Fifteen research reports related to mathematics education are abstracted and analyzed. Six of the reports deal with aspects of learning theory, four with areas in mathematics instruction (calculus, elementary mathematics for students of economics, and planning for topics for kindergarten children), and two with assessment or prediction of mathematics achievement. Remaining reports deal with adolescent attitudes towards mathematics, the effect of computer assisted instruction (CAI) on student attitudes, and the effectiveness and efficiency of a mathematics laboratory in improving student learning and attitude. Research related to mathematics education which was reported in CIJE and RIE between July and September 1974 is also listed. (MF)
INVESTIGATIONS
IN
MATHEMATICS
EDUCATION

Expanded Abstracts
and
Critical Analyses
of
Recent Research

Center for Science and Mathematics Education
The Ohio State University
in cooperation with
the ERIC Science, Mathematics and Environmental Education Clearinghouse
Callahan, Walter J.  Adolescent Attitudes Toward Mathematics.  
Abstracted by LEWIS R. AIKEN 15

Abstracted by THEODORE EISENBERG 19

Abstracted by JAMES M. SHERRILL 23

Abstracted by MARY E. MONTGOMERY 27

Abstracted by OTTO C. BASSLER 29

Abstracted by JOHN C. PETERSON 33

and  
Abstracted by JOHN G. HARVEY 37
ED 087 983  House, Peggy A. and Moore, Arnold J. The Learning Environment as a Predictor of the Academic Self-Concepts of Ninth Grade Mathematics Students. 9p. MF and HC available from EDRS.

ED 088 /10  A Pilot Mathematics Assessment of Arizona Sixth-Grade Students. 13p. Not available from EDRS. Available from ERIC/SMEAN, Ohio State University, 1200 Chambers Road, Columbus, Ohio 43212.

ED 088 712  Walzl, F. Neil. Using a Potential Method of Maintaining the Basic Skills of Arithmetic Through the Summer Months to Reduce the Review Time in the Fall. 28p. MF and HC available from EDRS.

ED 088 714  Ronshausen, Nina L. The Effect on Mathematics Achievement of Programmed Tutoring as a Method of Individualized, One-to-One Instruction. 19p. MF and HC available from EDRS.

ED 088 715  Mayer, Richard E. Acquisition Processes for Mathematical Knowledge. 13p. MF and HC available from EDRS.

ED 088 716  Uprichard, A. Edward and Collura, Carolyn. The Effect of Emphasizing Mathematical Structure in the Acquisition of Whole Number Computation Skills (Addition and Subtraction) By Seven- and Eight-Year Olds: A Clinical Investigation. 13p. MF and HC available from EDRS.


ED 088 915  Durward, M. Lynne. Computer-Assisted Instruction in Arithmetic at South Hill Elementary School. 33p. MF and HC available from EDRS.

ED 088 931  Klausmeier, H. J. and others. An Individually Administered Test To Assess Level of Attainment and Use of the Concept Equilateral Triangle. 38p. MF and HC available from EDRS.

ED 088 941  Austin, Gilbert R. and Postlethwaite, T. Neville. Cognitive Results Based on Different Ages of Entry to School: A Comparative Study. 23p. MF and HC available from EDRS.

ED 088 981  Thorndike, Robert L. The Relation of School Achievement to Differences in the Backgrounds of Children. 17p. MF and HC available from EDRS.
ED 089 785 Mitzel, Harold E. and Nortia, Kenneth H. The Development and Presentation of Four Different College Courses by Computer Teleprocessing. Interim Report. 112p. MF and HC available from EDRS.


ED 089 824 Grunes, Leslie S. A Critical Study of the Attrition in the Mathematics Courses at Mercer County Community College. 53p. MF and HC available from EDRS.

ED 089 874 Robinson, Violet B. An Investigation of the Performance of Kindergarten Children on Quantitative Class Inclusion Tasks. 12p. MF and HC available from EDRS.

ED 089 943 Cook, J. Marvin. Learning and Rate of Forgetting When Teachers are Informed of Behavioral Objectives. Final Report. 63p. MF and HC available from EDRS.


ED 089 991 Kerr, Donald R., Jr. Final Report, Gary SEED Evaluation Project. 108p. MF and HC available from EDRS.

ED 089 992 Schall, William E. and others. Developing Mathematical Processes (DMP): Field Test Evaluation, 1972-73. 59p. Not available from EDRS. Available from ERIC/SMEA, Ohio State University, 1200 Chambers Road, Columbus, Ohio 43212.

ED 089 997 Aiken, Lewis R., Jr. Affective Variables and Sex Differences in Mathematical Abilities. 11p. MF and HC available from EDRS.

ED 089 998 Fennema, Elizabeth. Mathematics, Spatial Ability and the Sexes. 17p. MF and HC available from EDRS.

ED 089 999 Denny, Rita T. The Mathematics Skill Test (MAST) as Rostering and Diagnostic Tools. 10p. MF and HC available from EDRS.
ED 090 023  Prekeges, Demitrios P.  Relationship Between Selected Teacher Variables and Growth in Arithmetic in Grades Four, Five and Six.  Final Report.  184p.  MF and HC available from EDRS.

ED 090 026  Geeslin, William E.  An Analysis of Content Structure and Cognitive Structure in the Context of a Probability Unit.  33p.  MF and HC available from EDRS.

ED 090 035  Shavelson, Richard J.  Some Methods for Examining Content Structure and Cognitive Structure in Mathematics Instruction.  16p.  MF and HC available from EDRS.

ED 090 037  Eisen, Francine Abrams.  The Relationship Between Activation and Logical Inference Performance.  23p.  Only MF available from EDRS.

ED 090 038  Lindvall, C. M. and Light, Judy A.  The Use of Manipulative Lessons in Primary Grade Arithmetic in a Program for Individualized Instruction.  16p.  MF and HC available from EDRS.


ED 090 041  Smith, Emma Drucilla Breedlove.  The Effects of Laboratory Instruction Upon Achievement in and Attitude Toward Mathematics of Middle School Students.  142p.  Not available from EDRS.  Available from University Microfilms (74-417).


ED 090 044  Kaspi, Moshe.  Consistency of Results in Measuring Growth Patterns of School Achievement in Longitudinal Studies.  104p.  Not available from EDRS.  Available from University Microfilms (74-1541).


ED 090 048  Sigmund, Thomas F.  *Design and Demonstration of a Methodology for Diagnostic Teaching by a Teaching Team.*  188p.  Not available from EDRS.  Available from University Microfilms (74-2112).


ED 090 050  McGlone, Virginia E.  *The Mathematics Laboratory: An Integral Part of the Pre-service Training of Elementary School Teachers.*  31p.  MF and HC available from EDRS.

ED 090 279  Kulm, George.  *The Effects of the Two Summative Evaluation Methods on Achievement and Attitudes in Individualized Seventh-Grade Mathematics.*  18p.  MF and HC available from EDRS.

ED 090 315  Ross, G. Robert, Jr. and Fletcher, Harold J.  *Some Personality Correlates of Logical Reasoning Ability.*  30p.  MF and HC available from EDRS.

ED 090 456  Anderson, Lorin W.  *Student Involvement in Learning and School Achievement.*  28p.  MF and HC available from EDRS.

ED 090 927  Hulten, Burma H.  *Games and Teams: An Effective Combination in the Classroom.*  20p.  MF and HC available from EDRS.

ED 090 970  Danforth, Douglas G. and others.  *Learning Models and Real-Time Speech Recognition.*  53p.  MF and HC available from EDRS.


Beers, Morris Irving. An Evaluation of Diagnosis and Remediation Based on a Mathematics Readiness Test for Entering First Grade Students. 145p. Not available from EDRS. Available from University Microfilms (74-4615).

Matthews, Frank F. An Investigation of the Feasibility of the Use of Students' Perceived Needs to Control the Rate of Instruction. 12p. MF and HC available from EDRS.

Suydam, Marilyn N. and Weaver, J. F. Research on Mathematics Education (K-12) Reported in 1973. 83p. MF and HC available from EDRS.

Stanton, George and Pelavin, Sol. Representation of Subject Matter in Teachers' and Students' Memories. 23p. MF and HC available from EDRS.

McIntosh, Jerry A. and Crosswhite, F. Joe. A Survey of Doctoral Programs in Mathematics Education. 48p. MF and HC available from EDRS.

Reiss, Douglas J. and others. The Effects of Contingent Use of Recess Activities on the Academic Performance of a Third Grade Classroom. 12p. MF and HC available from EDRS.

Peterson, John C. and Hancock, Robert R. Developing Mathematical Materials for Aptitude Treatment Interaction. 34p. MF and HC available from EDRS.
Mathematics Education Research Studies
Reported in Journals as Indexed
by Current Index to Journals in Education

EJ 093 384  Sheppard, John L. Concrete operational thought and developmental aspects of solutions to a task based on a mathematical three group. Developmental Psychology, v10 n1, pp116-123, Jan 74.


EJ 093 514  Harris, Margaret L.; Romberg, Thomas A. An analysis of content and task dimensions of mathematics items designed to measure level of concept attainment. Journal for Research in Mathematics Education, v5 n2, pp72-86, Mar 74.


EJ 095 292  Strong, David H. The effect of class size upon drop-out rate. MATYC Journal, v8 n2, pp7-8, Spr 74.

EJ 095 293  Van Druff, John C. Prediction of success of community college students in calculus in the state of Washington. MATYC Journal, v8 n2, pp8-10, Spr 74.


EJ 096 156  Melnick, Gerald; And Others. Piagetian tasks and arithmetic achievement in retarded children. Exceptional Children, v40 n5, pp358-361, Feb 74.

EJ 096 160  Spradlin, Joseph E.; And Others. Performance of mentally retarded children on pre-arithmetic tasks. American Journal of Mental Deficiency, v78 n4, pp397-403, Jan 74.


EJ 096 853  Austin, Joe Dan; Austin, Kathleen A. Homework grading procedures in junior high mathematics classes. School Science and Mathematics, v74 n4, pp269-272, Apr 74.

