This guide was prepared as an instructional aid for teachers of a first-year course in Algebra. It was designed to be applicable to the wide range of Algebra 1 programs being offered in the junior and senior high schools. In this guide, the content of the Algebra 1 program has been divided into 11 major units, each unit organized around student performance objectives. It is intended that the performance objectives serve as guidelines for teachers to design the specific instructional program to meet the unique developmental needs of their students. Each of the 11 major units includes the following features: (1) a list of performance objectives; (2) a vocabulary list; (3) a list of key skills; (4) a list of textbook references; (5) four assessment tasks for each performance objective; (6) an answer key for the assessment tasks; (7) a list of entering performance objectives covering skills which should be reviewed before beginning the unit; (8) a list of assessment items for entering performance objectives; (9) an answer key; (10) a list of suggestions to the teacher, including the number of instructional days per unit; and (11) a list of suggested minimal, average, and maximal objectives for adjusting the course to different ability levels. (Author/MK)
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Board of Education of Montgomery County
Rockville, Maryland
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ACKNOWLEDGMENTS

The Algebra 1 Instructional Guide has been constructed under the direction of Dr. Ellen L. Hocking, coordinator of secondary mathematics. The following teachers were responsible for constructing the guide:

Ralph B. Balliet  
Charles E. Barkley  
Russell L. Fleury  
Donald S. Mieczkowski  
Martha C. Price

Tilden Junior High School  
Belt Junior High School  
Montgomery Blair High School  
Sherwood High School  
Wootton High School

The typing of the guide has been done by Jean Bursky and Esther Caputo.
A provocative activity which teachers often use with pupils at various levels is trying to imagine what a world without numbers would be like. Such a world is difficult to imagine. The idea of number continues to play an important role in virtually all aspects of our world; and mathematics, therefore, constitutes a program of considerable importance in the schools.

As a discipline, mathematics is truly the art and science of abstraction. Characteristics of the physical world are converted into abstract ideas and symbols; these are then manipulated through mathematical operations to produce information and theorems about less easily observed aspects of the world. Recent evidence supports the contention that children's experiences with concrete materials are vital to later conceptual development. The school program thus proceeds from the concrete to the abstract.

The concepts of mathematics acquire greater meaning when they can be applied to the world in which we live. Because the variety and extent of mathematical applications have grown so rapidly in recent years, it is impossible for any one person to be conversant with the entire field. The school program must therefore be developed so that mathematical applications are selected and presented as efficiently as possible and with the intent of challenging pupils at all levels to see mathematics as an independent discipline as well as a tool for the advancement of other disciplines.

The Montgomery County mathematics program is designed and implemented to take into account the logical and relatively sequential nature of mathematics. Equally important, too, is the realization that the rate at which individual students learn mathematics varies significantly. The mathematics program is structured to encourage various approaches which allow students to progress at their individual rates.

The pre-algebra objectives range over six areas of mathematics and are arranged according to 14 different levels of achievement. Assessment measures have been constructed for each objective so that individual progress can be measured in a variety of categories. Enrichment activities are available for both the able student and the student who needs reinforcement.

Several options are available to the student who has completed the pre-algebra objectives. Differentiated paths through a variety of courses are available to the student, as can be seen in the Mathematics Program Patterns Chart, on page xv. Each student has available a sequence of courses which can be suited to his/her interests and abilities.

Computer literacy is addressed at several levels of the mathematics program; career information is incorporated as appropriate throughout. Consumer applications are taught as mathematical skills are developed; the mathematics of consumerism is further emphasized in an elective senior high course.

In general terms, the instructional program in mathematics should help each student to:

- Develop basic skills in using the vocabulary and symbols of mathematics
- Develop skills in recognizing common geometric shapes
Develop basic skills in computing.

Develop basic skills in working with geometric shapes.

Develop basic skills in measuring, graphing, and using tables and charts.

Develop understanding of the vocabulary and symbols of mathematics.

Develop understandings necessary for translating among mathematical symbols, words, and the physical world.

Develop concepts related to common geometric shapes.

Develop understanding of computation.

Develop understanding of measurement.

Develop an understanding of basic principles related to the structure of mathematics.

Develop understanding and basic skills in problem solving.

Apply the principles of mathematical reasoning to the solution of problems.

Appreciate the significance of mathematics in daily living, and its contribution to our cultural heritage.

Use mathematics as needed in daily living.
OVERVIEW

PHILOSOPHY

The beginning algebra student is seen on the initial rung of the ladder leading to an understanding of the developmental structure of a mathematical system. Throughout the course, the development of an ability to think mathematically is emphasized as the student is guided toward an appreciation of the orderliness of mathematics. This appreciation greatly facilitates the acquisition of skills and techniques used to analyze and solve both simple and complex problems.

The study of Algebra 1 affords the student the opportunity to organize his/her own knowledge of mathematics and to expand skills previously learned. Mastery of algebra is seen as the passport for a student's journey through the complexities of higher mathematics.

INTENT

This guide has been prepared as an instructional aid for teachers of a first year course in algebra. It has been designed to be applicable to the wide range of Algebra 1 programs being offered in the junior and senior high schools. In this guide, the content of the Algebra 1 program has been divided into eleven major units, each unit organized around student performance objectives. It is intended that the performance objectives serve as guidelines for teachers to design the specific instructional program to meet the unique developmental needs of their students.

ORGANIZATION AND FEATURES

Each of the eleven major units and four enrichment units deals with the development of the skills and concepts of a topic of first year algebra. With each unit, the following features are included:

- A list of performance objectives for the unit, each objective categorized by level of Bloom's taxonomy (See page xix.)
- A vocabulary list for the unit
- A list of key skills for end-of-course testing (See page xxii.)
- A list of textbook references for each performance objective
- Four assessment tasks for each performance objective, which can be used for testing or review
- An answer key for the assessment tasks
- A list of entering performance objectives covering skills which should be reviewed before beginning the unit (Note: These objectives include pre-algebra skills as well as skills from prior units.)
- A list of assessment items for each entering performance objective
An answer key for the entering performance objectives assessments

A list of suggestions to the teacher including the number of instructional days per unit. (Note: (1) The suggested number of instructional days does not include provisions for review or testing. (2) Memory aids included in the suggestions are intended to facilitate retention, not to take the place of learning the mathematical theory.)

A list of suggested minimal, average, and maximal objectives for adjusting the course to different levels of student ability

SEQUENCE

In examining all Algebra 1 books currently approved for use in the county, it was observed that they each developed the material in a slightly different order. After much discussion, the units of this guide were placed in order so that the study of linear relations and the language of algebra (Units I-VI) would constitute the first semester of the course, and the development of higher order relations and their applications would form the central theme of the second semester.

Many teachers prefer to introduce polynomials, factoring, and rational algebraic expressions (Units VII, VIII and IX) before graphing and systems of equations (Units V and VI). This guide reverses that order. It is left to the individual teacher to decide which sequence is best adapted to his/her particular needs.

KEY SKILLS FOR END-OF-COURSE TESTING

A list of twenty-eight summary skills, the testing of which would be a measure of the mastery of the Algebra 1 objectives as described in the MCPS Program of Studies, can be found on pages xxii and xxiii. The first fourteen skills cover the objectives of Algebra 1A; the last fourteen cover the objectives of Algebra 1B.

In each unit of the Algebra 1 Instructional Guide, the key skills pertinent to that unit are listed on the page listing the performance objectives for the unit.
LEVELS OF PERFORMANCE OBJECTIVES

Each performance objective in each unit has been classified as measuring knowledge (I), comprehension (II), or application (III) according to Bloom's taxonomy. While these classifications are highly subjective and open to interpretation, they have been included as an indication of the nature of the responses necessary to demonstrate attainment of the objectives. The following list of verbs was used in classifying each objective.

<table>
<thead>
<tr>
<th>I. Knowledge</th>
<th>II. Comprehension</th>
<th>III. Application</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Emphasis:</strong> Recall</td>
<td><strong>Emphasis:</strong> Grasp of meaning, intent, or relationship</td>
<td><strong>Emphasis:</strong> Applying appropriate principles or generalizations</td>
</tr>
<tr>
<td>choose from a list (judgment not involved)</td>
<td>classify</td>
<td>apply</td>
</tr>
<tr>
<td>define (give a dictionary definition)</td>
<td>define (in student's own words)</td>
<td>collect information</td>
</tr>
<tr>
<td>fill in the blank (or complete)</td>
<td>describe</td>
<td>(supply correct equation or formula)</td>
</tr>
<tr>
<td>follow directions</td>
<td>explain</td>
<td>compute</td>
</tr>
<tr>
<td>identify</td>
<td>express in other terms</td>
<td>construct</td>
</tr>
<tr>
<td>indicate</td>
<td>find (as in math)</td>
<td>convert (in math)</td>
</tr>
<tr>
<td>label</td>
<td>measure</td>
<td>draw</td>
</tr>
<tr>
<td>list</td>
<td>paraphrase</td>
<td>determine (calculate)</td>
</tr>
<tr>
<td>locate (on a map or a given document)</td>
<td>put in order</td>
<td>demonstrate</td>
</tr>
<tr>
<td>match</td>
<td>rewrite</td>
<td>derive</td>
</tr>
<tr>
<td>name</td>
<td>simplify</td>
<td>differentiate between</td>
</tr>
<tr>
<td>select (judgment not involved)</td>
<td>suggest</td>
<td>discuss</td>
</tr>
<tr>
<td>Math</td>
<td>summarize</td>
<td>distinguish between</td>
</tr>
<tr>
<td>add (find the sum)</td>
<td>apply</td>
<td>expand</td>
</tr>
<tr>
<td>balance</td>
<td>collect information</td>
<td>express in a discussion</td>
</tr>
<tr>
<td>calculate</td>
<td>(supply correct equation or formula)</td>
<td>estimate</td>
</tr>
<tr>
<td>compute (using a given formula)</td>
<td>compute</td>
<td>find (implies investigation)</td>
</tr>
<tr>
<td>divide (find the quotient)</td>
<td>construct</td>
<td>interpret</td>
</tr>
<tr>
<td>factor</td>
<td>convert (in math)</td>
<td>investigate</td>
</tr>
<tr>
<td>find square root or raise to power</td>
<td>draw</td>
<td>illustrate (give examples not previously specified)</td>
</tr>
<tr>
<td>multiply (find the product)</td>
<td>determine (calculate)</td>
<td>graph</td>
</tr>
<tr>
<td>perform operations on numbers</td>
<td>demonstrate</td>
<td>keep records</td>
</tr>
<tr>
<td>subtract (find the difference)</td>
<td>derive</td>
<td>locate (information)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>make</td>
</tr>
<tr>
<td></td>
<td></td>
<td>participate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>plan</td>
</tr>
<tr>
<td></td>
<td></td>
<td>predict (from known factors)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>prepare</td>
</tr>
<tr>
<td></td>
<td></td>
<td>present</td>
</tr>
<tr>
<td></td>
<td></td>
<td>prove (in math)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>solve (problems expressed in words)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>use</td>
</tr>
<tr>
<td></td>
<td></td>
<td>trace (development, history, process)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>translate</td>
</tr>
</tbody>
</table>
MCP$\text{ PROGRAM OF STUDIES. OBJECTIVES FOR ALGEBRA 1

Algebra 1 - Grades 9, 10, 11, 12.

Prerequisite: Attainment of the pre-algebra objectives in Elementary and Pre-Algebra Mathematics Objectives, Bulletin 285

3100 1 year 1 credit

Emphasis in Algebra 1 is on the development of an understanding of the basic structure of algebra related to the real number system; a recognition of the techniques of algebra as structures of this system; facility in applying algebraic concepts and skills, perception of the role of deductive reasoning in algebra; and an appreciation of the need for precision of language.

As techniques are developed, applications to appropriate, relevant problems are made; e.g., problems involving practical geometry, number theory, weather, air navigation, and money are included. Opportunities for enriching work in greater depth occur throughout the program.

Algebra 1 is taught as a one-year course or as a two-year course as student needs indicate; pupil placement depends on a decision made cooperatively by the student, the parent, and the school staff.

Algebra 1-A

Prerequisite: Attainment of the pre-algebra objectives in Elementary and Pre-Algebra Mathematics Objectives, Bulletin 285

3111 1 semester 1/2 credit

Upon completion of Algebra 1-A, the student should be able to:

. demonstrate the use of the roster, rule, and graphing methods in representing sets

. compute the value of a numerical expression involving symbols of inclusion and the order of operations rules

. add, subtract, multiply, and divide directed numbers

. apply properties of equality and inequality and the concepts of additive and multiplicative inverse to solve equations and inequalities

. apply the fundamental operations to solve open sentences and word problems involving real numbers

. graph the solution sets of linear equations and inequalities
add, subtract, multiply, and divide polynomials

- factor polynomials completely
- solve equations involving polynomials
- solve word problems involving the factoring of polynomials

Algebra 1-B

Prerequisite: Attainment of the objectives of Algebra 1-A

Upon completion of Algebra 1-B, the student should be able to:

- rewrite a linear equation into slope-intercept forms and graph its solution set
- determine an equation of a line
- solve a system of open sentences into variables
- solve word problems involving open sentences in two variables
- add, subtract, multiply, and divide algebraic fractions
- write radical expressions in simplest form
- solve quadratic equations
- solve equations and word problems involving irrational numbers

Algebra 1, Parts I and II

<table>
<thead>
<tr>
<th>Course</th>
<th>Duration</th>
<th>Credit Per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>3101 (Part I)</td>
<td>1 year</td>
<td>1 credit (9-12)</td>
</tr>
<tr>
<td>3102 (Part II)</td>
<td>1 year</td>
<td></td>
</tr>
</tbody>
</table>

Algebra 1, Parts I and II, is a two-year program for achieving the Algebra 1 objectives. This program is offered for students who have satisfactorily completed necessary pre-algebra objectives but who need additional instruction and time to achieve understanding of abstract concepts. Textbooks written especially for a two-year program are provided. The objectives for Algebra 1, Part I, are those for Algebra 1-A. The objectives for Algebra 1, Part II, are those for Algebra 1-B.
KEY SKILLS FOR END-OF-COURSE TESTING

As identified in the Algebra 1 Instructional Guide and the credit for examination, the student should be able to:

1. Demonstrate the use of the roster, rule, and graphing methods in representing sets
2. Compute the value of numerical expression involving symbols of inclusion and the order of operations rules
3. Evaluate algebraic expressions and open sentences by substituting for the variable
4. Add and subtract directed numbers
5. Multiply and divide directed numbers
6. Combine similar terms
7. Solve linear equations in one variable by applying the properties of equality
8. Solve linear inequalities in one variable by applying the properties of inequality
9. Solve various types of word problems, utilizing an organized approach
10. Graph linear equations in the coordinate plane
11. Graph linear inequalities in the coordinate plane
12. Determine an equation of a line, given the slope and a point or two points of the line
13. Solve a system of equations in two variables
14. Solve word problems involving two variables and a system of equations
15. Add and subtract polynomials
16. Multiply polynomials
17. Divide polynomials
18. Solve linear equations involving polynomials
19. Factor polynomials completely
20. Solve equations by factoring
21. Solve word problems involving factoring
22. Simplify an algebraic fraction
23. Multiply and divide algebraic fractions
24. Add and subtract algebraic fractions
25. Solve fractional equations
26. Write radical expressions in simplest form
27. Solve radical equations
28. Solve a quadratic equation by completing the square or applying the quadratic formula
BASIC TEXTBOOKS FOR ALGEBRA I AND THE TWO-YEAR ALGEBRA COURSE


ALGEBRA 1, PART I AND PART II. SETS


OUTLINE OF COURSE CONTENT

BASIC UNITS

I. SETS
II. FUNDAMENTAL CONCEPTS
III. REAL NUMBERS: PROPERTIES AND OPERATIONS
IV. SOLVING OPEN SENTENCES AND WORD PROBLEMS
V. GRAPHING
VI. SYSTEM OF OPEN SENTENCES
VII. POLYNOMIALS
VIII. FACTORING
IX. RATIONAL ALGEBRAIC EXPRESSIONS
X. RADICAL EXPRESSIONS
XI. QUADRATIC EQUATIONS

SUGGESTED ENRICHMENT UNITS

XII. NUMERICAL TRIGONOMETRY
XIII. PERIMETERS, AREAS, AND VOLUMES
XIV. RATIO, PROPORTION, AND PERCENT
XV. FLOWCHARTS
UNIT I - SETS

PURPOSE

This unit is designed to provide the students with a background in the use of sets. The emphasis is on the application of terminology and set notation as a building block for the study of elementary algebra. While some texts include sets in a unit on fundamental concepts, it has been separated in this guide to provide a short, initial unit for the opening days of school.

OVERVIEW

Familiarization with set notation and the methods of specifying sets are encompassed in the first several objectives. Recognition of various types of sets and set relationships is expected, as is a knowledge of the operations on sets.

SUGGESTIONS TO THE TEACHER

Instructional Days: 4-5
Minimal Course Objectives: #1-5, 10, 11
Average Course Objectives: #1-11
Maximal Course Objectives: ALL

Writing accurate descriptions of sets given in roster form can be difficult for some students. The students should be given a wide range of non-numerical as well as numerical sets to describe.

Emphasis should be given to the concepts of union and intersection of sets since these ideas are necessary for future units.

For many students, this unit is a review and should be treated as such.

VOCABULARY

between complement coordinate counting numbers element empty set equal sets factor finite set graph inclusive infinite set intersection integer member natural numbers null set number line one-to-one correspondence prime factor prime numbers real numbers roster rule set subset union universal set whole numbers
Add whole numbers.
1. 97
   68
   165

2. 507
   946
   1453

3. 4284
   6998
   11282

Subtract whole numbers.
4. 759
   508
   251

5. 725
   284
   441

6. 50,901
   26,978
   23,923

Multiply whole numbers.
7. 80
   32
   2560

8. 473
   65
   30745

9. 843
   726
   60568

Divide whole numbers.
10. 38 ) 419
    11. 46 ) 3198
    12. 29 ) 1889

Add decimals.
13. 11.84
    + 4.95
    16.79

14. 7.305 + 8.45

15. 9.6 + 12.07 + 15. + 23.297

Subtract decimals.
16. 85.24
    + 15.77
    101.01

17. 26 - 3.2

18. 38.29 - 0.7

Multiply decimals.
19. 8.8 x 7.7
    68.76

20. 4830 x .073
    352.19

21. 0.924 x 34.5
    32.246

Divide decimals.
22. 8 ) 62.4

23. 6.9 ) 33.45
    33.45

24. .068 ) 333.2

Reduce fractions to lowest terms.
25. \( \frac{24}{27} \)

26. \( \frac{18}{54} \)

27. \( \frac{45}{60} \)
Change mixed numbers to improper fractions.

28. \( \frac{72}{5} \)
29. \( \frac{27}{3} \)
30. \( \frac{53}{4} \)

Change improper fractions to mixed numbers.

31. \( \frac{27}{5} \)
32. \( \frac{31}{12} \)
33. \( \frac{8}{3} \)

Multiply fractions.

34. \( \frac{7}{11} \times \frac{5}{9} \)
35. \( \frac{7}{8} \times \frac{3}{4} \)
36. \( \frac{5}{6} \times \frac{9}{10} \)

Divide fractions.

37. \( \frac{4}{7} \div \frac{3}{5} \)
38. \( \frac{5}{9} \div \frac{2}{7} \)
39. \( \frac{5}{8} \div \frac{5}{6} \)

Add fractions.

40. \( \frac{1}{5} + \frac{3}{4} \)
41. \( \frac{7}{5} + \frac{3}{10} \)
42. \( \frac{25}{6} + \frac{4}{3} \)

Subtract fractions.

43. \( \frac{11}{15} - \frac{1}{2} \)
44. \( \frac{5}{8} - \frac{1}{4} \)
45. \( \frac{3}{8} - \frac{5}{6} \)
# PRE-ENTRY DIAGNOSTIC ARITHMETIC TEST ANSWER KEY

<table>
<thead>
<tr>
<th>1.</th>
<th>165</th>
<th>24.</th>
<th>4,900</th>
<th>43.</th>
<th>7/30</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>1453</td>
<td>25.</td>
<td>8/9</td>
<td>44.</td>
<td>1/8</td>
</tr>
<tr>
<td>3.</td>
<td>11,282</td>
<td>26.</td>
<td>1/3</td>
<td>45.</td>
<td>7/24</td>
</tr>
<tr>
<td>4.</td>
<td>251</td>
<td>27.</td>
<td>3/4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>441</td>
<td>28.</td>
<td>47/5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>23,923</td>
<td>29.</td>
<td>34/9</td>
<td></td>
<td></td>
</tr>
<tr>
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UNIT I - SETS

PERFORMANCE OBJECTIVES

1. Identify the following symbols: \( \in, \notin, \emptyset, \{ \}, \cap, \cup, \subseteq \). (I)
2. State whether a given number is an element of a given set. (II)
3. Write a roster for a set when given its rule. (II)
4. Determine a rule for a set when given its roster. (III)
5. Graph a given set on a horizontal number line. (III)
6. List all subsets of a given set. (II)
7. Distinguish between finite and infinite sets. (III)
8. Determine whether a one-to-one correspondence exists between two given sets. (III)
9. Determine whether two given sets are equal. (III)
10. Find the intersection of two given sets. (III)
11. Find the union of two given sets. (III)
12. Construct Venn diagrams to show the relationship between sets. (III)
13. Find the complement of a given set. (III)

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KEY SKILLS FOR END-OF-COURSE TESTING

1. Demonstrate the use of the roster, rule and graphing methods in representing sets.
## CROSS REFERENCES

### TEXTS (BY AUTHOR)

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I-7
UNIT I - SETS

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PERFORMANCE OBJECTIVE I-1

Identify the following symbols: \(=, \neq, \in, \emptyset, \subseteq, \cup, \cap\).

a) Which of the following statements is incorrect?
   A. \(\{1, 2\} \subseteq \{1, 2\}\)
   B. \(2 \in \{1, 2, 3\}\)
   C. \(\{3\} \in \{1, 2, 3\}\)
   D. \(\{a, b, c\} \neq \{1, 2, 3\}\)
   Answer ____________________

b) Place one of the symbols \(=, \neq, \in, \subseteq, \cup, \cap\), in the blank to make each statement true.
   (1) \(3 \_ \{1, 2, 3\}\)
   (2) \(\{3\} \_ \{1, 2, 3\}\)
   (3) \(\{1, 2, 3\} \_ \{3, 1, 2\}\)
   (4) \(\{a, b, c\} \_ \{c, d, e\} = \{c\}\)

c) Match each of the following symbols with its description.
   _____(1) \(\cup\) A. is a subset of
   _____(2) \(\subseteq\) B. is an element of
   _____(3) \(\in\) C. union
   _____(4) \(\cap\) D. intersection

d) Match each of the following symbols with its description.
   _____(1) \(\subseteq\) A. the empty set
   _____(2) \(\emptyset\) B. is not equal to
   _____(3) \(\neq\) C. is a subset of
   _____(4) \(\in\) D. is an element of
PERFORMANCE OBJECTIVE I-2

State whether a given number is an element of a given set.

a) Place either ε or ≠ in the blank to make each statement true.

(1) 3 _____ {1, 2, 3}
(2) 1 _____ {prime numbers}
(3) 0 _____ {natural numbers}

b) Which of the following is false?

A. 3 ε {odd numbers}
B. 4 ε {prime numbers}
C. 0 ∉ {counting numbers}
D. Rockville ∉ {counties in the state of Maryland}

Answer ___________

c) Given the following:

(1) 0 ε {whole numbers}
(2) 0 ∉ {natural numbers}

Which of the following statements is correct?

A. only #1 is true
B. only #2 is true
C. both #1 and #2 are true
D. both #1 and #2 are false

Answer ___________

d) True or false

_____ (1) 1 ∈ {prime numbers}
_____ (2) 0 ∈ {natural numbers}
_____ (3) 3½ ∈ {whole numbers}
_____ (4) 8 ∈ {whole numbers between 3 and 8}

Answer ___________
PERFORMANCE OBJECTIVE I-3

Write a roster for a set when given its rule.

a) Write the roster for \{\text{states bordering Washington, D.C.}\}.
   Answer

b) Write the roster for \{\text{grade levels in this school}\}.
   Answer

c) Write the roster for \{\text{whole numbers between 1 and 2}\}.
   Answer

d) Which of the following is the correct roster for \{\text{positive integral multiples of 5}\}?
   A. \{5, 10, 15, 20, \ldots\}
   B. \{5, 10, 15, 20\}
   C. \{0, 5, 10, 15, 20, \ldots\}
   D. \{\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \ldots\}
   Answer
PERFORMANCE OBJECTIVE I-4

Determine a rule for a set when given its roster.

a) State a rule that describes the following set: \{2, 4, 6, 8\}.

Answer ____________ 

b) State a rule that describes the following set: \{January, June, July\}.

Answer ____________ 

c) State a rule that describes the following set \{2, 3, 5, 7\}.

Answer ____________ 

d) Which of the following best describes the set \{1, 3, 5, 7, \ldots \}?

A. \{odd numbers less than 8\}
B. \{odd numbers greater than zero\}
C. \{odd numbers between zero and 8\}
D. None of the above

Answer ____________
PERFORMANCE OBJECTIVE I-5

Graph a given set on a horizontal number line.

a) Graph \{1, 3, 5\} on the number line.
   Answer

b) Graph \{whole numbers between 1 and 3\} on the number line.
   Answer

c) Graph \{real numbers between 0 and 4\} on the number line.
   Answer

d) Graph \{counting numbers between -3 and 5\} on the number line.
   Answer
PERFORMANCE OBJECTIVE I-6

List all subsets of a given set.

a) List all the subsets of \(\{1, 2, 3\}\).
   Answer

b) The set \(\{a, b, c\}\) has \(\_\_\_\_\_\) subsets.
   A. 3
   B. 6
   C. 7
   D. 8
   Answer

c) Which of the following sets is not a subset of \(\{1, 2, 3\}\)?
   A. \(\{3, 1\}\)
   B. \(\{1, 2, 3\}\)
   C. \(\{0\}\)
   D. \(\{1, 2\}\)
   Answer

d) Which is not a subset of (whole numbers)?
   A. \{natural numbers\}
   B. \{2, 4, 6, 8, \ldots\}
   C. \{1, 1, 2, 3, 5, 8, 13, \ldots\}
   D. \{real numbers\}
   Answer
PERFORMANCE OBJECTIVE 1-7

Distinguish between finite and infinite sets.

State whether each of the following is finite or infinite.

a) \{students in this school\}  
   Answer

b) \{real numbers between 2 and 4\}  
   Answer

c) \{grains of sand on the beach at Ocean City, Maryland\}  
   Answer

d) \{2, 4, 6, ... 20\}  
   Answer

e) \{whole numbers between 3 and 4\}  
   Answer

f) \{fractions between \(\frac{1}{4}\) and \(\frac{1}{2}\)\}  
   Answer
PERFORMANCE OBJECTIVE 1-8

Determine whether a one-to-one correspondence exists between two given sets.

a) Write a set that could be in a one-to-one correspondence to \{even numbers between 0 and 10\}.
   Answer

c) Which of the following sets could be in a one-to-one correspondence with \{\sigma, \Delta, \Box\}?
   A. \{1, 2, 3\}
   B. \{\sigma, \Delta\}
   C. \{3\}
   D. None of the above
   Answer

d) Which of the following diagrams shows a one-to-one correspondence between the two sets?
   A.
   B.
   C.
   D.
   Answer
Determine whether two sets are equal.

a) Which of the following sets is equal to \{ letters in the word \text{that} \}? 
   A. \{2, 3, 5, 7\} 
   B. \{letters in the word hat\} 
   C. \{a, b, c, d\} 
   D. \{a, b, c\} 
   Answer ________

c) Which of the following sets is equal to \{prime factors of 12\}? 
   A. \{1, 2, 3, 4, 6, 12\} 
   B. \{2, 3\} 
   C. \{1, 2, 3\} 
   D. \{2, 3, 6\} 
   Answer ________

d) Which of the following sets is equal to the set \{A, C, E\}? 
   A. \{1, 2, 3\} 
   B. \{C, E, A\} 
   C. \{C, 2, 3\} 
   D. All of the above 
   Answer ________

b) Write a set which is equal to \{ letters in the word Mississippi \}. 
   Answer ________

d) Which of the following sets is equal to the set \{A, C, E\}? 
   A. \{1, 2, 3\} 
   B. \{C, E, A\} 
   C. \{C, 2, 3\} 
   D. All of the above 
   Answer ________
PERFORMANCE OBJECTIVE I-10

Find the intersection of two given sets.

a) Determine the intersection of \{1, 2, 3, 4, 5\} and \{3, 4, 5, 6\}.
   Answer: 

b) Given: \(A = \{b, c, d\}\)
   \(B = \{e, f\}\),
   then \(A \cap B = \) 

c) Given \(A = \{1, 3, 5\}\)
   \(B = \{2, 4, 6\}\),
   then \(A \cap B = \) 

d) Given \(I = \{1, 2, 3, 4, 5\}\)
   \(J = \{2, 4, 6\}\)
   \(K = \{1, 3, 5\}\),
   then \(I \cap J = \), \(J \cap K = \),
   and \((I \cap J) \cap K = \) 

PERFORMANCE OBJECTIVE I-11

Find the union of two given sets.

a) Given: \( A = \{1, 2, 3, 4\} \)
\( B = \{3, 4, 5, 6\} \)
them \( A \cup B = \______ \)
A. \( \{3, 4\} \)
B. \( \{1, 2, 3, 3, 5, 6\} \)
C. \( \{1, 2, 3, 4, 5, 6\} \)
D. \( \emptyset \)

b) Given: \( A = \) letters in the word \textit{hot} \)
\( B = \) letters in the word \textit{shoot} \)
find \( A \cup B \).
Answer \______

c) Given: \( A = \{13, 17, 23, 29\} \)
\( B = \{\text{prime numbers less than 25}\} \)
find \( A \cup B \).
Answer \______

d) Given: \( A = \{4, 8, 12, 16\} \)
\( B = \{8, 16, 24\} \)
then \( A \cup B = \______ \)
A. \( \{8, 16\} \)
B. \( \{4, 8, 8, 12, 16, 16\} \)
C. \( \{4, 8, 12, 16, 24\} \)
D. \( \emptyset \)
Construct Venn diagrams to show the relationship between sets.

a) Construct a Venn diagram to show $A \cap B$.

Answer

b) Construct a Venn diagram to show $A \cap B = \emptyset$.

Answer

c) Construct a Venn diagram to show $A \subseteq B$.

Answer

d) Construct a Venn diagram to show $A \cup B$.

Answer
e) Construct a Venn diagram which shows the relationship between the following sets:

- \( A = \{1, 2, 3, 4, 5, 6\} \)
- \( B = \{4, 5, 6, 7, 8, 9\} \)
- \( C = \{3, 4, 5, 7, 10\} \)
- \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \)

Answer
PERFORMANCE OBJECTIVE 1-13

Find the complement of a given set.

a) Given that \( A \subseteq \{\text{whole numbers}\} \) and \( A = \{\text{even whole numbers}\} \), find the complement of \( A \).

Answer

b) Given: \( A \subseteq \{\text{whole numbers}\} \) and \( A = \{\text{positive and negative whole numbers}\} \), find the complement of \( A \).

Answer

c) Given that \( A \subseteq \{\text{real numbers}\} \) and \( A = \{\text{positive real numbers}\} \), find \( \bar{A} \).

Answer

d) Given: \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \), 
   \( A = \{2, 4, 6, 8, 10\} \), 
   \( B = \{1, 3, 5, 7, 9\} \), 
   \( C = \{2, 3, 4\} \), and 
   \( D = \{1, 2, 3, 7, 8, 9\} \), find:

(1) \( \bar{A} = \)

(2) \( \bar{B} = \)

(3) \( \bar{C} = \)

(4) \( \bar{D} = \)


UNIT I - SETS

Answers

1. a) C
   b) 1. $\in$

2. C

3. -

4. $\cap$

5. c) 1. C
   2. A
   3. B

4. D

d) 1. C
   2. A
   3. B
   4. D

6. a) $\{1, 2, 3\}$ $\{1, 2, 3\}$ $\{1, 2, 3\}$ $\{1, 2, 3\}$

b) D

c) C

d) D

7. a) finite
b) infinite
c) finite
d) finite
e) finite
f) infinite
UNIT I - SETS

Answers (continued)

8. a) Any set of 4 elements
   b) A
   c) A, B
   d) D

9. a) B
   b) \{ p, i, s, m \}
   c) B
   d) B

10. a) \{ 3, 4 \}
    b) \{ c, d \}
    c) \phi
    d) 1. \{ 2, 4 \}
        2. \phi
        3. \phi

11. a) C
    b) \{ s, h, o, t \}
    c) \{ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 \}
    d) C

12. a) [Diagram of sets A and B]
   b) [Diagram with empty set]
   c) [Diagram with set A inside B]
   d) [Diagram with overlapping sets A and B]
Answers (continued)

12. (continued)

e) 

13. a) \{ \text{odd whole numbers} \}
b) \{0\}
c) \{0, \text{negative real numbers} \}
d) 1. \{1, 3, 5, 7, 9\}
2. \{2, 4, 6, 8, 10\}
3. \{1, 5, 6, 7, 8, 9, 10\}
4. \{4, 5, 6, 10\}
UNIT II - FUNDAMENTAL CONCEPTS

PURPOSE
This unit serves as a foundation for the development of elementary algebra. Through mastery of the symbols, terminology, and computational conventions, the students are prepared to use the language of algebra.

