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ABSTRACT

As part of the "Educating Gifted and Talented Students" series, the booklet offers strategies for teaching mathematics to gifted and talented students. Methods for identifying mathematical ability are reviewed and a two stage method of identification is highlighted which involves screening students in an already chosen grade through two different measures of mathematical aptitude. General approaches are considered, including enrichment, busy work, general academic enrichment and cultural enrichment, specific academic enrichment, and educational acceleration.. Detailed are procedures for implementing teaching strategies--fast paced mathematics class and individualized classroom approach--found successful in the Study of Mathematically Precocious Youth. Also addressed are program evaluation, teacher selection, textbook selection, and other accelerative options. (SBH)

Teaching the Gifted and Talented in the Mathematics Classroom

by
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The Series Editor

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INTRODUCTION

Educators have focused a great deal of effort on developing programs for children labeled "mentally handicapped." However, they have often overlooked that gifted youths also need special academic programs. In particular, mathematically gifted youths should be given special opportunities that enable them to use their mathematical capabilities in the fullest possible way.

The facilitation of mathematically gifted youths can take many different forms, ranging from largely ineffective to extremely successful. The program ideas presented here have been used with success by various state and local agencies as well as by private organizations such as the Study of Mathematically Precocious Youth (SMPY) at The Johns Hopkins University. These ideas should be interpreted not as rigid guidelines, but as flexible methods of facilitation. Since each school and each student is unique, the models must be shaped to fit the individual situation.

Giftedness in mathematics is unique in comparison to giftedness in other subject areas. Until the student reaches the level of advanced college courses, the acquisition of mathematics follows a clearly defined sequence of courses. Thus the mathematically gifted individual who is learning at a rapid rate can follow an unambiguous path of courses. This aspect also facilitates the evaluation of the student's progress. Since the majority of existing curricula follow the same basic sequence, the student has a benchmark against which a comparison may be made. In addition, the sequential nature of mathematics means that the material in a given mathematics course builds upon and utilizes the concepts learned in the preceding courses of the sequence. An individual gifted in mathematics can learn mathematics rapidly because concepts are continually reinforced in this manner. Mathematics also differs from other subject areas in that it is not dependent on the chronological age and the associated life experiences of the student. Thus even very young students can excel.

Mathematically gifted students who are in a regular mathematics class waste countless hours. They spend much of their time going over topics already learned when they could be learning new concepts. However, the problem involves more than just wasted time. These students experience boredom that eventually may lead to resentment and alienation from the learning process. This alienation can manifest itself in the form of poor study habits, laziness, classroom behavior problems, daydreaming during class, and possibly arrogance. When "turned off" students are finally challenged, often they are unable to succeed. Thus these individuals become mathematically gifted college dropouts. Clearly, these youths must be identified and given alternatives that challenge their abilities. It is important that each individual be able to develop his or her potential to the fullest.

A mathematically gifted person should not be thought of as a computational wizard able to perform sizable numerical calculations mentally. Rather, the mathematically gifted student is one who exhibits extremely high mathematical aptitude. To qualify for entrance into a special program, the gifted youth should be in the upper one percent or one-half of one percent in mathematical reasoning in his or her age group. (21, 26) These gifted youths should not be confused with students in an "academically talented" or "honors" mathematics class. Typically, such a class consists of students in the top 10 to 20 percent in mathematical aptitude in that school. A class organized on this standard will not meet the needs of a mathematically gifted youth. The pupil gifted in mathematics is capable of learning mathematics much more thoroughly and at a significantly faster rate than the above-average student.

Some people expect the mathematically gifted student to be male, socially awkward, wear thick glasses, and have a calculator strapped to his belt. However, none of the common stereotypes is valid, as research has clearly shown. (17, 20, 28). Mathematically gifted youths are as diverse as any randomly selected group of people, except in the area of mathematical talent. Such a youth may be introverted, extroverted, athletic, clumsy, diligent, lazy, male, or female. It is important to remember that these people are individuals first. As a result, a procedure that is effective for one student may not suit another.

Mathematical giftedness is a specific aptitude; therefore, such individuals may or may not be gifted in other academic areas. (24) It is quite common to find, however, that students who excel in areas of pure and applied mathematics are often quite talented (and interested) in the sciences that use mathematics as a tool, such as physics, chemistry, and the engineering sciences.

Since mathematical giftedness is a specific aptitude, it should not be equated with a high intelligence quotient (IQ) as measured by a general abilities test such as the Stanford Binet or Wechsler. An individual's IQ is composed of many aptitudes and factors of which mathematical aptitude is only one. A person with a high IQ *may* have high mathematical aptitude and relatively low verbal aptitude; another person with the same high score may have low mathematical aptitude and extremely high verbal aptitude. The IQ is primarily a measure of verbal ability. The mathematical components of such a test tend to build on computational and recall skills, rather than on the reasoning component of mathematics. In addition, there are generally not enough items in the mathematics area to come up with a valid and definitive mathematics score. Put differently, the IQ score tends to mask an individual's strengths and weaknesses, making it an undesirable tool for identifying the mathematically gifted. (4, 16)

IDENTIFICATION

A variety of methods in addition to the IQ score have been used over the last quarter of a century to identify the mathematically gifted. (31) These methods include diagnostic testing, teacher nominations, standardized testing, nonverbal reasoning measures, peer nomination, and a combination of the aforementioned measures. Many of these methods have been found to be inadequate and/or subjective in nature. The SMPY has found that test measures that are nationally administered and pose sufficient challenge in the area of mathematical reasoning ability are one of the more successful alternatives. To date, it has screened over 10,000 youths via this method. (10, 11, 7, 1, 6) While specific aptitude tests are not a perfect indicator, when properly used these tests are often much better indicators than, for example, teacher nominations or grades. Pegnato and Birch demonstrated this point in a study that found that neither teacher nominations nor the honor roll were good indicators of high IQ students. (14) For other studies verifying this point see Terman (29) and Stanley (25). The problem with identification on the basis of grades or teacher evaluations is that these measures are based on achievement, motivation, and attitude. The mathematically gifted student may well be inattentive, a behavior problem, or a low achiever because of boredom. Teachers do not think highly of this type of student, and the student's grades reflect this. Renzulli has shown that classroom environment and teacher style will affect a student's performance negatively if the student's learning style is different from that of classmates. (30) In contrast, identification criteria based on appropriately difficult standardized measures will catch these alienated students—the ones in greatest need of special educational programs.

The first decision to be made in the identification process is degree of selectivity. What constitutes the criterion for entrance into a special program? There is a real danger of yielding to the common tendency of being underselective—that is, choosing students who are not sufficiently qualified. If some students are given a difficult test or placed in a class that is much too difficult for them, it could be highly damaging to their academic outlook. As a result, it is better to err in the direction of overselectivity. Put differently, it is wiser to limit the program to a smaller number of students who demonstrate a high probability of success, than to choose a larger number of students who show only marginal probabilities of success and put them in a situation where they may indeed fail. This consideration is especially crucial in a new program. Since there are so many details to handle, it is better to concentrate, at least initially, on a small, highly select group. In many cases there may be many more mathematically gifted youths than the program can accommodate. It is best to concentrate on the really exceptional ones instead of being diverted by slower students and underachievers.

The criterion of limiting selection to students who demonstrate mathematical reasoning ability in the upper one percent or upper one-half of one percent has worked extremely well for a number of state and local educational agencies. A small population base may necessitate less selectivity; it is unwise, however, to dip below the upper five percent for entrance into special programs in mathematics. If there is time and teachers are available, several levels of students may be selected. Expectations and curriculum design would vary from group to group. For example, in Chicago public schools, fast-paced and enriched mathematics programs were designed for the top students identified in an initial mathematics talent search. These classes met for one two-and-a-half-hour session each week. Students at the next level were placed homogeneously within learning centers across schools, but math was taught on a daily basis. The curriculum was taught at a level one year ahead of the normal content schedule. All students participating in the program were offered career education opportunities in the field of mathematics.

After determining the standard for selectivity, personnel must decide in which school grade to identify the mathematically gifted. Since mathematical giftedness is related to reasoning ability rather than to computational skill, special programs should generally start with subjects more conceptual in nature. These subjects (for example, algebra I and II, geometry) do require a certain mastery of basic computational skills. Therefore, the identification process should take place after the school grades in which arithmetic is taught but before the earliest grade in which algebra is taught. In most school systems, this means either at the end of the sixth grade or during the seventh and eighth grades. Before students reach the seventh grade the computational skills of even those who are mathematically gifted are usually not developed sufficiently to permit the learning of first-year algebra.

Of course, there are exceptions. However, if an elementary school student is deemed ready to learn algebra, her or his computational skills should be far superior to those of the other students in the same grade. These skills should at least match the skills of the "average" algebra student. Diagnostic testing via a standardized computational skills test, such as the Sequential Tests of Educational Progress (STEP), can be administered in order to determine the skill level of a particular student.

Two-Stage Method

The two-stage method has been one of the more successful methods of discovering mathematical talent. The procedure involves screening students in the already chosen grade through two different measures of mathematical aptitude. Using this approach, in six talent searches covering the states of Delaware, Maryland, Pennsylvania, Virginia, and West Virginia, as well as the District of Columbia, over 10,000 youths have been identified as mathematically talented. Although there are other models of identification, this method has been used successfully by a number of school systems to be cited later.