EJ 096 899  Merritt, Paul W. Some research support for a second chance for beginning algebra students. Two-Year College Mathematics Journal, v5 n2, pp50-54, Spr 74.


Descriptors—*Secondary School Mathematics, Student Attitudes, Attitude Tests, Grade 8, Mathematics Education, Surveys*

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, Sacred Heart College.

1. **Purpose**

   As a middle-school mathematics teacher, the author of this article felt a strong need to find ways to help his students enjoy mathematics and succeed in the subject. He was concerned with answers to the questions as to why some adolescents enjoy mathematics while others strongly dislike the subject, when attitudes toward mathematics develop, what adolescents like most and what they like least about mathematics, and how to prevent and change negative attitudes toward mathematics.

2. **Rationale**

   The results of an earlier survey by Dutton (School Review, Jan. 1956) of the attitudes toward mathematics of 459 junior-high school students are cited. Dutton's findings are interpreted as indicating that attitude has a great effect on the amount of work done, the effort expended, and the learning accomplished in junior-high mathematics classes. This investigation is a replication of Dutton's survey of the attitudes toward mathematics, reasons for liking or disliking the subject, and when the attitudes developed in a sample of eighth graders in 1956.

3. **Research Design and Procedure**

   The sample consisted of 186 girls and 180 boys in a Saratoga County, N.Y. junior high school. An inventory constructed by Wilbur Dutton, consisting of 22 Thurstone-type statements of attitude toward mathematics and a second section designed to obtain general information on the likes and dislikes of the respondents, was administered orally and anonymously by their mathematics teachers to the sample of 366 eighth graders.

4. **Findings**

   The results of the study are presented as the numbers and percentages of boys, girls, and total respondents checking each statement in the inventory. To give an indication of the findings, 6% of the students checked the statement "I detest math and avoid it at all times."
70% checked "I enjoy doing problems when I know how to work them well," and 54% checked "I am not enthusiastic about math, but I have no real dislike for it either." Respondents also checked one number on a scale ranging from 1 to 11 to indicate their general feelings toward mathematics. Among the results here were that 44 boys and 28 girls indicated a strong like but 12 boys and 7 girls indicated a strong dislike for mathematics. The investigator noted that the large percentage of respondents indicating that their general feeling toward mathematics was positive did not agree with the responses to specific items. Dutton's conclusion that pupils tend to overrate themselves on general feelings toward arithmetic was concurred with by the investigator in the present study.

In a third part of the survey, students were asked to check their strongest reason for liking or disliking mathematics, depending on whether they generally liked or disliked the subject, respectively. Among the findings were that 98 pupils indicated that they liked mathematics because they "Need math in life, future needs, practical applications." The largest number of students who generally disliked mathematics checked as their strongest reason "Not good in math, don't learn easily, not sure of myself" (36). The students were asked to estimate when their feelings toward mathematics developed. Of the 287 who felt that they knew, the modal grade level was seventh grade and the median grade level 6.4. The students were also asked to list the mathematics topics that they liked or disliked most and to answer "Yes" or "No" to the question "Have your feelings toward mathematics changed in the past year?" Fractions, addition, multiplication, geometry, and subtraction, in that order, were the most liked or disliked topics. Fifty-one percent (186) students indicated that their feelings toward mathematics had changed during the past year, whereas 48% (174) felt that they had not.

5. **Interpretations**

The investigator interpreted the results of this survey as showing that approximately 5% of the students had an extreme dislike for mathematics, 70% enjoyed doing problems when they knew how to work them well, and 66% felt that mathematics is as important as any other subject. It was concluded that the girls and boys were approximately equal in like or dislike for mathematics, although the girls showed a much stronger dislike for word problems. The main reasons given for liking mathematics were its practical aspects and the realization that it is needed in life. Those who disliked the subject gave as reasons their inability to do mathematics and the fact that it is boring and repetitious. Grades six and seven were reported as the most critical time for the development of attitudes toward mathematics.

**Abstractor's Notes**

This was a rather limited survey, involving only a single school. Consequently, although the results were undoubtedly interesting and
useful to the particular teachers and schools involved, it would be hazardous to overgeneralize the findings to other school systems. An especially serious shortcoming in selecting the sample is that no urban or inner-city schools were included. It is quite likely that the socioeconomic character of the sample was biased in favor of the middle class.

In addition to the survey sample, there are difficulties with the assessment instrument employed, the administration procedure, and the method of scoring. The fact that the questionnaires were administered by the students' mathematics teachers may have biased the responses in the favorable direction. There is a well-documented tendency for students to respond in a teacher-approved direction, unless they are greatly disturbed by the subject, the teacher, or the school situation. Among the shortcomings of the Thurstone-type instrument constructed by Dutton are: scale values based on twenty-year-old data; obvious failure of the scale values of statements to be equidistant when placed in order (the distribution of scale value intervals is highly positively skewed, ranging from .1 to .9); the instrument was standardized on a very limited sample and no norms are reported; the fact that students may check more than one statement introduces a response set of unknown effect. In scoring this scale, not only should the median of all items endorsed by each student have been computed for all students, but in computing these scores some provision should have been made to cope with the response set caused by the fact that different students checked different numbers of statements.

Inspection of Table 1 reveals that the range of responses (from like to dislike) is greater for boys than for girls, and the overall median for boys (6.40) is slightly higher than that for girls (6.23). Also, girls tend more than boys to check statements containing words or phrases such as "afraid, don't feel sure, like about as well as other subject," whereas boys more often than girls check statements containing "atest, never liked, not much value, avoid, have to do, are practical." Such findings are intriguing, especially in the light of current interest in sex differences in mathematics learning. Unfortunately, the investigator overlooked most of them and did little with the sex differences findings.

The abstractor's computations of the medians of results reported in Table 2 (estimates of general feelings toward mathematics) gives a median of 7.38 for boys, 7.09 for girls, and 7.26 total. These numbers are significantly higher than the medians computed from the Table 1 data: 6.40 for boys, 6.23 for girls, and 6.31 total, confirming the investigator's conclusion that the students probably overrated themselves on general attitude toward mathematics. This finding should not be surprising, however, it is a well-known result of rating scale research that an overall rating is usually higher than the average of the ratings on specific items of the scale. The leniency error apparently manifests itself more strongly in general ratings.
The finding that eighth graders tend to view the sixth or seventh grade as the time when their attitudes toward mathematics developed is consistent with the abstractor's own data. It would be careless to make too much of this finding, however. Eighth graders' memories of the sixth and seventh grades are undoubtedly better than their memories of pre-sixth grade feelings. And as any psychologist knows, retrospective accounts of development on the part of the developer himself are seldom veridical. Finally, the fact that approximately half of the respondents recognized a change in their attitudes during junior high school probably means little; a 50-50 split is a chance occurrence.

Lewis R. Aiken
Sacred Heart College


Expanded Abstract and Analysis Prepared Especially for I.M.E. by Theodore Eisenberg, Northern Michigan University.

1. **Purpose**

   Consider the following tasks:

   ![Diagram of classification tasks]

   Subjects were to choose correct responses for the indicated vacant cells from a circular array of objects.

   The authors developed a flow diagram of the thought processes involved in each of the above classification tasks. They reasoned that all behaviors needed to solve Task 1 were also needed, along with other behaviors, to solve Task 2, and those needed to solve Task 2 were prerequisite for Task 3. It was hypothesized that the optimal order for learning the three tasks would be 1, 2, 3, with Task 1 being the easiest to learn and Task 3 the hardest. Thus, the authors compared the order of instruction 1, 2, 3 with the reverse order 3, 2, 1.

   Another hypothesis was that their flow diagram of thought processes would be valid if students who had mastered Task 3 (regardless of the sequence of instruction) performed better on a 2x2 matrix with one vacant cell than students who failed to learn Task 3.

2. **Rationale**

   This experiment was concerned with sequencing and consequently Gagné's work with hierarchical learning structures. Much has been done in this area in the past. But as the authors state: "In the past, methods of task analysis for hierarchy studies have concentrated on relatively gross and largely overt aspects of the behaviors involved, and attempts to validate hierarchies have been most successful when relations have been sought among relatively large units of behavior rather than among finely differentiated tasks." The study explicated
In detail, via the flow chart, the hypothesized behavior of skilled performers of the tasks. The study is also tied in with Piaget's work on classification and seriation.

3. Research Design and Procedure

Twenty-six kindergarteners participated in the study. The children were from an urban school, predominantly black and from families on the low end of the economic scale. The experimenter was a white male and all instruction took place in a hallway outside the students' classroom.

By pretesting it was determined that all students in the study had the necessary requisites to learn the three tasks. Moreover, no student at the outset of the study could perform any of the required tasks.

Selection of the students and procedures of the study were as follows:

a) Students were given familiarization tasks which were designed to acquaint them with the nature of matrix boards. Similarities within various rows and columns were pointed out. Children who could not name the colors or shapes of the objects to be used in the experiment were dropped from the study.

b) A pretraining task shown in previous experiments to be necessary for Task 1 was given to all students. Students who did not reach criterion on this task (9 out of 10 correct responses) were dropped from the study.

c) The 2x2 matrix with one vacant cell was also used to screen subjects. Four such matrices were presented to the student. A criterion of no more than two correct was established for a failing score and acceptance into the study.

d) The twenty-six students chosen for the study were divided into two groups of 13 each. One group received instruction for solving the tasks in the order of 1, 2, 3. The other group received training in the reverse order 3, 2, 1.

e) Instruction proceeded in game format. "In this game someone has already taken away some of the pieces. I want you to point to the piece over here (E pointed to response choice array) that belongs here (E pointed to the missing cell)." Incorrect responses were examined and row and column similarities pointed out. The procedure was repeated until one of the following appeared: (1) the subject gave 9 out of 10 consecutive correct responses, (2) a total of 90 trials had elapsed or (3) a total of three training sessions of 15-20 minutes each had been completed. Subjects then moved to the next task. At the end of the sequence the 2x2 matrix was given to the student.
4. **Findings**

The number of subjects reaching criterion on Tasks 2 and 3 was significantly higher for those in the 1,2,3 sequence. No significant differences were observed between the two groups on Task 1.

