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ABSTRACT

This report summarizes a number of exploratory and pilot studies undertaken to investigate the relationship of individual differences among college mathematics students to discovery and expository instruction, and to Personalized Systems of Instruction (PSI). Research in education has often attempted to demonstrate the superiority of one type of instruction when compared with another. This research is based on the idea that no single treatment is satisfactory for all students, but that different pupils do best with different types of instruction, depending on the individual student's characteristics. Matching instructional treatments with student characteristics to maximize learning is called Aptitude-Treatment-Interaction (ATI). Significant ATI's were obtained in the reported studies with three different aptitude variables. Field dependence-independence, general reasoning, and locus of control appear to be the most promising variables for future research, particularly in mathematics education. A complete report of each study is provided in the appendices. (Author/MP)

Final Technical Report

The Role of Cognitive Style in the Learning of Mathematics: A Research Study

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15. Abstract

Research in education has often attempted to demonstrate the superiority of one type of instruction when compared with another. This research is based on the idea that no single treatment is satisfactory for all students, but rather, that different students will do best with different types of instruction, depending on the individual student's characteristics. Matching instructional treatments with student characteristics so as to maximize learning is called Aptitude-Treatment-Interaction (ATI). Difficulties associated with this type of research have involved selection of aptitude variables and treatments which produce different results with different students.

This report summarizes a number of exploratory and pilot studies undertaken to investigate the relationship of individual differences among college mathematics students to discovery and expository instruction, and to Personalized Systems of Instruction (PSI). Significant aptitude-treatment-interactions were obtained with three different aptitude variables. The variables of field-dependence-independence, general reasoning, and locus of control appear to be the most promising for future research, particularly in mathematics education. A complete report of each study is provided in Appendices A through F.

16. Descriptors

Cognitive Processes
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Discovery Learning
Interaction
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17. Identifiers

Aptitude-Treatment-Interaction (ATI); Personalized Systems of Instruction (PSI)

18. Field

Mathematics Education

19. Target Audience

Undergraduates

20. Availability

21. Supplemental Notes

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FINAL TECHNICAL REPORT

The Role of Cognitive Style in the Learning of Mathematics

Much of the research on instruction in mathematics has tried to prove that one kind of instruction is superior to another for all students. Such research generally has not been very conclusive. Rather than hypothesizing the superiority of a single treatment for all students, we believe that different students will do best in different types of instruction, depending on each student's characteristics. The problem of matching instructional treatments with student characteristics so as to maximize learning has come to be called the Aptitude-Treatment-Interaction (ATI) hypothesis. For a sample graph of an ATI, see Figure 1. As the figure indicates, a student with a high aptitude score seems to learn more in treatment 2, while low-aptitude students do better in treatment 1.

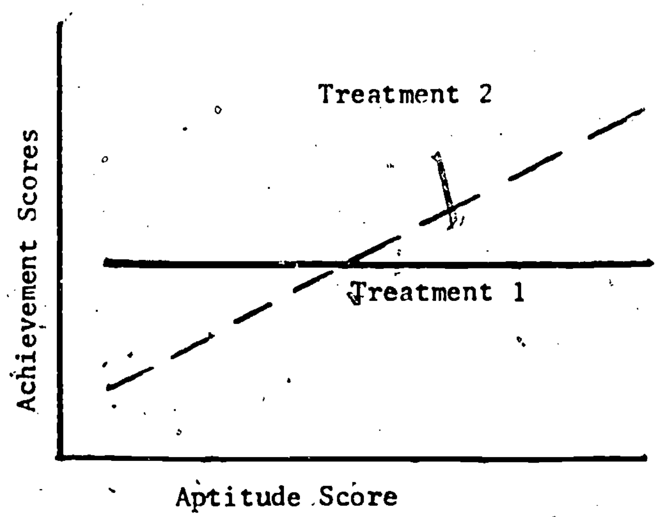

Figure 1. An example of regression lines that show an Aptitude-Treatment Interaction

ATI research, generally viewed as an outgrowth of the work of Cronbach (1957), has turned out to be somewhat more difficult than was first anticipated. The initial choices for aptitude variables frequently turned out to be inappropriate and unproductive. Also, it is difficult to produce treatments that are about equally effective, and still distinct enough to produce different results with different students. Nevertheless, Cronbach and Snow (1977), in their comprehensive review of the field, confirm the existence of ATI in a variety of settings, and conclude that "ATI has come of age" (p. 524).

Previous Research

Our ATI research program at San Diego State began by looking at treatment differences that are important in mathematics, especially dimensions of instruction that are related to discovery learning. Then we identified individual difference variables that seemed likely to predict differences in achievement in discovery or expository treatments. In our search for relevant aptitude variables, we tried to select only those that had a strong theoretical foundation. So far we feel that this approach has been quite successful.

We began by considering Witkin's work on field independence. This dimension of cognitive style has been the focus of a large number of research studies for 30 years, but its implications are just now becoming more widely known (Witkin, Moore, Goodenough, & Cox, 1977). In the latest revision of the theory, Witkin and Goodenough (Note 1) characterize field independence as "autonomy of external referents." This autonomy is expressed in terms

of two different types of ability, restructuring and interpersonal competencies. Field-independent students have greater personal autonomy, and they tend to do better in cognitive restructuring tasks, especially those that require disembedding a particular figure from the surrounding field. However, this autonomy in field-independent students seems to hamper the development of interpersonnal competencies and social skills. Field-dependent students, while they are less autonomous and weaker in cognitive restructuring, appear to be stronger in the area of interpersonal competencies.

Differences in abilities and preferences between field-independent and field-dependent students are related to at least some aspects of discovery learning in mathematics. In our first study, for example, we found a significant disordinal interaction between field independence and the level of guidance of mathematics instruction (McLeod, Carpenter, McCornack, & Skvarcius, 1978). As predicted by the theory, field-independent students did significantly better when the treatment provided minimal guidance, while field-dependent students seemed to learn best under conditions of maximal guidance. This result was confirmed in a later study using similar materials and procedures (McLeod & Adams, in press-b).

In another study, we investigated the relationship of field independence to a different dimension of discovery learning--the use of inductive and deductive sequences of instruction (McLeod & Briggs, in press). In this case we found an interaction between field independence and the sequence of instruction on only one of four dependent variables. Since the treatments used programmed materials (on the topic of equivalence relations), all students received a high level of guidance, which appeared to reduce the

interaction with field independence that we were expecting. However, there was a distinct pattern of interactions between the treatments and general reasoning ability. These interactions were in the same direction as that found by Eastman and Carry (1975). It is difficult to know at this point whether these interactions can really be attributed to general reasoning ability, or if they are the result of other variables like crystallized intelligence (Cattell, 1971) or differences in information processing strategies (Snow, Note 2). But the results of our first three ATI studies were certainly encouraging, and further research seemed appropriate.

Studies Conducted Under Grant #SED77-18531

During the period of the grant we continued to investigate the relationship of individual differences among college mathematics students to discovery and expository instruction. We also searched for interactions with Personalized Systems of Instruction (PSI), and conducted a number of exploratory and pilot studies. The main studies will be summarized briefly here; for a complete report of each study, see Appendices A through F. The investigations dealing with the relationship of individual differences to discovery and expository instruction will be discussed first.

Main Studies on Discovery Learning

The studies conducted under this grant concentrated on the relationship of field-dependence-independence and general reasoning ability to treatments that varied in dimensions of discovery learning. These studies were all preceded by pilot testing of the treatments and achievement tests, and

appropriate revisions of these materials. In all studies, students who were enrolled in a college mathematics course were randomly assigned to two treatment groups. The treatments were conducted during regular class time. Dependent variables generally included measures of immediate achievement and retention. Data were analyzed through the use of multiple regression techniques.

The first study to be discussed is "Aptitude-treatment Interactions in Mathematics Instruction Using Calculators" (see McLeod & Adams, Note 3, which is attached as Appendix A). In this study students were assessed on field independence and general reasoning, and randomly assigned to either discovery or expository instruction. The discovery treatment used an inductive sequence of instruction and provided as little guidance for the students as was feasible; calculators were also provided to help students discover concepts and rules independently. The expository treatment used a deductive sequence of instruction and provided maximal guidance for the students; this treatment was designed so that calculators were not needed. The topic of instruction involved errors in measurement and calculations with approximate data.

The hypothesis tested in this study was that both field independence and general reasoning would interact with the treatments, and that these interactions would be in opposite rections, as they were in the study by McLeod and Briggs (in press). The results confirmed only part of this hypothesis. There was an interaction with general reasoning, and it was in the predicted direction. Students with high scores in general reasoning did better in the deductive expository treatment than in the inductive dis-

covery treatment; for students with low scores in general reasoning, the situation was reversed (see Figure 1 in Appendix A), producing the desired disordinal interaction. There were no interactions with field independence, probably because the treatments provided a higher level of guidance than was originally intended.

Three other studies all used different versions of a unit on networks as the topic of instruction. The first of these studies, "The Interaction of Field-dependence-independence and the Level of Guidance of Mathematics Instruction", is reported in full in Appendix B. (See also Adams & McLeod, in press). This study tested the hypothesis that field independence would interact with treatments that differed in the use of high or low levels of guidance. Students were assessed on field independence and also on a pre-test that measured their achievement in a prerequisite course. This type of pretest can be considered a measure of crystallized ability or (possibly) general ability in mathematics.

Analysis of the data indicated that there were no interactions with field independence in this study. Instead, there was an interaction with the pretest. This interaction indicated that students who scored well on the pretest did better in the high guidance group where a more traditional expository mode of instruction was used. Students with low scores on the pretest achieved about the same in either treatment group. Snow (Note 2) has recently reported a number of similar interactions with measures of crystallized ability.

Since the high-guidance treatment seemed to be somewhat easier for most students than the low-guidance treatment, a second study on the topic of

networks was conducted (McLeod & Adams, in press-a, attached as Appendix C, "Individual Difference in Cognitive Style and Discovery Approaches to Learning Mathematics"). For this study the treatments were expanded to include more concepts and more problems, and slightly more structure was provided in the low-guidance treatment. Instead of using the pretest which had produced the interaction in the earlier study (Adams & McLeod, in press), students were assessed on measures of general reasoning and general ability, as well as on field independence. General reasoning was used because it had been an important variable in some earlier ATI studies, and because it could be assessed using only 15 minutes of class time in contrast to the pretest which took 50 minutes. General (or crystallized) ability was assessed by using SAT scores that were available in the records of the university.

There were no interactions with any of these aptitude measures. Although there was a tendency toward steeper regression slopes in the low-guidance treatment, the differences in slope were quite small. There were substantial differences between the treatment groups, however; students in the high-guidance treatment scored significantly higher on both the posttest and the retention test.

Following another revision of the unit on networks a third study was conducted, "The Interaction of Field Independence with Small Group Instruction in Mathematics" (McLeod & Adams, in press-c, in Appendix D). For this study, only one set of printed materials was prepared. This time the treatments differed only in whether the students worked together in small groups rather than working individually. The printed materials used

an inductive sequence of instruction which was designed to encourage student discovery. Students in the small-group treatment worked together in making discoveries; in the other treatment, students worked individually and asked the teacher for help when they had difficulty.

Since field-dependent students are more adept at working in groups than field-independent students, it was hypothesized that the small-group treatment would be more effective for field-dependent students. However, the results of the study did not support this hypothesis. Field-dependent students appeared to do better in the individual treatme[redacted] they had more guidance from the teacher. Several significant aptitude-treatment interactions occurred in the data with measures of both field independence and general ability. For all interactions, students with high aptitude scores did better in the small-group treatment, indicating that any interaction effect was probably due more to general ability than to field independence.

Students were also asked to evaluate the unit on networks. Since the printed materials were exactly the same, any differences in the evaluation should be due to the use of small-group as opposed to individual work. No interactions occurred when these student ratings were used as the dependent variable, but there was a significant treatment effect in favor of the small-group treatment.

There was substantial difficulty in all three network studies in finding the predicted interactions with field independence. When interactions did occur, they seemed to be due to general ability as much as to field independence. Part of the problem seems to be the similarity of the content

of the networks unit and the content of the items on the typical measures of field independence (the Group Embedded Figures Test and the Hidden Figures Test). Since similar skills are used in each case, field independence predicts achievement rather well in all kinds of treatments. Other problems in measuring field independence have been noted elsewhere (e.g., Cronbach & Snow, 1977); Witkin and his colleagues are aware of the difficulties in measuring field independence, and continue to work on this problem (Witkin & Goodenough, Note 1).

Personalized Systems of Instruction

Although the major work of the grant dealt with the interaction of aptitudes and treatments that differed in the use of discovery methods, we did conduct one study of possible interactions with Personalized Systems of Instruction (PSI). A PSI course which used self-pacing, frequent testing, and student tutors was compared to a traditional lecture-discussion approach to the same content. The study covered one semester's work in intermediate algebra as taught to college students. The aptitude variables that were used in the study were general ability, field independence, and locus of control. (See Appendix E for a complete report by McLeod & Adams, Note 4: "Individual Differences in Mathematics Learning Through Personalized Systems of Instruction.")

There is some support in the literature for an interaction effect between general ability and PSI. The nature of this interaction is that low-ability students tend to do better in a PSI treatment, where they get extra tutoring and other instructional support; high-ability students seem

to do equally well in either PSI or lecture classes. We predicted that field-independent students would do better in PSI than in lecture classes, since PSI requires students to do more work independently. Finally, we hypothesized that students with an internal locus of control would do better in PSI instruction, where students are expected to take responsibility for their own learning; students with an external locus of control were thought to be better suited to the traditional lecture class where the teacher takes more responsibility for student progress.

Although no significant interactions occurred in this study, there was some support for the predicted interactions with general ability and with locus of control. Differences between regression coefficients were in the predicted direction. There was no evidence of any interaction when field indeoendence was used as the aptitude.

Internal-External Locus of Control

Locus of control was investigated further in three other studies (see McLeod & Adams, Note 5, in Appendix F, "Locus of Control and Mathematics Instruction"). Students who participated in the three studies (see Appendices A, C, and D) were assessed on a measure of locus of control, and the data from those studies were reanalyzed using locus of control and general ability as the aptitude variables. Of the three dimensions of discovery learning that were used, only small-group instruction produced an interaction with locus of control. Students with an internal locus of control seemed to learn more in small-group instruction, but students with an external locus of control appeared to do better in individual instruction where they received help from the teacher.

There were no significant interactions when the treatments differed in level of guidance, but there was a trend in the expected direction. The regression coefficient for the locus-of-control scores was consistently greater in the low-guidance treatment, suggesting that students with an internal locus of control may learn more in a discovery-oriented treatment rather than under expository instruction.

Treatments differing in the use of inductive, rather than deductive, instruction produced no sign of an interaction with locus of control.

Pilot Studies

In addition to the main studies discussed above, a number of other pilot studies were conducted. The purpose of these studies was to test the feasibility of expanding our earlier work to new treatment dimensions or to new aptitude variables.

The first of these studies was an attempt to look at the relationship between field independence and an open-ended problem solving task. Students from five classes for prospective elementary school teachers participated in the study. Students were not randomly assigned to treatment groups and one instructor conducted the study aided by the regular classroom teachers.

The hypothesis was that relatively field-dependent students who received training prior to working on an open-ended problem solving task would do better than those who did not receive training. Relatively field-independent students were expected to do well whether or not they received any training and they were also expected to do better than relatively field-dependent students.

The open-ended problem solving task consisted of making as many different three-dimensional geometric figures (polyhedra) as possible from geometric shapes (triangles, squares, and rectangles) which had been provided for the student. Some of the students (N=50) were shown examples of prisms, pyramids, the regular polyhedra, and combinations of these prior to working on the task. These training sessions were done in a large-group setting. The rest of the students (N=39) were not given any training prior to the task. Some of the students in each class worked in small groups and others worked individually. Some students also participated in a similar task a few days prior to this one. These factors were ignored in the present analysis.