The first stage of the identification process consists of establishing a pool of talent from which the mathematically gifted can be chosen. The purpose of this step is to reduce the number of students to be screened further. Clearly, only the ablest students need to be examined further in order to discover those who are truly mathematically gifted. The initial screening is accomplished by examining student performance on nationally administered in-grade measures of mathematical aptitude or mathematical reasoning ability. To qualify, these screening measures must contain a separate mathematical reasoning or aptitude section that can be reported to the students. Preferably, these measures should provide in-grade national norms. Some commonly used screening measures are listed in Appendix A.

After choosing the instrument to use in the initial screening, one must decide what minimum percentile rank (on national norms) will qualify a student for the second stage of the identification process. This criterion will vary depending on the size of the population in a given locale and the general ability level of the students. The cutoff may range from the upper one or two percent to the upper five percent. In general, students below the 95th percentile should not be eligible for the second stage. Most students not scoring in the top five percent would find the second screening a depressing and possibly traumatic experience.

There should be room for flexibility in establishing the cutoff point. For instance, one may want to incorporate teacher nominations into the first screening. This can be done by allowing teachers to nominate for the second screening those students whose first screening measure scores are just below the cutoff point. Such nominations should not be haphazard, since the second stage screening may damage the egos of students who do not really excel in mathematics.

After the first stage is completed and a highly able pool of talent has been identified, it is necessary to perform a second screening of this group. The second screening is intended to separate the mathematically gifted students from those who are merely very able in mathematics. The in-grade measure of math aptitude used in the first screening did not adequately demonstrate individual differences. Since all scored in the upper few percent of their age group, the in-grade measure did not possess a sufficiently high ceiling to reveal differences in ability.

In the second screening, the test must be difficult enough to spread these students in the upper few percent out over the whole range of possible scores. Measures that are ideal for this purpose are those that are designed for students in higher grade levels. Since the identification process preferably occurs in a junior high grade, a suitable second screening measure would be one that is normally administered to and gives norms for a national sample of high school juniors and seniors. Some examples are listed in Appendix B. Note that all measures listed in Appendix B have a separate section dealing with mathematical aptitude.

It is a mistake to believe that a group of students who score within a few percentile points of each other on an in-grade measure of mathematical aptitude are quite homogeneous with respect to this ability. That may be true in a lower range of the scale, such as 64th through 67th percentile. However, a group of students in the upper few percent are actually extremely diverse. This is because there is no top to the measurement scale and no limit to how high an individual can be placed on the scale. Thus, two students who score in the upper two percent of mathematical reasoning ability can differ tremendously in their actual ability levels.

Thus a measure with a sufficiently high ceiling will spread the top few percent out over the whole range of scores. For example, consider the 1973 talent search conducted by the SMPY. The group being screened were seventh and eighth grade students. In the intital screening a score in the top two percent on national norms for an in-grade standardized measure of mathematical reasoning ability qualified students for the second screening. A group of 953 students were eligible for the second screening and were administered the Scholastic Aptitude Test (SAT) under standard conditions. On the mathematics (M) section, the scores of this seemingly uniform group of students were spread out over the whole range of ability levels for eleventh and twelfth grade students who take the SAT-M, from the lowest possible score of 200 up to a near perfect score of 800. In other words, a very able student who scored at the 99th percentile on the initial in-grade screening measure earned a score of 360 on SAT-M, whereas a mathematically gifted student who had the same rank on the in-grade measure scored 710 on SAT-M. It is for students in the latter category that we need to develop special programs in mathematics.

After obtaining the second screening scores, one must determine what will be the cutoff for eligibility to a special program. Although such a cutoff may seem arbitrary and unfair, limited resources often dictate this approach.

There is some flexibility in the choice of a cutoff score at this point, although one must be careful not to be underselective. Especially when a program is just getting underway, it is better to select only a few students who are sure to succeed. Several school districts that have used the SAT and have developed successful programs for junior high students require participants to have a score of at least 500 on SAT-M. This score corresponds to the 57th percentile on norms for college-bound seniors and corresponds to about the 80th percentile on norms derived for all eleventh and twelfth grade students. In its six talent searches, the SMPY has found that of the 10,000 seventh and eighth graders who scored in the upper three percent on the initial screening, about one-quarter scored at least 500 on SAT-M. This implies that the students who score 500 or more are at least in the approximately top one percent of their age group in mathematical reasoning ability. We educators should be concentrating our efforts toward this group of mathematically gifted youths.

Table 1 shows how some school systems have used this two-stage model for the identification process. Please note the different measures used at the second screening level(s).

TABLE 1.—SCREENING MEASURES USED AT THE SECOND LEVEL OF THE TWO-STAGE IDENTIFICATION PROCESS BY VARIOUS STATE AND LOCAL SCHOOL DISTRICTS

Program	Grade at time of testing	Screening measure used
State of Delaware (Rural, suburban, and urban)	7	Scholastic Aptitude Test—Mathematics (SAT-M)
State of Illinois (Rural, suburban, and urban)	7	School and College Ability Test—Quantitative (SCAT-Q)
Maryland		
Baltimore County (Suburban)	7	School and College Ability Test—Quantitative (SCAT-Q)
Charles County (Rural)	7, 8	Preliminary Scholastic Aptitude Test (PSAT)
Howard County (Suburban)	7	Orleans-Hanna Algebra Prognosis Test
Minnesota		
Minneapolis—St. Paul (Urban)	7, 8	School and College Ability Test—Quantitative (SCAT-Q)
Duluth (Urban)	7-9	School and College Ability Test—Quantitative (SCAT-Q)
Nebraska		
Educational Service Unit No. 11 (Six rural counties)	Pre-10, 11, and 12 Summer	1971-72 practice test in mathematics of the College Board Scholastic Aptitude Test
Omaha (Urban)	7	School and College Ability Test—Quantitative (SCAT-Q)
Oklahoma		
Bartlesville (Rural)	7	Orleans-Hanna Algebra Prognosis Test
Pennsylvania		
Montgomery County (Suburban)	8, 9	Scholastic Aptitude Test—Mathematics (SAT-M)
Wisconsin		
Eau Claire region (Suburban)	7, 8	Scholastic Aptitude Test—Mathematics (SAT-M)

At this point, the process of identifying mathematically gifted youths has been completed. It is important to realize that at any stage a student should have the option not to proceed further. A special program should only include students who are eager and highly motivated.

Before proceeding to a discussion of teaching strategies, there are some terms that need to be clarified. Concepts such as acceleration, enrichment, and creativity must be defined in order to avoid the misunderstanding that these terms often generate.

APPROACHES

Accelerated education is based on the concept of individual differences and connotes the idea of a person moving through one or more subject areas more quickly than in the standard program designed by the various school systems. (8) This alternative is suited for the upper few percent of students in a particular age group or grade. The purpose of acceleration is to meet the intellectual needs of gifted students—needs that are not met through the standard age-grade lockstep approach to education. The rate and need for acceleration varies from individual to individual. The rate of learning should be commensurate to the student's ability and motivation.

Enrichment of education implies that subject matter is presented in greater depth for the gifted student than for other students. Material is added to the standard course, but the grade or course placement of the gifted youth is neither advanced nor retarded.

Creativity is concerned with thinking in unique and interesting ways. For example, a creative thinker could prove a geometry theorem in an original way. Despite the classical definition of creativity, it seems that unique, interesting methods of learning in themselves do not generally stress the content or curriculum.

Enrichment

Whenever professionals discuss teaching strategies for mathematically gifted students, enrichment of curriculum rather than educational acceleration is the most popular alternative. The four most common forms of educational enrichment might be termed busy work, general academic enrichment, cultural enrichment, and specific academic enrichment. (20)

Busy Work

Some teachers use busy work to keep the brightest students occupied while the rest of the class proceeds in the normal manner. Busy work consists of having bright students do more work relating to a specific subject than the average students. It is an easy method to implement, since it requires virtually no extra effort from the teacher. This extra work is not designed to help students understand new concepts, only to give additional practice in performing work that has been learned previously. For instance, while most students in a mathematics class may be assigned every other problem in the textbook, a gifted individual might be asked to do every problem. Because mathematically gifted students are often forced to do extra work in a subject in which they are superior as compared to classmates, they can become bored and frustrated and lose interest in the subject. Actually mathematically gifted students need to do fewer problems than average students, since they can learn mathematical concepts far faster and with less practice. In the previous example, it would be better for the mathematically talented youngster to do every fourth problem, rather than every problem.

Another example might help amplify the dilemma faced by mathematically gifted youths. Mark, who had an IQ of 187 in kindergarten, was a 12-year-old eighth grader taking Algebra I. Hoping to challenge him, Mark's teacher asked him to work every problem in the book while the rest of the class was assigned every other problem. This mathematically gifted student already knew Algebra I quite well and needed very little practice in the material. He naturally resented the burdensome and boring task he was given.

Ironically, this situation motivated Mark to attain very high standards. Bored in Algebra I and encouraged by a mathematics specialist, in the spring semester of eighth grade Mark took a computer science course at The Johns Hopkins University and received an A. From then on he took the remainder of his mathematics coursework at the college level. The summer after eighth grade, Mark took a course in college algebra and trigonometry and received a B. In the following two years, he completed the college mathematics courses through calculus and linear algebra and took two chemistry courses, receiving all As. When Mark was only 15 years 2 months he entered JHU as a full-time student, with his freshman year and 30 percent of his sophomore year completed. Today he is a superb graduate student at Massachusetts Institute of Technology.

In a way, Mark's eighth grade algebra teacher did this brilliant young man a favor by causing him to find another way of challenging himself. However, without the special opportunities for educational acceleration in mathematics, Mark would have been forced to sit through a number of routine full-year mathematics courses far below his ability level.

