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ABSTRACT

This Pennsylvania Comprehensive Mathematics Plan (PCMP) was designed for the State Department of Education to chart the direction for mathematics education in the upcoming decade. Sections in this report cover: (1) the goals of PCMP, (2) the PCMP and instructional objectives, (3) the curriculum, (4) teachers' professional growth, (5) learning experiences, (6) an evaluation plan, (7) a summary, (8) a bibliography, and (9) four appendices. The first appendix presents some sources from literature that provide research support for PCMP. Appendix B is a list of selected references, Part C is a list of desired student outcomes in mathematics, and the fourth appendix reviews the Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES), a resource for educators to use in developing, implementing, and evaluating mathematics curricula. (MP)

pcmp

PENNSYLVANIA COMPREHENSIVE MATHEMATICS PLAN

Direction for the 80's

Commonwealth of Pennsylvania
Richard L. Thornburgh, *Governor*

Department of Education
Robert G. Scanlon, *Secretary*

Office of Basic Education
Ronald H. Lewis, *Commissioner*

Bureau of Curriculum Services
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PENNSYLVANIA COMPREHENSIVE MATHEMATICS PLAN: DIRECTION FOR THE 80's

Pennsylvania Department of Education 1980

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The Pennsylvania Comprehensive Mathematics Plan is the culmination of the research and thinking of nationally recognized mathematics educators. The final manuscript was designed and developed by M. Vere DeVault, University of Wisconsin, who selected, organized and synthesized the content suggestions from five preliminary papers.

From a five-person writing team came the initial form of the plan. The members: LeRoy Callahan, SUNY at Buffalo; Richard Gibboney, University of Pennsylvania; Ralph Heimer, The Pennsylvania State University; Bradley Seager, University of Pittsburgh; and Alex Tobin, School District of Philadelphia. Among the tasks assigned the team were defining the contents, suggesting the developmental process, producing preliminary outlines and narratives and promoting dialogue on each topic.

Appreciation is extended to Schuylkill Intermediate Unit 29 for the management of the Pennsylvania Department of Education contract to produce the Plan, and especially to Ralph Morgan who administered the project at the Intermediate Unit. Additionally, Carol Backman served as coordinator in managing the various activities of the Plan.

PENNSYLVANIA COMPREHENSIVE MATHEMATICS PROGRAM

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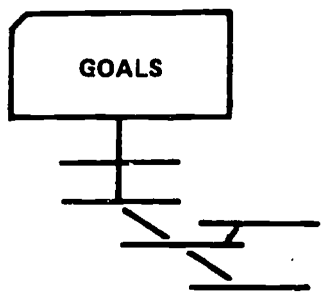
THE GOALS OF PCMP



School improvement that responds to the demands of the eighties will require sophisticated coordination of all available resources.

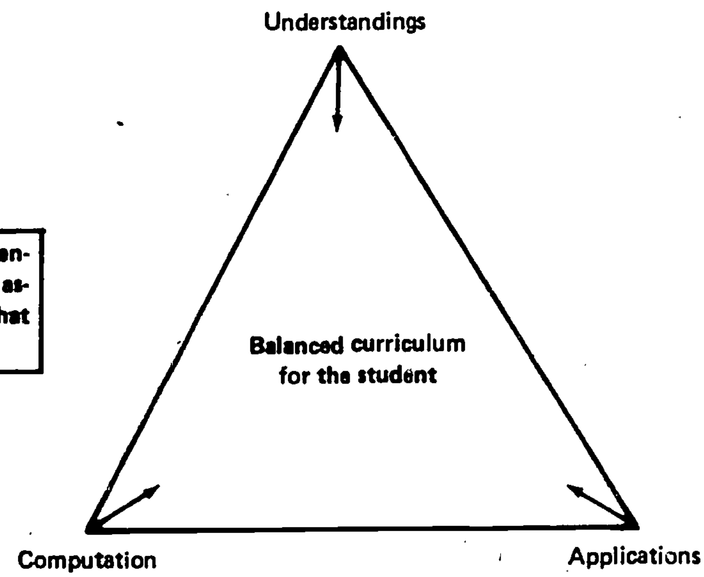
School mathematics programs throughout Pennsylvania are better now than they were three or four decades ago. The gap between what they are now and what they must become, however, continues to widen. This is because we live in an age that is increasingly technological—an age that demands mathematical competence at every turn.

The technology of the next two decades will require of our citizens *greater* competence in mathematics as well as *different* competence. In this time of stress, it is important that we avoid band-wagon fads. We need to renew our commitment to the basics in the mathematics programs, and to maintain a balance among them.

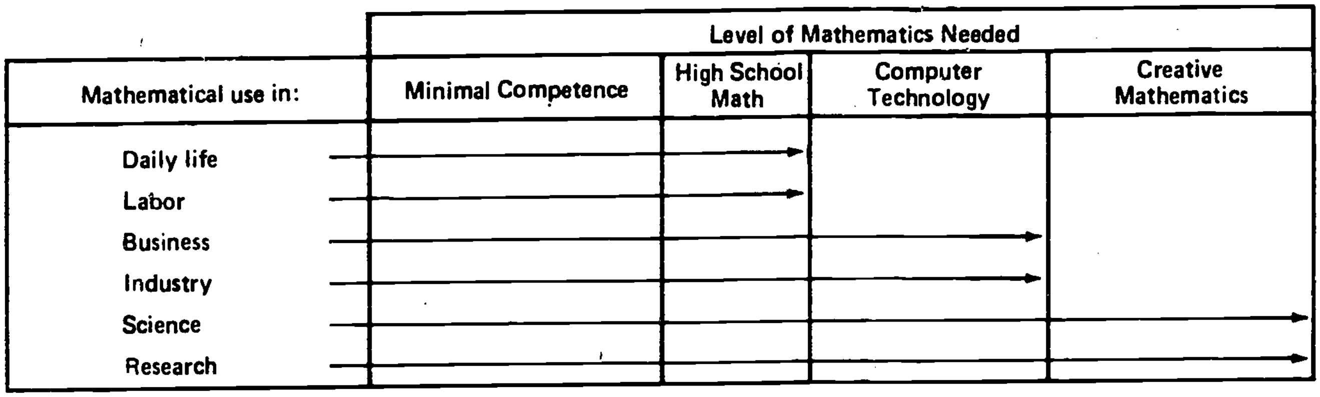
Fundamentally, the basics are the same as they have always been:

- *Understanding mathematical concepts
- *Computational skills
- *Applications and problem solving

Balance requires that in planning and in implementation comparable attention is given to all three aspects of the curriculum. The PCMP emphasizes that balanced focus.



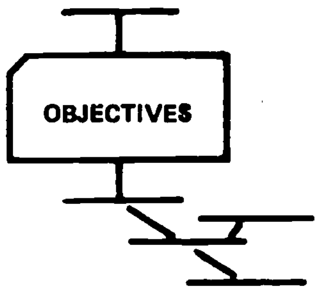
The goal of the PCMP is to prepare students throughout Pennsylvania with the mathematical competence required for society throughout the eighties. Effective living is more and more dependent on mathematical competence. Science and commerce are dependent on mathematical expertise in an increasingly substantial proportion of our population.



Society's Needs for Mathematical Competence

The PCMP encourages the improvement of student attitudes toward mathematics. The decade of the seventies was one in which we became increasingly aware of the importance of many factors on women's attitudes toward mathematics. There is no subject area in the school that is more important in determining eligibility for advanced study in many areas than is mathematics. Engineering, accounting, business, medicine, the sciences, and many more disciplines are closed to students without adequate and continuing study of mathematics throughout high school years. Teacher, parents, and community expectations for our women students must be raised, if women are to enjoy the same personal, economic, and professional advantages as their male counterparts.

THE PCMP AND INSTRUCTIONAL OBJECTIVES



The PCMP is based on the assumption that instructional objectives are an essential element of an effective mathematics program.

Though it is likely that instructional objectives were initially recommended for purposes of evaluation, current interest in the development of instructional systems has created a new and significant use for them.

Sets of objectives should be designed to serve the following purposes:

—Provide direction for the creation and selection of learning materials.

—Provide teachers with guides for determining instructional activities.

—Provide students with information about what is to be learned.

—Provide instructional systems designers with the necessary information for the development of instructional sequences and, hence, prescriptions for groups or individual students.

—Provide students with information about their progress in learning mathematics.

—Provide teachers with information about the progress of individual students and the class as a whole.

Instructional objectives take on meaning as they are grouped into related, sequential, or otherwise associated sets. The strength of the objectives-based curriculum is largely a function of the degree to which objectives having close relationships are packaged together into coherent teaching/learning units.

In an adaption of Bloom's levels of cognition, the School Mathematics Study Group identified five levels that are important to mathematics. All five should be represented in the set of objectives representing a comprehensive mathematics program.

*Knowledge of facts
*Computation
*Comprehension
*Application
*Analysis
*Synthesis

Instructional objectives should:

. . . be selected, prepared, and used as important realistic guidelines for instruction and evaluation.

