix}
\]

and \( P^* \) is a 3x3 matrix which governs transitions among the states of employment and out of the labor force in the absence of the program.
II. INTRODUCING THE PROGRAM

Introducing the program into the labor market requires defining a set of exit probabilities \( (P_{Gi}, i = E, U, N) \), a set of entrance probabilities \( (P_{iG}, i = E, U, N) \), and indicating how the elements \( (P_{ij}^{*}, i = E, U, N; j = E, U, N) \) of the pre-program transition matrix \( P^* \) are affected by the program.

If each individual spends \( \lambda \) periods in the program then the aggregate transition probability for exits from the program \( (P_{GO}) \) will be

\[
P_{GO} = \frac{1}{\lambda}
\]

Let \( Z_i, i = E, U, N \) denote the probability that a person who leaves the program will enter state \( i \). The three exit probabilities may then be written as:

\[
P_{Gi} = P_{GO} Z_i = \frac{Z_i}{\lambda}; \quad i = E, U, N.
\]

The entrance probabilities may be derived from information about the aggregate flow into the program and the distribution of that flow by labor market state occupied prior to entrance into the program. For a program with total slots \( G \), and aggregate exit probability \( P_{GO} \), the flow of terminations in any period \( (f^t) \) is

\[
f^t = G \cdot P_{GO}
\]

For a program in equilibrium, the flow out of the program \( (f^t) \) equals the flow in \( (F^I) \); thus we may write

\[
f^I = f^t = G \cdot P_{GO}
\]

If the fraction of the entrants coming from state \( i \) is denoted by \( S_i, i = E, U, N \),
U, N, then the entrance probabilities for the three states are

\[ p_{iG} = \frac{G_{iG}}{1} = \frac{G_{iG}}{1} ; \quad i = E, U, N \]

If the probability of entering the program in any period from any state is independent of the probability of moving to any other state or remaining in that state, then the introduction of the program scales down each of the transition probabilities in the Markov matrix \( P \) which described the labor market transitions without the program. The scaling factor is the probability of not entering the program. The new transition probabilities \( p_{ij}^* \) may be interpreted as the probability of moving from state \( i \) to state \( j \) conditional on not entering the program, i.e.

\[ p_{ij}^* = p_{ij}(1 - p_{iG}) \quad i = E, U, N ; \quad j = E, U, N \]

The full transition matrix for the labor market in equilibrium with the program in place is

\[
P^* = 
\begin{pmatrix}
p_{EE}^*(1 - G \frac{S^E}{E}) & p_{EU}^*(1 - G \frac{S^E}{E}) & p_{EN}^*(1 - G \frac{S^E}{E}) & G \frac{S^E}{E} \\
p_{UE}^*(1 - G \frac{S^U}{U}) & p_{UU}^*(1 - G \frac{S^U}{U}) & p_{UN}^*(1 - G \frac{S^U}{U}) & G \frac{S^U}{U} \\
p_{NE}^*(1 - G \frac{S^N}{N}) & p_{NU}^*(1 - G \frac{S^N}{N}) & p_{NN}^*(1 - G \frac{S^N}{N}) & G \frac{S^N}{N} \\
\frac{Z^E}{Z} & \frac{Z^U}{Z} & \frac{Z^N}{Z} & 1 - \frac{1}{Z}
\end{pmatrix}
\]

The equilibrium distribution of individuals across the four states is obtained by solving

\[ \Pi = \Pi P \]

where \( \Pi = (\Pi_E \ \Pi_U \ \Pi_N \ \Pi_G) \)
and \( \pi_i \) is the expected fraction of the population in state \( i \).

To illustrate how this approach may be used to estimate the impact of a program on a labor market, the following characteristics are assumed for a program:

\[
\begin{align*}
G & = 79,000 \\
L & = 6 \\
Z^E & = 0.6; Z^U = 0.2; Z^N = 0.2 \\
S^E & = 0.2; S^U = 0.6; S^N = 0.2
\end{align*}
\]

Prior to the introduction of the program, the labor market is assumed to be in equilibrium with the set of monthly transition probabilities given in Table 1. In Table 2, the distribution of the population before and after the introduction of the program is reported. As can be seen in Table 2, the employed, unemployed, and not in the labor force groups all are reduced as a result of the program. The impact of the program on the measured unemployment rate (i.e., unemployed as a fraction of the labor force) depends importantly on how the program participants are counted in the labor force statistics. When the participants are counted as out-of-the-labor force or employed, the unemployment rate falls as a result of program participation; when the participants are counted as unemployed, the unemployment rate rises as a result of the program.
### Table 1

Transition Probabilities

<table>
<thead>
<tr>
<th>Group</th>
<th>E</th>
<th>U</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.863</td>
<td>0.043</td>
<td>0.436</td>
</tr>
<tr>
<td>U</td>
<td>0.303</td>
<td>0.260</td>
<td>0.436</td>
</tr>
<tr>
<td>N</td>
<td>0.144</td>
<td>0.082</td>
<td>0.774</td>
</tr>
</tbody>
</table>
Table 2
Markov Distributions by Labor Force State

<table>
<thead>
<tr>
<th></th>
<th>Population</th>
<th>Program</th>
<th>Employment</th>
<th>Unemployment</th>
<th>Unemployment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-program</td>
<td>6962</td>
<td>0</td>
<td>3932</td>
<td>646</td>
<td>14.1</td>
</tr>
<tr>
<td>Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-program</td>
<td>6962</td>
<td>79</td>
<td>3896</td>
<td>637</td>
<td>14.0</td>
</tr>
<tr>
<td>Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15.5</td>
</tr>
</tbody>
</table>

1. Program participants counted as out of the labor force
2. Program participants counted as employed
3. Program participants counted as unemployed
III. SPECIAL CASE

There is a special case of this model which has an interesting property, namely, that the distribution of non-participants across the states of unemployment, employment, and not in the labor force is left unaffected by the presence or size of the program. That special case occurs when the distribution of the entrants by prior labor market state is identical to the distribution of terminees by subsequent state. In the notation of the model, \( S_i = Z_i \); \( i = E, U, N \). That is, the persons leaving the program on average return to the same states they occupied prior to the program. In the appendix, it is shown that in this special case, the model can be solved in two parts. First, the program participants may be subtracted from the population and assigned to the program. Then the non-participants (\( T-G \) in number) may be distributed across labor market states by applying the three state Markov matrix which governs transitions in the absence of the program. As a result of this procedure, the distribution of non-participants across labor market states will be identical to the distribution of all persons in the absence of the program since both distributions are based on \( P^* \). In this special case, the parameters \( \lambda, Z_i, \) and \( S_i \) no longer affect the equilibrium distribution.

