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ABSTRACT

This module introduces the student to a formal study of probability. The lessons progress from elementary notions of least likely to more complex considerations of the probability of occurrence of one or more possible outcomes. Probability is introduced in an empirical sense through experimentation and investigation. In the later experiments, the students are shown how mathematical probability differs from empirical probability. The last three experiments of the module demand some fairly mature thinking on the part of the students. The teacher may prefer to use a classroom setting if these three experiments are used. (Author/MK)

TOPICAL MODULE FOR USE
IN A
MATHEMATICS LABORATORY SETTING

TOPIC: Likelihood of Events – Probability

by

Lyle Andersen

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University of Denver
Mathematics Laboratory
Regional Center for
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Dr. Ruth I. Hoffman, Director

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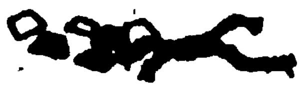
TEACHER'S GUIDE

Likelihood of events - Probability

This module may be used in a variety of ways.

1. It may be used with a pretest as suggested in the teacher's guide.
2. Groups of students, according to their past performance, may be assigned to various experiments.
3. The class may be divided so each group has a group leader who has the ability to help the other students.
4. Teachers may use a group-method of their own choosing which might be the entire class.

MATERIALS



.Contained in this Module:

1. Plastic bag containing 1 red. 4 black and 5 yellow chips. (Student Worksheet #1)
2. Cut-out spinners (Activity Cards #3 and #4)

.Teacher must provide:

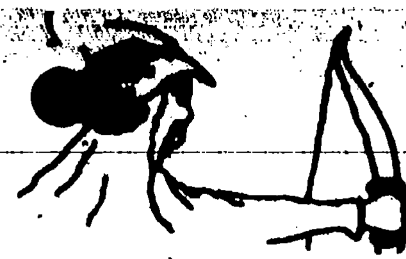
1. Tagboard that is opaque to writing (Intro. SW)
2. Thumb tacks or similar markers. (Activity Card #2)
3. Scissors for cutting our spinners. (Activity Cards #3 and #4)

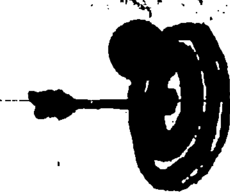
.Optional (Teacher provides)

1. Pair of dice (Student Worksheet #5)

TIME SCHEDULE

.During field testing, this module took an average of ______ fifty-minute class periods to complete.





OBJECTIVES

.Given an event, the student will list possible outcomes.

.Provided with the outcome of the event, the student will describe it as likely or not likely to occur.

.Given an event, the student will find the probability of various outcomes and will identify outcomes as most likely, least likely, certain or impossible.

OVERVIEW

.The student will be introduced to a rather formal study of probability. The lessons progress from elementary notions of least likely to more complex considerations of the probability of occurrence of one or more possible outcomes. Probability is introduced in an empirical sense through experimentations and investigation. In the later experiments, the students are shown how mathematical probability differs from empirical probability.

.The last three experiments of the module demand some fairly mature thinking on the part of the students. The teacher may prefer to use a classroom setting if these three experiments are used.

TESTING PROCEDURE

I. Pretest

A. The pretest checks the behavior entry level of the students to determine if they need additional preparation before beginning work on this module.

B. If the student is unsuccessful on the pre-test, then s(he) should be directed to pre-module materials.

II. Pre-Module Experiences

A. Students whose scores on the entry level test indicate they need additional preparation will likely profit most from the use of concrete materials such as drawing marbles from a vase or rolling dice.

B. It may be helpful to have another student help the student experiencing difficulty by utilizing elementary commercial materials.

C. If no pre-module materials are available, the student should begin with the introductory lesson in this module.

III. A. The posttest checks to see if the student is able to successfully complete the objectives established for this module.

OUTLINE

I. Introduction Lesson

A. Teaching Suggestions

1. This lesson requires the students to take a poll. This might be done as a class project, or students can take the poll outside of class. This lesson is highly motivaional. Students may need some direction.

2. Problems 3 and 4 may need additional explanation.

B. Materials supplied

1. Students will need the activity card and the introductory worksheet. 5 chips (2 cm x 2 cm tabs of cardboard) are needed for problem 4.

C. Work to be completed by the student.

1. Introductory worksheet which includes the taking of a poll.

Answer keys are provided for each lesson and experiment. You may prefer to cut them out, place on cardboard and laminate.

Answer Key - Introductory Lesson - Probability

1. Answers will vary depending on the individual poll.
2. Answers will vary. The event most likely to happen is the one which occurs most often in the poll. The event least likely to occur is the one which occurred least often in the poll.
3. Answers will vary. (Example: if 5 of the 15 chose A, then the humor index for A would be 5/15 or 1/3.)
4. Answers will vary. (Example: if ten 3-letter words were drawn and seven made sense, then the chance of forming a 3-letter word with meaning would be 7/10.)

II. Experiment #1

A. Teaching suggestions

1. Students should be encouraged to do this experiment in pairs. Some students may not know how to keep track of the draws. You may wish to demonstrate this experiment to the entire class before allowing them to work on it in pairs or individually.
2. This is the first introduction to the term <u>probability</u>. Allow the students to discover for themselves, if they can, the definition for simple probability.

B. Materials supplied

1. The plastic bag which is provided in the module is necessary for the completion of this experiment. The envelope contains 1 red, 4 black and 5 yellow chips.

C. Work to be completed by the student

1. Question on Activity Card #1.
2. Student Worksheet #1.
3. Have the students place their answers to the questions on the activity card on the back of Student Worksheet #1. (They should be encouraged not to write on the activity card since this module can be repackaged.)