General Academic Enrichment and Cultural Enrichment

General academic enrichment does not provide the specific type of advanced stimulation that a gifted student needs. Instead, it often consists of offering such subject matter as a special high-level social studies course for all high IQ youths, or perhaps something essentially nonacademic like chess or creative training (process-oriented). For the most part, these offerings are not related to specific subject matter and generally do not meet the particular needs or interests of mathematically gifted youths.

Cultural enrichment involves providing special cultural experiences beyond those offered in the usual school curriculum. Classes in music appreciation, the performing arts, and foreign languages may be offered. There is nothing wrong with this kind of program; however, these classes certainly do not meet the specific needs of a mathematically gifted youngster. Indeed, such experiences are often beneficial for all students regardless of ability level.

Specific Academic Enrichment

Finally, there is the form of educational enrichment termed specific academic enrichment. This approach would provide, for example, a special mathematics curriculum from kindergarten through seventh grade designed for those students whose abiilty to reason mathematically puts them in the upper 10 percent of their age group. But suppose that such a student goes through this program and then takes a regular Algebra I course in eighth grade. Obviously, the student will be bored and frustrated with the comparatively slow pace. Even if the special curriculum is continued through twelfth grade, there are still problems. If the enriched program is truly designed for the mathematically gifted, participants will have learned mathematics that nonparticipants will not learn until they attend college. Unless some provision is made for participants to earn high school and/or college credit for what they have learned, they will end up repeating coursework they have already covered. Thus the boredom and frustration are merely postponed. The resulting lack of stimulation and deceleration of potential may be worse than if no enrichment had taken place. Dr. Julian C. Stanley, Director of SMPY, states, "The more relevant and excellent the enrichment, the more it calls for acceleration of subject-matter or grade placement later. Otherwise, it just puts off the boredom awhile and virtually guarantees that eventually it will be more severe." (19)

Educational Acceleration

It follows then that relevant academic enrichment in mathematics is a good approach only if it is pursued with an eye to long-range planning and if the student receives credit for what is learned. When these conditions are met, then what is taking place may be called educational acceleration.

The best support for the implementation of educational acceleration is found in the many examples of mathematically gifted students who have accelerated their educations with good results. (28) As previously stated, mathematics is ideally suited for acceleration because the rate of learning is more dependent on intellectual ability than on chronological age and life experiences. Nevertheless, there must be strong student motivation. Educational acceleration must be the student's choice; it cannot be forced by parents, teachers, or administrators. If the student is not willing to accelerate, he or she will not want to do the amount of work necessary for success. But for those students who are eager to move ahead, educational acceleration has proven to be an excellent alternative.

For example, as of spring 1979, 250 of the students identified by SMPY (in a five-state region) had taken 589 college courses prior to entering college full-time. Their mean grade-point average was 3.52 out of 4.0 (A = 4, B = 3, etc.). The average age of this group at the time that the course was taken was 15 years. Although SMPY began in 1971 by facilitating seventh and eighth graders, already more than 220 young men and women identified by SMPY have entered college one to six years early.

Many highly talented professionals have accelerated their educations in some way. Norbert Weiner, the originator of cybernetics, received his doctorate at the age of 18. Charles Louis Fefferman became a full professor of mathematics at the University of Chicago when he was only 22 years old. Two years before he had received his Ph.D. He became the first winner of the National Academy of Science's $150,000 Waterman award when he was barely 27. (12) The list of successful early graduates is long and far outweighs the case of William James Sidis, who failed society's norms of accepatable behavior after having graduated from Harvard at the age of 16. (13)

Those opposed to educational acceleration are often concerned that the social-emotional growth and creative development of these students are hindered. This concern goes to the extreme of ignoring the other crucial factor, namely intellectual development. There are numerous cases of accelerated students who have made extremely good social adjustments and have not been creatively disadvantaged by acceleration. (27, 33) Furthermore, no studies show empirically that acceleration is bad, while many studies demonstrate that it is good. (2)

This is not to say that acceleration is a perfect solution to the problem of the mathematically gifted. In nearly all cases, there is no perfect solution. One must determine whether if this direction is not taken, a serious problem may result. The danger is that when a student's learning rate does not keep pace with his or her ability, there is a great risk of diminishing or even extinguishing a student's academic motivation in areas where he or she is quite talented. Again, the decision must be made for each individual student; an educational program that is right for one individual may not meet another's needs.

TEACHING STRATEGIES

From the previous discussion, it is clear that any teaching strategy geared to meet the needs of the mathematically gifted student must be based on altering the rate at which students progress through the typical mathematics sequence. This rate must not only exceed that of an average mathematics class, it must also be faster than the rate of a typical honors class consisting of students in the upper 20 percent in mathematical reasoning ability. The focus must be on the upper few percent. This select group is able to learn mathematics at such a great speed that a class designed for the upper 20 percent will only bore and frustrate the mathematically gifted. An excellent example of the tremendous speed at which mathematically gifted youths can learn precalculus mathematics is demonstrated by a special class conducted from June 1973 to August 1974. (9) During 14 months of meeting for one two-hour class per week, 16 ninth graders from the Baltimore area completed Algebra II, Algebra III (college algebra), plane geometry, trigonometry, and analytic geometry. Thus by the beginning of tenth grade these students were prepared to take a rigorous course in calculus.

A number of techniques pioneered by programs such as SMPY have proven to be extremely successful with mathematically gifted youths. Prior to presenting some of these techniques, it is important to note several strategies that should be avoided. Above all, it is clear that something must be done for the mathematically gifted. Ignoring the special needs of these youths often results in boredom, frustration, and possible behavior problems. Without sufficient challenge these students may demonstrate poor study habits and little ambition and initiative. These students eventually "turn off" to academics and never realize their full potential. (32)

As mentioned previously, busy work is not a solution. Self-pacing or independent study is another technique that is not very successful with this group. There must be teacher-pacing, accompanied by a program of evaluation and feedback. Any student who is able to pace himself or herself through an independent study course without feedback clearly has no need even to attend school.

The Fast-Paced Mathematics Class

The accelerated mathematics class is one strategy that SMPY has found very successful with gifted mathematics students. The accelerated mathematics class is specially designed for those in the top few percent in mathematical reasoning ability and speeds up the rate of learning to a pace that is consistent with students' abilities. In a regular mathematics class, much of the class time is spent going over and over a topic. The pace usually is geared to the middle or lower ability range so most everyone in the class can keep up. The average student often takes a long time to learn a single topic, then forgets it and must relearn it, and cannot build upon that topic when working further along the sequence. A youngster gifted in mathematics does not need repetition. He or she can learn a topic quickly and can progress further in the sequence of mathematics because one topic builds on another.

One type of accelerated mathematics program is the fast-paced, enriched mathematics class in which all students move at the same rapid pace. (5) In order to organize such a class, the gifted students must be homogeneous in terms of ability and level of knowledge in mathematics.

Instructors who teach a fast-paced, enriched mathematics class should gear their teaching to the top one-third or one-quarter of the class. When this group of students grasps a concept, the instructor moves on to the next topic. It is essential that this pace be maintained. The instructor must move ahead before every student understands a concept. These highly talented students generally can learn those topics not mastered in class by doing the homework. Homework, as a result, becomes a learning tool as well as a concept reinforcer. It is important not to slow down the pace of instruction for the sake of a few students. Students who truly belong in the class will be able to keep up with the pace so long as they are willing to spend extra study time outside the classroom.

The style of instruction should not be strictly lecture. Student participation should be maximized. Rather than present fully explained topics to the class, the instructor should try to involve the students in the development of a concept. For instance, in the course of a geometric proof, the instructor can continually solicit suggestions as to how to proceed. Or the instructor can ask the class to deduce the definition of a previously undefined geometric term such as a triangle. Frequently a student will solve a problem or prove a theorem in a way that is different from the technique the instructor would have chosen. In addition, students will constantly be jumping ahead to the next topic. The instructor must be flexible enough to allow this creativity and should not try to make students conform to her or his way of thinking. The goal of the instructor is to encourage the students to be actively involved in the instructional process. A skillful teacher, however, will not allow this kind of spontaneous participation to sidetrack the class in irrelevant directions.

The interaction of the gifted students is further facilitated by periodic blackboard work. By having the students work a particular problem at the board, the instructor obtains immediate feedback on concept mastery. This approach also facilitates individual help from the teacher and from classmates. One of the chief benefits of a fast-paced, enriched mathematics class is the chance to have these brilliant students interact with their peers in doing board work and in contributing to the overall lesson.

Homework is an essential aspect of any fast-paced mathematics program. The material is covered in class so quickly that it must be reinforced by extensive homework assignments. The amount of homework should not be so great that it places a burden on the student or becomes repetitive or busy work. Nevertheless, homework can help a gifted youth make the most of his or her abilities. Students participating in such a program cannot do all of the homework at the last minute. They must develop good study habits in order to space out the homework over the interval between classes. The discipline of pacing themselves through the homework assignment also leads to better recall of the material.

The instructor should spend very little classtime discussing homework—perhaps 10 or 15 minutes for a two-hour class. Even then the teacher should wait until after grading the homework, unless some students have experienced serious conceptual problems while attempting to do the assignment and are unable to understand further materials. While grading the homework, the instructor should be on the lookout for conceptual errors. Classtime should be spent discussing only those concepts that caused problems for a significant number of the class. It is also beneficial to discuss a few challenging problems that stumped many of the students. If, for instance, only one student solved a particular problem, ask that student to explain it to the rest of the class. Problems or concepts that are missed by small numbers of students can be handled by making appropriate notes on the homework or through extra help outside of class. Therefore, the cycle of a homework assignment is to make the assignment during one class, collect the work at the next class, and go over parts of it at the beginning of the following class (two classes after it was assigned).

