Research on routine problem solving (e.g. the typical "story" problem) was reviewed to facilitate the identification and dissemination of promising practices for teaching routine problem solving, and to provide suggestions and directions for further research in the area. Promising teaching practices which were identified included giving attention to processes involved in solving routine problems (e.g., write an equation, make a chart) and devoting time to developing the meanings of mathematical vocabulary and symbols. Areas identified as warranting further research included studies that examine the role of language variables (both syntactic and semantic) in the "decoding" phase of solving a routine problem. Appendix C contains a coded bibliography which may be of great value to researchers in problem solving. (MK)
A REVIEW OF RESEARCH ON SOLVING ROUTINE PROBLEMS IN PRE-COLLEGE MATHEMATICS

Larry Sowder
Jeffrey C. Barnett
Kenneth E. Vos

1979
SUMMARY

This project reviewed the research on routine problem solving (e.g., the typical "story" problem) with an aim toward (a) identifying and disseminating promising practices for teaching routine problem solving and (b) suggesting directions for further research in the area. The investigators surveyed relevant dissertations, journal articles, and files of research studies.

Products of the work include a chapter in a National Council of Teachers of Mathematics yearbook oriented toward teachers, and chapters in two monographs oriented toward educational researchers. Various talks at meetings for teachers and researchers have also been scheduled.

Practices which might improve the teaching of routine problem solving include these:

1. Give attention to processes involved in solving routine problems (e.g., write an equation, make a chart).

2. Devote time to developing the meanings of mathematical vocabulary and symbols.

3. Teach that reading of a mathematical problem is different from reading less technical prose, and requires multiple readings with attention to vocabulary and relationships among variables.

4. Have the learners make up, and solve, their own word problems.

Areas in which further research and development seem warranted include these:

5. Instrumentation is needed for process-analysis studies, both for protocol coding and process measurement.

6. Studies that examine the role of language variables (both syntactic and semantic) in the "decoding" phase of solving a routine problem should contribute to our knowledge of teaching problem solving.

7. Whether different ways of presenting problems--objects, pictures, words--help children of different ages and mental characteristics needs examination.
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A Review of Research on Solving Routine Problems in Pre-College Mathematics

Objective

The aims of the project were as follows:

1. Review and evaluate research on routine problem solving in pre-college mathematics.

2. To identify directions for further research on routine problem solving in mathematics.

3. To make the findings about routine problem solving available to classroom teachers and researchers.

None of the terms in the objectives do not have standard meanings. Here are explanations of how the terms were used in the project:

**Problem**—A problem is a task which does not immediately suggest to the solver a systematic procedure for resolving the task (i.e., an algorithm). Thus, a person's knowledge of algorithms determines whether a given task is a problem for that person. Multiplying with multi-digit numerals would be a problem for third graders but not a problem for most seventh graders.

**Routine vs nonroutine**—The routine nature of a problem may, in a rough way, be defined by the nature of its solution. The solution of a nonroutine problem requires considerable analysis, synthesis, and perhaps some novelty of approach. On the other hand, the solution of a routine problem requires only a relatively small amount of analysis and no unusual insights. The typical verbal problem in pre-college mathematics books is an example of a routine problem. Such problems usually require only the selection of an appropriate computation. In elementary school books, problems described as "challenges" or "brainbusters," are usually nonroutine problems.

Rationale

The rationale for the project was based on four things. First, attention to routine problems in mathematics is important. Effective citizenship as a consumer, as a wage earner, as a taxpayer, requires an ability to solve a myriad of routine problems. Checking purchases, calculating interest costs, evaluating budgets, determining best buys, planning meals—all these are sample routine problems.
Second, unfortunately many students do not solve problems well. The availability of handheld calculators is of no value if a person does not know which buttons to press. It is well known that students are not usually fond of "story" problems, and the first National Assessment of Educational Progress (1975) offered evidence that people are not very proficient in solving routine problems. If existing research evidence suggests that certain procedures for teaching routine problem solving are promising, these should be identified and disseminated.