OVERVIEW
Computations involving order of operations and symbols of inclusion rules are stressed. These skills are applied to the evaluation of algebraic expressions for specific replacements of the variable. These two concepts are then integrated into the translation of word phrases into algebraic expressions. In this unit and subsequent units, open sentence refers to both equations and inequalities. The word coefficient refers to the numerical coefficient.

SUGGESTIONS TO THE TEACHER
Instructional Days: 6-7
Minimal Course Objectives: ALL
Average Course Objectives: ALL
Maximal Course Objectives: ALL

The concepts of sets developed in unit one should be extended to include the ideas of domain and solution set.

The parallel between the strategies of learning algebra and learning a foreign language is helpful.

An analogy can be made between the use of punctuation marks in English and the use of symbols of inclusion in mathematics. Mnemonic devices and other memory aids can be used to help students retain some key ideas (example: My Dear Aunt Sally can help students remember that multiplications and divisions are performed before additions and subtractions in the order of operation).

VOCABULARY

<table>
<thead>
<tr>
<th>algebraic expression</th>
<th>evaluate</th>
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UNIT II - FUNDAMENTAL CONCEPTS

ENTERING PERFORMANCE OBJECTIVES

1. Compute the value of an arithmetic expression using the order of operations.

2. Compute the value of an arithmetic expression containing symbols of inclusion.

3. Place the proper symbol, <, >, or =, between two numerical expressions to make a true statement.

4. Simplify arithmetic expressions containing exponents.

Assessment Tasks

1. Simplify each expression:
   a) $27 - 5 \cdot 3 + 24 \div 3$
   b) $18 \div \frac{1}{2} + 16 \cdot \frac{1}{2}$
   c) $25 \times 4 + 20 \div 4$
   d) $5 \times 8 \div 2 + 3 \times 6$

2. Simplify each expression:
   a) $17 + [ (8 + 14) \div 2 ] - 9$
   b) $\frac{19 \times 3 - 10 \cdot 3}{4 + 5}$
   c) $[ (15 + 20) \div 5 ] + 6 \cdot 2^2$
   d) $7 \{ (51 - 27) \div [2 (9 + 3)] \}$

3. Insert <, >, or = to make a true statement:
   a) $4 \cdot 0$ ________ $0 + 4$
   b) $\frac{18 + 26}{2}$ ________ $\frac{5 \times 6 + 11 \times 13}{7 - 4}$
   c) $10 + 3^2$ ________ $4^2 + 8$
   d) $\frac{429}{3} + 7$ ________ $5 \times .5 \times 6$
4. Simplify each expression:
   a) \( \frac{2^2 + 3}{3^2 - 2^3} \)
   b) \( 2^4 - 2^2 - 2 \)
   c) \( 2 \cdot 3^3 - 2 \cdot 3^2 - 3^2 + 5 \)
   d) \( 6^3 - 3^2 - 2^2 + 7 \)
UNIT II - FUNDAMENTAL CONCEPTS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) 20  
   b) 44  
   c) 35  
   d) 38

2. a) 19  
   b) 3  
   c) 31  
   d) 7

3. a) <  
   b) >  
   c) <  
   d) =

4. a) 1  
   b) 13  
   c) 32  
   d) 13
PERFORMANCE OBJECTIVES

1. Identify the following:
   a) Factor
   b) Term
   c) Coefficient
   d) Exponent
   e) Base
   f) Power
   g) Variable
   h) Constant  (I)

2. Compute the value of a numerical expression using the order of operations rules. (II)

3. Compute the value of a numerical expression punctuated with symbols of inclusion. (II)

4. Evaluate algebraic expressions when given numerical replacements for the variables. (II)

5. Place the proper symbol (=, >, <) between two numerical expressions to make a true statement. (II)

6. State the number of terms in a given algebraic expression. (II)

7. Translate word phrases into algebraic expressions. (III)

8. Determine the solution set of an open sentence by replacing the variable with elements of a given domain. (III)

   Minimal    Average    Maximal
   All        All        All

KEY SKILLS FOR END-OF-COURSE TESTING

2. Compute the value of a numerical expression involving symbols of inclusion and the order of operations rules.

3. Evaluate algebraic expressions and open sentences by substituting for the variable.
### CROSS REFERENCES

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### CROSS REFERENCES

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### PERFORMANCE OBJECTIVE II-1

<table>
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<th>Identify the following:</th>
<th>a) factor</th>
<th>d) exponent</th>
<th>g) variable</th>
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<tr>
<td></td>
<td>b) term</td>
<td>e) base</td>
<td>h) constant</td>
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<td></td>
<td>c) coefficient</td>
<td>f) power</td>
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#### a) The coefficient of $x$ in $3x + 2y^2 + 6$ is:
- A. 1
- B. 2
- C. 6
- D. 3

Answer ________________

#### b) Use the expression $3x^2 + 7$ to answer the following.
1. The coefficient is ________.
2. The exponent is ________.
3. The variable is ________.
4. The constant is ________.
5. The base is ________.
6. The power of the base is ________.

#### c) Which of the following expressions contains an exponent of 2?
- A. $x + 2$
- B. $2x$
- C. $x^2$
- D. None of the above

Answer ________________

#### d) Which of the following expressions contains a coefficient of 1?
- A. $x$
- B. $x + 1$
- C. $x^3$
- D. All of the above
- E. None of the above

Answer ________________
Compute the value of a numerical expression using the order of operations rules.

a) Simplify the following expression \(20 \div 2 - 20 \div 10\).
   A. -1
   B. 8
   C. 20
   D. undefined
   Answer __________

b) Simplify the following expression \(28 + 0 \div 4 - 10 \times 2\).
   A. 8
   B. -6
   C. 36
   D. 13
   Answer __________

c) Give the first step in simplifying \(16 + 2 \times 10 + 13\).
   A. Add 16 and 2.
   B. Multiply 2 and 10.
   C. Add 10 and 13.
   D. None of the above
   Answer __________

d) Simplify the following:
   1. \(8 \div \frac{1}{2} + 3 \times 2\)
      Answer __________
   2. \(7 \times 2 - 2^2 \times 2 \div 4 + 2\)
      Answer __________
   3. \(\frac{2 + 3 \times 5^2}{12 \times 3 \div 4} + 2\)
      Answer __________
PERFORMANCE OBJECTIVE II-3

Compute the value of a numerical expression punctuated with symbols of inclusion.

ASSESSMENT TASKS

a) Simplify $36 + 4\left[1 + (12-8) \times 2\right]$. b) $4(7 + 9)$ means

A. 72
B. 76
C. 360
D. 400

Answer ________________   Answer ________________

b) Simplify.

1. $9 \times (11 + 3) \div 60$

   Answer ________________

2. $13 + [99 - (13 \times 7)]$

   Answer ________________

3. $(5 \times 7) + (40 \div 2) \div 7 + (16 \div 4)$

   Answer ________________

4. $5\left\{\left[4 \times (36 \div 3)\right] + 2\right\}$

   Answer ________________

   Answer ________________

   Answer ________________
PERFORMANCE OBJECTIVE II-4

Evaluate algebraic expressions when given numerical replacements for the variables.

a) Find the value of each expression if \( a = 1, \; b = 2, \; c = 12, \) and \( d \neq 0 \).

1. \( 5a + 3b \)
   Answer

2. \( 3b^3 + 3c \)
   Answer

3. \( \left( (a + b)(b + c) \right)d \)
   Answer

4. \( \frac{b + c}{2a - b} \)
   Answer

b) Find the value of each expression if \( a = 1, \; b = 2, \; c = 3, \; x = 12, \; y = 0, \) and \( z = \frac{1}{2} \).

1. \( cx - a \)
   Answer

2. \( bx + a \)
   Answer

3. \( (2b + 1)^2(a + c) \)
   Answer

4. \( ay + x^2 - bz \)
   Answer

c) Find the perimeter of a rectangle whose length is 7.3 meters and width is 4.2 meters, using the formula \( P = 2l + 2w \).

Answer

d) Use the following two formulae to find equivalent temperatures:

\( F = \frac{9}{5} C + 32^\circ \)

\( C = \frac{5}{9} (F - 32^\circ) \)

1. \( C = 20^\circ, \; F = \) ________

2. \( F = 95^\circ, \; C = \) ________
PERFORMANCE OBJECTIVE II-5

Place the proper symbol (<, >) between two numerical expressions to make a true statement.

Replace each ? with =, <, > to make each statement true.

a) \[ \frac{10 + 2}{2} \ ? \ 10 + 1 \]
   Answer ____________

b) \[ 3 \frac{1}{3} \ ? \ 2 + 1 \frac{3}{9} \]
   Answer ____________

c) \[ \frac{1}{2} \ ? \ 1/3 \]
   Answer ____________

d) \[ 11 \times (6 + 7) \ ? \ \{5 \times 5\} \times 6 \]
   Answer ____________

e) \[ 50 \div (2 \times 6) \ ? \ (16 \div 4) \div 2 \]
   Answer ____________

f) \[ (9 + 45) \div (18 \div 3) \ ? \ (72 \div 9) - 3 \]
   Answer ____________
PERFORMANCE OBJECTIVE II-6

State the number of terms in a given algebraic expression.

State the number of terms in each expression.

a) \( xy - wz \)  
   Answer

b) \( \frac{4(3xy - 15)}{73} \)  
   Answer

Multiple-choice. Determine the number of terms in the following expressions.

c) \( (x - y) + \frac{x}{bc} \)  
   A. 1  
   B. 2  
   C. 3  
   D. 4  
   Answer

d) \( \frac{ab - bc}{xy + yz} \)  
   A. 1  
   B. 2  
   C. 3  
   D. 4  
   Answer
PERFORMANCE OBJECTIVE II-7

Translate word phrases into algebraic expressions.

a) Match each word phrase with the correct algebraic expression.

1. sum of two numbers
   A. \( xy \)
   B. \( p/q \)
   C. \( r - s \)
   D. \( s + r \)

2. difference of two numbers
   A. \( 2n \)
   B. \( 2n + 1 \)
   C. \( 2n - 1 \)
   D. \( 2n < 1 \)

3. product of two numbers
   A. \( \)  
   B. \( \)  
   C. \( \)  
   D. \( \)  

4. quotient of two numbers
   A. \( \)  
   B. \( \)  
   C. \( \)  
   D. \( \)  

b) Choose the open expression which describes the word phrase, "One less than twice the number of field goals."

For each word phrase, write an algebraic expression.

c) the sum of five and twice \( w \)
   Answer \( \)  

d) three more than the product of seven and \( t \)
   Answer \( \)  

e) the quotient of \( x \) and seven, decreased by five
   Answer \( \)  

f) fifteen decreased by \( n \)
   Answer \( \)  

g) 6 less than \( x \)
   Answer \( \)  

II-1563
PERFORMANCE OBJECTIVE II-8

Determine the solution set of an open sentence by replacing the variable with elements of a given domain.

Using the domain \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, find the solution set of the equation.

a) \(2x + 1 = 11\)
Answer

b) \(\frac{3}{2}x + 4 = 10\)
Answer

c) \(2x - 1 < 7\)
Answer

d) \(2x + 3 \leq x + 4\)
Answer
UNIT II - FUNDAMENTAL CONCEPTS

Answers

1. a) D
   b) 1. 3
   2. 2
   3. x
   4. 7

2. a) B
   b) A
   c) B
   d) 1. 22
      2. 56
      3. 7

3. a) A
     b) B
     c) 1. 2 + 3(4 - 2) = 8
        2. 5 + [(6 - 4) x 3] ≠ 11
        3. (3 + 4 - 6) x 0 = 0
        4. [(6 + 5) x 3 - 7] ÷ 13 = 2

4. a) 1. 11
   2. 60
   3. 0
   4. undefined

5. x

6. a) 2
   b) 1
   c) B
   d) A

7. a) 1. D
    2. C
    3. A
    4. B

8. a) 5
    b) Ø
    c) 1, 2, 3
    d) 1

   23m
   60
   1. 35
   2. 324
   3. 100
   4. 143

   1. 68
   2. 50
   3. 20
   4. 76
UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

PURPOSE

Unit III provides the students with an introduction to the rules and properties that govern the language of algebra under the system of real numbers.

These properties are integrated with the basic operations on directed numbers to prepare the student to solve open sentences. These ideas add to the base upon which the course will be built.

OVERVIEW

A working knowledge of the basic properties of real numbers is an expected outcome. In a minimal or average course, emphasis should be placed on the applications of these properties; while in a maximal course, a more comprehensive understanding of the theory should be stressed.

Computations with directed numbers are reviewed, and the concept of absolute value is introduced. It is mandatory that all students demonstrate minimal competencies in working with directed numbers before proceeding to the next unit.

SUGGESTIONS FOR THE TEACHER

Instructional Days: 8-9
Minimal Course Objectives: #4-11, 13
Average Course Objectives: ALL
Maximal Course Objectives: ALL

Most texts relate real numbers to the horizontal number line. The concept of operations on positive and negative numbers can also be illustrated by examples dealing with up and down, gain (profit) and loss, right and left, above and below, north and south, east and west, or good and bad.

The term directed number is used in this unit instead of real number to emphasize the positive-negative aspects without restricting problems to integers or involving irrational numbers.

The intention of Objective 3 (identify irrational numbers) is that the student realize there exist some real numbers which are not integers, fractions, terminating decimals, or repeating decimals and that students will be able to give some examples of irrationals, such as \( \sqrt{2} \), \( \pi \), .12345678910, ...

It is important to stress absolute value and its use in addition and subtraction of directed numbers.

Some tricks to help students remember the rules for multiplying and dividing directed numbers may be useful. One such trick involves using the idea of
"good" to represent positive numbers and "bad" to represent negative numbers. Hence, the product of a negative number and a positive number could be remembered by the saying: "If bad things happen to a good person, that's bad." Similar sayings are possible for the other permutations.

It is important to realize that the subtraction sign, negative sign, opposite sign, and opposite of a sum often are difficult concepts for students to understand.

Fractions and decimals can be reviewed by including them in practice problems.

**VOCABULARY**

absolute value  
additive identity  
additive inverse (opposite)  
associative property of addition  
associative property of multiplication  
axiom  
closure property of real numbers  
commutative property of multiplication  
distributive property of multiplication with respect to addition  
integers (Z)  
irrational numbers (Ir)  
like terms  
magnitude  
multiplicative identity  
multiplicative inverse (reciprocal)  
multiplicative property of zero  
non-terminating, non-repeating decimals  
postulate  
property  
rational numbers (Q)  
real numbers  
reflexive property of equality  
repeating decimals  
similar terms  
substitution principle  
symmetric property of equality  
terminating decimals  
transitive property of equality
UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

ENTERING PERFORMANCE OBJECTIVES

1. Add, subtract, multiply, and divide directed numbers.

2. Evaluate algebraic expressions.

Assessment Tasks

1. a) \(-11 + 5\)  
   b) \(9 + (-3)\)  
   c) \(-7 + (-11)\)  
   d) \(-5 + (-32)\)  
   e) \(-\frac{6}{2} - (-\frac{3}{2})\)  
   f) \(7\frac{1}{2} - 9\frac{1}{2}\)  
   g) \(300 - (-200)\)  
   h) \(3 \cdot 12\)  
   i) \((-0.25) \cdot (-8)\)  
   j) \(-7 \cdot (21)\)  
   k) \((-2) \cdot (3\frac{1}{2})\)  
   l) \((-2) \cdot (-5) \cdot (-3)\)  
   m) \(18 ÷ (-2)\)  
   n) \((-26) ÷ (-8)\)  
   o) \((-12) ÷ \frac{3}{2}\)  
   p) \((-\frac{5}{9}) ÷ (-\frac{9}{5})\)

2. If \(a = -5\), \(n = 7\), \(t = 10\) and \(m = -3\), find the value of each of the following:  
   a) \(\frac{a + t + 4}{m} + n\)  
   b) \(a^2 + n^2 + 1\)  
   c) \(\frac{a^2 - m^2}{-2n^2}\)  
   d) \(\frac{1}{2}mt^2\)

III-3
UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) - 6
   b) - 6
   c) - 18
   d) - 37
   e) - 3
   f) - 2
   g) - 500
   h) - 9
   i) - 2
   j) - 147
   k) - 7
   l) - 30
   m) - 9
   n) - 3 1/4
   o) - 24
   p) - 25/81

2. a) - 4
   b) - 3
   c) - 1/7
   d) - 150
UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

PERFORMANCE OBJECTIVES

1. Determine whether a given set of numbers is closed under a given operation. (III)

2. Identify the following properties:
   a) reflexive property of equality
   b) symmetric property of equality
   c) transitive property of equality
   d) commutative (+)
   e) commutative (x)
   f) associative (+)
   g) associative (x)
   h) distributive property of multiplication with respect to addition
   i) additive identity
   j) multiplicative identity
   k) additive inverse
   l) multiplicative inverse
   m) multiplicative property of zero (I)
   n) substitution principle

3. Distinguish between rational and irrational numbers. (III)

4. State the absolute value of a given real number. (II)

5. Add directed numbers. (II)

6. State the additive inverse (opposite) of a given real number. (II)

7. Subtract directed numbers. (II)

8. Multiply directed numbers. (II)

9. State the multiplicative inverse (reciprocal) of a given real number. (II)

10. Divide directed numbers. (II)
11. Simplify mathematical expressions involving directed numbers. (II)
12. Simplify numerical expressions containing absolute values. (II)
13. Simplify algebraic expressions by combining similar (like) terms. (II)

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KEY SKILLS FOR END-OF-COURSE TESTING

4. Add and subtract directed numbers.
5. Multiply and divide directed numbers.
6. Combine similar terms.
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### UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

#### CROSS REFERENCES

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III-8
Determine whether a given set of numbers is closed under a given operation.

Determine whether the given set is closed under the given operation.

a) \{1, 2\}, addition
b) \{0, 1\}, multiplication
c) \{2, 4, 6, \ldots\}, division

Under which operation(s) (+, -, *, /) are the following sets closed?

d) \{0, 2, 4, 6\}
e) \{3, 6, 9, 12, \ldots\}
f) \{Integers\}
PERFORMANCE OBJECTIVE III-2

Identify the following properties: reflexive, symmetric, and transitive; commutative property of addition, commutative property of multiplication; associative property of addition, associative property of multiplication; distributive property of multiplication with respect to addition; additive inverse, multiplicative inverse; multiplicative property of zero; substitution principle; additive identity, multiplicative identity.

a) Match each property with the appropriate example:

1. If $5 + 0 = 5$, then $5 = 5 + 0$
   A. Substitution principle

2. $5(x + y) = 5x + 5y$
   B. Commutative property of addition

3. $(r + 3)19 = 19(r + 3)$
   C. Commutative property of multiplication

4. $x + (7 + 6x) = x + (6x + 7)$
   D. Symmetric property of equality

5. $97(100 + 1) = 97(101)$
   E. Distributive property of multiplication with respect to addition

b) State algebraically the following properties:

1. Associative property of addition

2. Transitive property of equality

3. Multiplicative identity
Identify the following properties: reflexive, symmetric, and transitive; commutative property of addition, commutative property of multiplication; associative property of addition, associative property of multiplication; distributive property of multiplication with respect to addition; additive inverse, multiplicative inverse; multiplicative property of zero; substitution principle; additive identity, multiplicative identity.

c) Name the property which justifies each step in the following:

1. \(16 + (27 + 14) = 16 + (14 + 27)\)
2. \(16 + (14 + 27) = (16 + 14) + 27\)
3. \((16 + 14) + 27 = 30 + 27\)
4. \(30 + 27 = 57\)


d) For all real numbers a and b:

1. \((a + 3) + (5 + b) = [(a + 3) + 5] + b\)
2. \(a + (3 + 5) + b\)
3. \((a + 8) + b\)
4. \((8 + a) + b\)
5. \(8 + (a + b)\)
Distinguish between rational and irrational numbers.

a) True or false?

1. \{irrational numbers\} \subset \{rational numbers\}
2. \{irrational numbers\} \subset \{real numbers\}
3. \{0\} \subset \{irrational numbers\}
4. \{integers\} \subset \{irrational numbers\}

Identify each number as rational (Q) or irrational (Ir).

b) 0

c) \sqrt{2}

d) \frac{1}{3}

e) \ldots .1211211121112\ldots
PERFORMANCE OBJECTIVE III-4

State the absolute value of a given real number.

Simplify:

a) \(|-28.5| = \ldots\)

b) \(|0| = \ldots\)

c) \(|16| = \ldots\)

d) True or false?

___ 1. The absolute value of a negative number is positive.

___ 2. The absolute value of a positive number is negative.

___ 3. The absolute value of 0 is 0.

___ 4. The absolute value of a positive number is positive.
Add directed numbers.

a) Add each of the following:

1. \(-17 + 8\)  
2. \(-28\)  
3. \(-28\)  
4. \(-15 + 42\)

b) Add each of the following:

1. \(+27 + (-9 + -13)\)
2. \(-7.5 + 8.8\)
3. \(-5\frac{1}{4} + 8\frac{1}{2}\)
4. \(-6 + -4\)

c) True or false?

1. \(-6 + -9 = 15\)
2. \(9 + -3 > -6\)
3. \((-3 + 4) + -1 = -3 + (4 + -1)\)
4. \(5 + -3 > -5 + 3\)

d) Add the following:

1. \(-98 + (-2 + 53)\)
2. \(-101 + (-99 + 28)\)
3. \((19 + -16) + (11 + -23)\)
4. \((-1.2 + -5.8) + 4\)
PERFORMANCE OBJECTIVE III-6

State the additive inverse of a given real number.

Give the opposite of each number:

<p>| | |</p>
<table>
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</table>
a) 1. 13 |   |
| 2. -16 |   |
b) 1. 0 |   |
| 2. -\( \frac{1}{3} \) |   |
c) 1. a |   |
| 2. -b |   |
d) 1. \( \frac{1}{3} \) |   |
| 2. -\( \frac{1}{2} \) |   |
PERFORMANCE OBJECTIVE III-7

Subtract directed numbers.

a) Match each difference with the corresponding sum:

1. 9 - 6
   - A. -9 + 6
2. -9 - 6
   - B. -9 + -6
3. -9 - -6
   - C. 9 + -6
4. 9 - -6
   - D. 9 + 6

Subtract:

b) -45 + 17
   - Answers

c) -38 - (-12)
   - Answers

d) +15 - +18
   - Answers
PERFORMANCE OBJECTIVE III-8

Multiply-directed numbers.

a) Find the value for x that makes each statement true.

1. \(-3x = 9\)
2. \(-x = 4\)
3. \(-2x (-1) = 10\)
4. \(x(-3)(-2) = -30\)

Multiply the following:

b) 1. \((-13)(+3) = \)
2. \((-6)(0) = \)

c) 1. \(-4(-3) = \)
2. \(-6(2) = \)

d) 1. \((-4)3 = \)
2. \(2(-2) = \)

Answers

III-17 82
PERFORMANCE OBJECTIVE III-9

State the multiplicative inverse of a given real number.

State the multiplicative inverse for each of the following:

Answers

1.

2. -1

b) 1. \( \frac{5}{3} \)

2. -\( \frac{7}{2} \)

c) 1. \( \frac{1}{2} \)

2. 0

d) 1. \( \frac{1}{4} \)

2. -2
PERFORMANCE OBJECTIVE III-10

Divide directed numbers.

a) Write each quotient as a product:

1. \(-36 ÷ -2\)
2. \(36 ÷ -\frac{1}{2}\)
3. \(\frac{\frac{14}{2}}{3}\)
4. \(\frac{\frac{1}{2}}{5}\)

Divide each of the following:

b) \(-144 ÷ +9\)

c) \(-75 ÷ -15\)

d) \(+100 ÷ +20\)
PERFORMANCE OBJECTIVE III-11

Simplify mathematical expressions involving directed numbers.

Let \( a = 2, \ b = -7, \ c = 6, \ d = -5, \ e = 4 \) and \( f = -1 \). Evaluate each of the following expressions.

\[
\begin{align*}
\text{a) } & \quad abc - def \\
\text{b) } & \quad \frac{abc}{e} \\
\text{c) } & \quad \frac{ab + cd}{ef} \\
\text{d) } & \quad \frac{af - de}{b + c}
\end{align*}
\]
PERFORMANCE OBJECTIVE-III-12

Simplify numerical expressions containing absolute values.

Simplify both a) and b).

Answers:

a) 1. \(-|-9|\)
2. \(-|-7|\)
3. \(-|0|\)
4. \(-(|-6|)\)

b) 1. \(7|-6|\)
2. \(3|-4|+|-3|\)
3. \(-1|-11+7|\)
4. \(4|-7|+|-12|\)

c) State whether x or y represents the greater real number.

1. The absolute value of x is greater than the absolute value of y.
   Both x and y are positive.

2. The absolute value of x is less than the absolute value of y.
   Both x and y are negative.

3. The absolute value of x is greater than the absolute value of y.
   Both x and y are negative.

d) Choose two numbers, m and n, such that the absolute value of m is greater than the absolute value of n and m is less than n.

Answer
PERFORMANCE OBJECTIVE III-13

Simplify algebraic expressions by combining similar terms.

Simplify by combining similar terms.

\[ a) \ 1. \ 9r - 5r \\
2. \ -6m + m \]
\[ b) \ 1. \ 8x + 5y - 3x + y \\
2. \ -7x + 7 + 3x -3 \]
\[ c) \ 1. \ -3r + 3s + 2r -2s \\
2. \ -2x + 2(2x - 4) \]
\[ d) \ 1. \ -r - r -3r \\
2. \ 4r - 3(r + 1) + 3 \]
UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

Answers

1. a) not closed
   b) closed
   c) not closed
   d) none
   e) +, x
   f) +, -, x

2. a) 1. D
   2. E
   3. C
   4. B
   5. A
   b) 1. \( x + (y + z) = (x + y) + z \)
   \[ 2. \text{If } a = b \text{ and } b = c \text{ then } a = c \]
   3. \( 1(x) = x \)
   c) 1. Commutative (+)
   2. Associative (+)
   3. Substitution
   4. Substitution
   d) 1. Associative (+)
   b) 1. +5

3. a) 1. F
   2. T
   3. F
   4. F
   b) Q
   c) Ir
   d) Q
   e) Ir

4. a) 28.5
   b) 0
   c) 16
   d) 1. T

5. a) 1. -9
   2. -64
   3. -28
   4. +27
   b) 1. +5
   2. 1.3
   3. -31/4
   4. -10
   5. associative (+)
UNIT III - REAL NUMBERS: PROPERTIES AND OPERATIONS

Answers (continued)

5. c) 1. T
    2. T
    3. T
    4. T
    d) 1. -47
    b) 1. -39
    2. -172
    3. -9
    4. -3.0

6. a) 1. -13
    2. 16
    b) 1. 0
    2. 1 3
    c) 1. -a
    2. b
    d) 1. -1 3
    2. 1 2

7. a) 1. C
    2. B
    3. A
    4. D
    b) -62
    c) -26
    d) -3

8. a) 1. -3
    2. -4
    3. 5
    4. -5

9. a) 1. 1
    2. 1 3
    c) 1. -a
    2. b
    d) 1. -1 3
    2. No reciprocal

10. a) 1. -36 x - 1 2
    2. 36 x -2
    3. 14 x 3 2
    4. 1 1 x 1 2
    b) -16
    c) 5
    d) -5

11. a) -104
    b) -21
    c) 11
    d) -18

12. a) 1. -9
    2. 7
    3. 0
    4. -6
    b) 1. 42
    2. 15
    c) 1. x
    2. y
    d) Answer varies

13. a) 1. 4 8
    2. -5 8
    b) 1. 5x + 6y
    2. -4x + 4
    c) 1. -r + s
    2. 2x - 8
    d) 1. -5r
    2. r
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

PURPOSE

This unit provides students with the opportunity to apply the skills mastered in previous units. Emphasis is on the techniques for solving open sentences. Students are introduced to equations and inequalities that require a sequence of transformations. A systematic approach to work problem analysis and solution is developed. This unit is the keystone of the course.

OVERVIEW

Basic transformations for solving open sentences are presented. They are applied to equations of varying complexity.

Inequalities are treated in a similar manner. Emphasis is placed on the difference between the multiplication and division transformations for equality and the corresponding transformations for inequality.

Specific techniques for analyzing and solving word problems are discussed. These techniques are applied to the solution of consecutive integer, age, geometry, uniform motion, and mixture problems.

SUGGESTIONS TO THE TEACHER

Instructional Days: 20-25
Minimal Course Objectives: #1-8, 10, 12-15
Average Course Objectives: ALL except #11
Maximal Course Objectives: ALL

Due to the length of this unit, it is suggested that at least two tests be given. Emphasize to students that transformed equations are always equivalent to the original equations. They have the same solution set.

Students frequently have difficulty in deciding what order to apply transformations to solve equations of the form $ax + b = c$. A technique to use in explaining the solution of these is the following analogy to unwrapping a birthday present. In opening a birthday present that is wrapped in paper and tied with a ribbon, one could remove either first. However, it would be much easier to remove the ribbon prior to removing the paper. In like manner, in equations of the form $ax + b = c$, it is frequently easier to remove the constant term before removing the coefficient.

Flow charts can be very useful in teaching the procedure for solving equations. It is strongly suggested that a check of all solutions be required.

The importance of proficiency in the reading of word problems cannot be over-emphasized. The SQ3R method (survey, question, read, recite, and review) can be a valuable approach to use.
Objectives #12-15 suggest an approach for attacking word problems. Teachers may want to use their own strategies, and suggestions would be welcomed by the math coordinator.

The use of calculators may be desirable to facilitate computations in the solution of word problems, particularly when decimals are involved.

VOCABULARY

addition property of equality
addition property of inequality
angle
comparison property
complementary angles
conjunction
consecutive integers
degree
disjunction
distance
division property of equality
division property of inequality
equation
equivalent equation
equivalent inequality
inequality

left member
linear equation
multiplication property of equality
multiplication property of inequality
open sentence
rate
ray
right member
root
solution set
subtraction property of equality
subtraction property of inequality
supplementary angles
transformations
transition property of inequality
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

ENTERING PERFORMANCE OBJECTIVES

1. Add, subtract, multiply, and divide directed numbers.
2. Name the opposite and reciprocal of a number.
3. Combine similar terms.
4. Graph a set on a number line.
5. Multiply algebraic expressions using the distributive property.