The Hidden Figures Test (HFT) was used as the measure of field-independence and the number of three-dimensional figures created by each student was used as the dependent variable. Treatments were defined as training and no training. In a multiple regression analysis of the data using vectors for HFT, treatment, and the interaction of treatment and HFT, no interaction occurred and only about 5% of the variance was accounted for by the HFT. The regression equation for the treatment which included training was $G = 7.97 + .24H$ where G is the number of geometric figures created and H is the HFT score. The regression equation for the treatment which did not include training was $G = 7.67 + .18H$.

Results from this study were not conclusive. Although the HFT and performance had a positive correlation of .22, a spatial visualization test might be more appropriate for predicting performance in this type of task. Greater detail about procedures used in this study can be found in the 1979 master's project paper prepared by June Dandliker at San Diego State University.

Another pilot study investigated the possibility of developing instructional units on the topic of "Mathematical Systems". Concepts included in the unit were related to properties of finite mathematical groups. The two treatments that were prepared used either discovery or expository approaches to the same content. The discovery treatment used minimal guidance and physical materials to enhance student discovery. The expository treatment provided maximal guidance and presented all concepts symbolically. Preliminary tryouts of these materials indicated that the presentation was too difficult for our students, and a major study was not attempted.

A third pilot study was conducted to determine the possible utility of measures of state and trait anxiety in ATI studies in mathematics. Cronbach and Snow (1977) have noted that measures of anxiety have produced a number of important results in ATI studies, and they have recommended further research using anxiety as an aptitude variable. There are several measures of anxiety that are now available, but the instrument that seems to have the strongest theoretical support is the State-Trait Anxiety Inventory (STAI) of Spielberger, Gorsuch, and Lushene (1970). The STAI consists of two scales; the A-Trait scale is designed to measure a general disposition to perceive circumstances as threatening, and the A-State scale assesses feelings of apprehension or tension associated with a particular situation.

The STAI was administered to 30 students, including 15 who had identified themselves as being anxious about mathematics and 15 who were identified as not being anxious. The math-anxious group was randomly selected from the participants in the Mathematics Anxiety Clinic at San Diego State University in the Spring Semester, 1978. Participation in the

Clinic was entirely voluntary and open to any students who identified themselves as math-anxious--i.e., one who avoids mathematics and is fearful of mathematics classes. The other group included 15 students from the fourth semester of a sequence of courses in mathematics for elementary school teachers. Since this course was not required, these students were not avoiding mathematics and were not generally fearful of the subject.

The A-Trait and A-State scales of the STAI were administered to both groups on a voluntary basis. The A-Trait scores resulted in a mean of 41 and standard deviation of 12 for the math-anxious group compared to a mean of 33 and standard deviation of 7.2 for the non-anxious group. This difference in scores was significant, $\underline{t}(1,28) = 2.43$, $\underline{p} = .01$. Compared to other groups of college students, where the mean on this scale is typically about 38 (Spielberger et al., 1970), the math-anxious group also scored relatively high.

On the A-State scale, which measures the student's level of anxiety in the environment of the mathematics classroom, the math-anxious group had a mean of 52 with a standard deviation of 14, while the other group had a mean of 32 with a standard deviation of 7.3. In this case the differences between the two means was also significant, $\underline{t}(1,28) = 4.83$, $\underline{p} < .001$. Even though we recognize the fact that these significance tests are being applied to data that do not satisfy all of the assumptions of the statistical model, these results seem reasonable and help to confirm the usefulness of the STAI in identifying students who suffer from anxiety in mathematics classes. Therefore, we suggest that the STAI be used in ATI studies in mathematics, along with other measures of anxiety that prove to be appropriate.

Comparison of Work Completed with Work Proposed

Figure 2 is taken from the original proposal which resulted in Grant #SED77-18531. The figure summarizes the studies that we proposed to do under this grant. In this section we will indicate the extent to which we met our proposed goals.

Component A, Discovery with Calculators, was completed as planned. The pilot study was relatively successful and the main study (McLeod & Adams, Note 3) is reported in Appendix A. This paper has been accepted for presentation at the Annual Meeting of the American Educational Research Association in San Francisco during April 1979, and it has also been submitted to both the ERIC system and a research journal.

Under Component B, Discovery with Physical Materials, we completed a pilot study (discussed above) that indicated our materials were too difficult for the students who were intended to be our subjects. As a result, no further studies were conducted in Component B.

Three studies were completed as a part of Component C, Networks. Reports of these studies are included in Appendices B, C, and D. Two of these papers were accepted for presentation at the 1978 and 1979 Annual Meetings of the National Council of Teachers of Mathematics (in San Diego and Boston, respectively) and all three have been accepted for publication (Adams & McLeod, in press; McLeod & Adams, in press-a, in press-c).

We completed more work on Component D than the original proposal called for. The report of this study is included in Appendix E, and it has been submitted to a journal for publication (McLeod & Adams, Note 4).

No replications were completed as a part of Component E. Treatments

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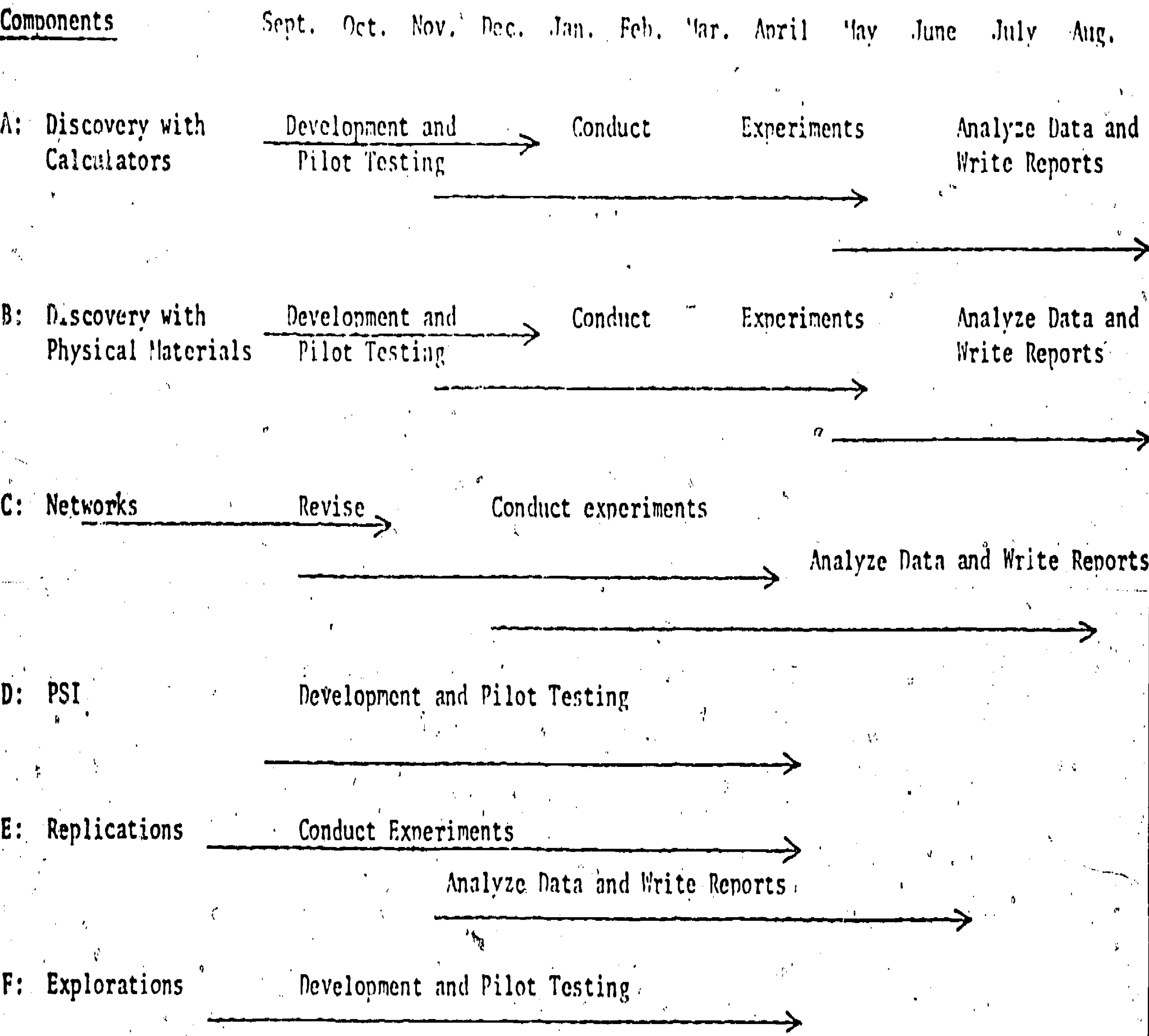

Figure 2. Schedule of activities.

from other ATI studies were either not available or not appropriate for our students.

A number of exploratory and pilot studies were completed as a part of Component F. The pilot studies of problem solving and measures of mathematics anxiety are discussed above. Also, three studies of locus of control are reported in Appendix F; this paper has also been submitted to a journal (McLeod & Adams, Note 5).

In summary, work in four of the six components (A, C, D, F) outlined in Figure 2 met or exceeded the goals set in the proposal. This work has resulted in six papers, including four accepted for publication in professional journals or for presentation at national meetings, and two submitted for publication, but not yet accepted.

Conclusions and Recommendations

The cognitive style variable of field-dependence-independence has produced a number of significant interactions, but not consistently in all studies. Given the difficulties of doing ATI research, some lack of regularity in reaching a .05 level of significance is to be expected. Since studies with a reasonable amount of power are prohibitively expensive, and since so many instructional variables are difficult to control, ATI research will continue to have problems with replications. Nevertheless, ATI research on field independence and level of guidance seems promising. It appears that studies in which the treatment uses non-geometric content are likely to be more successful than those using geometric content. However, when more adequate measures of field independence are developed, the content of the treatments

may become less important.

Another difficulty in doing research on field independence is its ill-defined relationship with general ability and fluid ability. More research on this topic is currently being done by Witkin and his colleagues, and the results should provide alternative instruments for measuring field-dependence-independence that will be useful in ATI studies.

The interactions that have occurred between general reasoning and inductive instruction provide another fruitful line for further research. The results of these studies are subject to varying theoretical interpretations. It is possible that crystallized ability, rather than general reasoning, is actually the operative variable in these studies. It is also possible that tests of general reasoning may actually be described more accurately in terms of an information processing model. For a more thorough discussion of these possible theoretical positions, see Appendix A. In any case, more research on the relationship of general reasoning and inductive instruction is certainly needed.

Finally, internal-external locus of control is another variable that warrants further study. Again, there are difficulties in assessing this dimension, but the instrument developed through this grant seems particularly appropriate for assessing locus of control in mathematics classrooms. Differences in locus of control seem important in small-group instruction and in PSI classes; other instructional variables such as level of guidance may also interact with locus of control. More research on this variable is needed to assess its influence in mathematics instruction.

For a more thorough discussion of the work of this project in relation

to other ATI studies, see Appendix G, "Recent Research on Aptitude-treatment Interactions". This paper was presented at the 1979 meeting of Project Directors sponsored by the National Science Foundation Division of Science Education Development and Research (McLeod, Note 6).

In summary, this project has produced significant aptitude-treatment interactions with three different aptitude variables. Further research is needed on all of these aptitude variables, and on others (e.g., anxiety) which have not yet been thoroughly investigated in the context of the mathematics classroom. In the last two decades the difficulties of doing ATI research have become all too clear, and many researchers in mathematics education have come to believe that no progress can be made on the ATI problem of matching instructional treatments and student characteristics so as to optimize achievement. But the results of this project, along with current studies that refine our conception of aptitudes, show that ATI research still holds great promise as a means of improving our understanding of the teaching and learning of mathematics.

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APPENDIX A

Aptitude-treatment Interaction in Mathematics Instruction Using Calculators

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Running head: Aptitude-treatment Interaction

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Presented at the Annual Meeting of the American Educational Research Association, San Francisco, April 1979.

Abstract

Students in three mathematics classes were assessed on two aptitudes, field independence and general reasoning, and randomly assigned to either an expository or a discovery treatment. The expository treatment used a deductive sequence of instruction and provided maximal guidance for the students. The discovery group used an inductive sequence with minimal guidance, and provided calculators to help students discover concepts and rules independently. The topic of instruction involved errors in measurement and calculations with approximate data. There was a significant interaction with general reasoning on the retention test, as predicted. There were no interactions with field independence.

Aptitude-treatment Interaction in Mathematics Instruction Using Calculators

Aptitude-treatment interaction (ATI) research, generally viewed as an outgrowth of the work of Cronbach (1957), has turned out to be more difficult than originally expected. Simple hypotheses about matching student abilities with appropriate treatments have proven difficult to substantiate. Nevertheless, Cronbach and Snow (1977), in their comprehensive review of the field, confirm that ATI do exist and are important to educational practice.

Cronbach and Snow (see also Snow, 1977) state that the most stable interactions occur with general ability. However, there are a number of interactions in the literature, especially with inductive and deductive instruction (Cronbach & Snow, 1977, p. 320, 371), that do not seem to be related to general ability. General reasoning is one of the aptitude variables that is frequently involved in these more specific interactions.

In mathematics education research, several studies have reported ATI between general reasoning and treatments that differed in the use of an inductive or a deductive sequence of instruction (Eastman & Carry, 1975; McLeod & Briggs, in press). There are also studies that have failed to find the expected interactions (Behr & Eastman, 1975; Eastman & Behr, 1977), but this may have been because the level of difficulty of the treatments was not appropriate for the students.

The theoretical framework for these interactions with general reasoning is not well established. Cronbach and Snow (1977) note that measures of general reasoning are closely related to general ability in mathematics.

In ATI studies, however, general reasoning seems to function quite differently from general ability. For example, tests of general reasoning seem to do a better job of predicting success in a more expository deductive treatment than in an inductive treatment, the reverse of what one usually finds for measures of general ability. To explain these interactions, Cronbach and Snow have suggested that a test of general reasoning might be a measure of crystallized ability, or achievement in traditional school subjects; therefore, it could be expected to produce steeper regression slopes in more traditional deductive instruction (Snow, Note 1). Carroll (1976) has analyzed the aptitude of general reasoning from a different perspective, using the concepts of information processing theory. From this point of view, general reasoning assesses the ability to perform serial operations, which seems to correspond to the more direct sequence (rules followed by examples) of deductive instruction.

One reason that Cronbach and Snow (1977) attribute most ATI to general ability is that it is difficult to separate the effects of a specific aptitude from general ability. The difficulties with traditional aptitude constructs led Glaser (1972) to call for research with "new aptitudes", including dimensions that are related to personality variables such as cognitive styles. One cognitive style variable, field independence, has received considerable attention in educational research (Witkin, Moore, Goodenough, & Cox, 1977). In a recent revision of cognitive style theory, Witkin and Goodenough (Note 2) suggest that cognitive restructuring ability and personal autonomy are the two characteristics on which field-dependent and field-independent students differ. Treatments that provide minimal

structure and guidance should be appropriate for field-independent students, since they can provide their own structure and work autonomously. Field-dependent students, however, should excel in a highly structured treatment which provides careful guidance. Some studies in mathematics education have found ATI that support this theoretical position (McLeod, Carpenter, McCornack, & Skvarcius, 1978; McLeod & Adams, in press), but other studies have not produced significant interactions.