D. Variations to the lesson

1. Provide envelopes containing various numbers of chips.

Answer Key - Student Worksheet #1 - Probability

1. 10 2. 1, 4 and 5

3. a) $\frac{1}{10}$ or $\frac{1}{5}$ b) $\frac{4}{10}$ or $\frac{2}{5}$

4. a) $\frac{2}{9}$ b) $\frac{4}{9}$ c) $\frac{3}{9}$ or $\frac{1}{3}$

5. Answers will vary. (Example: If a black chip was drawn on the first draw, then prior to this draw the probability was 4/10. Now suppose a black chip is drawn on the second draw. Prior to this draw the probability for a black on the second draw was 3/9, etc.)

III. Experiment #2

A. Teaching suggestions

1. This activity requires considerable movement and involvement on the part of the students. You may prefer having them work in small groups. Students usually enjoy completing Student Worksheet #2 in small groups. (You may prefer to have them drop a cardboard tab rather than a thumb tack.)
2. The triangle board can be drawn on an acetate sheet and shown on the overhead and used as a class demonstration.

B. Materials supplied

1. Students will use Activity Card #2 as a game board. They need to be provided with a marker such as a thumb tack.
2. They are provided with Student Worksheet #2.

C. Work to be completed by the student

1. Students will determine by investigation the empirical probability of the experiment.
2. Complete Student Worksheet #2.

Answer Key - Student Worksheet #2 - Probability

1. Answers will vary. Students should compare results.
2. Answers will vary. The empirical probability of a hit is the ratio of hits to total number of trials.
3. Answers will vary.
4. Likely it will lower the probability of a hit.

A. Teaching suggestions

1. This experiment provides additional practice with empirical probability. The material is highly motivational and should require only minimal supervision.
2. Some students may require help with the number pair concept which is introduced.
3. It is important that students complete the entire worksheet.
4. You may prefer to use glued toothpicks instead of pencils in the tops.

B. Materials supplied

1. Students will need scissors to cut out the pencil tops from Activity Card #3.
2. Activity Card #3 and Student Worksheet #3.

C. Work to be completed by the student

1. Students will complete Student Worksheet #3.
2. Students must complete the tables by collecting the data from their experiments.

Answer Key - Student Worksheet #3 - Probability

1. Answers will vary. The sum of the fractional part of total trials should equal 1.
2. Answers will vary. Students should be encouraged to share answers.
3. Answers will vary. The sum should equal the sum of the number pair for that trial.
4. Since this is empirical probability, the answers will vary. Again encourage students to compare answers.
5. Answers will vary. $P(1) = 0$
   $P(9) = 0$.

V. Experiment #4

A. Teaching suggestions

1. It may prove helpful to place the maze on an acetate sheet and demonstrate on the overhead how a "hamster" finds its way through the maze prior to allowing the students to complete this experiment.
2. Students may prefer working together on this experiment.
3. Mathematical probability is informally introduced in this lesson. You may choose to expand on this concept if students go on to experiments 5, 6 and 7.
4. The difference between empirical probability and mathematical probability should be made clear to the students.

B. Materials supplied

1. Students will need a marker to show location on the maze.
2. They will need a scissors to cut out the spinner on Activity Card #4.
3. They will need Activity Card #4 and Student Worksheet #4.

1. Students will collect their own data by actual experiment.

2. They will complete Student Worksheet #4.

Answer Key - Student Worksheet #4 - Probability

1-4. Students should verify their answers by comparing their work with classmates. If their results are not consistent, they should check their results and decide who is right.

5. a) $\frac{1}{4}$ b) $\frac{1}{8}$ c) $\frac{1}{16}$ d) $\frac{1}{32}$

e) $\frac{1}{64}$ f) $\frac{1}{512}$ g) $\frac{1}{256}$ h) $\frac{1}{256}$

i) $\frac{1}{256}$ j) $\frac{1}{8}$ k) $\frac{1}{16}$ l) $\frac{1}{8}$

m) $\frac{1}{8}$ n) $\frac{1}{16}$ o) $\frac{1}{512}$.

6. Sum of fractions equals 1.

VI. Experiment #5 (Optional)

A. Teacher suggestions

1. This experiment requires fairly abstract thinking on the part of the students. You may prefer to use this lesson as a teacher demonstration experiment. The coin tossing in Problem 2 may need close supervision.

2. Students should be encouraged to check the mathematical probability by using empirical methods.

3. You may prefer to develop the problems in Student Worksheet #5 empirically.

B. Materials supplied

1. Activity Card #5 and Student Worksheet #5.

2. Coins and dice are needed if the teacher approaches this lesson empirically.

C. Work to be completed by the student

1. Students will complete Student Worksheet #5.

2. Problem 2 requires students to make random tosses with three coins.

Answer Key - Student Worksheet #5 - Probability

1. $\frac{1}{6}$; 2   2. $\frac{2}{8}$ or $\frac{1}{4}$ 3. $\frac{6}{36}$ or $\frac{1}{6}$

4. $\frac{3}{4}$ 5. $\frac{1}{32}$.

VII. Experiment #6 (Optional)

A. Teaching suggestions

1. Since mathematical probability of independent events is the topic of this experiment, the students are required to complete this experiment without physical models.

2. Students having difficulty should be encouraged to make tables similar to the ones constructed in Experiment 5.

3. For those who wish, the examples may be solved empirically by providing the necessary models.

B. Materials supplied

1. Activity Card #6 and Student Worksheet #6.

2. If an empirical approach is used, three chips, two coins, and a deck of cards need to be provided.

C. Work to be completed by the student

1. Students will complete Student Worksheet #6.
2. You may wish to have the students make tables to show all possible events and use these tables to aid in solving the problems.