. . . be viewed as minimal attainments; not as ceilings or as final levels of achievement for all learners.

. . . be so clearly written that their intent cannot be mistaken.

. . . be feasible for students at the level for which they are intended.

. . . be written in terms of student behaviors.

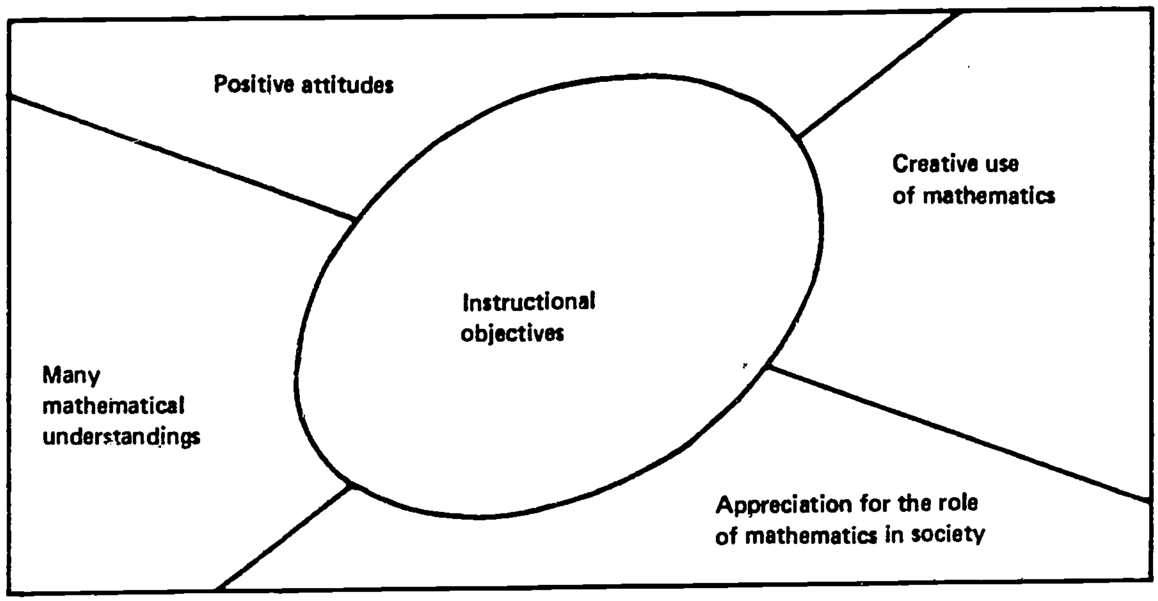
It should not be assumed, however, that all goals of the mathematics program can be represented by instructional objectives. The following areas represent those in which instructional objectives are probably inappropriate though the importance of these goals to the PCMP must be emphasized.

*Positive attitudes

*Creative uses of mathematics

*Many mathematical understandings

*Appreciation for the role of mathematics in society



In the PCMP, instructional objectives are viewed as the core of the program. They are the necessary, but not sufficient, elements of an effective program for the eighties. Embedded in an instructional environment that fosters understanding, creativity, appreciation, and positive attitudes toward mathematics, instructional objectives effectively serve many instructional/learning functions.

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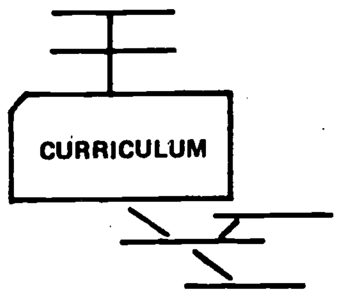
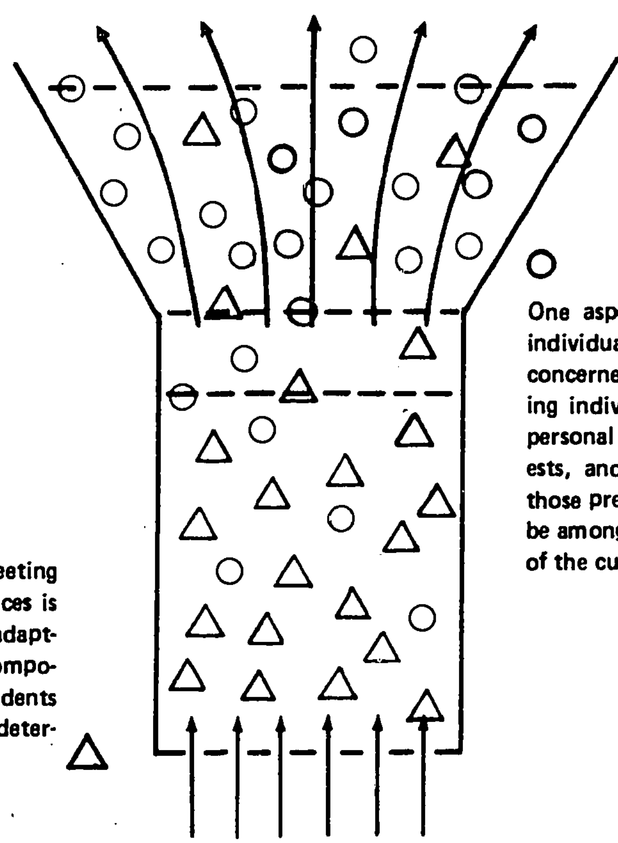
THE PCMP CURRICULUM



The PCMP curriculum is based on the assumption that all students can and should learn mathematics from elementary school through high school.

Students leave the PCMP with great variability in understanding, skill, and appreciation for mathematics.



One aspect of meeting individual differences is concerned with extending individual students' personal talents, interests, and skills beyond those pre-determined to be among the objectives of the curriculum.

One aspect of meeting individual differences is concerned with adapting program components to assist students in attaining pre-determined objectives.

Students enter the PCMP with various needs, interests, aptitudes for mathematics.

For all students, meeting individual differences means . . .

. . . Finding learning strategies and content sequences that assure the attainment of minimum competencies that have been identified in the curriculum. Though initially students may be very different in their competence with mathematics, instruction is designed to assure their similarity in terms of minimal competence that has been identified.

. . . Finding mathematical experiences in the curriculum that extend students' mathematical competencies in terms of their own unique talents and interests.

. . . The course of study for each student is success-oriented. It is never assumed that a student is incapable of further significant learning of mathematics, and the course of study for each student is designed to enable each to reach the goal of functional competence as an adult user of mathematics. The curriculum for each student challenges the student to exceed the minimal competence for graduation and will foster the attitude that the time spent in learning mathematics is well spent.

. . . Experiencing an enriched curriculum that

arises from activities designed and initiated by the students themselves.
is made possible through the availability of resources provided to each teacher through the local district.
is viewed by teachers, students, parents, and others as important as is the essential curriculum.

The PCMP curriculum is determined by the local school district in accordance with sound principles of curriculum development and adapted by the teacher to respond to the needs, interests and abilities of individual students.

The curriculum is described through three major resources made available to every teacher.

1. **Scope and Sequence Chart**
   Prepared locally with the help of resources from the Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES) or some other system of comparable quality.

2. **Curriculum Guide for Grades K-12**
   Based on the district's scope and sequence chart
   Includes instructional objectives given top priority for each content area at each grade level
   For each area of mathematical content
   - sample test item
   - reference to textbook pages
   - listing of appropriate supplementary materials

3. **Minimum Standards of Mathematical Content Attainment**
   Listing of specific requirements for High School graduation

The essential mathematics curriculum will be related to other concurrent learning experiences of the students.

The mathematics curriculum is broadly based. Just as every teacher is a reading teacher, so too, is every teacher a teacher of mathematics. Beyond the school, parents and others also teach much mathematics. Many concurrent daily experiences contribute to the students' understanding and appreciation of mathematics.

In school At home In the community	Formal Informal	Concrete Abstract	Other school subjects	Trivial Significant	Theoretical Practical	The language of mathematics	Life experiences Career choices

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The curriculum should contain a balance of breadth and depth for each student and should include most of the following topics with the depth of each topic determined by each student's needs, interests, and abilities.

1. Sets, members of a set, equivalent sets, and non-equivalent sets, subsets, the empty set, counting, cardinal numbers, and ordinal numbers.
2. Operations with whole numbers and fractional numbers including addition, subtraction, multiplication, and division.
3. Comparison of whole numbers and fractional numbers.
4. Reading and writing whole numbers and fractional numbers, including place value.
5. Recognizing shapes and geometric patterns both in the environment and abstractly.
6. Points, lines, line segments, intersecting and parallel lines, rays, angles, polygons, and general properties of two and three dimensional figures.
7. Congruence and symmetry; measurement of line segments and angles; perimeter and area of plane figures.
8. Metric and customary systems of measure for length, area, capacity, and weight.
9. Money and time.
10. Prime and composite numbers; greatest common factor; least common factor; ratios; percent.
11. Graphs; coordinate systems; average; function.
12. Mathematical sentences; solution of linear equations; estimation.
13. Probability and statistics.
14. Logic flowcharting and computer concepts.
15. Algebra and the theory of equations.
16. Solid geometry and trigonometry.
17. Number theory and the field properties.
18. History of mathematics.