As an example, the impact of the following type of program is estimated:

\[
G = 79,000 \quad Z_i = S_i \text{ for } i = E, U, N
\]

The transition probabilities are those given in Table 1. The equilibrium distribution before and after the program are given in Table 3. Observe that the post-program unemployment rate is identical to the pre-program rate when non-participants are counted as out of the labor force. This result is implied by the fact that the distribution of non-participants
Table 3

Distribution of the Population by Labor Force State Before and After the Introduction of a Program (in thousands)

<table>
<thead>
<tr>
<th>Program participant status</th>
<th>Pre-program Distribution</th>
<th>Post-program Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Population (G)</td>
<td>Employment (E)</td>
</tr>
<tr>
<td></td>
<td>6962</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6962</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Program participant status

1. out-of-the-labor-force
2. employed
3. unemployed
after the program is identical to the distribution of the entire population prior to the program. The identity between these two distributions will be quite important in contrasting the Markov equilibrium estimation with alternative procedures.
IV. COMPARISON WITH PRIOR ESTIMATION TECHNIQUES

Previous studies that report attempts to adjust the unemployment rate for the statistical impact of employment and training programs have been made by Cohen (1959), Small (1972), and Killingsworth and Killingsworth (1978). They all use a similar methodology, but its clearest statement and justification is given in Killingsworth and Killingsworth (1978). The approach is, in essence, to construct an estimate of what the unemployment rate would have been in the absence of the program and to compare the existing unemployment rate with that estimate of "what would have been." The estimated unemployment rate in the absence of the program is constructed by assigning individuals to labor market states (i.e., employed, unemployed, or not-in-the-labor force) according to the following rules: (1) program participants are assigned to the state which they occupied prior to entering the program; (2) program non-participants are assigned to the state which they occupy with the program in place. Cohen and Small state that the procedure assumes that in the absence of the program individuals occupy the same state that they did prior to entering the program. Killingsworth and Killingsworth emphasize that the assumption as stated by Cohen and Small is too strong and that a weaker assumption that program participants occupy "on average" the same states that they did prior to entering the program is sufficient for the procedure to give unbiased estimates.

In this statement of the estimation procedure (which they call the AREK procedure), Killingsworth and Killingsworth make it clear that only short-term direct impacts of the program are being estimated. Specifically, they state that the procedure ignores any induced effects resulting from the reaction of the rest of the economy to program enrollment changes or
the reaction of the rest of the economy to any changes in the economic environment which accompany and are the consequence of such enrollment changes. The authors claim that because "induced" effects are ignored, the procedure will understate any reduction in the unemployment rate. Their argument rests on assertions about such induced changes as increases in vacancy rates resulting from employed persons entering programs and increases in aggregate demand resulting from program expenditures.

A number of questions might be raised about the appropriateness of this method even considering its limited purpose. First, what implied assumptions are being made about labor market equilibrium before and after the program? Second, how can the procedure ignore the impact of program terminations on the labor market since no assumption is made about what states participants occupy when they leave the program? Third, under what conditions will the procedure give an unbiased estimate of impact on equilibrium unemployment for a suitably defined equilibrium.

Applying the Markov model introduced in sections II and III gives insights into the properties of the AHEM estimation procedure. In particular, it may be shown that a set of restrictions on a Markov model exist which are necessary and sufficient for the AHEM procedure to give unbiased estimates of the impact of a program on labor market equilibrium. These conditions are the following: (1) intake into the program is independent of the labor market state occupied prior to entering the program; (2) enrollees who leave the program return on average to the same labor market states that they occupied prior to entering the program; and (3) transition probabilities governing transitions between the three non-program states of employment, unemployment, and not-in-the-labor force for non-participants in the program are not affected by the program and are constant through time.
Assumption 2 is a property of the special case discussed in section III. Assumption 3 has been maintained in all of the models discussed in sections II and III. By imposing assumption 1 on the model in section III, it is possible to show that assumptions 1-3 are sufficient for the AHEM procedure to give unbiased estimates.

The proof is based on a property of Markov models meeting assumptions 2 and 3 that only such models possess, namely, that the equilibrium distribution of non-participants by labor market state with the program in place is identical to the distribution of all persons by labor market state without the program in place. In the appendix, it is shown that assumptions 2 and 3 are both necessary and sufficient for this property to hold. The proof in the appendix follows directly from the separability property implied by assumption 2 (which is sufficient and necessary for separability) which allows the equilibrium conditions to be stated in terms of a three state Markov process which does not involve the program state and from the constancy of the transition probabilities. The intuitive sense of this result is that a "non-participant" can expect the same labor market experiences after the introduction of the program as all individuals could have expected without the program. The program does not change the labor market experiences of non-participants.