Assessment Tasks

1. a) \((-2 \frac{1}{3}) + 2\)  
b) \((-8) + 4 \frac{1}{2}\)  
c) \(143 + (-482)\)  
d) \(-6.4 + 16.6 + (-12.9) + 8.4\)  

2. a) \(-0.6 - 0.4\)  
b) \(-0.50 - (-0.20)\)  
g) \(-10 - 32\)  
h) \(12 - [-(5 + 3)]\)  
i) \(-3(0.321)\)  
j) \((- \frac{1}{5})(\frac{3}{4})\)  
k) \((-3)(-5)(2)\)  
l) \((-0.5)^3\)

3. m) \(3.5 : (-.7)\)  
n) \(-3\frac{3}{5} : (-\frac{9}{10})\)  
p) \(108 : (-6)\)  
q) \(-.170 : (-.05)\)
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. a) State the opposite of each of the following numbers:
   (1) -5
   (2) 6.37
   (3) -4
   (4) 0

b) State the reciprocal of each of the following numbers:
   (1) 7
   (2) \(-\frac{3}{4}\)
   (3) .7
   (4) x

c) Which of the following numbers is the reciprocal of 1.5?
   (1) -1.5
   (2) \(\frac{3}{2}\)
   (3) \(\frac{2}{3}\)
   (4) \(1 \frac{1}{2}\)

   Answer:

   d) If x is a negative number, which of the following is the opposite
      of x?
   (1) x
   (2) \(\frac{1}{x}\)
   (3) \(-\frac{1}{x}\)
   (4) -x

   Answer:
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

3. Combine similar terms.

a) \((4 - 9t^2 + 15t^4) + (3t^4 + 2t^2 - 1)\)

b) \((8x^3 - x^2 - 5x + 6) + (14x^3 - 7x^2 + 10x + 15)\)

c) \((17m^2 - 13m - 6) - (3 + 7m + 17m^2)\)

d) \((2x + 7) - \left(\frac{1}{2}x + 9\right)\)

4. Graph the following sets on a horizontal number line:

a) \{whole numbers less than 8\}

b) \{integers between -5 and 3\}

c) \{real numbers greater than -2\}

d) \{real numbers between -7 and 3, including 3\}

5. Simplify each expression.

a) \(7(x^2 + y) + 3(x^2 + y) + 9y\)

b) \(8(2x^2 - 4x + 7) + 3(12x^2 - 5x)\)

c) \(2(3x^2 - 5x - 2) - 3(8 - 2x^2 + 7x)\)

d) \(-3(2m^2 - m + 4) - (2m^2 + 3m - 8)\)
ENTRÉER PERFORMANCE OBJECTIVES

Answers

1. a) \(- \frac{1}{3}\)  
   b) \(-3 \frac{1}{2}\)  
   c) \(-339\)  
   d) 5.7  
   e) -1.0  
   f) -0.30  
   g) -42  
   h) 20  
   i) -0.963  
   j) -\(\frac{3}{20}\)  
   k) 30  
   l) -0.125  
   m) -5  
   n) 4  
   o) -18  

2. a) (1) 5  
    (2) -6.37  
    (3) 4  
    (4) 0  
   b) (1) \(\frac{1}{7}\)  
    (2) \(-\frac{4}{3}\)  
    (3) \(\frac{10}{7}\)  
    (4) \(\frac{1}{x}\)  

3. a) \(18t^4 - 7t^2 + 3\)  
    b) \(22x^3 - 8x^2 + 5x + 21\)  
    c) \(-20m - 9\)  
    d) \(\frac{3}{2}x - 2\)  

4. a) \[\begin{array}{cccccccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0
\end{array}\]
   b) \[\begin{array}{cccccccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0
\end{array}\]
   c) \[\begin{array}{cccccccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0
\end{array}\]
   d) \[\begin{array}{cccccccccccc}
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0
\end{array}\]

5. a) \(10x^2 + 19y\)  
    b) \(5x^2 - 47x + 56\)  
    c) \(12x^2 - 3x - 28\)  
    d) \(-8m^2 - 3\)
PERFORMANCE OBJECTIVES

1. Identify the addition, subtraction, multiplication, and division properties of equality. (I)

2. Solve equations of the form \( x + a = c \), using the addition (subtraction) property of equality. (III)

3. Solve equations of the form \( ax = c \), using the multiplication (division) property of equality. (III)

4. Solve equations of the form \( ax + b = c \), using the appropriate properties. (III)

5. Solve equations of the form \( ax + b = cx + d \), during the appropriate properties. (III)

6. Solve equations involving the distributive property of multiplication with respect to addition. (III)

7. Identify the addition, subtraction, multiplication, and division properties of inequality: (I)

8. Solve inequalities, using the appropriate properties. (III)

9. Solve compound inequalities by applying the concepts of union and intersection of sets. (III)

10. Graph the solution set of an inequality or compound inequality on the real number line. (III)

11. Solve open sentences involving absolute value. (III)

12. Translate word statements into open sentences. (III)

13. Write algebraic expressions representing the unknown information in a word problem. (IV)

14. Write an open sentence expressing the relationship stated in a problem. (IV)

15. Solve various types of word problems, utilizing an organized approach. (IV)

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KEY SKILLS FOR END-OF-COURSE TESTING

7. Solve linear equations in one variable applying the properties of equality.

8. Solve linear inequalities in one variable applying the properties of inequality.

9. Solve various types of word problems utilizing an organized approach.
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

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## UNIT IV - SOLVING OPEN SENTENCES' AND WORD PROBLEMS

### CROSS REFERENCES

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PERFORMANCE OBJECTIVE IV-1

Identify the addition, subtraction, multiplication and division properties of equality.

Identify the property of equality that justifies the following transformations.

a) 1) If \( a + x = y \) then \( a + x + 5 = \frac{y + 5}{y + 5} \)
   Answer

b) 1) If \( x = y \) then \( x + a = y + a \)
   Answer

2) If \( a = b \) then \( -3a = -3b \)
   Answer

2) If \( a + x = b \) then \( \frac{a}{x} + \frac{a}{x} = \frac{a}{b} \)
   Answer

c) Match the following properties of equality with their respective names.

   1) If \( a = b \) then \( ac = bc \)
      A. Addition Property

   2) If \( a = b \) then \( a + c = b + c \)
      B. Subtraction Property

   3) If \( a = b \) then \( \frac{a}{c} = \frac{b}{c} \)
      C. Multiplication Property

   4) If \( a = b \) then \( a - c = b - c \)
      D. Division Property

   d) State the property that justifies each step.

   Given: \( 5x - 3 = 2x + 6 \)

   1) \( 5x - 2x - 3 = 2x - 2x + 6 \)
      Answer

   2) \( 3x - 3 = 6 \)
      Answer

   3) \( 3x - 3 + 3 = 6 + 3 \)
      Answer

   4) \( 3x = 9 \)
      Answer

   5) \( \frac{3x}{3} = \frac{9}{3} \)
      Answer

   6) \( x = 3 \)
      Answer
PERFORMANCE OBJECTIVE IV-2

Solve equations of the form \( x + a = b \) using the addition (subtraction) property of equality.

(a) Identify the following pairs of equations as equivalent or not equivalent.

1) \( x + 2 = 7 \)
   \( x + 3 = 8 \)

2) \( y = -4 \)
   \( 7 + y = 3 \)

3) \( -2 + x = 8 \)
   \( x = 6 \)

4) \( 5 + m = 19 \)
   \( (5 + m) + 2 = 19 + 2 \)

Solve each equation. Show all work including a check.

(b) \( m + 7 = 13 \)   Answer:

(c) \( x + 3\frac{1}{2} = 7 \)   Answer:

(d) \( 3.7 = a + 11.2 \)   Answer:
PERFORMANCE OBJECTIVE IV-3

Solve equations of the form ax = c, using the multiplication (division) property of equality.

a) State what number each phrase must be multiplied by to get x as the result.

   1) 3x
   2) -\frac{1}{2}x
   3) ax
   4) \frac{2x}{3}

Solve each equation. Show all work including a check.

b) 13 t = -52  Answer:

c) -8 = -1\frac{1}{5} b  Answer:

d) -14k = 84  Answer:
PERFORMANCE OBJECTIVE IV-4-

Solve equations of the form \( ax + b = c \), using the appropriate properties.

a) Number the following steps in correct order to solve \( 4x - 5 = 3 \).

- A. \( \frac{4x}{4} = \frac{8}{4} \)
- B. \( 4x - 5 + 5 = 3 + 5 \)
- C. \( x = 2 \)
- D. \( 4x = 8 \)

b) Given the equation \( 3x + 4 = 7 \), the most efficient first step in the solution would be:

- A. Multiply each side by \( \frac{1}{3} \)
- B. Divide both sides by 3
- C. Divide \( 3x \) and 7 by 3
- D. Subtract 4 from each side

Answer: 

Solve each equation. Show all work including a check.

c) \( 5x - 9 = -19 \) Answer:

d) \( 8 = 5x + 33 \) Answer:
PERFORMANCE OBJECTIVE IV-5

Solve equations of the form $ax + b = cx + d$, using the appropriate properties.

a) Tell what phrase you would add to each side of the equation to eliminate the variable from the right side.

1) $2x + 3 = 4x - 1$
2) $3 + 5x = 5 - 2x$
3) $x + 7 = 2 - \frac{3}{2}x$
4) $7 - 8x = x + 2$

Solve each equation. Show all work including a check.

b) $7x + 5 = 2x + 35$ Answer:

c) $7y - 5 = 9y + 29$ Answer:

d) $16 + 4y = 10y - 20$ Answer:
Solve equations involving the distributive property of multiplication with respect to addition.

Solve each equation. Show all work including a check.

a) \( 5(3y - 2) = 5 \)  
Answer:

b) \( 7(x + 2) = 5(x + 4) \)  
Answer:

c) \( 3d + 2(6d - 5) = 5 \)  
Answer:

d) \( 3(x + 2) + 3x + 2 = 74 \)  
Answer:
PERFORMANCE OBJECTIVE IV-7

Identify the addition, subtraction, multiplication, and division properties of inequality.

a) Identify each pair of inequalities as equivalent or not equivalent.
   1. \( x + 5 < m \)  
      \( x > m - 5 \)
      Answer ____________
   2. \( 5x < m \)  
      \( x < m/5 \)
      Answer ____________
   3. \( -5x < m \)  
      \( x < -\frac{m}{5} \)
      Answer ____________
   4. \( \frac{x}{5} < \frac{m}{5} \)  
      \( x < 5m \)
      Answer ____________

b) Match the following properties of inequality with their respective names.
   
   1. Addition Property
   2. Subtraction Property
   3. Multiplication Property
   4. Division Property

   In the following statements \( a, b, c \in \mathbb{R} \).
   
   A. If \( a < b \) and \( c > 0 \), then \( \frac{a}{c} < \frac{b}{c} \)
      If \( a < b \) and \( c < 0 \), then \( \frac{a}{c} > \frac{b}{c} \)
   B. If \( a < b \) then \( a + c < b + c \)
   C. If \( a < b \) and \( c > 0 \), then \( ac < bc \)
      If \( a < b \) and \( c < 0 \), then \( ac > bc \)
   D. If \( a < b \) then \( a - c < b - c \)

c) Explain the difference between the multiplication property of equality and the multiplication property of inequality.
   Answer: ____________

d) State the property which justifies each step.
   Given: \( 6 - 3x < 9 \)
   
   1. \( 6 - 6 - 3x < 9 - 6 \)
      \( -3x < 3 \)
      Answer ____________
   2. \( -3x < 3 \)
      \( \frac{-3x}{-3} > \frac{3}{-3} \)
      \( x > -1 \)
      Answer ____________
   3. \( \frac{-3x}{-3} > \frac{3}{-3} \)
      \( x > -1 \)
      Answer ____________
   4. \( x > -1 \)
      Answer ____________

IV-105
Solve inequalities, using the appropriate properties.

a) In order to form pairs of equivalent sentences, tell whether < or > should be used as missing symbols.

1. \( x < -3; 46 \) ? \( 2x \)
2. \( 5y > 45; 9 \) ? \( y \)
3. \( \frac{1}{3} y \geq -7; y \) ? \( -21 \)
4. \( -4a < 24; -6 \) ? \( a \)

Solve each inequality. Show all work.

b) \( b + 60 < -100 \)
   Answer:

c) \( -12x + 1 \geq 25 \)
   Answer:

d) \( 16 - 4n \leq 6n - 24 \)
   Answer:
Solve compound inequalities by applying the concepts of union and intersection of sets.

Solve each open sentence. Show all work.

a) \( x + 6 > 8 \) and \( x - 1 < 4 \)
   Answer:

b) \( x - 3 \leq -4 \) or \( x - 3 \geq 4 \)
   Answer:

c) \( -2 \leq x + 6 \leq 3 \)
   Answer:

d) \( y + 6 > 7 \) or \( 3y + 5 < 3 \)
   Answer:
Graph the solution set of an inequality or compound inequality on the real number line.

**PERFORMANCE OBJECTIVE IV-10**

a) Which of the following is the graph of $2x > 6$?

A. 

B. 

C. 

D. 

Answer: 

b) Which of the following is the graph of $2 < y < 6$?

A. 

B. 

C. 

D. 

Answer: 

c) Solve and graph the solution set of $6x + 2 - 8x < 14$

Answer: 

d) Solve and graph the solution set of $5x \leq 10 + 2(3x - 4)$

Answer:
Solve open sentences involving absolute value.

Solve each open sentence. Show all work:

a) \(|2x + 3| = 11\)
   Answer:

b) \(|x + 7| - 3 \geq 9\)
   Answer:

c) \(5 + |2x - 5| \leq 7\)
   Answer:

d) \(|3x + 4| < 13\)
   Answer:
Translate word statements into open sentences.

a). Match the word phrase with its corresponding open sentence.

1. Twice a number is six.
   - A. \( x + y = xy \)
2. Eight decreased by a number is 10.
   - B. \( x - 4 < 0 \)
3. The sum of two numbers is their product.
   - C. \( 2x = 6 \)
4. 4 less than a number is less than 6.
   - D. \( 8 - x = 10 \)

Translate each word statement into an open sentence.

b) Five times a number is 6 more than twice a number.
   Answer ___________________

c) Seven times a number is less than twice the sum of the number and seven.
   Answer ___________________

d) The product of 2 and 7 more than a number is equal to the sum of twice the number and 14.
   Answer ___________________
PERFORMANCE OBJECTIVE IV-13

Write algebraic expressions representing the unknown information in a word problem.

For each word problem answer the questions immediately following it.

a) Two cars started from the same point and traveled in opposite directions, at rates of 30 km/h and 40 km/h. In how many hours will the cars be 2100 kilometers apart?

1. Identify a variable to represent what you are trying to find.
2. Write an expression to represent the distance traveled by the 30 km/h car.
3. Write an expression to represent the distance traveled by the 40 km/h car.

b) Find two consecutive integers whose sum is 57.

1. Identify a variable to represent the first integer.
2. In terms of this variable write an expression to represent the second integer.

b) Dr. D left his home by car, traveling on a certain road at the rate of 45 mph. Two hours later, his son Wes left home and started after him on the same road, traveling at the rate of 55 mph. Wes overtook his father in x hours.

1. Represent in terms of x the number of hours Dr. D traveled.
2. Represent in terms of x the distance Dr. D traveled.
3. Represent in terms of x the distance Wes traveled.

d) Find 3 consecutive even integers such that 4 times the first decreased by the second is 12 more than twice the third. Let x represent the first even integer.

1. In terms of x, represent the second even integer.
2. In terms of x, represent the third even integer.
Write an open sentence expressing the relationship stated in a word problem.

For each of the following, write an algebraic sentence to solve the problem.

a) Find two consecutive integers whose sum is 57.
   Answer __________________

b) Two cars started from the same point and traveled in opposite directions at rates of 30 mph and 40 mph. In how many hours will the cars be 2100 miles apart?
   Answer __________________

c) How many pounds of $2.50 coffee and $2.87 coffee must a dealer mix to produce 70 pounds of coffee to sell for $2.61 per pound?
   Answer __________________

d) Find the two greatest consecutive even integers whose sum is less than 80.
   Answer __________________
PERFORMANCE OBJECTIVE IV-15

Solve various types of word problems utilizing an organized approach.

Solve each word problem. Show all work.

a) Barry owns one more than twice as many books as Elaine. If Barry owns 59 books, how many books does Elaine own?

Answer:

b) A football team won twice as many games as it lost. It won 18 games; how many did it lose?

Answer:

c) The school store sold 348 notebooks, some for 25 cents each and the rest at 38 cents each. The total receipts for notebooks was $100.91. How many of each kind was sold?

Answer:

d) At 8 a.m. two planes leave St. Louis. One flies west at 350 km/h. The other flies east at 400 km/h. At what time will they be 1500 kilometers apart?

Answer:
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

Answers

1. a) 1) Addition Property of Equality
c) \( x + 3\frac{1}{2} = 7 \)
   \[ x + 3\frac{1}{2} - 3\frac{1}{2} = 7 - 3\frac{1}{2} \]
   \[ x = 3\frac{1}{2} \]
   Check: \( 3\frac{1}{2} + 3\frac{1}{2} = 7 \)

2) Multiplication Property of Equality

b) 1) Addition Property of Equality
d) \( 3.7 = a + 11.2 \)
   \[ 3.7 - 11.2 = a + 11.2 - 11.2 \]
   \[ -7.5 = a \]
   Check: \( 3.7 = -7.5 + 11.2 \)
   \[ 3.7 = 3.7 \]

2) Multiplication Property of Equality

2. a) 1) Equivalent

b) \( m + 7 = 13 \)
   \( m + 7 - 7 = 13 - 7 \)
   \( m = 6 \)
   Check: \( 6 + 7 = 13 \)
   \[ 13 = 13 \]

b) 1) Equivalent

c) \( 1/3 \)

2) A

3) D

4) B

d) 1) Subtraction Property of Equality

b) \( 13t = -52 \)
   Check: \( 13(-4) = -52 \)

2) Substitution

b) \( 13t = -52 \)
   \[ \frac{13t}{13} = \frac{-52}{13} \]
   \[ t = -4 \]

3) Addition Property of Equality

4) Substitution

b) \( -8 = \frac{-1b}{5} \)
   Check:
   \[ (-5)(-8) = (-5)(\frac{-1b}{5}) \]
   \[ -8 = \frac{-1b}{5} \]
   \[ 40 = b \]
   \[ -8 = -8 \]

5) Division Property of Equality

b) \( (-5)(-8) = (-5)(\frac{b}{5}) \)
   \[ -8 = \frac{-1(40)}{5} \]
   \[ -8 = -8 \]

6) Substitution

b) \( -14k = 84 \)
   Check: \( -14(-6) = 84 \)
   \[ 84 = 84 \]

2. a) 1) Equivalent

2) Equivalent

3) Not Equivalent

4) Equivalent

b) \( m + 7 = 13 \)
   \( m + 7 - 7 = 13 - 7 \)
   \( m = 6 \)
   Check: \( 6 + 7 = 13 \)
   \[ 13 = 13 \]

b) \( m + 7 = 13 \)
   \( m + 7 - 7 = 13 - 7 \)
   \( m = 6 \)
   Check: \( 6 + 7 = 13 \)
   \[ 13 = 13 \]

4. a) A. 3

B. 1

C. 4

D. 2
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

Answers (continued)

4. b) D

c) $5x - 9 = -19$

$5x = -10$

$x = -2$

Check: $5(-2) - 9 = -19$

$-10 - 9 = -19$

$-19 = -19$

d) $8 = 5x + 33$

$8 - 33 = 5x + 33 - 33$

$-25 = 5x$

$-5 = x$

Check: $8 = 5(-5) + 33$

$8 = -25 + 33$

$8 = 8$

5. a) 1) $-4x$

2) $2x$

3) $\frac{1}{2}x$

4) $-x$

b) $7x + 5 = 2x + 35$

$7x + 5 - 2x = 2x + 35 - 2x$

$5x + 5 = 35$

$5x = 30$

$x = 6$

c) $7y - 5 = 9y + 29$

$7y - 7y - 5 = 9y + 29 - 7y$

$-5 - 29 = 2y + 29 - 29$

$-34 = 2y$

$-17 = y$

Check: $7(-17) - 5 = 9(-17) + 29$

$119 - 5 = -153 + 29$

$-124 = -124$

d) $16 + 4y = 10y - 20$

$16 + 4y - 4y = 10y - 20 - 4y$

$16 = 6y - 20$

$16 + 20 = 6y - 20 + 20$

$36 = 6y$

$6 = y$

Check: $16 + 4(6) = 10(6) - 20$

$16 + 24 = 60 - 20$

$40 = 40$
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

Answers (continued)

6. a) \[5(3y - 2) = 5\]
\[15y - 10 = 5\]
\[15y = 15\]
\[y = 1\]
Check: \[5(3 \cdot 1 - 2) = 5\]
\[5(1) = 5\]
\[5 = 5\]

b) \[7(x + 2) = 5(x + 4)\]
\[7x + 14 = 5x + 20\]
\[7x + 14 - 5x = 5x + 20 - 5x\]
\[2x + 14 = 20\]
\[2x + 14 - 14 = 20 - 14\]
\[2x = 6\]
\[x = 3\]
Check: \[7(3 + 2) = 5(3 + 4)\]
\[7(5) = 5(7)\]
\[35 = 35\]

c) \[3d + 2(6d - 5) = 5\]
\[3d + 12d - 10 = 5\]
\[15d - 10 = 5\]
\[15d - 10 + 10 = 5 + 10\]
\[15d = 15\]
\[d = 1\]

6. c) (continued)
Check: \[3(1) + 2(6 \cdot 1 - 5) = 5\]
\[3 + 2(1) = 5\]
\[3 + 2 = 5\]
\[5 = 5\]

d) \[3(x + 2) + 3x + 2 = 74\]
\[3x + 6 + 3x + 2 = 74\]
\[6x + 8 = 74\]
\[6x + 8 - 8 = 74 - 8\]
\[6x = 66\]
\[6x = 66\]
\[x = 11\]
Check: \[3(11 + 2) + 3(11) + 2 = 74\]
\[33 + 33 + 2 = 74\]
\[74 = 74\]

7. a) 1. Not Equivalent
2. Equivalent
3. Not Equivalent
4. Equivalent

b) 1. B
2. D
3. C
4. A

c) Multiplying both sides of an equation by a negative constant has no effect on the equation. Multiplying both sides of an inequality by a negative constant reverses the inequality.
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

Answers (continued)

7. d) 1. Subtraction Property of Inequality
2. Substitution
3. Division Property of Inequality
4. Substitution

8. a) 1. >
2. <
3. <
4. <

b) \[ b + 60 < -100 \]
   \[ b + 60 - 60 < -100 - 60 \]
   \[ b < -160 \]

c) \[ -12x + 1 \geq 25 \]
   \[ -12x + 1 - 1 \geq 25 - 1 \]
   \[ -12x \geq 24 \]
   \[ \frac{-12x}{-12} \leq \frac{24}{-12} \]
   \[ x \leq -2 \]

d) \[ 16 - 4n \leq 6n - 24 \]
   \[ 16 - 4n + 4n \leq 6n - 24 + 4n \]
   \[ 16 \leq 10n - 24 \]
   \[ 16 + 24 \leq 10n - 24 + 24 \]
   \[ 40 \leq 10n \]
   \[ \frac{40}{10} \leq \frac{10n}{10} \]
   \[ 4 \leq n \]

IV-28
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

Answers (continued)

9. a) $x + 6 > 8$ and $x = 1 < 4$
   $x + 6 - 6 > 8 - 6$ $x - 1 + 1 < 4 + 1$
   $x > 2$ and $x < 5$.
   b) $x - 3 \leq -4$
   $x - 3 + 3 \leq -4 + 3$ $x - 3 + 3 \geq 4 + 3$
   $x \leq 1$, or $x \geq 7$.
   c) $-2 \geq x + 6 \leq 3$
   $-2 \leq x + 6$ and $x + 6 \leq 3$.
   $-2 - 6 \leq x + 6 - 6$ $x + 6 - 6 \leq 3 - 6$
   $-8 \leq x$ and $x \leq -3$
   $-8 \leq x \leq 3$.
   d) $y + 6 > 7$
   $y + 6 - 6 > 7 - 6$ $3y + 5 - 5 > 3 - 5$
   $y > 1$, $3y < 2$.
   $3y < -2 \frac{2}{3}$
   $y < -\frac{2}{3}$.

10. a) C
   b) D
   c) $6x + 2 - 8x < 14$
   $-2x + 2 < 14$
   $-2x + 2 - 2 < 14 - 2$
   $-2x < 12$
   $\frac{24}{-2} > \frac{12}{-2}$
   $x > -6$
   d) $5x \leq 10 + 2 (3x - 4)$
   $5x \leq 10 + 6x - 8$
   $5x \geq 2 + 6x$
   $5x - 6x \leq 2 + 6x - 6x$
   $-x \leq 2$
   $\frac{-x}{-1} \geq \frac{2}{-1}$
   $x \geq -2$. 
Answers (continued)

11. a) \[ |2x + 3| = 11 \]
   \[ 2x + 3 = 11 \quad \text{or} \quad 2x + 3 = -11 \]
   \[ 2x = 8 \quad \text{or} \quad 2x = -14 \]
   \[ x = 4 \quad \text{or} \quad x = -7 \]

b) \[ |x + 7| = 9 \]
   \[ x + 7 \geq 12 \quad \text{or} \quad x + 7 \leq -12 \]
   \[ x \geq 5 \quad \text{or} \quad x \leq -19 \]

b) \[ |x + 7| = 9 \]
   \[ x + 7 \geq 12 \quad \text{or} \quad x + 7 \leq -12 \]
   \[ x \geq 5 \quad \text{or} \quad x \leq -19 \]

12. a) 1. C
   2. D
   3. A
   4. B

b) \[ 5x = 6 + 2x \]
   \[ 7x < 2(x + 7) \]
   \[ 2(x + 7) = 2x + 14 \]

13. a) 1. Let \( x = \) number of hours
   2. \( d = 30x \)
   3. \( d = 40x \)

b) 1. Let \( x = 1st \) integer
   2. Let \( x + 1 = 2nd \) integer

b) 1. Let \( x = 1st \) integer
   2. Let \( x + 1 = 2nd \) integer

1. Mr. Roberts = \( x + 2 \)
   2. Mr. Robert's distance
      = \( 45(x + 2) \)
   3. Tony's distance = \( 55x \)

1. \( x + 2 = 2nd \) even integer
   2. \( x + 4 = 3rd \) even integer
UNIT IV - SOLVING OPEN SENTENCES AND WORD PROBLEMS

Answers (continued)

14. a) Let \( x = \) 1st integer
   \[ x + 1 = \) 2nd integer
   \[ x + (x + 1) = 57 \]

b) Let \( w \) = number of hours
   \[ 30(w) = \) slow car's distance
   \[ 40(w) = \) fast car's distance
   \[ 30w + 40w = 2100 \]

c) Let \( x = \) pounds of $2.50 coffee
   \[ 70 - x = \) pounds of $2.87 coffee
   \[ .50(x) + .87(70-x) = .61(70) \]

d) Let \( x = \) 1st integer
   \[ x + 2 = \) 2nd integer
   \[ x + (x + 2) < 80 \]

15. a) Let \( x = \) Elaine's books
   \[ 2x + 1 = \) Barry's books
   \[ 2x + 1 = 59 \]
   \[ 2x = 58 \]
   \[ x = 29 \]
   Elaine has 29 books

b) Let \( x = \) games lost
   \[ 2x = \) games won
   \[ x = 9 \]
   Lost 9 games

c) Let \( x = \) number of 25c notebooks
   \[ 348 - x = \) number of 38c notebooks
   \[ .25 x + .38 (348 - x) = 109.91 \]
   \[ .25 x + 132.24 - .38 x = 109.91 \]
   \[ .13 x = 31.33 \]
   \[ x = 241 \]
   \[ 348 - x = 107 \]

d) Let \( x = \) number of hours
   \[ 350x = \) distance west
   \[ 400 x = \) distance east
   \[ 350 x + 400 x = 1500 \]
   \[ 750 x = 1500 \]
   \[ x = 2 \]
   Time: 10:00 a.m.
UNIT V - GRAPHING

PURPOSE

Graphing forms a bridge between the study of algebra and geometry. This section of the course introduces the students to Cartesian coordinates and their application to linear open sentences. The knowledge of linear functions and their characteristics provides students with alternative methods for solving open sentences.

OVERVIEW

Students become acquainted with the basic terminology of rectangular coordinates and the location of points in the system. Graphing of linear equations is developed so that a functional level of understanding of equations and their graphs is obtained. The relationships between a line and the equation which defines it are discussed. The coordinate system is also used to graph linear inequalities and higher order functions.

SUGGESTIONS TO THE TEACHER

Instructional Days: 9-12
Minimal Course Objectives: §1-10, 14
Average Course Objectives: ALL
Maximal Course Objectives: ALL

This unit is an excellent one for incorporating multimedia techniques such as the overhead projector and colored chalk to stimulate student interest.

There are numerous supplementary resources available which enable students to obtain identifiable pictures as a result of graphing ordered pairs. Advanced students may enjoy drawing original pictures on graph paper (using only straight lines) and then determining the equation for each line.

When discussing the slope of a line, it is helpful to speak constantly of the line going from left to right.

Games such as "Battleship" can be readily adapted for practice in coordinate graphing.

VOCABULARY

abscissa (x-coordinate) plane
Cartesian coordinates quadrant
coordinates range
domain relation
function rectangular coordinates
half plane slope
ordered pair x-axis (horizontal axis)
ordinate (y-coordinate) y-axis (vertical axis)
origin y-intercept
UNIT V - GRAPHING

ENTERING PERFORMANCE OBJECTIVES

1. Add, subtract, multiply, and divide directed numbers.
2. Graph an ordered pair of numbers on the coordinate plane.
3. Evaluate algebraic expressions.
4. Solve equations for a given variable.

Assessment Tasks

<table>
<thead>
<tr>
<th>Task</th>
<th>Equation</th>
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<tbody>
<tr>
<td>a)</td>
<td>-7 + (-12)</td>
</tr>
<tr>
<td>b)</td>
<td>-2(\frac{2}{3}) + (-1(\frac{1}{3}))</td>
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<tr>
<td>c)</td>
<td>-4(\frac{1}{4}) + 2(\frac{3}{8})</td>
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<tr>
<td>d)</td>
<td>-6.4 + 16.6 + (-12.9) + 8.4</td>
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<td>e)</td>
<td>3(\frac{1}{3}) + (-11(\frac{2}{3}))</td>
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<td>f)</td>
<td>-5.8 - 8.8</td>
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<td>g)</td>
<td>8.4 - 12.9</td>
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<td>h)</td>
<td>1(\frac{7}{8}) - 7(\frac{3}{8})</td>
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<tr>
<td>i)</td>
<td>-12(\frac{1}{3}) - (-3(\frac{1}{3}))</td>
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<tr>
<td>j)</td>
<td>(\frac{7}{12}) - (\frac{2}{5})</td>
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<td>k)</td>
<td>(5)(-7)(-3)</td>
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<td>l)</td>
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<td>m)</td>
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<td>n)</td>
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<tr>
<td>o)</td>
<td>(2.5608) : (-0.8)</td>
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<td>p)</td>
<td>(-75) ÷ 15</td>
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<tr>
<td>q)</td>
<td>(-3(\frac{4}{3})) ÷ (-(\frac{1}{3}))</td>
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<tr>
<td>r)</td>
<td>(-144) ÷ (-9)</td>
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V-3
UNIT V - GRAPHING

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. Graph the following ordered pairs of numbers, and label each point with its coordinates.
   a) (5,0)  e) (3,4)  i) (7,3)
   b) (0,7)  f) (-2,6)  j) (-8,2)
   c) (-3,0)  g) (-3,-8)  k) (-5,-7)
   d) (0,-4)  h) (3,-5)  l) (7,-4)

3. If \( x = 2, y = -3, \) and \( z = -1, \) find the value of each of the following:
   a) \( xy + xz - yz \)
   b) \( x^3y - x^2z + x + 20 \)
   c) \( x^2yz - 2z \)
   d) \( x^3y^2 + x^2z + xy - 4 \)

4. Solve each equation for \( x, y, \) or \( z. \)
   a) \( 2x + 5w = 1 \)
   b) \( P = 2x + 2w \)
   c) \( s + ry = t \)
   d) \( 4v - 3y = 6 \)
Answers

1. a) -19  
   b) -4  
   c) $-\frac{7}{8}$  
   d) 5.7  
   e) $-8\frac{1}{3}$  
   f) -14.6  
   g) -4.5  
   h) $-\frac{5}{2}$  
   i) $-\frac{5}{6}$  
   j) $\frac{59}{60}$  
   k) 105  
   l) 7.25  
   m) $-\frac{49}{4}$ or $-12\frac{1}{4}$  
   n) $\frac{1}{24}$  
   o) -3.201  
   p) -5  
   q) $11\frac{2}{5}$  
   r) 16

2. 