Answer Key - Student Worksheet #6 - Probability

1. $\frac{1}{8}$

2. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4^5} = \frac{1}{1024}$

3. $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \frac{1}{4}$

4. $\frac{4}{52} \times \frac{4}{51} = \frac{16}{52 \times 51} = \frac{4}{13 \times 51} = \frac{4}{663}$

5. $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} = \frac{1}{270,725}$

VIII. Experiment #7 (Optional)

A. Teaching suggestions

1. Since mathematical probability of mutually exclusive events is the topic of this experiment, the students are required to complete this experiment without physical models.
2. Students may need close teacher direction and explanation when working on this experiment.
3. You may wish to use models to show all possibilities and then determine the probability from this data.

B. Materials supplied

1. Activity Card #7 and Student Worksheet #7.
2. If an empirical approach is used, then a pair of dice and a deck of cards will need to be provided.

C. Work to be completed by the student

1. Students will complete Student Worksheet #7.
2. You may wish to have the students make tables to show all possible events and use these tables to aid in solving the problems.

Answer Key - Student Worksheet #7 - Probability

1. $\frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

2. $\frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

3. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

4. $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

5. $\frac{6}{11} \cdot \frac{5}{10} \cdot \frac{4}{9} + \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} = \frac{4}{33} + \frac{2}{33} = \frac{6}{33}$ or $\frac{2}{11}$

IX. Enrichment Cards

A. Teacher suggestions

1. If the pretest was used, those students who scored 90% or higher (depending on teacher discretion) may be given an enrichment card to investigate.

2. These may be used as extension work for students who have completed the unit.

B. Answer Key

1. The answer key has been provided at the bottom of the enrichment card. These may be cut off before the student is given the card.

Answer Key - Probability

Pretest

1. a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{2}{3}$

2. a) $\frac{1}{2}$ b) $\frac{1}{4}$

3. a) $\frac{3}{10}$ b) $\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{12}$

4. a) $\frac{3}{6}$ b) $\frac{1}{6}$ c) $\frac{12}{30} = \frac{2}{5}$

Posttest

1. a) not likely b) likely c) not likely

2. a) $\frac{2}{4}$ or $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{1}{4}$

3.

H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

<u>Note:</u> Answers may be in different order than shown.

4. a) $\frac{2}{9}$ b) $\frac{2}{9}$ c) $\frac{2}{9}$

5. a) $\frac{5}{20}$ or $\frac{1}{4}$ b) $\frac{13}{20}$

6. a) $\frac{3}{6}$ or $\frac{1}{2}$ b) $\frac{2}{6}$ or $\frac{1}{3}$ c) $\frac{12}{30}$ or $\frac{2}{5}$

Pretest - Probability

1. After lunch, Clarence can shoot his bow and arrow, or wash dishes, or do his mathematics. If all three possibilities are equally likely, find the probability that Clarence will

   a. shoot his bow and arrow.
   b. not wash dishes.
   c. do his mathematics or wash the dishes.

2. What is the probability of:

   a. flipping one coin and having it land "heads up"?
   b. flipping two coins and having both coins land "heads up"?

3. A vase contains 10 marbles; three are red, two are green and five are blue. Assuming you don't look, what is the probability of:

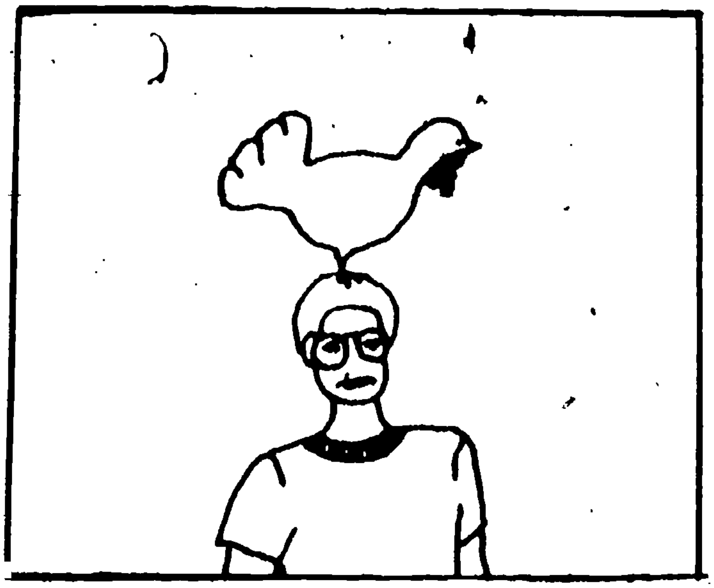
   a. pulling out a red marble on the first draw?
   b. pulling out three blue marbles on three draws when <u>not</u> replacing them after each draw?

4. If you have three pennies, two nickels and one dime in your pocket, what is the probability that:

   a. the first coin you pull out is a penny?
   b. a dime will be drawn first?
   c. if you remove two of the coins, their value will be 11¢?

Poll-taking

Using the table (Fig. 1) on the introductory worksheet, take a poll to determine the "humor-index" of 15 people. Do this by showing the cartoon (one person at a time) and then recording the caption (A-E) that they think best fits the picture. From the results of your poll, complete the introductory worksheet.



