This paper discusses several instructional issues, defines the benefits of algorithmic prescriptions, and examines their effectiveness and efficiency. An experimental study asked 77 undergraduates to master and utilize one of three forms of an algorithm: a rose discourse, a flowchart procedure, or a mastery-inducing flowchart technique. Also examined were the effects of the presence or absence of the algorithm in subsequent problem solving and the learners' ability to retain the algorithmic content over a period of time. Both test scores and times were subjected to analyses of variance. The results provided strong data for the overall effectiveness of algorithms, showing from 52 percent to over 80 percent accuracy following an average of less than 15 minutes of instruction and practice. Almost no decrement (only one percent) in performance was observed after a one week delay. Subjects did perform better when allowed continued use of the algorithmic representation, and the flowchart group was much more efficient than the other groups. The interactions suggested differential desirability of instructional strategy depending on the training needs.

(Author/JEG)
ALGORITHMS IN EDUCATION
SOME EMPIRICAL CONSIDERATIONS

by
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An instructional designer who relies heavily on the use of algorithms in a training program may be doing so prematurely. The state-of-the-art regarding algorithms as instructional devices is that we "think" they work and that algorithmic instruction appears well worth very close examination. Beyond that, however, there is apparently no data which provides answers to the major questions of good instructional systems design. In the following paper, we respond to some of these questions by identifying some of the problems pertinent to algorithms, we provide data for testing several important hypotheses concerning algorithmic instruction, and offer suggestions as to how and where the concept of algorithmic instruction might advance.

The first aim of this study was to establish the effectiveness of an algorithmic approach to the teaching of mathematical/conceptual problems. By comparing the effectiveness of some new type of instruction with a traditional method, researchers usually are able to demonstrate that the experimental group (e.g., some innovative technique) performed "significantly better" than the control. The question arises, however, "Better than what?" If a standard method of teaching produces 25% mastery, a new approach yielding 35% mastery indicates a marked improvement. However, when emphasis is placed on criterion referenced training, even an improved level of performance is unacceptable if its ceiling is only 35% mastery. Therefore, the following study avoids the traditional and well-established procedure of comparing an innovative method with an already poorly ranked alternative. Rather, mastery of a specific learning process is observed, and absolute, not relative, levels of performance are assessed.

A second critically important issue in instructional systems design is the extent to which the learners can retain information. The amount of effort put into a learning task is usually directly proportional to the amount retained, given that all other factors are held constant (Jenkins, Note 1). The intent of any instructional innovation is to make learning "easier" without loss of speed, accuracy, or quality of learning and/or performance. Therefore, "easier" must be carefully defined. First, extensive research as well as intuition tells us that the lesson content must be somewhat familiar to the learner and that the instructional input must be meaningful (Ausubel, 1968; Haviland & Clark, 1974; Schmid, Note 2). That is, the instruction must be sufficiently elementary. Second, instruction must contain only that information which is relevant to mastery of the task (Breche, Gerlach, & Schmid, 1976). Irrelevant or nonessential information leads to confusion and processing overload. That is, instruction must lead the

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learner unambiguously to the correct result. Finally, instruction should always be applicable to an array of interrelated problems. The interrelation is best defined as the problem context, which consists of a small set of cognitive and psychomotor skills (Gagne', 1965) and the environmental input. Processes of transfer, generalization, and learning set are products of these kinds of general psychological schemata. Some educators fear that algorithmic instruction is so proceduralized that its use will reduce the effectiveness of a training system by impeding the development of such processes as transfer. Retention, therefore, depends upon the learner's prior knowledge, the clarity and precision of the instruction, and the contribution any given unit of information will provide for overall learning.

The third area of interest explored in this study is the problem of instructional technique. Actually using algorithms in instruction constitutes an upper level concern of instructional technique. How algorithms are presented to the student represents a lower, more pragmatic level. Naturally, an adequate test of the algorithmic approach can be made only when several potentially effective methods of algorithmic teaching have been developed. In the present study, three standard classroom techniques were employed to teach an algorithm for solving tax problems. In this way, a wider range of instructional sampling was implemented for evaluating the effectiveness of algorithms in general.

Lastly, we were interested in the extent to which learners depend upon the physical presence of the algorithmic prescription. Because algorithms have been used for years in the form of job performance aids (e.g., recipes; instructions for using telephones, vending machines), there is a question regarding their applicability beyond providing instructions not necessarily committed to memory (Landa, 1974, 1960). The following study examines this factor for both immediate and delayed performance. It may be that algorithms are extremely effective and efficient when readily available, but difficult to internalize or retain as a systematic solution procedure.