The pace of the class is so rapid that it is sufficient to meet only once a week for two to two-and-a-half hours. Appropriate pacing will keep students busy between classes as they learn the concepts they missed or reinforce the concepts the rest of the class might have missed.

It is important to realize that the fast-paced, enriched mathematics class will function properly only if the group of participating students is quite homogeneous. Frequently, in a group of gifted mathematics students, some students know more mathematics than the others, despite being in the same grade. In addition, individual learning rates may differ significantly. In such a situation, one does not want to use the fast-paced approach with the whole group. Some students might be bored, and some would be forced to drop out. In short, these gifted students would be no better off than they were in the regular mathematics class. If the pool of students is small, it is impractical to break them up into smaller, homogeneously grouped classes that function independently. Such a situation calls for an individualized classroom approach.

Individualized Classroom Approach

SMPY has formulated a technique that accomplishes the goal of individualized instruction: Diagnostic Testing followed by Prescriptive Instruction (DT $\rightarrow$ PI). The DT $\rightarrow$ PI model has been replicated by several school districts. Its use as a classroom strategy can best be explained by first detailing its application to a situation in which there is only one mathematically gifted youth.

The Single Gifted Student

Suppose that it has been established that a certain youth has an extremely high ability to reason mathematically. Furthermore, assume that this youth has just completed seventh grade and has had no formal course entitled Algebra I. It is highly probable that this student already knows a significant amount of Algebra I. By administering a diagnostic test to assess the student's knowledge of first-year algebra, the student's mentor (a special kind of tutor) is able to discover just how much the student actually does know. The mentor then designs a program of prescriptive instruction to help the student learn rapidly only those concepts the student does not know. When the mathematics curriculum is streamlined to teach only what the student does not already know, the rate of learning increases dramatically. Note, however, that the mentor must use many of the same instructional techniques used by the teacher of a fast-paced, enriched mathematics class.

It is especially helpful if there are two parallel forms of each diagnostic test used to assess knowledge of Algebra I, II, III, plane geometry, trigonometry, analytic geometry, and calculus. This permits pre- and post-evaluation of knowledge learned. Another useful tool, if available, is an analysis chart that breaks down the items on each test into topics. Such charts are especially helpful in deciphering what a student does and does not know. One example of a diagnostic test series that meets all these requirements is the Cooperative Mathematics Test series developed by the Educational Testing Service.

Following is an example of the DT → PI procedure as used by a hypothetical mentor who is facilitating one hypothetical mathematically gifted seventh grader who has had no formal Algebra I course. This outline is adapted from a short article that appeared in *ITYB* (*Intellectually Talented Youth Bulletin*). (22)

1. Administer to the student a diagnostic test of first-year algebra knowledge. Observe carefully the instructions that come with the test, giving special attention to the time limit. Provide plenty of scratch paper and extra pencils. Do not answer any questions about the content

of the items. Let the student know when half the testing time is over and when five minutes are left. In addition to notifying the student in advance of the time limit and number of items, the following points of test-taking strategy should be explained.

a. Since the final score on the test is the number of correct answers, the student should mark an answer for every item, even if it is a guess.

b. The student should work as quickly and efficiently as possible, not spending too much time on any one item.

c. The student should place a question mark on the answer sheet next to those items of which he or she is unsure. If time permits, the student can return to the marked items for further study. The student should make all marks on the answer sheet or scratch paper, not in the actual test booklet.

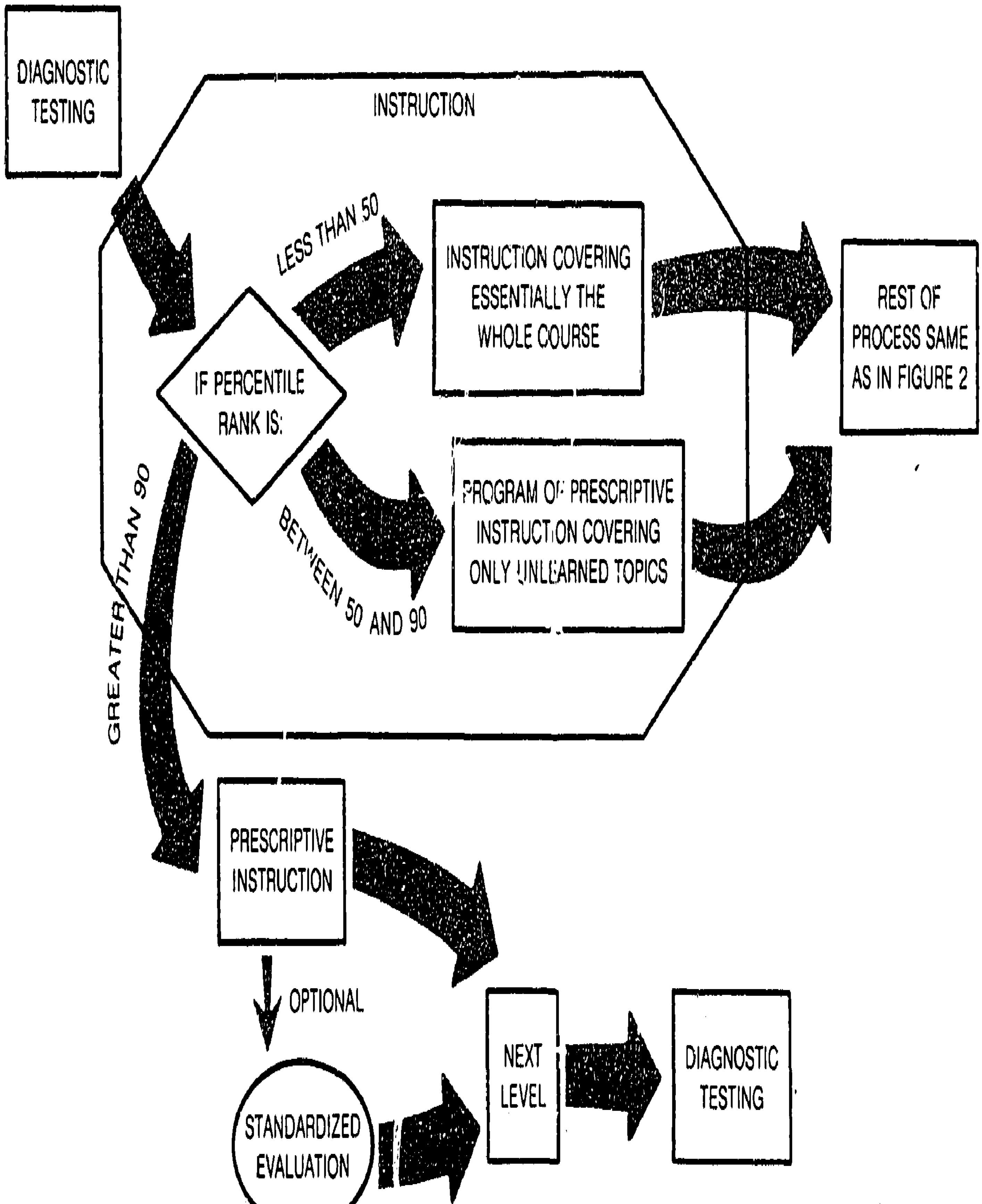
2. When the testing time expires, the mentor collects all of the diagnostic materials (test booklet, answer sheet, scratch paper). In order to protect the security of the measure, the scratch paper is destroyed. Carefully score the answer sheet in another room away from the student. Determine the percentile rank of the score (if available) from a table of national norms for Algebra I knowledge.

   a. If the percentile rank is below 50, then the student clearly knows very little of Algebra I. If this is the case, it is best to start at the beginning of Algebra I and efficiently cover all of first-year algebra at a rapid pace (see Figure 1).

   b. If the percentile rank is at least 50, proceed to the next steps.

3. Give the student a list of the missed items, the test booklet, and ask the student to try those items again on scratch paper. Do not let the student know how she or he answered the items on the answer sheet or, for that matter, what are the correct responses. Have the student attempt these items again without imposing a time limit. This time, the student should leave blank those items she or he has no idea of how to solve.

FIGURE 1: FLOW CHART FOR DT ⟶ PI MODEL



4. After the student has finished all of these items, the mentor should again collect the diagnostic materials and mark the answers. The mentor should make sure that the items now answered correctly are actually understood and were not mere guesses. This can be done by querying the student about the method of solution. The items that are answered correctly at this time were probably missed under standard conditions due to carelessness or lack of time. These are hardly valid reasons for teaching the student those concepts. The same certainty should be sought for those items answered correctly under standard conditions but which had a question mark next to them.

5. The items which the student has missed with unlimited time should be examined carefully by the mentor. By referring to the student's answers each time (i.e., were they the same or different both times?) and by querying the student, the mentor should be able to deduce which method of solution was used. Sometimes it will be clear that the student had no idea how to solve an item. If available, an item analysis chart should be utilized in the following way:
   a. Label the sheet with the student's name and score under standard conditions.
   b. Circle the number of all items missed under standard conditions.
   c. Check off those items that were answered correctly the second time.
   d. The grouping of the remaining items gives a clear picture of the concepts the student does not know.

6. At this point, the mentor knows enough about the points of ignorance in the student's knowledge of Algebra I to be able to devise a program of prescriptive instruction. Using a good Algebra I text, the mentor should guide the student through those concepts that need to be covered. It is imperative that the student *not* work through the entire book, but cover only those points not already well understood.