A. Hey Doc, I got this guy attached to my foot.

B. Go ahead and jump. You won't be the first one.

C. I said I wanted to be paid by check, not chick.

D. This time the humane society went too far.

E. With the price of meat, don't knock it.

1. Use this table to record the results of your poll.

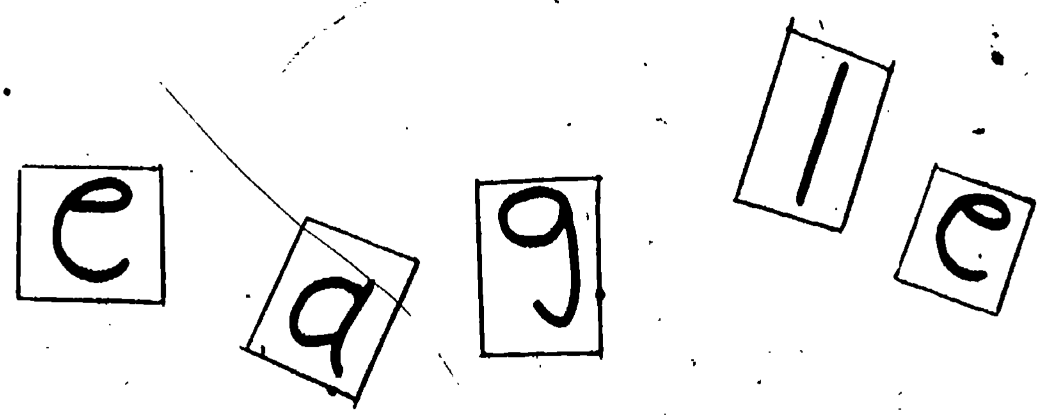
Caption Letter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total No. of squares darkened
A																
B																
C																
D																
E																

The activity card shows five (5) possible captions. The captions are examples of possible outcomes. Uncertainty of events is often expressed in conversation, such as least likely, most likely or certain.

2. From the results of your poll, which event is most likely to occur?

3. The "humor index" might be described as the ratio of the number choosing that event or caption to the total number choosing. This may be described as a fraction or a decimal. From the results of your poll, determine the "humor index" of each caption. A–F. (Note you can then tell the people you polled what their humor-index is.)

4. Print the letters of the word "eagle" on five identical tabs of paper that you cannot see through. Turn each slip face down and thoroughly mix the slips. Draw any three of the slips. Try to form a word with meaning using the three letters you picked. Repeat the experiment several times. Describe the chance of forming a word with meaning.



What are the Chances?

Materials: Envelope A containing 1 red, 4 black and 5 yellow chips.

When an outcome is likely to occur, its probability is high; if not likely, the probability is low.

Without looking, reach into envelope A and take out a chip. Replace it, then draw out another chip. Do this 10 times and always replace the chip before the next draw. From your experiments, do you think the probability of drawing a red chip is likely or unlikely?

Complete Student Worksheet #1.

Experiment I

1. How many chips are in envelope A?

2. How many of the chips are red? _____ black?_____ yellow? _____

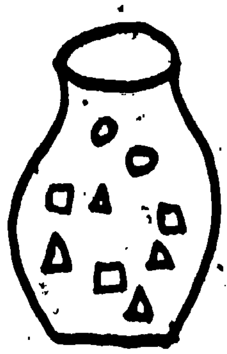
The probability of an event occurring is the ratio of the number of favorable outcomes compared to the total number of possible outcomes. This ratio is often expressed as a fraction.

3. Since 5 of the 10 chips are yellow, the probability of yellow is 5/10. We write P (Yellow) = 5/10 = 1/2

Answer the following.

a. P (red) = _____ b. P (black) = _____

The chance of drawing a triangular shape from the bowl is 5/5 or 1. The chance of drawing a circular shape is 0/5 or 0.

4. 

The vase contains rectangular, triangular and circular chips. Answer the following by referring to the vase.

a. P(circular shape) = _____

b. P(triangular shape) = _____

c. P(square shape) = _____

5. Place the 10 chips back in the envelope. Then remove them one at a time without replacing any chips. After each draw, determine what the probability was of that particular draw happening. Show your results below.

P (1st draw) = _____ P (6th draw) = _____

P (2nd draw) = _____ P (7th draw) = _____

P (3rd draw) = _____ P (8th draw) = _____

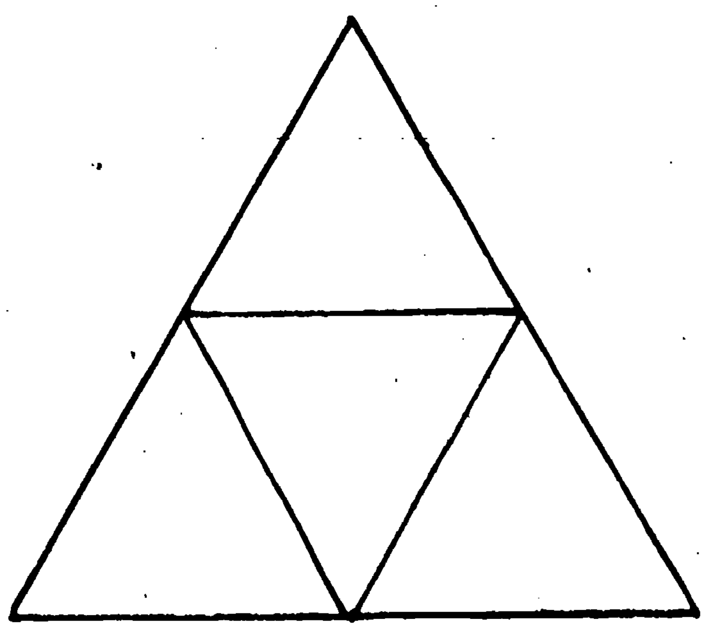
P (4th draw) = _____ P (9th draw) = _____

P (5th draw) = _____ P (10th draw) = _____

Bomb-out a Probability

terials: Thumb-tack

Sometimes the only basis for finding the probability of an outcome is through experimentation. This is an example of <u>empirical probability</u>.



Drop a thumb tack from a height of 5 cm. above the center-most triangle. The thumb tack hits if it stops completely within one of the small triangles. The probability of a hit on one trial is the ratio of the number of hits to the number of trials, after a sufficient number of trials: $P\text{ (hit)} = \frac{\text{number of hits}}{\text{number of trials}}$

Complete Student Worksheet #2.