Similarly, students in the 1,2,3 sequence mastered Task 3 in fewer trials than those in the 3,2,1 sequence.

In no case did a person master a higher level task and fail to master a lower level task.

Students who had mastered Task 3 performed better on the 2x2 matrix than those who failed to learn Task 3.

5. **Interpretations**

The task dependency hypothesis was supported. Mastery of Task 1 was prerequisite for mastery of Task 2 which in turn was necessary for Task 3. The data supports the 1,2,3 sequence for mastering the classification tasks.

However, the authors question the validity of their flow chart illustrating the thinking processes involved. "...Aspects of the present data suggest that the hypothesized solution process for Task 2 might not have been the one actually used by the Ss, despite training in it." It was hypothesized that students would have more trouble in moving from Task 1 to Task 2 than in moving from 2 to 3; but the opposite occurred. An alternative flow chart was discussed.

**Abstractor's Notes**

It is well established that Gagnèan type task analyses can facilitate cognitive skill development. The worth of this study is not in validating the task dependency hypotheses of the exercises, but rather in the authors' attempt of flow charting the thinking processes employed in solutions to the exercises. Unfortunately, the authors seemed to lose site of this.

Although the experimenters were meticulous in their procedures, several questions arise which might even cast a shadow upon the validity of their hierarchy.

1. The subjects were predominantly black from supposedly an inner city school. The experimenter was white. Could this racial difference have affected the training sessions?

2. No order was mentioned in the presentation of the familiarization tasks. If the tasks were presented in the 1,2,3 sequence, might not the reverse sequence students have been disadvantaged?
3. Why was verbalization of colors and shapes necessary for inclusion in the experiment? Subjects apparently pointed to solutions.

4. The experiment was carried out over a 10 week session. What evidence is there that the subjects did not interact?

5. The fact that the same tasks were used for both pre- and post testing might have influenced the results.

6. Was the experimenter cognizant of the hypotheses of the study? If so, could not have unconsciously put more into the training sessions of the 1,2,3 sequence?

This article was based upon a master's thesis written by the first author under the direction of the second. Perhaps the questions raised above are addressed in the thesis proper.

Theodore Eisenberg
Northern Michigan University

Descriptors—Mathematics, Measurement, Instructional Programs, Aptitude Tests, High School Students, Mathematics Teachers, Tables (Data), Data Analysis, [Scholastic Aptitude Test]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James M. Sherrill, University of British Columbia.

1. Purpose

To test the differential susceptibility of the Quantitative Comparisons (QC), Data Sufficiency (DS), and Regular Mathematics (RM) formats of Scholastic Aptitude Test - Mathematics items to an intensive program of short-term instruction. Of secondary interest was the relative susceptibility of geometry and nongeometry items within these three formats.

2. Rationale

Although it has been found in several studies of special instruction for aptitude tests that expected score gains from coaching are negligible, a widespread interest in special instruction and concern about its possible effectiveness persists. If special instruction were found to influence Scholastic Aptitude Test (SAT) scores substantially, the validity of the test would be open to question, since it is intended to be a measure of relatively stable attributes developed over a long period of time. The effects of special instruction again became a basic concern when the College Entrance Examination Board (CEEB) proposed to change the SAT mathematics test by replacing the RM and DS item formats with a new format, QC.

3. Research Design and Procedure

High school junior volunteers (n = 559) from each of 12 high schools were randomly assigned to one of two experimental groups or a control group. In each school one experimental group was instructed for the new QC format only and the other for either the RM or DS format, while a control group received no special instruction during the experimental period. The control group was assured of and received special instruction at a later date. The list of volunteers was screened to eliminate subjects who were either too mathematically sophisticated or deficient to be likely to benefit from the instruction.

On the Saturday prior to the instructional period all subjects took as a pretest one of two parallel forms of a special aptitude test. Both experimental groups then attended seven three-hour instruction sessions (one each Saturday). On the Saturday following the last instructional session all subjects took as a posttest a parallel form of the pretest (509 subjects remained through the posttest).
Each of the 7 sessions consisted of general test-taking strategies and mathematical content, plus strategies, content, and practice specific to the appropriate item format. Each instructor was provided with a detailed lesson outline for each of the six lessons. Lesson 7 was devoted to a review of the material covered in the first six lessons.

Parallel test forms (A and B) were constructed to use as pre- and post-measures. The posttest was administered so the subjects who took Form A as a pretest received Form B as a posttest and vice versa. The tests yielded seven separate pretest scores and seven corresponding posttest scores (QC geometry, QC nongeometry, DS geometry and nongeometry, EM geometry and nongeometry, and SAT-V).

The data were analyzed in a two-way (treatment x sex) multivariate analysis of covariance. The seven pretest scores were all retained as covariates. Once significant differences were observed among the treatment by sex groups with respect to the dependent variable set, multiple group discriminant analysis was used to generate a set of discriminant function weights for each canonical variate of the analysis.

4. Findings

a. Instruction for each of the three formats was effective. Subjects instructed for a given format gained more on a dimension characterized mainly by subtests composed exclusively of items of that format than either of the other two groups.

b. QC format was most affected by special instruction, followed in order by DS and EM formats.

c. Males obtained higher scores on each of the six dependent variables.

d. The two instructed groups did not differ significantly with respect to gains on tests composed of items for which neither group was coached.

5. Interpretations

If the SAT-M were made up entirely of either QC or DS item formats, the intensive program of instruction could be expected to produce changes in the scores that would result in different admission decisions for many of the students.

The question of whether the effectiveness of the special instruction would have an effect on the validity of the SAT-M is still unanswered.
The Evans and Pike study was both carefully planned and implemented. Due to the reasonably tight controls of the study, the results do present a strong statement of the differential susceptibility of QC, DS, and RM formats of the SAT-M items to an "intensive program of short term instruction." The instruction program was different for each of the groups. If the mathematics content was the same (or very similar), then the QC-vs-DS or RM difference is of some importance. The experimental-vs-control differences certainly could be at least partially explained by the control group getting instruction in mathematics content.

Since the CEEB was proposing to replace the DS and RM formats with the QC format, the results become more important. The QC format was shown to be the most affected by the instruction of the three formats. The instruction on the QC format enabled the QC group to gain 0.94 standard deviations on QC format items while the control group gained 0.27 standard deviations in terms of pretest units. How much of this rather dramatic increase is explained by the differential in instruction in mathematics content can only be a matter of speculation.

Finally, the study should not be interpreted as making any claims about the SAT-M in the normal DS/RM format.

James M. Sherrill
University of British Columbia


Expanded Abstract and Analysis Prepared Especially for I.M.E. by Mary E. Montgomery, University of Wisconsin-Madison.

1. Purpose

The purposes of this study were: (1) to test experiences designed to facilitate children's acquisition of conservation of discontinuous quantities; (2) to test experiences designed to facilitate children's ability to take different social roles; and (3) to test the hypotheses that successfully training a child to conserve will improve his ability to take different social roles, and conversely.

2. Rationale

Feffer and Gourevitch (1) have shown that there is a strong, positive relation between a child's ability to analyze physical perspective and his ability to assume different social perspectives. Therefore, one might hypothesize that successfully training a child to utilize conservation will also improve his ability to assume different roles, and conversely.

3. Research Design and Procedure

The population was 103 children from seven day-care and kindergarten centers in Dayton, Ohio. Their ages, at the beginning of the study, ranged from 45 to 64 months.

The design was test, train, test. The pretest consisted of a battery of ten tests on conservation field dependence, role-taking, physical perspective analysis as well as an IQ test. The population was divided into seven training groups (11 ≤ N ≤ 15) and a control group (N = 8). Each of the training groups were from two centers and were led in small group activities in a different combination of one or more of the following: reversibility-reciprocity, physical perspective-taking and social role-playing. The posttest consisted of a subtest of the battery given at pretesting.
4. **Findings**

1. Significant differences (Wilcoxon test) were found for each training group as well as the control group on at least one of the pretest comparisons of the four conservation tests. The results also indicate that the reversibility-reciprocity training group improved more on the conservation tests than did other groups.

2. Results of Wilcoxon tests of significance between pretesting and posttesting of role-taking and physical perspective-taking were inconclusive and appeared negative.

3. Correlation of performance on role-taking tests with performance on conservation showed only one test of conservation related to role-taking. This was true for correlation done between pretest scores as well as between posttest scores.

5. **Interpretations**

The findings indicated that the control group showed significant improvement on four of the six comparisons between pre- and posttesting. Thus, one may question whether results were due to natural acquisition, training, or testing.

**Abstractor's Notes**

As the investigator notes in his discussion the design did not allow for answers to his questions. First, the design did not account for differences due to natural acquisition or to testing. Second, once it was established that the training was not successful in producing positive changes in a child's ability for role-taking, it is difficult to test the hypothesis that successful training in role-taking improves a child's ability to conserve. In fact, the analysis used would not have tested this hypothesis even if the training had been successful. Third, the design made it impossible to ascertain the effects of the multiple condition training as compared with the single condition training. From reading the objectives of the study, it is not clear why physical perspective training and testing were included.

The investigator is to be commended for pointing some of these out, as well as other, weaknesses; but is to be questioned for not taking seriously the title of his study ... "A Pilot ... ." It is a shame that the care taken in pointing out the weaknesses was not reflected in the rationale and design of the study.

Mary E. Montgomery
University of Wisconsin-Madison

THE RELATIVE EFFECTIVENESS OF A SYMBOLIC AND A CONCRETE MODEL IN 
LEARNING A SELECTED MATHEMATICAL PRINCIPLE. Fennema, Elizabeth H., 
Journal for Research in Mathematics Education, v.3 n4, pp. 233-238, 
Nov. 1972. 
Descriptors---*Elementary School Mathematics, *Instruction, 
*Learning, *Manipulative Materials, *Research, Activity Learning, 
Multiplication, Mathematics Education, Mathematical Models, 
Symbolic Learning

Expanded Abstract and Analysis Prepared Especially for I.M.E. by 
Otto C. Bassler, George Peabody College for Teachers.