In summary, ATI research in mathematics education has found two aptitude variables, general reasoning and field independence, that have produced significant interactions with two dimensions of discovery learning, level of guidance and inductive instruction. The purpose of this study was to search for ATI between these two aptitude variables and treatments that differed in both level of guidance and in use of an inductive or deductive sequence of instruction. The treatment that provided a minimal level of guidance and used an inductive sequence was labeled the discovery treatment; the expository treatment provided maximal guidance with a deductive sequence of instruction. Based on the theoretical background for these two aptitude variables, it was predicted that field-independent students would do best in the discovery treatment, while students who scored well on tests of general reasoning would be better off in the expository group. Rephrasing this hypothesis in terms of regression slopes, it was predicted that the regression of achievement on field independence would be steeper in the discovery group than in the expository group, but the regression on general reasoning would be steeper in the expository group.

Method

Subjects

Students from three sections of a mathematics course for prospective elementary school teachers participated in the study. All three classes met in the afternoons for 75 minutes on two days each week. About 87% of the 60 students in the classes were women. Complete data were obtained for 47 subjects, 24 in the expository group and 23 in the discovery group. Other students were absent for one or more days of instruction and testing. The rate of student absenteeism did not appear to be related to differences in the treatment groups.

Treatments

Two instructional units were prepared on the topic of errors in measurement and their effect on calculations with approximate data. This topic was suggested by the Report of the Conference on Needed Research and Development on Hand-held Calculators in School Mathematics (1976). The treatments included such concepts as precision of measurements, significant digits, and their relationship to adding, subtracting, multiplying, and dividing approximate data. Both treatments covered exactly the same concepts, and students were given about the same amount of practice in solving problems. However, the concepts were presented in different ways in the two treatments.

In the expository treatment, instruction proceeded in a deductive sequence, with definitions and rules followed by examples. Students were given maximal guidance; sample problems were worked out completely before students were asked to do similar problems. The problems were chosen so that they could be worked easily without a calculator. In the discovery treatment,

however, concepts were presented in an inductive sequence. Students first worked out several examples, using a hand-held calculator when it was needed. Students were then encouraged to generalize and produce rules that would follow the examples. Although the students were given an opportunity to discover the rules, the materials did provide the rules to students who did not discover them independently. In both treatment groups, the teacher was available to help answer student questions.

Tests

Field independence was measured using the Group Embedded Figures Test (GEFT) and a version of the Hidden Figures Test (HFT). The GEFT (Witkin, Oltman, Raskin, & Karp, 1971) is the most appropriate group measure of field independence. The version of the HFT that was used (Hidden Figures 2--Form 271) was adapted by the National Longitudinal Study of Mathematical Abilities (NLSMA) from the original of the Educational Testing Service (French, Ekstrom, & Price, 1963). For a complete discussion of this test, see the appropriate NLSMA reports (Romberg & Wilson, 1969; Wilson, Cahen, & Begle, 1968).

The time allowed for the GEFT and HFT was adjusted for this study. Since the GEFT is relatively easy for college students, subjects were given four minutes for each part, rather than five. The version of the HFT that was used was rather difficult, so students were given 15 rather than 10 minutes for that test.

The HFT was used along with the GEFT in order to provide a second measure of field independence, a procedure in line with the multitrait-multimethod approach to measuring aptitude that is recommended by Cronbach

and Snow (1977).

The most common measure of general reasoning in ATI studies is the Necessary Arithmetic Operations (NAO) test (French, Ekstrom, & Price, 1963). In order to distinguish between scores on the NAO test and general ability, students were asked to allow the university to release their SAT scores. Most students agreed to this request, but only 28 of those subjects actually had SAT scores on file.

A 20-item posttest that covered all of the concepts in the unit was used to measure immediate achievement. A subset of 10 items was used to measure retention. The retention test covered only the parts of the unit that had been completed by most participants. Fifteen minutes was allowed for the posttest, and seven minutes for the retention test.

The KR-20 reliability coefficients were judged to be satisfactory on all tests. They ranged from .61 on the posttest to .82 on the NAO.

<u>Procedures</u>

The HFT and NAO tests were administered during the first week of class as a part of the regular course procedures. During the middle of the term, 90 minutes of class time was devoted to the study.

Students were randomly assigned to treatment groups within each class. Students assigned to the discovery treatments were asked to go to a room equipped with calculators. Students in the expository group stayed in the regular classroom. They were told that they would get their chance to work with the calculators later, since there were not enough calculators for the entire class to use them at the same time. Since no calculators were needed for the expository treatment, the lack of a calculator caused no problems

for that group.

At the beginning of the treatments, students were given a brief introduction to the materials and were encouraged to work independently, directing their questions to the teacher. At the end of the first day of the study, the materials were collected and graded. Most students were not able to complete the treatments in the 75 minutes allowed. The posttest was administered two days later at the next class meeting. Four weeks later students were assessed again to measure retention. On the same day, students took the GEFT.

Results

Descriptive statistics are presented in Tables 1, 2, and 3. Table 1 includes the means and standard deviations for all tests; scores ranged widely among students, but there were no large differences between groups. Table 2 presents the correlation matrix for the aptitude and achievement tests. Correlations between the NAO test and the two measures of field independence were somewhat higher than one usually expects. Also, there was a strong correlation between the posttest and retention test. Table 3 presents the regression equations for each group, using HFT and NAO as predictors. Substitution of the other measure of field independence (GEFT) for the HFT scores produced similar results.

Insert Tables 1-3 about here

Tests for Interaction

The data were analyzed using multiple regression techniques. The two dependent variables were treated separately. For the main analyses, the full model included vectors for field independence (HFT, GEFT, or their sum), NAO, treatment, and the interaction of treatment with each of the aptitude vectors. As these vectors entered the equation (in the specified order), the change in R^2 due to each interaction vector was calculated. On the retention test, the interaction of NAO and treatment was significant (see Table 4) and in the predicted direction.

Insert Table 4 about here

Figure 1 presents the interaction of NAO and treatment for the retention test. In the figure, the regression equations are calculated for each group using the NAO scores as the only predictor. The slope for the expository group was .42; in the discovery group it was .09. This difference in slopes is significant, $F(1, 43) = 6.96$, $p = .011$.

Insert Figure 1 about here

The data were analyzed further in several different ways. Scatterplots of each aptitude variable with the two achievement measures were constructed; in each case the use of linear models seemed appropriate.

Other measures of field independence (GEFT, the sum of HFT and GEFT) were included in the main analysis along with NAO. The results were essentially the same as those reported in Tables 3 and 4. There was still an interaction with NAO on the retention test, but not on the posttest. There were no interactions at all with field independence.

Since there was no interaction on the posttest, it was appropriate to test for a difference between treatment group means, when using HFT and NAO as covariates. No difference was found, $F(1, 43) = .67$, $p = .418$.

The importance of class effects has been emphasized by Cronbach (Note 3), so the data were reanalyzed taking into account the student's class and possible interactions of class with treatment, NAO, and the treatment-by-NAO interaction. On the retention test, the interaction with NAO occurred consistently across classes. On the posttest, only one class produced this type of interaction effect; in the other two classes the NAO slopes were about the same in both treatment groups.

Source of the Interaction

The data were analyzed further to determine whether the interaction with NAO could be attributed to general reasoning alone, or whether it should be thought of as an interaction with general ability or crystallized ability. The analysis began by considering the 28 subjects on which SAT data were available. The sum of the verbal and quantitative parts of the SAT were used as a measure of general ability. There was no evidence of any interaction with SAT, either by itself or in conjunction with the other aptitude variables. When SAT and NAO were put in the same regression equation with the retention test as the dependent variable, the NAO-by-treatment vector

accounted for about 3% of the variance, substantially more than the 1% due to the SAT-by-treatment vector. Of course, neither of these interactions was significant, since there were only 28 subjects in this analysis. However, these data provide some support for attributing the interaction with NAO to the aptitude of general reasoning rather than to general ability.

Further information on the nature of the NAO interaction was obtained by considering the difference of the standardized scores for HFT and NAO. Cronbach and Snow (1977, p. 84) state that two predictors behave differently if their standard-score difference interacts with the treatment dimension. The interaction between treatments and difference scores was not significant, $\underline{F}(1, 43) = 1.97$, $\underline{p} = .168$. The sum of the standardized scores for NAO and HFT, however, did interact with treatment, $\underline{F}(1, 43) = 4.804$, $\underline{p} = .034$. Since the combination of NAO and HFT should act more like general ability than general reasoning, the analysis of sum and difference scores provides some support for attributing the interaction to general ability rather than to the more specific aptitude of general reasoning.

Regions of Significance

Regions of significance for the interaction represented in Figure 1 were calculated in two ways. Following Cronbach and Snow (1977), confidence intervals were computed about each of the regression lines, using a confidence level of 68%. The confidence intervals overlapped for NAO scores of 13 to 17; therefore, the regions of significance for this interaction were for NAO scores of less than 13 and more than 17. These two regions included 55% of the students. Students with NAO scores of 17 or more did better in the expository group, as predicted, while students who scored less than 13

achieved more in the discovery group.

The Johnson-Neyman technique (Borich, Godbout, & Wunderlich, 1976) is another method of calculating regions of significance. For a level of significance of .10, this technique found the regions of significance for the interaction in Figure 1 to be almost the same as in the analysis using confidence intervals. For the Johnson-Neyman analysis, the upper region of significant differences included scores of more than 18. The lower region was found to be the same as in the analysis using confidence intervals. The regions of significance in the Johnson-Neyman analysis included 49% of the students.

Discussion

This study tested the hypothesis that ATI would occur between two aptitudes, field independence and general reasoning, and treatments that differed in dimensions of discovery learning in mathematics. Field independence was expected to interact with the treatments since they differed in the level of guidance provided to the students. General reasoning was expected to interact with the treatments since they differed in the use of deductive or inductive sequences of instruction. The ATI with general reasoning occurred as predicted on one of the two dependent variables. Therefore, this study helps to confirm the existence of an ATI that has appeared in several other studies (Cronbach & Snow, 1977; Eastman & Carry, 1975; McLeod & Briggs, in press).

Although a number of studies have found ATI with general reasoning, as measured by the NAO test, it is still not clear whether this interaction can be attributed to this specific aptitude, or whether it is the result of general or crystallized ability (Cattell, 1971). Data from the present

study were not conclusive on this point. Further investigation using an information processing approach may help to explain the effects of this aptitude variable. It seems likely that sequence differences in treatments may be related to fixed, as opposed to flexible, sequences of information processing. In this study, it appeared that students with high NAO scores were less flexible in terms of adapting to instruction using an inductive sequence where students were supposed to make generalizations with the assistance of hand-held calculators. In this interpretation, the ATI of this study fits nicely into Snow's recent work (Snow, Note 1) on the relationship of crystallized ability to ATI. Since the interaction occurred only on the retention test, it may be that these differences in information processing are only important when they involve retrieval from long-term memory.

The expected ATI with field independence did not occur. The major reason for this appeared to be that the treatments provided more guidance than was originally intended. This extra guidance was provided partly because the students requested, even demanded, considerable help from the instructor in the classroom. Also, treatments frequently need to be "tuned" in order to produce ATI, and appropriate revisions of the treatments used in this study could result in instruction that provides sufficient, but minimal, support. Such a revision might produce the expected interaction with field independence.

In summary, this study identified the expected ATI with general reasoning (as measured by the NAO test) but not with field independence. Further research on the topic seems appropriate. It used to be sufficient in ATI

research just to find an interaction; no one worried a great deal about whether the ATI could be attributed to a specific aptitude as opposed to general ability. But now more detailed information is necessary as researchers try to build a theory of aptitudes and interactions. These higher expectations seem to be a sign that ATI research is making substantial progress.

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Table 1

Means and Standard Deviations of

All Tests for Each Treatment Group

Test	Maximum possible score	Range	Discovery		Expository	
			Mean	SD	Mean	SD
HFT	16	0-16	5.6	3.6	4.9	3.5
GEFT	18	0-18	9.8	4.5	8.8	5.7
NAO	30	3-24	13.8	3.9	13.8	4.5
Posttest	20	0-15	5.9	2.6	6.3	3.1
Retention	10	0-9	4.6	1.5	4.4	2.7

Table 2

Correlation Matrix for All Tests

Test	Correlation 1	2	3	4	5
1. HFT	1.00	.54	.53	.39	.43
2. GEFT		1.00	.43	.50	.41
3. NAO			1.00	.61	.54
4. Posttest				1.00	.72
5. Retention					1.00

Table 3

Regression Equation Data for Each Dependent Variable

Dependent variable	Group	Intercept	Regression coefficients HFT	NAO
Posttest	Discovery	.66	.07	.35
	Expository	.08	.12	.41
Retention	Discovery	3.40	.12	.04
	Expository	-1.34	.08	.38

Table 4

Tests for Interaction

Dependent variable	R^2 for full model	Source	Change in R^2	F	p
Posttest	.391	HFT X Treatment	.003	.20	.657
		NAO X Treatment	.001	.10	.753
Retention	.419	HFT X Treatment	.020	1.25	.270
		NAO X Treatment	.076	5.33	.026

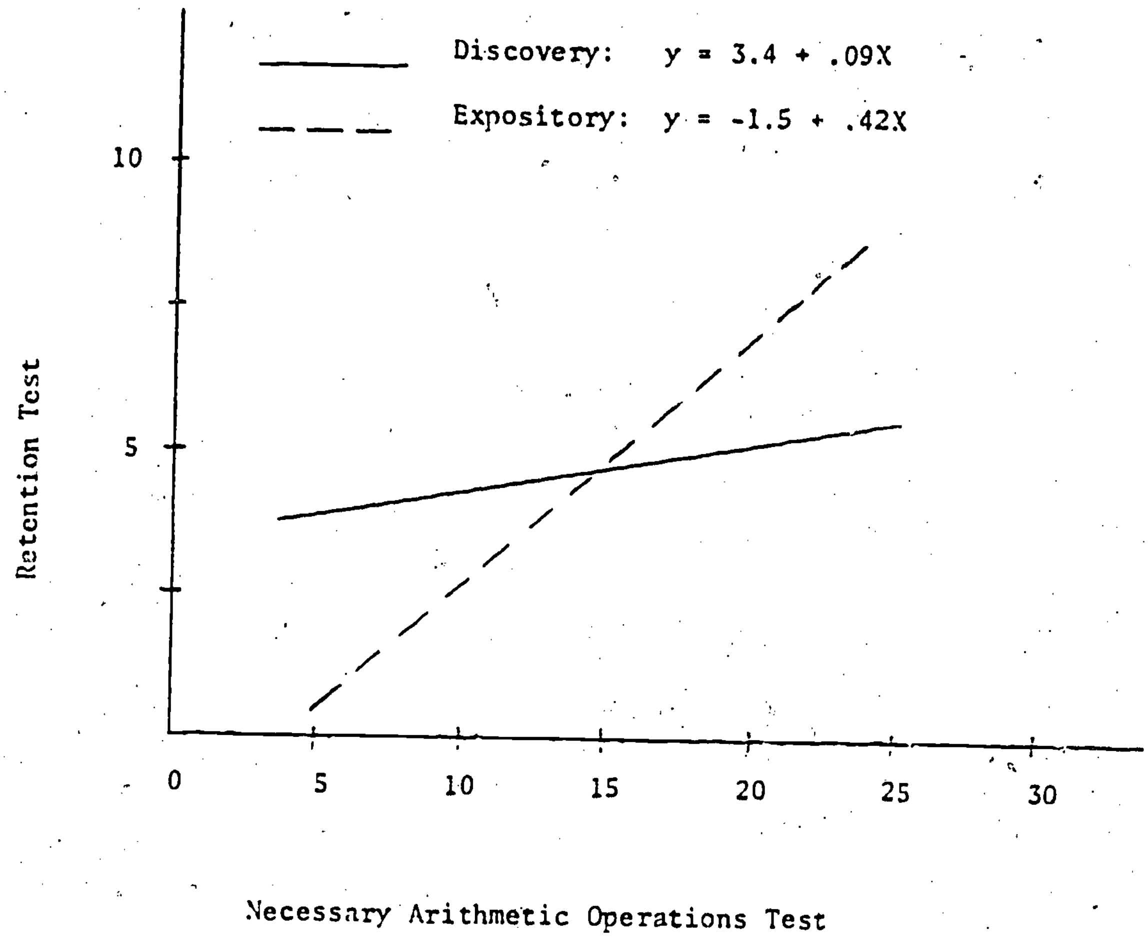

Figure 1. Interaction of NAO test with discovery and expository treatments on the retention test.