Third, at least a few such promising practices are identified in the literature but do not appear to be widely known. For example, VanderLinde (1964) found a positive effect on problem solving from spending time on developing meanings for symbols and on studying quantitative vocabulary.

Finally, directions for further research on routine problem solving in mathematics might be identified by an analysis and critique of the existing research.

Procedures

The investigators--Barnett, Vos, and Sowder--identified as many studies of routine problem solving in pre-college mathematics as they could, through searches of dissertation abstracts and ERIC files and through examination of journals deemed most likely to contain such studies. The studies were categorized with an adaptation of Kilpatrick's taxonomy of variables in problem solving research (1978). The most promising dissertations, reports, and articles were studied in full. On the basis of this work, manuscripts were prepared (see Products below). In addition, Dr. Edward Silver agreed to consult with the project and prepared a project paper on memory aspects of routine problem solving (see Appendix A).
Limitations. The project was necessarily limited in scope. Which journals should be concentrated on? How far afield from routine problems in mathematics should we explore? The homely when-do-we-stop-reading-and-start-writing question demanded an answer, dictated by manuscript deadlines in our case. Hence, we cannot claim to have accomplished a comprehensive search. For example, the information-processing approach to routine problem solving was slighted. Work along these lines is currently popular, perhaps too current for a dispassionate critique or for an even moderately thorough survey. Silver's project paper (Appendix A) does draw on work in information processing, however. Another attractive but unexplored body of work was in problem solving in fields other than mathematics—e.g., science, therapy, business. It may well be that important implications for mathematics education lie in studies in such domains.

Products

Serendipitously, the dissemination objective of the project was realized through the post-proposal appearance of plans for two works on problem solving—a monograph on "applied" problem solving (R. Lesh & D. Mierkiewicz, Eds.) and a National Council of Teachers of Mathematics yearbook on problem solving (S. Krulik, Ed.). Proposals and drafts of chapters for these two forthcoming works were prepared and accepted. In addition, Barnett prepared a chapter for a monograph on task variables in mathematical problem solving (G. Goldin & E. McClintock, Eds.), soon to appear. Citations for these publications are summarized in Appendix B. The intent to reach the teacher audience through the Arithmetic Teacher was abandoned on the appearance in the November, 1977, issue of an excellent problem-solving article along the lines planned (Suydam & Weaver); the yearbook chapter served as a teacher-oriented article.
Presentations based on the project have aided, or will aid, in the dissemination of the major findings: Barnett (state meeting of the Illinois Council of Teachers of Mathematics, October, 1978); Sowder (regional Illinois meeting, March, 1979; National Council of Teachers of Mathematics regional meeting, March, 1980); Vos (California Mathematics Council, Southern Section, November, 1979); Barnett, Vos, and Sowder (National Council of Teachers of Mathematics national meeting, April, 1980).

Finally, one product of the project was not planned in the original project but may be of great value to researchers in problem solving: the coded bibliography (see Appendix C). This bibliography will be made available to interested researchers through contacts in the Special Interest Group on Research in Mathematics Education of the American Educational Research Association, and through ERIC.

Selected research recommendations

The following condensed excerpts from the manuscript for the Lesh monograph represent the flavor of our recommendations for further research in routine problem solving in mathematics:

1. For studies of the processes involved in routine problem solving, test instruments that emphasize such process variables must be developed, as well as a protocol scoring-coding scheme that is both elegant and efficient.

2. Studies that attempt to determine the role of syntax and semantic variables in the decoding process in the first stage of problem solving are of particular importance.

3. Another area of needed research is that concerned with the improvement of instruction in reading and its relationship to improved problem solving ability.

4. Although the linear regression model has shown some promise as a research technique in the area of language variables and routine problem solving, it is clear that in its present form it falls short of being able to predict problem solving success. Improvements in the model might include different criteria of importance; it would be helpful for studies to provide data on several dependent measures used with several measures of importance.
5. The relative effects of different formats—words, pictures, objects—for problems should be investigated, particularly as they relate to learner characteristics.