Seven characteristics of the PCMP school

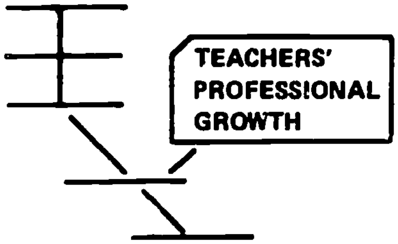
Program

1. Instructional objectives are at the core of the program but are surrounded and embedded in a set of broad goals for mathematical competence that include understandings, attitudes, creativity, and appreciation.
2. Balance is maintained among the three major emphases of the program: computation, understanding, and applications.
3. There is a commitment on the part of each teacher to personalized/individualized instruction.

Implementation

4. The faculty is committed to the improvement of clearly defined prioritized components of the mathematics program.
5. Parents, faculty, and administration are committed to the implementation of the PCMP.
6. An active four-way working relationship exists between the school, the LEA, the Intermediate Unit, and the Department of Education.
7. There is continuous monitoring and appraisal of the successes and problems associated with the implementation of each component of the PCMP.

THE PCMP AND TEACHERS' PROFESSIONAL GROWTH



School improvement envisioned through the PCMP requires both curriculum innovation and its associated professional growth of teachers.

The PCMP is designed to improve existing practices in many instances, but it is also designed to add new dimensions to the mathematics program K-12. In both instances, there is a continuing need for inservice education designed to help teachers develop and maintain those competencies essential to the effective implementation of the PCMP.

Just as school improvement in all aspects of the PCMP cannot be attained all at once, necessary development of all teacher competencies to implement the PCMP cannot be sought simutaneously either. Curriculum development and inservice education, therefore, should be precisely coordinated so that improvement is accomplished step-by-step.

Each of the major characteristics of the PCMP implies needs and opportunities for teacher growth.

The first PCMP characteristic

Instructional objectives are at the core of the program and are surrounded and embedded in a set of broad goals for mathematical competence that include understandings, attitudes, creativity, and appreciation.

Professional growth needs. As mathematics programs have become increasingly objectives-based, there has been reason to fear their increasing narrowness. Improved teacher competence is needed if activities are to be integrated in such a way as to foster understandings, positive attitudes, creativity, and appreciation of the role of mathematics in society, while at the same time meeting the requirements or goals established by the program objectives.

The second PCMP characteristic

Balance is maintained among the three major emphases of the program: computation, understandings, and applications.

Professional growth needs. In both computation and understandings, inservice needs are for sharing among teachers and for renewing commitments to excellence in these goals. Recent research concerning the importance of time on task—the actual time a student gives to the study of a given task—should receive considerable attention by practicing teachers. Understandings and the role of concrete materials in their development requires renewed commitment. For many teachers, however, the recent new emphases on problem solving and applications, K-12, require special study and explorations. Though these three emphases can be studied in isolation, an additional major task is to determine what balance among the three means for the instructional program.

The third PCMP characterisitic

The commitment to personalized/individualized instruction requires a sophisticated instructional management system.

Professional growth needs. One of the dangers with instructional management systems and commitments to personalized/individualized instruction is the reduction that frequently follows in the significance of instructional roles assumed by teachers. The teacher role has always been one of integrating instruction and management. With the advent of management systems, frequently computerized, the teacher should now be increasingly freed to assume more time for instruction. That instruction will be with the whole class, but also with small groups and with individuals. Each requires a different set of teaching skills.

Planning for inservice education

The involvement of teachers is crucial to the success of the PCMP inservice program.

Teacher involvement occurs at three points:

1. Planning the goals and activities
2. Sharing expertise as a part of the inservice program
3. Determining the nature and content of one's own participation

Planning goals and activities. Priorities need to be set in keeping with those priorities established for the local PCMP. Not all things can be accomplished at once. Teachers are in the best position to determine their needs if established goals are to be achieved.

Sharing expertise. As in the concept of the Teacher Center, teachers frequently have within their group experts who can provide the best kind of inservice help. Teachers know others who are experts in management, in content, in the use of games to develop skills in computation, and in the use of hand-held calculators or micro-computers for instruction in mathematics. These people are identified and at the request of teachers, serve as leaders of inservice education activities.

Determining one's own participation. Just as the PCMP puts a premium on personalization/individualization of instruction for children, it also does for teachers. Within the set of priorities established for the local school, the individual teacher exercises a variety of options as to the manner in which competence is to be enhanced.

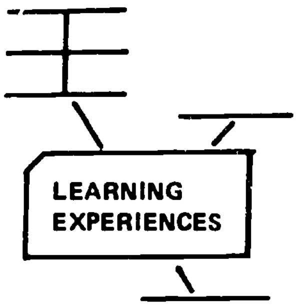
The substantive focus of inservice activities is specific though the activity options for teachers are varied.

Specific focus. The topics of study for inservice education are directly related to the primary focus of developing PCMP within a local school. If the focus of the program is on instructional management, for instance, then the inservice education program would be centered around problems associated with instructional management.

Activity options. Though the planning of inservice activities is planned cooperatively with those teachers expected to participate, there is, nonetheless, still opportunity for teachers to choose from among options such as:

—independent study
—sharing with and by other teachers
—school system provisions
—college or university courses

LEARNING EXPERIENCES IN THE PCMP



In the PCMP, learning experiences are designed to respond to the unique talents, interests, and abilities of individual students.

In responding to the individuality of each student, consideration is given to:

—variety of experiences for each student
  variety in learning materials
  variety in learning pace
  variety in grouping structure

—focus on learning goals
  focus on instructional objectives
  focus on attitudes
  focus on understandings
  focus on creative uses of mathematics
  focus on appreciation for mathematics in society

—concern for humanistic values in the classroom
  concern for interpersonal relations
  concern for the quality of verbal interaction among peers and between students and teachers
  concern for appropriateness of prescription in terms of success probabilities

The PCMP recognizes the uniqueness of each teacher and hence, the uniqueness of instructional style in each classroom.

Only as the uniqueness of each teacher is recognized and appreciated, can one expect that students' uniquenesses will also be appreciated.

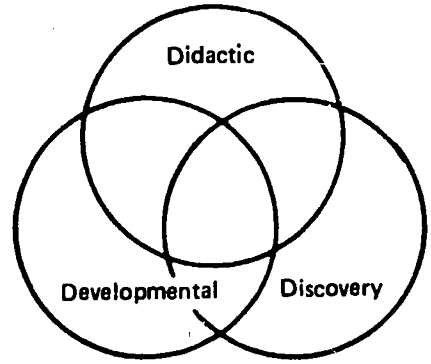
. . . group instruction with one teacher differs from that same kind of instruction by another.

. . . individual one-to-one instruction has a different impact on students with different teachers.

. . . a didactic presentation by one teacher has a different kind of enthusiasm and creates different kinds of motivation than does another.

Teachers employ a balanced variety of instructional methods.

In whatever organizational structure teaching occurs, three instructional modes are used to provide balanced, instructional experience in mathematics.



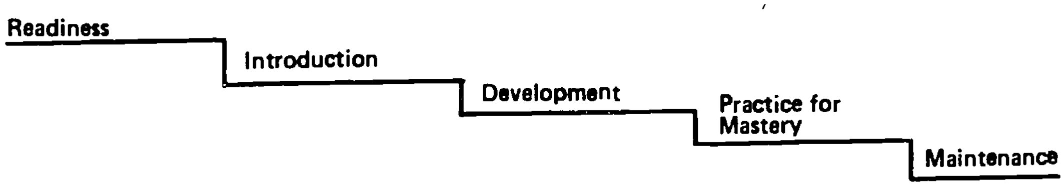
Instructional Methods

Didactic. There are times, topics, students and teachers for which didactic (direct presentation) instruction is appropriate.

Developmental. There are opportunities for developmental activities appropriate for readiness experience or exploration of materials and ideas prior to practice for mastery.

Discovery. Discovery experiences are provided to enhance interest and understanding and to share with learners one of the major inquiry modes in the development of mathematical ideas.

The PCMP makes a clear distinction among the five steps in the learning sequence.



Readiness experiences are given a high priority at all levels of instruction. Teachers understand curricular expectations of the next two or three years and find ways of incorporating experiences that are as harbingers of curricular expectations yet to come.

Instruction that introduces new topics may be minimized as greater amounts of instructional/learning time are devoted to developmental activities.

Concentrated and spaced drill and practice provide for skill mastery.