APPENDIX B

The Interaction of Field-Dependence-Independence and the Level of Guidance of Mathematics Instruction

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Running head: The Interaction of Field-Dependence-Independence

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Abstract

This study investigated the relationship between the cognitive style variable of field-dependence-independence and instructional treatments using high or low guidance in a unit on networks. The 97 subjects, all prospective elementary teachers, were pretested on cognitive style (using the Group Embedded Figures Test) and on mathematical achievement (a measure of crystallized ability), and randomly assigned to treatments. Following instruction, students were tested for immediate achievement and then retested 5 weeks later. There were no interactions with field-dependence-independence, but there was a significant ($p < .05$) interaction with crystallized ability on the retention test.

The Interaction of Field-Dependence-Independence and the Level of Guidance of Mathematics Instruction

Attempts to individualize instruction have traditionally involved varying the rate of instruction and relatively little attention has been given to adapting the method of instruction to student characteristics. Cronbach (1957) recommended that researchers try to find aptitudes which interact with variations in instructional treatments and to design instructional treatments to fit particular aptitudes of groups of students. The search for ways of adapting instructional treatments to individual differences is known as aptitude-treatment-interaction (ATI) research.

In general, the results accumulated from ATI studies have been less than satisfactory, and few significant interactions have been found. Cronbach and Snow (1977) state that aptitude-treatment interactions do exist and that while no interactions have been confirmed well enough to be used as guides in making decisions about instruction, much has been learned. They feel that what the results do indicate is that it will take more than just a few years of research on a limited scale to produce both solid theory and useful generalizations about aptitudes and instruction. For reviews of ATI research, see Berliner and Cahen (1973), Cronbach (1975), Cronbach and Snow (1977), and Tobias (1976).

In mathematics education, some recent studies successful in finding ATI's have used the cognitive style of field-dependence-independence as an aptitude variable. Individual differences in field-dependence-

independence are identified on a continuum determined by the extent a person perceives analytically. Students who are relatively field-dependent find it difficult to restructure a situation in order to solve a problem or to impose structure on material when structure is lacking. On the other hand, field-independent students are more capable of taking a critical element out of context and restructuring a problem in order to use that element in a different context.

Another aspect of field-dependence-independence which may be important in developing instructional materials is that the effect of cue salience is greater for field-dependent than for field-independent students. Field-dependent students also favor a spectator approach to learning while field-independent students favor a more active approach. Witkin, Moore, Goodenough, and Cox (1977) summarize the educational implications of the field-dependent-independent cognitive styles.

An investigation of the relationship between field-dependence-independence and expository vs. discovery learning was done by McLeod, Carpenter, McCornack, and Skvarcius (1978). Treatments were based on two levels of guidance crossed with two levels of abstraction; the topic was numeration systems. The results support the hypothesis that field-independent students will perform best when allowed to work independently while field-dependent students perform best when given extra guidance.

McLeod and Adams (Note 1) attempted to replicate the above study by investigating the interaction between field-dependence-independence and manipulative materials used in a discovery mode vs symbolic materials used in an expository mode. Again, a significant interaction was found on a

posttest where questions were presented symbolically.

McLeod and Briggs (Note 2) used an inductive vs. deductive approach with field-dependence-independence as one of two aptitude variables. A significant interaction between field-dependence-independence and the sequence of instruction based on inductive and deductive approaches was found on only the transfer test. While field-dependence-independence seemed to interact dependably with level of guidance in two earlier studies, its interaction with sequence of instruction appeared to be less consistent.

For a thorough review of ATI research, including a discussion of field-dependence-independence, see Cronbach and Snow (1977). They indicate that field independence could represent fluid ability and that field dependence may represent a deficit rather than a cognitive style. However, Witkin and Goodenough (Note 3) feel that field-dependence-independence is a dimension of individual differences related to the individual's reliance on internal and external referents and conforms to the concept of style rather than the concept of ability. Witkin and Goodenough suggest that field-dependence-independence is bipolar and that field-independent persons are more adaptive in situations requiring cognitive restructuring skills; field-dependent individuals, on the other hand, are more adaptive in situations which involve social skills.

In the present study using the topic of networks, the interaction between levels of guidance and field-dependence-independence was investigated. Also, a pretest was used as a measure of general mathematical abilities. Two levels of guidance, low and high, were chosen varying the amount of structure, cue salience, and active involvement by the student. Materials were prepared on

the basis of the models suggested by Salomon (1972). The high-guidance treatment (HG) was designed as a compensatory treatment for field-dependent students. The low-guidance treatment (LG) was designed as a preferential treatment for field-independent students. It was expected that the slope for the regression line from the preferential treatment would be much greater than that of the slope from the compensatory treatment. The slope from the compensatory treatment was expected to be nearly level and in that way a significant interaction would be obtained.

Method

Subjects

Students from four sections of Math 210B, the second semester of a course designed for prospective elementary school teachers, participated in the study. Although the majority of students were juniors and seniors, there were a few freshmen, sophmores, and graduate students enrolled. Also, the majority of students in these classes were women. Only 17% were men.

A total of 132 students were originally enrolled in the four sections. There were 16 students that dropped the course before the study was completed and 19 students that were absent on one of the three days used to conduct the study and retention test. Of the 97 students completing the study, 51 were in the low-guidance group and 46 were in the high-guidance group.

Materials

Two treatments, both inductive, were developed on the topic of networks.

Concepts presented included equivalence of networks, traversability, and applications. In the treatment for the high-guidance group, partially completed tables and rules were included in order to compensate for the field-dependent students' inability to provide structure. Double spacing and underlining of key words were used since the field-dependent student seems to need help in identifying relevant cues. The low-guidance treatment did not provide tables nor provide help with discovering the rules. It did include short questions throughout the treatment in order to keep the students actively involved in the treatment. These were omitted from the high-guidance materials where students were given the same information in an expository fashion. Both treatments presented the same content on networks, used the same problems, and provided about the same amount of practice.

The Pretest consisted of 27 multiple choice questions on concepts normally covered in the first semester of the course. In this study, the Pretest was used as another aptitude variable along with the measure of field-dependence-independence. The Group Embedded Figures Test (GEFT) was used to measure field-dependence-independence (Witkin, Oltman, Raskin, & Karp, 1971). The first section of the GEFT is used for practice. The second and third sections each have nine figures and students are allowed five minutes for each part. In the present study, the combined score for the last two sections was used as the GEFT score.

The Posttest and Retention Test contained three subsections intended

to measure comprehension, applications, and analysis.

Procedure

The study was run during regular class periods early in the semester. All classes met twice a week for 75 minutes. One class met on Mondays and Wednesdays and three classes met on Tuesdays and Thursdays. All were afternoon classes.

Students were randomly assigned to two groups within each class. The two instructors for the four sections participated in the study and were randomly assigned to groups with the restriction that each have two low-guidance groups (LG) and two high-guidance groups (HG).

On the first day of class, the game of Sprouts was played. While this was not part of the study, it served as introductory material to the study and later proved to introduce some set breaking problems for the students.

Euler's formula was discussed the second day of class. Again while it was not part of the study, it later proved to introduce set breaking interference with the study. The last 50 minutes of class time on the 2nd day were used to administer the Pretest.

On the third day of class, the classes split into two groups. The LG group remained in the classroom and the HG group went to the math lab. The study was introduced by telling the students that it was a lesson related to what they had been doing but with a different approach. They were asked to work on the materials by themselves and turn in their papers at the end of the period. They were told that their papers would be returned to them the next period.

Papers were returned on the fourth day of class with comments marked on them about the errors. In the HG group, errors were corrected by the grader. In the LG group, students were not told the correct answers; instead they were asked to look for different patterns and to try to use the vocabulary used in the lesson. Students were told that there would be a quiz over the material and were given the opportunity to review and then take the Posttest after 15 minutes if they were finished. The rest of the class started the test after 30 minutes. All were told that the test would not be part of their grade for the course and that they were to try to do their best. Students were allowed 30 minutes to complete the test.

After the study and Posttest were completed, the materials were not discussed in class until after the Retention test was administered five weeks later. On that day, the GEFT was administered first, and then the Retention test and the answers to the Retention test were discussed. Also, the teacher answered student questions about the materials.

Results

Multiple regression techniques (Kerlinger & Pedhazur, 1973) were used for analysis of the data. A separate analysis was completed for the Posttest and each of its subtests as well as the Retention test and each of its subtests. KR-20 reliability coefficients were computed for all tests except the GEFT and item analyses were completed for both the Posttest and Retention test. Tests for differences between the means were computed when appropriate.

Descriptive Statistics

Table 1 presents the means and standard deviations for all tests. On the Pretest, scores ranged from 7% to 85%. On the GEFT, scores ranged from 9% to 100%. The Posttest scores ranged from 22% to 94% while the Retention test scores ranged from 17% to 78%. Table 2 presents the correlation matrix for all tests. Table 3 provides information about the regression equations for each group.

Insert Tables 1 through 3 about here

The KR-20 reliability coefficients for the Pretest, Posttest, and Retention test were .78, .50, and .48 respectively. A reliability estimate of .82 for the GEFT was reported by Witkin et al. (1971).

Tests for Interaction

The interaction vector Pretest X Treatment was checked for significance first for both the Posttest and the Retention test. If it was not significant, it was dropped from the model and the interaction vector GEFT X Treatment was then checked. Table 4 gives information for the tests for interaction showing the squared multiple correlation for the full model, the drop in $\underline{R}^2$, and the value of $\underline{F}$ for the interaction on both the Posttest and Retention test. On the Retention test, the Pretest X Treatment vector was significant; however, the GEFT X Treatment vector was not. Neither of the interaction vectors was significant on the Posttest; therefore, intercepts were checked and were found to be significantly different,

$F(1, 93) = 5.75$, $p < .05$.

Insert Table 4 about here

Since there was a significant interaction on the Retention test, the Johnson-Neyman technique was applied to find the region of significance (Borich, Godbout, & Wunderlich, 1976). Using the Pretest as the only predictor, the left region of significance was not definable within the range of data. The right region of significance was bounded by 15.8 and 23. There were 40 students (41%) who had pretest scores which were in this region of significance.

Interactions were checked for each of the subtests for both the Posttest and the Retention test. None were found to be significant for the Posttest; however, there was a significant interaction between the Pretest and treatments on the applications subtest for the Retention test. In this case, R^2 for the full model was .184; the change in R^2 was .054, $F(1, 91) = 5.99$, $p < .05$.

Discussion

Cronbach and Snow (1977) report that general ability is the most reliable source of ATI. According to their hypothesis, students with low general ability should do well in treatments that provide extra support and guidance. In such a treatment, the regression of achievement measures onto general ability is expected to be relatively flat. In a discovery-oriented treatment

that provides minimal guidance, steeper regression slopes are expected. If the pretest of this study is taken as a measure of general ability in mathematics, the results contradict Cronbach and Snow's hypothesis.

In the present study, students with high pretest scores tended to do better in the HG group while the pretest did not seem to be related to performance in the LG group. This interaction reached significance only on the Retention Test, but the direction was the same on the Posttest. The interaction can be interpreted through the construct of crystallized intelligence (GC), a dimension of general ability (Cattell, 1971). School achievement scores correlate well with GC and pretests like the one used in the present study are frequently used to measure this dimension.

Since GC measures aptitude for learning in a school setting, it seems reasonable that it should be a good predictor of achievement in the HG group, where instruction was more similar to traditional expository mathematics teaching. In the LG group, with its use of a non-traditional discovery approach, there was no relationship between GC and achievement.

Based on the data from this study, it seems appropriate to hypothesize that students high in GC will do best in traditional expository settings, while students low in GC may do just as well in discovery as in expository instruction. In his recent papers, Snow (Note 4) has accumulated substantial evidence in support of this hypothesis.

The original hypothesis that this study was designed to test involved the interaction of field-dependence-independence and the level of guidance. While the interaction due to GEFT scores that was expected did not occur, the slope for the LG group on the Posttest was steeper than that for the

HG group, as the theory predicts. This difference in slopes did not recur on the Retention Test, however.

Several things may have affected the results. The GEFT scores were skewed to the left. This would not be expected in the population of students that make up these classes (prospective elementary school teachers). Since the GEFT did not discriminate well for the high scores, this could easily have affected the slope of the regression line.

A second factor was observed when examining treatment materials. Previous work with the Sprouts game, and in particular the exercise about Euler's formula that was discussed with it, introduced a set breaking factor which could have affected the slope of the regression line for the LG group. In the LG group, for example, only four people showed evidence of counting the number of even and odd vertices as was necessary in order to discover the rules on traversability. Fourteen people counted vertices, edges and/or faces as was done in the work with Euler's formula. This set breaking factor appears to have affected mainly middle-and high-aptitude scorers and would appear to contradict theory about field-dependence-independence. It can be speculated, however, that since the work with Euler's formula was in a highly discovery-oriented situation, that only high-aptitude scorers discovered the rule. Thus it created a set breaking problem only for them. This extra difficulty for high-aptitude students could also have contributed to the direction of the interaction with the pretest.

Another problem with the study was the relatively low reliability of the Posttest and the Retention Test. The lack of reliability makes it more difficult to get a significant interaction; however, it does not affect

the Interpretation of ATI (Cronbach & Snow, 1977, p. 34).

Since there were no significant interactions on the Posttest, it was appropriate to test for differences between the treatments. Using the two aptitude variables as covariates, there was a significant difference between the two treatment groups with the more expository HG group getting higher scores on the Posttest. As is typical in studies of discovery learning, the initial advantage for more expository treatments on an immediate posttest had disappeared by the time of the retention test five weeks later.

In summary, this study found a significant interaction between Pretest and achievement scores that may be interpreted as an interaction between crystallized intelligence and treatments that differ in terms of traditional expository vs. non-traditional discovery techniques. This interaction fits nicely into the pattern of ATI with crystallized ability that Snow (Note 4) has recently identified. The expected interaction between field-dependence-independence and achievement failed to occur. However, detailed analysis of the data revealed a number of trends consistent with the theory of field-dependence-independence. Further investigation of both aptitudes in future ATI studies seems appropriate.