3. a) -11  
   b) 2  
   c) 14  
   d) 58

4. a) $\frac{-5w + 1}{2} = x$  
   b) $x = \frac{P - 2w}{2}$  
   c) $y = \frac{t - s}{r}$  
   d) $y = \frac{4v}{3} - 2$
UNIT V - GRAPHING

PERFORMANCE OBJECTIVES

1. Distinguish among axes, origin, quadrants, and ordered pairs when given an illustration of a rectangular coordinate system. (I)

2. Graph ordered pairs on a coordinate plane. (II)

3. State the coordinates of a given point. (II)

4. Graph a linear equation, using a table of values. (III)

5. Determine the slope of a line from its graph. (II)

6. Graph a line, given its slope and the coordinates of a point on the line. (III)

7. Compute the slope of a line algebraically. (II)

8. Rewrite a given equation in slope-intercept form. (II)

9. Graph a linear equation using the slope and y-intercept. (III)

10. Write the equation of a line, given its slope and y-intercept. (II)

11. Determine the equation of a line, given its slope and the coordinates of a point on the line. (III)

12. Determine the equation of a line, given the coordinates of two points on the line. (III)

13. Determine whether a given relation is a function. (II)

14. Graph a linear inequality. (III)

15. Graph a quadratic equation, using a table of values. (III)

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KEY SKILLS FOR END-OF-COURSE TESTING

10. Graph linear equations in the coordinate plane.

11. Graph linear inequalities in the coordinate plane.

12. Determine an equation of a line, given the slope and a point, or two points of the line.
## CROSS REFERENCES

### TEXTS (BY AUTHOR)

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PERFORMANCE OBJECTIVE V-1

Distinguish among axes, origin, quadrants, and ordered pairs when given an illustration of a rectangular coordinate system.

a) Indicate, by the correct letter(s), illustrations of each of the following in the diagram at the right.
   1. axes ____________
   2. origin ____________
   3. quadrants ____________
   4. ordered pairs ____________

b) Match each name to its correct illustrations by writing the letter of the correct answer on the blank.
   __ 1. axes A. D,E,F,H
   __ 2. origin B. B,I
   __ 3. quadrants C. C,G
   __ 4. ordered pairs D. A
PERFORMANCE OBJECTIVE V-1

Distinguish among axes, origin, quadrants, and ordered pairs when given an illustration of a rectangular coordinate system.

c) In the drawing at the right, R and B represent which of the following?
   A. axes
   B. origin
   C. quadrants
   D. ordered pairs
   Answer ________________________

d) In the drawing at the right, C, F, and X represent which of the following?
   A. axes
   B. origin
   C. quadrants
   D. ordered pairs
   Answer ________________________
PERFORMANCE OBJECTIVE V-2

Graph ordered pairs on a coordinate plane.

a) On the graph below, plot each of the following points; and label with the appropriate letter.
   A. (-1,3)    C. (2,4)
   B. (0,0)     D. (3,-1)

b) On the graph below, plot each of the following points; and label with the appropriate letter.
   A. (5,0)     C. (2,5)
   B. (0,-3)    D. (-4,-1)

c) Identify the quadrant that contains each of the following points.

<table>
<thead>
<tr>
<th>Point</th>
<th>Quadrant</th>
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<tr>
<td>1. (6,3)</td>
<td></td>
</tr>
<tr>
<td>2. (-2,4)</td>
<td></td>
</tr>
<tr>
<td>3. (3,-1)</td>
<td></td>
</tr>
<tr>
<td>4. (-2,-2)</td>
<td></td>
</tr>
</tbody>
</table>

d) On the graph below, plot the following vertices of a triangle. Label each vertex and draw the sides.

<table>
<thead>
<tr>
<th>Point</th>
<th>Quadrant</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. (-4,0)</td>
<td></td>
</tr>
<tr>
<td>C. (2,-2)</td>
<td></td>
</tr>
<tr>
<td>B. (0,3)</td>
<td></td>
</tr>
<tr>
<td>D. (2,5)</td>
<td></td>
</tr>
</tbody>
</table>
PERFORMANCE OBJECTIVE V-3

State the coordinates of a given point.

a) State the coordinate of each point graphed below.
1. point A = __________ 4. point D = __________
2. point B = __________ 5. point E = __________
3. point C = __________

b) From the diagram, name the coordinates of each point described below.
1. point in Quadrant I ________
2. point in Quadrant IV ________
3. point in Quadrant III ________
4. point on the y-axis ________

c) Name the coordinates of each point described below.
   1. the point located 4 units up from the origin
   2. the point located on the x-axis 4 units to the left of the origin
   3. the point 2 units to the right of (3,-1)
   4. the point 3 units below (2,-3)

d) State the coordinate of each point shown on the graph below.
1. point A ________ 4. point D ________
2. point B ________ 5. point E ________
PERFORMANCE OBJECTIVE V-4

Graph a linear equation, using a table of values.

Complete the following table and then use the values to draw the graph.

a) \( x + y = 6 \)

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

b) \( 2x = y \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>-1</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

c) \( 2y - 3x = 10 \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

d) Make a table of three pairs of values for \( x \) and \( y \) that satisfy the equation \( 2x + y = 10 \).
PERFORMANCE OBJECTIVE V-5

Determine the slope of a line from its graph.

a) Determine the slope of the line.
   from its graph.

   ![Diagram a]

   Answer _______________________

b) Determine the slope of the line from its graph.

   ![Diagram b]

   Answer _______________________

c) Determine which diagram below satisfies the conditions described.

   A   B   C   D

   1. a positive slope
   2. a negative slope
   3. slope of zero
   4. no slope

   Answer _______________________

d) Determine the slope of the line shown below.

   ![Diagram d]

   Answer _______________________
Graph a line, given its slope and the coordinates of a point on the line.

a) Draw the graph of a line that has a slope of 3 and passes through the point (0,2).

b) Draw the graph of a line that has a slope of \(-\frac{2}{3}\) and passes through the point (0,0).

c) Draw a line which passes through the point (5,-3) and has no slope.

d) Determine two other points on the line which passes through (-2, -3) and has a slope of 3.

Answers: __________  __________
Compute the slope of a line algebraically.

a) Find the slope of a line passing through the points (4,0) and (2,-1).
Answer ________________________

b) What is the slope of a line passing through the points (a,0) and (0,a)?
Answer ________________________

c) The slope of a line is -2. It passes through the points (1,4) and (5,?). Find the missing coordinate.
Answer ________________________

d) Which of the following is a formula for determining the slope of a line?
A. \( M = \frac{x_2 - x_1}{y_2 - y_1} \)
B. \( M = \frac{y}{x} \)
C. \( M = \frac{y_2 - y_1}{x_2 - x_1} \)
D. None of the above.
Answer ________________________
PERFORMANCE OBJECTIVE V-8

Rewrite a given equation in slope-intercept form.

a) Which of the following is the correct slope-intercept form of the equation $3x + 2y = 3$?
   A. $y = 3x + 3$
   B. $y = \frac{3}{2}x + 1$
   C. $y = -\frac{3}{2}x - 1$
   D. none of the above
   Answer ____________________

b) Rewrite the following equation in slope-intercept form: $x + 2y = 6$.
   Answer ____________________

c) What is the slope of the line defined by the equation $2x - y = 3$?
   Answer ____________________

d) Solve the following equation for $y$: $3x + 4y = 8$.
   Answer ____________________

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Graph a linear equation, using the slope and y-intercept.

a) Which of the following is the graph of the line for the equation $2x - 3y = 3$?

b) Graph the line of the equation $3x + y = 0$ by using the slope and y-intercept.

c) Draw the graph of the equation $-2x + y = 6$ by using the slope and y-intercept.

d) Graph the following equation: $5y + 10 = -2x$. 
PERFORMANCE OBJECTIVE V-10

Write the equation of a line, given its slope and y-intercept.

a) A line has a slope of 3 and a y-intercept of $\frac{1}{2}$. The equation that describes the relationship between x and y coordinates of the graph is:

A. $\frac{1}{2}x + 3 = y$
B. $3x + y = \frac{1}{2}$
C. $y = 3x + \frac{1}{2}$
D. $y = \frac{1}{2}x - 3$

Answer ______________________

Write the equations for the graphs with the following characteristics.

b) y-intercept of (0,4) and slope of 2.

Answer ______________________

c) Slope of zero and y-intercept of (0,2).

Answer ______________________

d) Slope of $-\frac{3}{2}$ and y-intercept at the origin.

Answer ______________________
PERFORMANCE OBJECTIVE V-11

Determine the equation of a line, given its slope and the coordinates of a point on the line.

a) Given a line passing through the point (3,0) and slope of $\frac{1}{2}$, write an equation for the line.
   Answer ______________________

b) Given a line crossing the y-axis at -2 and a slope of 3, write an equation for the line.
   Answer ______________________

c) Given a line passing through the origin and a slope of $-\frac{3}{2}$, write an equation for the line.
   Answer ______________________

d) Determine the equation of the line graphed below by finding its slope and y-intercept.

Answer ______________________
PERFORMANCE OBJECTIVE V-12

Determine the equation of a line, given the coordinates of two points on the line.

a) Write the equation of a line passing through the points (2,3) and (3,2).
   Answer ________________________

b) A line passes through the origin and the point (-3,5). Find the equation of the line.
   Answer ________________________

c) Find the equation of the line passing through the points (3,4) and (2,-5).
   Answer ________________________

d) What is the equation of the line passing through the points (-6,3) and (3,-3)?
   Answer ________________________
PERFORMANCE OBJECTIVE V-13

Determine whether a given relation is a function.

a) Which of the following illustrate a function?

A

B

C

b) Which of the following sets of ordered pairs is a function?

A. \{(2,3), (3,4), (4,5)\}
B. \{(3,0), (0,2), (3,-1), (2,0)\}
C. \{(-1,2), (1,2), (2,3), (2,-3)\}
D. None of the above

Answer ________________

c) Which equations define a function?

A. \(y = 3x - 4\)
B. \(x = -8\)
C. \(y = -9\)
D. None of the above.

Answer ________________

d) Which of the following sets of ordered pairs represent functions?

A. \{(1,6), (2,4), (3,2), (4,0), (6,-2)\}
B. \{(-1,-1), (0,0), (3,1), (0,-2), (3,-3)\}
C. \{(1,-4), (0,-3), (2,-3), (1,0), (3,0)\}
D. \{(0,0), (1,-6), (-1,4), (2,-8), (-2,0), (3,0), (-3,-18)\}
E. All of the above

Answer ________________
PERFORMANCE OBJECTIVE V-14

Graph a linear inequality.

a) Which of the following is the graph of the inequality $y - x > 4$?

- A
- B
- C
- D

Answer ____________________________

c) Graph the inequality $2x + y \geq -3$.

d) Graph the inequality $y < -\frac{1}{2}x - 1$. 
Graph a quadratic equation, using a table of values.

a) Find the appropriate points by completing the table, and use them to graph the equation \( y = x^2 + 2x - 3 \).

<table>
<thead>
<tr>
<th>x</th>
<th>( x^2 + 2x - 3 )</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
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<td>-4</td>
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<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

b) Determine which equation defines the graph below.

A. \( y = x^2 - 4x + 4 \)
B. \( y = x^2 - 4 \)
C. \( y = x^2 + 4x + 4 \)
D. None of the above.

Answer ________
PERFORMANCE OBJECTIVE V-15

Graph a quadratic equation, using a table of values.

c) Graph the quadratic function \( y = x^2 - 4 \). Use the accompanying chart.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x^2 - 4 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


d) Complete the following table of values, and graph the equation
\( y - 6x = -x^2 - 9 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Answers

1. a) 1. A, C
2. B
3. E, F, G, H
4. D
b) 1. C
2. D
3. A
4. B
c) C
d) D

2. a)

b)

c) 1. I
2. II
3. IV
4. III

UNIT V - GRAPHING

3. a) 1. (2,2)
2. (2,-3)
3. (-3,0)
4. (-1,-5)
5. (-2,4)
b) 1. (2,1)
2. (3,-2)
3. (-2,-2)
4. (0,-4)
c) 1. (0,4)
2. (-4,0)
3. (5,-1)
4. (2,-6)
d) 1. (2,4)
2. (-4,1)
3. (-3,-4)
4. (6,-2)
5. (3,0)
UNIT V - GRAPHING

Answers (continued)

4. a) \[
\begin{array}{cccc}
 x & 2 & 3 & 6 \\
 y & 4 & 3 & 0
\end{array}
\]

b) \[
\begin{array}{cccc}
 x & 0 & 1 & 3 \\
 y & 0 & 2 & 6
\end{array}
\]

c) \[
\begin{array}{cccc}
 x & -4 & -2 & 0 & -6 & 2 \\
 y & -1 & 2 & 5 & -4 & 8
\end{array}
\]

d) \[
\begin{array}{cc}
 x & y \\
 3 & 4 \\
 2 & 6 \\
 0 & 10 \\
 5 & 0
\end{array}
\]

5. a) -1

b) \(\frac{3}{2}\)

c) 1. A

2. D

3. C

4. B

d) \(-\frac{1}{2}\)

6. a)
Answers (continued)

6. c)  
   \[ y = 2x + 6 \]
   
   d) \((-1, 0)\)
   \((0, 3)\)

7. a) \( \frac{1}{2} \)
   b) \(-1\)
   c) \(-4\)
   d) C

8. a) D
   b) \( y = -\frac{1}{2}x + 3 \)
   c) 2
   d) \( y = -\frac{3}{4}x + 2 \)

9. a) C
   b) \( 3x + y = 0 \)
      \[ y = -3x \]

10. a) C
    b) \( y = 2x + 4 \)
    c) \( y = 2 \)
    d) \( y = -\frac{3}{2}x \)

11. a) \( y = \frac{1}{2}x - \frac{3}{2} \)
    b) \( y = 3x - 2 \)
    c) \( y = -3x \)
    d) \( y = \frac{2}{3}x + 2 \)

UNIT V - GRAPHING
Answers (continued)

12. a) \( y = -x + 5 \)
   b) \( y = -\frac{5}{3}x \)
   c) \( y = 9x - 23 \)
   d) \( y = -\frac{2}{3}x - 1 \)

13. a) B, C
   b) A
   c) A, C
   d) A, D

14. a) C
   b) \( y > 3x - 4 \)
   c) \( 2x + y \geq -3 \)

15. a) \[
\begin{array}{|c|c|c|}
\hline
x & x^2 + 2x - 3 & y \\
\hline
0 & 0^2 + 2(0) - 3 & -3 \\
1 & 1^2 + 2(1) - 3 & 0 \\
-3 & (-3)^2 + 2(-3) - 3 & 0 \\
-1 & (-1)^2 + 2(-1) - 3 & -4 \\
2 & 2^2 + 2(2) - 3 & 5 \\
\hline
\end{array}
\]
UNIT V - GRAPHING

Answers (continued)

15. c) | x | x^2 - 4 | y |
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0^2 - 4</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>1^2 - 4</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>2^2 - 4</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-1^2 - 4</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
<td>-2^2 - 4</td>
<td>0</td>
</tr>
</tbody>
</table>

15. d) | x | x^2 + 6x - 9 | y |
<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>0</td>
<td>-0^2 + 6(0) - 9</td>
<td>-9</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
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<td>-4^2 + 6(4) - 9</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>-5^2 + 6(5) - 9</td>
<td>-4</td>
</tr>
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</table>
UNIT VI - SYSTEMS OF OPEN SENTENCES

PURPOSE

This unit provides the students with alternative methods for solving systems of two equations in two variables. The techniques involved necessitate the integration of most concepts contained in the first semester of the course.

OVERVIEW

Students draw upon their knowledge of graphing techniques to solve systems of equations. Systematic algebraic manipulations provide alternative methods for finding the solutions to systems of open sentences. Word problems that involve the use of two variable solutions are discussed.

SUGGESTIONS TO THE TEACHER

Instructional Days: 9-12
Minimal Course Objectives: #2-5
Average Course Objectives: #1-7
Maximal Course Objectives: ALL

Emphasize the availability of several alternative methods for solving a system of open sentences, and stress the selection of the most efficient method.

Linear programming is a valuable topic for advanced students. It involves the utilization of algebraic skills in the solution of applied problems.

The Gauss Elimination Method for solving systems of equations may be a good strategy for reviewing operations with fractions and directed numbers.

VOCABULARY

coinciding lines
intersecting lines
linear programming
parallel lines
system of inequalities
system of simultaneous equations
ENTERING PERFORMANCE OBJECTIVES

1. Determine the LCM of two arithmetic numbers.
2. Graph the equation of a line.
3. Write an equation in the slope-intercept form.
4. Determine whether a given point satisfies the equation of a line.
5. Write an equivalent equation for a given equation by applying the multiplication property of equality.

Assessment Tasks

1. a) The lowest common multiple of 12 and 16 is:
   A. 4
   B. 24
   C. 32
   D. 48
   Answer

1. b) The lowest common multiple of 6, 9, and 15 is:
   A. 15
   B. 30
   C. 90
   D. None of the above
   Answer

1. c) Find the lowest common multiple of 45 and 50.
   Answer

1. d) Find the lowest common multiple of 8, 12, and 20.
   Answer
UNIT VI - SYSTEMS OF OPEN SENTENCES

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. Graph the lines represented by the following equations:
   a) \( x + y = -4 \)
   b) \( y = \frac{2}{3}x - 5 \)
   c) \( y = 3x + 1 \)
   d) \( y = \frac{4}{3}x + 2 \)

3. a) Which of the following is the slope-intercept form of \( 2x + 3y = -6 \)?
   A. \( 2x = -3y - 6 \)
   B. \( x = \frac{3y}{2} + 3 \)
   C. \( y = -\frac{2}{3}x - 2 \)
   D. \( y = \frac{3}{2}x - 2 \)
   Answer

   b) Which of the following is the slope-intercept form of \( x + 2y = 4 \)
   A. \( x = \frac{1}{2}y + 2 \)
   B. \( x = -2y + 4 \)
   C. \( y = 2x + 4 \)
   D. \( y = -\frac{1}{2}x + 2 \)
   Answer

   c) Write the equation \( 3y - 4x = 6 \) in slope-intercept form.
   Answer

   d) Write the equation \( 3y + x = -6 \) in slope-intercept form.
   Answer

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ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

4. a) Which of the following equations does the ordered pair (3,4) satisfy?
   A. \(3x + 2y = 6\)
   B. \(x + y = -5\)
   C. \(\frac{2}{3}x = \frac{1}{2}y\)
   D. \(x - 2y = -12\)

   Answer ______

   b) Which of the following points satisfy the equation \(3x - 5 = y?\)
   A. \((2,1)\)
   B. \((1,2)\)
   C. \((-1,2)\)
   D. \((-1,-2)\)

   Answer ______

   c) Does the point \((-2,1)\) satisfy the equation \(\frac{1}{2}x + 7y - 6 = 0?\)

   Answer ______

   d) Does the point \((\frac{1}{2},3)\) satisfy the equation \(6x - y = 3?\)

   Answer ______

5. a) Which of the following is not an equivalent equation for \(3x - 4y = -1?\)
   A. \(-6x + 8y = 2\)
   B. \(-3x = 4y + 1\)
   C. \(9x + 12y = -3\)
   D. \(-3x + 4y = 1\)

   b) Multiply the equation \(3y - x = 4\) by \(-4\) to obtain an equivalent equation.

   Answer ______
UNIT VI - SYSTEMS OF OPEN SENTENCES

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

5. c) Which of the following is an equivalent equation for $2x + 3y = 0$?
   
   A. $4x + 6y = 2$
   B. $-2x - 3y = -1$
   C. $-6x - 9y = 0$
   D. $8x - 12y = 0$

   Answer __________

d) Multiply the equation $5y - 2x = 3$ by $-2$ to obtain an equivalent equation.

   Answer __________
UNIT VI - SYSTEMS OF OPEN SENTENCES

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) D
   b) C
   c) 450
   d) 120

2. a) 

3. a) C
   b) D
   c) \( y = \frac{4}{3}x + 2 \)
   d) \( y = -\frac{1}{3}x - 2 \)

4. a) C
   b) A
   c) yes
   d) no

5. a) B
   b) \(-12y + 4x = -16\)
   c) C
   d) \(-10y + 4x = -6\)
UNIT VI - SYSTEMS OF OPEN SENTENCES

PERFORMANCE OBJECTIVES

1. Determine whether the graphs of a system of equations are parallel, intersecting, or coinciding lines by examining slopes and y-intercepts. (II)
2. Solve a system of equations graphically. (III)
3. Solve a system of equations by the addition (subtraction) method. (III)
4. Solve a system of equations using multiplication with the addition (subtraction) method. (III)
5. Solve a system of equations by the substitution method. (III)
6. Solve word problems involving two variables and a system of equations. (III)
7. Solve a system of linear inequalities graphically. (III)
8. Solve linear programming problems. (IV)

<table>
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<tr>
<th>Minimal</th>
<th>Average</th>
<th>Maximal</th>
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</thead>
<tbody>
<tr>
<td># 2-5</td>
<td>#1-7</td>
<td>ALL</td>
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</table>

KEY SKILLS FOR END-OF-COURSE TESTING

13. Solve a system of equations in two variables.
14. Solve word problems involving two variables and a system of equations.
UNIT VI - SYSTEMS OF OPEN SENTENCES

CROSS REFERENCES

TEXTS (BY AUTHOR)

<table>
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UNIT VI - SYSTEMS OF OPEN SENTENCES

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PERFORMANCEOBJECTIVEVI-1

Determine whether the graphs of a system of equations are parallel, intersecting, or coinciding lines by examining slopes and y-intercepts.

Tell whether the graphs of each of the following systems of equations are parallel lines, intersecting lines, or coinciding lines

a) \[ y = -2x + 3 \]
   
   \[ y = -5x + 11 \]

b) \[ y = 2x + 7 \]
   
   \[ y' = 2x - 3 \]

b) \[ y = \frac{1}{2}x + 3 \]
   
   \[ 2y = x + 6 \]

d) Which of the following equations has a graph which coincides with the graph of the equation \[ y = 3x - 4 \]:

   A. \[ y = \frac{1}{3}x - 4 \]
   
   B. \[ 2y - 6x + 8 = 0 \]
   
   C. \[ 2y = 6x + 8 \]
   
   D. None of the above

   Answer: ________
PERFORMANCE OBJECTIVE VI-2

Solve a system of equations graphically.

1. a) Match each system of equations with its graph.

   1) \( y = x + 2; \ y - x = 4 \)
   2) \( x - y = 2; \ x + y = 4 \)
   3) \( x + y = 1; \ 2y = -2x + 2 \)

   A. \[ \begin{align*}
   &\text{1.} \quad y = x + 2 \quad \text{Graph A} \\
   &\text{2.} \quad y = x + 4 \quad \text{Graph B} \\
   &\text{3.} \quad y = -x + 2 \quad \text{Graph C}
   \end{align*} \]

b) Determine the solution set of the following systems of equations by graphing.

   1) \( x + 2y = 14 \quad y - 2x = 2 \)
   2) \( 2x + 3y = 9 \quad y = x - 7 \)
   3) \( x + y = 5 \quad y = 3 - x \)

   \[ \begin{align*}
   &\text{1.} \quad \text{Graph D} \\
   &\text{2.} \quad \text{Graph E} \\
   &\text{3.} \quad \text{Graph F}
   \end{align*} \]
PERFORMANCE OBJECTIVE VI-3

Solve a system of equations by the addition (subtraction) method.

1. Solve simultaneously by the addition (subtraction) elimination method.

   a) \[3r + 2s = 10\]
   \[5r + 2s = 10\]
   Answer: 

   b) \[3x + 2y = 12\]
   \[-3x - 2y = 6\]
   Answer: 

   c) \[2x + 3y = 15\]
   \[-2x + 9y = 21\]
   Answer: 

   d) \[-x + 2y = 6\]
   \[2y - x = 6\]
   Answer: 

PERFORMANCE OBJECTIVE VI-4

Solve a system of equations using multiplication with the addition (subtraction) method.

1. Solve, using multiplication with the addition method.
   a) \(5p + 2q = 5\)
      \(15p - 3q = 15\)
      Answer: 
   b) \(17r + 12s = -19\)
      \(13r - 3s = 22\)
      Answer: 
   c) \(3a + 2b = 10\)
      \(2a + 3b = -5\)
      Answer: 
   d) \(5p - 2q = 16\)
      \(3p + 5q = 22\)
      Answer: 

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VI-14
PERFORMANCE OBJECTIVE VI-5

Solve a system of equations by the substitution method.

1. Solve the following systems of equation by substitution
   a) \( x + 2y = 5 \)
      \( 3x + 2y = 3 \)
      Answer: ______________________
   b) \( 4p + q = 10 \)
      \( 2p - 3q = 12 \)
      Answer: ______________________
   c) \( 2x + 3y = 2 \)
      \( 4x - y = 18 \)
      Answer: ______________________
   d) \( y = \frac{2}{3}x + 6 \)
      \( 3y - 2x = 8 \)
      Answer: ______________________
1. Solve each word problem:

   a) Elaine bought eight fuzzy peaches and one cucumber for $2.43. Les bought eight cucumbers and one fuzzy peach for $11.25. Find the cost of a cucumber.

   Answer: 

   b) The sum of the digits of a two-digit number is 13. The number with digits interchanged is 14 more than 20 times the original tens digit. Find the original number.

   Answer: 

   c) A canoeist who took 3 hours to paddle 9 miles upstream was able to return to his starting point in 30 minutes. At what rate could he paddle in still water?

   Answer: 

   d) Eleven years ago Mrs. Alva was three times as old as her daughter Rose. Five years from now Mrs. Alva will be 5 years less than twice as old as Rose. How old is each now?

   Answer: 
PERFORMANCE OBJECTIVE VI-7

Solve a system of linear inequalities graphically.

1. Solve each system of inequalities graphically
   a) \(x + y \geq 6\)
      \(y - x > -8\)
      Answer: 
   b) \(x + y < 5\)
      \(2y - x \geq -8\)
      Answer: 
   c) \(x \leq -4\)
      \(x - 2y \geq -2\)
      Answer: 
   d) \(y > -6\)
      \(2y + 2 < -x\)
      Answer: 

VI-17
1. Solve each problem.

   a) A regional park commission hopes to build campsites in two locations. At one location in the mountains, each campsite costs $750.00 and requires 36 man-hours of labor. At the other location, nearer a major city, each campsite costs $1500.00 and requires 12 man-hours of labor. The commission can spend at most $150,000.00 and use at most 4800 man-hours of labor in building the campsites. How many campsites should be built at each location in order to maximize the number of sites available?
   Answer: 

   b) An automobile assembly plant has a maximum production capacity of 1200 cars per day. The sales force predicts it can sell at most 600 Cloudstars and 800 Dachsunds per day. The profit for a Cloudstar is $650.00 and the profit for a Dachsund is $500.00. How many of each car should be produced in order to make the greatest profit?
   Answer: 

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PERFORMANCE OBJECTIVE VI-8 (continued)

Solve linear programming problems.

c) A student gets a job in a Tastee-Freez stand. Each day he has 80 units of milk, 70 units of ice cream, and 30 units of syrup. The specialty of the house is the "Last Stand" which consists of 2 units of milk, 3 units of ice cream and 1 unit of syrup. Another specialty, the "Vanilla Plains," consists of 2 units of milk, 1 unit of ice cream, and 1 unit of syrup. The "Last Stand" sells for $.80 and the "Vanilla Plains" sells for $.50. How many sales of each would be most profitable?
Answer: ________

d) Each week the McKay Trucking Co. needs at least 650 gallons of diesel fuel, 34 gallons of gasoline, and 48 gallons of oil to keep its fleet of trucks in operation. Pacific Petroleum Co. can deliver 130 gallons of diesel fuel, 36 gallons of gasoline, and 4 gallons of oil for a wholesale rate of $60.00. A similar plan costing $75.00 is available from Overseas Oil Company for 65 gallons of diesel fuel, 54 gallons of gas, and 12 gallons of oil. How many standing orders should the McKay Co. place with each firm in order to meet its petroleum needs at the smallest cost?
Answer: ________
UNIT VI - SYSTEMS OF OPEN SENTENCES

VI-1

a) intersecting
b) parallel
c) coinciding
d) B

VI-2

a)
1) B'
2) A
3) C
b)

VI-3

a) r = 0
   s = 5
b) φ
   c) y = 3
   x = 3
   d) {All x, y such that \(-x + 2y = 6\)}

VI-4

a) p = 1
   q = 0
b) r = 1
   s = 3
   c) a = 8
   b = -7
   d) p = 4
   q = 2
   {All x, y such that \(-x + 2y = 6\)}

VI-5

a) x = -1
   y = -3
b) r = 3
   q = -2
   c) x = 4
   y = -2
   d) φ

VI-20
UNIT VI - SYSTEMS OF OPEN SENTENCES

VI-6

a) A cucumber cost $1.39.
b) Original number is 49.
c) Canoeist could paddle $10\frac{1}{2}$ mph in still water.
d) Mrs. Alva is 44.
Rose is 22.

VI-7 (continued)

a) Let $x =$ number of campsites in the mountains
$y =$ number of campsites near the city
$x \geq 0$
$y \geq 0$
$750x + 1500y \leq 150,000$
$36x + 12y \leq 4800$
They should build 120 campsites in the mountains and 40 near the city.
UNIT VI - SYSTEMS OF OPEN SENTENCES

Answers (continued)

VI-8 (continued)

b) Let $x =$ number of Cloudstars produced each day $\quad y =$ number of Dachsunds produced each day

(1) $x \leq 600$  
(2) $y \leq 800$  
(3) $x + y \leq 1200$  
(4) $x \geq 0$  
(5) $y \geq 0$

To maximize profit, 600 of each car should be built each day.

c) Let $x =$ the number of "Last Stands" $\quad y =$ the number of "Vanilla Plains"

(1) $x \geq 0$  
(2) $y \geq 0$  
(3) $2x + 2y \leq 80$  
(4) $3x + y \leq 70$  
(5) $x + y \leq 30$

He should sell 20 "Last Stands" and 10 "Vanilla Plains" to make the greatest profit.
Answers (continued)

VI-8. (continued)

d) Let \( x = \) number of orders from Pacific Petroleum

Let \( y = \) number of orders from Overseas Oil

(1) \( x \geq 0 \)

(2) \( y \geq 0 \)

(3) \( 130x + 65y \geq 650 \)

(4) \( 36x + 54y \geq 324 \)

(5) \( 4x + 12y \geq 48 \)

To meet their needs at the smallest cost, McKay Trucking Co. should place 3 standing orders with Pacific Petroleum Co. and 4 with Overseas Oil Co.
UNIT VII - POLYNOMIALS

PURPOSE

This unit introduces the students to higher-order, non-linear relations. It is one of three interrelated units which concentrates on positive integral exponents. Students apply the laws of exponents to simplify polynomial expressions.

OVERVIEW

Students are introduced to the basic terminology of polynomials. The concept of combining similar terms is expanded to include the addition and subtraction of polynomials. The laws of exponents are combined with the addition and subtraction properties to simplify polynomial expressions. Some basic applications of polynomials are investigated through the solution of open sentences and word problems.

SUGGESTIONS TO THE TEACHER

Instructional Days: 16-19
Minimal Course Objectives: #1-12, 16-19
Average Course Objectives: #1-14, 16-19, 21, 22
Maximal Course Objectives: ALL

Due to the length of this unit, it is suggested that at least two tests be given.

Although many books discuss special products in relation to factoring, they are included as part of the multiplication objectives in this unit.

The FOIL method (First, Outer, Inner, Last) may be introduced as a procedure for multiplying two binomials.

Objectives containing the phrase "by inspection" are intended to require students to simplify expressions quickly without paper and pencil.

Some teachers feel that this unit is the most appropriate place to introduce zero and negative exponents.

Advanced students should be exposed to the most difficult exercises throughout this unit.

VOCABULARY

binomial
increasing order
decreasing order
monomial
degree of a monomial in a variable
polynomial
degree of a monomial
simple form
degree of a polynomial
trinomial

VII-1
UNIT VII - POLYNOMIALS

ENTERING PERFORMANCE OBJECTIVES

1. Simplify arithmetic expressions containing exponents.
2. Add, subtract, multiply, and divide integers.
3. Compute long division problems.
4. Compute three place multiplication problems.
5. Simplify expressions using the distributive property.
6. Combine similar terms.
7. Solve equations.