Experiment II

1. Drop the thumbtack 40 times and record the results in the table.

	1st 10 trials	2nd 10 trials	3rd 10 trials	4th 10 trials	Total
hits	____	____	____	____	____
misses	____	____	____	____	____
				Total trials	____

2. What is the empirical probability of a hit?

3. Try it 40 times with your eyes closed. What is the empirical probability of a hit?

4. Complete 1 and 3 from a height of 10 cm. What does this do to the probability?

Experiment III

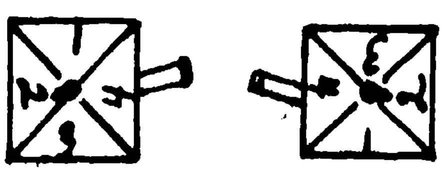
"Spinning-out" Outcomes

1. Spin a "pencil top" 50 times. Use the tally space to tally each outcome and complete the table.

Outcome	Cumulative Tally Score	Sum of Tallies	Fractional Part of Total Trials
1			
2			
3			
4			
	Total		

2. Did the outcome turn out as you expected? If you took the class average for each outcome, what would you expect?

3. Now spin both tops. Record them as a number pair. Always record the encircled numeral first. Determine the number pairs' sum. Example:



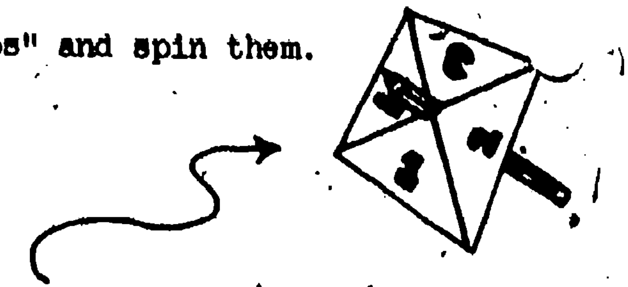
This is recorded as (3, 1), the sum is 4.

Now complete this table.

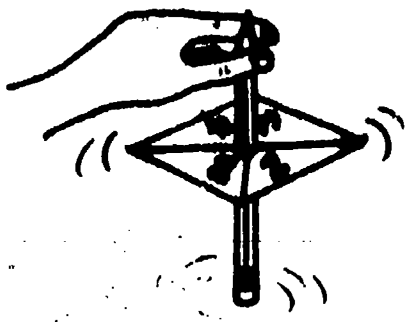
Trial No.	Number Pair	Sum
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

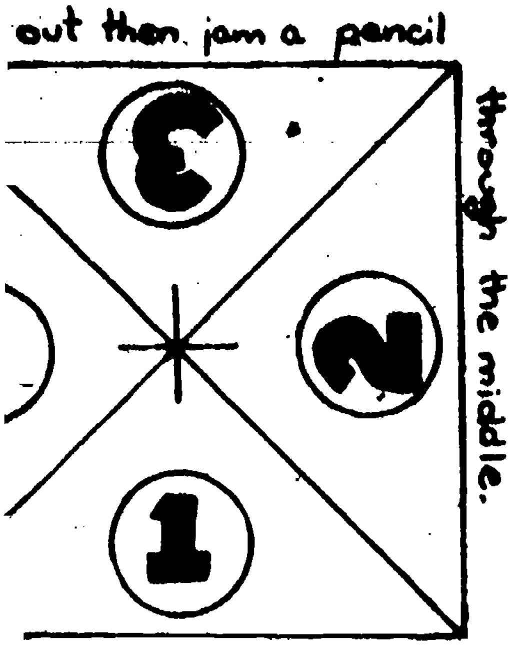
"Spinning-out" Outcomes

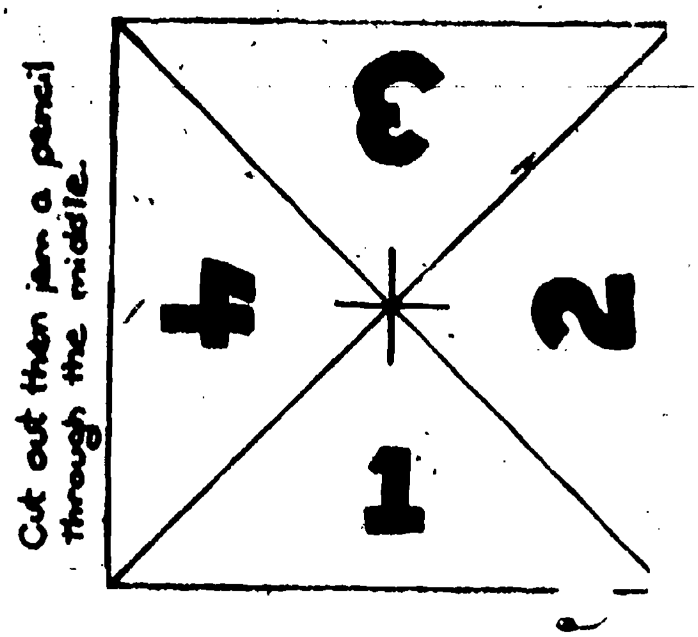
In this experiment, you will build two "pencil tops" and spin them. The outcome is the numeral that lands down.



This time the outcome was 1.