1. **Purpose**

To investigate the relative effectiveness of a concrete 
instructional model as opposed to an abstract model.

2. **Rationale**

There has been much theoretical discussion concerning the value of 
concrete manipulative materials in enhancing learning. Manipulation of 
these materials provide action experiences which permit young children 
to add new ideas to their cognitive structure. The empirical evidence 
concerning the efficacy of concrete instructional materials is far 
from conclusive.

3. **Research Design and Procedure**

Subjects were 95 second grade students who had reached criterion 
on a background knowledge test. These subjects were then randomly 
assigned to one of eight groups - four of which were designated concrete 
and four, abstract. N's per group ranged from 11 to 13.

The learning task was multiplication defined as the union of 
equivalent disjoint subsets (products ≤ 10). Instruction in the 
concrete treatment utilized Cuisenaire rods. Subjects were taught to 
solve problems such as $5 \times 2 \rightarrow \square$ by placing 5 two-rods end to end and 
then finding a single rod of equivalent length. In the symbolic treat-
ment, no concrete or semi-concrete models were used. Rather, subjects 
were taught to solve $5 \times 2 \rightarrow \square$ by adding 5 twos.

The same teacher taught all groups for 14 instructional sessions, 
the same worksheets were used, the same problems were solved, and the 
same drill games were used.

Following instruction, a test of recall and two tests of transfer 
were administered. The Recall Test contained problems like those 
considered during instruction, that is, Find either a, b or c given the
other two in \(a, b + c\) where the rule to be applied is multiplication and \(c \leq 10\). The Transfer Tests contained problems \(a, b + c\) where \(c\) is the product and \(11 \leq c \leq 16\). On Transfer Test I subjects were permitted to use as aids to problem solving the same aids they had used in the instructional sessions. On Transfer Test II, administered one week after Transfer Test I, all subjects were permitted to use counters to solve the problems. All tests had reliability greater than .9 as determined by the Hoyt reliability formula.

Data from each test were analyzed by a one-way ANOVA using group means as the data points.

4. Results

The mean percents of correct responses on the Recall Test were 95% for the symbolic treatment group and 89% for the Concrete treatment group. Analogous percents for Transfer Test I were 77% and 62% and for Transfer Test II were 80% and 58%.

When mean differences were tested for significance the following probability levels were obtained:

\[
\begin{align*}
\text{Recall Test} & \quad p < .090 \\
\text{Transfer Test I} & \quad p < .053 \\
\text{Transfer Test II} & \quad p < .003
\end{align*}
\]

All differences favored the symbolic treatment group.

5. Interpretations

Two conclusions were drawn. They were:

1. It is possible for children of 7-8 years of age to learn a mathematical idea to the point of direct recall by using either a concrete or symbolic representational model.

2. When learning is defined as transfer or extension of a specified mathematical idea certain groups of children who use a symbolic model perform at a higher level than those who use a concrete model.

These conclusions are accepted tentatively within limitations such as the particular learning tasks, the type of concrete aids that were used, and the definition of transfer. In any case the study does not refute the necessity of using action experiences in learning mathematical principles. It does, however, indicate that concrete models are not always necessary or more effective than symbolic models.
Abstractor's Notes

The problem of this investigation is still of interest to mathematics education and still unresolved. Results of this carefully conducted study will contribute to the growing body of literature pertaining to the use of concrete aids in instruction. The conclusions are warranted by the data.

The author's use of group means rather than individual student scores may have been overly cautious. No rationale was given for this procedure which may tend to raise the probability levels for the comparisons of treatment means.

The number of instructional sessions (14) seems to provide a longer period of time to teach the learning task (26 multiplication facts) than was necessary. The result of too much training may be rote learning of the multiplication facts by both treatment groups. This in turn would produce the high scores on the Recall Test and would tend to suppress any differences between the treatments.

Otto C. Bassler
George Peabody College for Teachers
1. Purpose

The purpose of this study was to attempt to ascertain the relationship between and among the cognitive variables thought necessary to ensure success in a second-year computational mathematics course and actual achievement in that course.

2. Rationale

The instructors of a second-year computational science class were concerned about the recurring bi-modal distribution of the achievement scores. The distributions about the two distinct modes for the last two years had been approximately normal. The instructors were interested in knowing what abilities differentiated the high achievers from the low achievers.

3. Research Design and Procedures

Instructors of the computational mathematics class were asked, by the Es, to identify some of the cognitive abilities they felt were necessary to successfully handle the class requirements. Based upon this list of abilities the Es selected a battery of tests. A meeting with the instructors was held at which the description of each of the tests was presented and their opinions as to the suitability of the instruments were ascertained. The instruments finally selected included seven tests taken from the Kit of Reference Tests for Cognitive Factors (Hidden Patterns, Thing Categories, Letter Sets, Locations, Figure Classification, Object-Number, and Seeing Problems) together with five subtests of the Watson-Glaser Test of Critical Thinking (CT).

The selected instruments were administered during the first two weeks of lectures. The final mark (criterion) consisted of a combination of a final common examination, a mid-term examination, and a mark on projects. The final examination had a weight of .7 and the mid-term and projects' marks together were weighted .3.

Ss were 119 students enrolled in a computational sciences course.
Raw scores on the seven cognitive factors tests, the five CT subtests, and the final achievement mark for the course were obtained. The inter-correlation matrix on the resulting 13 variables was computed by the Pearson product-moment procedure. The iterative principal factor solution of the correlation matrix was rotated to the varimax criterion. In order to understand better the relationships among the variables, a Schmid-Leiman hierarchical factor analysis was carried out.

From an inspection of the intercorrelation matrix, four cognitive and five CT variables were used in step-wise regression analysis with achievement scores as the dependent variable. The results of the step-wise regression analysis resulted in the selection of four potential predictors from the original set of predictors.

Since it was intended to identify cognitive abilities on which successful and unsuccessful students were similar and different, the sample was divided into two distinct groups on the basis of achievement scores. The low group (LO) was defined as those students who scored at and below 65 and the high group (HI) as those students who scored above 65. The LO group had an N = 38 and the HI group an N = 81. A one-way multivariate analysis of variance (MANOVA) was performed on the four variables selected on the basis of the above-mentioned regression analysis.

After the intercorrelation matrix among the 13 variables was determined, it was factor analyzed using both the iterative and noniterative principal factor procedures. On examination of the results, Es chose the iterative solution for varimax and promax rotations. The iterative factor solution was used as input for the Schmid-Leiman hierarchical factor solution. After the relationship among the various cognitive variables and achievement by means of factor analytic methods had been examined, the relationship between the various cognitive variables and achievement was investigated by regression analysis.

4. Findings

As a result of the factor analyses, seven factors were found. Factors I and II were a general cognitive factor and a specific rote memory factor. Factor III was essentially a critical-thinking factor. Factor IV was an achievement factor; Factor V emphasized quantity and not quality of ideas; Factor VI was an inductive-deductive factor; and Factor VII corresponded to Factor II.

From an inspection of the intercorrelation matrix, three subtests were excluded from the set of variables to be used in the stepwise regression analysis because of their low correlation with achievement. The set of five CT subtests was kept intact in spite of three relatively low correlations with achievement in order to examine the relative predictive power of the five subtests of the CT test against the total CT score as well as against the relevant cognitive abilities subtests.
The selected set of nine covariates accounted for 25.6 percent of the variance in achievement. Four potential covariates, Locations, Hidden Patterns, and Seeing Problems from the reference tests, and Deduction from the CT test, were chosen. The set of four covariates accounted for 23.1 percent of the variance.

The four potential predictors of achievement in the computational science class having been determined, an answer to the question of whether the cognitive abilities identified did differentiate between the successful and nonsuccessful students was sought. A one-way (HI-LO) MANOVA was performed using the four identified cognitive variables as dependent variables. The mean-vectors of the HI and LO groups were significantly different (p < .05). The F-ratios for the corresponding univariate ANOVA on Hidden Patterns, Locations, Seeing Problems, and Deduction were respectively 5.08 (p < .05), 12.0 (p < .05), 3.0 (p < .10), and 10.9 (p < .05).

5. Interpretations

It would appear that the four selected variables are not only good predictors of success in the computational science course, but also significantly differentiate between the successful and non-successful students.

The strategy and solution of the problem presented here is unique in the sense that no new instrument was constructed, but instead an intuitive analysis of the underlying necessary abilities required for success in the criterion variable was carried out. On the basis of this intuitive analysis, the existing predictor tests were identified and then examined using both correlational and inferential techniques. Together the two techniques resulted in the identification of the requisite cognitive abilities necessary for success in the criterion task.

Abstractor's Notes

The researchers should be congratulated for trying to use available instruments to predict success in a computational science course and for basing the instrument selection upon an intuitive analysis of the underlying abilities needed for successful completion of the course. This enabled them to attack the problem immediately with reliable instruments without embarking on the arduous task of developing an instrument (probably) based upon these same intuitive abilities.

The authors were not clear about whether the final achievement score was the same one used by course instructors or was their own weighting of the final exam, mid-term exam and projects marks. Also, since a score of 65 was used to separate HI (successful) and LO (unsuccessful) students, were these the same scores used by the university to determine whether a student had successfully completed the course?
The authors were not clear how the four variables would be used to predict success in the course or how this would affect the recurring bimodal distribution of the achievement scores in the course. Will students be given these tests the first few class meetings and then advised whether they should continue or drop the course? Would a course (or four mini-courses) be developed to help students who did poorly on these tests improve their abilities in these four areas so that they could successfully complete the course?

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Eastern Illinois University
A PILOT STUDY ON TEACHING THE DERIVATIVE CONCEPT IN BEGINNING CALCULUS
BY INDUCTIVE AND DEDUCTIVE APPROACHES. Lackner, Lois M. School Science

Methods, College Mathematics, Mathematics, Programed Materials,
Research, Secondary School Mathematics, Teaching Methods

and

TEACHING OF LIMIT AND DERIVATIVE CONCEPTS IN BEGINNING CALCULUS BY
COMBINATIONS OF INDUCTIVE AND DEDUCTIVE METHODS. Lackner, L. M.