Assessment Tasks

1. Simplify each of the following expressions:
   a) \(4 \cdot 3^2 \cdot 4^2 \cdot 3^2\) 
   b) \(\frac{25}{2^3}\)
   c) \(5^2 + 9^2\)
   d) \(15 \div 25 - 110\)

2. a) \(4 + (-6) + (-7)\)
   b) \(814 + (-857) + 311\)
   c) \(1.033 + (-0.1) + (-10.066)\)
   d) \(1\frac{1}{3} + (-2\frac{3}{4})\)
   e) \(308 - 684\)
   f) \(-177 - 643\)
   g) \(-20.5 - 856.175\)
   h) \(11\frac{1}{3} - 12\frac{2}{5}\)
   i) \((96)(-65)\)
   j) \((-1.3)(-3.5)\)
   k) \((-56)(-79)\)
   l) \((-3\frac{6}{7})(2\frac{1}{3})\)
   m) \((-208) \div (-16)\)
UNIT VII - POLYNOMIALS

ENTRING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. (continued)
   n) $21.707 \div (-.07)$
   o) $(-6240) \div 65$
   p) $(-1\frac{3}{8}) \div (-5\frac{1}{2})$

3. a) $58)21,808$
    b) $72)60,048$
    c) $29)16,625$
    d) $126)38,430$

4. a) $257$
    b) $592$
    c) $728$
    d) $364$

   x917
   x795
   x238
   x419

   a) _________
   b) _________
   c) _________
   d) _________

5. Simplify each of the following expressions:
   a) $7(x + 4 - 6y)$
   b) $-5(3x - 9 + 7y)$
   c) $8(9x + 5y - 8)$
   d) $-1(2x^2 - 3x - 9)$

6. Combine similar terms.
   a) $23 + 5t + 7y + t + y + 27$
   b) $6x^4 + 3x^3 - 1 + 4x^4 - 2x^3 + 5$
   c) $5m^3 - 6m^2 + 4m - 6m^3 + 6m^2 - 5m$
   d) $2x^2 - 6x - 15 - 11x^2 + 7x - 6$

7. Solve each of the following equations:
   a) $7(y + 2) + 4(y - 1) + 12 = 0$
   b) $3(8n - 5) - 3(1 - n) = 9$
   c) $2 - 7(m - 1) = 3(m - 2) - 5(m + 3)$
   d) $6(2x + 1) - 3(4x - 3) - (6x + 10) = -(4x - 3) + 3$
UNIT VII - POLYNOMIALS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) 5184  b) 4  c) 106  d) 32

2. a) −9  b) 268  c) −9.133  d) −1\frac{5}{12}
   e) −376  f) 466  g) −876.675  h) −1\frac{1}{15}
   i) −6240  j) 4.55

3. a) 376  b) 834  c) \frac{573}{28}  d) 305

4. a) 235,669  b) 470,640  c) 173,264  d) 152,516

5. a) 7x + 28 - 42y  b) −15x + 45 - 35y
   c) 72x + 40y - 64  d) −2x^2 + 3x + 9

6. a) 6t + 8y + 50  b) 10x^4 + x^3 + 4
   c) −m^3 − m  d) −9x^2 + x - 21

7. a) y = −2  b) n = 1  c) m = 6  d) x = −\frac{1}{2}
UNIT VII - POLYNOMIALS

PERFORMANCE OBJECTIVES

1. Classify a polynomial according to the number of terms: monomial, binomial, trinomial. (II)

2. State the degree of a monomial. (II)

3. State the degree of a polynomial. (II)

4. Arrange a polynomial in increasing or decreasing order.

5. Add polynomials. (II)

6. Subtract polynomials. (II)

7. Multiply a monomial by a monomial. (II)

8. Determine the power of a product. (II)

9. Simplify expressions involving powers of products, multiplication of monomials, and combining similar terms. (II)

10. Multiply a polynomial by a monomial. (II)

11. Multiply two binomials. (II)

12. Multiply two binomials of the form \((a + b)(a - b)\) by inspection.

13. Find the square of a binomial by inspection. (II)

14. Multiply a trinomial by a binomial. (II)

15. Multiply two polynomials each containing at least three terms. (II)

16. Solve open sentences involving operations with polynomials. (III)

17. Divide a monomial by a monomial. (II)

18. Divide a polynomial by a monomial. (II)

19. Simplify expressions that involve dividing by a monomial and combining similar terms. (II)

20. Simplify expressions involving zero and negative exponents. (II)

21. Divide a polynomial by a polynomial. (II)

22. Solve word problems involving polynomials. (III)
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**KEY SKILLS FOR END-OF-COURSE TESTING**

15. Add and subtract polynomials.
17. Divide polynomials.
18. Solve linear equations involving polynomials.
### UNIT VII - POLYNOMIALS

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PERFORMANCE OBJECTIVE VII-1

Classify a polynomial according to the number of terms: monomial, binomial, or trinomial.

Identify each polynomial as a monomial, binomial, or a trinomial.

a) \( 3r^2s + 15 \)
   Answer __________

b) \( x^3 - 2xy + y^2 \)
   Answer __________

c) \( 4x^2y^3z^4 \)
   Answer __________

d) \( -5 + 2v^2w + 5x^3 \)
   Answer __________
PERFORMANCE OBJECTIVE VII-2

State the degree of a monomial.

a) State the degree of $-3x^2y^3z$.
   Answer __________

b) State the degree of $5x^3y$.
   Answer __________

c) State the degree of $11x^2yz^5$.
   Answer __________

d) The degree of $-5x^2y^7z$ is:
   A. 9
   B. 5
   C. 14
   D. 10
   Answer __________
PERFORMANCE OBJECTIVE VII-3

State the degree of a polynomial.

1. State the degree of each of the following:
   a) $4x^2yz + 7x^3z + 13x^5y^2z^2 + 17x^2y$
      Answer ________
   b) $5x^3yz^4 + 7x^5yz + 9xyz + 9x^2y^2z^2$
      Answer ________
   c) $6x^2y^2z^3 + 9xyz^2 + 11x^4y^5z^3 + 8x^3yz$
      Answer ________
   d) $7y^2x^2z + 15z^2y^2x^5 + 11x^3z^8y + 9zxy$
      Answer ________
   e) The degree of $-15x^3yz^5 + 75x^2y^2z + 15xy^7z^{11}$ is:
      A. 9
      B. 5
      C. 19
      D. 11
      Answer ________
PERFORMANCE OBJECTIVE VII-4

Arrange a polynomial in increasing or decreasing order.

a) Arrange the following polynomial in decreasing order of the degree of x:
   \[ 4xy^2z + 7x^3z + 13x^5y^2z + 17x^2y \]
   Answer

b) Arrange the following polynomial in increasing order of the degree of y:
   \[ 5x^3y^3z^4 + 7x^5yz + 8xy^4z + 9x^2y^2z^2 \]
   Answer

c) Arrange the following polynomial in decreasing order of the degree of z:
   \[ 9xyz + 11x^3yz^8 + 7x^2y^2z^3 + 15x^5y^2z^2 \]
   Answer

d) Arrange the following polynomial in increasing order of the degree of x:
   \[ 8x^3yz^2 + 13x^5z + 2x^2y^2z^2 + 11x^4yz \]
   Answer
PERFORMANCE OBJECTIVE VII-5

Add polynomials.

Simplify:

a) \((x + 5) + (4x - 7) + (5x - 8)\)
   Answer ______

b) \((8x^3 - 40x^2 + 50x) + (-20x^2 + 10x - 125)\)
   Answer ______

c) \((5x - y + z) + (3x - 8 - y) + (3y + 7)\)
   Answer ______

d) \((y^2 + 4y + 6) + (y^2 - 4y - 12) + (2y + 6 - y^2)\)
   Answer ______
PERFORMANCE OBJECTIVE VII-6

Subtract polynomials.

Simplify:

a) \((5a - 6b + c) - (3a + b - c)\)
   Answer __________

b) \((8ab - 5ax) - (9ax + 11ab)\)
   Answer __________

c) \((a^2 - a + 4) - (5 + 2a - a^2)\)
   Answer __________

d) \((3a^2 + 5ab - 7b^2) - (3ab - a^2 - b^2)\)
   Answer __________
PERFORMANCE OBJECTIVE VII-7

Multiply a monomial by a monomial.

Find the product of each pair of monomials:

a) \((2a^4)(5a^2)\)   Answer __________

b) \((-3m^2n)(7mn^2)\)   Answer __________

c) \((2ax^2y)(-5a^4xy^3)\)   Answer __________

d) \((-3a)(-2a^2b)(-a^3b^2)\)   Answer __________

e) \((a^m)(a^n)\)   Answer __________
Performance Objective VII-8

Determine the power of a product.

Simplify:

a) \((3a^2)^3\)
   Answer
b) \((-5m^4n^2)^3\)
   Answer
c) \((-3a^3b^4)^2\)
   Answer
d) \((2b^4c^5)^3\)
   Answer
e) \(m_n\)
   Answer
PERFORMANCE OBJECTIVE VII-9

Simplify expressions involving powers of products, multiplication of monomials, and combining similar terms.

Simplify:

a) \((3x^2y)^2 + (3x^2y^2)(2x^2)\)

Answer __________

b) \((3u)(u^2v)^3 + (2u)^2(-u^5v^3)\)

Answer __________

c) \((-6y^2z^3)^2(2yz) - (3yz^2)^3(-2y^2z)\)

Answer __________

d) \((5m^2n)^2(3m^3) - (3m^2)^2(2m^3n^2)\)

Answer __________
PERFORMANCE OBJECTIVE VII-10

Multiply a polynomial by a monomial.

Simplify:

a) \( 7x^2(x^3 - 2x^2 + 11) \)
   Answer

b) \( 4x(5 - x - 10x^2) \)
   Answer

c) \( -2abc(5a + 2b - 3c + 8) \)
   Answer

d) \( 2xy(3x^2 - 8xy + 5y^2) \)
   Answer
PERFORMANCE OBJECTIVE VII-11

Multiply two binomials.

Simplify:

a) \((3x + 4) \cdot (2x + 5)\)
   Answer

b) \((2x - 7) \cdot (x - 8)\)
   Answer

c) \((3x - 2) \cdot (2x + 3)\)
   Answer

d) \((-2x + 3) \cdot (3x - 2)\)
   Answer
PERFORMANCE OBJECTIVE VII-12

Multiply two binomials of the form \((a + b) (a - b)\) by inspection.

Find the product of the following binomials by inspection:

a) \((3x + 2)(3x - 2)\)
   
   Answer

b) \((y^2 - 5)(y^2 + 5)\)
   
   Answer

c) \((w - x)(w + x)\)
   
   Answer

d) \((5x + \frac{3}{7})(5x - \frac{3}{7})\)
   
   Answer
PERFORMANCE OBJECTIVE VII-13

Find the square of a binomial by inspection.

1. Find the square of the following binomials by inspection:
   a) \((x + 3)^2\)
      Answer __________________________
   b) \((2x - 3)^2\)
      Answer __________________________
   c) \((5w^2 + 6t^3)^2\)
      Answer __________________________
   d) \((xy^2 - w^3v)^2\)
      Answer __________________________
PERFORMANCE OBJECTIVE VII-14

Multiply a trinomial by a binomial.

Find each product:

a) \( m^2 + 5m - 6 \)

Answer

b) \( 5x^2 - 6x + 7 \)

Answer

c) \( (3c^2 - c + 5)(c^2 - 1) \)

Answer

d) \( (x^2 + 2xy + y^2)(x + y) \)

Answer
Multiply two polynomials each containing at least 3 terms.

Simplify the following:

a) \((x^2 + 3x - 2) (x^2 - 5x + 3)\)
Answer __________

b) \((2x^2 - x + 7) (3x^2 + 2x - 5)\)
Answer __________

c) \((3x^2 + 2xy - y^2) (2x^2 - 3xy + 5y^2)\)
Answer __________

d) \((a + 2b + c) (2a + 3b - c)\)
Answer __________
PERFORMANCE OBJECTIVE VII-16

Solve open sentences involving operations with polynomials.

Solve the following open sentences:

a) \(-10(3 - 4n) - 7(5n + 3) = -51\)
   Answer

b) \(6k - 5(3k + 2) = 5(k - 1) - 8\)
   Answer

c) \(3(x - 2) + 4(x + 6) = 3(k + 14) - 2\)
   Answer

d) \(4(2m + 5) - 2(5 - 6m) < 12 - 3(10 - 8m)\)
   Answer
PerformancE objective. VII-1.7

Divide a monomial by monomial.

Simplify:

a) \( \frac{42x^8y^3w}{6x^5y^3} \)

Answer

b) \( \frac{35a^4b^3c^2}{-5ab^5c} \)

Answer

c) \( \frac{36m^6n}{45m^5n} \)

Answer

d) \( \frac{-27a^7b^9c^5}{-3a^5b^7c^8} \)

Answer

e) \( \frac{a^m}{a^n} \)

Answer
PERFORMANCE OBJECTIVE VII-18

Divide a polynomial by a monomial.

1. Simplify:
   a) \(-\frac{9a^2b + 15ab^2 - 21ab}{-3ab}\)
      \(\text{Answer } \underline{\text{_______}}\)
   b) \(\frac{ax^3y^4 - bxy^3 + cx^2y^2}{x^2y^2}\)
      \(\text{Answer } \underline{\text{_______}}\)
   c) \((4a^2 - 6a^3 + 4a^4) \div (2a^2)\)
      \(\text{Answer } \underline{\text{_______}}\)
   d) \((3mx^3 + 18nx^3 - 12x^3y) \div 3x^3\)
      \(\text{Answer } \underline{\text{_______}}\)
PERFORMANCE OBJECTIVE VII-19

Simplify expressions that involve dividing by a monomial and combining similar terms.

Simplify:

a) \[ \frac{38p^3q^5}{19pq^2} - \frac{15p^4q^4}{-3p^2q} \]
Answer

b) \[ \frac{25y^3 - 15y^2 + 30y}{-5y} + \frac{8y^5 - 3y^3}{y^3} \]
Answer

c) \[ \frac{40cd^2 - 32c^2d + 24c^2d^2}{-8cd} + \frac{24c^3d - 12c^2d^2}{3c^2d} \]
Answer

d) \[ \frac{12m^5n^4 - 18m^6n^5}{6mn^2} - \frac{9m^7n^2 + 3m^6n^2}{3m^2} \]
Answer
Simplify expressions involving zero and negative exponents.

Simplify:

a) \( \frac{-5x^3y^{-5}w^2}{-75x^{-4}y^{-3}} \)
   Answer

b) \( \frac{3^{-2}x^2y^0}{9^{-1}x^{-2}y} \)
   Answer

c) \( \frac{x^{-2}y^0z^2}{x^{2}y^{-1}z} \)
   Answer

d) \( (x^2y^0z^{-3})^{-\frac{1}{3}} \)
   Answer

e) \( a^{-n} \)
   Answer
PERFORMANCE OBJECTIVE VII-21

Divide a polynomial by a polynomial.

Simplify:

a) \( x - 6 \) \( \overline{x^2 - 12x + 25} \)

b) \( x + y \) \( \overline{2x^2 + 3xy + y^2} \)

c) \( \frac{x^3 - y^3}{x - y} \)

Answer

d) \( \frac{10x^2 + 7 - 19x}{2x - 1} \)

Answer
PERFORMANCE OBJECTIVE VII-22

Solve word problems involving polynomials.

Solve each problem. Show all work.

a) The squares of two consecutive integers differ by 75. Find the integers.
Answer ________

b) A rectangular pool is 5 feet longer than it is wide. If a concrete walk 2 feet wide is placed around the pool, the area covered by the pool and the walk is 156 square feet greater than the area covered by the pool alone. What are the dimensions of the pool? (Hint: a sketch may help.)
Answer ________

c) A square and a rectangle have the same area. The length of the rectangle is three inches more than a side of the square. The width of the rectangle is 2 inches less than a side of the square. Find the side of the square. (Hint: a sketch may help.)
Answer ________

d) A rectangular piece of glass, whose dimensions are 17 cm by 12 cm, must be trimmed by cutting an equal amount from the length and width, so that its area is 104 cm². How much must be cut from the length and width?
Answer ________
UNIT VII - POLYNOMIALS

Answers

VII-1

a) binomial
b) trinomial
c) monomial
d) trinomial

VII-2

a) 6
b) 4
c) 9
d) d

c) 11
d) 12
e) c

VII-3

a) 9
b) 8
c) 11
d) 12
e) c

VII-4

a) 13x^5y^2z + 7x^3z + 17x^2y + 4xy^2z
b) 7x^5yz + 9x^2y^2z^2 + 5x^3y^2z^2 + 8xy^4z

VII-5

a) 10x - 10
b) 8x^3 - 60 + 60x - 125
c) 8x + y + z - 1
d) y^2 + 2y

VII-6

a) 2a - 7b + 2c
b) -3ab - 14ax
c) 2a^2 - 3a - 1
d) 4a^2 + 2ab - 6b^2

VII-7

a) 10a^6
b) -21m^3n^3
c) -10a^5x^3y^4
d) -6a^6b^3
e) a^m + n

VII-8

a) 27a^6
b) -125m^{12}n^6
c) 9a^6b^8
d) 8b^{12}c^5
e) \frac{amn}{b}

VII-34
UNIT VII - POLYNOMIALS

Answers (continued)

VII-9

a) $15x^4y^2$

b) $-u^7v^3$

c) $126y^5z^7$

d) $57m^7n^2$

V-10

b) $20x - 4x^2 - 40x^3$

c) $-10a^2bc - 4ab^2c + 6abc^2 - 16abc$

d) $6x^3y - 16x^2y^2 + 10xy^3$

VII-11

b) $2x^2 - 23x + 56$

c) $6x^2 - 13x + 6$

d) $-6x^2 + 13x - 6$

VII-12

a) $9x^2 - 4$

b) $y^4 - 25$

c) $w^2 - x^2$

d) $25x^2 - \frac{9}{49}$

VII-13

a) $x^2 + 6x + 9$

b) $4x^2 - 12x + 9$

c) $25w^4 + 60w^2t^3 + 36t^6$

d) $x^2y^4 - 2w^3vxy + w^6v^2$

VII-14

a) $2m^3 + 3lm^2 - 7m - 6$

b) $15x^3 - 23x^2 + 27x - 7$

c) $3c^4 - c^3 + 2c^2 + c - 5$

d) $x^3 + 3x^2y + 3xy^2 + y^3$

VII-15

a) $x^4 - 2x^3 - 14x^2 + 19x - 6$

b) $6x^4 + x^3 + 9x^2 + 33x - 35$

c) $6x^4 - 5x^3y + 7x^2y^2 + 13xy^3 - 5y^4$

d) $2a^2 + 7ab + 6b^2 + ac + bc - c^2$

VII-16

a) $n = 0$

b) $\frac{3}{14} = k$

c) $x = \frac{5}{2}$

d) $m > 7$

VII-17

a) $\frac{7x^3w}{8}$

b) $\frac{-7a^3c}{b^2}$

c) $\frac{4m}{5}$

d) $\frac{9a^2b^2}{c^3}$

e) $a^{m-n}$
UNIT VII - POLYNOMIALS

Answers (continued)

VII-18

a) $3a - 5b + 7$

b) $\frac{-ax}{y} + \frac{by}{x} - c$

c) $2 - 3a + 2a^2$

d) $m + 6n - 4y$

VII-19

a) $7p^2q^3$

b) $3y^2 + 3y - 9$

c) $-9d + 12c^3 - 3cd$

d) $m^4n^2 - 6m^5n^3$

VII-20

a) $\frac{x^{13}}{15y^2w^2}$

b) $\frac{x^4}{y}$

c) $\frac{y}{x^2z}$

d) $\frac{z^9}{x^6}$

e) $\frac{1}{a^n}$

VII-21

a) $x - 6 - \frac{11}{x - 6}$

b) $2x + y$

c) $x^2 + xy + y^2$

d) $5x - 7$

VII-22

a) 37, 38

b) 15' x 20'

c) Side = 6

d) 4 cm

VII-36

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UNIT VIII - FACTORING

PURPOSE

The concepts contained in this unit provide the students with a technique for expressing polynomials as indicated products. When polynomials are written in this form, the division property of equality can be used to simplify algebraic fractions and the zero product rule can be used to solve higher-order equations.

OVERVIEW

Algebraic factoring is introduced through a review of the concept of prime factorization. Various techniques for factoring polynomials are presented and applied to the solution of quadratic equations and word problems.

SUGGESTIONS TO THE TEACHER

Instructional Days: 12-15
Minimal Course Objectives: 1-8
Average Course Objectives: ALL
Maximal Course Objectives: ALL

An alternative method for factoring quadratic trinomials is offered:

A trinomial of the form $ax^2 + bx + c$ can be expressed as $ax^2 + rx + sx + c$ where $r + s = b$ and $rs = ac$.

The trinomial $6x^2 + 19x - 7$ can be written $6x^2 - 2x + 21x - 7$ where $-2 + 21 = 19$ and $-2 \cdot 21 = 6 \cdot 7 = -42$.

To complete the factoring, $6x^2 - 2x + 21x - 7 = 2x(3x - 1) + 7(3x - 1) = (3x - 1)(2x + 7)$.

The following procedure could be used for general factoring of polynomials:

1. Check for common factors.
2. Check for difference of squares.
3. Check for a trinomial square.
4. Factor as a product of sums, a product of differences, or a product of a sum and a difference.
5. Check each factor to see whether it can be factored by one of the methods in steps #2-4.
VOCABULARY

binomial difference
binomial sum
composite
constant term
cubic equation
difference of squares
factor
greatest common factor
linear term

polynomial equation
prime factorization
prime number
product
quadratic equation
quadratic term
trinomial square
zero product property
UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

1. Identify prime and composite numbers.
2. Prime factor given whole numbers.
3. Determine the greatest common factor of two or more whole numbers.
4. Multiply a polynomial by a monomial.
5. Multiply a binomial sum by a binomial difference.
6. Square a binomial.
7. Solve equations.

Assessment Tasks

1. a) Identify each of the following as being prime or composite:
   A. 17
   B. 26
   C. 13
   D. 68
   E. 57

b) Which of the following is a prime number?
   A. 0
   B. 1
   C. 19
   D. 87

Answer ________

c) Which of the following is a composite number?
   A. 5
   B. 11
   C. 20
   D. 53

Answer ________
II la

UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2.  a) $2^3 \cdot 5$ is the prime factorization of which of the following numbers?
   
   A. 30
   B. 40
   C. 1000
   D. None of the above.

   Answer ________

b) The prime factorization of 72 is:
   
   A. $2 \cdot 3 \cdot 12$
   B. $8 \cdot 9$
   C. $2^4 \cdot 3^2$
   D. $2^3 \cdot 3^2$

   Answer ________

c) The prime factorization of 300 is:

   A. $2^2 \cdot 3^3 \cdot 5^2$
   B. $2^3 \cdot 2^2 \cdot 5$
   C. $2^2 \cdot 3 \cdot 5^2$
   D. None of the above.

   Answer ________

d) Prime factor each of the following numbers:

   A. 36
   B. 54
   C. 84
   D. 105
   E. 140
   F. 180

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VIII-4
UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

3. a) What is the greatest common factor of 24 and 36?
   Answer
   b) The greatest common factor of 12 and 16 is:
      A. 2
      B. 4
      C. 6
      D. 8
   Answer
   c) What is the greatest common factor of 15, 25, and 30?
      Answer
   d) The greatest common factor of 30, 54, and 72 is:
      A. 6
      B. 9
      C. 12
      D. 27
   Answer

4. Simplify each of the following:
   a) \(-8x (3x^2 - 5x - 1)\)
   b) \(2x^3y (-8x^2y + 11xy^2)\)
   c) \(a^4 (a^3 - 2a^2 + 2)\)
   d) \(5s^2t (2t^2 - 3s^2t^2 + 6s^2)\)

5. a) \((8x - 7y)(8x + 7y)\)
   b) \((xy + 9)(xy - 9)\)
   c) \((x + 3)(x - 3)\)
   d) \((3x - 1)(3x + 1)\)
UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

6. a) Which of the following is the square of \((7x + 8y)\)?
   A. \(49x^2 + 56xy + 64y^2\)
   B. \(49x^2 + 56xy + 64y\)
   C. \(49x^2 + 112xy + 64y^2\)
   D. \(49x^2 + 64y^2\)
   Answer ________

   b) Which of the following is the square of \((9x - 4y)\)?
   A. \(81x^2 + 72xy - 16y^2\)
   B. \(81x^2 - 72xy + 16y^2\)
   C. \(81x^2 - 16y^2\)
   D. \(9x^2 - 13xy + 16y^2\)
   Answer ________

   c) \((3x - 1)^2\)
   Answer ________

   d) \((2x + 9)^2\)
   Answer ________

7. Solve each of the following equations:
   a) \(\frac{1}{2}x + 6 = 0\)
   b) \(\frac{2}{3}x = 0\)
   c) \(5t + 9 - (3t + 1) = 0\)
   d) \(3(5x - 7) + 66 = 0\)
   a) ________
   b) ________
   c) ________
   d) ________
UNIT VIII - FACTORING

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) A. prime
   B. composite
   C. prime
   D. composite
   E. composite
   b) C
   c) C

2. a) B
   b) D
   c) C
   d) A. $2^2 \cdot 3^2$
   B. $2 \cdot 3^3$
   C. $2^2 \cdot 3 \cdot 7$
   D. $3 \cdot 5 \cdot 7$
   E. $2^2 \cdot 5 \cdot 7$
   F. $2^2 \cdot 3^2 \cdot 5$

3. a) 12
   b) B
   c) 5
   d) A

4. a) $-24x^3 + 40x^2 + 8x$
   b) $-16x^5y^2 + 22x^4y^3$
   c) $a^7 - 2a^6 + 2a^4$
   d) $10s^2t^3 - 15s^4t^3 + 30s^4t$

5. a) $64x^2 - 49y^2$
   b) $x^2y^2 - 81$
   c) $x^2 - 9$
   d) $9x^2 - 1$

6. a) C
   b) B
   c) $9x^2 - 6x + 1$
   d) $4x^2 + 36x + 81$

7. a) $x = -12$
   b) $x = 0$
   c) $t = -4$
   d) $x = -3$

VIII-7
UNIT VIII - FACTORING

PERFORMANCE OBJECTIVES
1. Determine the GCF of two or more monomials with integral coefficients.
2. Factor a polynomial by isolating the greatest common factor.
3. Factor a polynomial that is the difference of two perfect squares.
4. Factor a trinomial that is the square of a binomial.
5. Factor a trinomial of the form $x^2 + bx + c$.
6. Factor or a trinomial of the form $ax^2 + bx + c$.
7. Factor a polynomial completely.
8. Solve equations by factoring.
9. Solve word problems involving factoring.

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KEY SKILLS FOR END-OF-COURSE TESTING
19. Factor polynomials completely.
20. Solve equations by factoring.
21. Solve word problems involving factoring.
CROSS REFERENCES

TEXTS (BY AUTHOR)

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## CROSS REFERENCES

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VIII-10
PERFORMANCE OBJECTIVE VIII-1

Determine the greatest common factor of two or more monomials with integral coefficients.

a) The greatest common factor of the terms of the polynomial
   \[ 24x^5 + 60x^4 - 108x^3 \]
   A. 24
   B. 6x^3
   C. 12x^3
   D. 12x^3
   E. x^3
   Answer ________

b) State the greatest common factor of the terms of each polynomial:
   b) \[-30z^6 + 18z^3 - 24z^2\]
   Answer ________

c) \[11ab + 23a^2b^3 - 69a^2b^5\]
   Answer ________

d) \[75m^7n^3 - 60m^6n^2 - 45m^6n\]
   Answer ________

PERFORMANCE OBJECTIVE VIII-2

Factors a polynomial by isolating the greatest common factor.

a) Which of the following is the prime factorization of

$$50x^2y^5 - 100x^2y^2 + 125x^3y^3$$

A. $$25x^2y^2(2y^3 - 4x^3 + 5xy)$$
B. $$25xy(2xy^4 - 4x^4y + 5x^2y^2)$$
C. $$25(2x^2y^5 - 4x^5y^2 + 5x^3y^3)$$
D. None of the above

Answer

b) Determine the missing monomial factor given a polynomial factor and

the product.

1) ____ $$(2x^2 + 5x - 9)x = 24x^5 + 60x^4 - 108x^3$$.
2) ____ $$(5z^4 - 3z + 4) = -30z^6 + 18z^3 - 24z^2$$.
3) ____ $$(11 + 23ab^2 - 30a^2b^4) = 11ab + 23a^2b^3 - 30a^3b^5$$.
4) ____ $$(5m^3n^2 + 4m^2n - 3) = 75m^2n^3 + 60m^2n^2 - 45m^2n$$

c) Factor:

1) $$9x^3 + 18x^2 + 24x$$
   Answer __________
2) $$72mn^2 - 48mn$$
   Answer __________
3) $$17a^3 + 9a^2 + 7a$$
   Answer __________
PERFORMANCE OBJECTIVE VIII-2 (continued)

d) Factor:
1) \(24a^3b + 36a^2b + 18ab\)
Answer \(_______\)
2) \(6x^3y^2z^2 + 14x^2y^2z + 22x^2yz^2\)
Answer \(_______\)
3) \(5abc^2 - 10ab^2c - 25a^2bc\)
Answer \(_______\)
PERFORMANCE OBJECTIVE VIII-3

Factor a polynomial that is the difference of two perfect squares.

a) Identify the binomials that are the difference of two perfect squares by writing DS in the blank provided. (Write NOT if it is not a difference of squares.)

1) $x^2 - 9$
2) $-9 + 4y^2$
3) $m^6 - n^2$
4) $-(x^2 + 36)$
5) $6x^2 - 36$
6) $-16y^2 + 1$
7) $x^2a - 4$
8) $x^8 - 16$

b) Factor:

1) $x^2 - 4$

Answer

2) $49x^2 - 9$

Answer

c) Factor:

1) $-4x^2 + 1$

Answer

2) $9x^2 - 25y^2$

Answer

d) Factor:

1) $y^{2n} - 1$

Answer

2) $n^6 - m^6$

Answer
PERFORMANCE OBJECTIVE VIII-4

Factor a trinomial that is the square of a binomial.

a) Which of these is factored incorrectly?
   A. \(a^2 + 4a + 4 = (a + 2)(a + 2)\)
   B. \(x^2 - 2x + 1 = (x + 1)(x + 1)\)
   C. \(4x^2 - 12x + 9 = (2x - 3)(2x - 3)\)
   D. \(1 + 6y + 9y^2 = (1 + 3y)(1 + 3y)\)
   Answer

b) To be factored as the square of a binomial, the missing term of the polynomial \(9x^2 + \boxed{} + 4\) must be:
   A. \(36x\)
   B. \(72x\)
   C. \(12x\)
   D. \(6x\)
   Answer

c) Factor:
   1) \(x^2 + 2x + 1\)
      Answer
   2) \(a^2 + 2ab + b^2\)
      Answer

d) Factor:
   1) \(9 - 12x + 4x^2\)
      Answer
   2) \(x^{2a} + 2x^a + 1\)
      Answer
PERFORMANCE OBJECTIVE VIII-5

Factor a trinomial of the form \( x^2 + bx + c \).

a) Which of the following have a common binomial factor?

1. \( x^2 + 3x - 54 \)
2. \( x^2 - 3x - 18 \)
3. \( x^2 + 2x - 24 \)

A. (a) 1 and 2
B. (b) 2 and 3
C. (c) 1 and 3
D. (d) 1, 2, and 3
E. (e) None of the above

Answer:

b) What is the common binomial factor of the following polynomials?

\( x^2 - 7x + 12 \)
\( x^2 - 10x + 24 \)
\( x^2 + 5x - 36 \)

A. (x + 4)
B. (x - 4)
C. (x - 2)
D. (x - 6)
E. None of the above

Answer:
PERFORMANCE OBJECTIVE VIII-5 (continued)

c). Factor completely:

1) \( x^2 + 10x + 24 \)

Answer

2) \( x^2 + 14x + 40 \)

Answer

d) Factor completely:

1) \( x^2 - 5x - 36 \)

Answer

2) \( x^2 - 13x + 12 \)

Answer
PERFORMANCE OBJECTIVE VIII-6

Factor a trinomial of the form $ax^2 + bx + c$.

a) Which of the following 3 trinomials have a common binomial factor?
   1. $3x^2 - 16x - 12$
   2. $6x^2 - 23x - 18$
   3. $6x^2 + 5x - 6$
   A. 1 and 2
   B. 1 and 3
   C. 2 and 3
   D. 1, 2; and 3
   E. None of the above

Answer __________

Factor the following trinomials:
   b) $6x^2 + 19x + 10$
      Answer __________
   c) $8x^2 - 34x + 21$
      Answer __________
   d) $20x^2 + 7x - 6$
      Answer __________
PERFORMANCE OBJECTIVE VIII-7

Factor a polynomial completely.

Factor as completely as possible:

a) $15x^4 - 10x^3 - 25x^2$
   Answer __________

b) $a^8 - 256$
   Answer __________

c) $2y^4 - 15y^2 - 27$
   Answer __________

d) $m^4 - 13m^2 + 36$
   Answer __________
PERFORMANCE OBJECTIVE VIII-8

Solve equations by factoring.

a) The sum of the roots of the equation \( x^2 + 6x + 8 = 0 \) is:
   
   A. 6  
   B. 8  
   C. -8  
   D. -6  

   Answer ________

b) If \((x + 2)(x - 3) = 0\), which of these four conclusions follows?
   