Method

Design and subjects. Three factors, Tax Law Availability, Instructional Representation, and Test Interval were combined factorially to form six treatment groups. Test Interval was varied as a within-subject factor. The design was thus a 2 Availability (with tax law vs. without) X 3 Representation (prose vs. flowchart vs. faded flowchart) X 2 Test (immediate vs. delay). Analyses of variance, with repeated measures on Test interval, were employed.

Subjects consisted of 77 undergraduate volunteers from Arizona State University. Eleven subjects were dropped from the experiment for failure to follow procedures, leaving 11 per factorial cell. Subjects were run in groups during normal classroom sessions. Materials were prepared beforehand and shuffled, so that assignment to treatments was completely randomized.
Materials. The instructional task was adapted from Horabin's Algorithms (1974). The task consisted of generating the solution to tax problems involving the purchase and sale of shares of stock. The authors created two sample problems for each of six possible solutions, from which two sets of problems were created. Instruction on how to apply the tax law took one of three representational forms: the prose condition received a verbal description of the law, which followed an if/then format; the flowchart condition received the same information in a flowchart form, such that each decision (discriminator) was binary, leading to another discriminator, until the terminal solution box (operator) was reached; the faded flowchart was exactly the same as the flowchart treatment, except that for each problem completed, one of the discriminators necessary for the solution of subsequent problems was deleted. The deleted information was made available only if the subject was unable to recall it. Corrective feedback was supplied in separate booklets. The questionnaire following the treatment asked about the learner's strategies and reactions to the instruction. The posttest contained six additional randomly ordered problems of the same type, without feedback. The delayed posttest and the immediate posttest were identical, except for the order of the problems.

Procedure. The experimental sequence consisted of (a) an orientation, (b) a practice session, (c) an immediate posttest, (d) a questionnaire, (e) a one-week delay posttest, and (f) a questionnaire. In addition to completing six practice and six posttest problems, subjects recorded the time in spaces provided at the start and finish of each problem. Following the introduction read aloud by the experimenter, subjects completed all phases at their own pace. During both posttests, only half the subjects from each condition were supplied the tax law for solving the problems.

Results

Achievement

All protocols were scored for number correct, with one point for the dollar amount and one for the tax status (charged, allowed, or no tax). Omissions were counted as errors.

Three Representation x two Availability analyses of variance were conducted on the practice and immediate posttest sessions. A repeated measures ANOVA using the delayed test data was then performed. The repeated measures ANOVA was performed twice, (a) using groups as represented by the with/without distinction on the immediate posttest, and (b) using groups separated according to the Availability factor on the delayed test.

As expected, no differences were found between groups during the practice session. However, posttest performance was significant for Representation, $F(2, 60) = 3.20, p < .04$, and Availability, $F(1, 60) = 21.82, p < .001$. Scheffé tests on the Representation factor ordered the groups: prose = flowchart > faded flowchart.
The repeated measures anova on original groups yielded significance for the Availability main effect, \( F(1, 54) = 16.83, p < .001 \), and for the Availability x Test interaction, \( F(1, 54) = 8.17, p < .006 \). Not surprisingly, subjects allowed to use the procedure during the test session performed significantly better than the group from whom the procedure was withheld. Of greater interest, however, was the interaction. Individual comparisons showed that learners who were allowed to use the procedure during initial testing performed significantly worse on the delayed test, due to the fact that the procedure was withheld from half the subjects during the delay. On the other hand, significant increment in performance was noted for subjects not allowed to use the procedure during initial testing, an effect likely attributable to the renewed availability of the procedure for half those subjects.

The repeated measures anova utilizing a regrouping of subjects on the delay according to procedure availability yielded significance for the Availability main effect, \( F(1, 54) = 64.77, p < .001 \), and the Representation x Availability interaction, \( F(2, 54) = 3.37, p < .04 \). Subjects using the procedure produced higher scores than those without. The significant interaction demonstrated that when subjects are supplied with the procedure, the flowchart is the most effective representation for solving the problems. However, when the procedure is removed, the fading group fell significantly below the prose and flowchart groups, which did not differ. These effects reiterate the ineffectiveness of the fading instructional strategy, especially emphasized by the fact that the flowchart and faded flowchart groups did not differ in any way following the practice portion of the experiment. This interaction is shown in Figure 1.

**Time**

Time data were generated by computing the mean number of seconds taken per problem solution. Omitted problems were not included in the estimates.

Three Representation x two Availability analyses of variance were conducted on the practice and immediate posttest sessions. A repeated measures anova using the delayed test data was then performed. As with the test scores, the repeated measures anova was performed twice, (a) using the immediate posttest Availability distinction, and (b) using the delayed posttest Availability regrouping.

Times taken during the practice session differed significantly, \( F(2, 60) = 8.44, p < .001 \), with Scheffe' tests ordering the means: flowchart < prose = faded flowchart. These initial differences, however, disappeared during the immediate posttest.