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Table I

Means and Standard Deviations for All Tests

Test	Maximum Possible Score	Mean	Standard Deviation
Pretest	27		
Low Guidance		14.16	4.12
High Guidance		14.13	5.06
Both Groups		14.14	4.56
GEFT	18		
Low Guidance		12.88	4.96
High Guidance		11.57	4.51
Both Groups		12.26	4.77
Posttest	18		
Low Guidance		9.14	2.13
High Guidance		10.98	2.45
Both Groups		10.01	2.46
Retention	18		
Low Guidance		9.27	2.23
High Guidance		9.93	2.63
Both Groups		9.59	2.44

Table 2

Correlation Matrix for All Tests

Test	Correlation 1	2	3	4
1. Pretest	1.00	.30	.30	.30
2. GEFT		1.00	.38	.36
3. Posttest			1.00	.50
4. Retention				1.00

Table 3

Regression of Posttest and Retention Test

on GEFT and Pretest for Each Treatment

Test	Intercept	Regression Coefficients GEFT	Pretest
Posttest			
Low Guidance	5.60	.24	.03
High Guidance	7.14	.15	.15
Retention			
Low Guidance	7.37	.17	-.02
High Guidance	4.98	.18	.20

Table 4

Tests for Interactions

Dependent variable	R^2 for full model	Source	Change in R^2	F
Posttest	.374			
		Pretest X Treatment	0.010	1.454
		GEFT X Treatment	0.003	0.434
Retention	.245			
		Pretest X Treatment	0.038	4.580*
		GEFT X Treatment	0.000	0.000

*$p < .05$

APPENDIX C

Individual Differences in Cognitive Style and Discovery Approaches to Learning Mathematics

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Running head: Individual Differences in Cognitive Style

ABSTRACT

In order to test the hypothesis that field independence would interact with level of guidance, students in five mathematics classes were randomly assigned to either a low guidance or a high guidance treatment group for a week of instruction. Both treatments used an inductive sequence of instruction on the topic of networks. Students were assessed on two aptitudes, field independence and general reasoning. Achievement was significantly better ($p < .05$) in the high guidance group than the low guidance group on both the Posttest and the Retention Test. No significant interactions with the aptitude variables were found.

Individual Differences in Cognitive Style and Discovery Approaches to Learning Mathematics

Attempts to adapt instructional treatments to student characteristics as suggested by Cronbach (3) are known as Aptitude-Treatment-Interaction (ATI) studies. No single instructional treatment is likely to maximize achievement for all students; thus, instead of looking for one treatment, ATI studies attempt to match different instructional strategies with different student characteristics. This matching is difficult to accomplish, but Cronbach and Snow in their complete review of ATI research (4) indicate that interactions do exist. Furthermore, these interactions have important implications for individualizing instruction.

One variable used in several ATI studies in mathematics which have found significant interactions is the cognitive-style variable of field independence (6,7). Considered to be a rather stable trait related to the performance of cognitive tasks and to personality characteristics, field independence has received a lot of attention in educational research. When restructuring or reorganizing of the content is required for success at a task, field-independent students are expected to do better than field-dependent students. They are also expected to work more autonomously. Field-dependent students, on the other hand, are more adaptive than field-independent students in social situations which require interpersonal skills (12,13).

It is hypothesized that differences between field-independent and field-dependent students are related to the level of guidance of instruction, an aspect of discovery-oriented instruction. Two studies (6,7) support this

hypothesis; field-independent students excelled in treatments that provided minimal guidance and maximal opportunity for discovery while field-dependent students performed best in expository treatments which provided a great deal of structure.

The purpose of the present study was to investigate further the effect of the level of guidance on the learning of field-dependent students in inductive instruction. Two treatments were developed varying the level of guidance. It was expected that relatively field-dependent students would perform best in the high guidance treatment which provided guidance in the form of partially completed tables and rules, underlined definitions, and extra details in examples. Relatively field-independent students were expected to excel in the low guidance treatment. Thus, when achievement was regressed on field independence, the slope of the regression line for the low guidance treatment was expected to be greater than that of the high guidance treatment.

Method

Subjects

Students from five sections of a mathematics class for prospective elementary school teachers were randomly assigned to the two treatment groups within each section. About 18% of the students were men.

Of the 99 students participating in the study, 38 were absent on one of the days of the treatment or on days used for aptitude testing or retention testing. The treatments did not appear to influence the rate of absenteeism in any way. Of the 61 students for which complete data were obtained, 36 were in the low guidance group and 25 were in the high guidance group.

Treatments

Materials from an earlier study on networks (1) were expanded to include the concepts and applications of equivalent networks, traversability of networks, and separating edges; Euler's formula; and other related topics. Both treatments used an inductive sequence of instruction, presenting examples and then having students generalize rules about the concepts. The treatments covered the same content, used the same examples, and included about the same amount of practice. In the high guidance materials, partially completed tables and rules were included, definitions were underlined, and greater detail in the examples was used. No underlining was used in the low guidance materials and students were expected to make their own tables and discover their own rules.

Tests

A version of the Hidden Figures Test (HFT) was chosen as the measure of field-dependence-independence for this study. This test was adapted from the original publication of the Educational Testing Service (5) for use in the National Longitudinal Study of Mathematical Abilities (NLSMA). The appropriate NLSMA reports (8,11) provide greater detail about this test. In this study, students were allowed 15 minutes for the test rather than 10 minutes as used in the NLSMA studies since NLSMA data indicated that the test statistics were influenced by a speed factor.

The total score (verbal plus quantitative) of students' SAT scores was used as a measure of general ability when it was available (N = 34). Since many students had not taken the SAT, the Necessary Arithmetic Operations (NAO) test was administered and used as a second measure of general ability.

Although the NAO test is considered a measure of general reasoning (5), it correlates well with general ability in mathematics (9).

The Posttest and Retention Test each consisted of 20 multiple choice questions. Students were allowed 15 minutes for each of these tests. The KR-20 reliability coefficients for the Posttest, Retention Test, NAO, and HFT were .66, .74, .76, and .83, respectively.

Procedure

Students and the two instructors were randomly assigned to treatment groups for each of the five sections participating in the study. One instructor had three low guidance and two high guidance groups; the other instructor was assigned the alternate groups. Two of the classes were morning classes and two were afternoon classes; they met twice a week for 75-minute periods. The other class met in the afternoon for 50 minutes three times a week.

The treatments were completed by the students during the last half of the semester and were included in the class as part of the regular course work. Students were told that they would be working on a unit not included in their text and that the class would be split into two groups for the week of instruction. The instructor and students assigned to the high guidance treatment went to the mathematics laboratory to work on the materials while the low guidance group remained in the regular classroom. Students were encouraged to work independently and to direct their questions to the instructor. They were told that they were to turn in their papers at the end of the period and that their answers would be checked. Papers were returned at the next class meeting with comments marked on them about errors and with suggestions on how to look for the correct answer.

At the end of the week of instruction, students were given a 15-minute multiple choice posttest. The majority of the students did not complete all of the material in the treatments and students were not given time to review prior to taking the Posttest.

The Retention Test, which was identical to the Posttest, was administered four weeks after students completed the treatments. Students took the HFT immediately after the Retention Test, and the NAO was administered at the end of the semester.

Results

Descriptive statistics are presented in Tables 1 and 2. Table 1 includes the means and standard deviations for all tests. Table 2 presents the correlation matrix for all tests.

Insert Tables 1 and 2 about here

As suggested by Cronbach and Snow (4), scatterplots of each aptitude variable with both the Posttest and the Retention Test were examined to determine whether or not the linear model was appropriate. It was concluded that the linear model could be used. Analyses of the data treating the Posttest and the Retention Test as separate dependent variables were completed using multiple regression techniques. The full model included vectors for field independence, general ability (NAO scores or SAT scores), treatment, and the interaction of each aptitude variable with the treatment. When NAO scores were used, complete data for the model were available on 61 subjects. When SAT scores were used, this number was reduced to 34.

In all analyses for the full model, the general ability vector was entered first, then the field-independence vector, followed by the treatment vector. Each interaction vector was then tested to see if it made a significant contribution when added to the model. If there were no significant interactions, treatment effects were checked by dropping the treatment vector from the model.

Tests for Interaction and Treatment Effects

Tables 3 and 4 report the regression equations for each group and the tests for interactions using HFT and NAO as predictors. Since there were no significant interactions, treatment effects were checked and found to be significant for both the Posttest, $F(1,57) = 14.460$, $p < .001$, and the Retention Test, $F(1,57) = 12.891$, $p < .001$.

Insert Tables 3 and 4 about here

Further analyses of the data were completed using HFT as the only predictor and using SAT scores as the measure of general ability with HFT instead of NAO. In all cases, the interactions were not significant, but the treatment effect was. Class effects were also checked as suggested by Cronbach (2). No consistent pattern of differences in regression slopes occurred.

Discussion

This study tested the hypothesis that the cognitive style of field independence would interact with treatments that differed in level of guidance. The expected interactions did not occur. Although the slopes for HFT were slightly greater for the low guidance group than for the high guidance group

as predicted, they were not significantly different and were in the same direction as those for general ability as measured by the NAO. The slopes for NAO were greatest in the low guidance treatment supporting the interpretation of NAO scores as a measure of general ability.

Several authors have discussed the difficulty in distinguishing between measures of field independence and general ability (4,12). This difficulty may be particularly important when dealing with geometric topics such as those used in this study. In ATI studies where the geometric treatment and the aptitude measure for field independence depended on the same ability for completion, slopes tended to be relatively steep in both treatment groups (1,10) just as if the aptitude were general ability. In contrast, when treatments using an arithmetic topic were used, there were substantial differences between slopes when achievement was regressed on field independence (6,7). In particular, the slope of the regression line in the high guidance treatment tended to be close to zero, or even negative in some cases. Therefore, it appears that tests like the HFT behave like general ability when the treatments cover geometric content and like field independence when the treatments present arithmetic concepts.

In the present study, the possibility of finding an ATI was made more unlikely by the presence of a substantial treatment effect. The achievement scores in the high guidance group were consistently greater than in the low guidance treatment. The topic appeared to be too difficult for these particular students to master without a substantial amount of guidance. Also, the students involved in this study tended to be relatively field dependent, as is generally the case for prospective elementary school teachers (13).

As a result, most of the students had a cognitive style that was not well suited to the low guidance treatment. If the same treatments were used with students that were, on the average, more field independent, then an ATI would be more likely to occur.

In summary, the cognitive style of field independence did not interact with the level of guidance of instruction in this study, even though such an interaction has occurred in other studies using different content. Further research on interactions with cognitive style seems warranted.

NOTE

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Table 1-Means and Standard Deviations of All Tests for Each Treatment Group

Test	Maximum Possible Score	Mean	Standard Deviation	Number of Cases
HFT	16			
Low Guidance		5.0	3.4	39
High Guidance		5.9	3.7	32
NAO	30			
Low Guidance		14.1	3.8	36
High Guidance		13.9	3.6	25
Posttest	20			
Low Guidance		7.4	2.5	39
High Guidance		10.2	3.2	32
Retention	.0			
Low Guidance		5.8	2.7	39
High Guidance		8.4	3.1	32

Table 2-Correlation Matrix for All Tests

Test	Correlation 1	2	3	4
1. HFT	1.00	.23	.28	.20
2. NAO		1.00	.34	.44
3. Posttest			1.00	.81
4. Retention				1.00

Table 3-Regression of Posttest and Retention Test on HFT and NAO for Each Treatment

		Regression Coefficients	
Test	Intercept	HFT	NAO
Posttest			
Low Guidance	1.59	.15	.36
High Guidance	7.85	.13	.10
Retention			
Low Guidance	-.43	.10	.40
High Guidance	3.62	.02	.32

Table 4-Tests for Interaction

Dependent Variable	$\underline{R}^2$ for Full Model	Source	Change in $\underline{R}^2$	$\underline{F}$	$\underline{p}$
Posttest	.352	HFT	.002	.128	.722
		NAO	.023	1.988	.164
Retention	.357	HFT	.003	.262	.611
		NAO	.003	.274	.603

APPENDIX D

The Interaction of Field Independence with Small Group Instruction in Mathematics

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Running head: The Interac on of Field Independence

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Presented at the Annual Meeting of the National Council of Teachers of Mathematics, Boston, April 1979.

Abstract

This study tested the hypothesis that the cognitive style of field independence would interact with treatments that differed in the use of small groups as opposed to individual instruction. Students (N = 111) were assessed on field independence and general ability and randomly assigned to treatments for a week of instruction. Achievement was measured by an immediate posttest and a delayed retention test, and student ratings of instruction were obtained. There was a significant ($p < .05$) interaction with measures of field independence when achievement was the dependent variable, but not when student ratings were used. However, the interaction appeared to be due more to general ability than to cognitive style. Also, students gave significantly higher ratings to small-group instruction.

The Interaction of Field Independence with Small Group Instruction in Mathematics

Research on mathematics teaching indicates that no one instructional treatment is likely to maximize learning for all students. Instead of looking for a single treatment that will be best for all learners, Cronbach (3) has suggested that different instructional strategies should be used for students with different characteristics. Attempts to adapt instructional treatments to student characteristics are known as Aptitude-Treatment-Interaction (ATI) studies.

A complete review of ATI research has recently been completed by Cronbach and Snow (4). Although there have been many difficulties in conducting ATI studies, Cronbach and Snow conclude that interactions do exist and that ATI research has important implications for individualizing instruction.

Recently several ATI studies in mathematics have found interactions using the cognitive-style variable of field independence (8,9). Field independence is a rather stable trait that is related to both the performance of cognitive tasks and to personality characteristics. Field-independent students tend to do well at tasks that require restructuring or reorganization of the content, especially if the content deals with mathematics or science. They also seem to have more personal autonomy than field-dependent students. Field-dependent students, on the other hand, are more adept in social situations where they seem to exhibit greater interpersonal skills. However, they

have more difficulty with tasks that require restructuring ability (15,16).

These differences between field-independent and field-dependent students seem to be related to performance in discovery-oriented instruction. For example, in two studies (8,9) there was a significant interaction between field independence and the level of guidance of instruction. Field-independent students excelled in the treatment that provided minimal guidance and maximal opportunity for discovery. Field-dependent students, however, learned more in an expository treatment where lots of structure was provided.

In addition to its interaction with level of guidance, field independence seems to be related to the use of small-group instruction. Cognitive-style theory suggests that field-dependent students should learn more in a small-group setting, where their greater social skills would be an advantage. Field-independent students, on the other hand, would be expected to excel in individual work, while learning in a small-group setting might hold them back. Some support for this hypothesis comes from studies (4,6,7) where more student discussion seemed to help field-dependent students. In those studies the treatments used an inductive sequence of instruction, or guided-discovery methods, and encouraged student discussion of the problems.

The purpose of this study was to investigate further the effect of social interaction on the learning of field-dependent students in inductive instruction. Treatments differed in their use of small-group as opposed to independent work. The hypothesis to be tested was that these treatments would interact with measures of field independence. Field-dependent students were expected to do better in small-group instruction, while field-independent students might be held back by small-group work. Also,

field-dependent students were expected to have a more positive attitude toward small-group instruction, while individual instruction was thought to be preferred by field-independent students.

Method

Subjects

The participants in the study came from five sections of a mathematics class for prospective elementary school teachers. As is typical of these classes, the majority of the students were women; only 13% were men. Originally, 111 students completed the treatments and took the posttest. However, some of the students were absent on the day of retention testing or missed one of the days when the aptitude tests were administered. As a result, most analyses were done on 92 subjects, 49 in small groups and 43 in individual instruction. Treatment differences did not seem to be related to student absenteeism.

Materials

Both treatments used the same written material; they differed by having students work individually or in small groups. The unit presented a variety of concepts related to the study of networks, including equivalence of networks, traversability and its applications, Euler's formula, and related topics. The unit encouraged students to make discoveries through its use of an inductive sequence of instruction, presenting first a number of examples from which students could then generalize to obtain a rule.

Students were assessed on field independence using the Hidden Figures Test (HFT) and the Group Embedded Figures Test (GEFT). The version of the HFT that was used (Hidden Figures 2-Form 271) was adapted by the National

Longitudinal Study of Mathematical Abilities (NLSMA) from the original published by the Educational Testing Service (5). For a complete discussion of this version of the HFT, see the appropriate NLSMA reports (10,14). Students were allowed 15 minutes to complete the test.

The GEFT (17) is recommended by Witkin and his colleagues as the best measure of field independence. In the present study, the combined score for the last two sections of the GEFT was used as the student's score. Students were allowed four minutes for each section. The use of two measures of field independence is in line with recommendations by Cronbach and Snow (4) for multiple assessment of aptitude variables.