Methods, *Deductive Methods, *Calculus, High School Students,
Programed Texts, Analysis of Variance, Criterion Referenced Tests,
Tables (Data)

Expanded Abstract and Analysis Prepared Especially for I.M.E. by
John G. Harvey, University of Wisconsin-Madison.

(Editor's Note: These two articles report on a pilot-study and main-
study of a single research problem. The following abstract refers to both
articles.)

1. Purpose

The purpose of the pilot study was "...to present the derivative
concept in beginning calculus by both an inductive and a deductive approach,
to determine if there was a significant difference between the effectiveness
of the two approaches ..." (1971, p. 563).

The programmed text materials developed for the pilot study were
refined and used in the main study. The purpose of the main study was
"... to compare the teaching of the limit and derivative concepts by
inductive and deductive methods to advanced high school students ..." (1972, p. 51).

2. Rationale

During the 60's and 70's many claims have been made for the advantages
of discovery methods in teaching mathematics. However, there is very little
research at present which supports these claims or refutes them, especially
at the upper secondary and collegiate levels. Most notable is the research
of Cummins (1960), Larsen (1960), Shelton (1965), Davidson (1970), and
Lucas (1972). The two Lackner studies contribute to research in this area
in that they compare an inductive treatment with a deductive treatment of
concepts included in beginning calculus courses for high school seniors or
college freshmen.
3. Research Design and Procedure

For the pilot study Lackner developed an inductive treatment and a deductive treatment. The content of both of the treatments was the same and included the definition of the derivative as the limit of a difference quotient, the standard theorems for finding the derivatives of sums, differences, products and quotients of functions, and the derivatives of some of the standard functions. Each of the treatments were programmed units and were developed using the model espoused by Markel (1964). The deductive unit followed a rule-to-example programming technique while the inductive unit was developed using the example-to-rule technique. The deductive treatment was piloted with eight high school students in Urbana, Illinois, while the inductive treatment was tried with an unspecified number of high school students from Midlothian, Illinois. Each student who participated in the pilot was also given (1) a pretest pertaining to knowledge required to work through a unit on limits, (2) a programmed unit on limits developed by Shelton (1965), and (3) a posttest on the limit unit. Finally, when each student concluded study of one of the derivative treatments, a criterion test, developed by the investigator, was administered to that student. Using a method proposed by Suits (1957), the treatment was considered as a dummy variable, and the complete matrix of zero-order correlation coefficients among the four variables of the pilot study (treatment, pretest, limit, and derivative) was found.

The main study (1972) used the inductive and deductive derivative treatments developed for the pilot study and refined on the basis of that study. In addition, the main study also used the inductive and deductive limit treatments developed by Shelton. Each of the limit treatments consisted of 309 frames; there were 346 frames in the inductive derivative unit and 359 in the deductive derivative unit.

Four hundred forty-nine students from eight Chicago area high schools were considered as one population for this study; four hundred of these students were randomly selected to participate in the study. On the basis of pretest scores, Ss were assigned to a high or low achievement level. Within each level, the inductive and deductive limit and derivative treatments were randomly assigned to Ss. Thus there were eight treatment groups for the main study; Table 1 displays the design.
Table 1
Treatments X Levels Experimental Design
For Total Treatment Study

<table>
<thead>
<tr>
<th>Levels</th>
<th>Treatments</th>
<th>Keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>A-H</td>
<td>A = Inductive limit-inductive derivative</td>
</tr>
<tr>
<td></td>
<td>B-H</td>
<td>B = Inductive limit-deductive derivative</td>
</tr>
<tr>
<td></td>
<td>C-H</td>
<td>C = Deductive limit-inductive derivative</td>
</tr>
<tr>
<td></td>
<td>D-H</td>
<td>D = Deductive limit-deductive derivative</td>
</tr>
<tr>
<td>Low</td>
<td>A-L</td>
<td>H = High</td>
</tr>
<tr>
<td></td>
<td>B-L</td>
<td>L = Low</td>
</tr>
<tr>
<td></td>
<td>C-L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D-L</td>
<td></td>
</tr>
</tbody>
</table>

In each of the twenty-two junior and senior mathematics classes from which the Ss were chosen, the teachers distributed and collected the appropriate treatments for each student each class meeting of the two week period during which the study was conducted. These teachers were allowed to answer individual student questions, but not to conduct a general discussion. Individual questions were at a minimum due to the programmed nature of the materials.

The following were administered to each S:

1. a pretest, written by Shelton;
2. a limit criterion test, upon completion of the limit unit;
3. a derivative criterion test, upon completion of the derivative unit.

The limit criterion test had split-half reliability coefficients of .60 and .79; these coefficients were derived from data collected by Shelton for his two independent limit studies. The derivative criterion test had a split-half reliability coefficient of .78 with an odd item-total score correlation of .87 and an even item-total score correlation of .93.

The limit criterion test scores were used in the analysis of variance for the limit study. The derivative criterion test scores were used in the analysis of variance for the derivative and total treatment studies.
4. **Findings**

**Pilot Study**

1. There was a significant \( p < 0.10 \) difference between the pretest mean of the deductive treatment group and the pretest mean of the inductive treatment group. \((D > I)\)

2. Using the pretest and the limit scores as covariates, there is a significant \( p < 0.05 \) difference between the means of the deductive and inductive groups on the criterion derivative test. \((D < I)\)

**Main Study**

1. On the limit test there was a significant \( p < 0.0001 \) difference between the performance of high and low achievement groups. \((H > L)\)

2. On the limit test the hypotheses that there is no difference between the mean achievement test scores for the two treatments and that there is no interaction between treatments and levels were not rejected.

3. On the derivative test there was a significant \( p < 0.003 \) difference between treatments \((D > I)\) and a significant \( p < 0.001 \) difference between the high and low levels \((H > I)\).

4. The hypothesis that there was no interaction between treatments and levels was not rejected.

5. For the total treatment the analysis of variance shows a significant difference between treatments \( p < 0.021 \) and levels \( p < 0.001 \). Using Scheffé's method for post-hoc comparisons it was found that this difference in treatment effects was significant between the inductive limit-inductive derivative and deductive limit-deductive derivative with the later superior at each of the two levels.

6. The hypothesis that there was no interaction between the four treatments and the two levels was not rejected.

5. **Interpretations**

On the basis of the pilot study the investigator concluded that "... a student's performance on the derivative test cannot be predicted from his pretest score" and "... the degree to which a student has mastered the limit unit appears to have a powerful effect on mastery of the derivative unit, independent of the pretest and form of the derivative unit."

For the main study Lackner concluded that the deductive method of teaching was superior to an inductive approach in teaching the derivative concept in beginning calculus and that the deductive method is superior in teaching the limit and derivative concepts together.
Abstractor's Notes

Considered together these two studies describe a worthwhile and well planned attack on a significant problem in mathematics education. The investigator is to be lauded for her careful attention to design and methodology in the main study and for the restraint demonstrated in making conclusions. However one wonders why the investigator attempted to make comparisons of the two treatments based on data collected in the pilot study since only eight Ss read the deductive derivative treatment for that study. It would have been more appropriate to consider that study as a preliminary tryout of the two derivative treatments and to have collected data which revealed the weaknesses of those treatments so that they could be refined for use in the main study.

The main study is well planned and faithfully executed. However the report of that study could have been improved if some sample frames from the inductive and deductive derivative treatments had been included so that readers could judge the validity of the claim that the treatments were indeed inductive and deductive. Also a more complete description of the test items on the criterion derivative instrument should have been included for much the same reason. Finally, since the mean scores on the derivative posttest were not high, it would appear that both of the instructional units were not very effective and that inferences based on these scores may not be supportable.

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Expanded Abstract and Analysis Prepared Especially for I.M.E. by William E. Geeslin, University of New Hampshire.

1. Purpose

To obtain a non-normative operational definition of adjustment and determine whether adjustment could account for variance in mathematical performance independently of intelligence and anxiety.

2. Rationale

In past studies adjustment has been defined in normative terms. That is, adjustment was conceived of as a unitary trait which has a desirable end point that each individual should move toward in order to be "well adjusted." This criterion would be the same for all individuals and carries a good versus bad connotation in terms of socially acceptable behavior. Past research has obtained differing and contradictory results concerning the relationship between adjustment and scholastic performance. At least part of the difficulty lies in the varied operational definitions of adjustment, particularly those which rely on normative connotations of adjustment and/or anxiety scales as a measure of adjustment.

A more appropriate definition of adjustment is suggested by Rogers. He states adjustment is the "congruence" between self and experience. This definition is non-normative in the sense that two individuals could be equally adjusted but be at quite different points on a socially acceptable continuum. Adjustment in this study is defined as the degree to which self-concept and ideal self-concept correspond. This definition is felt to be consistent with Rogers' work and distinct from past definitions.

Measures of anxiety have been used frequently as operational definitions of adjustment. That is, less well adjusted individuals were thought to exhibit higher levels of anxiety. The concepts of facilitating anxiety, debilitating anxiety, state anxiety (a transitory emotional state due to a specific situation), and trait anxiety (relatively stable individual differences in anxiety proneness) suggest anxiety may not be a single construct. Additionally, it is conceivable that two individuals could be equally adjusted and yet have quite differing levels of the various types of anxiety with respect to specific academic situations.
The semantic differential is an instrument that could eliminate both the problems of using a normative definition and using an anxiety scale to determine adjustment. The semantic differential lends itself to Rogers' theory in that it is non-normative and can be used to determine the correspondence between "myself" and "the person I would like to be." The semantic differential eliminates the problems of faking and response sets found in normative instruments. It also offers a method of measuring adjustment that can be regarded as distinct from various anxiety scales.