   A. \((x + 2) = 0\) and \((x - 3) = 0\)  
   B. \(x = 2\) and \(x = -3\)  
   C. \(x^2 - x = -6\)  
   D. Either \((x + 2) = 0\) or \((x - 3) = 0\)  

   Answer ________

c) Solve by factoring:
   \[ x^2 - 3x - 18 = 0 \]

   Answer ________

d) Solve by factoring:
   \[ 6x^2 - 15x + 6 = 0 \]

   Answer ________

e) Solve by factoring:
   \[ 4x^3 - 9x = 0 \]

   Answer ________
PERFORMANCE OBJECTIVE VII-9

Solve word problems involving factoring.

a) In a school auditorium the number of seats in each row is 3 fewer than the number of rows. How many seats are in each row if the auditorium seats 810 persons?
Answer _______

b) The Centerville Municipal Pool measures 500 feet by 40 feet. How wide must a concrete walk around the pool be if the walk is to cover an area of 1000 square feet?
Answer _______

c) Find two consecutive integers such that their product is 72 less than twice the square of the first integer.
Answer _______

d) A rectangular piece of sheet metal is 12 cm wide and 17 cm long. An equal amount is to be cut from the length and width. How much must be cut off from the length and width to leave an area of 84 cm$^2$?
Answer _______
UNIT VIII - FACTORING

Answers

VIII-1

a) D
b) $-6z^2$ or $6z^2$
c) ab
d) $15m^4n$

VIII-2

a) A
b) 1) $12x^3$
   2) $-6z^2$
   3) ab
   4) $15m^4n$
c) 1) $3x(3x^2 + 6x + 8)$
   2) $24mn(3n - 2)$
   3) $a(17a^2 + 9a + 7)$
d) 1) $6ab(4a^2 + 6a + 3)$
   2) $2x^2yz(3xyz + 7y + 11z)$
   3) $5abc(c - 2b - 5a)$

VIII-3

a) 1) DS
   2) DS
   3) DS
   4) NOT
   5) NOT
   6) DS
   7) DS
   8) DS
b) 1) $(x + 2)(x - 2)$
   2) $(7x + 3)(7x - 3)$
c) 1) $(1 + 2x)(1 - 2x)$
   2) $(3x + 5y)(3x - 5y)$
d) 1) $(y^n + 1)(y^n - 1)$
   2) $(n^3 + m^3)(n^3 - m^3)$
UNIT VIII - FACTORING

Answers (continued)

VIII-4
a) B
b) C

c) c) 
1) \((x + 1)^2\)
2) \((a + b)^2\)

d) d) 
1) \((2x - 3)^2\)
2) \((x^a + 1)^2\)

VIII-5
a) A
b) B

c) c) 
1) \((x + 6)(x + 4)\)
2) \((x + 10)(x + 4)\)

d) d) 
1) \((x - 9)(x + 4)\)
2) \((x - 12)(x - 1)\)

VIII-6
a) A
b) \((3x + 2)(2x + 5)\)

c) \((4x - 3)(2x - 7)\)

d) \((4x + 3)(5x - 2)\)

VIII-7
a) \(5x^2(x + 1)(3x - 5)\)

b) \((a^4 + 16)(a^2 + 4)(a + 2)(a - 2)\)

VIII-8

a) \((2y^2 + 3)(y + 3)(y - 3)\)

b) \((m + 2)(m - 2)(m + 3)(m - 3)\)

c) \{−3, 6\}

d) \{\frac{1}{2}, 2\}\}

e) \{−\frac{11}{2}, 0, \frac{11}{2}\}

VIII-9

a) There are 27 seats in each row

b) The walk must be 5 feet wide

c) The integers are 9 and 10 or −8 and −7

d) Five cm must be cut from the length and width so that 84 cm² are left.

VIII-23
UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

PURPOSE

In this unit, techniques for simplifying and factoring polynomial expressions are applied to algebraic fractions. Rational algebraic expressions and fractional equations are utilized in the solution of related word problems.

OVERVIEW

Simplification of algebraic fractions and the basic operations with algebraic fractions are stressed. Solutions of fractional equations and word problems involving algebraic fractions are discussed.

SUGGESTIONS TO THE TEACHER

Instructional Days: 11-14
Minimal Course Objectives: #1-7, 10
Average Course Objectives: #1-6, 10-12
Maximal Course Objectives: ALL

The parallel between the techniques for simplifying arithmetic fractions and algebraic fractions should be stressed.

VOCABULARY

algebraic fraction
complex fraction
denominator
extremes
extraneous roots
fractional equations
lowest terms

means
mixed expressions
nonrational
numerator
proportion
ratio
rational algebraic expression
reciprocal
UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

1. Reduce arithmetic fractions.
2. Multiply arithmetic fractions.
3. Divide arithmetic fractions.
4. Change mixed numbers to improper fractions.
5. Change improper fractions to mixed numbers.
6. Add and subtract fractions with like denominators.
7. Add and subtract fractions with unlike denominators.
8. Solve equations which equal zero.

Assessment Tasks

1. Simplify each fraction.
   a) \[
   \frac{17}{34}
   \]
   b) \[
   \frac{24}{36}
   \]
   c) \[
   \frac{24}{32}
   \]
   d) \[
   \frac{15}{45}
   \]
   e) \[
   \frac{20}{32}
   \]
   f) \[
   \frac{16}{18}
   \]
   g) \[
   \frac{54}{81}
   \]
   h) \[
   \frac{20}{45}
   \]
   i) \[
   \frac{57}{95}
   \]
   j) \[
   \frac{36}{48}
   \]
UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. a) \( \frac{1}{8} \times \frac{1}{3} \)
   b) \( \frac{15}{8} \times \frac{1}{2} \)
   c) \( \frac{3}{8} \times \frac{4}{5} \)
   d) \( \frac{4}{3} \times \frac{2}{13} \)

3. a) \( 3 \div \frac{5}{8} \)
   b) \( \frac{7}{9} \div \frac{1}{6} \)
   c) \( \frac{1}{2} \div \frac{3}{14} \)
   d) \( \frac{13}{3} \div \frac{1}{13} \)

4. Change to improper fractions.
   a) \( \frac{8}{9} \)
   b) \( \frac{2}{3} \)
   c) \( \frac{5}{6} \)
   d) \( \frac{12}{4} \)

5. Change to a mixed number.
   a) \( \frac{14}{5} \)
   b) \( \frac{35}{6} \)
   c) \( \frac{37}{3} \)
   d) \( \frac{95}{17} \)
UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

6. a) \( \frac{9}{10} - \frac{7}{10} \)  
   \( \frac{2}{5} + \frac{4}{5} \)  
   \( \frac{7}{15} - \frac{11}{15} \)  
   \( \frac{19}{20} + \frac{9}{20} \)

6. b) a) \( \frac{9}{10} - \frac{7}{10} \)  
   b) \( \frac{2}{5} + \frac{4}{5} \)  
   c) \( \frac{7}{15} - \frac{11}{15} \)  
   d) \( \frac{19}{20} + \frac{9}{20} \)

7. a) \( \frac{3}{4} - \frac{1}{2} \)  
   b) \( \frac{3}{5} + \frac{4}{9} \)  
   c) \( \frac{7}{12} + \frac{1}{18} \)  
   d) \( \frac{7}{8} - \frac{3}{5} \)

7. b) a) \( \frac{3}{4} - \frac{1}{2} \)  
   b) \( \frac{3}{5} + \frac{4}{9} \)  
   c) \( \frac{7}{12} + \frac{1}{18} \)  
   d) \( \frac{7}{8} - \frac{3}{5} \)

8. Solve each of the following equations:
   a) \( x^2 + 5x - 24 = 0 \)
   b) \( 3x = 0 \)
   c) \( x^2 - x - 6 = 0 \)
   d) \( x(x + 3)(x - 4) = 0 \)
ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) $\frac{1}{2}$
   b) $\frac{2}{3}$
   c) $\frac{3}{4}$
   d) $\frac{1}{3}$
   e) $\frac{5}{8}$
   f) $\frac{8}{9}$
   g) $\frac{2}{3}$
   h) $\frac{4}{9}$
   i) $\frac{3}{5}$
   j) $\frac{3}{4}$

2. a) $\frac{1}{24}$
   b) $\frac{15}{16}$
   c) $\frac{11}{10}$
   d) $\frac{19}{3}$ or $6\frac{1}{8}$

3. a) $\frac{24}{5}$ or $\frac{44}{5}$
   b) $\frac{14}{3}$ or $\frac{42}{3}$
   c) $\frac{2}{7}$
   d) 10

4. a) $\frac{26}{9}$
   b) $\frac{14}{3}$
   c) $\frac{35}{6}$
   d) $\frac{51}{4}$

5. a) $\frac{24}{5}$
   b) $\frac{5}{6}$
   c) $12\frac{1}{3}$
   d) $\frac{10}{17}$

6. a) $\frac{1}{5}$
   b) $\frac{6}{5}$ or $1\frac{1}{5}$
   c) $\frac{4}{15}$
   d) $\frac{7}{5}$ or $1\frac{2}{5}$

7. a) $\frac{1}{4}$
   b) $\frac{47}{45}$ or $\frac{2}{45}$
   c) $\frac{23}{36}$
   d) $\frac{11}{40}$

8. a) $x = -8$ or 3
   b) $x = 0$
   c) $x = 3$ or $-2$
   d) $x = 0, -3$ or 4
UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

PERFORMANCE OBJECTIVES

1. Determine which value(s) of a variable make the denominator of a given fraction equal to zero. (II)

2. Simplify an algebraic fraction. (II)

3. Multiply algebraic fractions. (II)

4. Divide algebraic fractions. (II)

5. Simplify expressions containing both multiplication and division of algebraic fractions. (II)

6. Add (subtract) algebraic fractions with like denominators. (II)

7. Add (subtract) algebraic fractions with unlike denominators. (II)

8. Write a mixed expression as a single fraction. (II)

9. Simplify a complex algebraic fraction. (II)

10. Solve open sentences with fractional coefficients. (III)

11. Solve fractional equations. (III)

12. Solve word problems that involve the use of algebraic fractions. (III)

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KEY SKILLS FOR END-OF-COURSE TESTING

22. Simplify an algebraic fraction.

23. Multiply and divide algebraic fractions.


25. Solve fractional equations.
## UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

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IX-8

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### Cross References

#### Texts (By Author)

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PERFORMANCE OBJECTIVE IX-1

Determine which value(s) of a variable make the denominator of a given fraction equal to zero.

a) Determine the values of the variable which make the denominator of the fraction \( \frac{x+3}{x^2-5x-14} \) equal zero.

The sum of these values is:

A. -5
B. 0
C. 5
D. 7
E. none of the above

Answer

Determine the values of the variables which make the denominators of the following fractions equal zero.

b) \( \frac{2y^2+10y+8}{y^3-y} \)

Answer

j) \( \frac{n^2-7n}{n^2-8n+7} \)

Answer

Answer
PERFORMANCE OBJECTIVE IX-2

Simplify an algebraic fraction.

a) Which of the following fractions have the same simplest form?

1. \( \frac{5x^2+10x}{5x} \)
2. \( \frac{x^2-4}{x^2-4x+4} \)
3. \( \frac{x^2+5x+6}{x+3} \)

A. 1 and 2  
B. 1 and 3  
C. 2 and 3  
D. 1, 2, and 3  
E. one of the above

Answer

Write each of the following fractions in simplest form.

b) \( \frac{y^2-3y+2}{1-y} \)

c) \( \frac{m^2-5m+6}{m^2-9} \)

d) \( \frac{x+3}{x^2-4x-21} \)
PERFORMANCE OBJECTIVE IX-3

Multiply algebraic fractions.

a) What is the simplest form of the expression

\[
\frac{3x+3y}{30x} \cdot \frac{5x-5y}{x^2-y^2} \]

A. \( \frac{1}{2} \)

B. \( \frac{1}{2x} \)

C. \( \frac{x+y}{10} \cdot \frac{5}{x+y} \)

D. \( \frac{5}{x} \)

Answer

b) What is the simplest form of the expression

\[
\frac{a^2-2a-8}{a^2+2a-3} \div \frac{a^2-2a-8}{a^2+3a-4} \div \frac{a^2-6a+8}{a^2+6a+8} \]

A. \( \frac{a+2}{a-2} \)

B. \( \frac{a-1}{a+4} \)

C. 1

D. \( \frac{a+2}{a+3} \)

Answer

c) Simplify

\[
\frac{a^2+2a-15}{a^2+7a+10} \div \frac{3a^2+12a+12}{5a^2-5a-30} \]

Answer

d) Simplify

\[
\frac{p^2-4}{4p+8} \div \frac{4p-8}{p^2-4p+4} \]

Answer
PERFORMANCE OBJECTIVE IX-4

Divide algebraic fractions.

a) What is the simplest form of the quotient

\[
\frac{x^2-16}{x+2} \div \frac{x-4}{2x+4} \tag{?}
\]

A. \( \frac{2x+4}{x+2} \)
B. \( 2x-16 \)
C. \( \frac{x^2-2x-12}{2x-2} \)
D. \( 2x+8 \)

Answer:

b) What is the simplest form of the quotient

\[
\frac{p^2-2p-8}{p^2-p-12} \div \frac{p^2-6p+8}{p^2+p-6} \tag{?}
\]

A. \( 1 \)
B. \( p+2 \)
C. \( \frac{p+2}{p-4} \)
D. None of the above

Answer:

c) Simplify each quotient.

1. \( \frac{y^2-25}{y+1} \div \frac{6y+30}{18x^2y} \)

Answer:

2. \( \frac{3a^2-48}{a^2-a-20} \div \frac{a^2-a-12}{a^2-2a-15} \)

Answer:

d) Simplify each quotient.

1. \( \frac{x}{x^2-2x+1} \div \frac{1}{1-x^2} \)

Answer:

2. \( \frac{2m^2+4m-6}{m^2-9} \div \frac{5m^2+30m-35}{m^2-10m+21} \)

Answer:

IX-13

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PERFORMANCE OBJECTIVE IX-5

Simplify expressions containing both multiplication and division of algebraic fractions.

a) \( \frac{x}{x + 1} \cdot \frac{6x + 18}{5x^2} \div \frac{x^2 - 9}{5x^3 + 5x^2} \)

Answer

b) \( \frac{m^2 + 2m - 15}{m^2 - m - 6} \cdot \frac{m^2 - 25}{m^2 - 4m - 5} \div \frac{m^2 + 5m + 6}{m^2 - 1} \)

Answer

c) \( \frac{10i + 5k}{j^2 - jk - 2k^2} \cdot \frac{3j - 6k}{4j^2 - k^2} \div \frac{15j - 15k}{j^2 - k^2} \)

Answer

d) \( \frac{r^2 + 11r + 18}{r^2 + 4r - 5} \div \frac{r^2 - 7r - 8}{r^2 + 2r - 15} \div \frac{r^2 - 6r + 7}{r^2 + 8r + 12} \)

Answer
PERFORMANCE OBJECTIVE IX-6

Add (subtract) algebraic fractions with like denominators.

Simplify the following

a) \( \frac{8}{5x} - \frac{2}{5x} + \frac{4}{5x} \)

b) \( \frac{h^2}{h+j} - \frac{1^2}{h+j} \)

Answer \[ \frac{h^2 - 1^2}{h+j} \]

Answer \[ \frac{h^2 - 1^2}{h+j} \]

c) \( \frac{k^2}{k-3} + \frac{9}{k-3} - \frac{6k}{k-3} \)

d) \( \frac{4a+10}{2a^2+a-3} - \frac{2a+7}{2a^2+a-3} \)

Answer \[ \frac{k^2 + 9 - 6k}{k-3} \]

Answer \[ \frac{4a+10 - 2a+7}{2a^2+a-3} \]
PERFORMANCE OBJECTIVE IX-7

Add (subtract) algebraic fractions with unlike denominations.

a) What is the sum in simplest form of

\[
\frac{3}{b^2 - 2b - 8} + \frac{2}{4 - b^2}
\]

A. \[
\frac{1}{(b + 2)(b - 4)}
\]

B. \[
\frac{-b - 2}{-1(b - 4)(b + 2)(b - 2)}
\]

C. \[
\frac{1}{-1(b + 2)(b - 4)}
\]

D. \[
\frac{1}{(b - 2)(b - 4)}
\]

Answer: [Blank]

b) Simplify the following:

\[
\frac{3}{2m + 18} + \frac{27}{m^2 - 81}
\]

Answer: [Blank]

c) \[
\frac{5}{2z^2 - 7z - 4} - \frac{2}{2z^2 + 9z + 4}
\]

Answer: [Blank]

d) \[
\frac{2q + 3}{q - 8} - \frac{q^2 + 6q + 5}{q^2 - 7q - 8}
\]

Answer: [Blank]

IX-16
PERFORMANCE OBJECTIVE IX-8

Write a mixed expression as a single fraction.

a) Express the following expression as a single fraction $5 + \frac{m + n}{m - n}$.

A. $\frac{5 + m + n}{m - n}$

B. $\frac{6m}{m - n}$

C. $\frac{6m - 4n}{m - n}$

D. None of the above

Answer ________

Express each of the following expressions as a single fraction.

b) $s + 3 + \frac{1}{s + 1}$

Answer ________

c) $e + 2f - \frac{2ef}{e-f}$

Answer ________

d) $3d - 2 + \frac{9}{d+3}$

Answer ________
PERFORMANCE OBJECTIVE IX-9

Simplify a complex algebraic fraction.

a) What is the simplest form of the fraction \( \frac{m^2 + 4m + 4}{m^2 - m - 6} \) Simplify each fraction.

\( a^2 - 1 \)

h) \( \frac{a^2 - 6a + 5}{a^2 - 4a - 5} \)

\( \frac{a^2 - 10a + 25}{a^2 - 4a - 5} \)

Answer

\( \frac{k - 1}{k - 2} \)

c) \( \frac{k^2 + k - 2}{k^2 - k - 2} \)

Answer

d) \( \frac{1 - \frac{2}{c}}{1 - \frac{1}{c} - \frac{2}{c^2}} \)

Answer
PERFORMANCE OBJECTIVE IX-10

Solve open sentences with fractional coefficients.

a) Solve the equations \( \frac{b}{12} + \frac{b - 4}{12} = 5 \) and

\[ \frac{4z + 3}{15} - \frac{2z - 3}{9} = \frac{3z + 2}{3} - z \]

then \( b + z \) is:
A. \( \frac{71}{2} \)
B. \( 31\frac{6}{7} \)
C. 32
D. 35
E. 29

Answer __________

Solve the following equations.

b) \( \frac{4x + 1}{6} + \frac{3x}{4} = \frac{2x - 4}{3} \)

Answer __________

c) \( \frac{b + 5}{9} - b - 3 = \frac{2b + 2}{3} \)

Answer __________

d) \( \frac{m + 2}{6} - \frac{m - 7}{3} = 8 \)

Answer __________
Solve fractional equations.

a) If you solve each of the equations \( \frac{x}{2} + \frac{x}{5} = 35 \), \( \frac{3}{x} + \frac{2}{5} = \frac{19}{10} \), the difference between their roots is:

A. 52  
B. 48  
C. 3  
D. None of the above  

Answer __________

Solve the following equations:

b) \( \frac{5q}{q + 1} - \frac{q}{q + 6} = 4 \)

Answer __________

c) \( \frac{3x - 1}{x + 3} - \frac{4x}{x - 3} = -3 \)

Answer __________

d) \( \frac{n + 2}{n - 2} = \frac{2}{n + 2} - \frac{7}{3} \)

Answer __________
Solve each problem. Show all work.

a) Mr. Smithers can paint his fence in 6 hours, but it takes his daughter 9 hours to paint it. How long will it take to paint the fence if both work together?
   Answer ________

b) Fred can finish his paper route in 3 hours. If his sister Jill helps him, they both can deliver the papers in 2 hours. How long would it take Ned to deliver the papers by himself?
   Answer ________

c) The distance from Cooper to Asheville is 60 miles. Ms. Pawning drove from Cooper to Asheville and returned. The entire trip took 5 hours. She averaged 10 mph more while returning from Asheville. How fast did she travel each way?
   Answer ________

d) A freight train travels 160 miles in the same time that a trailer truck travels 75 miles. If the rate of the train is 18 mph greater than the rate of the truck, find the rate of each.
   Answer ________
UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

Answers

IX-1
a) C
b) $y \neq 0, 1, -1$
c) $x \neq 2, -2$
d) $n \neq 1, 7$

IX-2
a) B
b) $-y + 2$
c) $\frac{m - 2}{m + 3}$
d) $\frac{1}{x - 7}$

IX-3
a) B.
b) C
c) $\frac{3}{5}$
d) 1

IX-4
a) D
b) C
c) $1. \frac{3x^2y(y - 5)}{y + 1}$
d) $2. \frac{-x(x + 1)}{x - 1}$

IX-5
a) $\frac{6x}{x - 3}$
b) $\frac{m + 3}{m - 1}$
c) $\frac{1}{2j - k}$
d) $\frac{(r + 9)(r - 7)(r - 3)}{(r - 1)(r + 6)(r - 8)}$

IX-6
a) $\frac{2}{x}$
b) $h - j$
c) $k - 3$
d) $\frac{1}{a - 1}$

IX-7
a) D
b) $\frac{3}{2m - 18}$
c) $\frac{3z + 28}{(2z + 1)(z - 4)(z + 4)}$
d) $\frac{q - 2}{q - 8}$

IX-22
$2z^2 - 54$
UNIT IX - RATIONAL ALGEBRAIC EXPRESSIONS

Answers (continued)

IX-8

a) C

b) \( \frac{s^2 + 4s + 4}{s + 1} \)

c) \( \frac{e^2 - ef - 2f^2}{e - f} \)

d) \( \frac{3d^2 + 7d + 3}{d + 3} \)

IX-9

a) B

b) 1

c) \( \frac{k + 1}{k + 2} \)

d) \( \frac{c}{c + 1} \)

IX-10

a) D

b) \( x = -2 \)

c) \( b = -2 \)

d) \( m = -32 \)

IX-11

a) B

b) \( q = 24 \)

c) \( x = -1 \) or \( x = 12 \)

d) \( n = \frac{2}{5} \) or \( n = -1 \)

IX-12

a) It will take them \( 3\frac{3}{5} \) hours if they work together.

b) It would take Ned 6 hours to deliver the papers himself.

c) Going to Asheyleille, Ms. Pawning averaged 20 mph and returning she averaged 30 mph.

d) The truck averages 54 mph and the train averages .72 mph.
UNIT X - RADICAL EXPRESSIONS

PURPOSE

This unit introduces square roots and other irrational expressions. It extends the techniques for combining similar terms to combining similar radical expressions. Proficiency in simplifying radical expressions is necessary for geometry and advanced algebra.

OVERVIEW

Conditions for writing radicals in simplest form and techniques for simplifying radical expressions are developed.

Radical expressions and extraneous roots are applied to the solution of open sentences.

SUGGESTIONS TO THE TEACHER

Instructional Days: 10-12
Minimal Course Objectives: #2 - 5, 7, 9
Average Course Objectives: #1 - 11
Maximal Course Objectives: ALL

- Computation of approximate square roots could be performed by the standard square root algorithm or by the divide-and-average method.
- For advanced students, the concepts of fractional exponents and higher order roots could be introduced.
- Hand calculators could be a valuable aid to this unit.

VOCABULARY

conjugate
distance formula
hypotenuse
index
leg
Pythagorean Theorem
radicand
rationalize the denominator
similar radicals
square root
UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

1. Identify square factors of a whole number.
2. Identify numbers which are perfect squares.
3. State the square root of a square number.
4. Subtract directed numbers.
5. Combine similar terms.
7. Multiply binomials.
8. Solve equations by factoring.

Assessment Tasks

1. a) Which of the following are square factors of 36?
   A. 2
   B. 3
   C. 4
   D. 6
   E. 9
   Answer

   b) Which of the following is the largest square factor of 180?
   A. 4
   B. 9
   C. 18
   D. 36
   Answer

   c) Name the greatest square factor of 54.
   Answer

X-3
UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES
Assessment Tasks (continued)

1. d) Which of the following is the largest square factor of 250?
   A. 5
   B. 10
   C. 25
   D. 100
   Answer __________

2. a) Which of the following is not a perfect square?
   A. 1
   B. 2
   C. 4
   D. 9
   E. 121
   Answer __________

b) Which of the following is a perfect square?
   A. 20
   B. 40
   C. 50
   D. 81
   Answer __________

c) Which of the following is a perfect square?
   A. 15
   B. 21
   C. 75
   D. None of the above
   Answer __________

d) Name the numbers between 50 and 150 which are perfect squares.
   Answer __________
UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

3. a) \( \sqrt{81} = \) ________ a) ________
   b) \( \sqrt{36} = \) ________ b) ________
   c) \( \sqrt{400} = \) ________ c) ________
   d) \( \sqrt{9} = \) ________ d) ________

4. a) \( 5 - 11 = \) ________ a) ________
   b) \( -3 - (-15) = \) ________ b) ________
   c) \( -8 - 6 = \) ________ c) ________
   d) \( 4 - (-13) = \) ________ d) ________

5. Combine similar terms.
   a) \( 2a + 8 - 2d + 4a - 13 - d = \) a) ________
   b) \( 4k^3 - 5k^2 - 3 + 2k^3 - 3k^2 - 6k = \) b) ________
   c) \( 3m^2 - 6m + 7 - 4m^2 - m - 9 = \) c) ________
   d) \( x - 7y + 2z - x - 11y - 8z = \) d) ________

6. Solve each of the following equations:
   a) \( 4(h + 5) + h = 35 = \) a) ________
   b) \( 3(z + 1) = 2(z - 2) = \) b) ________
   c) \( 5y - 30 = \) c) ________
   d) \( .07x + .04(9000 - x) = 450 = \) d) ________

7. a) \( (3x + 2)(5x - 7) = \) a) ________
    b) \( (7x - 2y)(3x + 8y) = \) b) ________
    c) \( (3 - 2m)(7 - 6m) = \) c) ________
    d) \( (3x - 7y)^2 = \) d) ________

X-5 255
UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

8. Solve each of the following equations:
   a) \( x^2 - x - 56 = 0 \)  
   b) \( 15 + x = 2x^2 \)  
   c) \( 3x^2 = 10x - 3 \)  
   d) \( \frac{y}{3} + 1 = \frac{6}{y} \)
UNIT X - RADICAL EXPRESSIONS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) C, E
   b) D
   c) 9
   d) C

2. a) B
   b) D
   c) D
   d) 64, 81, 100, 121, 144

3. a) 9
   b) 6
   c) 20
   d) 3

4. a) -6
   b) 12
   c) -14
   d) 17

5. a) 6a - 3d - 5
   b) 6k^3 - 8k^2 - 6k - 3
   c) -m^2 - 7m - 2
   d) -18y - 6z

6. a) h = 3
   b) z = -7
   c) y = 6
   d) x = 3000

7. a) 15x^2 - 11x - 14
   b) 21x^2 + 50xy - 16y^2
   c) 21 - 32m + 12m^2
   d) 9x^2 - 42xy + 49y^2

8. a) x = 8 or -7
   b) x = \frac{5}{2} or 3
   c) x = \frac{1}{3} or 3
   d) x = -6 or 3
UNIT X - RADICAL EXPRESSIONS

PERFORMANCE OBJECTIVES

1. Compute the approximate square root of a number by using a square root algorithm.

2. Write radicals in simplest form using the product and quotient properties of square roots.

3. Multiply and divide radical expressions.

4. Combine similar radicals. (II)

5. Multiply binomials containing radicals. (II)

6. Simplify a radical expression with a binomial denominator. (II)

7. Solve radical equations. (III)

8. Solve word problems involving the use of radical expressions. (III)

9. Determine the length of the unknown side of a right triangle, using the Pythagorean Theorem. (III)

10. Compute the distance between two points, using the distance formula. (II)

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KEY SKILLS FOR END-OF-COURSE TESTING

26. Write radical expressions in simplest form.

27. Solve radical equations.
### CROSS REFERENCES

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**UNIT X - RADICAL EXPRESSIONS**

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PERFORMANCE OBJECTIVE X-1

Compute the approximate square root of a number by using a square root algorithm.

Find the following square roots correct to the nearest tenth.

a) \( \sqrt{1200} \)
   Answer __________

b) \( \sqrt{1977} \)
   Answer __________

c) \( \sqrt{1320} \)
   Answer __________

d) \( \sqrt{2079} \)
   Answer __________
PERFORMANCE OBJECTIVE X-2

Write radicals in simplest form using the product and quotient properties of square roots.

a) Which of the following expresses \( \frac{\sqrt{a^2}}{8} \) in simplest form?
   A. \( \frac{a}{2} \)
   B. \( a\sqrt{2} \)
   C. \( \frac{a\sqrt{2}}{2} \)
   D. None of the above

Answer ________

b) Simplify.
   i. \( \sqrt{\frac{3}{4}} \)
   Answer ________
   ii. \( \sqrt{54t^7} \)
   Answer ________
   iii. \( \frac{4}{\sqrt{3}} \)
   Answer ________

c) Simplify.
   i. \( \sqrt{20x^2y^3} \)
   Answer ________
   ii. \( \sqrt{\frac{25r^2}{12s^3}} \)
   Answer ________
   iii. \( \sqrt{\frac{8}{49}} \)
   Answer ________
PERFORMANCE OBJECTIVE X-2 (continued)

d) Simplify.

1. \( \frac{a}{\sqrt{a}} \)

   Answer

2. \( \frac{5}{\sqrt{24}} \)

   Answer

3. \( -5\sqrt{\frac{9}{7}} \)

   Answer
PERFORMANCE OBJECTIVE X-3

Multiply and divide radical expressions.

a) Perform the indicated operation and leave answers in simplest form.

1. \( \frac{2\sqrt{3}}{\sqrt{6}} \)
   Answer: 

2. \( 3\sqrt{5} \cdot 5\sqrt{5} \)
   Answer: 

3. \( \frac{\sqrt[6]{123}}{7\sqrt{5}} \)
   Answer: 

b) Perform the indicated operation and leave answers in simplest form.

1. \( \sqrt[3]{3ab^3} \cdot \sqrt[3]{18ab} \)
   Answer: 

2. \( (5\sqrt{3m})^2 \)
   Answer: 

3. \( \frac{12\sqrt{20}}{3\sqrt{5}} \)
   Answer: 

X-14
PERFORMANCE OBJECTIVE X-3 (continued)

c) Perform the indicated operation and leave answers in simplest form.

1. $\frac{5}{6} \cdot \frac{2}{3} \sqrt{15}$
   Answer: __________

2. $\frac{\sqrt{24x}}{10 \sqrt{6x}}$
   Answer: __________

3. $\frac{\sqrt{27} + \sqrt{75}}{\sqrt{3}}$
   Answer: __________

d) Perform the indicated operation and leave answers in simplest form.

1. $\sqrt{3xy^5} \cdot \sqrt{12x^7y^3}$
   Answer: __________

2. $\frac{\sqrt{48a^3b}}{10 \sqrt{3ab}}$
   Answer: __________

3. $\frac{6\sqrt{27} + 12\sqrt{15}}{3\sqrt{3}}$
   Answer: __________
PERFORMANCE OBJECTIVE X-4

Combine the similar radicals.

a) The result of simplifying \( \sqrt{4x} - \sqrt{x} - \sqrt{36x} \) is:
   A. \(-5\sqrt{x}\)
   B. \(-\sqrt{33x}\)
   C. \(-5x\)
   D. None of the above

   Answer ________

b) Simplify.
   1. \(3\sqrt{5} - 5\sqrt{5} + 9\sqrt{5}\)
      Answer ________
   2. \(\sqrt{12} - \sqrt{27} + \sqrt{48}\)
      Answer ________
   3. \(\sqrt{150} - 5\sqrt{24} + 11\sqrt{54}\)
      Answer ________

c) Simplify.
   1. \(\sqrt{7} - 4\sqrt{7} + 9\sqrt{7}\)
      Answer ________
   2. \(9\sqrt{12} + 3\sqrt{48} - 7\sqrt{27}\)
      Answer ________
   3. \(\sqrt{\frac{2}{3}} + 5\sqrt{\frac{1}{6}} - 3\sqrt{\frac{3}{2}}\)
      Answer ________
PERFORMANCE OBJECTIVE X-4 (continued)

d) Simplify.