4. From the 10 trials, what appears as the most likely sum? least likely sum?

5. From the 10 trials, determine the empirical probability for each of the following sums.

   a. P(1) =

   b. P(2) =

   c. P(3) =

   d. P(4) =

   e. P(5) =

   f. P(6) =

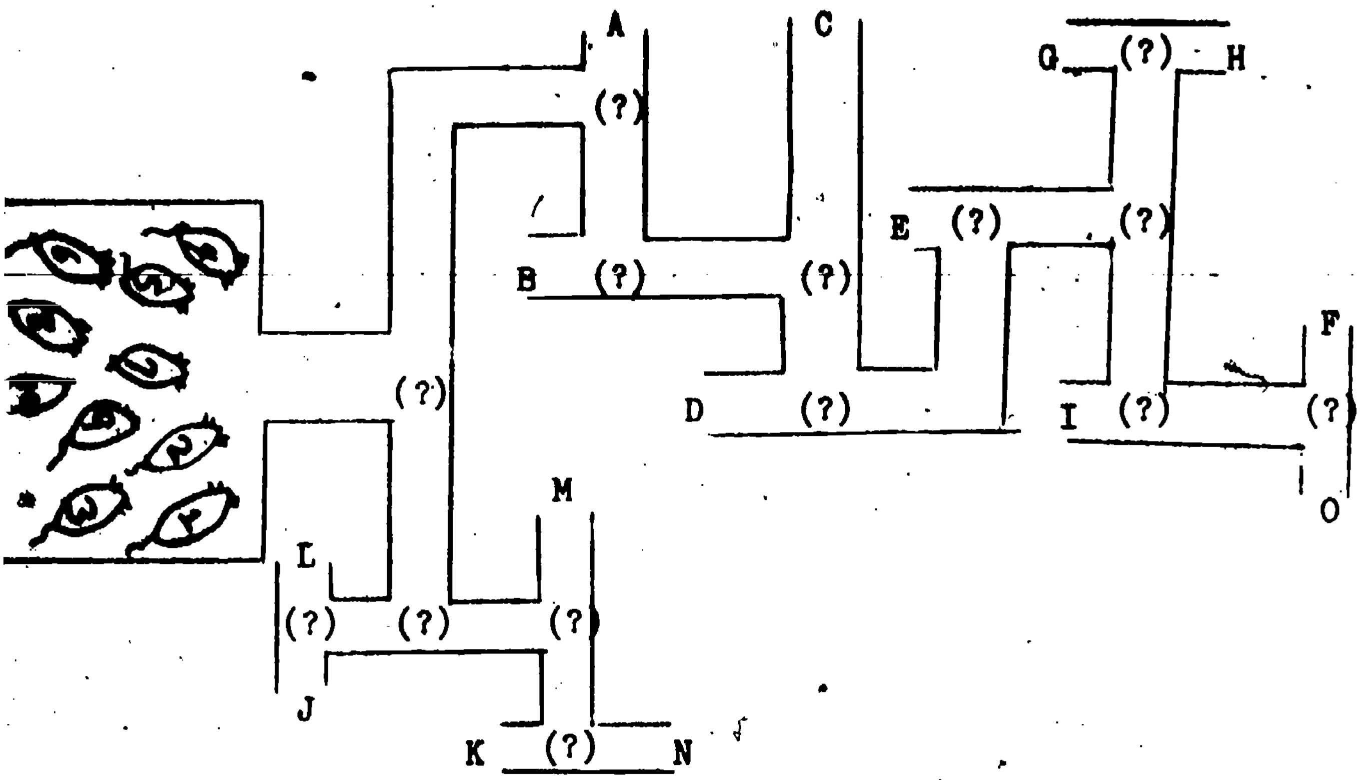
   g. P(7) =

   h. P(8) =

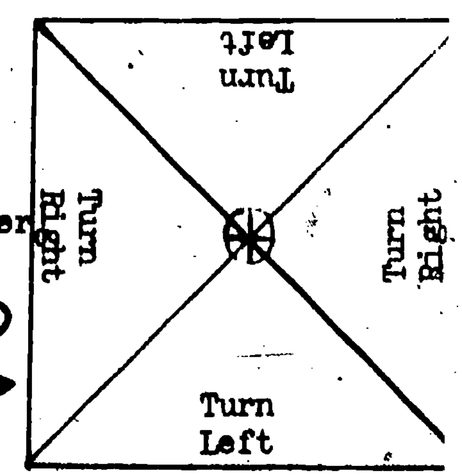
   i. P(9) =

Catching Hamsters Is More Than Luck!

The hamster cage doors have been lost. The hamsters will all escape through the tunnels. You have four cages to place at four different tunnel openings. Place them where you think you will catch the most hamsters. Indicate where the traps are placed by recording the letters (on SW #4) which mark these openings. Each hamster has a number on his back for experimental purposes.



Cut out and jam a pencil through the center of the spinner.



Catching Hamsters Is More Than Luck!

1. Name the four openings where you placed the traps. ____, ____, ____, ____.

2. To determine the path of each hamster, place a marker on the student activity card at start (a coin might be used for a marker). Spin the "pencil top" and turn left or right at that (?) depending on what the spinner indicates. Then advance to the next (?). Continue advancing in this way until the "simulated hamster" has escaped through one of the lettered openings. In the table below, show which opening the hamster "escaped" through. Repeat this experiment for each of the 10 hamsters.

Hamster Number

Tunnel Openings	1	2	3	4	5	6	7	8	9	10
A										
B										
C										
D										
E										
F										
G										
H										
I										
J										
K										
L										
M										
N										
O										

3. How many of the escaping hamsters did you recapture? Compare your results with a friend.

4. From your results, determine the empirical probability of each of the following.

a. P (A) =   b. P (B) =   c. P (C) =   d. P (D) =

e. P (E) =   f. P (F) =   g. P (G) =   h. P (H) =

i. P (I) =   j. P (J) =   k. P (K) =   l. P (L) =

m. P (M) =   n. P (N) =   o. P (O) =

(Optional)

5. Use mathematical probability to work problem 5. Unlike empirical probability, mathematical probability is not determined by experimentation. Assume at each (?) that 1/2 of the remaining hamsters go each direction. To determine the mathematical probability of the hamsters escaping through opening A, we would assume 1/2 of the hamsters go toward A. Then, we would assume that 1/2 of this 1/2 would turn out at A. Thus, 1/2 of 1/2 or 1/4 go out opening A.