The AHEM procedure creates a hypothetical distribution of persons by labor market state in the absence of the program by summing two frequency distributions. The distribution of program participants by prior state is added to the distribution of non-participants by current state with the program in place. This may be expressed algebraically by

\[
\Pi_T = \Pi_C + \Pi_N(T-G)
\]
where $T$ is total population, $G$ is the number of program participants, $\Pi^N$ is a state vector for non-participants with the program in place, $S_1$ is a state vector for participants, and $\Pi$ is a state vector for all persons in the absence of the program. (Each element of the state vector is the expected fraction of its population in a particular state).

It has already been shown that assumptions 2 and 3 are necessary and sufficient for $\Pi = \Pi^N$. Imposing this constraint on the AHEM equation (11) implies that $S_1 = \Pi = \Pi^N$. Conversely, when the equality $S_1 = \Pi$ holds, it follows that $\Pi = \Pi^N$. The equality $S_1 = \Pi$ implies random sampling from the population since the distribution of program participants by prior state is identical to the distribution by state for all persons. Therefore, assumptions 1 (random selection), 2, and 3 are necessary and sufficient for the AHEM procedure to be unbiased.

The AHEM procedure will give biased estimates of the change in Markov equilibrium distribution produced by a program except in the case of random selection into the program. The direction of the bias can be seen in an example. If a program draws in unemployed workers disproportionately, then the distribution of program participants by pre-program state will favor the unemployed more than the equilibrium distribution of non-participants. Thus, the AHEM procedure will produce an estimated unemployment rate in the absence of the program that exceeds the unemployment rate of non-participants. Since assumptions 2 and 3 imply that the equilibrium distribution of all persons in the absence of the program is identical to the distribution of non-participants with the program in place, the AHEM estimated unemployment rate in the absence of the program is biased upward in this case. The resulting estimate of program impact in reducing the unemployment rate is therefore overstated. For programs that draw in the non-unemployed
disproportionately, the reverse holds: the program impact is understated. Since most employment and training programs favor the unemployed, the AHEM procedure is more likely to overstate impacts on the unemployment rate in a world in which assumptions 2 and 3 hold.

This bias is not discussed by Killingsworth and Killingsworth as they devote an appendix to discussing a different question, namely, what is the true statistical impact of employment and training programs. The question examined here has been under what circumstances will the AHEM procedure give an unbiased estimate of the true impact. It is true that the actual statistical impact of a program on employment understates the total impact since effects such as vacancy creation are ignored. (The same does not necessarily hold for impacts on unemployment since the size of the labor force responds to changes in vacancy rates). However, the AHEM procedure may overestimate this statistical impact because its estimate of what the labor market would be like in the absence of the program may overstate employment. The question of whether the AHEM procedure gives unbiased estimates is never directly addressed in Killingsworth and Killingsworth because the partial equilibrium model that they use does not fix an equilibrium unemployment rate. The AHEM procedure allows the estimate of the unemployment rate existing without the program to differ from the unemployment rate of non-participants with the program in place. A Markov general equilibrium model constrains these two rates to be equal and therefore produces different estimates of program impact.

To illustrate let us use the example in section III with the data in Table 3. The Markov estimates of the post-program unemployment rate were 14.1, 13.9, and 15.6 depending on whether the program participants were counted as not-in-the labor force, employed, or unemployed. These correspond to impacts of 0, -0.2, and +1.5, respectively. Starting with the post-program
distribution data in Table 3, assuming that 60 percent of the enrollees were unemployed prior to entering the program, 20 percent were employed, and the rest were out of the labor force, and applying the AHEM procedure to derive a pre-program distribution gives an estimated pre-program unemployment rate of 14.9 (compared to the equilibrium 14.1). Thus, the AHEM estimated impacts are −0.8, −1.0, and +0.7. The complete AHEM pre-program distribution is compared with the Markov distributions in Table 4. As can be seen in this example, the potential bias in the AHEM procedure is large even when only 60 percent of the enrollees were unemployed prior to entering the program.
Table 4

AHEM and Markov Estimates

<table>
<thead>
<tr>
<th></th>
<th>Population</th>
<th>Program (G)</th>
<th>Employment (E)</th>
<th>Unemployment (U)</th>
<th>Unemployment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>True.</strong></td>
<td></td>
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<tr>
<td>Pre-program Distribution</td>
<td>6962</td>
<td>0</td>
<td>3932</td>
<td>646</td>
<td>14.1</td>
</tr>
<tr>
<td>Post-program Distribution</td>
<td>6962</td>
<td>79</td>
<td>3887</td>
<td>637</td>
<td>14.1&lt;sup&gt;1&lt;/sup&gt;</td>
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<td></td>
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<td>13.9&lt;sup&gt;2&lt;/sup&gt;</td>
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<td></td>
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<td></td>
<td>15.6&lt;sup&gt;3&lt;/sup&gt;</td>
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<tr>
<td><strong>AHEM</strong></td>
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<tr>
<td>Estimate of Pre-program Distribution</td>
<td>6962</td>
<td>0</td>
<td>3903</td>
<td>686</td>
<td>14.9</td>
</tr>
</tbody>
</table>

1. Program participants counted as out-of-the-labor force
2. Program participants counted as employed
3. Program participants counted as unemployed
V. VARIABLE TRANSITION PROBABILITIES

So far the only Markov models examined have been those with constant transition probabilities. This is obviously an undesirable property since turnover rates are known to vary with demand and supply conditions in the labor market. In this section program impacts will be estimated in the context of a model in which transition probabilities respond to the state of the labor market. The types of programs considered will be like those in section III in the sense that the enrollees return on average to the states that they occupied prior to entering the program. The relation between job vacancies and transition probabilities will be borrowed from a monthly dynamic labor market model described by Smith (1977). Smith’s model relates transition probabilities for sixteen age, race, sex groups to seasonal dummy variables, a time trend, and the ratio of the Conference Board’s help wanted index to aggregate unemployment. It was estimated using time series data on transition probabilities estimated from the Current Population Survey. Unfortunately, there is no disaggregation of job vacancies in Smith’s model. This limitation makes it difficult to simulate programs that target jobs for particular groups of workers. Also, there is no estimate of the total number of job vacancies since the model uses an index. However, in simulations reported by Smith (1977) an estimate of total job vacancies has been derived by assuming that the total number of job vacancies is 20,000 times the value of the index. This gives an estimate of about 2 million job vacancies for the period 1967-1973.