1. \( \sqrt{2} - \sqrt{3} + 4\sqrt{3} - \sqrt{2} \)
   Answer \( ( \) 

2. \( 3\sqrt{x} + 5\sqrt{y} - 7\sqrt{x} - 5\sqrt{y} \)
   Answer \( ( \) 

3. \( \frac{1}{3} \sqrt{\frac{2}{27}} - \frac{1}{6} \sqrt{\frac{1}{3}} \)
   Answer \( ( \)
PERFORMANCE OBJECTIVE X-5

Multiply binomials containing radicals.

a) Which of the following pairs of expressions are conjugates?
   A. \((2 + \sqrt{5}) (2 + \sqrt{5})\)
   B. \((2 - \sqrt{5}) (2 + \sqrt{5})\)
   C. \((3 + \sqrt{7}) (3 - \sqrt{7})\)
   D. \((\sqrt{2} + \sqrt{5}) (\sqrt{2} + \sqrt{5})\)

   Answer

b) Express each in simplest form.
   1. \((3 + \sqrt{5}) (3 - \sqrt{5}) = \) ________
   2. \((1 + \sqrt{6}) (2 - \sqrt{6}) = \) ________
   3. \((\sqrt{2} + \sqrt{3}) (\sqrt{2} - \sqrt{3}) = \) ________
   4. \((2\sqrt{3} + 1)^2 = \) ________

c) Evaluate \(x^2 + 2x - 1\) for \(x = (\sqrt{2} - 1)\)
   Answer ________

d) Simplify.
   1. \((4\sqrt{3} - 3\sqrt{5}) (3\sqrt{3} + \sqrt{5}) = \) ________
   2. \((2\sqrt{3} + \sqrt{2})^2 = \) ________
PERFORMANCE OBJECTIVE X-6

Simplify a radical expression with a binomial denominator.

a) To write \( \frac{1}{\sqrt{7} - 2} \) in simplest form:

A. Multiply the denominator by \( \sqrt{7} \)
B. Multiply by \( \frac{\sqrt{7}}{\sqrt{7}} \)
C. Multiply by \( \frac{\sqrt{7} + 2}{\sqrt{7} + 2} \)
D. Already in simplest form

Answer

b) The simplest form of \( \frac{\sqrt{3}}{\sqrt{3} + 6} \) is:

A. \( \frac{1}{6} \)
B. \( \frac{1}{3} \)
C. \( \frac{-1 + 2\sqrt{3}}{11} \)
D. \( \frac{\sqrt{6}}{-11} \)

Answer

c) Simplify \( \frac{\sqrt{3} + \sqrt{5}}{2\sqrt{5} + \sqrt{5}} \)

Answer

d) Simplify \( \frac{1}{3\sqrt{5} + 4} \)

Answer
Solve radical equations.

a) Find the solution to the equation $\sqrt{7} + 2x = \sqrt{5}$.
   A. 1
   B. $\frac{\sqrt{5} - 1}{2}$
   C. $\frac{\sqrt{5}}{2}$
   D. None of the above
   Answer __________

b) Find the solution to the equation $\sqrt{7} + 3x = -4$
   A. 3
   B. -3
   C. 0
   D. None of the above
   Answer __________

c) Solve the following equations.
   1. $\sqrt{5x} = 15$
      Answer __________
   2. $\sqrt{3a} = 8$
      Answer __________
   3. $8 = \sqrt{x} - 9$

d) Solve the following equations.
   1. $\sqrt{t} = t - 6$
      Answer __________
   2. $\sqrt{x} - x = -1$
      Answer __________
PERFORMANCE OBJECTIVE X-8

Solve word problems involving the use of radical expressions.

Solve each word problem. Show all work.

a) Twice the square root of a number is 44. Find the number.
   Answer __________

b) When 9 is added to a number, the square root is 10. Find the number.
   Answer __________

c) If an object is dropped from a certain height, the time it takes to fall can be found by the formula \( t = \sqrt{\frac{s}{16}} \), where \( t \) is time and \( s \) is the distance in feet. Find the number of seconds it takes a baseball that is hit to a height of 200 feet to fall to the ground from its highest point.
   Answer __________

d) Heron's (Hero's) formula, \( A = \sqrt{s(s-a)(s-b)(s-c)} \), is used to find the area of a triangle when only the lengths of the sides \( (a, b, \text{ and } c) \) are known and \( s \) is the value of one-half the perimeter. Find the area of a triangle whose sides are 12 cm, 16 cm, and 20 cm (use a square root table).
   Answer __________
PERFORMANCE OBJECTIVE X-9

Determine the length of the unknown side of a right triangle using the Pythagorean Theorem.

Solve each problem.

a) Find the hypotenuse of a right triangle if the lengths of the shorter sides are 18 cm and 24 cm.
   Answer

b) A television antenna tower is 350 m tall. If a cable 500 m long were used to support the tower 300 m from the ground, how far from the base of the tower will the cable be anchored to the ground?
   Answer

c) Find the length of the diagonal of a rectangle that is 9 inches long and 4 inches wide.
   Answer

d) Romeo placed the bottom of his ladder 8 feet from the wall of Juliet's house. The top of the ladder just reached the bottom of Juliet's window which was 15 feet above the ground. How long was Romeo's ladder?
   Answer
PERFORMANCE OBJECTIVE X-10

Compute the distance between two points using the distance formula.

a) Find the distance between (-3, 1) and (5, 4).
   Answer

b) Find the distance between (3, 3) and (9, -5).
   Answer

c) The vertices of a triangle are (0, 0), (3, 4), (80). Find its perimeter.
   Answer

d) The vertices of a diamond are:
   A. (-5, 1)
   B. (-2, 5)
   C. (1, 1)
   D. (-2, -3)
   Find its perimeter.
   Answer
UNIT X - RADICAL EXPRESSIONS

Answers

X-1

a) 34.6  
b) 44.5  
c) 36.3  
d) 45.6  

X-2

a) B

b) 1. \( \frac{1}{2} \sqrt[3]{3} \)
    2. \( 3t^3 \sqrt[6]{6t} \)
    3. \( \frac{4\sqrt[6]{3}}{3} \)

c) 1. \( 2xy \sqrt[5]{5y} \)
    2. \( \frac{5x \sqrt[6]{3x}}{6x^2} \)
    3. \( \frac{8x \sqrt[2]{2}}{7} \)

d) 1. \( \sqrt[n]{a} \)
    2. \( \frac{2\sqrt[6]{6}}{2} \)
    3. \( \frac{-15\sqrt[7]{7}}{67} \)

X-3

a) 1. \( 6\sqrt[2]{2} \)
    2. \( 75 \)
    3. \( \frac{3\sqrt[7]}{7} \)

b) 1. \( 3ab \sqrt[6]{c} \)
    2. \( 75m \)
    3. \( 8 \)

X-3 (continued)

c) 1. \( 10\sqrt[10]{10} \)
    2. \( 1 \)
    3. \( 8 \)

d) 1. \( 6x^4 \sqrt[4]{y^4} \)
    2. \( x \)
    3. \( 6 + 4\sqrt{5} \)

X-4

a) A

b) 1. \( \sqrt[5]{5} \)
    2. \( 3\sqrt[3]{3} \)
    3. \( 28\sqrt[6]{6} \)

c) 1. \( 5\sqrt[7]{7} \)
    2. \( 9\sqrt[3]{3} \)
    3. \( \frac{1}{3} \sqrt[6]{6} \)

d) 1. \( 6\sqrt[2]{2} - 3\sqrt[3]{3} \)
    2. \( 4\sqrt{x} \)
    3. \( \frac{2\sqrt[6]{6} - 3\sqrt[3]{3}}{54} \)

X-5

a) B, C

b) 1. \( 4 \)
    2. \( -4 + \sqrt{6} \)
UNIT X - RADICAL EXPRESSIONS

Answers (continued)

X-5 (continued)

3. -1
4. $13 + 4\sqrt{3}$
c) 0
d) 1. $-24 + 7\sqrt{15}$
2. $14 + 4\sqrt{6}$

a) C
b) C
c) $7 - \sqrt{15}$
d) $\frac{3\sqrt{5} - 4}{29}$

X-7

a) B.
b) C
c) 1: $x = 45$
2. $a = 21\frac{2}{3}$
3. $73 - x$
d) 1. (9)
2. (9)

X-8

a) $x = 484$
b) $r = 91$
c) $t = \frac{5\sqrt{2}}{2}$ sec
d) $A = 96 \cdot cm^2$

X-9

a) 30 cm
b) 400 m
c) $\sqrt{97}$ inches
d) 17 feet

X-10

a) $d = \sqrt{73}$
b) $d = 10$
c) Perimeter = $13 + \sqrt{41}$
d) Perimeter = 20
UNIT XI - QUADRATIC EQUATIONS

PURPOSE

For students who will continue their study of mathematics in geometry and advanced algebra, a knowledge of the general solution for any quadratic equation is important. The main emphasis of this unit is placed on the quadratic formula and its application.

OVERVIEW

Three methods for solving a quadratic equation, graphing, completing the square, and the quadratic formula are presented. (The factoring method was introduced in Unit VIII.) Quadratic functions and their graphs are also discussed. As a culminating activity for this unit, students solve word problems involving quadratic relations.

SUGGESTIONS TO THE TEACHER

Instructional Days: 8-10
Minimal Course Objectives: #1, 2, 3
Average Course Objectives: #1, 2, 3, 5
Maximal Course Objectives: ALL

Familiarization with the quadratic formula is a minimal outcome of this unit. It is strongly suggested that advanced students learn the derivation of the quadratic formula.

VOCABULARY

- completing the square
- discriminant
- nature of the roots of a quadratic equation
- parabola
- quadratic equation
- quadratic formula
- quadratic relation
- symmetry
- vertex
UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

1. Add fractions with unlike denominators.
2. Multiply whole numbers and fractions by \( \frac{1}{2} \).
3. Square arithmetic fractions.
4. Square a binomial.
5. Factor a trinomial square.
7. Evaluate algebraic expressions containing square roots.
8. Solve equations by factoring.
9. Graph a quadratic function.

Assessment Tasks

1. (a) \( \frac{3}{8} + \frac{3}{4} \)  
   (b) \( \frac{1}{2} + \frac{1}{3} \)  
   (c) \( \frac{4}{5} + \frac{2}{3} \)  
   (d) \( \frac{1}{6} + \frac{3}{4} \)  
   (e) \( \frac{13}{36} + \frac{3}{4} \)  
   (f) \( \frac{19}{25} + \frac{3}{10} \)  
   (g) \( \frac{8}{49} + \frac{9}{14} \)  
   (h) \( 3 + \frac{1}{2} \)  
   (i) \( 5 + \frac{1}{9} \)  
   (j) \( -1 + \frac{25}{4} \)

2. (a) \( \frac{1}{2} \cdot \frac{3}{4} \)  
   (b) \( \frac{3}{4} \cdot \frac{1}{8} \)  
   (c) \( \frac{1}{2} \cdot 11 \)  
   (d) \( 15 \cdot \frac{1}{2} \)  
   (e) \( 24 \cdot \frac{1}{2} \)  
   (f) \( \frac{5}{6} \cdot \frac{1}{2} \)  
   (g) \( \frac{5}{7} \)  
   (h) \( \frac{1}{3} \)  
   (i) \( \frac{2}{3} \)  
   (j) \( \frac{2}{5} \)
UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

2. g) \( \frac{9}{17} \cdot \frac{1}{2} \)  
   h) \( \frac{1}{2} \cdot \frac{4}{5} \)  
   i) \( 12 \cdot \frac{1}{2} \)  
   j) \( \frac{1}{2} \cdot \frac{16}{25} \)

3. a) \( \left( \frac{2}{3} \right)^2 \)  
   b) \( \left( \frac{3}{4} \right)^2 \)  
   c) \( \left( \frac{5}{6} \right)^2 \)  
   d) \( \left( \frac{1}{2} \right)^2 \)  
   e) \( \left( \frac{3}{4} \right)^2 \)  
   f) \( \left( \frac{2}{3} \right)^2 \)  
   g) \( \left( \frac{16}{7} \right)^2 \)  
   h) \( \left( \frac{2}{5} \right)^2 \)  
   i) \( \left( \frac{17}{5} \right)^2 \)

4. a) \( (2x - 3)^2 \)  
   b) \( (2x + 1)^2 \)  
   c) \( (5x - 4)^2 \)  
   d) \( \left( \frac{1}{2}x + 5 \right)^2 \)
UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

5. a) $25x^2 + 30x + 9$
   b) $49a^2 - 28ab + 4b^2$
   c) $9x^2 - 42xy + 49y^2$
   d) $25x^2 - 10x + 1$

6. Simplify each of the following expressions.
   a) $\sqrt{(2x + 3)^2}$
   b) $\sqrt{81(x - 25)^2}$
   c) $\frac{6 + \sqrt{52}}{2}$
   d) $\frac{-26 + \sqrt{144}}{4}$

7. If $a = 3$, $b = -2$, $c = -1$, and $d = 12$, find the value of each of the following expressions:
   a) $\sqrt{-2a(-d)}$
   b) $\sqrt{-9}$
   c) $\sqrt{8abc}$
   d) $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$

8. Solve each of the following equations:
   a) $a^2 - 8a = 20$
   b) $2h^2 = 210 + 16h$
   c) $x = \frac{40}{x - 3}$
   d) $x = \frac{8}{x + 2}$
UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

Assessment Tasks (continued)

9. Graph each of the following functions:
   a) \( y = x^2 + 1 \)

   ![Graph of \( y = x^2 + 1 \)]

   a) \( \begin{array}{c|c}
   x & y \\
   \hline
   0 & \phantom{0} \\
   1 & 2 \\
   2 & 5 \\
   3 & 10 \\
   -1 & 2 \\
   -2 & 5 \\
   -3 & 10 \\
   \end{array} \)

   b) \( y = \frac{1}{2} x^2 + 7 \)

   ![Graph of \( y = \frac{1}{2} x^2 + 7 \)]

   b) \( \begin{array}{c|c}
   x & y \\
   \hline
   0 & 7 \\
   1 & \frac{9}{2} \\
   2 & 8 \\
   3 & 11.5 \\
   -1 & \frac{9}{2} \\
   -2 & 8 \\
   -3 & 11.5 \\
   \end{array} \)

   c) \( y = \frac{1}{2} x^2 - 3 \)

   ![Graph of \( y = \frac{1}{2} x^2 - 3 \)]

   c) \( \begin{array}{c|c}
   x & y \\
   \hline
   0 & -3 \\
   1 & \frac{1}{2} \\
   2 & 1 \\
   3 & \frac{7}{2} \\
   -1 & \frac{1}{2} \\
   -2 & 1 \\
   -3 & \frac{7}{2} \\
   \end{array} \)
UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

Answers

1. a) \frac{9}{8} \text{ or } 1 \frac{1}{8}
   b) \frac{5}{6}
   c) \frac{22}{15} \text{ or } 1 \frac{7}{15}
   d) \frac{11}{12}
   e) \frac{10}{9} \text{ or } 1 \frac{1}{9}
   f) \frac{53}{50} \text{ or } 1 \frac{3}{50}
   g) \frac{79}{98}
   h) 3 \frac{1}{2}
   i) 5 \frac{1}{9}
   j) 21 \frac{1}{4} \text{ or } 5 \frac{1}{4}

2. a) \frac{3}{8}
   b) 4
   c) \frac{11}{2} \text{ or } 5 \frac{1}{2}
   d) \frac{15}{2} \text{ or } 7 \frac{1}{2}
   e) 12
   f) \frac{5}{12}
   g) \frac{9}{34}
   h) \frac{2}{5}
   i) 6
   j) \frac{8}{25}

3. a) \frac{4}{9}
   b) \frac{9}{16}
   c) \frac{25}{36}
   d) \frac{9}{4}
   e) \frac{9}{16}
   f) \frac{64}{9}
   g) \frac{256}{49}
   h) \frac{49}{25}
   i) \frac{289}{25}

4. a) 4x^2 - 12x + 9
   b) 4x^2 + 4x + 1
   c) 25x^2 - 40x + 16
   d) \frac{1}{4}x^2 + \frac{25}{4}x + 25

5. a) (5x + 3)^2
   b) (7a - 2b)^2
   c) (3x - 7y)^2
   d) (5x - 1)^2

6. a) \frac{2x + 3}{4}
   b) 9(x - 25)
   c) 3 + \sqrt{13}
   d) \frac{7}{2} \text{ or } -3 \frac{1}{2}
UNIT XI - QUADRATIC EQUATIONS

ENTERING PERFORMANCE OBJECTIVES

Answers (continued)

7. 
   a) 6
   b) \( \frac{1}{2} \)
   c) 2
   d) 1

8. 
   a) \( a = 10 \) or \( -2 \)
   b) \( h = 15 \) or \( -7 \)
   c) \( x = 8 \) or \( -5 \)
   d) \( x = -6 \) or \( 4 \)

9. a) 
   b) 
   c)
UNIT XI - QUADRATIC EQUATIONS

PERFORMANCE OBJECTIVES

1. Solve quadratic equations by applying the square root property of equality. (III)
2. Solve quadratic equations by completing the square. (III)
3. Solve quadratic equations using the quadratic formula. (II)
4. Solve quadratic functions graphically. (III)
5. Solve word problems that involve the quadratic formula. (III)

Minimal

Average #1-3, 5

Maximal ALL

KEY SKILLS FOR END-OF-COURSE TESTING

28. Solve a quadratic equation by completing the square or applying the quadratic formula.
CROSS REFERENCES

UNIT XI - QUADRATIC EQUATIONS

TEXTS (BY AUTHOR)

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PERFORMANCE OBJECTIVE XI-1

Solve equations by applying the square root property of equality.

Solve the following equations by applying the square root property of equality.

- a) 1. \(5x^2 = 500\)  
   Answer: 
   
   2. \((x - 3)^2 = 49\)  
   Answer: 
   
   3. \(x^2 + 10x + 25 = 121\)  
   Answer: 

- b) 1. \(2x^2 - 3 = 0\)  
   Answer: 
   
   2. \((x + \frac{1}{3})^2 = \frac{25}{49}\)  
   Answer: 
   
   3. \(9x^2 - 6x + 1 = 16\)  
   Answer: 

- c) 1. \(z^2 + .01 = .37\)  
   Answer: 
   
   2. \((y - 9)^2 = 64\)  
   Answer: 
   
   3. \((z - 4)^2 = 11\)  
   Answer: 

- d) 1. \(7x^2 = 4x^2 + .75\)  
   Answer: 
   
   2. \((x + 5)^2 = 44\)  
   Answer: 
   
   3. \(4x^2 - 12x + 9 = 169\)  
   Answer: 

XI-11

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PERFORMANCE OBJECTIVE XI-2

Solve quadratic equations by completing the square.

Solve the following equations by completing the square.

a) \( x^2 + 4x - 12 = 0 \)
   Answer

b) \( x^2 - 4x + 2 = 0 \)
   Answer

c) \( 2x^2 = -7x - 3 \)
   Answer

d) \( 3x^2 - 6x = 2 \)
   Answer
PERFORMANCE OBJECTIVE XI-3

Solve a quadratic equation using the quadratic formula.

Use the quadratic formula to solve the following equations.

a) \(2x^2 - 7 = 0\)
   Answer ________

b) \(x^2 = 3 - 3x\)
   Answer ________

c) \(2x^2 + 5x = -2\)
   Answer ________

d) \(8x + 1 = -3x^2\)
   Answer ________
PERFORMANCE OBJECTIVE XI-4

Solve quadratic functions graphically.

a) Solve graphically $x^2 - 4 = y$ by making a table of values.

<table>
<thead>
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<tbody>
<tr>
<td>-2</td>
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<td>-1</td>
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b) Solve graphically $x^2 - 6x + 9 = y$ by making a table of values.

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<thead>
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c) Solve graphically $(x + 2)^2 - 1 = y$ by making a table of values.

<table>
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<th>y</th>
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<tr>
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<tr>
<td>-2</td>
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<tr>
<td>-3</td>
<td></td>
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<tr>
<td>-4</td>
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</tbody>
</table>

d) Given the graph of $y = x^2 + 8x + 12$, find the values for $x$ that would make $y = 0$.

Answer: $x = -4, -3$
PERFORMANCE OBJECTIVE XI-5

Solve word problems that involve the quadratic formula.

a) The area of the Student Government Association bulletin board is 24 square feet. If it is 2 feet longer than it is wide, find the dimensions.
Answer ________

b) In an apartment building there are 8 fewer apartments per floor than there are floors. If the building has 609 units, how many floors are in the building?
Answer ________

c) In computing the total cost, c, of setting up a factory to manufacture mag wheels, A.J. came up with the formula $C = 500 + 10x + x^2$, where $x$ is the number of wheels produced. How many wheels can be produced at an initial cost of $3500.00?
Answer ________

d) The Vera City School District has instituted a voucher system under which each student goes to the school of his choice. The city pays the school $(36 + 4x)$ dollars per student per month. How many students, $x$, would a school have to have on its rolls to be paid $1600 per month?
Answer ________
UNIT XI - QUADRATIC EQUATIONS

Answers

1. a) $x = \pm 10$
   b) $x = \frac{\pm 6}{2}$
   c) $z = \pm \frac{6}{2}$

2. $x = 10$ or $x = -4$
   b) $x = \frac{22}{21}$ or $x = -\frac{8}{21}$
   c) $y = 17$ or $y = -1$

3. $x = 6$ or $x = -16$
   b) $x = \frac{5}{3}$ or $x = -\frac{2}{3}$
   c) $z = 4 \pm 3$
   d) $x = \pm 5$

2. a) $x = -6$, or $x = 2$
   b) $x = 2 + \sqrt{2}$
   c) $x = -3$ or $x = -\frac{1}{2}$
   d) $x = 3 + \sqrt{15}$

3. a) $x = \pm \frac{\sqrt{14}}{2}$
   b) $x = -3 \pm \frac{\sqrt{21}}{2}$
   c) $x = -2, \pm \frac{1}{2}$
   d) $x = -4 \pm \frac{\sqrt{13}}{3}$

4. a) $x = -2, 0$
   b) $x = 2, 4, 6$
   c) $x = -1, 1, 3$
   d) $x = -4, 0$

5. a) $x = 4$
   b) $x = 29$
   c) $x = 50$
   d) $x = 16$
PURPOSE

Familiarity with the basic trigonometric ratios and right triangles will be a valuable aid to the student who will continue his/her mathematical studies in geometry and trigonometry. This unit will provide a background in the rudiments of right triangle trigonometry. The ideas discussed are simple enough to enable slower classes to grasp the basic concepts, if desired.

OVERVIEW

Students are introduced to the basic terminology of the right triangle. Emphasis is placed on the sine, cosine, and tangent ratios and their relationships to the sides of a right triangle. Students are expected to use these ratios and a table of trigonometric values to find the missing parts of a right triangle.

SUGGESTIONS TO THE TEACHER

Instructional Days: 6-8.
Minimal Course Objectives: None
Average Course Objectives: 1, 5, and 8
Maximal Course Objectives: ALL

Students will find it especially handy to memorize the values of sine, cosine, and tangent of 0°, 30°, 45°, 60°, and 90°. Hand calculators will greatly simplify the computation of this unit.

VOCABULARY

acute angle
adjacent leg
angle of depression
angle of elevation
cosine
hypotenuse
legs of a right triangle
obtuse angle
opposite leg
Pythagorean Theorem
right angle
right triangle
similar triangles
sine
tangent
trigonometric ratio
trigonometry
PERFORMANCE OBJECTIVES

1. State the sine, cosine, and tangent ratios in terms of the sides of a right triangle. (I)

2. Determine the value of the sine, cosine, or tangent of a given angle of a right triangle when given the lengths of the sides of the triangle. (II)

3. Locate in a table of trigonometric values the sine, cosine, or tangent of a given angle. (I)

4. Determine the measure of an angle from a table of trigonometric values when given the value of its sine, cosine, or tangent. (I)

5. Determine the measure of a given angle of a right triangle from a table of trigonometric values when given the lengths of any two sides of the triangle. (II)

6. State from memory the values of the following:
   - \( \sin 0^\circ \), \( \sin 30^\circ \), \( \sin 90^\circ \)
   - \( \cos 0^\circ \), \( \cos 60^\circ \), \( \cos 90^\circ \)
   - \( \tan 0^\circ \), \( \tan 45^\circ \), \( \tan 90^\circ \) (I)

7. Demonstrate that the following statements are true:
   - \( \cos A \tan A = \sin A \)
   - \( \tan A = \frac{\sin A}{\cos A} \)
   - \( \cos A = \frac{\sin A}{\tan A} \)
   - \( \sin^2 A + \cos^2 A = 1 \)

8. Determine the length of a specified side of a right triangle when given the length of another side and the measure of one of the acute angles. (III)

9. Solve word problems involving right triangles, utilizing the sine, cosine, or tangent ratios. (III)

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## CROSS REFERENCES

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UNIT XII - NUMERICAL TRIGONOMETRY

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PERFORMANCE OBJECTIVE XII-1

State the sine, cosine, and tangent ratios in terms of the sides of a right triangle.

a) The sine of an angle of a right triangle is the ratio of:
   - **A.** \( \frac{\text{side adjacent to the angle}}{\text{hypotenuse}} \)
   - **B.** \( \frac{\text{side opposite the angle}}{\text{side adjacent to the angle}} \)
   - **C.** \( \frac{\text{side adjacent to the angle}}{\text{side opposite the angle}} \)
   - **D.** \( \frac{\text{side opposite the angle}}{\text{hypotenuse}} \)

Answer

b) \[
\begin{align*}
A & \quad \text{C} \\
& \quad b \\
& \quad a \\
& \quad B
\end{align*}
\]

\[
\sin A = \quad \cos B = \quad \tan B =
\]
PERFORMANCE OBJECTIVE XII-I (continued)

c) \[\tan Y = \frac{?}{?}\]

A. \[\frac{XY}{YZ}\]
B. \[\frac{YZ}{XZ}\]
C. \[\frac{XZ}{XY}\]
D. \[\frac{XZ}{YZ}\]

E. None of the above

Answer \[
\]

d) State the following in terms of: 1) the side opposite the angle, 2) the side adjacent to the angle, and 3) the hypotenuse, of a right triangle.

1) sine of an angle: 

2) cosine of an angle 

3) tangent of an angle
PERFORMANCE OBJECTIVE XII-2

Determine the value of the sine, cosine, and tangents ratios of an angle of a right triangle when given the lengths of the sides of the triangle.

Refer to the drawings below to complete the following: (Write each answer as a decimal correct to the nearest hundredth.)

\[ a) \ \sin A = \ \cos A = \ \tan A = \]

\[ b) \ \sin B = \ \cos B = \ \tan B = \]

\[ c) \ \sin X = \ \cos X = \ \tan X = \]

\[ d) \ \sin Y = \ \cos Y = \ \tan Y = \]
PERFORMANCE OBJECTIVE XII-3

Locate in a table of trigonometric values the sine, cosine, or tangent of a given angle.

Use a table of trigonometric values to find the following:

a) $\sin 15^\circ = \underline{\quad}$
   $\cos 79^\circ = \underline{\quad}$
   $\tan 80^\circ = \underline{\quad}$

b) $\sin 78^\circ = \underline{\quad}$
   $\cos 23^\circ = \underline{\quad}$
   $\tan 41^\circ = \underline{\quad}$

c) $\sin 64^\circ = \underline{\quad}$
   $\cos 89^\circ = \underline{\quad}$
   $\tan 13^\circ = \underline{\quad}$

d) $5 \sin 42^\circ = \underline{\quad}$
   $3.5 \cos 13^\circ = \underline{\quad}$
   $9 \tan 39^\circ = \underline{\quad}$
PERFORMANCE OBJECTIVE XII-4

Determine the measure of an angle from a table of trigonometric values when given the value of its sine, cosine, or tangent.

State the following to the nearest degree:

a) If \( \sin \theta = .3090 \), then \( \theta = \) 

b) If \( \cos \theta = .4848 \), then \( \theta = \) 

c) If \( \tan \theta = 2.0503 \), then \( \theta = \) 

d) If \( \cos \theta = .8310 \), then \( \theta = \)
PERFORMANCE OBJECTIVE XII-5.

Determine the measure of a given angle of a right triangle from a table of trigonometric values when given the lengths of any two sides of the triangle.

a) 

\[ \triangle ABC \]

\[ A \]

\[ B \]

\[ C \]

Find the measure of \( \angle A \) to the nearest degree.

b) 

\[ \triangle PQR \]

\[ P \]

\[ Q \]

\[ R \]

Find the measure of \( \angle Q \) to the nearest degree.

c) 

\[ \triangle PHG \]

\[ P \]

\[ H \]

\[ G \]

Find the measure of \( \angle F \) to the nearest degree.

d) In \( \triangle ABC \) with right angle \( C \), \( AB = 13'' \) and \( BC = 5'' \). Find the measure of \( \angle A \) to the nearest degree.
PERFORMANCE OBJECTIVE XII-6

State from memory the values of the following:

\[
\begin{align*}
\sin 0^\circ & \quad \sin 30^\circ & \quad \sin 90^\circ \\
\cos 0^\circ & \quad \cos 60^\circ & \quad \cos 90^\circ \\
\tan 0^\circ & \quad \tan 45^\circ & \quad \tan 90^\circ \\
\end{align*}
\]

a) \(\sin 0^\circ = ?\)
   A. 0
   B. \(\cos 90^\circ\)
   C. \(\tan 0^\circ\)
   D. All of the above
   E. None of the above
   Answer: ________

b) \(\sin 30^\circ = ?\)
   A. \(\cos 60^\circ\)
   B. \(\tan 45^\circ\)
   C. \(\cos 90^\circ\)
   D. All of the above
   E. None of the above
   Answer: ________

c) Complete the following:
   1) \(\cos 0^\circ = \) ________
   2) \(\tan 0^\circ = \) ________
   3) \(\cos 60^\circ = \) ________

d) Complete the following:
   1) \(\tan 45^\circ = \) ________
   2) \(\sin 90^\circ = \) ________
   3) \(\sin 30^\circ = \) ________
PERFORMANCE OBJECTIVE XII-7

Demonstrate that the following statements are true:

\[
\begin{align*}
\cos A \tan A &= \sin A \\
\sin A &= \tan A \\
\tan A &= \cos A
\end{align*}
\]

\[
\sin^2 A + \cos^2 A = 1
\]

a) Show that \(\cos A \tan A = \sin A\)

b) Show that \(\frac{\sin A}{\cos A} = \tan A\)

c) Show that \(\frac{\sin A}{\tan A} = \cos A\)

d) Show that \(\sin^2 A + \cos^2 A = 1\)

a) Find AC.

b) Find PR.

c) Find YZ.

d) In \( \triangle ABC \), \( \angle C = 90^\circ \), \( \angle A = 27^\circ \) and \( BC = 10.19" \). Find AC.
Solve word problems involving right triangles utilizing the sine, cosine, or tangent ratios.

a) Dr. Doobee, a ham radio operator, builds an antenna 60 feet tall for his radio. He mounts a support cable between the ground and the top of the antenna. If the support cable makes an angle of 70° with the ground, how long is the cable? How far from the base of the antenna is the cable anchored in the ground? (Find answers to the nearest tenth.)

b) Franklin Benjamin is flying a kite at the end of a 300' long string. If the kite is flying at a height of 200', what angle (to the nearest degree) does the string make with the ground?

c) At the end of his act, the Great Mitch Donkowski, a circus tightrope walker, walks down a cable from a platform 50 feet high to the ground. If the cable makes an angle of 25° with the ground, how long is the cable? (Find answer correct to the nearest tenth.)

d) The rope tow at Mrs. Gurr's Ski Lodge makes a 42° angle with the horizontal. If the rope tow is 3000 feet long, how many feet higher is the top of the slope than the bottom of the rope tow?
UNIT XII - NUMERICAL TRIGONOMETRY

Answers

1. a) D
   
   b) \( \sin A = \frac{a}{c} \)
   \( \cos B = \frac{a}{c} \)
   \( \tan B = \frac{b}{a} \)
   
   c) D
   
   d) 1) sine of an angle = \( \frac{\text{side opposite the angle}}{\text{hypotenuse}} \)
   2) cosine of an angle = \( \frac{\text{side adjacent to the angle}}{\text{hypotenuse}} \)
   3) tangent of an angle = \( \frac{\text{side opposite the angle}}{\text{side adjacent to the angle}} \)

2. a) \( \sin A = .88 \)
   \( \cos A = .47 \)
   \( \tan A = 1.88 \)
   
   b) \( \sin B = .47 \)
   \( \cos B = .88 \)
   \( \tan B = .53 \)
   
   c) \( \sin X = .28 \)
   \( \cos X = .96 \)
   \( \tan X = .29 \)
   
   d) \( \sin Y = .96 \)
   \( \cos Y = .28 \)
   \( \tan Y = 3.43 \)

3. a) \( \sin 15^\circ = .2588 \)
   \( \cos 79^\circ = .1908 \)
   \( \tan 80^\circ = 5.6713 \)
Answers (continued)

3. b) \( \sin 78^\circ = .9781 \)
   \( \cos 23^\circ = .9205 \)
   \( \tan 41^\circ = .8693 \)
   c) \( \sin 64^\circ = .8988 \)
   \( \cos 89^\circ = .0175 \)
   \( \tan 13^\circ = .2509 \)
   d) \( 5 \sin 42^\circ = 3.3455 \)
   \( 3.5 \cos 13^\circ = 3.4104 \)
   \( 9 \tan 39^\circ = 7.2882 \)

4. a) \( \theta = 18^\circ \)
   b) \( \theta = 61^\circ \)
   c) \( \theta = -64^\circ \)
   d) \( \theta = 34^\circ \)

5. a) \( m_\angle A = 62^\circ \)
   b) \( m_\angle Q = 74^\circ \)
   c) \( m_\angle F = 37^\circ \)
   d) \( m_\angle A = 23^\circ \)

6. a) D
   b) B
   c) 1) \( \cos 0^\circ = 1 \)
   2) \( \tan 0^\circ = 0 \)
   3) \( \cos 60^\circ = .5 \)
   d) 1) \( \tan 45^\circ = 1 \)
   2) \( \sin 90^\circ = 1 \)
   3) \( \sin 30^\circ = .5 \)
Answers (continued)

7. a) \( \cos A \tan A = \frac{b}{c} \cdot \frac{a}{b} = \frac{ab}{bc} = \frac{a}{c} = \sin A \)

b) \( \frac{\sin A}{\cos A} = \frac{a}{c} \cdot \frac{b}{c} = \frac{ab}{bc} \)

\( = \frac{a}{c} \cdot \frac{b}{c} \)

\( = \frac{a}{c} \cdot \frac{c}{b} = \frac{ac}{bc} = \frac{ac}{bc} = \frac{a}{b} = \tan A \)

\( c) \frac{\sin A}{\tan A} = \frac{a}{c} \cdot \frac{a}{b} = \frac{a}{c} \cdot \frac{a}{b} \)

\( = \frac{a}{c} \cdot \frac{b}{a} = \frac{ab}{ac} = \frac{b}{c} = \cos A \)
UNIT XII - NUMERICAL TRIGONOMETRY

Answers (continued)

d) \( \sin^2 A + \cos^2 A = \left( \frac{a}{c} \right)^2 + \left( \frac{b}{c} \right)^2 \)

\[
= \frac{a^2}{c^2} + \frac{b^2}{c^2}
\]

\[
= \frac{a^2 + b^2}{c^2}
\]

\[
= \frac{c^2}{c^2}
\]

\[
= 1
\]

8. a) \( \sin 33^\circ = \frac{AC}{24} \)

\[
24 \cdot \sin 33 = AC
\]

\[
24 \cdot 0.5446 = AC
\]

\[
13.0704 = AC
\]

b) \( \tan 67^\circ = \frac{PR}{19} \)

\[
19 \cdot \tan 67^\circ = PR
\]

\[
19 \cdot 2.3559 = PR
\]

\[
44.7621 \text{ cm} = PR
\]

c) \( \cos 70^\circ = \frac{YZ}{23} \)

\[
23 \cdot \cos 70^\circ = YZ
\]

\[
23 \cdot 0.3420 = YZ
\]

\[
7.866 \text{ m} = YZ
\]

d) \( \tan 27^\circ = \frac{10.19}{AC} \) or \( \tan 63^\circ = \frac{AC}{10.19} \)

\[
AC \cdot \tan 27^\circ = 10.19
\]

\[
10.19 \cdot \tan 63^\circ = AC
\]

\[
10.19 \cdot 1.9626 = AC
\]

\[
19.998894 = AC
\]

\[
AC = \frac{10.19}{\tan 27^\circ}
\]

\[
AC = \frac{10.19}{0.5095}
\]

\[
AC = 20''
\]

XII-19
UNIT XII - NUMERICAL TRIGONOMETRY

Answers (continued)

9. a) \( \sin 70^\circ = \frac{60}{x} \)
\( x \sin 70^\circ = 60 \)
\( x = \frac{60}{\sin 70^\circ} \)
\( x = \frac{60}{.9397} \)
\( x = 63.9 \)

\( \tan 70^\circ = \frac{60}{x} \)
\( x \tan 70^\circ = 60 \)
\( x = \frac{60}{\tan 70^\circ} \)
\( x = \frac{60}{2.7475} \)
\( x = 21.8 \)

b) \( \sin \theta = \frac{200}{300} \)
\( \sin \theta = .6667 \)
\( \theta = 42^\circ \)

c) \( \sin 25^\circ = \frac{50}{x} \)
\( x \sin 25^\circ = 50 \)
\( x = \frac{50}{\sin 25^\circ} \)
\( x = \frac{50}{.4226} \)
\( x = 118.3' \)

d) \( \sin 42^\circ = \frac{x}{3000} \)
\( 3000 \cdot \sin 42^\circ = x \)
\( 3000 \cdot .6691 = x \)
\( 2007.3' = x \)
PURPOSE

Students completing algebra in the 8th grade often miss additional information about perimeter, area, and volume. Exposure to such material is very important for the students to have in order to progress in geometry. This unit provides much of this background information.