   Using this type of reasoning, determine the mathematical probability for each of the following:

   a. P(A) = b. P(B) = c. P(C) = d. P(D) =

   e. P(E) = f. P(F) = g. P(G) = h. P(H) =

   i. P(I) = j. P(J) = k. P(K) = l. P(L) =

   m. P(M) = n. P(N) = o. P(O) =

6. If you worked problem 5 correctly, then the sum of the probabilities in 5 will equal 1. Check by adding. Show your work.

Chance and Certainty

People often say that a certain event will or will not take place: "It won't snow tomorrow"; "the bus is likely to be late"; "I doubt if it is the bank's mistake"; and so on. Probability is often viewed from two different approaches,

(1) The so-called classical probability, and

(2) The statistical or empirical probability.

When a coin is tossed, we feel it can land in only one of two ways - either heads or tails. This intuitive idea is expressed mathematically by saying that the chance of obtaining a "head" is one out of two or 1/2. The chance of obtaining a tail is equally likely, also, one out of two, or 1/2. This example illustrates a simple but fundamental rule of probability; namely, that of several equally likely events, the probability that a given event will happen is the ratio of the number of favorable possibilities to the total number of possibilities.

Expressed using symbols, we say that probability (P) of an event happening is equal to the possible successes (S) divided by the total number of trials (T).

$$P = \frac{S}{T}.$$

When an event is certain to happen, $S = T$ and $P = 1$. When it is certain not to happen, $S = 0$ and $P = 0$. Refer to this activity card when completing Student Worksheet #5.

Chance and Certainty

1. What is the probability of rolling 3 on one toss of a die? (Remember, a die has six faces marked from 1–6.)

$$P = \frac{S}{T} = \text{—}$$

How many 4's would you expect if you rolled the die 12 times? Try it. What happened?

2. If three coins are tossed randomly at one time, what is the chance there will be 2 heads and 1 tail? The table shows you all possible outcomes.

Make eight throws, tossing three coins each time. How many times were 2 heads and 1 tail tossed? What is the empirical probability for this experiment?

How does the empirical probability compare with the classical probability?

1st Coin	2nd Coin	3rd Coin
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

3. If 2 dice are tossed, what is the probability that a total of 7 will turn up? You may need to complete the table before you can determine your answer.

First Die

Second Die	+	1	2	3	4	5	6
	1	2					
	2					7	
	3						
	4			7			
	5				9		
	6						12

4. Two coins are tossed. What is the probability that at least one tail will turn up? (Refer to the Table in Problem 2). Hint: 2 coins can come up in 4 possible ways, HH, HT, TH, TT.

5. If 5 coins are tossed at one time, what is the probability that all 5 will fall heads?

Probability of Independent Events

If two events are of such a nature that the happening or failing of one of them has no influence upon the happening or failing of the other, they are called independent events. The probability that two independent events will occur simultaneously or in succession is called a compound probability. The compound probability of two independent events equals the product of the two events taken singly.

Thus, for n independent events whose respective probabilities are $P_1, P_2, P_3, \ldots, P_n$, the compound probability (P) that all the events will simultaneously (or in succession) happen, is given

$$P = P_1 \cdot P_2 \cdot P_3 \cdot \ldots P_n .$$

Example: What is the probability of obtaining a 4 on the first throw of a die, followed by a 6 on the second throw? The probability of the first event is 1/6; that of the second event is also 1/6. Hence the probability of both happening is 1/6 x 1/6 = 1/36. Refer to this activity card when completing Student Worksheet #6.

Probability of Independent Events

1. Three chips, each marked with an H on one side and a T on the other side, were tossed at the same time. What is the probability that all three came up with the T showing? You may wish to make a table showing all possible outcomes.

2. If on a multiple choice test each question has four choices, what is the probability that a student will answer the first five questions correctly if s(he) simply guesses at each question? (The answer is $\frac{1}{1024}$.) Show the work involved in determining this answer.

3. What is the chance of throwing 2 tails in a single throw of 2 coins? in two successive throws of 1 coin?

4. What is the probability of drawing a king and then a queen from a deck of 52 cards containing 4 kings and 4 queens? (This is a dependent event.)

$$P_{king} = \frac{4}{52} \text{ and } P_{queen} = \frac{4}{51}.$$

So, the probability of both events is

$$= \frac{4}{52} \times \frac{4}{51} = \_\_\_\_\_.$$

5. What is the probability of drawing 4 queens in 4 draws from a deck of 52 cards. The queens are <u>not</u> replaced after each draw. (Another dependent event.)

Probability of Mutually Exclusive Events

It sometimes happens that when one event has occurred, the possibility of another event is excluded during that trial. Such events are known as <u>mutually exclusive</u> events. For example, flipping a coin can yield a "heads" or a "tails" but not both for the same toss. It can be proved that if the separate probabilities of two mutually exclusive events are P_1 and P_2, then the probability that one of these events will happen when either could happen is $P_1 + P_2$.