7. The program of instruction must be mentor-paced and not student-paced. More will be said about this later.
8. The goal of the mentor is to pace the student through the unlearned material as rapidly as is appropriate while preparing that individual to achieve an almost perfect score on a different evaluative measure of Algebra I (for example, an alternate form of the diagnostic test).
9. If the student obtains a very high score, say above the 90th or 95th percentile on national norms, then it may not be necessary for the student to take another Algebra I test. Nevertheless, the missed items should be reviewed using the above procedure. If it is clear that the student has no holes in his or her knowledge of Algebra I, the mentor may proceed directly to the next content area, probably Algebra II.
10. When the student is ready to have his or her knowledge of Algebra I evaluated, the mentor may want to readminister the original diagnostic measure. This can be used as a partial indicator of the student's progress, especially if the first score was very low and a few weeks have passed since the first evaluation.
11. When Algebra I has been mastered, the same diagnostic technique is applied to the next subject, preferably Algebra II. The mentor may decide to skip the diagnostic testing in Algebra II and commence instruction immediately. This is a valid alternative if the student scored very poorly, say below the 50th percentile on national norms, on the initial Algebra I test. The judgment of the mentor at this point is crucial. If it seems likely that a great deal of Algebra I has been learned, diagnostic testing at this level is a must. Certainly the mentor should use diagnostic testing in Algebra II if the student scored well (for instance, a percentile rank of 85) on the initial Algebra I test administered prior to any instruction.

The DT $\rightarrow$ PI procedure described above can be modified for use with a good Algebra I text that has a review test at the end of each chapter. In this scheme, the mentor asks the student to complete a sampling of the Chapter 1 test (for example, the odd-numbered items) without referring back to Chapter 1. The mentor analyzes the student's answers in the same fashion as with the standardized test. Those items answered incorrectly illustrate which sections of Chapter 1 need to be learned. When the student has mastered those concepts, the mentor assigns the even-numbered items in the Chapter 1 test. The student should achieve a nearly perfect score on these items. Once the student has mastered Chapter 1, the mentor proceeds in the same way with Chapter 2. When the student has progressed through the Algebra I material in this fashion, the mentor may then give the student a standardized Algebra I test. Notice that this modification is consistent with the model of teaching only what the student does not already know. The modification is especially useful when a student scores very poorly on the initial Algebra I test.

It is important to note that the DT → PI model described earlier focused on a seventh grader who had not had a formal Algebra I course. If, however, the student being considered is in ninth grade and has had Algebra I, the mentor would start the diagnostic testing with Algebra II. The mentor always begins one level higher than the last mathematics course the student has taken formally in school.

When using the DT → PI model, the rate of progression through mathematics is rapid enough that the mentor needs to meet with the student only once a week for two to three hours. The composition of this class is similar to that of the fast-paced, enriched mathematics class. As before, the homework is to be extensive without being repetitive, and it must be done thoroughly and well. Needless repetition can create unnecessary frustration for the student. For example, consider the case of John, an eighth grader who scored 40 out of 40 items on a diagnostic measure of Algebra I. Why should this pupil be forced to sit through 180 50-minute Algebra I classes? As an eighth grader, John wanted to be in a ninth grade Algebra I class, but the Algebra I teacher refused. John was sure he knew Algebra I, so he took an Algebra I diagnostic test. True to his expectations, he achieved a perfect score. John also scored in the high 30s on 40-item tests of Algebra II and III. After hearing that, the Algebra I teacher relented and told John he could take

Algebra I. John decided instead to take a college calculus course, and he did very well in it.

Teaching Small Groups of Gifted Students

The DT → PI model can be used by an instructor who is teaching heterogeneous groups of mathematically gifted students. Initially the instructor goes through the diagnostic testing procedure with each student. The results lead to the formation of small, homogeneous groups. Some students, especially those who are much more talented than the others, may have to be instructed individually. The teacher works with each group separately, pacing the group through the material using the techniques described previously. Several groups can be functioning at once, but the teacher must be careful not to be sidetracked by any one student or any one group.

The instructor works with a group for a short time, pacing students through the material or explaining a concept. Then the instructor's attention shifts to another group. While one group is working with the instructor, the other groups must be active. For instance, the instructor can explain a topic to group A, assign a few problems to check their understanding, and then go work with group B while group A is solving the problems. The teacher later returns to group A to check the problems and pace them through another section.

Obviously, there will be gaps, as when a group of students is waiting for assistance from the instructor. This is why it is important to choose a good text. The text must be clear and succinct enough that students can rapidly teach themselves without constant help from the teacher. Thus if the teacher is unavailable at some point during the class, the students can still progress through the material by reading the text. This approach also permits some individuals to move faster than others.

The purpose of the small groups is to encourage interaction as students ask each other questions and assist one another. The composition of the groups must be flexible; the brighter students should be able to move ahead and out of the group, possibly into another group.

As with previous strategies, this technique is also teacher-paced, not student-paced. The instructor's chief responsibility is to stimulate the students to move through the material at a pace consistent with their abilities. The teacher must guide students through only the necessary parts of the text and not force them to relearn concepts already known. The students should know at all times what point is to be learned and how fast it is to be mastered.

For an accelerated mathematics class, the best sequence for covering precalculus mathematics is Algebra I, Algebra II, Algebra III, geometry, trigonometry, analytic geometry, and calculus. The rationale behind this sequence is that each course is reinforced by the next. Mathematics that is learned quickly must be reinforced by building upon it immediately, as Algebra III builds upon and reinforces Algebra II.

Some believe that holes develop in a student's knowledge when mathematics is learned rapidly. A skilled instructor will not allow this to occur. The feedback generated from homework and testing should be scrutinized carefully to make sure that no holes develop. An accelerated pace of instruction does not imply that material is omitted. It means only that the students' level of reasoning ability is high enough that the instructor does not need to predigest material, give lengthy explanations about concepts, or go over points more than once or twice. In fact, because the youths in an accelerated mathematics class are so brilliant, the instructor can cover topics in greater depth than an average class is able to handle. Ironically it is more likely that gaps in knowledge will occur when a mathematically gifted student is placed in an average class. Because the student already knows many sections of the material, the course seems repetitive and slow. The student will frequently become inattentive and miss points that he or she genuinely needs to learn.

These two strategies for teaching mathematically gifted youths—the fast-paced, enriched mathematics class and the DT → PI model—have each been implemented successfully in various secondary school systems as well as in special programs such as SMPY. The following examples should help instructors appreciate the effectiveness of these two strategies.

IMPLEMENTATION

The Fast-Paced, Enriched Class Model

There are a number of programs that have been based on the homogeneously grouped fast-paced, enriched mathematics class model. One such class sponsored by SMPY lasted for 14 months, from June 1972 until August 1973. The class began with 19 students from the Baltimore area who had just finished sixth grade or were of that age. Six students left the class by the end of the summer. In September three more students were added. All 16 of these students quickly completed Algebra I and II. Ten of the 16 students then completed Algebra III and trigonometry. Nine of the 10 continued in the class and completed analytic geometry. Eight of these nine studied further material during the summer of 1973 and completed plane geometry. These eight students were thus prepared to begin calculus in the fall of 1973. (3, 23)

Throughout the duration of the program, this class met for two hours each week on Saturday mornings. Therefore, the total amount of instructional time for eight students who remained in the class for all 14 months was approximately 120 hours. These eight completed the entire precalculus sequence in 120 hours. Students in the regular school curriculum take 180 50-minute classes for each of four-and-a-half years! Even the students who left the program before its conclusion made astounding progress through part of the precalculus sequence when compared to the average secondary school mathematics student.

In Minnesota (Minneapolis-St. Paul and Duluth areas) fast-paced mathematics programs are currently in their fourth year of existence. The programs are designed as a two-year bridge of the limited precalculus course offerings available in junior high school. In approximately 120 hours of instruction spaced over a two-year period, students cover Algebra I and II and plane geometry. When these students complete junior high school, they are then permitted to continue at the next appropriate level in the mathematics sequence taught in the high school. The suburban school district of Howard County (Maryland) and schools in the state of Delaware rely on this approach. In each case instructors have found it an excellent way to bridge the age-grade gap in mathematics content between elementary school and senior high school.

In rural Charles County (Maryland), a fast-paced mathematics program that starts in the second semester of the seventh grade has resulted in the development of a calculus program for the school system. Until this fast-paced, enriched mathematics model was introduced, calculus was unheard of as a high school subject.

There is one factor common to all school districts, regardless of the talent population and transcending the rural, urban, and suburban classifications. Each district that has instituted the concept has found that students develop a new attitude towards mathematics, resulting in increased expectations as to what can be accomplished in a learning atmosphere that is both stimulating and challenging.

When mathematically gifted youths are placed together in one class they can learn calculus rapidly and at the rigorous level of a college course. Consider, for example, one calculus class that lasted from September 1974 to May 1975. (23) This special class, which met for two hours on Saturday mornings, supplemented the regular high school calculus classes each of the 13 students was taking. Most of the students previously had been in a fast-paced, enriched precalculus mathematics sequence. This supplemental calculus class delved into material in greater depth than the regular high school class. The purpose was to prepare students to take the more difficult (BC) level Advanced Placement Program (APP) exam in calculus, administered nationally each May. While many high schools offer calculus, only some offer it at a level high enough so that the students are prepared to take the less difficult (AB) level APP calculus exam. Very few high schools have the resources to offer a calculus course sufficiently advanced to prepare students to take the more strenuous BC level APP exam. However, due to the ability level of the students in this special calculus class, the group was able to cover thoroughly the advanced topics tested by the BC level exam.