Two measures of general ability were included in the study. SAT scores were obtained from the university records (with student approval) and the total score (verbal plus quantitative) was used as a measure of general ability. Since many students had never taken the SAT, a second test, Necessary Arithmetic Operations (NAO), was administered to all students. The NAO test is believed to measure the aptitude of general reasoning (5), but it also correlates well with general ability in mathematics (11).

A 19-item posttest was used to measure student achievement; 15 minutes were allowed for this test. The same test was readministered later to assess retention. The KR-20 reliability coefficients were .68 for measures of student achievement and around .80 for the aptitude tests.

Procedures

During the first week of the semester students took the HFT and NAO tests. The study was conducted six weeks later, and retention testing occurred eight weeks after instruction. The GEFT was administered at the

timo of retention testing.

Within each section, subjects were randomly assigned to either small-group or independent instruction. The only difference between the two treatment groups was that in one, the students were assigned to work in [illegible] of four, and in the other students were asked to work independe[illegible] The small-group treatment was conducted in the mathematics laboratory, while the students who worked independently stayed in the regular classroom. Four of the sections met twice a week for 75-minute periods and one section met three times a week for 50-minute periods.

After students were randomly assigned to treatment groups, students were assigned to small groups according to their scores on a measure of field independence, the HFT. Each small group contained one student from the highest quartile, one from the lowest quartile, and two from the middle 50% of the scores. This kind of heterogeneous grouping tends to promote higher overall achievement (13).

The two instructors were randomly assigned to treatments for each section. They told the students that they were to try to discover rules concerning networks by working through the written materials. Students working individually were told to work by themselves and if they needed help to ask the instructor. Students working in small groups were encouraged to work together page-by-page and help each other clarify points as they came up. If the group could not figure something out, then they were to ask the instructor for help. The instructors answered questions by encouraging students to look for patterns and to make discoveries.

A week of class time was used by the students to complete the treatments

and to take the posttest. Most students were not able to complete the material in the time allowed. Ten minutes prior to the 15-minute posttest, students were told they would be taking a quiz and that they could take a few minutes to look through the materials to review what they had covered. After the quiz, students were asked to rate the instructional unit on a scale of one to five, and to write any comments about the unit that they wished.

There were no major difficulties in carrying out the treatments as planned. However, in the small-group treatment, slower students sometimes had trouble keeping up with the pace set by their group. In individual instruction, the students were able to set their own pace. But students in the individual treatment asked for much more help from the teacher, while the small groups seemed to resolve all of their questions themselves. An unexpected event that occurred during the treatments was a city-wide blackout that left the last class of the day in complete darkness for a few minutes until the teacher could find a classroom with natural lighting. Although this blackout was a surprise to both the students and the teachers, there appeared to be no differences in achievement in that class.

Results

Table 1 presents descriptive statistics for each aptitude and achievement measure in each treatment group, small-group instruction and individual instruction. The correlation matrix is found in Table 2.

Insert Tables 1 and 2 about here

The data were analyzed using multiple regression techniques (4), and the posttest and retention test were treated as separate dependent variables. For the main analyses, the full model included vectors for field independence, general ability, treatment, and the interaction of each of the aptitude variables with the treatment. The aptitude vectors were entered in the regression first, followed by the treatment. Then each interaction vector was tested to see if it made a significant contribution when added to the model. If there was no significant interaction, then the treatment vector was dropped from the model to see if there were significant treatment effects.

Data analyses were run with HFT or GEFT as the measure of field independence, and with NAO or SAT used as an indicator of general ability. When NAO was used, complete data for the model were available on 92 subjects. When SAT was used, this number was reduced to 53.

Tests for Interaction

Table 3 presents the regression equation for each treatment group. The tests for interaction, using HFT and NAO as predictors, are presented in Table 4. The initial regression analysis used HFT scores, NAO scores, and the treatment vector to predict the posttest scores. When the HFT interaction vector was added to this model, R^2 increased by .047; this increase was significant, $F(1, 87) = 6.36$, $p = .013$, as indicated in Table 4. The NAO interaction vector did not cause a significant increase in R^2 by itself (see Table 4), but when HFT and NAO vectors were both added to the model, their joint effect was again significant, $F(2, 86) = 3.58$, $p = .032$. None of the interactions was significant on the retention test, although the regression coefficients were still larger for the small-group treatment.

Insert Tables 3 and 4 about here

When the data were analyzed again using the GEFT as the measure of field independence, none of the interactions was significant. The regression coefficients still were larger in the small-group treatment, but the differences were less pronounced.

Further analyses of the data were completed using SAT scores as the measure of general ability. As Table 5 indicates, the same pattern continued with the regression coefficients, which were always larger in the small-group treatment. However, this time there was no significant interaction on either the posttest or the retention test (see Table 6). For example, when the HFT interaction vector was added to the model after the HFT scores, SAT scores, and treatment vector, the increase in R^2 was .030, $F(1, 48) = 1.96$, $p = .168$.

Insert Tables 5 and 6 about here

Since there were only 53 subjects in the analysis presented in Table 6, it seemed likely that the drop in the number of subjects could have caused the disappearance of the significant interaction with HFT on the posttest. Therefore, a number of regressions were run with a single aptitude variable and with just the 53 subjects on which SAT data were available. These regressions included only one aptitude, the treatment, and the corresponding

interaction vector. In this situation, the interaction with HFT on the post-test was still significant, $F(1, 49) = 4.33$, $p = .043$. Moreover, on the retention test, the interaction vector also contributed substantially: For GEFT, $F(1,49) = 4.11$, $p = .048$; for SAT, $F(1,49) = 3.77$, $p = .058$; and for HFT, $F(1,49) = 2.90$, $p = .095$. However, when the two aptitudes (field independence and general ability) were combined in the same regression equation, none of the interactions was significant.

Regions of Significance

Since there was a significant interaction with HFT on the posttest, the Johnson-Neyman technique (1) was used to calculate the regions of significant differences between the treatments. For this analysis, the HFT scores were used as the only predictor in each treatment group. With 92 subjects, the slope was .55 for small-group instruction and .15 in the individual treatment. This difference in slopes was significant, $F(1,88) = 6.87$, $p = .010$. Using a level of significance of .10, small-group instruction was significantly better for students with HFT scores of 13 or more, and individual work was better for students with HFT scores of 5 or less. Approximately 50% of the students had HFT scores in these two regions, almost all of them in the lower region.

Student Ratings

Students were asked to rate the unit on networks, and further analyses were conducted using these ratings as the dependent variable. There were no interactions with either field independence or general ability in these analyses. However, there was a consistent treatment effect, with most students giving higher ratings to small-group instruction. For example, when HFT and SAT were used as aptitudes (covariates), the treatment vector was significant, $F(1,45) = 5.547$, $p = .023$. In other analyses of student ratings, the F ratio for the treatment effect was even larger.

Other Analyses

Following Cronbach and Snow's recommendations (4), scatterplots of each aptitude variable with both the posttest and the retention test were constructed. Inspection of these scatterplots indicated that the linear

model was appropriate.

Cronbach (2) has pointed out that teacher and class effects can be crucial in ATI research. In this study, however, there was no evidence of any differences due to the two teachers who conducted the treatments. Similarly, other multiple regression analyses revealed no significant differences due to the effects of the student's classroom.

Discussion

This study tested the hypothesis that the cognitive style of field independence would interact with treatments that differed in the use of small groups as opposed to individual instruction. This interaction was expected to occur in both the assessment of student achievement and in the student ratings of the instructional unit.

When achievement was the dependent variable, there were significant interactions with field independence that appeared in the data. However, these interactions were in the opposite direction from the predictions of cognitive style theory. The interactions with field independence were in the same direction as the interaction with general ability, as measured by SAT scores. And when SAT scores were included in the regression along with measurements of field independence, all interactions disappeared. As a result, it seems reasonable to conclude that the interactions that occurred were unstable and probably not due to field independence. Instead, they should be attributed to general ability and viewed with some caution.

Close inspection of the data provides support for attributing the interactions to general ability. Students in the small-group treatment who scored well tended to have taken more mathematics courses than other

students. This trend was less evident in the individual treatment, where low-ability students got more help from the teacher.

Although there were very different patterns of social interaction in the two treatments, that difference did not appear to be related to the significant ATI that were identified. Instead, the relevant differences in the treatments seemed to be the level of guidance obtained from the teacher. In small-group instruction, the students expected and received all of their guidance from the printed materials and from fellow students. Low-ability students sometimes did not get the help that they needed in the small groups since the majority of the group was frequently more interested in pushing ahead quickly than in helping other members understand the concepts.

While some low-ability students appeared to receive insufficient help from the small groups, students who worked individually requested and received help from the teacher. These differences in the nature of the support provided to students appeared to cause the ATI with general ability and with measures of field independence. The direction of the interactions was the same as for most ATI with general ability (4).

Although small-group instruction did not help students learn any more overall, they clearly gave it higher ratings than individual work. Even here, though, the results are tempered by observation of the groups themselves. Most students certainly did give higher ratings to small-group instruction, but there were a number of students who gave it very low ratings. Usually these students worked more slowly than the rest of the group and got too far behind to profit from the group discussion.

Occasionally students had difficulty keeping up because they spoke English as a second language. These types of students seemed more comfortable working individually.

The difficulty of distinguishing between field independence and general ability has been noted by a number of authors (e.g., 4), and Witkin and his colleagues are also concerned about this problem (15). They hope to develop new instruments that will make it easier to separate field independence from general ability, as well as from fluid and general visual ability (11). The difficulties involved in interpreting these aptitudes seem to be particularly crucial in instruction on geometric topics. When the aptitude measures (such as HFT and GEFT) depend on the same ability that is needed for the geometric treatments, the slopes tend to be relatively steep in both treatment groups (12). Therefore, it is difficult to produce the kind of ATI effect that was predicted.

The use of small groups represents an important instructional variable, but conducting research on this variable is difficult. One of the difficulties is statistical; as Cronbach (2) points out, it is very difficult to separate the effects of the treatment from the effects of the different small groups themselves. Other difficulties include controlling student and group characteristics, as well as the nature of the social interaction in the groups (13). Even though it is difficult, further research is needed to clarify the effects of small-group instruction.

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Table 1

Means, Standard Deviations and Number of Cases for All Tests

Test	Maximum possible score	Mean	Standard deviation	Number of cases
HFT	16			
Small groups		5.5	3.4	57
Individual		5.9	3.6	55
GBFT	18			
Small groups		10.6	5.2	50
Individual		10.6	3.9	44
NAO	30			
Small groups		14.6	4.4	57
Individual		14.5	3.9	55
SAT	1600			
Small groups		891	181	33
Individual		868	116	34
Posttest	19			
Small groups		8.8	3.1	57
Individual		9.0	2.8	54
Retention	19			
Small groups		6.3	3.3	49
Individual		6.3	3.0	44
Evaluation	5			
Small groups		3.8	1.2	52
Individual		3.0	1.2	48

Table 2

Correlation Matrix for All Tests

Test	Correlation 1	2	3	4	5	6
1. HFT	1.00	.52	.40	.36	.43	.51
2. GEFT		1.00	.42	.59	.45	.40
3. NAO			1.00	.56	.50	.34
4. SAT				1.00	.43	.37
5. Posttest					1.00	.71
6. Retention						1.00

Table 3

Regression of Posttest and Retention Test on Two Predictors

for Each Treatment

			Regression coefficients	
Test	Treatment	Intercept	HFT	NAO
Posttest	Small groups	2.06	.38	.29
	Individual	6.55	.07	.16
Retention	Small groups	1.52	.45	.15
	Individual	4.12	.36	.01

Table 4

Tests for Interaction with HFT and NAO

Dependent variable	R^2 for full model	Source of interaction	F^a	p
Posttest	.366	HFT	6.36	.013
		NAO	3.48	.065
Retention	.293	HFT	.99	.323
		NAO	1.50	.224

[a]Degrees of freedom are (1,87) in each case.

Table 5

Regression of Posttest and Retention Test on Field Independence and General Ability

			Regression coefficients	
Test	Treatment	Intercept	HFT	SAT
Posttest	Small groups	2.10	.29	.0060
	Individual	6.70	.03	.0030
Retention	Small groups	-.01	.38	.0055
	Individual	6.12	.18	-.0009

Table 6

Tests for Interaction with HFT and SAT

Dependent variable	R^2 for full model	Source of interaction	F^a	p
Posttest	.266	HFT	1.96	.168
		SAT	.98	.326
Retention	.315	HFT	1.54	.221
		SAT	2.51	.120

[a] Degrees of freedom are (1,48) in each case.

APPENDIX E

Individual Differences in Mathematics Learning through Personalized Systems of Instruction

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Running head: Individual Differences in Mathematics Learning

ABSTRACT

Individual differences in general ability, locus of control, and field independence were investigated among 121 college students enrolled in either Personalized Systems of Instruction (PSI) or lecture sections of a course in intermediate algebra. At the end of one semester, there were no significant aptitude-treatment interactions; however, the PSI approach seemed particularly effective for students who were low in general ability and who had an internal locus of control. Final exam scores were significantly higher ($p < .05$) in the PSI group when field independence and locus of control were used as covariates, but not when general ability was entered in the regression.

Individual Differences in Mathematics Learning through Personalized Systems of Instruction

The importance of individual differences in learning is frequently overlooked in the evaluation of a new instructional strategy. Instead, researchers tend to emphasize the overall effect of the strategy compared to other instructional techniques. This has been true of research on the effectiveness of Personalized Systems of Instruction (PSI).

PSI is characterized by self-pacing, frequent testing, immediate feedback, a unit-mastery requirement, and the availability of tutors. Recent reviews (5, 10) suggest that PSI is generally superior to lecture-discussion approaches, although differences in the rate of withdrawal in the two types of instruction raise questions about the superiority of PSI classes for at least some students.

In the search for ways to optimize achievement, it seems reasonable to try to identify students who are most likely to succeed in PSI courses and to separate them from students who may do better in a lecture-discussion approach. The attempt to identify characteristics of these different types of students is part of Aptitude-Treatment-Interaction (ATI) research. ATI research, generally viewed as an outgrowth of the work of Cronbach (2), has turned out to be quite difficult. Many studies have not found the predicted interactions. Nevertheless, in a recent review of the field, Cronbach and Snow (3) report a number of interactions that show promise and confirm the importance of ATI research.

A number of ATI studies have been conducted using PSI as one of the treatment dimensions. Some of these (7, 8, 9) have found support for an interaction with general ability as the aptitude, but the evidence is not yet conclusive (4). The nature of the hypothesized interaction with PSI is similar to other interactions with general ability (3); low-ability students seem to profit more from the extra support and individual help that they can obtain in a PSI setting, and high-ability students do equally well in either type of instruction.

Although the relationship of general ability to PSI is important, there are other characteristics of students that also seem relevant to student performance in PSI courses. For example, several studies (10) have found interesting relationships with Rotter's locus-of-control variable (6, 12). According to Rotter's theory, individuals who perceive reinforcement as the result of their own behavior are said to have an internal locus of control; those who believe that their success or failure is due to luck, fate, or other people are said to have an external locus of control.

The relationship of locus of control to PSI instruction in mathematics has not been established. However, it seems reasonable to hypothesize that students who are more internal should be matched with PSI instruction, where they are expected to take more of the responsibility for their own learning. Students with a more external orientation should find that the traditional lecture-discussion approach is more appropriate for them.

Another variable that may be related to PSI is the cognitive style of field independence (16, 17). In the latest revision of cognitive-style theory, field independence is characterized as autonomy of external referents.