Examples:

1. What is the chance of throwing either a 2 or a 3 in a single throw of a die.

$$P = 1/6 + 1/6 = 2/6 = 1/3$$

2. What is the probability of drawing either a queen, a king or an ace from a deck of 52 cards?

$$1/52 + 1/52 + 1/52 = 3/52$$

3. What is the chance of tossing a head or a tail on one toss of a coin?

$$1/2 + 1/2 = 2/2 = 1$$

Probability of Mutually Exclusive Events

1. What is the probability of drawing either an ace or a king from a deck of 52 cards?

2. What is the chance of throwing an even number or a 3 on one throw of a die?

3. What is the probability of either a 1, 2 or a 6 in a single throw of a die?

4. If two dice are thrown, what is the chance that both will turn up 6?

5. From a shelf containing 6 mathematics books and 5 science books, 3 books are chosen at random. What is the probability that that the three books chosen are either all mathematics books or all science books?

Find A Way To Show Results From Tossing Number Cubes

Materials needed:

2 wood or styrofoam cubes

Make a pair of dice.

a. How many possible sums are there when the two dice are "rolled"?

b. Make an addition table to show all possible sums.

c. What sum is most likely? least likely?

Answers 1

a. Eleven different sums are possible (2 - 12)

b.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

c. 7 since 6/36 or 1/6 of the sums are equal to 7.

2 and 12 - in both cases the the probability is 1/36

Birthdays Have Many Surprises

Have each student secretly write the month and day of their birthday on a piece of paper. Before you check the birthdays, determine what the chances are that two students have written the same date.

Answer

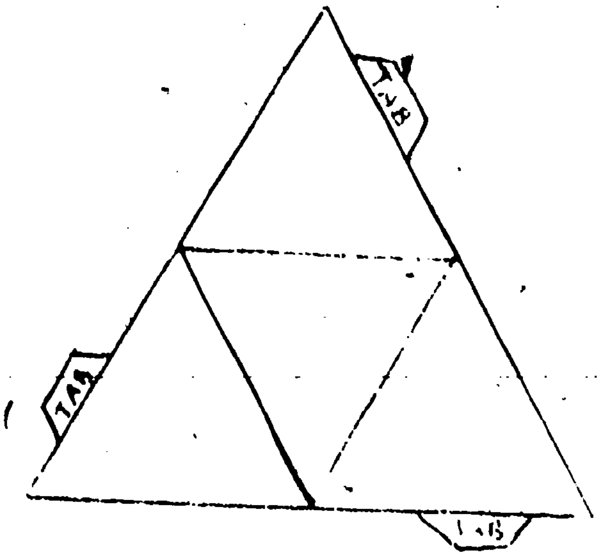
If there are 23 students, it is more likely than not that a duplication will occur.

Flipping Tetrahedrons (Optional)

Materials Needed:

Construction paper, scissors and glue or tape.

Make three (3) tetrahedrons from the pattern having its faces numbered with a different sequence. The sequences are (1, 1, 1, 2), (1, 1, 2, 2) and (1, 2, 2, 2). Using each tetrahedron separately, find the sum of the faces showing when thrown. Figure our for each tetrahedron what the probability is for all possible sums.



Answers

For the (1, 1, 1, 2) tetrahedron, one-fourth of the time the sum will be 3 and three-fourths of the time the sum will be 4.

For the (1, 1, 2, 2) tetrahedron, one half of the time the sum will be 4 and one-half of the time the sum will be 5.

For the (1, 2, 2, 2) tetrahedron, three-fourths of the time the sum will be 5 and one-fourth of the time the sum will be 6.

Probability 4

What Letter Is Used Most Often

Materials Needed:

Newspaper or Magazine

Pick out at random a paragraph from a newspaper or magazine. Determine how often each letter of the alphabet is used. Use this information to determine what letter is used most often.

Answer

Compare your results with others in the class. E is usually the most-often occurring letter.

Shooting Free Throws

Shoot 25 free-throws keeping track of how many you make. Now predict how many you would expect to make if you shot 100. Try it and see what happens.

Answers

(Answers will vary.)

Example, if they made 12 out of 25, they would expect to make 48 out of 100. Point out that it would be unlikely they would make exactly the number predicted.

What's Wrong Here?

Tell why incorrect conclusions are often drawn from information:

One-third of the children in the world are Chinese. Therefore, you or the person in front or behind you is Chinese.

Heads come up five times, so tails will be next for sure.

Homerun Lyle hasn't had a hit in 4 times at bat. He's sure to get a hit today.

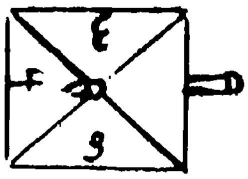
Answer

Probability indicates the likelihood of an event. It does not indicate certainty. The solution must be a member of the sample space.

Posttest - Probability

1. Describe the following as likely or not likely.

   a. A lion cub grows up to be a tiger.
   b. You know how to spell your own name.
   c. A coin, when flipped, will stop standing on its edge.

2. 

   When the spinner is spun, it comes to rest on one of the "numbers". Find the probability that the spinner lands on:

   a. 3
   b. 4
   c. 3 or 4
   d. an even number.

3. Complete the table, showing all possibilities when three coins are flipped. (H means heads, T means tails.)

1st Coin	2nd Coin	3rd Coin
H	H	H
H	H	
H	T	
H	T	
T		
T		
T		
T		

4. Joe's pets are a horse, a canary, and a dog. Connie's pets are a parrot, a canadian goose, and a lamb. Each day Connie and Joe bring one pet to work. Find the probability that both pets will:

   a. have only two legs.
   b. have wings.
   c. eat bird food.

5. Assuming that a bowl contains seven yellow, five red, and eight green chips, what is the probability that:

   a. a red chip is drawn on the first draw?
   b. the first chip drawn will be a red or green chip?

6. If you have three nickels, two dimes, and one quarter in your pocket, what is the probability that:

   a. the first coin you pull out is a nickel?
   b. a dime will be drawn first?
   c. if you remove exactly two of the coins, their value will be 30¢?