In order to measure student progress, six instructor-designed tests were administered. In May all 13 of the students took the BC level calculus examination. Nine students earned scores of 5, three received 4s, and one received a 3 (on a 1 to 5 scale, where 5 is highest). The significance of these scores is that a 4 or 5 on level BC is worth two semesters of calculus at most selective universities, enabling participants to get a head start on their college educations. (At Johns Hopkins University, two semesters of calculus are worth eight credits, where 15 credits is the normal course-load for one semester.) It also means that high-scoring students have the opportunity, while still in high school, to take more advanced college courses that build upon the calculus, thereby offering even greater stimulation and challenge.

The DT → PI Model

The classroom strategy of diagnostic testing followed by prescriptive instruction has not been used as much as the strategy of homogeneously grouped fast-paced mathematics classes. It has, however, been extremely successful in the situations where it has been implemented. SMPY used this model with two groups of students that met for eight weeks during the summer of 1978. The older group consisted of 44 mathematically brilliant youths who had just completed the eighth grade. They met with skilled mentors on Tuesdays and Thursdays for three to five hours each day. A summary of the courses completed by this group is given in Table 2.

The younger group consisted of 33 mathematically gifted students who had just completed the seventh grade. This group met only on Wednesdays during the eight weeks, for five hours each day. A summary of the accomplishments of this group is also given in Table 2.

In each of these two groups, many students already had an understanding of various sections of the precalculus sequence. Nevertheless, almost none of the Wednesday group had taken any formal school courses beyond Algebra I, if that much. Likewise, the majority of the Tuesday-Thursday group had not progressed beyond Algebra II in formal coursework, if that far. Despite this, these youngsters had learned much advanced precalculus. They were able to progress quickly through several subjects because they were taught only the concepts they had not already

TABLE 2.—RESULTS OF VARIOUS PROGRAM IMPLEMENTATIONS OF THE DT → PI MODEL

Program	Number of students	Grade(s) when in program	Typical amount of coursework covered	Maximum amount of precalculus coursework covered	Maximum hours of instruction
SMPY's Summer 1978 Wednesday Program	33	7-8	2	4	40
SMPY's Summer 1978 Tuesday-Thursday Program	44	8-10	2	4	48
SMPY's Summer 1979 Program	94	7-8	2	4	40
Nebraska's 1978 Summer Program for ESU #11	8	9-11	2	3	60*
Nebraska's 1979 Summer Program for ESU #11	16	8-12	2	3	60*

* This includes homework time, since the Nebraska summer program was residential. Homework and instructional time is included in their six hours each day.

learned. As a result, 12 out of the 33 in the Wednesday group were certified as having completed the precalculus sequence. Students in the Wednesday classes, meeting for a total of approximately 40 hours, successfully completed an average of 1.94 years worth of precalculus mathematics. The Tuesday-Thursday group, which met for a total of 48 to 60 hours, successfully completed an average of 2.20 years worth of precalculus mathematics. If these students had not had this opportunity, they typically would have had to sit through a full year each of routinely paced precalculus subjects, repeating many concepts they already understood.

The DT → PI model was also used in the mathematics segment of the first two Summer Honors Programs sponsored by the Educational Service Unit (ESU) #11 in rural south central Nebraska (see Table 2). The classes met six hours a day for ten days during two-week periods in June 1978 and June 1979. A total of 24 students, who ranged from end of eighth grade to end of twelfth grade, participated in the two classes. Under the direction of two trained mentors, the students in these two classes typically completed two precalculus courses in approximately 60 hours of instructional time.

No matter which of the described teaching strategies is used to teach mathematically gifted students, the learning rate is so rapid that a class of extended duration needs to meet only once a week for two to two-and-a-half hours. Depending on the situation, one model may be better suited than the other. For instance, the fast-paced mathematics model is applicable only where there is a large enough population base to enable identification of a group of homogeneous students—for example, in a metropolitan area in which a county-wide program is implemented. The DT $\rightarrow$ PI model is applicable to all situations, but especially useful in locales where the population base is so small that any class-size group of mathematically gifted students would be quite heterogeneous. The DT $\rightarrow$ PI model could be implemented with a whole class on a district-wide basis, or in individual schools where in many cases less than 10 students comprise a regular mathematics class. In a small class of mostly average students, it is quite feasible to use the DT $\rightarrow$ PI model with the one or two mathematically gifted students present.

Other Activities

In addition to offering mathematically gifted youths a program of instruction, it is beneficial to provide career education in the areas that make great use of mathematics. This feature may take the form of inviting speakers who have careers in mathematics or in areas of applied mathematics. The speakers provide excellent role models and will stimulate student interest in fields that call for special mathematical talents.

Math teams are particularly popular with youths who reason well mathematically. Teams can be organized on a school-wide, district-wide, or even county-wide basis. Statewide or even broader competitions may be organized in addition to local contests.

EVALUATION

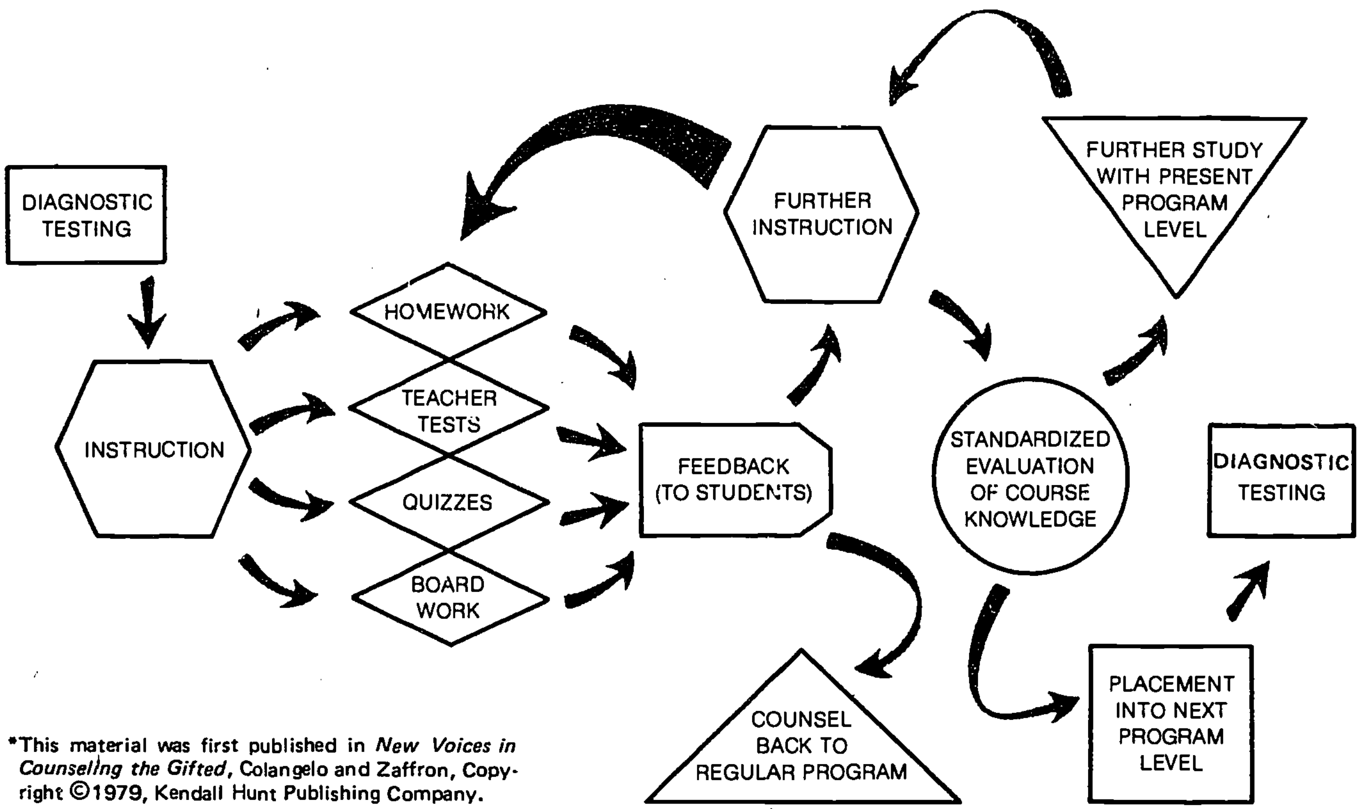
In an accelerated mathematics program, it is essential that the teacher closely monitor progress through effective techniques of evaluation. There are general evaluative guidelines that should be followed for the two classroom models that have been presented.

A flow chart of the evaluative process for the homogeneous fast-paced, enriched mathematics class is shown in Figure 2. The first step of this process occurs before the actual instruction begins. The students who are enrolled in the class should take a diagnostic test to determine which concepts they already know in the subject to be taught. Concepts most class members know can then be covered very quickly or even skipped. If a few students do not understand a topic that has been mastered by the others, they can be handled individually outside of class. The initial diagnostic testing may be accomplished through standardized measures (which are particularly effective when item analysis sheets are used, as previously explained) ; teacher-designed tests; or textbook tests, such as chapter review tests, which frequently are broken down according to topic.

When actual classroom instruction begins, progress should be continuously measured and reported to the students. The use of board work and homework as evaluative tools have been discussed previously. Periodically, the instructor should administer teacher-designed tests and quizzes. However, frequent quizzes use up too much valuable instructional time. Homework and tests, that indicate to the teacher how well an individual is learning the classroom material, should be returned to the student with evaluative comments. Once the student is given this feedback, instruction continues. This cycle occurs over and over again.