3. Research Design and Procedure

Subjects in the experiment were 316 boys and 305 girls from 18 seventh grade classes at different schools in Melbourne, Australia. Selected schools were representative of the types of schools found in Melbourne. This study is an analysis of the relationship between anxiety, intelligence, mathematics achievement, and adjustment. Instruments used were the Test Anxiety Scale for Children (TASC), an intelligence test of the Otis type, the State-Trait Anxiety Inventory (STAI), and achievement on the mid-year examination in mathematics. Adjustment was assessed by using a semantic differential (SD) consisting of eleven different concepts judged on eighteen bipolar adjectival scales. The SD was factor-analyzed to isolate those scales which clearly defined an evaluative factor. D-scores of profile congruence on the evaluative factor for the concepts of "myself" and "the person I would like to be" (M-I) were calculated and used as the measure of adjustment. The TASC, STAI, SD, and intelligence test were administered several weeks prior to the mid-year examination.

Using the D-scores, two groups of subjects were formed by splitting these scores at the median. Similarly, the state anxiety scale scores (from the STAI) were used to form two groups. The data were analyzed using multivariate ANOVA with a 2x2x2 design (sex x D-score level x state anxiety level). The dependent variables were intelligence, trait anxiety, TASC, and mathematics scores. (Mathematics scores within each class were converted to T-scores.) Univariate F tests were completed for sex, M-I, and state anxiety level. Three separate analyses of covariance were performed with mathematics scores as the dependent variable and intelligence, trait anxiety, and TASC, respectively as the covariant to determine if sex, adjustment, or state anxiety effects were present.

4. Findings

The multivariate ANOVA revealed no interactions but all three main effects were significant (p < .001). Univariate F tests revealed the following significant differences (for most F's p < .001): 1) boys and girls on TASC; 2) M-I groups on intelligence, trait anxiety, TASC, and mathematics performance; and 3) state anxiety groups on intelligence, trait anxiety, TASC, and mathematics performance. The ANCOVA's revealed that significant differences between M-I groups on mathematics performance still existed when mathematics scores were adjusted separately for intelligence (p < .044), trait anxiety (p < .001), and TASC (p < .008).
5. **Interpretations**

The most important results were the maintenance of significant differences between adjustment (M - I) groups on adjusted means of mathematics performance. The data suggested that intelligence and adjustment are quite distinct constructs. Additionally, adjustment is a construct distinct from trait anxiety and TASC. The relationship between state anxiety and adjustment was unclear. Finally, "the semantic differential can be used as a non-normative operational definition of adjustment unconfounded by the effects of normative measures of anxiety and intelligence."

**Abstractor's Notes**

As mathematics educators we often tend to focus our research on the effects of various treatments on achievement. The Naylor and Gaudry article is refreshing in that it begins to explore other variables that may be a part of the psychology of learning mathematics. It is plausible that variables such as anxiety or adjustment may affect achievement as much as treatment or interact with the treatment effect. In a study of this type, the reader should expect no "cause and effect" discussion nor much reference to information of practical use to the teacher. Naylor and Gaudry have included neither and are to be congratulated for resisting the temptation. From this type of study one can gain both information for future research and a notion of important variables the researcher/teacher might consider in his work. This study has attempted to assess important variables that are often overlooked.

One shortcoming of the report is noting that this study was a portion of a larger study without referencing or describing the larger study. Did the larger study have experimental treatments of any sort? If so, can we expect the relationships between variables to hold up under non-experimental conditions? Does the fact that the portion of the SD used in this study was imbedded in a larger instrument have any effect on the result?

A second difficulty for this reader was the term "non-normative." At one point "context free" was substituted for non-normative and this should have been done throughout the study. One could obtain "norms" for the M - I variable. The essential distinction of the SD is that it refers to no particular situation and greatly reduces the possibility of a subject faking a response due to social desirability. The magnitude of the M - I score might still be related to social desirability.

The report needed more explanation concerning the reasons various analyses were selected. At points the statistical procedures appeared overly complicated. Correlation matrices and partial correlation matrices would be very interesting. How large were the correlations? What other relationships existed among the variables? Why not use multiple comparison procedures as opposed to univariate F tests. Why only use ANCOVA with one covariate? Is the distinction between state anxiety and trait anxiety sufficiently clear to allow using them quite differently in the analyses? What portion of the variance in mathematics scores was accounted for by
the variables? Was the same mathematics test given in each class? If not, are we dealing with achievement relative only to one's class?

Finally, an examination of the means indicates that group differences were generally less than two points (mean = 50). Is this difference large enough to be educationally significant?

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Expanded Abstract and Analysis Prepared Especially for I.M.E. by John G. Harvey, University of Wisconsin-Madison.

1. **Purpose**

The purpose of the investigation was to compare the effectiveness of four instructional treatments in elementary mathematics offered to first year economics undergraduates at Cambridge University. The four treatments were:

A) Lectures, computer class, course booklet and weekly exercises.

B) Self-instructional materials, optional problem class, course booklet, and a shorter set of exercises than in Treatment (A).

C) Lectures and the same set of exercises used in Treatment (A).

D) No instruction in elementary mathematics.

2. **Rationale**

The investigation was undertaken to try to develop a course in elementary mathematics for economics undergraduates which would better

1) provide basic skills so as to allow ordinary students to follow mathematical arguments and to prepare them for the compulsory second year statistics course,

2) provide a bridge to specialized papers in mathematics, mathematical economics and theoretical statistics for the more mathematically advanced students, and

3) economize on lecturers' time.

Treatment (C) was the one which had been used previously in this course at Cambridge. In hopes of improving motivation, computer access and a course booklet applying the mathematics being taught to economics were added to this treatment; the result was Treatment (A).
Because of some evidence that self-instructional materials are successful in courses of this type and in order to attain goal (3), materials of this type were prepared for Treatment (B). To try to improve the effectiveness of the self-instructional materials, a weekly problem class was included in this treatment as was the course booklet used in Treatment (C).

3. Research Design and Procedure

In 1971, 172 people at Cambridge read Part I economics. These students each elected a treatment; 14 chose Treatment (A), 29 Treatment (B), 57 Treatment (C), and 72 Treatment (D).

A pretest was given to 128 first year economics students, and 113 of these students completed the posttest. Both of these tests were based on the format of those used in the Accelerated Mathematics Project at Lancaster University. Of those taking the posttest, six were in Treatment (A), 18 in Treatment (B), 24 in Treatment (C), and 65 in Treatment (D).

Additional data was gathered by questionnaires whose purpose and content is unstated. These questionnaires were administered to an unspecified number of the Ss at the beginning and end of the term. Finally, "course cards" were sent to each student weekly; return of those cards was voluntary. On these cards the student checked boxes to indicate the mathematics they had covered and registered their comments. No report is made of the weekly return rate of the course cards.

4. Findings

The mean score of each treatment group was higher on the posttest than on the pretest for that group. The pretest and posttest means for each of the Treatment groups (A), (B) and (C) was at least four points lower than the pretest mean for Treatment group (D). An analysis of covariance of the data indicated no significant differences (p < .05) between the treatment groups on the posttest. The covariate was the score on the pretest.

Feedback obtained from the questionnaires and course cards shows the following:

1) The majority in all groups indicated that they did not feel that the economic context of the mathematics taught had been adequately stressed.

2) Everyone in Treatment Group (A) had only 0-level mathematics and found the lectures and course booklet too difficult. This group questioned the value of the computer as a teaching aid at their level.

3) The four Ss in Treatment (B) who had only 0-level mathematics found the self-pacing aspect of the materials advantageous but considered the content to be too difficult. These Ss found the classes helpful and attended consistently.
4) Most of the 13 A-level Ss in Treatment (B) found the level of the material about right and particularly liked those with high economic content.

5) While the majority of the Ss in Treatment (B) considered they had too many programs to complete they agreed that self-instructional materials together with an optional problem class was a satisfactory method of learning elementary mathematics.

6) The Ss in Treatment (C) found the mathematical terminology confusing and considered they were given too many exercises each week.

5. Interpretations

This study is very specific to the situation at Cambridge. Because of this and the problems of design and control the investigation acknowledge that the results are not definitive. But they do conclude that all undergraduates reading economics at Cambridge should take a diagnostic test in basic mathematics relevant to economics. This test would be used to divide economics students into three groups. One group would be exempt from mathematics the first year or would take the second year course in their first year; a second group would be given the self-instructional materials and optional problem class and the third group, of poor mathematical achievement, would be given special classes.

Abstractor's Notes

Research in the development of college mathematics curriculum should be encouraged; far too little has been or is being done. For this reason the initiators of this study are to be lauded. However, there are some serious problems of both developmental and methodological nature associated with this study. Some distinguished by the abstractor or the authors are:

1. The study implies that the self-instructional materials and the course booklet were developed at the same time as the treatments were progressing. It would seem that before attempting to compare instructional treatments the materials should have been developed, tested and refined.

2. The analysis of covariance applied to the pretest and posttest data is not appropriate. Since each S chose a treatment, no randomness was present. Because of the varying prior mathematical experiences of the Ss a stratified random assignment of Ss to treatments is suggested.

3. The dropout rate in Treatments (A), (B), and (C). Only six Ss (42%) completed Treatment (A); 18 (62%) Treatment B; and 24 (42%) Treatment (C).
It is not surprising that no significant differences between treatments were detected. The investigators were wise to concentrate on the information gathered by the questionnaires and course cards.

John G. Harvey
University of Wisconsin-Madison
1. Purpose

The purpose of this article is twofold. First, to present "a summary of findings related to entering kindergartener's knowledge in measurement, money, number, and geometry." The findings are based on data gathered in 1968 from 727 subjects in the metropolitan area of St. Louis, Missouri. Second, to provide "information . . . which teachers might use in planning appropriate learning activities for youngsters entering school."

2. Rationale

The implied rationale for the investigation was to refute two assumptions about kindergarteners: 1) that these subjects demonstrate "no prior acquisition of skills and knowledge," and 2) that "instruction . . . should be exclusively aimed at social and educational readiness."