This autonomy is expressed in terms of two different types of ability, restructuring and interpersonal competencies. Field-independent students have greater personal autonomy and they tend to do better at tasks that require them to restructure problems independently. Field-dependent students are less autonomous, but appear to get along better with others in group situations.

These differences in cognitive style may also be related to PSI. It seems reasonable to hypothesize that field-independent students should be more suited for PSI instruction, where they work more independently, while field-dependent students are likely to do better in the more traditional social setting of the lecture-discussion classroom. Although field-dependence-independence and locus of control are similar in certain respects, they are not highly correlated and should be treated as different variables (6).

The purpose of this study was to identify individual difference variables that might interact with mathematics instruction using PSI or lecture-discussion approaches. Individual differences in general ability, field independence, and locus of control were of primary interest. Also, differences in field independence and locus of control appear to be sex-related in at least some studies, so sex was also included as a predictor variable. Dependent variables included both achievement and rate of withdrawal from the course.

Method

Subjects

Participants included 121 college students who were registered in Inter-

mediate Algebra, a regular course offered by a large state university. All students were enrolled in morning classes, with 55 in PSI and 66 in the lecture group. Each of the two treatment groups was about evenly divided according to sex; 48% of the students were female.

Materials

The two treatment groups used the same textbook; in addition, the PSI group received supplementary study guides to assist them in working independently. The same final exam was administered in both groups; it was a 50-item multiple choice test that was generated from a computerized test bank. The test was graded by machine.

Students who did not take the final exam were counted as having withdrawn from the course. Therefore, the rate of withdrawal was measured by noting whether or not a student took the final.

Field independence was assessed using the Hidden Figures Test (HFT) as adapted by the National Longitudinal Study of Mathematical Abilities (NLSMA). For a detailed description of this test, see the appropriate NLSMA reports (11, 15). In this study the KR-20 reliability coefficient for the HFT was .81.

Rotter's locus of control dimension was assessed with the Mathematics Achievement Questionnaire (MAQ), a test based on an earlier instrument developed by Crandall, Katkovsky, and Crandall (1). As Rotter (13) and Lefcourt (6) point out, the usual measures of locus of control are broad in scope and may not be appropriate for classroom research carried out in a particular subject such as mathematics. Therefore, they recommend the development of more specific measures of locus of control. The MAQ is one

such instrument; it deals with the participants' views on mathematics, mathematics teachers, and fellow students in mathematics classrooms. The KR-20 reliability coefficient for the MAQ was .60, which compares favorably with the reliability estimates for other personality measures, including those of Rotter (12).

Procedures

On the first day of class, students in both types of instruction (PSI and lecture) were invited to participate in the study. They were asked to sign a consent form that would allow the university to release student SAT scores for use in this study; most of them did so. Then the HFT and MAQ were administered, allowing 15 minutes for each.

After the first day, the students in the PSI group worked at their own pace following regular PSI procedures. The students in the lecture group were expected to attend class regularly. The teacher for the two lecture classes, an experienced graduate teaching assistant, followed the typical mathematics classroom sequence. First, students were encouraged to ask questions over the previous material; this was followed by a lecture and discussion of new material, along with a specific assignment for the next class.

At the end of the semester, both groups took the same final exam at the same time, except for a few students in the PSI group who were allowed to finish the course and the final exam early.

Results

Table 1 presents the descriptive statistics for all tests, and Table 2

includes the correlation matrix. The two treatment groups appeared to be about evenly matched; there were no significant differences between the treatment groups on any of the aptitude measures, although scores for the PSI group tended to be slightly higher in all cases. The withdrawal rate was somewhat higher in the PSI group, where 58% of the students dropped out before the final exam; in the lecture group, the corresponding figure was 45%. This difference in the rate of withdrawal was reflected in the course grades of those who took the final exam. Of the 36 people in the lecture group who completed the final exam, 25 passed the course. In the PSI group, all but one of the people who took the final exam passed the course. Since weaker students were more likely to take the final exam if they were in the lecture group, their scores tended to lower the mean and increase the standard deviation of final exam scores in the lecture group. Therefore, all statistical results should be interpreted with caution.

Insert Tables 1 and 2 about here

The data were analyzed using multiple regression procedures as outlined by Cronbach and Snow (3). Scatterplots of the final exam score with each aptitude variable indicated that a linear model was appropriate. The aptitude vectors were entered into the regression equation first, followed by the treatment vector, and then the interaction vectors. If none of the interactions vectors contributed significantly to the proportion of the variance that was accounted for by the regression, then the interaction

vectors were dropped from the model and analysis of covariance was used to test for treatment effects.

The aptitude variables to be included in the multiple regression analyses were chosen from among field independence, locus of control, general ability, and sex. Preliminary analyses of the data indicated that sex differences did not make an important contribution to any of the regression analyses, so data on sex were eliminated from further consideration.

The most important dependent variable was final exam scores. In the first regression model, final exam scores were used as the dependent variable, and field independence (HFT) and locus of control (MAQ) were used as aptitude variables. Data were available on 59 students for this model, and these two aptitudes accounted for 22% of the variance. Neither of the aptitudes produced a significant interaction, but there was a significant treatment effect, $\underline{F}(1,55) = 6.24$, $\underline{p} = .016$, favoring the PSI group.

Table 3 presents the regression equations from this analysis for the two treatment groups. Although there was no significant interaction, the differences between regression coefficients were substantial for the MAQ test. As predicted by the theory, students with a more internal locus of control (high MAQ scores) tended to learn more in the PSI group.

Insert Table 3 about here

When course grade was used as the dependent variable, the regression coefficients for HFT and MAQ were similar to those in Table 3. Again, there

were no significant interactions with these two aptitude measures. In addition, there was no significant difference between the treatments when course grade was the dependent variable.

The data were analyzed further with SAT scores added to the model along with HFT and MAQ scores. Since a number of students had never taken the SAT test, complete data were obtained on only 34 students, and the model accounted for 35% of the variance in the fir exam scores. There were no significant interactions in this analysis, and wi.. scores as one of the covariates, the difference between treatments was not significant, $F(1,29) = .99$, $p = .328$.

Although there was no significant interaction with SAT, students with low SAT scores appeared to do slightly better in the PSI treatment. For example, when final exam scores were regressed on SAT scores, the slope was .012 in the PSI group and .042 in the lecture group. This difference in slopes was not significant, as one would expect given the lack of power in this case. However, the difference is in the same direction as that obtained in other studies (4,7,8).

Frequently studies of PSI have used the quantitative portion of the SAT test, rather than total score, in their analyses. Making that change in this study produced essentially the same results as those obtained using SAT total scores. The correlation between SAT-Total and SAT-Quantitative for this study was .63.

The data were also analyzed using rate of withdrawal as the dependent variable. There were no significant interactions between any of the aptitude variables and treatment for the rate of withdrawal. And even though the rate of withdrawal appeared to be higher in PSI instruction,

analysis of covariance (using SAT as the covariate) indicated that the difference between treatments was not significant, $\underline{F}(1,69) = .44$, $\underline{p} = .509$.

Discussion

The purpose of this study was to investigate the relationship of individual difference variables to PSI instruction in mathematics. The most important of these variables continues to be general ability. Although this study produced no significant interactions with general ability, the direction of the results was consistent with that of earlier studies (7,8,9). Students who were low in general ability tended to do better if they were in the PSI group, but the differences were not large.

There was also some support in this study for the hypothesis that students with a more internal locus of control tend to learn more in PSI instruction. Although the interaction was not significant, further research on locus of control seems appropriate.

There was no support from this study for the hypothesized interaction between field independence and PSI instruction. When HFT was used as the measure of field independence, there was very little evidence of any interaction. The differences in regression slopes that did occur tended to be in the same direction as the expected interaction with general ability. The difficulties of separating field independence from general or fluid ability have been noted by Cronbach and Snow (3), among others; these difficulties may have been related to the lack of an interaction effect with field independence. Witkin and his colleagues are aware of the difficulties involved in separating the effects of field independence from those of general ability

(16), and their continuing efforts in this area may help to alleviate the problem.

Although students in the PSI group appeared to perform better on the final exam, this difference was not significant when general ability, as measured by SAT scores, was used as a covariate. PSI studies frequently show higher achievement in the PSI group (5,10), but it is difficult to identify the source of this difference. At least part of the time it may be related to the higher rate of withdrawal by low-ability students in the PSI treatment.

Determining the overall effectiveness of PSI instruction in mathematics is an important goal of educational research. But it is also important to try to assess the impact of this type of instruction on individuals with different characteristics. Based on the results of this study and others, it appears that general ability and locus of control are important in assessing the effects of PSI, as opposed to lecture classes, in mathematics. The cognitive style of field independence, however, does not seem to be related to individual differences in achievement in PSI courses.

NOTE

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Table 1. - Standard Deviations and Number of Cases for All Tests

Test	Maximum possible score	Mean	Standard deviation	Number of cases
HFT	16			
PSI		7.3	3.6	55
Lecture		5.3	3.5	66
MAQ	20			
PSI		13.5	2.5	55
Lecture		12.4	3.0	65
SAT	1600			
PSI		880	137	31
Lecture		833	145	41
Final exam	100			
PSI		76.8	6.8	24
Lecture		67.4	12.2	36

Table 2. - Correlation Matrix for All Tests

Test	Correlation 1	2	3	4
1. HFT	1.00	.17	.19	.32
2. MAQ		1.00	.05	.20
3. SAT			1.00	.50
4. Final exam				1.00

APPENDIX F

Locus of Control and Mathematics Instruction: Three Exploratory Studies

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Running head: Locus of Control

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Abstract

The relationship between locus of control and three dimensions of discovery learning was investigated in a series of studies. Mathematics students were randomly assigned to treatments that differed in level of guidance, inductive or deductive sequencing, or use of small groups. A significant aptitude-treatment interaction occurred between locus of control and small-group instruction on the topic of networks. The other studies did not produce significant interactions, although varying the level of guidance did produce a trend in the predicted direction. Using inductive or deductive sequences of instruction did not appear to interact with locus of control.

Locus of Control and Mathematics Instruction: Three Exploratory Studies

The importance of individual differences in the learning of mathematics is widely recognized, and in recent years student personality characteristics, as well as cognitive aptitudes, have begun to play a prominent role in research on learning. The search for interactions between student characteristics (aptitudes) and instructional strategies (treatments), which concentrated originally on cognitive measures, has now broadened its scope to include more research on personality dimensions. This change in emphasis is reflected in recent reviews of aptitude-treatment-interaction (ATI) research (Cronbach & Snow, 1977; Snow, 1977).

One personality dimension that has recently received some attention in ATI research is Rotter's locus-of-control variable (Rotter, 1966). According to Rotter's theory, individuals with an internal locus of control perceive the outcomes of their actions as being due to their own behavior; those who are classified as external, however, tend to attribute the consequences of their actions to chance or to fate. An individual's locus of control, then, is a measure of belief about whether one's rewards and successes (or punishments and failures) can be attributed to internal or external causes. For a more general description of locus of control in the context of attribution theory, see Bar-Tal (1978). Recent research on locus of control has been reviewed by Lefcourt (1976).

Most ATI studies using locus of control have focused on the relationship of this personality dimension to the organization of instruction.

Daniels and Stevens (1976), for example, found an interaction between locus of control and treatments that differed in whether or not they used student-teacher contracts as a means of organizing instruction. In another study, there was an interaction between locus of control and instruction in computer programming where treatments varied in the level of structure provided to the students (Parent, Forward, Canter, & Mohling, 1975). Robin (1976) discusses several studies where locus of control appeared to interact with "behavioral instruction" (such as Keller's Personalized System of Instruction) as opposed to more traditional lecture-discussion classes. All of these interactions were in the direction predicted by the theory; students with an internal locus of control tended to learn more in the treatment that gave them more responsibility for their own learning, but students with an external locus of control seemed to do better in a more traditional class where the teacher took responsibility for student learning.

The purpose of the three present studies was to extend these earlier results on the organization of instruction to treatments that differed in various dimensions of discovery learning in mathematics. The dimensions included level of guidance of instruction; use of inductive, as opposed to deductive, sequences of instruction; and the use of small groups, rather than individual work. Treatments using a low level of guidance, inductive sequences, or small groups were designed to encourage student discovery. The conjecture was that students with an internal locus of control would do best in treatments that required that they discover concepts independently, and students with an external orientation would learn most in an expository setting.

Method

Three ATI studies were conducted to search for interactions between locus of control and three different dimensions of discovery learning. The design of each study was the same. Students were assessed on a measure of locus of control and randomly assigned to one of two treatment groups. After instruction, students were tested for immediate achievement and then for retention several weeks later. After the studies were completed, SAT scores were obtained from university records and used as a measure of general ability.

Students who participated in the three studies were enrolled in mathematics classes for prospective elementary school teachers. In each study about 90% of the students were female. Students were dropped from a study if they missed one or more days of instruction; the distribution of absences appeared to be random and not related to the differences in the instructional treatments. Also, many students did not have SAT scores in their student record file, so they were dropped from analyses that used the SAT as a measure of general ability.

Locus of control was assessed using the Mathematics Achievement Questionnaire (MAQ), an instrument based on an earlier measure of locus of control that was developed by Crandall, Katkovsky, and Crandall (1965). Lefcourt (1976) and Rotter (1975) have noted that many measures of locus of control are too general to be of use in classroom research on the learning of a particular discipline such as mathematics. Instead of using a general measure of locus of control, they suggest that more narrow and specific

measures of locus of control are needed for special situations. Following this suggestion, the MAQ was developed to assess locus of control in the specific environment of the mathematics classroom.

The MAQ used 20 items to ask students about their views of mathematics, problem solving, their mathematics teachers, and their fellow mathematics students. Students responded to each item by choosing one of two alternatives that represented either an internal or an external point of view. Scores from all of the studies ranged from 4 to 19, where a high score represented an internal orientation; the overall mean was 12. The KR-20 reliability coefficients for the MAQ were in the range of .5 to .6 for the three studies; although these figures are not high, they compare favorably with the reliability estimates for other measures of locus of control (Rotter, 1966).

Reliability coefficients were also obtained for the posttests and retention tests. These coefficients ranged from .6 to .8 in the three experiments, and were judged to be satisfactory in each case.

Experiment 1

Students in each of five classes were randomly assigned to two instructional treatments on networks. Two instructors were randomly assigned to treatments for each class. Each treatment discussed the same topics, including equivalence of networks, traversability and its applications, Euler's formula, and related concepts. The treatments differed in the level of guidance provided to the students. In the high-guidance treatment, much of the work was done for the students; they only had to read the material,

finish filling in some of the tables, and draw some fairly obvious conclusions. In the low-guidance treatment, however, students were given problems to solve with very little direction from the materials or the teacher. In this treatment, students were expected to gather relevant data, organize it, and test hypotheses about networks. When students asked the teacher for answers, the teacher told them to look for patterns in the data that would help them discover the answers independently.

In each treatment, materials were collected at the end of each class period, checked, and then returned to the students at the beginning of the next class period. A total of 150 minutes (one week) of class time was used for the treatments and a 15-minute posttest. After the posttest was completed, networks were not discussed in class until after the retention test was administered four weeks later.

Results

Descriptive statistics for the measures of student achievement are included in Table 1. As indicated in the table, students who received a higher level of guidance performed better on both the posttest and the retention test.

Insert Table 1 about here

The data were analyzed using multiple regression techniques (Cronbach & Snow, 1977). Scatterplots were inspected to insure that a linear model was appropriate. The analysis was done separately for each of the dependent variables, the posttest and the retention test. The independent variables

in the regression included general ability and locus of control as aptitudes, the treatment, and the interaction of each aptitude with treatment. The aptitudes and the treatment were entered into the regression first; then each of the interaction vectors was tested to see if it contributed significantly (at the .05 level) to the regression. If neither interaction vector was significant, both were dropped from the regression, and the treatment vector was tested to see if it made a significant contribution beyond that of the two aptitude variables.