6. What within-format variations make a difference?

7. Studies with positive results should be replicated—for example, Keil's 1964 study, in which student-generated and student-solved problems apparently led to improved problem solving.

8. Would concentrated attention to routine problems give the same striking results as in Bramhall's 1939 study (8 months growth in 2.5 months)?

9. Cooperation among researchers interested in routine problem solving must increase so that common problems, similar instruments, and shared data analysis can be more easily facilitated.

References


Appendix A

Project Paper

A Review of Selected Literature
on the Role of Memory in
Arithmetic and Algebra Word Problem Solving

by
Edward A. Silver
San Diego State University
When I recently mentioned to a colleague that I was writing a paper on the role of memory in solving mathematics problems, she remarked, "Well, it certainly helps!" The fact is, however, that there are three distinct ways in which memory might interact with problem-solving performance. First, information from previous problems might not be available either because the solver has not encoded it or because it has been encoded in a fashion that makes it difficult or impossible to retrieve. In this case, memory would have little or no effect on problem-solving performance. The second way in which memory might interact with problem solving is to have a negative effect. Gestalt psychologists (e.g., Duncker, 1945; Luchins, 1942; Wertheimer, 1959) have examined extensively the instances in which past experience can negatively affect present problem-solving performance. The third way, of course, is the one to which my colleague was referring, in which information gained from previous problem-solving encounters is successfully recalled and used to solve a new problem.

Psychologists have studied the role of memory in general problem-solving activities, and the classical theories have differed greatly in the importance given to memory. The present review has been greatly influenced by the modern information-processing view of problem-solving (Newell & Simon, 1972). The reader who is unfamiliar with information processing psychology can find excellent discussions in Mayer (1977) or Norman (1976).

The human information-processing model can be divided into two general components: perception and memory. Memory
is described in the literature in a variety of ways using various models and analogies. It is not possible in this paper to summarize the various models, but the reader will find an excellent summary of hypothesized memory structures and theories in Gagne and White (1978) and a briefer, but highly readable, summary in Shavelson and Porton (Note 1).

In writing this paper, I have attempted not to duplicate the work of Greeno (1973), in which he applied the general information-processing view of problem solving in reviewing studies relating memory and problem solving. Therefore, this review has generally confined its attention to studies conducted since Greeno's excellent review and to studies that deal in some way with the concerns of mathematical word problem solving. While no claims are made for completeness of the review, it is hoped that the reader will become acquainted with the dominant theories, major results, and possible future directions for research on the role of memory in solving mathematics word problems.

Arithmetic Problem Solving

Although the arithmetic problem-solving competence of children has been of great interest to researchers, their primary focus has been the product (i.e. correct/incorrect answer) rather than the process. In their recent work, Jim Greeno, Joan Heller, and Mary Riley (e.g. Greeno, Note 3; Heller, Note 4; Heller & Greeno, Note 5; Riley, Note 6; Riley & Greeno, Note 7) have applied the information-processing viewpoint to arithmetic problem solving.
Heller and Greeno (Note 5) have developed a model of arithmetic word problem solving that emphasizes semantic processing as the primary component of problem understanding. Although the work of Tom Carpenter, Jim Moser, and their associates at the University of Wisconsin suggests that the Heller-Greeno model is both incomplete and partially incorrect (see Carpenter, Hiebert, & Moser, Note 8; Carpenter & Moser, Note 9), the model deserves careful consideration here since it points to the important role of memory in the solution of arithmetic word problems, especially by young children.

In the Heller-Greeno model, initial understanding of a problem is viewed as a process of constructing an integrated semantic representation of the general quantitative relations in the problem situation. Subsequent selection of the correct operation is based on a direct association between this semantic representation (corresponding to one of three fundamental schemata in the Heller-Greeno model) and the operators (available and associated with the given schema). Carpenter and Moser (Note 9) have suggested that there are more than three fundamental schemata and that children do not appear to reduce all problems to instances of a particular type and apply a single strategy. Nonetheless, their work suggests the fundamental importance of semantic processing in arithmetic word problem solving.