The simulations reported here will utilize the separability property of Markov models in which the pre- and post-program distribution of participants by labor force state are identical (section III). For programs
which return participants on average to the same states that they occupied prior to entering, the labor market equilibrium for persons outside of the program may be derived independently of that for program participants.

To determine a baseline for the simulation average values of monthly transition probabilities for the period October 1976–September 1977 (fiscal year 1977) were computed for each of sixteen demographic groups. These are reported in Table 5. These transition probabilities were then used to determine a pre-program equilibrium distribution of the population by labor force state. These distributions for each population group are given in Table 6. A simulation of the impacts of a set of programs similar to those authorized in the Youth Employment Demonstration Projects Act of 1977 was carried out. The results reported here extend those described in Toikka (1978). The simulation assumed a net increase in the number of jobs or training slots of 220,600 which was distributed across eight demographic groups as described in Table 7. On the assumption that the transition probabilities were constant, and the program drew 30 percent of its enrollees from the employed, 34 percent from the unemployed, and the rest from out of the labor force, simulations of program impact on unemployment rates were carried out using the Markov model. Estimates of these impacts were then made using the AHEM procedure. Both the Markov and AHEM program impact estimates are reported in Table 8.

The assumption of constant transition probabilities was then relaxed and the probabilities \( P_{tij} \), for the \( j^{th} \) transition probability for the \( i^{th} \) group, in month \( t \) were allowed to deviate from their baseline 1977 value according to the following relation:

\[
\ln P_{tij} = \ln P_{ij}^* + \beta_{ij} [\ln(V_t/U_t) - \ln(V^*/U^*)]
\]

Here \( P_{ij}^* \), \( V^* \), are 1977 fiscal year averages for the \( j^{th} \) transition probability.
Table 5
Averse Transition Probabilities
Fiscal 1977

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EN</td>
<td>.0957 .0285</td>
<td>.1078 .0470</td>
<td>.1653 .0415</td>
<td>.1568 .0589</td>
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<td>EU</td>
<td>.0436 .0292</td>
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<td>EE</td>
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<td>.8596 .9332</td>
<td>.7632 .9083</td>
<td>.7849 .9064</td>
</tr>
<tr>
<td>NE</td>
<td>.1429 .1579</td>
<td>.1029 .0738</td>
<td>.0993 .1124</td>
<td>.0453 .0605</td>
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<tr>
<td>NU</td>
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<td>.0679 .0643</td>
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<tr>
<td>NN</td>
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<td>.8119 .7664</td>
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<tr>
<td>UE</td>
<td>.2932 .3367</td>
<td>.2720 .2795</td>
<td>.1640 .1897</td>
<td>.1444 .1447</td>
</tr>
<tr>
<td>UN</td>
<td>.2379 .1153</td>
<td>.2797 .2498</td>
<td>.3748 .1495</td>
<td>.4348 .3300</td>
</tr>
<tr>
<td>UU</td>
<td>.4689 .5480</td>
<td>.4483 .4707</td>
<td>.4612 .6608</td>
<td>.4208 .5253</td>
</tr>
</tbody>
</table>
Table 6

Markov Equilibrium and Actual Values
for Employment and Unemployment in Fiscal 1977
(in thousands)

<table>
<thead>
<tr>
<th>Sub-Group</th>
<th>Population</th>
<th>Markov Equilibrium</th>
<th>Actual</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Employment</td>
<td>Unemployment</td>
<td>Unemployment Rate</td>
<td>Employment</td>
</tr>
<tr>
<td>WM 16-19</td>
<td>6959</td>
<td>3885</td>
<td>679</td>
<td>14.9</td>
<td>3709</td>
</tr>
<tr>
<td>20-24</td>
<td>7962</td>
<td>6342</td>
<td>616</td>
<td>8.8</td>
<td>6236</td>
</tr>
<tr>
<td>WF 16-19</td>
<td>7017</td>
<td>3369</td>
<td>577</td>
<td>14.6</td>
<td>3182</td>
</tr>
<tr>
<td>20-24</td>
<td>8356</td>
<td>5145</td>
<td>520</td>
<td>9.2</td>
<td>5081</td>
</tr>
<tr>
<td>NM 16-19</td>
<td>1204</td>
<td>369</td>
<td>171</td>
<td>9.6</td>
<td>325</td>
</tr>
<tr>
<td>20-24</td>
<td>1184</td>
<td>728</td>
<td>200</td>
<td>21.5</td>
<td>723</td>
</tr>
<tr>
<td>NF 16-19</td>
<td>1285</td>
<td>278</td>
<td>142</td>
<td>33.3</td>
<td>256</td>
</tr>
<tr>
<td>20-24</td>
<td>1457</td>
<td>664</td>
<td>169</td>
<td>20.3</td>
<td>655</td>
</tr>
</tbody>
</table>