The repeated measures anova on the original groups yielded differences for the Representation main effect, \( F(2, 54) = 3.74, p < .03 \), and the within subject Test factor, \( F(1, 54) = 12.46, p < .001 \). Scheffe' tests ordered the Representation means: flowchart = prose < faded flowchart. More time was spent on the immediate posttest than the delayed test.
Figure 1. Representation× availability interaction from the repeated measures ANOVA on test scores regrouped on delayed test availability.
When subjects were regrouped according to procedure availability on the delayed test, significant effects were found for the Representation factor, $F(2, 54) = 3.83, p < .03$, the test/retest factor, $F(1, 54) = 11.98, p < .001$, and the Representation x Availability interaction, $F(2, 54) = 6.16; p < .004$. Scheffe tests ordered the representational treatment means: flowchart = prose < faded flowchart. Less time was spent on the delayed items than the immediate test items. The interaction produced some very curious results: the flowchart group spent the same amount of time either with or without the procedure, the prose group spent significantly less time without, and the faded flowchart subjects spent significantly more time without the procedure present. Further, the flowchart representation subjects worked significantly faster than the prose version subjects when the procedure was available, whereas the opposite held true when the procedure was removed. A graphic representation appears in Figure 2.

**Discussion**

The results of this investigation supply convincing answers to the primary questions raised in the rationale. Overall effectiveness of the algorithmic prescriptions was quite high, ranging from 80% to 52% accuracy. Surprisingly, performance decreased only one percentage point after one week. These effects persisted in spite of the fact that 1/4 of the subjects were not allowed to examine the procedure at any time after the practice session, and 1/4 did not see the procedure after the immediate test. Nevertheless, the availability of the procedure during the immediate test still facilitated learning significantly, in that the group receiving the procedure achieved superior scores on the delayed test, as indicated by the Availability x Test interaction.

These data suggest two conclusions. First, the effectiveness of the algorithm was demonstrated by the high performance not only immediately following instruction, but also one week later. The classic forgetting curve appears to have been defied, even without the presence of a ceiling effect. Thus, the absolute value of algorithms was demonstrated. This effect is compounded by the high efficiency of instruction (mean learning time, approximately 15 minutes). Within the task requirement of this study, algorithms were extremely good instructional devices.

The second conclusion drawn from these data is that although learners can indeed memorize the content of an algorithm for later use, enabling them to continue using the procedure is significantly more effective. When the procedure was made available, 80% performance was found on both immediate and delayed tests. The Availability x Test interaction graphically shows the potency of the presence of the algorithm. The significant convergence from both immediate test means to the nonsignificant difference in the delay test suggests that the presence of the procedure is equally critical regardless of the subjects' counterbalancing membership.
Figure 2. Representation x availability interaction from the anova of test times with subjects regrouped for delayed test availability.
The next question referred to specific instructional techniques. Although a pilot study did not yield differences between the three algorithmic representations, the revision in the main study yielded some interesting and unexpected results. The flowchart and prose groups performed equally well on the immediate test, supporting the contention that both produce equivalent information and that both are equally algorithmic. The mode of representation apparently did not differentially effect processing. However, the faded flowchart group, contrary to expectations, performed significantly worse than the other groups, despite the fact that its instruction was derived from the identical information base. The special processing demands required of the fading condition apparently interfered with task acquisition, as indicated by the significant performance decrement in the Representation x Availability interaction. These differences were especially strong when the procedure was withheld. Thus, the technique developed to increase learning actually hindered overall performance. One possible explanation for these results is that the procedure placed undue emphasis on the discriminators, or decision-making aspect, and too little on the operators, which actually formed the solutions. It may also be that this strategy requires more practice than the other treatments (Schmid & Gerlach, Note 3). A tradeoff with efficiency may yield better effectiveness.

In an overall assessment of efficiency, we noted not only the short period of instruction required, but also that efficiency for the algorithmic approach improved 28% over the time delay. While any recommendations to teachers or instructional developers must be qualified, these results suggest that the best technique for task performance would be to supply the learner with a flowchart which remains available for reference.

In summary, algorithms were found to be both effective and efficient in absolute, rather than comparative, terms. They seem especially resistant to forgetting. Differences between the flowchart and prose treatments remain somewhat unclear, each having shown distinct advantages depending upon procedure availability and the retention interval. Our present conclusion is that the same underlying learning processes are occurring regardless of the representation. We are now completing several studies which are examining the structural and cognitive characteristics of algorithms by means of cybernetics and structural learning theory. Finally, forcing the processing of a flowchart in the manner employed here is detrimental to both the effectiveness and efficiency of algorithmic instruction. Further experimentation must distinguish between the mathemagenic and conceptual acquisition effects responsible for these data.
Reference Notes


References