The Heller-Greeno characterization is especially interesting because it contrasts with the earlier information-processing model for word problem solving proposed by Bobrow (1968), in which the problem text is interpreted phrase-by-phrase, using
syntactic function tagging, and directly transformed into an equation or system of several simultaneous equations representing the problem situation. Support for Heller & Greeno's emphasis on semantic processing may be found in studies that collectively suggest that the ability to represent a word problem in the form of an equation or a system of equations is not a necessary condition for successful solution of the problem. For example, several studies (Buckingham & Maclatchy, 1930; Carpenter, Hiebert, & Moser, Note 8) have found that young children can correctly solve some word problems before receiving any formal instruction in equation writing or the translation process. Furthermore, Riley and Greeno (Note 7) reported that second-grade children sometimes found it difficult or impossible to write equations for problems they had already solved. Additional support for the importance of semantic processing comes from reports of successful problem solvers and their characteristics (e.g., Larkin, Note 10; Paige & Simon, 1966; Simon & Simon, 1978); discussion of these reports is found later in this paper.

Therefore, the data suggest that the crucial understandings in the process of solving a problem are those involving "making sense" of the problem situation; i.e. applying to the problem at hand real world or technical domain-specific semantic knowledge that is stored in LTM. The typical instruction given to students who are learning to solve word problems usually encourages such semantic processing, but the usual emphasis is on syntactic procedural mechanisms.
For example, two reasonably well known procedures taught to children are the "Wanted-Given" approach and the "Action-Sequence" approach (Wilson, 1964). Both approaches emphasize a certain amount of semantic processing, in that students are trained to "look for" the wanted-given relationship or the imagined action-sequence embedded in a problem. Nevertheless, the major emphasis of instruction in either procedure is on the composition of an equation, often in a rather rote fashion that seems somewhat independent of the initial semantic processing that is presumed to occur.

Unfortunately, at this time, we know very little about how children "see" word problems. For example, what is it that suggests that a given problem is a subtraction problem, and how is that realization associated with the production of an appropriate equation or operational sequence?

One particularly fruitful line of research would appear to be the identification of the fundamental units of children's understanding of arithmetic concepts and problems. The work of Carpenter and his associates is noteworthy in this regard. Another approach is being taken by Alan Rudnitsky at Smith College. Rudnitsky has been interviewing children to determine the "primitives" (basic elements) of their arithmetic schemata. Such work can be seen as extending the seminal studies of Erlwanger (1975) and Ginsburg (1977) on children's understanding of arithmetic concepts and principles.
Algebra Problem Solving

If a competition were held to determine the most influential and popular memory construct in the area of algebra problem solving, there is no doubt that the current winner would be the notion of "schema" (taken here to be equivalent to notions such as "frame" or "script"). A memory schema, as it is typically conceptualized today, is a cluster of knowledge-concepts, procedures, and relations among these - that defines a more complex and frequently encountered concept or phenomenon.

Schemata have been variously defined and discussed in the current literature on memory models (e.g., Bobrow & Norman, 1975; Rumelhart & Ortony, 1977), but certain common properties are invariant across the different definitions. For example, a schema represents a prototypical abstraction of a complex concept, and the schema is derived from past experience with numerous exemplars of the complex concept. Furthermore, a schema can guide the organization of incoming information into clusters of knowledge that are "instantiations" of the schema (Thorndyke & Hayes-Roth, 1979). The notion of schema was first proposed in connection with algebra word problems by Hinsley, Hayes, and Simon (1977) and has been recently adopted by Bob Davis and his colleagues (Davis, Note 11; Davis, Jockusch, & McKnight, 1978) in discussing algebra problem solving in general.

Hinsley, et al. found that their subjects used two different procedures in solving algebra word problems. One approach involved
a line-by-line direct translation procedure, such as the one proposed by Bobrow (1968) and discussed previously. The second approach involved reading the entire problem before formulating any equations or writing any relations among variables. This second approach - the "schema" approach - emphasized the fundamental importance of semantic knowledge and major decisions occurring early in the comprehension process. The data provided by Hinsley, et al. demonstrate that the "schema" approach is typically used by successful solvers and that the line-by-line procedure is a default process used only if the problem is not successfully matched to one of the solver's available problem category schemata.