WM = white male
WF = white female
NM = Nonwhite male
NF = Nonwhite female
Table 7
Changes in Number of Job or Training Positions
Fiscal 1977-78
(thousands of person-years)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CETA Title 1</td>
<td>8.2</td>
<td>5.7</td>
<td>11.8</td>
<td>8.0</td>
<td>3.1</td>
<td>1.9</td>
<td>5.7</td>
<td>3.9</td>
</tr>
<tr>
<td>CETA Titles II &amp; VI</td>
<td>8.8</td>
<td>8.4</td>
<td>7.7</td>
<td>7.6</td>
<td>5.7</td>
<td>4.5</td>
<td>3.9</td>
<td>4.3</td>
</tr>
<tr>
<td>Job Corps</td>
<td>0.9</td>
<td>-0.1</td>
<td>0.1</td>
<td>0</td>
<td>2.0</td>
<td>-0.1</td>
<td>0.5</td>
<td>-0.1</td>
</tr>
<tr>
<td>SPDY²</td>
<td>2.6</td>
<td>1.2</td>
<td>2.2</td>
<td>1.1</td>
<td>3.2</td>
<td>1.6</td>
<td>2.7</td>
<td>1.4</td>
</tr>
<tr>
<td>YETP³</td>
<td>14.7</td>
<td>1.8</td>
<td>15.1</td>
<td>1.8</td>
<td>12.1</td>
<td>1.5</td>
<td>12.4</td>
<td>1.5</td>
</tr>
<tr>
<td>YIEPP⁴</td>
<td>0.5</td>
<td>0</td>
<td>0.6</td>
<td>0.1</td>
<td>2.7</td>
<td>0.3</td>
<td>3.0</td>
<td>0.4</td>
</tr>
<tr>
<td>YCCIP³</td>
<td>3.2</td>
<td>0.1</td>
<td>0.9</td>
<td>0</td>
<td>2.9</td>
<td>0.1</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>YACC³</td>
<td>8.1</td>
<td>0.6</td>
<td>4.6</td>
<td>0.4</td>
<td>7.1</td>
<td>0.5</td>
<td>3.9</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>47.0</td>
<td>17.7</td>
<td>43.0</td>
<td>19.0</td>
<td>38.8</td>
<td>10.3</td>
<td>32.9</td>
<td>12.0</td>
</tr>
</tbody>
</table>

1. Comprehensive Employment and Training Act
2. Summer Program for Disadvantaged Youth
3. Youth Employment and Training Program
4. Youth Incentive Employment Pilot Projects
5. Youth Conservation and Community Improvement Program
6. Young Adult Conservation Corps
<table>
<thead>
<tr>
<th>Group</th>
<th>Pre-program Markov Equilibrium</th>
<th>Post-program Markov Equilibrium</th>
<th>AHEM Estimates of Pre-program Distribution</th>
<th>Impact on Unemployment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Employment</td>
<td>Unemployment</td>
<td>NILF</td>
<td>UR^2</td>
</tr>
<tr>
<td>WM 16-19</td>
<td>3885</td>
<td>679</td>
<td>2295</td>
<td>14.9</td>
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<tr>
<td>WM 20-24</td>
<td>6312</td>
<td>616</td>
<td>1004</td>
<td>8.2</td>
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<tr>
<td>WF 16-19</td>
<td>3669</td>
<td>577</td>
<td>3071</td>
<td>14.6</td>
</tr>
<tr>
<td>WF 20-24</td>
<td>5145</td>
<td>520</td>
<td>2691</td>
<td>9.2</td>
</tr>
<tr>
<td>MF 16-19</td>
<td>369</td>
<td>171</td>
<td>664</td>
<td>31.6</td>
</tr>
<tr>
<td>MF 20-24</td>
<td>728</td>
<td>200</td>
<td>256</td>
<td>21.5</td>
</tr>
<tr>
<td>WF 16-19</td>
<td>278</td>
<td>142</td>
<td>665</td>
<td>33.6</td>
</tr>
<tr>
<td>WF 20-24</td>
<td>664</td>
<td>169</td>
<td>624</td>
<td>20.3</td>
</tr>
</tbody>
</table>

1. Not in the Labor Force
2. Unemployment Rate
3. Unemployment Rate with Program Participants Counted as Employed
4. Unemployment Rate with Participants Counted According to Official Definitions as Reflected in Table 10 for the Distribution of Enrollees by Program Given in Table
for the $i^{th}$ group) and the help wanted index, respectively. $U^*$ is the pre-
program equilibrium aggregate unemployment stock in fiscal 1977 (from Table
6). The parameter $\beta_{ij}$ is taken from econometric estimates of the relation
between $j^{th}$ transition probability for the $i^{th}$ group and the logarithm of the
ratio of the help wanted index to aggregate unemployment. The help wanted in-
dex $V_t$ is determined from a relation between the total stock of jobs ($J$) and
the aggregate employment stock ($E$) [$J$ and $E$ are measured in thousands].

$$V_t = K(J_t - E_t)$$

where $K$ is a scaling factor ($K = .05$). The aggregate unemployment level is
determined as an endogenous variable from the previous period's stocks and
the transition probabilities.

In performing the simulation, the labor market for non-program partici-
pants was modelled. All program job slots were assumed to be filled immedi-
ately (since the timing was of less interest that the resulting equilibrium).
The program participants were subtracted out from the population totals for
each group. Twenty percent of the participants were drawn from among the
employed, thirty-four from among the unemployed, and the rest from out of
the labor force. The job stock for the first period of the simulation was
then set equal to the sum of the 1977 fiscal year estimated vacancies ($V^*/K$)
plus 1977 fiscal year equilibrium aggregate employment (from Table 6), plus
estimated new job vacancies assumed to be created by program participants
leaving jobs to enter the program. The number of new job vacancies was
assumed to be 20 percent of the total stock of program jobs (the number of
workers who left jobs to enter the program). The simulation program was
then run until a flow equilibrium was obtained for the non-participant's
labor market. The resulting distributions for the sixteen groups with the
implied unemployment rates are reported in Table 9. Four unemployment rates
### Table 9

Markov Estimates  
(with variable transition probabilities)