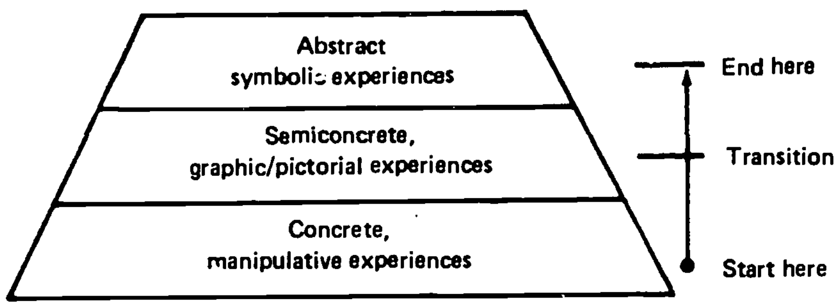
Maintenance of skills is a continuing part of the mathematics programs throughout grades 4-12.

In the PCMP, learning of mathematical concepts begins, where possible, with concrete real-world forms of the idea, and only then moves toward more abstract, symbolic representations.

In all phases of instruction, the idea of moving from the concrete to the abstract has an important role to play in the development of understanding mathematical ideas. One of the by-products of the idea, however, is the mathematics lab approach to teaching.

> As defined by Webster, a laboratory is "a place equipped for experimental study in a science or for testing and analysis; a place providing opportunity for experimentation, observation, or practice in a field of study."

In general, this definition provides an apt description of the nature and function of the mathematics laboratory. The laboratory learning approach emphasizes the concept of learning by doing, of actual manipulation of apparatus. This "hands-on" experience with physical-world objects serves a basic need in the child's concept development. The presence of concrete learning experiences helps to bring meaning to the symbolic representation necessary to embody mathematical concepts.



The diagram suggests the desired learning sequence. As indicated, the teacher should arrange first to have the child engage in concrete, manipulative types of learning experiences; then move the child to a semiconcrete learning environment; and finally introduce the abstract forms of the ideas to be learned. Thus, when a child is working with mathematical symbols, the symbols have concrete referents in the child's experience. Such is the basis for meaningful learning.

A wide variety of instructional materials characterize learning experiences in the PCMP.

1. *The teacher as instructor.* Any discussion of instructional materials in the PCMP begins with the teacher. The teacher is in charge; the teacher makes the decisions; and the teacher is the student's primary source of knowledge, motivation, and advice concerning an individual's program of learning experiences.
2. *The teacher as manager.* A wide variety of options for the teacher and for individual learners are continuously available. It is the teacher who determines when and in what manner these many options will be used either with the entire class, with small groups, or with individual students.
3. *The textbook.* Of central importance to the PCMP is the carefully selected mathematics textbook that is correlated closely with the school system's curriculum guide. Though the textbook is the most important single piece of instructional material, its judicious use is accompanied by experiences provided by the teacher, by a wide range of concrete materials, multi-media aids, field trips, projects, and computer facilities.

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4. *Other instructional materials.* The PCMP assumes the use of many instructional materials of a wide variety at all levels. Though such materials are traditionally used more at the early grades than at grades beyond four or five, PCMP assumes a major role for instructional materials, other than the textbook, throughout the middle and high school years. Such materials include:

Displays	Manipulatives
Filmstrips and films	Tapes
Games	Calculators
Counters	Metric measurement devices
Kits	

5. *Computer facilities.* Hands-on experiences with computers are seen as an important part of the PCMP at appropriate levels. Such facilities provide for students an opportunity to:

   Acquaint themselves with fundamentals of program logic
   Acquire computer literacy
   Program problem solutions
   Simulate situations that test hypotheses
   Study probability and statistics
   Test, drill, and tutor in mathematics

The management of learning experiences is a major undertaking. Current developments in micro-computer technologies make computer-managed instruction increasingly available. Even without computers, recent considerations of the many and varied problems associated with instructional management have provided many solutions to classroom management problems.

Management systems should be designed to accomplish the following:

... Maintain information about the curriculum, including objectives, sequences and learning materials associated with sets of objectives.

... Maintain information about each learner, including work completed and test scores attained for.that work.

... Provide instructors with means of adding to, deleting from, or otherwise changing the nature of the curriculum so as to make it appropriate for a class or for individuals within the class.

... Provide for the input of information concerning individual student progress, including recent units completed, test data obtained.

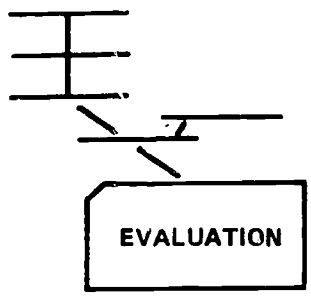
... Provide prescriptions for individual learners or for groups of learners based on the information in the computer concerning the curriculum and its sequences and information about the learners and their past histories.

... Provide grouping of learners according to their individual needs either in terms of next units for which they are ready or in terms of past effc :s with units on which these particular learners need additional assistance.

... Provide instructors with information about the learning materials, texts, and supplementary materials that are available for use by groups or by individual learners.

Time on task has been shown by numerous recent research studies to be the most important factor associated with achievement in mathematics. Teachers, with or without computer assistance, should be constantly aware of the implications of this finding.

THE PCMP EVALUATION PLAN



Evaluation is an important component of the PCMP A comprehensive program for evaluation employs a variety of assessment devices with different populations. Evaluation is also concerned with the continual assessment of program goals.

The past two decades have seen substantial progress in the development of tests, testing modes, and test applications; yet, there remains controversy over test use in the schools.

The PCMP is based on the following set of assumptions:

1. Testing is an essential element of an effective mathematics program.
2. Test construction is, at best, a fledgling science with many problems and issues confronting those who construct and use them.
3. Information about individual learners must come from a variety of sources if that information is to be valid and if it is to encompass the broad and varied goals established in the PCMP.
4. Formal test strategies such as standardized tests and curriculum related (criterion-referenced) tests must be supplemented with informal measures made through interviews, observations, and self-evaluations.
5. Program evaluation is based on the information obtained from this array of formal and informal measures supplemented by informal teacher reactions made through the use of interviews and rating questionnaires.

Evaluation Forms

Standardized tests. Teachers use standardized (norm-referenced) tests to compare individual progress with the progress of other learners in appropriate reference groups. It is useful to know how a student's progress compares with that of other students, but the choice of appropriate reference groups can be a matter of controversy. There is a tendency for many people to overgeneralize and oversimplify the interpretation of students' scores on norm-referenced tests. Such tests should be analyzed to determine the extent of overlap between the objectives measured on the test and the objectives of the student's curriculum. Also, some students perform less well on standardized tests and in other formal examination settings than they do in informal settings, and this difference (where it exists) should be taken into account in interpreting test scores.

Curriculum-related tests. Teachers use curriculum-related (criterion-referenced) tests to compare individual learning with objectives specified in the essential curriculum for each student. Sometimes commercially available tests are suitable as criterion-referenced tests, because there is a close match between the objectives on the test and those in the student's curriculum. When this is not the case, the teacher can construct (with the help of the mathematics specialist, if necessary) criterion-referenced tests. These tests will show whether the students have mastered specific curriculum objectives.

Diagnostic tests. Teachers use diagnostic tests (pretests) to help determine students' readiness for mastery of specific objectives in the curriculum. A criterion-referenced test can serve a diagnostic function by showing whether a student has the prerequisite concepts and skills that will enable the student to master new concepts or skills in the curriculum. For example, if a student demonstrates skill in prime factorization, it is likely that the student is ready to develop the concept of the greatest common factor of two numbers. Some topics in mathematics can be more clearly seen as hierarchical and sequential than others. Diagnostic tests are based on the assumption that the objectives they measure are hierarchical and sequential.

Test Type	Population: SCHOOL	CLASS	GROUP	INDIVIDUAL
STANDARDIZED	Comparing achievement levels at various grades with national norms	Determining average achievement growth over a year's time		Growth profile across years for permanent record
CURRICULUM RELATED		Provide a topic-by-student skills attainment profile	Determine skill development resulting from group study	Determine which specific skills have been mastered to create student mastery profile
DIAGNOSTIC			Determine specific topics for which all students in group need additional study	Determine specific topics for which the student needs additional study
INTERVIEWS				Determine understanding and attitudes of individual students
OBSERVATIONS				Determine style and quality of individual student's work

Major Functions of Different Evaluation Forms with Varying Populations

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Interviews. Teachers use interviews to determine the conceptual level of the child's understanding, to get a picture of the child's attitudes toward mathematics, and to learn more about the child's best learning style or at least preferred learning style. The interview brings the teacher in touch with the child's perspective of mathematics in its broadest sense.

Observations. Teachers use observations to determine the child's work habits, to see what kind of exploratory activities the student employs and to encourage creative expressions in mathematics. The broader concepts and understandings are difficult for the more formal measures to tap and hence, the informal interviews and observations. Children aware that such interviews and observations are a part of their mathematics instruction are more likely than others to extend their mathematics investigations to include ideas and activities that cannot otherwise be reported or shared with teachers or with other students.