Since the Hinsley, et al. study, further evidence of the existence of problem category schemata has been produced involving algebraically naive subjects (Silver, 1977; Silver, Note 12; Silver, Note 13), college students solving physics problems (Larkin, Note 10), and a wide variety of mathematical topics and students of various ages (Davis, Jockusch, & McKnight, 1978). The results of these studies suggest that problem schemata not only exist but are used by successful problem solvers in planning their approach to solving a given problem.

Larkin (Note 10) analyzed the protocols of college students solving rather complex physics problems. She found evidence that successful problem solvers performed an initial "qualitative analysis" before writing any equations. In the early stages of a problem solution, successful solvers constructed representations of the physical situation described in the problem, and they subsequently modified and elaborated the representation.
by including supplementary information required for a complete understanding of the problem situation but not given explicitly in the problem's written statement.

Larkin's protocols provide evidence that successful solvers retrieve from memory preliminary "chunks" or "schemata" of related physics concepts and principles and apply the "chunks" to some aspect of their problem representation. Problem features are elaborated further if necessary in relation to the "chunk" under consideration as the solver attempts to determine the applicability of the knowledge cluster to the problem representation or the solver exits from the problem solution episode. Upon finding a "chunk" that adequately "fits" the problem representation, the solver generates a solution procedure. The findings of Hinsley, et al. and Larkin suggest that problem schemata exist and may play a critical role in solving certain classes of problems, such as algebra word problems. Far less is known about the mechanisms of schema construction; i.e. how students form problem schemata.

Research conducted by Krutetskii (1976), Chartoff (1977), and Silver (1977) has suggested several dimensions along which students might form schemata. Silver asked eighth grade students to sort a set of word problems into groups of problems that were "mathematically related"; Chartoff asked students to rate problem pairs on a continuous scale, ranging from extremely dissimilar to extremely similar. The two investigators independently identified three similarity dimensions perceived by the students: mathematical structure, contextual (cover story) details, and the nature of the question asked. In addition,
Chartoff found that students could recognize generalizations and specializations, and Silver identified a tendency to form clusters of problems on the basis of a common measurable quantity, such as age or weight.

The findings of Chartoff and of Silver, together with the observation by Krutetskii that good problem solvers tend to notice and recall a problem's structure, whereas poor solvers notice and recall only the details of a problem's statement, suggest that students apprehend the important aspects of a problem in different ways. This initial processing is clearly influenced by existing problem schemata, if any exist for the solver, and form the basis for construction of new schemata. Recent work by Silver (Note 12, Note 13) suggests that students cluster recall of problem information abound existing schemata, that they use information from previously solved problems when solving what they perceive to be related problems, and that good and poor problem solvers exhibit qualitatively different clustering and recall performances. These findings will be discussed in more detail in a later section of the paper.

Whereas the investigations cited above involved no direct schema-forming instruction, it is common for algebra word problem instruction to organize problems into "types"; such as "age" problems, "mixture" problems, and "work" problems. The emphasis on "types" may lead to the students' forming problem schemata on the basis of those categories. Hinsley, et al. found that their college subjects did organize algebra word problems into groups that conformed to the stereotypic groupings typically taught to first year algebra students. Nevertheless, it is evident that
not all students who receive the same instruction form the same problem schemata.

Instruction involving problem "types" was prevalent in the Soviet Union in the 1930s and 1940s. The usual pedagogical style involved teaching students to identify problem "types", to recall "model" solutions and to ignore the influence of unfamiliar settings or extraneous data. Russian school psychologists thus had an opportunity to study the process by which a student forms the concept of a problem type. Although their paradigms differ from the modern information-processing viewpoint, their findings are germane.

Kalmykova (1947/1969) reported that the extensive use of model problems tends to reduce the act of problem solving to a choice of conditions of the problem. Menchinskaya (1946/1969) also expressed the view that typification leads students to search their memories for models to "fit" the given problem. She reported that such instruction led students to search their memories to reconstruct a previously encountered problem to serve as a model, rather than examine the problem's conditions effort to construct an appropriate solution.