<table>
<thead>
<tr>
<th>Group</th>
<th>Program</th>
<th>Employment</th>
<th>Unemployment</th>
<th>NILF (^1)</th>
<th>UR(^2)</th>
<th>UR(^3)</th>
<th>UR(^4)</th>
<th>(\Delta UR(^1))</th>
<th>(\Delta UR(^2))</th>
<th>(\Delta UR(^3))</th>
<th>(\Delta UR(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>WM 16-19</td>
<td>47</td>
<td>3918</td>
<td>638</td>
<td>2356</td>
<td>14.0</td>
<td>13.9</td>
<td>14.9</td>
<td>14.1</td>
<td>-0.9</td>
<td>-1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>WM 20-24</td>
<td>18</td>
<td>6385</td>
<td>558</td>
<td>1001</td>
<td>8.0</td>
<td>8.0</td>
<td>8.3</td>
<td>8.1</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-0.5</td>
</tr>
<tr>
<td>WF 16-19</td>
<td>43</td>
<td>3391</td>
<td>553</td>
<td>3030</td>
<td>14.0</td>
<td>13.9</td>
<td>15.0</td>
<td>14.1</td>
<td>-0.6</td>
<td>-0.7</td>
<td>+0.4</td>
</tr>
<tr>
<td>WF20-24</td>
<td>19</td>
<td>5176</td>
<td>493</td>
<td>2668</td>
<td>8.7</td>
<td>8.7</td>
<td>9.0</td>
<td>8.7</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.2</td>
</tr>
<tr>
<td>NM 16-19</td>
<td>39</td>
<td>365</td>
<td>161</td>
<td>639</td>
<td>30.6</td>
<td>28.5</td>
<td>35.4</td>
<td>29.7</td>
<td>-1.0</td>
<td>-3.1</td>
<td>+3.8</td>
</tr>
<tr>
<td>NM 20-24</td>
<td>10</td>
<td>736</td>
<td>184</td>
<td>253</td>
<td>20.0</td>
<td>19.8</td>
<td>20.9</td>
<td>19.9</td>
<td>-1.5</td>
<td>-1.7</td>
<td>-0.6</td>
</tr>
<tr>
<td>NF 16-19</td>
<td>33</td>
<td>273</td>
<td>135</td>
<td>844</td>
<td>33.0</td>
<td>30.6</td>
<td>38.0</td>
<td>32.1</td>
<td>-0.8</td>
<td>-3.2</td>
<td>+4.2</td>
</tr>
<tr>
<td>NF 20-24</td>
<td>12</td>
<td>664</td>
<td>168</td>
<td>613</td>
<td>20.2</td>
<td>19.9</td>
<td>21.3</td>
<td>20.2</td>
<td>-0.1</td>
<td>-0.4</td>
<td>+1.0</td>
</tr>
</tbody>
</table>

---


2. \(UR\(^1\), UR\(^2\), UR\(^3\), are unemployment rates measured with program participants counted as out-of-the-labor force, employed, and unemployed, respectively. \(UR\(^4\)\) is the unemployment rate measured with program participants counted as in the official statistics for the distribution of participants by program type given in Table 10.

3. \(\Delta UR\(^1\), \Delta UR\(^2\), \Delta UR\(^3\), and \(\Delta UR\(^4\)\) are the differences between the post-program unemployment rate (e.g., \(UR\(^1\)\)) and the pre-program Markov equilibrium unemployment rate (in Table 8).
were computed. Three of them, UR1, UR2, and UR3, correspond to counting program participants as out-of-the-labor force, employed, or unemployed. The fourth statistic, UR4, represents the unemployment rate which would result if program participants were counted as in the official statistics and the mix of programs was that which obtained in the expansion of youth programs which occurred in fiscal 1978. The assumed distribution of participants for each program across demographic group is provided in Table 7 and the assumptions regarding how the participants were classified are reported in Table 10.

As can be seen from a comparison of the data in Table 9 with the data in Table 8, allowing the transition probabilities to respond to changes in labor market conditions has reduced the unemployment rates for all groups. Unemployment rates U2 and U4 in Tables 8 and 9 are computed under the assumptions that program participants are counted as employed and counted as in the official statistics, respectively. The reductions in official unemployment rates, reported in Table 8 for transition probabilities constant, ranged from a reduction of 1 percentage point for non-white teenage males and females to increases of 0.1 percentage point for several groups (WM 16-19, WM 20-24, WF 16-19). With transition probabilities allowed to vary, equilibrium unemployment rates were lower than the rates obtained in the simulation with constant probabilities for all groups. As reported in Table 9, the changes ranged from reductions of 1.9 percentage points for non-white males 16-19 to 0.5 percentage points for white females 16-19 and 20.24.

In comparing the results in the two Tables 8 and 9, it appears that reductions in unemployment rates (∆U4) reported in Table 8 are concentrated in two groups, non-white males and females aged 16-19. The direct effects on the other groups are negligible, with some groups even experiencing
Table 10

Classification of Participants
Program

<table>
<thead>
<tr>
<th></th>
<th>Percent</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Employed</td>
<td>Unemployed</td>
<td>Not-in-the Labor Force</td>
</tr>
<tr>
<td>FY 1977</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CETA Title I</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>CETA Titles II &amp; VI</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job Corps</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>SPDY</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FY 1978</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CETA Title I</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>CETA Titles II &amp; VI</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job Corps</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>YETP</td>
<td></td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>YICCCIP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>YIEPP</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YACC</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Program participants in public or private jobs or receiving training allowances or classified as employed except for Job Corps participants who are classified as out-of-the labor force; participants in classroom training or receiving transition services are classified as unemployed.
slight increases in measured unemployment. However, the figures in Table 9 reveal quite a different picture. While the two non-white teenage groups lead in terms of reductions, non-white males aged 20-24 also experience substantial gains. Even the white groups which experienced no reductions in unemployment from direct impacts seem to gain substantially from the indirect effects, with reported reductions ranging from 0.5 to 0.8 percentage points.