3. Research Design and Procedure

This investigation was a status study where data was gathered by administering the "Comprehensive Mathematics Inventory" (CMI) to 727 students from 30 selected kindergarten classes during the first week of school in the 1968-69 school year. The test contained at least 178 items, was administered in two parts and took between 30 and 40 minutes to give. No further information is reported about the test and how it was administered (although it must have been given individually). Also, no additional information is given about the sampling. However, the authors do caution the reader not to overgeneralize their findings. And finally, while other data such as age and parent's education was gathered in an attempt to account for variability the use of that information is not reported in this paper. (It has been reported elsewhere.) The only data presented are item p-values grouped by topical category for 136 of the 178(?) items. No overall means for categories (or variances) are reported.
4. Findings

The results of the test administration are summarized in four tables (corresponding to the four categories of number, money, measurement and geometry) subdivided into 17 topics and later re-summarized into 28 areas (rather confusing). Each table is discussed in detail but adequate evidence of knowledge is assumed if over half the subjects have answered an item correctly. I have summarized their results in the following tables and have added a more stringent (80%) mastery level for emphasis.

Table 1

Summary of Number of Items in Each Category Mastered by 50% and 80% of the Kindergarten Population

<table>
<thead>
<tr>
<th>Category</th>
<th>No. of Items</th>
<th>No. of Items p-value &gt; .50</th>
<th>No. of Items p-value &gt; .80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numeral Identification</td>
<td>12</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Sequence</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Cardinal</td>
<td>20</td>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>Ordinal</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Group Comparison</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Money</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identification</td>
<td>9</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Value Comparison</td>
<td>6</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Making Change</td>
<td>7</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Measurement</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Clock</td>
<td>7</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Calendar</td>
<td>7</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Ruler</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Thermometer</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Geometry</td>
<td>34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape Matching</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>9</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Spatial Relations</td>
<td>9</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Geometric Reproduction</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
5. **Interpretations**

Because more than 50% of the subjects answered 84 of the 136 items correctly the authors conclude that "the data clearly negate the assumption of no prior knowledge and skill, and raise at least some doubt concerning an exclusively social skills experience. ... it seems desirable to build upon such beginnings rather than postpone further development."

**Abstractor's Notes**

This paper is clearly one of several written by the authors to report data from a large study. It is understandable that multiple papers are written, but this article suffers in part from lack of appropriate information obviously given in other reports. Except for this limitation the authors are to be commended for their clarity of presentation and cautiousness in generalizing their results.

Their first purpose, "to present a summary . . .", has been met. Clearly, for this population these subjects knew something about numbers, money, measurement, and geometry.

However, for the second purpose, I see no way that teachers might use the data to change practices. First, I judge that the extent of demonstrated knowledge for number (only familiarity with 1,2,3,4), money (only identify penny, nickel and dime), and measurement (only identify gross weight-size relationship and a clock) to be insufficient to change what is done in those areas. Second, even for geometry where a high percent of mastery is shown, the items not mastered are vocabulary items (name a triangle, circle, rectangle, etc.) which are precisely what is usually taught in kindergarten. Third, unless the item data is used as a diagnostic tool to group students who have similar learning needs classroom teachers cannot profit from this study (this is the intended use of the CMI battery).

The findings of this study rest on the validity of the CMI test. After examining the items I doubt they adequately measure what would be useful to kindergarten teachers. They only measure surface characteristics of several underlying dimensions of subjects' concrete-operational thought processes. A more useful approach would have been to identify theoretic categories and have items which reflect those processes.

Finally, I am curious about the appropriateness of the data (gathered in 1968) to today's students. I suspect that because of "Sesame Street," "The Electric Company" and changes in mass education, the profile of scores in 1975 would be quite different.

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53
Impact of Computer-Assisted Instruction on Student Attitudes.


Expanded Abstract and Analysis Prepared Especially for I.M.E. by Harold Schoen, University of Iowa.

1. Purpose

The purpose of this study was to investigate the effect of computer-assisted instruction (CAI) on student attitudes of self-concept, locus of control, and level of aspiration.

2. Rationale

Attitude variables were found to be strongly related to school achievement by Coleman (in his 1966 report). Since CAI has been shown to increase achievement in selected school subjects by many researchers, it was reasoned that CAI may contribute to positive changes in student attitudes.

3. Research Design and Procedure

The impact of the Stanford mathematics drill-and-practice program on three attitude variables was investigated. The sample consisted of 320 students from a junior high school in the San Francisco Bay Area. The subjects, 75% with Mexican-American backgrounds, were performing an average of 2 to 3 years below grade level in arithmetic. The treatment (CAI) group consisted of all students assigned by their teachers to the drill-and-practice program. The investigator had no control over which students used the CAI program. The control (non-CAI) group was selected from the classes of the math teachers of the CAI students. This was done to minimize differential teacher effect. Using this procedure the boys in the CAI group out-numbered the girls two to one.

Sears Self-Concept Inventory, Coopersmith Self-Esteem Inventory, and Crandall Locus of Control Instrument were the measures of self-concept, level of aspiration and locus of control, respectively. The three instruments were administered as a pretest before the CAI program had begun operation and again 11 weeks later as a posttest. There were 115 CAI and 125 non-CAI students who completed both pretest and posttest.
4. **Findings**

a. The posttest scores on the self-concept scales of the CAI group were found to be less predictable from their pretest self-concept scores than were those of the non-CAI group, although there was not a significant increase in pretest to posttest mean self-concept scores.

b. A similar pattern of findings was recorded for the locus-of-control scales.

c. The initial aspiration levels of the CAI students were found to be closely related, in terms of means and correlation coefficients, to their actual performance.

d. The pattern of results for students using CAI for their second successive year did not differ from that of beginning CAI students.

e. A modest relationship ($r = .25$) was found between math self-concept and achievement in the CAI program after 9 weeks, whereas the relationship on the pretest was nonsignificant.

5. **Interpretations**

The following limitations are cited:

a. Subjects were not randomly allocated to treatments.

b. The subjects were performing 2 or 3 years below grade level in arithmetic.

c. The CAI students were approximately 75% Mexican-American, a group found by Coleman to be systematically different from other ethnic groups.

d. Only one CAI program was used in this study.

e. Self-report attitude measures were used in this study to the exclusion of such methods as interview or behavior-rating.

Within the study's limitations, the following conclusions and implications are drawn:

a. The CAI experience brings students' math self-concepts more into line with their math achievement.

b. For this reason, and since it avoids fear of failure, CAI is an efficient and effective form of remedial instruction.

c. CAI does not decrease student attitude, contrary to many articles in the popular press reporting the dehumanizing influence of computer technology. In fact, CAI appears to have a positive effect on many individual students' attitudes.
CAI is an efficient and effective form of remedial instruction. This is well established by previous research using achievement criteria, and this study provides further evidence. Nonetheless, CAI is not being used in very many schools and is not likely to be adopted on a wide scale in the near future. The reasons are many, none of which have to do with lack of research supporting its effectiveness. Cost, political difficulties in local school systems, lack of personnel trained in CAI, lack and incompatibility of software, and a basic fear of the machine are some of the practical problems that must be overcome. While Smith's findings are interesting, it will take a much wider effort primarily outside the research field to have any real impact on CAI usage in the schools. Perhaps the massive PLATO and Mitre Corporation projects will have such an effect.

The positive effect of CAI on attitudes found in this study is worthy of further investigation. What is it about the CAI experience that caused the stabilization of attitudes and the pattern of more realistic self-concepts? Does the interaction of the teacher's personality with individual student personalities cause some students to "waver?" Is this effect peculiar to CAI or would other individualization media have the same effect? Will more personal attention from a teacher, in fact, have a positive effect on all or even most students?

Harold Schoen
University of Iowa
1. **Purpose**

The investigation focused on the effectiveness and efficiency of a mathematics laboratory in improving student learning and attitude.

2. **Rationale**

Data on student outcomes of labs is needed to help evaluate the role of math labs in teaching mathematics.

3. **Research Design and Procedure**

Students from seven Grade 7 and seven Grade 8 classes in a large, urban junior high school were randomly assigned to three groups:

- **Math Lab:** Pairs of students, following written instruction, worked directly with physical materials.
- **Class Discovery:** Teachers demonstrated the lab activities with the physical materials to whole classes.
- **Control:** Students studied the regular mathematics program.

The experimental groups spent one class out of every four (one class per week) in the experimental settings. The control group continued with the regular instruction during these periods.

The lab activity consisted of ten lessons, each employing a different device and/or set of physical materials. An "explore-formalize-practice" sequence was used in each lesson. For the math lab group, the teacher responded only to student questions. Each pair of students completed the ten lessons in predetermined random order. The class discovery group saw demonstrations of the ten activities. The three teachers involved in the study each supervised at least one lab and taught at least one class discovery group.
Evaluation included: (1) an achievement test covering the regular program; (2) review sheets given at the end of each activity in the experimental groups; (3) a cumulative achievement test and a test of higher level thinking and problem solving (both based on the experimental program); (4) a divergent thinking test; (5) two attitude scales; and (6) personal reactions to the details of the experimental settings (experimental groups only). The tests in (3) incorporated parallel items. The first test measured knowledge of a result or an activity like that described in the item; and the second, knowledge of a result of the inverse activity using different materials. (Sample items are given in the paper.) Test items gave all information necessary for solution.

As a measure of divergent thinking students listed as many different mathematical properties or problems that could be illustrated by a set of 40 cards, numbered from 1 to 10 in each of four different colors. One attitude scale was specially constructed, and one was "a more standard instrument." Student reactions were obtained by asking students to agree/disagree with items in parallel sets of statements.

4. Findings

There were no significant differences among the three groups at either grade level on the achievement test (1). For average and low ability Grade 7 students only, the class discovery group scored significantly higher (p < .05) on the review sheets than the math lab group. On evaluation item (3), the experimental groups performed significantly better than the control group, but the two experimental groups did not differ significantly. On the divergent thinking test, the mean number of acceptable responses was highest for the class discovery group and lowest for the control group. The math lab group appeared to have a "slightly better" attitude toward mathematics, as measured by both attitude scales. Significantly more students in the math lab group than in the class discovery group felt they gained something from the lab activities.