Self-Evaluations

Tests, interviews, and observations form the bulk of the evaluation program from the teacher's point of view. Self-evaluations can do much to extend both the assessment program and the instructional program. Students who are involved in extensive self-evaluation procedures are much more aware of the goals and objectives of the program, their progress toward those goals, and the manner in which their present study is moving them toward desirable program objectives.

Self-evaluation includes:

. . . students' setting the objectives for their learning

. . . students' assurance that objectives are realistic in terms of their needs, interests, and abilities

. . . students' knowledge of the explicit criteria for evaluating their achievement

. . . students' opportunities to practice self-evaluation and to improve their skills in self-evaluation

. . . respect for students' self-evaluation

. . . students' access to other forms of evaluation to reinforce their self-evaluations and to build confidence in their growing skills of self-evaluation.

PCMP Evaluation

The success of the implementation plan for the PCMP requires continuous monitoring of the program to determine the success that has been achieved thus far and to set goals for continued implementation plans.

Program evaluation will include:

. . . a careful and continuous comparison between stated program objectives and the actual results of the program as it functions in the school.

Illustrations

If increased competence in problem solving and applications was viewed as a need, and hence, an objective of the local PCMP, what evidence is there that students are experiencing more problem solving activities and are achieving greater success with problem solving?

If computer applications in the curriculum was one of the goals of the local PCMP, what progress has been made in the implementation of computer uses in the school mathematics program?

. . . a careful review of the results of formal and informal evaluations of student achievement to determine the extent to which the achievement goals and objectives of the program have been met.

Illustrations

Are subgroups within the student population achieving in accordance with the goals of the local PCMP?

Are the broad topics in mathematics equally represented in the success profiles of students and classes?

. . . various means of obtaining reactions from teachers to determine their perceptions of the success of the PCMP and their feeling of success as a result of the implementation of the PCMP.

SUMMARY

GOALS: School improvement that responds to the demands of the eighties will require sophisticated coordination of all available resources.

OBJECTIVES: The PCMP is based on the assumption that instructional objectives are an essential element of an effective mathematics program.

CURRICULUM: The PCMP curriculum is based on the assumption that all students can and should learn mathematics from elementary school through high school.

TEACHERS' PROFESSIONAL GROWTH: School improvement envisioned through the PCMP requires both curriculum innovation and its associated professional growth of teachers.

LEARNING EXPERIENCES: In the PCMP, learning experiences are designed to respond to the unique talents, interests, and abilities of individual students.

EVALUATION: Evaluation is an important component of the PCMP. A comprehensive program for evaluation employs a variety of assessment devices with different populations. Evaluation is also concerned with the continual assessment of program goals.

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APPENDIX A—RESEARCH SUPPORT FOR PCMP

Introduction: The body of empirical evidence accumulated in mathematics education over the past century is neither sufficiently comprehensive in scope or stable in structure to be used exclusively in generating the curriculum for PCMP. The empirical evidence must be liberally laced with authoritative thought and experience that has accumulated over many hundreds of years of teaching mathematics to a society's children and youth. This section presents some sources from the literature that support the PCMP's balanced perspective regarding curriculum, instruction, evaluation, and the professional growth of teachers and the leadership roles of personnel at various levels of authority. The sources should help sharpen the focus on the parameters that bound these areas and support schools in establishing a mathematics education stance that is sufficiently flexible to respond to changing social conditions, yet sufficiently rigid to withstand fickle and passing presses constantly buffeting general education. The development of a balanced program is essential to insure the growth and development of student's knowledge and insight in mathematics.

I. Curriculum

A. *What mathematics should be included in the general education of all students?*

Two lines of force have traditionally influenced mathematics in general education. These spring from the dual nature of mathematics. On the one hand there is the view of mathematics as a logical system of abstract related ideas. From this perspective a premium is placed on the study of mathematics as a logical system of related ideas. The systematic study of this elegant product of the human mind is viewed as essential for the generally educated person. Support for inclusion of this aspect of mathematics in the curriculum can be found in the literature (Bruner, 1960; Goals for School Mathematics, 1963; Judd, 1926; McConnell, 1941; Piaget, 1973).

On the other hand, there is an instrumentalist view of mathematics. That perspective places a premium on mastery of socially and vocationally useful mathematical skills and their application to relevant situations. The mastery of essential mathematical skills and abilities and proficiency in applying them in every day situations is viewed as important for the generally educated person. Support for this aspect of mathematics can be found in the literature (Bell, 1974; Braunfeld, 1975; Brueckner, 1941; Kline, 1973; National Assessment of Educational Progress, 1979; Wilson, 1951).

The PCMP promotes a balanced view regarding these two lines of thought. Programs should reflect a concern for providing students with opportunities to glimpse the logical structure of mathematics. This consideration for the relationships, interrelationships, and generalizations of mathematics contributes to understanding and skills in student learning. Likewise, opportunities must be provided for mastery of essential skills and abilities and their application to socially relevant situations. There is extensive support for such a balanced perspective in the literature (Brownell, 1956; Conference Board of the Mathematical Sciences, 1975; Glennon, 1965; Skemp, 1976).

B. *What principles should guide curriculum organization?*

One view on this matter places central importance on the objective analysis of the mathematical content into discrete learning units; the arrangement of these units into a logically sequenced order; and the systematic presentation of the units to be learned in orderly sequence (Gagné, Mayor, Garstens, & Paradise, 1962; Resnick, Wang & Kaplan, 1970). Another view places central importance on student-driven, wholistic experiences in mathematics. Needed mathematical skills and abilities should be oriented around significant and purposeful problems being explored by students and teachers (Brown, 1973; Harap & Mapes, 1939; Reys & Post, 1973). The PCMP affirms the need for a broad, well-structured and sequenced mathematics curriculum organization for school systems. This will assure comprehensive learning opportunities for all students. However, within this broad structure, opportunities for students to make in-depth or in-breadth exploration, unrestrained by any objectively imposed factors, should be provided.

C. *Will the curriculum need to be accommodated to address student differences?*

There is little doubt that accommodations must be made. Evidence has accumulated over the years that points to student differences in development of arithmetic abilities (Ilg, Ames, Haines and Gillespie, 1978; Washburne, 1928); in mathematical concept development (Almy, 1966; Carpenter, 1979; Lovell, 1971); in different learning styles (Erlwanger, 1973; Ginsburg, 1974). In addition accommodations must be made for exceptional learners such as gifted and talented (Glennon, 1963; Krutetskii, 1976; Stanley, 1974); the mentally retarded (Cawley and Vitello, 1972; Connally, 1973; Cruickshank, 1946), the learning disabled (Greenes, 1978; Johnson, 1979; Sharma 1979) and students with other handicapping conditions (Suppes, 1974).

II. Learning Experiences

A. *Methodology*

Two lines of force have influenced teaching methodology. One is the direct didactic teaching of the subject by telling the student. The contemporary teacher is aided in the use of this method by different media. There are textbooks, videotapes, and computer programs that tell students about mathematics. The essential method of teaching remains the same, however, regardless of media. Support for this method can be found in the literature (Ausubel, 1968; Carroll, 1968; Engelmann, 1968).

The other method is the more indirect discovery, or guided discovery, approach to teaching mathematics. The essence of this method involves the learner in discovery of the mathematics through the use of various heuristic procedures. Support for this approach can also be found in the literature (Bruner, 1961; David, 1966; Hendrix, 1961).

The PCMP advocates a balanced perspective in methods of teaching mathematics. No one method has been shown to be consistently superior. It is hardly likely that any one method can fit all teaching situations. Some reasonable variety in methods based on content, student, and teacher considerations would appear to be defensible (Friedlander, 1965; Shulman, 1970; Wittrock, 1966).

B. *Classroom organization*

Various procedures have been formulated over the years to optimize teaching/learning efficiency and quality through classroom organization schemes. There are procedures that highlight the importance of individualized or personalized instruction, while others highlight the importance of the group dynamic in learning. It is highly unlikely that any one organization scheme will address the complex instructional needs of all students and teachers. Some reasonable and well planned procedures that incorporate variety into a building unit's organization procedure would seem to be very defensible (Clark and Ramsay, 1973; Henderson, 1972; Lipson, 1974; Rising, Brown and Meyerson, 1977; Romberg, 1977; Shoen, 1976)

C. *Classroom practice: some selected considerations*

1. **The use of manipulative materials.** Evidence points to the importance of concrete manipulative materials at stages in the learning process (Bruner, 1964; Dienes, 1970; Piaget, 1973; Williams, 1963). There may be a need not only at a certain stage of intellectual development, but also with each new area or field of mathematics studies (Skemp, 1971).

2. **The use of class time for developmental and drill/practice activities.** Evidence would suggest that students learn mathematics skills better by spending less time on drill and practice and more time on developmental activities (Milgram, 1969; Shipp and Deer, 1960; Zahn, 1966). Recent evidence, however, suggests that more class time is spent on written seatwork (practice) than any other activity (Conference Board of the Mathematical Sciences, 1975). Teachers should be encouraged and assisted in planning developmental activities in their daily lessons.