The only group which does not appear to benefit from the indirect effects is the older non-white women (20-24). They experience no change in unemployment rate from the direct impacts, chiefly because of the small number of jobs targeted at this group (12 thousand); however, when the indirect effects are added in they still experience a reduction in unemployment rate of only 0.1. This situation is in sharp contrast to that for older non-white males who gain markedly from the indirect effects. Non-white males aged 20-24 experience a reduction in their unemployment rate of only 0.1 percentage points from direct effects (Table 8), but when indirect effects are added, the reduction jumps to 1.6 percentage points.

This difference between the two older non-white groups is partly a result of the fact that the employment situation is improved for males more than for females by indirect effects but also a result of the fact that female labor force participation rate increases by more than the male rate does. For example, as a result of indirect effects, the employment to population ratio increases for older non-white males from .617 to .629, while the increase for females is only from .458 to .462; the female labor force participation rate increases from 57.5 to 57.9 while the male rate increases slightly from 78.5 to 78.6.
VI. SUMMARY AND CONCLUSION

In this paper a Markov transition probability model has been applied to estimate the direct and indirect impacts of employment and training programs on unemployment. Impacts are defined in terms of changes in a Markov equilibrium. Such an equilibrium can be defined with and without the presence of a program or set of programs. To introduce a program, a fourth Markov state was defined in addition to the labor market states of employment, unemployment, and not in the labor force states. Transition probabilities into and out of the program state were defined in terms of the distribution of program enrollees by labor market state prior to entering and after leaving the program and the average length of time spent in the program. The general case of the Markov equilibrium was then derived.

A special case of the general model in which the distribution of non-participants was independent of the size or existence of the program was shown to exist if enrollees return on average to the same states occupied before they entered the program. In this case, the resulting equilibrium distribution of non-participants was shown to be independent of the length of time spent in the program and the pre- and post-program distribution of enrollees by labor market state.

The Markov model results were compared with an alternative estimation procedure used by previous investigators called the AHEM procedure. The AHEM procedure was found to give unbiased estimates only when selection into the program was independent of labor market state occupied, enrollees return on average to the same states occupied prior to entrance into the program, and transition probabilities are constant. The AHEM procedure was
found to overestimate a program's impact if intake favored the unemployed and transition probabilities were constant.

Finally, the impact of allowing transition probabilities to respond to changing conditions in the labor market was investigated. The introduction of a program tightens the labor market by increasing job vacancies as employers replace program enrollees who had jobs prior to entering the program and the number of unemployed job seekers declines as the unemployed enter the program. Labor force participation and hiring rates were both expected to increase as a result of program induced tightening of the labor market. These impacts were estimated using a monthly econometric model of the U.S. labor market. Programs were found to have a greater effect in reducing unemployment rates when the transition probabilities were allowed to change them when they were held constant for all sixteen demographic groups examined.

The programs simulated were designed to replicate the Department of Labor youth programs introduced under the Youth Employment and Demonstration Projects Act of 1977. With transition probabilities constant, the estimated impact on unemployment rates ranged from +0.1 to -1.0 percentage points while with the probabilities variable, the impacts ranged from -0.1 to -2.1. The greatest gains were for non-white youth aged 16-19. When direct effects alone were considered, the impact was concentrated among two groups: non-white males and female teenagers. When indirect effects were also considered, the impacts were more broadly diffused substantially affecting all groups but non-white females aged 20-24.

The Markov model has been found to be a useful tool in assessing program impacts. Indirect impacts can be very simply assessed under two assumptions: (1) transition probabilities are constant and (2) enrollees return on average to the same states occupied prior to entering the program. Unlike the AHEM
procedure, the Markov method requires no information on the prior labor market state of program enrollees when these two assumptions are met. By using an econometric model, the assumption of constant transition probabilities may be relaxed and indirect program effects on the labor market investigated.
Appendix

In this appendix it will be demonstrated that under certain assumptions the equilibrium conditions governing the distribution of non-participants across states for the general four state model discussed in section II are identical to those which hold in the 3 state model without the program in place. First, let us re-write the equilibrium condition for the four state model as it appears in section II, in matrix algebra with the vectors and matrices partitioned:

\[(A1) \quad \Pi = (\Pi_1 \vert \Pi_2) = (\Pi_1 \vert \Pi_2) \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \]

where \( \Pi_1 = (\pi_E \pi_U \pi_N) \)
\( \Pi_2 = \pi_G \)
\( P_{11} = P^* (I - \ell^{-1} \Pi_2 RS) \)
\( P_{12} = \ell^{-1} \Pi_2 RS_i \)
\( P_{21} = \ell^{-1} i Z \)
\( P_{31} = 1 - \ell^{-1} \)

\[(A2) \quad \Pi_1 i' + \Pi_2 = 1 \]

and \( I \) is a 3x3 identity matrix; \( i \) is a row vector of 1's; \( \ell \) is the length of stay variable discussed in section II; \( P^* \) is a 3x3 matrix of transition probabilities that determine transitions in the absence of the program; \( R \) is a diagonal matrix with the reciprocals of the elements of the state vector \( \Pi_1 \) on the diagonal.
\[
R = \begin{pmatrix}
\Pi^{-1}_E & 0 & 0 \\
0 & \Pi^{-1}_U & 0 \\
0 & 0 & \Pi^{-1}_N 
\end{pmatrix}
\]

S and Z are diagonal matrices defined as

\[
S = \begin{pmatrix}
S^E & 0 & 0 \\
0 & S^U & 0 \\
0 & 0 & S^N 
\end{pmatrix} \quad Z = \begin{pmatrix}
Z^E & 0 & 0 \\
0 & Z^U & 0 \\
0 & 0 & Z^N 
\end{pmatrix}
\]

and \(S^i\) and \(Z^i\) (i = E, U, N), are the parameters that determine the distribution of enrollees by pre- and post-program state as discussed in section II.