This evaluative process will indicate when a student falls behind the rest of the class, either because of poor homework, a poor test score, or both. If that happens, then a conference should be arranged with the student and/or parent(s). If it is determined that this student is below the ability level of the rest of class, she or he may need to be placed back in a regular mathematics classroom. It is best to make such moves at the end of a subject sequence, such as Algebra I. If, however, the struggling student is willing to work assiduously, seek extra help outside of class, and spend more time on homework, she or he may be able to remain in the class. If it is determined that the student is doing poorly because of an unwillingness to make a suffi-

FIGURE 2: FLOW CHART FOR A FAST-PACED, ENRICHED MATHEMATICS CLASS



*This material was first published in *New Voices in Counseling the Gifted*, Colangelo and Zaffron, Copyright ©1979, Kendall Hunt Publishing Company.

cient effort on the homework, this person should be given an ultimatum to either do the homework diligently or be placed back in the regular instructional classroom. Mathematically gifted youths should view participation in an accelerated mathematics class as a privilege and not a right, no matter how bright they may be.

When the instructor believes that a subject has been mastered by the class, a diagnostic measure (such as the Cooperative Mathematics Test series) of that particular subject should be administered. This phase of the evaluative process leads to one of two alternatives. In some cases, it may indicate that more instruction is needed at the present course level. For those students who score well (say, above the 90th percentile), this standardized evaluative measure becomes the basis for giving credit for the subject matter completed. Those students who are awarded credit then begin the next level in the course sequence.

A flow chart of the evaluative process for a class following the DT→ PI model has already been shown in Figure 1. The process is similar to that of the fast-paced, enriched mathematics class. The chief difference is that the initial diagnostic measure determines the level of instruction on an individual rather than on a group basis. The remainder of the process is unchanged. Again, it must be stressed that the teacher should hold the students to high standards of homework.

When grades are given for completed coursework, they should be seen as a reward and not as a penalty. These bright students would probably obtain an A if they were in a regular math class. Students who participate in an accelerated mathematics class complete more courses, do more work, and cover material at a higher level than do pupils in the regular mathematics classes. Since only those students who do put forth a strong effort are allowed to remain in the class, they should not be placed in "double-jeopardy" by receiving poorer grades than they would have received in a regular classroom situation. Therefore, students who complete a subject (i.e., score above the 90th percentile on the evaluative measure) should receive a grade of A for that course. In a few cases, a grade of B may be appropriate. However, no one should receive a grade lower than B.

Students deserving lower grades belong in a regular classroom.

TEACHER SELECTION

The choice of instructor is a major factor in the success of an accelerated mathematics class. Instructors should meet several *criteria:*

1. They should know mathematics very well (for example, at least through calculus if teaching precalculus). It is important that a teacher's knowledge level extend beyond the level at which they are teaching.
2. They should be flexible, bright, quick-thinking, and alert.
3. They should be able to relate well to the needs and personalities of their students. A good sense of humor is helpful in promoting teacher-student interaction.
4. They should have respect for students and their feelings/opinions.
5. They should desire to see their students reach maximum potential. This will require diligence on behalf of the instructor and may require extra time spent working with students outside the regular classroom situation.
6. They should be enthusiastic about mathematics.
7. They should be willing to teach mathematics fast, at a pace appropriate to the students' abilities. This implies challenging individuals to go one step beyond what they already grasp.

In addition to these criteria, there are certain *skills* that these instructors should exercise while teaching:

1. The lectures should be concise and at a quick but appropriate pace. This pace may vary as content becomes more difficult. As mentioned previously, instructors must be flexible during a lecture, allowing students to participate in the development of concepts.
2. They should facilitate interaction between students.
3. They should be sensitive to the students' level of under-

standing. In a fast-paced, enriched class, they should teach at a pace that challenges the top students in the class. Through wise homework selection and extra help outside of class, a skillful instructor will maintain the fast pace without allowing the less able students to fall hopelessly behind. In a class that follows the DT → PI model, the instructor must be a master pacer and stimulator, knowing the speed at which each student can learn new material.

4. They should not be threatened by the abilities of their students. They should not be intimidated by the fact that many mathematically gifted youths are capable of learning mathematics better and more quickly than they did. Teachers should not be threatened by students' questions. Students frequently ask difficult questions that may be beyond the scope of the present topic. These questions should be handled in a manner that further stimulates the questioner's curiosity. Inquiries should not be dismissed as irrelevant merely because the instructor has no answer. In such a situation, it is best not to fake an answer; admit ignorance and then obtain appropriate reference materials or point the questioner in the direction of such materials. A good teacher will allow time for a certain number of questions that may not relate directly to the current subject. This keeps the interest level high and expands students' knowledge. In-depth questions that require a lengthy answer may be pursued after class.

5. They should not spoon-feed material to the students. Mathematically gifted youths should learn self-teaching skills. Good instructors will not use class time to explain every detail of the concepts being taught. Instead, they should assign homework that helps pupils learn those points not covered in class. Through diligent effort on homework, bright students are able to master a concept that was covered rapidly or merely alluded to in class. This approach gives the instructor more time to explain and perform examples of more difficult concepts and problems. It also speeds up the class and reduces boredom.

6. As much as possible, teachers should not answer questions that deal directly and specifically with a topic already covered or reveal how to solve a problem that is being assigned for further study. Instead, instructors should answer with leading questions that will reveal the answer. These bright students probably have never had to work very hard at academics and have always done well. Therefore, when they begin to be challenged in mathematics, rather than make an extra effort to learn difficult material, they frequently attempt to use the teacher as a crutch who predigests the material for them. Instructors must not fall into a pattern of being used this way by students. These students can be quite clever at disguising laziness and lack of effort. They may say they are unable to understand a concept, when in fact they could understand it if they tried a bit harder. Teachers who answer a pupil's question with a question force that student to think things through and thus encourage a fuller use of capabilities.

From these lists of criteria and skills, it is clear that the instructor of an accelerated mathematics class should be quite bright in mathematics and possess masterful teaching techniques. Mathematically gifted college students have been used quite successfully as instructors by SMPY. Particularly those who themselves were students in accelerated mathematics classes possess a keen understanding of mathematics and can identify with their students. Mathematically gifted students have proven to be especially able as mentors of other bright youngsters in one-on-one situations using the DT → PI model. College professors often make good accelerated mathematics class instructors (and have been used as such by SMPY). (32)

TEXTBOOK SELECTION

From the discussion thus far, one may have inferred some criteria for selecting textbooks to be used in accelerated mathematics classes. Some of these criteria are:

1. The textbook should contain explanations sufficiently clear for a student to be able to learn many concepts

without teacher assistance. As has been stated, this is particularly important when using the DT $\rightarrow$ PI model.

2. It should be concise.
3. It should contain numerous examples of completely solved problems.
4. It should contain large numbers of problems that can be assigned for completion in class or for homework.
5. It should not provide the answers to all of the problems. It is helpful if the text gives answers to only the odd-numbered problems. The student would be able to check those answers, while being required to submit some of the even-numbered problems.
6. It is helpful if the text contains chapter tests that can be used as diagnostic and evaluative tools.

Once an accelerated mathematics class is underway, there are various ways to supplement the instruction. Periodically the teacher may wish to introduce more advanced topics that follow from the material then under consideration. However, as already mentioned, a subject should not be probed too deeply during class, especially if there is no provision for obtaining credit.

In selecting a textbook it is important to realize that a unified mathematics curriculum presents difficulties relevant to articulation. For example, suppose an individual is enrolled in an accelerated mathematics class, completes Algebra I and Algebra II, and then is counseled to go back into the regular mathematics sequence at his or her school. If the school has a unified mathematics curriculum it is difficult to place this student in the proper course, since Algebra I and Algebra II are integrated and blended with other subjects in mathematics over a period of several years. The same difficulty exists if the accelerated mathematics class follows a unified mathematics curriculum and the student's school does not.

OTHER ACCELERATIVE OPTIONS

There are a number of opportunities for educational acceleration that should be made available to mathematically gifted youths. (20) Sometimes mathematically gifted students who are not learning mathematics at an accelerated rate are instead able to start the precalculus sequence a year ahead of their classmates. For instance, if Algebra I is offered in eighth grade, some bright seventh grade students can be allowed to enroll in the class. In addition, these able students can be encouraged to take two mathematics courses (such as Algebra II and plane geometry) concurrently.

Some students learn mathematics at an accelerated rate outside of school, either through a special class or with a mentor. This often leaves an opening in the daily schedule for other courses. Thus acceleration in the mathematics sequence may lead to acceleration in other areas. Therefore, those bright individuals who are eager and of high ability should be allowed to skip a grade in school. This is done most inconspicuously if the affected student skips the final year in one school and begins attending another. For instance, in a junior high school (grades seven through nine), an eighth grade student would skip ninth grade and enter tenth grade at the senior high school.

Many mathematically gifted youths are quite able to enter college full-time a year or more ahead of their agemates. Students who are willing and able should be given the option of graduating from high school one year early (after eleventh grade) or leaving high school without a diploma a year or more ahead of their peers. Many school systems will grant these students their diplomas after satisfactory completion of one full year of college.

The Advanced Placement Program (APP) allows highly able students to earn college credit by examination before entering college on a full-time basis. Many high schools offer special courses specifically designed to cover the advanced material tested by these difficult examinations. Mathematically gifted students should be helped and encouraged to prepare for the more difficult APP calculus examination as well as for exams in any other areas in which they are talented, such as chemistry or physics.

Finally, these brilliant youths should be encouraged to take college courses on a part-time basis while still enrolled in junior or senior high school. (18) This can be done either on released time from school, in the evenings, or during the summer months. Through college courses and APP credits, many students just beginning a full-time college program already possess one, two, or more semesters' worth of college credits.