3. **The use of error analysis.** A number of studies have described the nature and frequency of errors commonly made by students in their mathematics work (Cox, 1975; Engelhardt, 1977; Lankford, 1974; Radatz, 1979). These data can be used in programs and by teachers to anticipate, and be alert to, areas of learning difficulties in mathematics.

4. **The use of calculators in mathematics programs.** A number of conferences, reports, and articles have addressed the use of calculators in the classroom (Bell, 1978; Bitter, 1978; Conference Board of the Mathematical Sciences, 1975; Rudnick and Krulick, 1976; Suydam, 1976). The PCMP concurs with the majority of conclusions drawn, namely:

   —calculators cannot (and must not) be ignored, their use in the schools must be carefully explored

   —the focus of attention must be on how the calculators can *best* be used to develop and reinforce mathematical skills and ideas

   —there must be long-term planning for mathematics curricula and instructional practices that incorporate optional uses of the calculator (Bell, Esty, Payne and Suydam, 1977).

5. Teaching word problems and other applications to students. Extensive research has been carried out on students solving of word problems. Programs should provide a variety of problems that reflect an awareness of linguistic structures that affect student performance (Caldwell and Golden, 1979; Carpenter, Hiebert and Moser, 1979; Jerman, 1973; Jerman and Sanford, 1974; Loftus and Suppes, 1972). Further, programs should provide students with some guidelines for choice of operations in solving word problems (Zweng, 1979; Wilson, 1967).

III. Evaluation

Evaluation can be viewed as the process of determining the degree to which outcomes agree with goals or objectives of the mathematics program. The comprehensive goals or objectives of mathematics programs in general education must be evaluated comprehensively. If mastery of arithmetic skills and abilities is an objective of a program, student performance on samples of those tasks must be evaluated. If development of meaning and understanding of mathematics as a system of related ideas is a goal of the program, then thinking processes must be evaluated for their maturity. Affective goals or objectives of the program must also be evaluated. These are often associated with outcomes that indicate less anxiety, less fear, and less avoidance of mathematics; or more self-confidence, more self-esteem, and more willingness to approach mathematics tasks.

At the state and federal level primary concern is with summative evaluation of programs. Focus is typically on a few broad, general program areas. Procedures for evaluating are usually restricted to standardized paper-pencil tests because of need for objectivity and ease in handling large amounts of data.

At the local classroom level evaluation is inextricably a part of the ongoing instructional process. Evaluation procedures can and should be quite diverse and would include informal observational procedures, interviews, diagnostic testing, and various types of paper and pencil tests. These procedures would be used for both formative and summative evaluation purposes. At the local level there is less concern for the restriction of handling large masses of data, and therefore evaluation programs can be more finely tuned and sensitive to specific outcomes of the program. When meshed properly with state and federal programs, in order to avoid duplication and excessive testing, the levels of evaluation can complement one another and provide a comprehensive evaluation program (Bloom, 1971; Brownell, 1941; Ebel, 1969; Freeman, Kuhs, Knappen and Porter, 1979; Haney and Madaus, 1978; Weaver, 1970; Wilson, 1971).

IV. Professional Growth and Leadership

A. *Professional growth of teachers*

The preservice and inservice education of teachers is a critically important component in a successful mathematics program. Competence is needed in the mathematics, *qua mathematics*, within the range of the teachers' professional practice. There must also be some awareness of content development beyond the upper and lower bounds of their professional practice. Of equal importance is development and growth in the teachers' repetoire of ideas for classroom instruction and management practices. Continuous growth and development in these areas will give their instruction a variety and dynamism which will make learning mathematics a more exciting undertaking for students.

The attainment, retainment, and continous professional growth of teachers requires close cooperation between local and state educational agencies, professional organizations, and the higher education institutions in the State (Commission on Preservice Education of Teachers, 1973; Glennon, 1965; Good and Grouws, 1979; LeBlanc, 1970; Osborne and Bowling, 1977; Sarason and Sarason, 1969).

B. *Professional leadership*

Strong, but sensitive, leadership is essential if schools are to blend curriculum, instruction, evaluation, and professional development into a progressive mathematics program. Strong leadership is the catalyst that gives form, substance, consistency and validity to the program. Leadership at the state level is important since it is at that level where there is articulation of directions, and the creation of support mechanisms to accomplish those directions. Leadership is of critical importance at middle management levels, e.g., school principals and supervisors, where the exercise of leadership is in terms of affecting the learning environments that intimately involve parents, children and teachers (Campbell, 1977; Goodlad, 1976; Harris, 1977; Leeper, 1977; Osborne and Bowling, 1977).

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APPENDIX B—THE STATE LEADERSHIP ROLE IN PCMP IMPLEMENTATION

The State's role in implementing the Pennsylvania Comprehensive Mathematics Program (PCMP) will be discussed under three topics:

I. **Assumptions Related to Implementation**
II. **Internal Actions within the Department of Education (PDE)**
III. **External Field Actions**

The narrative will provide more details on each topic, point out interrelationships among the topics as necessary, and, more generally, describe the logic of the implementation effort.

I. Assumptions Relating to the State's Role

Since educational change is more complex than many of us care to believe, it is important to make explicit some of the assumptions and value judgments on which the implementation rests.

A. Resources for implementation are limited to approximately two full-time equivalent Department of Education employees from the total Department of Education.

B. Participation by the Intermediate Units (IUs) and local school districts (LEAs) will be voluntary. This assumption is dictated partly by costs, but our primary concern lies with the quality of PCMP's implementation. Mandated actions encourage perfunctory performance by many LEAs. This attitude violates the spirit of the change that is mandated with a consequent loss of quality.

C. Quality of the implementation in LEAs is more important than the number of implementations in LEAs.

D. PCMP is supported by the top management levels of the Department of Education.

E. The implementation plan for PCMP (which reflects a substantive educational concern) is the best way to think about a state "administrative plan" because (1) it links the adminstrative process to a significant eduational plan, and (2) the implementation plan can easily be adapted to an administrative plan by the clarification of such delivery elements as the following:

1. Who does what?
2. When is it to be done?
3. To (with) whom?
4. At what cost?
5. By what process?
6. Is the "what" properly coordinated within various units of PDE and is it consistent with the philosophy of PCMP?

F. The plan outlined here includes enough specifics to offer guidance in implementation planning, but is also flexible enough so that important internal or external actions could be modified as future events dictate.

Two of the more important assumptions, more important in that they exert a direct influence on the PDE's internal and external actions, are those dealing with the voluntary participation of LEAs, and the intention to put the quality of implementations above the number of implementations.

II. Internal Actions Within the Department of Education

The internal actions within the Department will be discussed first because experience with state efforts to lead educational change over the past fifteen years indicates that without certain specific internal moves a single change effort is blunted through such factors as poor horizontal communication at the bureau and division levels, a lack of explicit sanction for a project at the Secretary and Commissioner levels, and the failure to pinpoint responsibility for the change effort at the bureau or divisional levels.

Experience suggests that the following major steps be taken within the Department to ensure that a complex organization with diverse functions does not lose the program needle in the haystack of daily operations.

A. Secure the enthusiastic approvals of key administrators, budget directors, deputy secretaries and Secretary of Education, and their key assistants.

B. Begin (or continue) general and more detailed informational-discussion briefings across appropriate bureau or division lines on what PCMP is, what it is trying to do, and how it will be implemented. A series of meetings between the PCMP staff and bureau directors and selected division heads among the units suggested would be helpful: Bureau of Press and Com-

munications; Bureau of School Improvement; Divisions of In-Service Education, Local District Services, Intermediate Unit Services, Nonpublic and State Owned/Related Schools Services, Educational Planning and Teacher Education. Meetings such as those suggested, while not dramatic, build knowledge about and confidence in the change effort, subvert rumors and the bureaucratic in fighting such rumors often spark, and offer units within PDE that are not primarily involved with PCMP a chance to offer suggestions for fitting PCMP to the needs of their several constituencies.

C. Every regularly scheduled PDE staff meeting, bureau meeting or divisional meeting will include a brief report on the status of the PCMP.

D. From the people attending the meetings suggested above, a smaller group of people would be identified whose support is critical to the success of PCMP. This group would receive individual communications, as necessary from the PCMP staff or director.

E. Identify individual staff members within the PDE who will agree, in all of their field contacts with administrators, supervisors, or teachers, to say a word about the current status of PCMP.

F. Solicit, and use if possible, constructive suggestions on PCMP which are certain to come from the give-and-take among horizontal units of the PDE.

G. Effect a functional link with the Bureau of Coordination and Field Liaison and use this link for PCMP communications.

With the leadership sanctions secured, communication channels established, and the support of other organizational units in hand, the Department is in a position to reach out to the more uncertain world of the IUs and LEAs.