Table 2 presents the regression equations for each dependent variable. As predicted by the theory, the regression coefficients for the MAQ scores were greater in the low-guidance treatment. However, there was no significant interaction with either the MAQ or SAT scores.

Insert Table 2 about here

Since neither interaction vector was significant, the data were analyzed further to test for treatment effects. There were significant differences in favor of the high-guidance treatment on both the posttest, $\underline{F}(1,25) = 13.0$, $\underline{p} < .002$, and the retention test, $\underline{F}(1,25) = 13.2$, $\underline{p} < .002$.

Experiment 2

Students in each of three classes were randomly assigned to two instructional treatments on measurement and approximate data. The treatments included such topics as precision of measurement, significant digits, and their

effect on calculations with approximate data. Although both treatments covered the same topics, the topics were presented differently. In the inductive treatment, students were given a number of examples to work, and then encouraged to generate a rule that was suggested by the examples. These students had access to calculators to help them work enough examples so that they could discover a pattern. In the deductive treatment, students were given the rules first, and then asked to apply the rules to some simple problems. These problems were constructed so that a calculator was not necessary for these students. In both treatments, the teacher was available to help students who were having difficulty.

Students were given 75 minutes to work on the materials. Two days later the posttest was administered, and four weeks later students were tested again for retention.

Results

Descriptive data on the achievement measures are found in Table 1, and the regression equations are in Table 2. The data from Experiment 2 were analyzed using the same multiple regression procedures as in Experiment 1. There was no significant interaction between either SAT or MAQ scores and the inductive-deductive treatment dimension, and in this study there was no consistent pattern in the regression coefficients. Since there was no interaction effect, the data were checked for treatment differences. The difference in favor of the deductive group was significant on the posttest, $F(1,27) = 5.47$, $p = .027$, but not on the retention test, $F(1,24) = 1.61$, $p = .217$.

Experiment 3

Students from five classes were randomly assigned to two instructional treatments on the topic of networks. The two instructors were randomly assigned to treatments for each class. In this experiment, the two treatment groups used exactly the same printed materials. The only difference was that students worked in small groups in one treatment; in the other, students worked individually.

The printed materials were written in an inductive mode, where students were encouraged to generalize a rule from a number of examples. In small-group instruction, students were asked to work together in groups of four as they discovered solutions to the problems. Students were told to ask the teacher for help if the group could not solve the problems. In the individual treatment, students were told to work by themselves and to direct their questions to the teacher; the teacher then helped students solve the problems. Students in the small groups asked the teachers very few questions. Students in the individual treatment, however, asked many questions and received considerable help from the teachers.

One week of class time was used for the treatments and a 15-minute posttest. Most students did not complete all of the treatment materials in this amount of time. The retention test was administered eight weeks later.

Results

Tables 1 and 2 include the descriptive data for Experiment 3. The same multiple regression procedures were used in this experiment as in the other two.

There was no interaction on the posttest in Experiment 3, but on the

retention test there was a significant interaction between locus of control and treatment, $F(1,49) = 4.73$, $p = .034$. This interaction was disordinal and in the predicted direction; students with an internal locus of control were better off in small-group instruction, and students with an external locus of control learned more in individual instruction where they received help from the teacher. The interaction between general ability and treatment was not significant, but the joint contribution of the two interaction vectors was significant, $F(2,48) = 3.20$, $p = .050$.

Since the direction of the interaction was the same for both SAT and MAQ scores, it is possible that the interaction could be due to only one trait that is measured by both tests. However, this interpretation seems unlikely. Measures of locus of control are not highly correlated with general ability (Rotter, 1975); in this study, for example, the correlation of MAQ and SAT scores was .07. Therefore, it seems appropriate to attribute the interaction with MAQ scores to locus of control and not to general ability.

Regions of significance for the interaction were calculated using confidence intervals, as recommended by Cronbach and Snow (1977). When the MAQ scores were used as the only predictor, the confidence intervals for the two regression lines did not overlap for scores of 13 or more. About 50% of the students were in this region of significance. Using the Johnson-Neyman technique (Borich, Godbout, & Wunderlich, 1976) resulted in the same region (for a significance level of .10).

When both MAQ and SAT scores were used as predictors, the confidence intervals for the regression planes did not overlap in two regions. These regions of significance included students with high scores on both tests

(13 or more on MAQ and 900 or more on SAT) and low scores on both tests (8 or less on MAQ and 700 or less on SAT). About 40% of the students were in these two regions, almost all of them in the region with high scores.

Discussion

In three ATI studies using locus of control as an aptitude variable, there was a significant interaction with discovery instruction only in the case where treatments differed in the use of small-group instruction as opposed to individual work. When treatments differed in level of guidance, the difference in regression slopes was in the predicted direction, but the interaction was not significant; this lack of significance could be due to the lack of power, since a sample size of 100 subjects per treatment is desirable in ATI research (Cronbach & Snow, 1977). When treatments differed in the use of an inductive or deductive approach to mathematics, there was no consistent pattern of differences in the regression slopes. Therefore, one dimension of discovery instruction that appears promising for ATI research with locus of control is the use of small groups; further work on the relationship of level of guidance to locus of control may also be profitable.

Research on the effectiveness of small groups is very difficult to analyze. It is hard to ascertain the effects of the dynamics of each separate small group, and to distinguish them from the effects of the treatment as a whole (Webb, Note 1). Moreover, the statistical questions that are involved in determining the effects of small groups raise problems that are not easy to resolve using the ATI model (Cronbach, 1976). Nevertheless, research on small groups is important and small-group instruction does appear to interact

with locus of control. Further research on locus of control and various dimensions of discovery instruction in mathematics seems appropriate.

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Table 1

Means and Standard Deviations for Posttests and Retention Tests

Test	Treatment	Range	Mean	Standard deviation	Number of cases
		Experiment 1			
Posttest	Low guidance	3-13	7.8	2.5	14
	High guidance	4-16	11.8	3.2	15
Retention	Low guidance	1-12	5.6	2.7	14
	High guidance	4-15	9.9	3.1	15
		Experiment 2			
Posttest	Inductive	2-12	6.2	2.6	17
	Deductive	3-15	7.9	3.1	14
Retention	Inductive	2-7	4.8	1.5	16
	Deductive	2-9	5.7	2.7	12
		Experiment 3			
Posttest	Small groups	6-16	9.7	3.1	33
	Individual	6-14	9.5	2.8	34
Retention	Small groups	2-14	7.3	3.3	26
	Individual	0-12	6.5	3.0	28

Table 2

Regression of Posttests and Retention Tests on General Ability and Locus of Control

Test	Treatment	Intercept	Regression coefficients SAT	MAQ
		Experiment 1		
Posttest	Low guidance	-1.1	.007	.24
	High guidance	8.8	.004	-.07
Retention	Low guidance	-3.0	.009	.04
	High guidance	9.3	.004	-.30
		Experiment 2		
Posttest	Inductive	-1.0	.014	-.45
	Deductive	3.7	.008	-.20
Retention	Inductive	1.6	.004	-.02
	Deductive	2.7	.005	-.12
		Experiment 3		
Posttest	Small groups	2.5	.008	.01
	Individual	6.6	.004	-.04
Retention	Small groups	-2.1	.009	.12
	Individual	9.4	.003	-.43

APPENDIX G

Recent Research on Aptitude-Treatment Interactions

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Recent Research on Aptitude-Treatment Interactions

Much of the research on instruction in mathematics has tried to prove that one kind of instruction is superior to another for all students. Such research generally has not been very conclusive. An alternative hypothesis would suggest that different students would do best in different types of instruction, depending on each student's characteristics. The problem of matching instructional treatments with student characteristics so as to maximize learning has come to be called the Aptitude-Treatment-Interaction (ATI) hypothesis. For a sample graph of an ATI, see Figure 1. As the figure indicates, a student with a high aptitude score seems to learn more in treatment 2, while low-aptitude students do better in treatment 1.

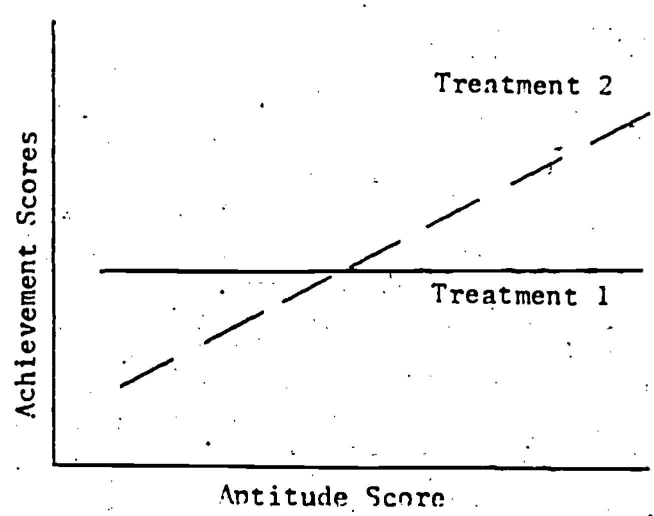

Figure 1. An example of regression lines that show an Aptitude-Treatment Interaction

ATI research, generally viewed as an outgrowth of the work of Cronbach (1957), has turned out to be somewhat more difficult than was first anticipated. Nevertheless, Cronbach and Snow (1977), in their comprehensive review of the field, confirm the existence of ATI in a variety of settings, and conclude that "ATI has come of age" (p. 524).

Since Cronbach and Snow completed their review, a number of new ATI studies have appeared in the mathematics education literature. The purpose of this paper will be to review the theoretical background of ATI research, to discuss the results of recent studies, and to provide a critique of current research in the area.

Theoretical Background

Originally, many researchers in mathematics education used Guilford's ideas about aptitudes in their search for ATI. The weaknesses of this approach have been described by Cronbach and Snow (1977), who prefer a more hierarchical model based mainly on the work of Cattell (1971).

Briefly, Cattell's theory is based on a factor analytic model of human abilities. One can think of Cattell's model as a pyramid, with general ability at the top. At the next level below general ability, there are a number of other general factors. The first of these is fluid intelligence, a factor which loads mainly on several perceptual, culture-fair tests. Another is crystallized intelligence, which is represented by tests of verbal and numerical ability that are similar to school achievement measures. Other general factors, such as speed and spatial visualization, are also found at

this level. More specific primary factors, such as flexibility of closure and general reasoning, are found at the next lower level. While these specific, primary factors continue to play a role in ATI research, Snow's recent work tends to put more emphasis on Cattell's more general concepts of fluid and crystallized intelligence (Snow, 1977b, Note 1).

Although Cattell's approach has much to recommend it, there has been considerable difficulty in separating out the effects of the various specific, primary factors from general ability (Glaser, 1972). In response to this problem, Glaser called for further study of the "new aptitudes" derived from information processing approaches to learning and from investigations of cognitive style and personality characteristics.

Several researchers have begun to apply concepts from information processing theory to the problems of ATI research. Carroll (1976) has gone back to the 1963 French kit to reanalyze those tests in terms of the cognitive processes which they require. Also, Snow (1977a) has given examples of how an information processing approach can be used to generate promising ATI hypotheses. While these ideas are not yet well formulated or widely used, information processing theory provides a promising new approach to ATI research.

Glaser's second suggestion for "new aptitudes", cognitive styles and personality characteristics, has also been an active research area. Witkin's cognitive style variable, field independence, has received considerable attention (Witkin, Moore, Goodenough, & Cox, 1977; Witkin & Goodenough, Note 2). According to Witkin's theory, field-independent students should do well when they are allowed to discover concepts with little guidance, while

field-dependent students will be better off in a more expository treatment. A number of recent studies have also investigated the personality variables of anxiety and achievement motivation (Snow, 1977b), and Rotter's locus-of-control variable (Rotter, 1975) has played a role in several ATI studies (Robin, 1976).

Recent Results

Since the appearance of Cronbach and Snow's book, a number of interactions with traditional aptitudes have been identified. Continuing an earlier series of studies, Eastman and Salhab (1978) reported another ATI between general reasoning and treatments that differed in the use of algebraic or geometric approaches to absolute value concepts. Eastman and Behr (1977), however, did not find an interaction with general reasoning in a similar study using concepts from logic. But when the treatment dimension was changed to inductive as opposed to deductive instruction, two studies again found an interaction with general reasoning (McLeod & Adams, Note 3; McLeod & Briggs, Note 4). The source of these interactions with general reasoning is not clear, but it may come from either general ability or crystallized intelligence instead of from general reasoning. Other studies have also reported ATI with measures of general ability or crystallized intelligence, sometimes when the treatments differed in use of discovery approaches (Adams & McLeod, Note 5) or in use of Personalized Systems of Instruction (Pascarella, 1978).

Several other studies have reported interactions with the "new aptitudes" that Glaser referred to. In two studies, there was an interaction between the cognitive style of field independence and the level of guidance provided

in a discovery approach to mathematics (McLeod, Carpenter, McCornack, & Skvarcius, 1978; McLeod & Adams, in press). In addition, Horak and Zweng (Note 6) found evidence of an interaction between field independence and treatments that apparently differed in student interaction patterns. However, in a later study designed to find an interaction between field dependence and small-group instruction, the pattern of ATI results was unstable and appeared to be due more to general ability than to cognitive style (McLeod & Adams, Note 7). In other studies, field independence did not appear to interact strongly with the inductive-deductive dimension of discovery learning (McLeod & Briggs, Note 4; Threadgill, Note 8).

Interactions between locus of control and the organization of instruction have occurred in several studies (Robin, 1976), and the interactions have been in the direction predicted by Rotter's theory. Students with an internal locus of control seemed to do better in a treatment where they had more responsibility for their own learning, but students with an external locus of control tended to learn more in a traditional class where the teacher took responsibility for student progress. Similar results have now been obtained in a study using college mathematics students, where there was an interaction between locus of control and small-group instruction (McLeod & Adams, Note 9).

In summary, there has been considerable success in identifying interactions with general reasoning, field independence, and locus of control. Sometimes, however, replications have been difficult to obtain. ATI studies using field independence, for example, seem to be less successful when the topic of instruction is geometric in nature (McLeod & Adams, Note 7).

Analysis and Critique

In their review of individual differences and the learning of mathematics, Fennema and Behr (in press) provide a thorough critique of ATI research and especially of the "new aptitudes" referred to by Glaser (1972). Fennema and Behr take a rather positive view of the impact of information-processing theory on ATI research, although Cronbach and Snow (1977) say that these "new aptitudes" are likely to look more "traditional" by the time they are applied in ATI research.

Research on personality characteristics is also viewed positively by Fennema and Behr. However, the use of cognitive style as a new aptitude variable receives substantial criticism, both from Fennema and Behr and from Cronbach and Snow. Field independence is criticized particularly because of the instruments used to measure it (Horn, 1976; Vernon, 1972). Recent work by Witkin & Goodenough (Note 2) that relates field independence to Cattell's work may help resolve some of these problems of measurement and interpretation.

Measurement problems have also caused difficulty for ATI studies using locus of control as an aptitude variable. Rotter (1975) has noted that many measures of locus of control are too general to be of use in classroom research; as a result, it is necessary to develop new instruments that are designed to assess locus of control in a specific environment, such as the mathematics classroom.

In summary, ATI research is difficult to conduct, but recent studies show great promise. Cronbach and Snow (1977) have discussed in detail the many ways in which ATI research can be improved, and studies conducted since their review show evidence that important progress is being made in the area.

A number of interactions with the aptitudes of general reasoning, field independence, and locus of control are of particular interest to the field of mathematics education. Further research should help to clarify the nature and source of these interactions.

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