No matter how successful an accelerated mathematics class is, the benefits will be negated if there is no coordination and articulation between grades and between schools. Any program of accelerated instruction should be organized with the cooperation of the participating schools. For example, students in an accelerated mathematics class should not in addition be required to attend the regular school mathematics class. Most important, the participating schools should give credit for mathematics courses completed in a special class. This ensures that students will not be asked to repeat a course if they are placed back into the standard mathematics curriculum in a later grade or when transferred to another school district. For instance, if students who are enrolled in an accelerated mathematics class in junior high find no continuation of the class in senior high, they should not be forced to repeat coursework that has already been covered in the special class.

CONCLUSION

If mathematics is truly the cornerstone for many of the sciences and for engineering and computer science, then we need to motivate and stimulate our mathematically talented youths. *Flexibility* and *alternative education* seem to be the key, based on the experiences of state and local educational agencies and on the insights gained through programs such as SMPY.

Educational acceleration is a critical variable. It has been found to be the method of choice for those youths who reason extremely well mathematically and are eager to move ahead educationally. Now is the time to examine the evidence. The experiences to date have shown that program development for the mathematically gifted is not costly. It is, indeed, a necessity if we are to maximize the potential of these individuals.

APPENDIX A

Commonly used in-grade nationally normed first-stage screening measures that have been used to identify the mathematically talented. (Listed in alphabetical order with the name of the publisher.)

1. California Achievement Tests (California Test Bureau)
2. California Basic Skills Test (California Test Bureau)
3. Cognitive Abilities Test (Houghton Mifflin)
4. Iowa Tests of Basic Skills (Houghton Mifflin)
5. Metropolitan Achievement Test (Harcourt, Brace, and World)
6. Science Research Associates Achievement Series (Science Research Associates)
7. Stanford Achievement Tests (Harcourt, Brace, and World).

APPENDIX B

Measures commonly used at the second level of the two-stage screening method for the mathematically gifted.

1. The Preliminary Scholastic Aptitude Test (PSAT)
2. The Scholastic Aptitude Test (SAT)
3. The School and College Ability Test (SCAT)
4. The Differential Aptitude Test (DAT)
5. The American College Testing Program (ACT)
6. The College Board Mathematics Achievement Tests—Levels I and II.

SELECTED REFERENCES

1. Cohn, S. J. "SMPY's Fifth Talent Search: A Time of Changes." *ITYB* (*Intellectually Talented Youth Bulletin*) 4, no. 10: 9–11, 1978.
2. Daurio, S. P. "Educational Enrichment Versus Acceleration: A Review of the Literature." *Educating the Gifted: Acceleration and Enrichment.* (Edited by W. C. George, S. J. Cohn, and J. C. Stanley.) Baltimore, Md.: The Johns Hopkins University Press, 1979. pp. 13–63.
3. Fox, L. H. "A Mathematics Program for Fostering Precocious Achievement." *Mathematical Talent: Discovery, Description, and Development.* (Edited by J. C. Stanley, D. P. Keating, and L. H. Fox.) Baltimore, Md.: The Johns Hopkins University Press, 1974. pp. 101–125.
4. George, W. C. "The Talent Search Concept: An Identification Strategy for the Intellectually Gifted." *Journal of Special Education* 13, no. 3: 221–237, 1979.
5. ———. "The Third D: Development of Talent (Fast-Math Classes)." *New Voices in Counseling the Gifted.* (Edited by N. Colangelo and R. T. Zaffran.) Dubuque, Iowa: Kendall/Hunt, 1979. pp. 119–132.
6. George, W. C., and Benbow, C. P. "SMPY's Sixth and Last Talent Search: It Broke the Barrier." *ITYB* 5, no. 10: 2–4, 1979.
7. George, W. C., and Cohn, S. J. "The 1976 Talent Search Results Are In!" *ITYB* 3, no. 6: 1–2, 1977.
8. George, W. C., Cohn, S. J., and Stanley, J. C., editors. *Educating the Gifted: Acceleration and Enrichment.* Baltimore, Md.: The Johns Hopkins University Press, 1979.
9. George, W. C., and Denham, S. A. "Curriculum Experimentation for the Mathematically Talented." *Intellectual Talent: Research and Development.* (Edited by D. P. Keating.) Baltimore, Md.: The Johns Hopkins University Press, 1976. pp. 103–131.
10. George, W. C., and Solano, C. H. "Identifying Mathematical Talent on a Statewide Basis." *Intellectual Talent: Research and Development.* (Edited by D. P. Keating.) Baltimore, Md.: The Johns Hopkins University Press, 1976. pp. 55–89.
11. Keating, D. P. "The Study of Mathematically Precocious Youth." *Mathematical Talent: Discovery, Description, and Development.* (Edited by J. C. Stanley, D. P. Keating, and L. H. Fox.) Baltimore, Md.: The Johns Hopkins University Press, 1974. pp. 23–46.
12. Montour, K. M. "Charles Louis Fefferman: Youngest American Full Professor." *ITYB* 2, no. 8: 2, 1976.
13. ———. "William James Sidis: The Broken Twig." *American Psychologist* 32, no. 4: 265–279, 1977.
14. Pegnato, C. W., and Birch, J. W. "Locating Gifted Children in Junior High Schools: A Comparison of Methods." *Exceptional Children* 25, no. 7: 300–304, 1959.
15. Renzulli, J. S. *The Enrichment Triad Model: A Guide for Developing Defensible Programs for the Gifted and Talented.* Wetherfield, Conn.: Creative Learning Press, 1977.
16. Solano, C. H. "The First D: Discovery of Talent, or Needles in a Haystack: Identifying the Mathematically Gifted Child." *New Voices in Counseling the Gifted.* (Edited by N. Colangelo and R. T. Zaffrann.) Dubuque, Iowa: Kendall/Hunt, 1979. pp. 89–106.
17. ———. "Teacher and Pupil Stereotypes of Gifted Boys and

Girls." *Talents and Gifts* 19, no. 4: 2–15, 21–26, 1977.
18. Solano, C. H., and George, W. C. "College Courses for the Gifted." *Gifted Child Quarterly* 20, no. 3: 274–285, 1976.
19. Stanley, J. C. "Identifying and Nurturing the Intellectually Gifted." *Phi Delta Kappan* 58, no. 3: 234–237, 1976.
20. ________. "Rationale of the Study of Mathematically Precocious Youth (SMPY) During Its First Years of Promoting Educational Acceleration." *The Gifted and the Creative: A Fifty-Year Perspective.* (Edited by J. C. Stanley, W. C. George, and C. H. Solano.) Baltimore, Md.: The Johns Hopkins University Press, 1977. pp. 75–112.
21. ________. "The Second D: Description of Talent." *New Voices in Counseling the Gifted.* (Edited by N. Colangelo and R. T. Zaffrann.) Dubuque, Iowa: Kendall/Hunt, 1979. pp. 107–118.
22. ________. "SMPY's DT → PI Mentor Model: Diagnostic Testing Followed by Prescriptive Instruction." *ITYB* 4, no. 10: 7–8, 1978.
23. ________. "Special Fast-Mathematics Classes Taught by College Professors to Fourth- Through Twelfth-Graders." *Intellectual Talent: Research and Development.* (Edited by D. P. Keating.) Baltimore, Md.: The Johns Hopkins University Press, 1976. pp. 132–159.
24. ________. "The Study and Facilitation of Talent for Mathematics." *The Gifted and the Talented: Their Education and Development* (The Seventh-eighth Yearbook of the National Society for the Study of Education). (Edited by A. H. Passow.) Chicago: The University of Chicago Press, 1979. pp. 169–185.
25. ________. "Test Better Finder of Great Math Talent Than Teachers Are." *American Psychologist* 31, no. 4: 313–314, 1976.
26. ________. "Use of Tests to Discover Talent." *Intellectual Talent: Research and Development.* (Edited by D. P. Keating.) Baltimore, Md.: The Johns Hopkins University Press, 1976. pp. 3–22.
27. Stanley, J. C., George, W. C., and Solano, C. H., editors. *Educational Programs and Intellectual Prodigies.* Baltimore, Md.: The Study of Mathematically Precocious Youth, The Johns Hopkins University, 1978.
28. Stark, E., and Stanley, J. C. "Bright Youths Dispel Persistent Myths about Intellectual Talent: Panel Discussion with Parents and Educators." *Gifted Child Quarterly* 22, no. 2: 220–234, 1978.
29. Terman, L. M. "Mental and Physical Traits of a Thousand Gifted Children." *Genetic Studies of Genius*, vol. I. Stanford, Calif.: Stanford University Press, 1925.
30. Tuttle, Jr., Frederick B. *Gifted and Talented Students.* Washington, D.C.: National Education Association, 1978.
31. Tuttle, Jr., Frederick B., and Becker, Laurence A. *Characteristics and Identification of Gifted and Talented Students.* Washington, D.C.: National Education Assn., 1980.
32. ________. *Program Design and Development for Gifted and Talented Students.* Washington, D.C.: National Education Assn., 1980.
33. Zuckerman, H. *Scientific Elite: Nobel Laureates in the United States.* New York: Free Press, 1977.

The program ideas presented in *Teaching the Gifted and Talented in the Mathematics Classroom* have been used successfully by various state and local agencies as well as by private organizations. The ideas should not be considered rigid guidelines, but flexible methods of facilitation. Since each school and each student is unique, the models should be shaped to fit each individual situation.

The ideas are developed through eight focal aspects: Identification, Approaches, Teaching Strategies, Implementation, Evaluation, Teacher Selection, Textbook Selection, and Other Accelerative Options. A carefully prepared group of Selected References provides excellent sources for further study.

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