III. External Field Actions to Implement PCMP

The process of implementation will be viewed as consisting of four phases within which the PDE, IUs, and LEAs will react in ways that can be anticipated. The four phases are perceived needs, awareness of needs and possible solutions, response by LEAs to PCMP, and implementation of PCMP which, in time, will be followed by additional needs (or problems) to which a resolution (or solution) may be sought.

The process can be more understood as "pictured" in the following diagram.

Needs (Problems) ⟶	Awareness ⟶	Response ⟶	Implementation
LEAs, IUs, PDE (congruent and incongruent needs)	PDE creates awareness of PCMP by LEAs, IUs	LEAs respond to PCMP as a solution to perceived problems (needs)	PCMP implemented according to plan

Needs—each organizational unit, LEAs, IUs, and the PDE, has perceptions of its own needs (significant problems) as well as perceptions of the problems faced by the other two units. The LEAs, for example, have probably not defined mathematics education as a top priority for 1980 although the PDE, in the perception of LEA needs, has defined mathematics as a significant problem. At this point the LEA-PDE perceptions are not congruent. Further, the LEA may well be looking to the PDE for "solutions" to other problems such as additional funds for court mandated integration or special education plans. This diffused and complex situation relative to different problem (needs) perceptions creates the need for the awareness stage.

Awareness—since the PDE has exercised its initiative in developing PCMP on something less than a petition by a majority of the LEAs, it must create as awareness of the problem within the LEAs and IUs most of which were gainfully occupied before PCMP came along.

Response—the LEAs and IUs will respond to the PDE efforts to create awareness of PCMP as a solution to significant LEA problems to the extent that they believe they have a valid need to which PCMP is a reasonable response.

Implementation—some LEAs and IUs respond positively to the PDE efforts and begin to implement PCMP.

Although a generalizable theory of dissemination is yet to be developed, the preceding rationale offers a few reasonable ideas that help to organize what would otherwise be a confusing mass of information, problems, and suggested solutions.

The more specific activities that will be taken under the awareness and response and implementation phases are suggested below:

A. Awareness Activities

The Department will create awareness of the PCMP through systematic actions such as the following:

1. PDE newsletter articles on PCMP
2. Briefings at IU Executive Director meetings
3. Regional and state-wide meetings of superintendents, principals, teachers, general and mathematics supervisors.
4. Secretary of Education and Commissioner for Basic Education would insert a paragraph or two about PCMP in all their speeches.
5. Professional meetings—the assistance of the Pennsylvania Council of Teachers of Mathematics (and the regional affiliates) and the Pennsylvania Council of Supervisors of Mathematics would be enlisted to meet with groups of administrators and general curriculum supervisors to explain the need for PCMP and to offer implementation support.

B. Response and Implementation Activities

The following recommendations will create a balanced and decentralized organization to facilitate positive LEA responses and, more important, to provide reasonable follow-up support to LEAs adopting PCMP. The plan is simple in order to reduce expense and to increase its practicality.

1. A PCMP Department inter-bureau task force will be set up to implement and evaluate the PCMP.
2. A comparable task force to the PDE group will be set up in five representative IUs for the 1980-81 year, the membership of which will be modified as necessary by the IU function; e.g., more teachers, supervisors, principals, and superintendents would be included at the IU level.
3. The implementation plan requires that the following major tasks be completed:
   This implementation plan is systematic and practical. It is realistically grounded on past experience and the resources available for PCMP support.
   a. The five IUs would receive the major amount of PDE resources for the 1980-81 year in which the PCMP will be experimentally tested and revised (other IUs could participate on other bases, less formal, to be developed by the state).
   b. The unit of participation will be the school (a principal and two or three teachers) with the support of the superintendent and two parents from each teacher's room.
   c. The PDE and IU teams will write up a simple but rigorous in-service program that stresses why X is done a certain way, the underlying values on which the PCMP is based, and the behavior required to teach PCMP; inservice work will be mandatory for all LEAs.
   d. If a thorough evaluation cannot be done during 1980-81 (including a documentation of the process and problems of implementation), a thorough monitoring-description of the process should be done adjusting the scope of monitoring and description to the funds available (doctoral students might assist with this activity).
   e. The PDE-IU teams (and/or the evaluation team) will continously feed back and receive information to LEAs, IU, PDE relative to the successes and problems encountered during implementation.
   f. Each IU will support the nuclear school unit throughout the year of implementation with a nonpaid (or nominally paid) part-time staff to help with problems arising on specific sites. Assistance could be given through telephone or personal conferences. This support staff would reside within a 25 mile radius from the center of the IU and could be drawn from school districts, IUs, teacher education institutions and professional organizations within the service area.
   g. Coordination and communication among the 25 school sites, the five IUs, and the PDE team would function through a very simple system: one person at each of the three levels would function as the sole initial point of contact up or down the system; after initial contact, the specified person could "switch" the message anywhere within the system.

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APPENDIX C—GOAL II: MATHEMATICS
STUDENT OUTCOMES

Quality education should help every student acquire skills in mathematics. In support of this goal, students should:

—Believe they can succeed in learning and using mathematics.

—Appreciate the contributions of mathematics and mathematicians to civilization.

—Demonstrate an interest in and an appreciation for the learning of mathematics and its many diverse applications.

—Demonstrate awareness of the interrelationships among and applications of mathematics to other disciplines.

—Recognize similarities and differences among objects.

—Recognize patterns and relationships in mathematics.

—Understand numeration systems and read and write numerals.

—Be able to add, subtract, multiply, and divide with whole numbers, decimal and fractional numbers.

—Be able to perform mental computations.

—Be able to understand and use verbal and numerical systems interchangeably.

—Recognize the components and determine the procedures necessary for problem solving.

—Be able to apply logic and reasoning to the solving of problems.

—Be able to apply mathematical knowledge and skills to the management of personal and family responsibilities.

—Be able to understand the use of percents in calculating simple interest, discounts, taxes, etc.

—Be able to apply formulas commonly used in everyday life.

—Acquire techniques for estimation and approximation and use these techniques to decide when a particular result is precise enough for the intended purpose.

—Inspect all results and check for reasonableness in terms of the original problem.

—Understand the basic concepts of measurement and monetary systems.

—Be able to measure distance, weight, time, capacity and temperature in both the English and metric system, and make conversions among units *within* each system.

—Understand basic geometric concepts such as point, line, parallel, and perpendicular.

—Know basic properties of geometric figures and forms.

—Be able to calculate areas and volumes of simple geometric figures.

—Understand how mathematics is used to predict events.

—Understand commonly used statistical terms such as mean and median.

—Be able to read and interpret tables, charts, graphs, maps, and other statistical materials.

—Be able to organize numerical or verbal information into tables, charts, graphs, and maps.

—Demonstrate awareness of the capabilities and limitations of the computer.

—Be able to develop a flow chart related to computers and also related to other events.

—Be able to use calculators, other devices and techniques that simplify computations.

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APPENDIX D—PRIMES: A RESOURCE FOR IMPLEMENTING PCMP

Overview

PRIMES, The Pennsylvania Retrieval of Information for Mathematics Education System, is a resource for educators to use in developing, implementing and evaluating mathematics curricula. The system enables individual school districts to tailor the curriculum to their particular needs in kindergarten through grade eight.

PRIMES is a total system consisting of two parts: data base and service. The major file of the data base has been developed from analysis of every page of every commercially available basal textbook. Other files contain information about tests, kits, audio-visual and manipulative materials currently available for elementary and middle school mathematics. Information is classified by a comprehensive set of authoritative concepts and skills with correlated performance objectives. Information is fully retrievable from the data base to meet the specifications of any school district. Any mathematics curriculum committee can now have a comprehensive concept/skill listing, with instructional and evaluation materials correlated to this listing, from which to build the precise curriculum it desires.

There is a singular goal for PRIMES: to bring about systematic curriculum and instructional change suited to a district's needs.

Although the initial purpose of developing the information system was to assist school districts in instructional materials selection, the long range objective was to provide reliable, readily available information for all curriculum and evaluation decision making. From its inception, PRIMES has taken the position that curriculum objectives should be defined and that instructional and evaluation materials should be selected to match the objectives. Consideration should be given to the qualifications of the teachers and the needs of the learners. Mathematics curriculum decisions are the responsibility of local school districts. The system is not intended to evaluate or to recommend or to assess materials, it is intended to analyze and to make information available for curriculum decision-making.

Services

PRIMES services include curriculum development, individualized instruction, teacher training, and assessment.

Curriculum Development—Designing a local curriculum is intended to: (1) provide a sound basis for instruction, materials acquisition and evaluation; (2) develop teacher competencies in the process of curriculum achievement and increase teacher knowledge of mathematics and (3) prepare the framework for individualizing instruction.

PRIMES offers districts a "systems approach" to curriculum development. Its procedures outline the number of people, time, and materials necessary to generate a curriculum. Following these procedures does not impose a pre-designed curriculum on a district; rather, it makes available an authoritative data base to facilitate local decision-making.