It would appear that schemata are important especially in the formulation of problems in which the contextual details, the semantics of the cover story, match the underlying problem structure in an expected way. For these problems, if the necessary schema is available to the solver, then a solution may be obtained; otherwise, the line-by-line default procedure must be used.

The data of Chartoff (1977), Krutetskii (1976), and Silver (1977) suggest that schemata might be formed along inefficient dimensions; i.e. with respect to non-structural problem characteristics. The
reports of Kalmykova (1947/1969) and Menchinskaya (1946/1969) suggest that, even if problem schemata are formed along the appropriate dimension of mathematical structure, they may not be useful in solving a problem when the solver fails to analyze carefully the conditions of the problem. Thus, we are reminded that the process of solving a typical algebra problem probably involves not only the recall of an appropriate schema but also the construction of an initial problem representation. The representation provides a framework to which the solver can apply the retrieved schema.

It is not at all uncommon to find first-year algebra students who can solve a problem when it matches exactly the "model" problem they have already solved but who cannot solve a similar problem that they perceive as different. One reasonable explanation for such behavior is the absence of semantic processing of problem information. In other words, the students may be searching his memory for a "model" problem to apply to the given situation and failing to find a "match". The failure may be due to the non-existence of an appropriate schema or the misdirection of the search due to the student's lack of problem representation to guide the search.

Construction of a meaningful problem representation involves the incorporation of semantic knowledge in the problem understanding process. The work of Larkin (Note 10) and Heller and Greeno (Note 5) discussed earlier suggest the critical importance of semantic processing in successful problem-solving performance. Further support for this view may be found in the work of Paige and Simon (1966) who reported that solvers who used a direct translation approach to solving problems containing containing contra-
dictory information were able to obtain "impossible" solutions and not perceive the contradiction. They found that subjects who constructed "auxiliary representations" of the problem situation (e.g. drawings) or who relied on semantic, substantive information in the solution process were considerably more successful at recognizing the presence of incongruities in the problem's conditions. Krutetskii (1976) also reported similar findings in his work with highly capable mathematics students. The findings of the studies reported in this section strongly suggest that future research pay specific attention to the mechanisms of schema construction and problem representation formulation.

Another focus for further research might be the nature of schema composition; i.e. what knowledge is embedded in one's problem schema? It seems reasonable to expect that successful problem solvers may exhibit certain process similarities, such as those discussed by Larkin (Note 10), but that they may possess different knowledge structures. For example, two solvers may be quite successful in solving typical Distance/Rate/Time problems, yet they may have different schemata for such problems. One solver might view these problems as being similar to other typical algebra problems, such as "mixture" and "coin" problems, since they all involve the general structural notion:

\[ \text{Total} = \text{Rate Per Unit} \times \text{Number of Units}. \]
Another solver's schema might include specific details regarding the assumptions of such problems; for example, uniform rate of travel, smoothness of surface, diversity of path, and instantaneous "turn around". Another solver might not have these details explicitly stated, but may operate with "default" values that are equivalent to the necessary assumptions.

In addition to the few examples given above, it is clearly possible to propose other possible individual differences in schema composition. If such differences do exist, it may be fruitful for researchers to examine not only the expert-novice distinctions that have captured our attention for the past decade, but also expert-expert and novice-novice distinctions with respect to processes and with respect to schema composition. By pursuing this line of research, we may learn if there are necessary and sufficient components of problem schemata for various classes of problems, and this information could be useful in guiding instruction.

Of course, not all problem solving behavior can be neatly described in terms of schemata. When subjects have little or no experience in solving a class of problems, the usefulness of schemata is limited. When solving a new problem, a successful problem solver presumably uses information, procedures, and more general notions that have been obtained from previous experience and training. As noted earlier in this paper, Gestalt psychologists have demonstrated that prior experience may have a negative effect in problem solving performance. In recent years, attention has been focused in identifying the circumstances under which positive transfer occurs.