Expanding equation (A1) we get

\[
(A3) \quad \Pi = (\Pi_1 | \Pi_2) = (\Pi_1 P_{11} + \Pi_2 P_{21} | \Pi_1 P_{12} + \Pi_2 P_{22})
\]

which implies the following equalities:

\[
(A3.1) \quad \Pi_1 = \Pi_1 P_{11} + \Pi_2 P_{21} = \Pi_1 P^* (I - \xi^{-1} \Pi_2 RS) + \Pi_2 \xi^{-1} i Z
\]

\[
(A3.2) \quad \Pi_2 = \Pi_1 \xi^{-1} \Pi_2 RS_i + \Pi_2 (1 - \xi^{-1})
\]

The labor market for the three non-program states will have an equilibrium that is independent of the program state if and only if that equilibrium is the same as what it would be in the absence of the program. Without the program, the equilibrium would be described by

\[
(A4) \quad \Pi^* = \Pi^* F^*
\]

where \(\Pi^* = (\Pi^*_E \Pi^*_U \Pi^*_N)\);

\[
(A5) \quad \Pi^*_i = 1
\]

If the distribution of persons among the non-program states in the 4 state model is identical to that for all persons in the 3 state model without the program, if and only if
(A6) \[ \Pi_1 = \delta \Pi^* \quad ; \quad \delta > 0 \]

by definition of a state distribution. The adding up constraints in the 3 state and 4 state model are consistent with one and only one value for \( \delta \), namely, \( \delta = 1-\Pi_2 \). This result follows from the identities

(A2) \[ \Pi_1 \mathbf{i}' + \Pi_2 = 1 \]

(A5) \[ \Pi^* \mathbf{i}' = 1 \]

Substituting (A6) and (A5) sequentially into (A2) and solving for \( \delta \) gives

(A7) \[ \delta = 1-\Pi_2 \]

Substituting (A7) into (A6) we see that relation between the state vectors \( \Pi_1 \) and \( \Pi^* \) must be:

(A8) \[ \Pi_1 = (1 - \Pi_2) \Pi^* \]

Necessary and sufficient conditions for the distribution of non-participants in the presence of the program to be identical to the distribution of all persons in the absence of the program are equations (A8), (A4), (A1), and (A2). The equations (A1) and (A4) state the conditions for flow equilibria; equation (A2) states the required adding up constraint for the 4-state model; equation (A8) gives the required relation between the two state vectors \( \Pi_1 \) and \( \Pi^* \) if the distributions are to be identical and the adding up constraints in both models are binding.

Substituting (A8) and (A4) into (A3) [the expanded version of (A1)], gives the following transformed equation for the sub-matrix \( \Pi_1 \):

(A3.1') \[ \Pi_1 = \Pi_1 (I - \lambda^{-1} \Pi_2 RS) + \Pi_2 \lambda^{-1} iZ \]
Necessary and sufficient conditions for (A3.1') will now be derived.

Expanding and rearranging terms simplifies (A3.1') to

\[(A9) \quad \lambda^{-1} \Pi_2 \Pi_1 RS - \lambda^{-1} \Pi_2 iZ = \emptyset\]

Since \(R\) is by definition a diagonal matrix of reciprocals, it follows that \(\Pi_2 R = i\); substituting this equality into (A9) and eliminating terms gives

\[(A10) \quad iS - iZ = \emptyset\]

Since \(S\) and \(Z\) are diagonal matrices, \(S = Z\) is sufficient and necessary for (A10).

These conditions are met only when the pre- and post-program distribution of enrollees is identical (i.e., \(S_i = Z_i\) for all \(i\)). Put less precisely, the condition is that program enrollees return on average to the same states they occupied prior to the program. It follows from the sufficiency of the condition that all Markov models with \(S = Z\) and constant \(P^*\) (assumptions 2 and 3 in the text) will have an equilibrium distribution of non-participants in the presence of the program which is identical to the equilibrium distribution of all persons in the absence of the program. It follows from the necessity that in the class of Markov models with constant \(P^*\) only those in which \(S = Z\) will produce the equality of the two distributions.
Footnotes

1. Alternatively, one can interpret $\lambda$ as the average length of time spent in the program with $P_{G0}$ constant for all. For large samples, the mean duration will approach $1/P_{G0}$. A more complex model would allow $P_{G0}$ and $\lambda$ to vary by type of participant. The types of models we will use in simulation will in fact allow variation in transition probabilities by age, race, and sex.

2. Alternatively, $P_{ij}^*$ can be thought of as the aggregation of transition probabilities for participants and non-participants. The fraction of persons in state $i$ who become participants is $P_{iG}$, and the fraction who do not is $(1-P_{iG})$; the transition probability for non-participants is $P_{ij}$ and that for participants is $0$.

\[
P_{ij}^* = P_{iG} \times 0 + (1-P_{iG}) \times P_{ij} = (1-P_{iG})P_{ij}
\]

3. For a detailed description of how those distributions were derived, see Toikka (1979).

4. These rates were computed by counting program participants as they would be counted in the official statistics. Based on the figures in Table 7, of 10 thousand non-white males aged 20-24, 1 thousand are counted as unemployed, while 9 thousand are counted as employed; the corresponding allocation for the 12 thousand non-white females is 10 thousand employed and 2 thousand unemployed. The tendency for males to be in jobs programs more frequently than females contributes in a small degree to the difference in gains from direct effects of programs between the two groups but should not affect the relative experience of each group in gaining from indirect effects since these result from non-program jobs. For example, when all program participants are counted as employed as in the computation of $U2$, older non-white males experience a change of -1.7 percentage points in their unemployment rate with indirect effects as compared with -0.2 with only direct effects; the corresponding figures for females are -0.4 and -0.3.

5. The remaining condition (A3.2) must hold if (A3.1') holds since with the distribution across 3 states given, the state element for the fourth state may be determined from an identity constraint (A2) which is always binding in a Markov model.
References