OVERVIEW

The emphasis in this unit is placed on the students being introduced to area and volume formulas and to calculating areas and volumes by substituting into these formulas. The derivation and proof of these formulas is left for study in a course on geometry.

SUGGESTIONS TO THE TEACHER

Instructional Days: 8-10
Minimal Course Objectives: Numbers 1-3
Average Course Objectives: Numbers 1-6
Maximal Course Objectives: ALL

The formulas may be used to review solving literal equations:

Example: Solve $SA = 2\pi r^2 + 2\pi rh$ for $h$

VOCABULARY

altitude, area, base, circumference, cone, cube, cylinder, depth, diameter, face, height, lateral area, length, parallelogram, perimeter, pi, polygon, prism, pyramid, radius, rectangle, regular, rhombus, slant height, sphere, square, surface area, trapezoid, triangle, vertex, volume, width
ENRICHMENT

UNIT XIII - PERIMETER, AREA, AND VOLUME

PERFORMANCE OBJECTIVES

1. Compute the perimeter of a given polygon. (II)

2. Compute the circumference of a circle. (II)

3. Compute the area of each of the following: triangle, square, rectangle, parallelogram, trapezoid, and circle. (II)

4. Compute the lateral area of each of the following: rectangular prism, cylinder, cone. (II)

5. Compute the surface area of the following: rectangular prism, cylinder, sphere, cone. (II)

6. Compute the volume of each of the following: rectangular prism, triangular prism, cylinder, sphere, rectangular pyramid, cone. (II)

7. Compute the area of a geometrical figure composed of triangles, squares, rectangles, parallelograms, trapezoids, and/or circles. (III)

8. Compute the volume of a geometrical solid composed of rectangular prisms, rectangular pyramids, cones, cylinders, spheres, and/or triangular prisms. (III)

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PERFORMANCE OBJECTIVE XIII-1

Compute the perimeter of a given polygon.

Compute the perimeter of each of the following:

a) 7 cm
   Answer _______

b) 5 m
   Answer _______

c) 1.7 yd.
   Regular hexagon
   Answer _______

d) _______
PERFORMANCE OBJECTIVE XIII-2

Compute the circumference of a circle.

Compute the circumference for each of the following:

a) Use $\pi = \frac{22}{7}$
   Answer __________

b) Use $\pi = \frac{22}{7}$
   Answer __________

c) Use $\pi = 3.14$
   Answer __________

d) Use $\pi = 3.14$
   Answer __________
PERFORMANCE OBJECTIVE XIII-3

Compute the area of each of the following: triangle, square, rectangle, parallelogram, trapezoid, circle.

Compute the area of each of the following:

a) Triangle
   ![Triangle Diagram]
   Answer

b) Square
   ![Square Diagram]
   Answer

c) Rectangle
   ![Rectangle Diagram]
   Answer

d) Parallelogram
   ![Parallelogram Diagram]
   Answer

e) Trapezoid
   ![Trapezoid Diagram]
   Answer

f) Use $\pi = 3.14$
   ![Circle Diagram]
   Answer
PERFORMANCE OBJECTIVE XIII-4

Compute the lateral area of each of the following: rectangular prism, cylinder, cone.

Compute the lateral area for each of the following:

a) 3.2 cm

b) Use \( \pi = 3.14 \)

Answer

Answer

Answer

Answer
Compute the surface area of the following: rectangular prism, cylinder, sphere, cone.

Compute the surface area for each of the following:

a) 

Answer

b) Use $\pi = \frac{22}{7}$

Answer

c) Use $\pi = \frac{22}{7}$

Answer

d) Use $\pi = 3.14$

Answer
PERFORMANCE OBJECTIVE XIII-6

Compute the volume of each of the following: rectangular prism, cylinder, sphere, rectangular pyramid, cone.

a) Answer 6

b) Answer

c) 
Answer

r = 7 cm

\( \pi = 3.14 \)

d) 
Answer

r = 35'
\( \pi = \frac{22}{7} \)

e) Answer

r = 7 in.
\( \pi = 3.14 \)

f) Answer

r = 3 in.
\( \pi = 3.14 \)
Compute the area of a geometrical figure composed of triangles, squares, rectangles, parallelograms, trapezoids and/or circles.

Compute the area for each of the following:

a)

b) Use $\pi = 3.14$

Answer

Answer

Answer

Answer

b) Use $\pi = 3.14$

Answer

Answer

Answer

Answer
PERFORMANCE OBJECTIVE XIII-8

Compute the volume of a geometrical solid composed of rectangular prisms, rectangular pyramids, cones, cylinders, spheres, and/or triangular prisms.

Compute the volume of the following:

a) Use \( \pi = \frac{22}{7} \)

\[
\text{Answer} \quad 56 \text{ cm}^3
\]

b)

\[
\text{Answer} \quad 60 \text{ m}^3
\]

c) Use \( \pi = \frac{22}{7} \)

\[
\text{Answer}
\]

d)

\[
\text{Answer}
\]
UNIT XIII - PERIMETER, AREA, AND VOLUME

Answers

1. \( a) \ 20 \text{ cm} \)
   \( b) \ 30 \text{ m} \)
   \( c) \ 10.2 \text{ yd.} \)
   \( d) \ 39' \)

2. \( a) \ 22'' \)
   \( b) \ 27\frac{1}{2} \)
   \( c) \ 62.8 \text{ mm} \)
   \( d) \ 157 \text{ cm} \)

3. \( a) \ 14 \text{ cm}^2 \)
   \( b) \ 2.89 \text{ m} \)
   \( c) \ 28\frac{7}{8} \text{ sq. ft.} \)
   \( d) \ 176\frac{1}{4} \text{ sq. in.} \)
   \( e) \ 30 \text{ m}^2 \)
   \( f) \ 1256 \text{ sq. yd.} \)

4. \( a) \ 35.84 \text{ cm}^2 \)
   \( b) \ 3516.8 \text{ sq. ft} \)
   \( c) \ 2411.52 \text{ mm}^2 \)
   \( d) \ 63 \text{ sq. in.} \)

5. \( a) \ 8214 \text{ mm}^2 \)
   \( b) \ 5104 \text{ sq. in.} \)
   \( c) \ 5544 \text{ sq. ft.} \)
   \( d) \ 32.1536 \text{ cm}^2 \)

6. \( a) \ 90 \text{ cu. in.} \)
   \( b) \ 25 \text{ m}^3 \)
   \( c) \ 2154.04 \text{ cm}^3 \)
   \( d) \ 179,666.66 \text{ cu. ft.} \)
   \( e) \ 74.6 \text{ m}^3 \)
   \( f) \ 65.94 \text{ cu. in.} \)

7. \( a) \ 133 \text{ m}^2 \)
   \( b) \ 3314 \text{ mm}^2 \)
   \( c) \ 44 \text{ sq. yd.} \)
   \( d) \ 20.41 \text{ sq. ft.} \)

8. \( a) \ 25,872 \text{ cm}^3 \)
   \( b) \ 9520 \text{ m}^3 \)
   \( c) \ 36,069\frac{1}{3} \text{ cu. ft.} \)
   \( d) \ 21,000 \text{ cu. ft.} \)
ENRICHMENT

UNIT XIV - RATIO, PROPORTION, AND PERCENT

PURPOSE

This unit is intended to provide a review of percent and to give students practice in solving practical consumer related problems. The information is especially important to 8th grade algebra students who would ordinarily receive additional instruction in these topics in the regular 8th grade curriculum.

OVERVIEW

Proportions and their relation to percent are the key to this unit. Discount, commission, and tax problems may be discussed by adapting the proportion \( \frac{\text{percentage}}{100} = \frac{\text{discount}}{\text{base}} \) to these kinds of problems or by the traditional method of converting the percent to a decimal and multiplying. The interest formula and its applications to simple and compound interest are also discussed.

SUGGESTIONS TO THE TEACHER

Instructional Days: 7-9
Minimal Course Objectives: Numbers 1-8
Average Course Objectives: ALL
Maximal Course Objectives: ALL

Administration of a pretest before beginning this unit could be important. It may be desired to integrate this unit with Unit IX on Rational Algebraic Expressions.

VOCABULARY

base
commission
compound interest
decimal numeral
discount
extremes
interest
means
percent
percentage
principal
proportion
ratio
simple interest
term
ENRICHMENT

UNIT XIV - RATIO, PROPORTION, AND PERCENT

PERFORMANCE OBJECTIVES

1. Write a proportion which represents the relationship between quantities in a given word problem. (III)

2. Solve word problems using proportions. (III)

3. Write a given percent as a ratio. (II)

4. Write a given percent as a decimal numeral. (II)

5. Find the percentage of a given number using the proportion:

$$\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$$  (III)

6. Find the base, using the proportion: $$\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$$  (III)

7. Find the percent, using the proportion: $$\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}$$  (III)

8. Compute simple interest. (II)

9. Compute compound interest. (II)

10. Solve word problems that involve discounts. (III)

11. Solve word problems that involve commissions. (III)

12. Solve word problems that involve taxes. (III)

13. Solve word problems that involve percent mixtures. (III)

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PERFORMANCE OBJECTIVE XIV-1

Write a proportion which represents the relationship between quantities in a given word problem.

a) Henry and Louise drove 510 miles one day of their vacation. They estimated this was done in $8 \frac{1}{2}$ hours. At the same average rate, how long will it take them to drive an additional 720 miles?

Answer

b) When buying groceries, Rose bought 6 limes for 39c. Ralph needed 10 limes to make punch. How much would they cost?

Answer

c) A crepe mix calls for $1 \frac{3}{4}$ cups of milk for 2 cups of mix. How much milk will be needed for 3 cups of mix?

Answer

d) A five acre field yields 93 bushels of peanuts. About how many bushels would an 18 acre field yield under similar conditions?

Answer
PERFORMANCE OBJECTIVE XIV-2

Solve word problems using proportions.

a) The straight line distance between Richmond and Baltimore is 140 miles. How far apart should they be placed on a map on which \( \frac{1}{2} \) inch represents 45 miles?
   Answer  

b) Mary Ella, the pharmacist, had placed an order with the warehouse for 810idal tablets which cost $11.34. How much will it cost the pharmacy when she fills Wes' order for 13 tablets?
   Answer  

c) The differential gear in an automobile turns twice for every 7 turns of the rear wheels. If the differential gear turns 358 times, how many times do the rear wheels turn?
   Answer  

d) In Mr. Smith's math classes, 13 students out of 35 fail. If Mr. Smith has 145 students this year, how many will fail (to the nearest student)?
   Answer
PERFORMANCE OBJECTIVE XIV-3

Write a given percent as a ratio.

a) 33% is a little more than ________.
   A. \(\frac{3}{8}\)
   B. \(\frac{2}{5}\)
   C. \(\frac{3}{10}\)
   D. None of the above

Answer ________

b) Write a fraction for each percent.
   A. 50% = ________
   B. 66\(\frac{2}{3}\)% = ________
   C. 16\(\frac{2}{3}\)% = ________

c) Which of the following percents is represented by the fraction \(\frac{1}{8}\)?
   A. 12\(\frac{1}{2}\)%
   B. \(\frac{1}{8}\)%
   C. 80%
   D. None of the above

Answer ________

d) 150% equals ________.
   A. \(\frac{3}{20}\)
   B. \(\frac{2}{3}\)
   C. \(\frac{3}{2}\)
   D. None of the above

Answer ________
PERFORMANCE OBJECTIVE XIV-4

Write a given percent as a decimal numeral.

a) 55% is a little less than:
   A. .549
   B. .6
   C. .5
   D. All of the above
   Answer ________

b) Write a decimal for each percent.
   A. \( \frac{2}{3} \) % = ________
   B. 25% = ________
   C. \( \frac{1}{3} \) % = ________

c) \( \frac{1}{2} \) % equals:
   A. \( \frac{0.5}{2} \)
   B. .5
   C. .005
   D. None of the above
   Answer ________

d) Which one of the following equals 250%?
   A. 2.5
   B. .25
   C. .025
   D. All of the above
   Answer ________
PERFORMANCE OBJECTIVE XIV-5

Find the percentage of a given number using the proportion:
\[
\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}
\]

a) 52% of 72 is what number?
   Answer __________

b) What is 45% of 135?
   Answer __________

c) 70% of 800 equals _________
   A. 56
   B. 560
   C. 5600
   D. None of the above
   Answer __________

d) 175% of 80 is _________
   A. 1.4
   B. .14
   C. 140
   D. None of the above
   Answer __________
Find the base using the proportion: \[ \frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}} \]

a) 38% of what number is 95?
   Answer ________

b) 96 is 24% of what number?
   Answer ________

c) 76 is 80% of what number?
   A. 9.5
   B. 95
   C. 950
   D. None of the above
   Answer ________

d) 120% of what number is 5.4?
   A. 4.5
   B. 450
   C. 4500
   D. None of the above
   Answer ________
PERFORMANCE OBJECTIVE XIV-7

Find the percent, using the proportion: \[
\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}
\]

a) 37 is what percent of 222?
   Answer __________

b) What percent of 112 is 70?
   Answer __________

c) 7.2 is what percent of 80?
   A. 9%
   B. 90%
   C. 900%
   D. None of the above
   Answer __________

d) 160 is what percent of 16?
   A. 1%
   B. 10%
   C. 100%
   D. 1000%
   Answer __________
PERFORMANCE OBJECTIVE XIV-8

Compute simple interest.

a) John deposits $700 in a bank which pays simple interest at a rate of 5% per year. How much interest will John receive after 9 months?

Answer: __________

b) What is the simple interest on $1700 after 1 year 8 months at a rate of 6% per year?

Answer: __________

c) In order to make a down payment on an apartment building, Pearl borrowed $10,000 from the Credit Union. It charges interest at a rate of 12% per year. How much simple interest will she be charged if she pays the money back after 1 month?

Answer: __________

d) Don Kowski borrows $500 from his good friend Kay for 8 months and agrees to pay her simple interest at an 8% rate. How much interest will Don owe?

Answer: __________
Compute compound interest.

a) Chuck deposited $300 in a savings account at the Price National Bank. If he receives interest at a rate of 5% per year compounded quarterly, how much will be in Chuck's account after 9 months?
Answer _______

b) Compute the total interest paid on a savings account of $10,000 after 1 year if interest is paid at a rate of 6% per year compounded quarterly.
Answer _______

c) When David was 16, his grandfather gave him $3,000. He deposited it in the MR National Bank. If the interest rate was 5% per year compounded semiannually, how much was in his account on his 18th birthday?
Answer _______

d) Find the total interest paid on a $500 savings account after 3 years at 5% per year, compounded semiannually.
Answer _______
PERFORMANCE OBJECTIVE XIV-10

Solve word problems that involve discounts.

a) Mary bought a mink coat on sale for 45% off. If the coat usually cost $3500, what did Mary pay for the coat?
Answer

b) At their going-out-of-business sale, Schmitt's Sporting Goods Store offered discounts of 30-70%. If Ruth bought a pair of skis at 60% off and they normally cost $150, how much did she pay for the skis?
Answer

c) In August one year all bathing suits at the Angel's Rest store were sold at 75% of their marked price. What was the rate of discount? What was the amount of discount on a Madame Elaine bathing suit marked $32?
Answer

d) At Weikel's TV shop, a Webber TV regularly priced $495 is on sale for 20% off. The same model at Toby's TV Tent has an original price of $455 but now is discounted 10%. Which television would be the better buy?
Answer
PERFORMANCE OBJECTIVE XIV-11

Solve word problems that involve commissions.

a) Wes sold shoes to pay his way through school. He was paid a salary plus a bonus commission of 5% of his sales over $200 each week. How much was his bonus in a week during which he sold $570 worth of shoes?
Answer __________

b) Ms. Rosas received a commission of 20% for obtaining a rare antique for Mr. Webb. If the antique cost $525, how much did Ms. Rosas receive for her commission?
Answer __________

c) One summer, Helen drove a Bad Joke Ice Cream truck. She was to receive a 2% commission on all sales over $350 each week. The last week of the summer, she had a total sales of $1126.32. What was her commission for the week?
Answer __________

d) Mr. Balet is an agent for the superstar basketball player Mitch Donkowski. For negotiating a $1,200,000 six year no-cut contract with the Bullets, Mr. Balet was given a 15% commission. How much did he receive?
Answer __________
Solve word problems that involve taxes.

a) At her summer job, Susan earned $183.05 in a week. From this, various amounts of money were withheld to cover her federal income tax, state income tax, and social security. 6.13% of her total weekly earnings were withheld to cover her social security payment. How much money was withheld for social security? (Round to nearest cent.)
Answer __________

b) In 1973 Martha earned $17,500. When she computed her federal income tax, she read in the tax table that people who earned over $16,000 but not over $18,000 had to pay $4,330 plus 42% of the amount they made over $16,000. How much federal income tax did Martha have to pay that year?
Answer __________

c) The state tax table in Maryland for the year 1978 requires that the amount of tax to be paid is $90 plus 5% of all taxable income over $5000. If Ralph's taxable income was $11,854, how much state tax did he pay?
Answer __________

d) Henry bought a Spanish television for $495 and a stand for $39.95. He lives in Maryland which has a 5% sales tax. What was the total amount paid?
Answer __________

XIV-17
PERFORMANCE OBJECTIVE XIV-13

Solve word problems that involve percent mixtures.

a) A chemist has a solution that is 35% acid. She wishes to increase the amount of acid to 75% by adding 90% acid. If she starts with 900 ml of the 35% solution, how much of the 90% acid must be added to have the desired result?
   Answer

b) Winter birdseed contains 60% sunflower seeds and summer birdseed contains only 25% sunflower seeds. How much of each would you use to make 28 kg of birdseed containing 50% sunflower seeds?
   Answer

c) How many grams of a 65% solution of sodium hydroxide must be mixed with 160 grams of a 30% solution of sodium hydroxide to make a 55% solution of sodium hydroxide?
   Answer

d) In creating his monster, Dr. Frankenstein kept the brain in a 6% salt solution. He sent his incompetent assistant Igor to the pharmacist to buy 500 ml of this solution. Igor bought 200 ml of 4% solution and 300 ml of 7% solution by mistake. How much of the 4% solution must he mix with the 300 ml of 7% solution to form a 6% solution?
   Answer
UNIT XIV - RATIO, PROPORTION, AND PERCENT

Answers

1. a) \(\frac{510}{8 \frac{1}{2}} = \frac{720}{x}\)
   b) \(\frac{6}{39} = \frac{10}{x}\)
   c) \(\frac{1 \frac{3}{4}}{2} = \frac{x}{3}\)
   d) \(\frac{5}{93} = \frac{18}{x}\)

2. a) \(\frac{140}{x} = \frac{45}{\frac{7}{2}}\)
    \(x = 1 \frac{5}{9}\)
   b) \(\frac{\$81}{\$11.34} = \frac{13}{x}\)
    \(x = \$1.82\)
   c) \(\frac{2}{7} = \frac{358}{x}\)
    \(x = 1253\)
   d) \(\frac{13}{35} = \frac{x}{145}\)
    \(x = 54\)

3. a) C
   b) A. \(\frac{1}{2}\)
   B. \(\frac{2}{3}\)
   C. \(\frac{1}{6}\)
   c) A
   d) C

4. a) B
   b) A. \(0.66\overline{6}\)
   B. \(0.25\)
   C. \(0.83\overline{3}\)
   c) C
   d) A

5. a) 37.44
   b) 60.75
   c) B
   d) C

6. a) 250
   b) 400
   c) B
   d) A

7. a) 16 \(\frac{2}{3}\) %
   b) 62 \(\frac{1}{2}\) %
   c) A
   d) D

8. a) $26.25
   b) $170
   c) $100
   d) $26.67
UNIT XIV - RATIO, PROPORTION, AND PERCENT

Answers (continued)

9. 
   a) $311.39
   b) $613.64
   c) $3311.44
   d) $79.85

10. 
    a) $1925
    b) $52.50
    c) $24
    d) Weikel's TV ($495 reg/$396 sale) is better buy.

11. 
    a) $18.50
    b) $105
    c) $15.53
    d) $180,000

12. 
    a) $11.22
    b) 4960
    c) $432.70
    d) $561.70

13. 
    a) 2400 ml of 90% acid
    b) 20 kg of 60% seed and 8 kg of 25% seed
    c) 400 g of 65% sodium hydroxide
    d) 150 ml of 4% solution
PURPOSE

Computers are becoming more and more a part of our everyday life. Many of today’s algebra students will be working with computers tomorrow. To begin preparing them to understand computers, this unit has been included. The ability to draw a flowchart for the step-by-step solution of a problem is a valuable aide to programming.

OVERVIEW

The four basic flowcharting symbols are introduced: Start/Stop, Input/Output, Operation, and Decision. (The symbols used in this unit are those used in Coan’s Basic BASIC and Golden’s Computer Programming in the BASIC Language. These books are commonly used in teaching programming in the county.) Instruction follows in order of increasing complexity from inserting the steps of a given algorithm into its flowchart to writing an algorithm and drawing its flowchart.

SUGGESTIONS TO THE TEACHER

Instructional Days: 7-9

Minimal Course Objectives: None

Average Course Objectives: 1-3

Maximal Course Objectives: ALL

VOCABULARY

algorithm
decision box
flowchart
input/output box
loop
operation
PERFORMANCE OBJECTIVES

1. Identify the four basic flowchart symbols: Start/Stop, Input/Output, Operation, and Decision. (I)

2. Complete a blank flowchart when given the necessary instructions out of sequence. (II)

3. Determine the output for a given input by following a given flowchart. (III)

4. Draw the flowchart for a given sequence of instructions. (III)

5. Construct a flowchart which specifies the procedure for accomplishing a given task. (III)

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### UNIT XV - FLOWCHARTS

#### CROSS REFERENCES

#### TEXTS (BY AUTHOR)

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---

XV-4
Identify the four basic flowchart symbols (Start/Stop, Input/Output, Operation, and Decision).

a) In each box, write the name of the symbol.  

b) Match each symbol with its name.

1.  
2.  
3.  
4.  
5.  
6.  

A: Decision  
B: Input/Output  
C: Operation  
D: Start/Stop
PERFORMANCE OBJECTIVE XV-1 (continued)

c) Draw the symbol for each one of the following:
   1. Start/Stop
   2. Operation
   3. Input/Output
   4. Decision

d) Which of the following is not a flowchart symbol?

A. 

B. 

C. 

D. 

Answer
PERFORMANCE OBJECTIVE XV-2

Complete a blank flowchart when given the necessary instructions out of sequence.

a) In the appropriate box, place the number corresponding to each instruction.

Making a Phone Call

1. Pick up receiver.
2. Say "Hello!"
3. Does someone answer?
4. Dial the number.
5. Hang up.
6. Do you know the number?
7. Find the number in the directory.
8. Start.
9. Stop.
PERFORMANCE OBJECTIVE XV-2 (continued)

b) In the appropriate box, place the number corresponding to each instruction.

Selecting a TV Program

1. Do you want to see this show?
2. Do you know what show you want to see?
3. Start.
4. Stop.
5. Change to next channel.
6. Turn on TV.
7. Turn to correct channel.
8. Look at program.
9. Settle, back, relax, and enjoy show.
c) In the appropriate box, place the number corresponding to each instruction.

How To Tell if a Number is Divisible by 6.

1. Number is not divisible by 6.
2. Add digits.
3. Number is divisible by 6.
4. Start.
5. Stop.
6. Does number end in 0, 2, 4, 6, or 8?
7. Is sum divisible by 3?
d) In the appropriate box, place the number corresponding to each instruction.

How To Buy a Softdrink from a Machine

1. Do you have enough money?
2. Push button.
4. Get enough money.
5. Will machine give change?
6. Read price.
7. Start.
8. Pick up can.
9. Stop.
10. Pick up money from slot.
11. Get correct change.
12. Deposit money.
14. Did drink come out?
15. Drink it.
PERFORMANCE OBJECTIVE XV-3

Determine the output for a given input by following a given flowchart.

a) Given the following flowchart and Input, find the Output.

```
Start.
   ┌─────── Input. ───────────────────────┐
   │                                   │
   │ Multiply by 3.                     │
   │                                   │
   │ Add 1.                             │
   │                                   │
   │ Is number greater than 30?         │
   │                                   │
   │ YES                                │
   │                                   │
   │ Subtract 7.                        │
   │                                   │
   │ EVEN number?                       │
   │                                   │
   │ YES                                │
   │                                   │
   └─ Output.                           ┘
   └─ Stop                              ┘

<table>
<thead>
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<td></td>
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<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>
```
b) Given the following flowchart and Input, find the Output.

```
<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>
```

```
Start,

Input,

Add 1,

Even number?

Yes

Number divisible by 5?

Yes

Output,

Stop.

No

Add 1,

No

Add 2,

Yes

Add 1,
```
c) Given the following flowchart and input, find the output.

```
Start

Input

Add 1

Even number? YES

Number less than 10? YES

Number divisible by 3? YES

Output

Stop

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>63</td>
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<td>2.</td>
<td>72</td>
</tr>
<tr>
<td>3.</td>
<td>84</td>
</tr>
</tbody>
</table>
```

Divide by 2.
d) Given the following flow chart and Input, find the Output.

```
<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>89</td>
</tr>
</tbody>
</table>
```
PERFORMANCE OBJECTIVE XV-4

Draw a flowchart for a given sequence of instructions.

a) Construct a flowchart for the following set of instructions:
   Start.
   Read input.
   Multiply the numbers.
   Is the product divisible by 2?
   Yes--Continue
   No--Add 1 to product
   Print output.
   Stop.

b) Construct a flowchart for the following set of instructions:
   How To Multiply Integers
   Start.
   Read Input.
   Multiply absolute values.
   Did the original numbers have the same sign?
   Yes - Continue
   No - Put "-" sign in front of answer.
   Print Output.
   Stop.
PERFORMANCE OBJECTIVE XV-4 (continued)

c) Construct a flowchart for the following set of instructions:

**How To Find the Perimeter of a Polygon**

Start.

Are all sides the same length?

Yes - Count the number of sides

Multiply the number of sides by the length of one side

Go to Print

No - Continue

Add the lengths of all sides.

Print Output.

Stop.

d) Construct a flowchart for the following set of instructions:

**How To Determine Whether a Number Is Divisible by 9**

Start.

Read Input.

Add digits of number.

Is the sum divisible by 9?

Yes - Continue

No - Number is not divisible by 9

Go to Print

Number is divisible by 9.

Print Output.

Stop.
PERFORMANCE OBJECTIVE XV-5

Construct a flowchart which specifies the procedure for accomplishing a given task.

Construct an appropriate flowchart for each of the following tasks:

a) Getting a date  
b) Adding a positive and negative integer

c) Buying a pair of slacks  
d) Finding the perimeter of a rectangle
UNIT XV - FLOWCHARTS

Answers
1. a) 1. Start
2. Input
3. Operation
4. Decision
5. Output
6. Stop
b) 1. C
2. A
3. D
4. B
c) 1. 
2. 
3. 
4. 
d) B
UNIT XV - FLOWCHARTS

Answers: (continued)

2. b)

1. NO

3

6

2

8

1

5

7

4

9
UNIT XV - FLOWCHARTS

Answers (continued)

2. c)
Answers (continued)

3. a) 1. 70
2. 32
3. 26
b) 1. 10
2. 20
3. 30
c) 1. 9
2. 6
3. 6
d) 1. 1
2. 6
3. 91

4. a) Start.
Read input.
Multiply the numbers.
Is the product divisible by 2?
Add 1 to product.
Print output.
Stop.

b) Start.
Read input.
Multiply the absolute values.
Did the original numbers have same sign?
Put "-" sign in front of product.
Print output.
Stop.

c) Start.
Are all sides the same length?
YES
Count the number of sides.
NO
Add lengths of all sides.
Multiply number of sides by length of one side.
Print output.
Stop.
Answers (continued)

4. c) Alternate Solution

Start.

Are all the sides the same length?

Add the lengths of all sides.

Count the number of sides.

Multiply the number of sides by the length of 1 side.

Print output.

Stop.

4. d)

Start.

Add digits.

Read input.

Is the sum divisible by 9?

Number is divisible by 9.

Print output.

Stop.

5. Answers will vary.