5. Interpretations

The experimental treatments did not seem to have an adverse effect on the learning of the textual material. The experimental group subjects did well with respect to the review sheets, even though "there was no guarantee that a pair working on a lab finished it or got its point." On the divergent thinking test the control group responded frequently (about 3/4 of answers) with arithmetic ideas, while the experimental groups responded with ideas of the special lessons (about 2/3 of answers). The most popular student-identified feature of the lab was working independently of the teacher. In using a lab a teacher "would have some hope that he was giving students a positive and an experimental outlook on mathematics, and an experience which promotes independence on the part of the learner, as well as increasing his range of mathematical ideas."
Abstractor's Notes

The authors state in the second paragraph that "rather than try to give a technical report, this paper will try to give a 'teacher's-eye' view of the study." The report is of some interest, therefore, merely as an example of the way in which serious research might be presented to teachers. There are some omissions in the report, however, which raise concerns about the study.

1. Sample: (a) Did teachers volunteer for the experiments? If so, this needs to be taken into account in the interpretations.
   (b) There is some confusion about the way students were assigned to treatments. "Students were randomly assigned to one of three groups," but in the class discovery group "lessons were presented to whole classes of students by their teachers." Was the random assignment done by individual students or by intact classes? If the former, were students in the class discovery group segregated so that they were taught only by their regular teacher?

2. Treatment: (a) The control group studied the regular mathematics program only. What was the nature of the regular program? Did it include much independent study? Were manipulative materials ever used? Were the experimental groups using manipulatives for the first time?
   (b) Apparently each piece of equipment was used only once during the experiment. Perhaps learning was affected by the novelty of the equipment.

3. Procedure: (a) For the lab group, the teacher served only as a resource person. How often was he/she called on to play this role?
   (b) In the lab group, "it was observed that pairs worked much better than trios." The definitions of treatments make no mention of the use of trios. When were such trios observed?
   (c) Was there a check on the extent to which the experimental techniques seeped into the regular instruction? If not, why was such a check deemed unnecessary?

4. Evaluation: (a) How long were the specially constructed tests? Were they reliable? Valid?
   (b) No measure was made of the teachers' attitudes toward the treatments. The students' attitudes may have reflected the teachers' attitudes rather than their own.

In spite of these questions, the study has several real strengths.

1. It was of long enough duration (ten weeks) to make the results believable. It approximated the real world more closely than most research.
2. The variety of evaluation (achievement, problem solving, transfer, attitudes) was not only exceptional but also appropriate since there was an attempt to approximate classroom practice and to gather data useful to teachers.

3. The authors provide enough information so that a teacher could develop a similar procedure. There is not enough information, however, to replicate the study. The treatments depend on the instructions written for the lab activities, and most of the evaluation instruments were specially constructed.

George W. Bright
Emory University
1. **Purpose**

Two major questions were investigated:

a) To what degree do second- and third-grade children learn transformational concepts under specific instructional conditions?

b) What are the effects of transformational geometry instruction upon children's spatial abilities?

2. **Rationale**

Various study groups have advocated the inclusion of geometry, particularly transformational geometry, instruction in the grades. Transformational geometry topics can be approached quite naturally through the manipulation of concrete objects or figure drawings. Piaget suggests an action component of cognitive functioning built upon sensori-motor actions. Eventually, after objects become distinct images, the child is able to perform mental transformations upon these images. Since imagery manipulations can be described in terms of geometrical transformations, it is natural to question the effects of a study of geometrical transformations upon one's imagery or spatial abilities.

3. **Research Design and Procedure**

A sample of 63 subjects from a population of 106 was evenly divided among two schools, two grade levels (2 and 3), two treatments (experimental and control) and two IQ levels (104-144 and 81-103). No significant initial differences were detected between experimental and control groups at either level of grade or IQ.

An experimental instructional unit, revised on the basis of a pilot study, was divided into three sections: congruent figures, rigid motion, and congruence and motion. In general, lesson activities progressed from those requiring manipulation of real objects to the manipulation of tracing illustrations, and finally to the mental manipulation of figure drawings.
An achievement test was constructed to measure the objectives of the instructional unit; it was designed to measure both comprehension and applications. A space test consisted of four 10-item subtests; it was designed to measure the ability to perform mental spatial manipulations.

The achievement and space tests were administered to all subjects before and after instruction. The experimental unit was administered in twelve 25- to 30-minute sessions held approximately three times a week over a period of four to five weeks. All experimental subjects were removed from their regular class and taught by the experimenter. About one week prior to the completion of the experiment, the control groups received a single lesson treatment dealing with terminology and a brief overview of the experimental unit.

An item analysis was performed on all dichotomous achievement items (there were 6 multiple choice items to test one objective). A point biserial coefficient, a difficulty index, and a Phi coefficient were computed for each item. Also an internal-consistency reliability coefficient was computed for all items in the item analysis. Program MUDAID was used for the multivariate and univariate analysis of variance of both the achievement and space-test data. Gain scores of the treatment group on the achievement and space tests were compared with those of the control group by using t tests.

4. Findings

The internal consistency reliability coefficient for the achievement tests was .80 for the pretest and .89 for the posttest. None of the items had a negative point biserial correlation indicating that no item was a negative discriminator. The Phi coefficients indicated that on the pretest, no significant differences existed between experimental and control on any of the items. On the posttest, no items favored the control group, but 39 items favored the experimental group. Also 15 items were answered by a significantly larger percentage of the experimental group; 3 of these were application items and the remaining 12 were comprehension items.

For the achievement posttest, the experimental group significantly outperformed the control group, but no other differences were detected. The t statistics indicated the experimental subjects gained significantly more than the control subjects. Although the third-grade experimental subjects scored significantly higher than the second-grade subjects on the achievement posttest, no significant grade or IQ gain score differences within the experimental group were detected.

Before instruction, significant grade (favoring Grade 3) and treatment (favoring experimental) differences existed on the space test; after instruction, grade and IQ differences occurred, but not treatment differences. A t test of the space gain scores revealed no significant differences between treatment groups.
5. **Interpretations**

Even though large differences were noted on the posttest achievement scores, item analyses revealed that only 15 of the 44 dichotomous items were answered correctly by a significantly larger percentage of experimental subjects. The fact that only 3 of those were "applications" items suggests the experimental subjects learned to perform certain skills to a greater degree than they were taught to apply such skills to the solution of more general exercises.

In terms of Piaget's theory, the subjects did not exhibit operations upon imagery. Instead, they were able to fashion reproductive images based upon the perception of original figures and upon manipulative techniques, rather than to produce anticipatory images resulting from operative thought.

**Abstractor's Notes**

This was a good example of a carefully designed and executed experimental study. My only wish is that the author had chosen a wider age range in his choice of experimental subjects. Rather than second and third graders, perhaps second and fourth, or second and fifth might have been better. The slightly older children could be good ones to use in a replication of this study.

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Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn N. Suydam, The Ohio State University.

1. **Purpose**

The study was undertaken to determine the validity of the criterion-referenced Tests of Achievement in Basic Skills (TABS)-MATH, Level B, Form 1, in predicting teachers' marks in grades 4, 5, and 6.

2. **Rationale**

This study parallels in design an earlier one to determine the predictive validity of the TABS, Level C. Although the shift in emphasis from norm- to criterion-referenced measurement has resulted in refinements of statistical techniques, in most situations standard techniques may be applied. Only in the case of the target population and determining an appropriate course of instruction paralleling test objectives would the distribution of scores theoretically differ substantially. Thus criterion-referenced tests are scaled at range extremes, while norm-referenced tests are scaled primarily at mid-range. Classical data analyses might seem appropriate for criterion-referenced tests at the beginning of instruction or for tests designed to serve the dual purposes of criterion- and norm-referenced tests.

3. **Research Design and Procedure**

Each of the two TABS-Level B forms contains 69 items arranged in three parts (Arithmetic Skills, Geometry-Measurement-Application, and Modern Concepts), representing year-end student achievement for grade 6 in the "typical school curriculum." The TABS was administered in October 1972 to 198 fourth graders, 316 fifth graders, and 245 sixth graders in two California schools, heterogeneously grouped although identified by the district at three ability levels. (Note: Table 3 indicated that N = 254 at grade 6.) Marks were accumulated in June 1973. Test results were not made available to teachers until after the assignment of grades, and thus were not used as a basis for instruction. Pearson product-moment correlations were computed at each grade level between teachers' marks in mathematics and each of the four predictor variables (total score and score from each part) and also among the predictors.
4. Findings

Correlations of the total TABS score with teachers' marks were .43, .62, and .73 for grades 4, 5, and 6, respectively. All correlations were significant (level not specified). Correlations of part scores with marks were .42, .33, and .40 at grade 4; .60, .56, and .53 at grade 5; and .65, .58, and .59 at grade 6.

5. Interpretations

The correlations "were considered to be at a level consistent with results obtained in previous studies relating test scores to achievement as reflected in teachers' marks." Scores from the test were "shown to be quite valid predictors at the fifth- and sixth-grade levels," while the grade 4 correlation was considerably lower. Since the TABS was focused on selected instructional objectives, it is of interest that correlations generated against traditional criteria remained at acceptable levels. Somewhat lower correlations were found between TABS part scores and marks, as would be expected.

"If a criterion-referenced test is to be used as a predictor for a target population, if teachers do not change their A to F marking procedure, and if the Pearson-product-moment correlation is used to estimate the predictive validity, even higher correlations than those found might be anticipated. However, if teachers do not force all students into a testing situation at the same time and if teachers were to give students A's when all items were mastered, lower correlation estimates would be anticipated and acceptable. Thus depending upon the achievement marking system employed, predictive validity (correlation) coefficients could be either high or low and still could be reasonable estimates." (pp. 912-913)

Abstractor's Notes

What can be said? The sample size is relatively adequate, the correlations are reasonable, the procedures are simple. Missing information (e.g., significance level, information on the criteria teachers used in marking), a printing error (size of grade 6 sample), and obscure writing at some points (e.g., the last paragraph, quoted above) made this report take longer to read than was warranted. For those who want to consider the feasibility of using a so-called criterion-referenced test as a (normative) predictor, it may provide some useful information.

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