The local curriculum committee can develop:

1. *Scope and Sequence:* The district designs its basic mathematics curriculum by deciding the concepts and skills to be taught and indicating for each: (a) importance, (b) when introduced, developed, mastered, and reviewed, and (c) the instructional sequence within grade/age level. *(See Figure 1)*
2. *Curriculum guide:* Building on the scope and sequence design the guide includes the list of concepts for each grade-age level and for each indicates: (a) suggested teaching behavioral objectives, (b) sample test items, (c) testbook references and (d) supplementary materials references. *(See Figures 2 and 3)*
3. *Instructional materials selection:* District-established concept priorities are matched against the computerized textbook file to identify the series that best "fit." The series are reviewed by the local curriculum committee for final selection.

Individualized Instruction—The instructional kit program was designed for learners with problems in mastering basic skills in mathematics. Developed for basic concepts, each kit provides for concrete, pictorial and symbolic learning experiences. The kits are learner-directed and can be used in a variety of instructional settings. Nationally recognized consultants prepared the behavioral objectives, test items, learning activities and materials, and a classroom management plan.

Teacher Training—The teacher training program is intended to: (1) develop competencies in diagnosing learning needs, prescribing appropriate instructional activities, providing alternate instructional strategies in varied educational settings and evaluating achievement; (2) implement a remedial program in a variety of classroom settings under the direction of the regular and/or remedial teaching staff and (3) provide the learner with a variety of instructional experiences tailored to his/her needs.

Assessment—the design of a comprehensive testing program includes: (1) diagnostic procedures, (2) achievement tests, (3) measures of pupil attitude and (4) informal interviews with teachers and pupils.

The testing program can be correlated with the Educational Quality Assessment program and can serve as a follow-up for districts that wish to improve their mathematics programs.

Criterion-referenced tests are correlated to a district's curriculum. Tests are machine-scored. Both computer and written reports are provided to analyze and interpret the data. These reports specify the pupil's degree of mastery for each concept tested.

The testing program is intended to identify the specific areas of strengths and weaknesses in a district's instructional program and be applied to diagnostic-prescriptive instruction. Teacher competencies are developed in (1) type of tests, (2) interpretation of data, (3) uses of data and (4) pupil record keeping.

WILLIAM PENN SCHOOL DISTRICT

			K			1			2			3			4			5			6			7			8		
Code	Concept	IMP.	B	M	E	B	M	E	B	M	E	B	M	E	B	M	E	B	M	E	B	M	E	B	M	E	B	M	E
0020	Developing cardinal number zero	1		I		M																							
0030	Developing cardinal numbers one through ten	1	I M																										
0035	Developing cardinal numbers beyond ten	1			I			M																					
0040	Developing ordinal number sense	1			I				M																				
0050	Associating the idea of number with the number line	1					I M																						
0060	Counting to find cardinal number of set	1		I		M																							
0070	Ordinal counting	1					I		M																				
0075	Sequence of numbers increasing by one	1	I						M																				
0080	Skip counting							I		M																			
0090	Other counting: backward, rote, etc.	1	I							M																			
0100	Ordering numbers as greater than, less than, equal to or not equal to and between;	1					I					M																	
0120	Addition developed by using the union of disjoint sets or joining action	1		I					M																				
0130	Addition developed by using the number line	1					I			M																			
0150	Commutativity, a property of addition	1						I				M																	
0160	Associativity, a property of addition	1										I M																	

: = Introduced
——— = Developed
M = Mastered
--------- = Maintained

1 = mastered by all students
2 = important, not to be mastered by all students
3 = enrichment

FIGURE 1: Scope and Sequence

38

Introduce	
Develop	B
Master	
Maintain	E

MARPLE NEWTOWN SCHOOL DISTRICT

MATHEMATICS CURRICULUM

0120

Content Statement

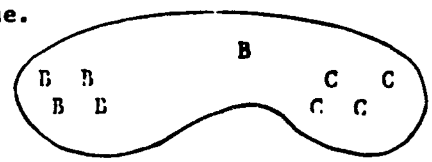
Addition developed by using the union of disjoint sets.

Behavioral Objectives

Given:	A pupil will be able to:
1. Two sets of objects or a diagram that illustrates the union of two sets	Write the number of elements in each set and the number of elements in the union. Sums to twelve.
2. An addition statement using numerals and a "+" sign and an "=" sign or "plus" and "equals", and a counting aid	Count the number of elements, and write the sum to complete each number sentence. Sums to twelve.

Example — Test Item

Use the pictured sets to fill in the blanks and make each number sentence true.



___ + 4 = ___

Textbooks

SILVER BURDETT, 1978
Mathematics for Mastery p. 24-31, 98-99, 103-105, 113-114, 119, 250-253

MIS Kits
0120

FIGURE 2: Curriculum Guide

0190

Clearfield Area School District
MATHEMATICS CURRICULUM
Grade—1

Kits

Introducing Numbers, BFA Educ. Media
Individualized Math Drill and Practice Kit, Random House
Starter Kit
Kit BB
Arithmetic Fact Kit, Addition, 1969, SRA

MIS Kits

0190
0200
0210

Audio-Visual Materials

Transparencies

Tapes

Mind & Math 2 - 1, 2, Ed. Resources
Mind & Math 3 - 1, 2, Ed. Resources
Let's Learn to Add, Educ. Insights
Computapes, Module 1, Add. & Sub., 1B, 2A, 2B, 3A, 3B, 6A, 9A, SRA
Module 2, Add. & Sub., 1B, 4B, 5A
Module 3, Mult. & Div., 3A, 7A

Films/Filmstrips

Using and Understanding 2 & 3 Place Nos., SVE
Addition - Objects and Symbols, SVE
Addition and Subtraction - 2 & 3 Place Nos., SVE
New Phases of Add. & Sub., SVE
Preparing for Addition and Subtraction, 3M
Addition and Subtraction Concepts, Eye Gate House
Set - Concepts, Symbols, Operations, SVE
How Many In All? How Many Are Left?, Eye Gate House

Games

Tortoise and the Hare
Quizmo Arithmetic: Add. & Sub.
Tumble Numble
Imma Whiz, Add. & Sub.

Devices

Math Wheel
Classroom Counting Frame

Other Material

Addison Wesley Duplicating Masters, 69, BK. 3 - 1, 4, 5
BK. 4 - 3, 5
BK. 5 - 2, 7
BK. 6 - 1
Self-Teaching Arithmetic Book 3, 4, 5

PRIMES

FIGURE 3: Curriculum Guide

Administration

In 197 , after a major portion of the research and development of PRIMES was completed and field-tested, the Pennsylvani Department of Education transferred PRIMES to the Continuing Education Department, West Chester State College. Th PDE is responsible for policy decisions, for evaluation, and research and development efforts. The college is responsible fc (1) maintaining and expanding the system and (2) providing the services to educational agencies. PRIMES is supported throug grants and school district and agency contracts.

Expansion of PRIMES

Research and development is being conducted in the following areas:

1. *Extension of the system to grades 9 through 12*—A preliminary list of concepts and skills has been developed for algebr geometry and trigonometry. The list will serve to classify instructional materials and to make curriculum decisions.
2. *Diagnostic-prescriptive-instruction*—An instructor's manual for D-P-I mathematics has been written. Topics include: (a background for the national D-P-I movement, (b) four levels of diagnosis, (c) content objectives, (d) performance objective (e) instructional materials and (f) learning activity types. There are several college/university professors who have bee trained to use the manual. More than 100 teachers have been trained in D-P-I mathematics. There is a statewide plan fc establishing and operating math labs/clinics and training personnel.
3. *A total curriculum information system*—PRIMES has served as a model for information systems for other curriculum area The major system and service components are applicable, with minor modifications, to other disciplines. The need to: (a identify curriculum objectives expressed as pupil skills or knowledge, (b) correlate instructional materials to achieve thes objectives, and (c) correlate methods for evaluating pupil needs and progress, exists in every discipline. The need can be me by an information system relating these three curriculum components.

Features

—Most comprehensive data base for mathematics education nationally.

—Authoritative documents of concepts and skills and competencies.

—Curriculum, instructional materials, and evaluation correlated.

—Proven system and services.

—Services tailored to client's needs.

—Alternate programs for learners with problems.

—Computer storage and retrieval

—Cost effective—number of staff and time.

—Compensatory and special education programs validated by the Department of Education.

Summary

PRIMES is consultation. PRIMES assists school districts in making mathematics curriculum decisions using systematic prc cedures.

PRIMES is technology. Comprehensive information about textbooks, diagnostic and achievement tests, and audio-visua materials has been collected, analyzed, and computer-stored.

PRIMES is experience. It is an organization that for 12 years has aided more than 250 school districts in meeting mathematic curriculum needs.

PRIMES is service. Services are provided in curriculum development, individualized instruction, teacher training, and asses ment tailored to each client's needs.

NOTES ON CONTRIBUTORS

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