Most of the work in this area has dealt with "puzzle problems", 
such as the Tower of Hanoi or the Missionaries-Cannibals problem. The classic study by Reed, Ernst, and Banerji (1974) suggested that positive transfer occurred only when subjects were told of the relationship between the problems and only when they solved the more difficult problem of the pair first. Kulm and Days (1979) used an information-theoretic approach to study transfer between problems with related structures. They reported that the solution of related problems appeared to help subjects focus on relevant strategies, but that different problem contexts appeared to interfere with transfer.

Silver (Note 12) has suggested that the potential transfer to a new problem is greatly influenced by the solver's initial perception of the problem's relationships to previously solved problems; furthermore, the initial perception is largely a function of what aspects of a problem the solver views to be mathematically relevant to its solution. In other words, the solver must not only recognize that the new problem is related to previously encountered problems but also identify the important mathematical considerations that are relevant to the relationship with previous problems. Of course the solver must also have the necessary information stored in long term memory.

The question of what gets remembered after a problem solution episode has been dealt with at length by Reed and Johnsen (1977) and to a lesser extent by Jacoby (1978). Unfortunately, the literature on this subject is sketchy and largely based on non-mathematical problems. In the next section, we will discuss the few studies that have dealt specifically with long-term retention of mathematical problems.
Individual Differences in Memory and Problem Solving

In studying individual differences in technical problem solving, many researchers have examined the differential processing characteristics of novices and experts (e.g. Chi & Glaser, Note 14; Larkin, Note 10, Simon & Simon, 1978). The data from these studies generally suggest that experts are capable of deeply processing problem information very early in the solution process, thus facilitating solution plan formulation for complex problems and essentially solving "immediately" simple problems.

Since, as Miller, Galanter, and Pribram (1960) have noted, the major source of new plans is old plans, the process differences noted early in the solution are likely indicators of differences in the memories of experts and novices. In fact, the classic work of de Groot (1966) on the memories of skilled and unskilled chess players has stimulated much of the research into expert-novice distinctions. The data from de Groot's study and subsequent studies (Chase & Simon 1973a, 1973b; Frey & Adesman, 1976) demonstrated that skilled chess players were considerably more successful than weaker players at reproducing meaningful chess situations, and that the results were not attributable to superior memory or better guessing on the part of the experts.

Individual differences in memory associated with mathematical problem solving is a largely unexplored area. Krutetskii (1976) noted that skillful problem solvers were able to recall accurately the structure of a mathematics problem even after long periods of time; whereas, poor problem solvers tended to recall, if anything, only the details of the problem's statement.

Recently, Silver (1977, Note 12, Note 13) has reported data
suggesting that good and poor problem solvers demonstrate qualitative differences in their recall of problem information and in their perception of problem relatedness. Regarding the latter, Silver (1977) had students sort a set of word problems into groups that were "mathematically related". The data indicated that good problem solvers tended to group the problems on the basis of mathematical structure, even when they lacked specific techniques designed to solve problems with the given structure. To examine differences in recall, Silver (Note 12, Note 13) asked students to reproduce all they could remember about a mathematical problem and its solution. Recall was examined on several occasions, both before and after presentation of problem solutions, and the data indicated superior structural recall by skillful problem solvers. Furthermore, the data indicated that skillful problem solvers were better able to transfer information from one problem solution to the solution of a structurally related problem (Silver, Note 12) and that skillful problem solvers tended to cluster related information from several problems in terms of problem structure, whereas, less skilled solvers tended not to cluster or to cluster in terms of problem details or cover story (Silver, Note 13).

Much more attention is needed to the issue of individual differences in mathematical problem-solving performance that may be related to memory. As Hunt (1978) has remarked, "Individual differences are undoubtedly due both to differences in peoples' mental machinery and to differences in how they program that machinery to bring it to bear upon the problems they face."
Solving Word Problems: A Final Word

Word problems have been the subject of much research activity by psychologists and mathematics educators. Since word problems require the solver to read and understand a written passage, to select and apply mathematical principles, algorithms or procedures in determining the value of one or more unknown quantities, and to interpret the mathematical solution with respect to the verbal information given in the problem, they represent a point of intersection of the concerns of those interested in mathematical competence and those interested in prose text comprehension. Thus it is fitting that some of the major conclusions of this review parallel results found in the literature on prose text comprehension.

For example, the influence and power of schemata in guiding encoding and retrieval of text information has been demonstrated by Anderson, Reynolds, Schallert, and Goetz (1977) and Mandler and Johnson (1977). Another parallel finding is the existence of differences between good and poor readers' recall of thematically relevant material (Smiley, Oakley, Worthen, Campione, & Brown, Note 2).

The major conclusions of this review are that the critical processes in mathematical word problem solving involve the solver in constructing an accurate representation of the problem and using that representation as a guide in recalling relevant and necessary information, often in the form of schemata, to solve the problem. We have seen that skilled and unskilled solvers demonstrate qualitative differences in the representations they construct and the structures from which they retrieve needed
information. Nevertheless, we have also seen that our knowledge of how memory is involved in mathematical problem solving is very incomplete. Perhaps this review has sharpened a few questions for further study.
Reference Notes


References


Appendix B

Publications Based on the Project


Appendix C

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<td>Clinical (C) or survey/status (S) study</td>
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<td>Process (forward, looking back also)</td>
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Non-letter codes: C=not studied, I=studied

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<td>Parents make a difference in teaching in urban schools. Arithmetic Teacher, October, 1975, 22(6), 410-413. (University Microfilms No. 75-448)</td>
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<td>Loftus, Elizabeth F., &amp; Suppes, Patrick.</td>
<td>Structural variables that determine problem-solving difficulty in computer-assisted instruction. Journal of Educational Psychology, December, 1972, 63(6), 511-542. (University Microfilms No. 72-23, 224)</td>
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PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address  
   Northern Illinois University  
   DeKalb, IL 60115

2. NSF Program  
   RISE

3. NSF Award Number  
   SED 77-19157

4. Award Period  
   From 9/1/77 To 10/31/79

5. Cumulative Award Amount  
   $35,000

6. Project Title  
   A Review of Research on Solving Routine Problems in Pre-College Mathematics

PART II—SUMMARY OF COMPLETED PROJECT (FOR PUBLIC USE)

This project reviewed the research on routine problem solving (e.g., the typical "story" problem) with an aim toward (a) identifying and disseminating promising practices for teaching routine problem solving and (b) suggesting directions for further research in the area.

Practices which might improve the teaching of routine problem solving include these:

1. Give attention to processes involved in solving routine problems (e.g., write an equation, make a chart).
2. Devote time to developing the meaning of mathematical vocabulary and symbols.
3. Teach that reading of a mathematical problem is different from reading less technical prose, and requires multiple readings with attention to vocabulary and relationships among variables.
4. Have the learners make up, and solve, their own word problems.

Areas in which further research and development seem warranted include these:

5. Instrumentation is needed for process-analysis study, both for protocol coding and process measurement.
6. Studies that examine the role of language variables (both syntactic and semantic) in the "decoding" phase of solving a routine problem should contribute to our knowledge of teaching problem solving.
7. Whether different ways of presenting problems—objects, pictures, words—help children of different ages and mental characteristics needs examination.

PART III—TECHNICAL INFORMATION (FOR PROGRAM MANAGEMENT USES)

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2. Principal Investigator/Project Director Name (Include)  
   Larry Sowder

3. Principal Investigator/Project Director Signature  
   [Signature]

4. Date  
   Dec 14, 1979
Appendix D

Project Collaborators

Co-investigators

Jeffrey C. Barnett, Associate Professor, Fort Hays State University, Hays, Kansas

Larry K. Sowder, Associate Professor, Northern Illinois University, DeKalb, Illinois

Kenneth E. Vos, Associate Professor, College of St. Catherine, St. Paul, Minnesota

Consultant

Edward A. Silver, Assistant Professor, San Diego State University, San Diego, California
# ERIC Accession Number Ranges (By Year)

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