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ABSTRACT

This is one in a series of SMSG supplementary and enrichment pamphlets for high school students. This series makes available expository articles which appeared in a variety of ma. ematical periodicals. Topics covered include: (1) Laplace; (2) Ca. Friedrich Gauss; (3) Wolfgang and Johann Bolyai; (4) Evariste Gal. s; and (5) Josiah Willard Gibbs. (MP)

Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

Mathematics is such a vast and rapidly expanding field of study that there are inevitably many important and fascinating aspects of the subject which do not find a place in the curriculum simply because of lack of time, even though they are well within the grasp of secondary school students.

Some classes and many individual students, however, may find time to pursue mathematical topics of special interest to them. The School Mathematics Study Group is preparing pamphlets designed to make material for such study readily accessible. Some of the pamphlets deal with material found in the regular curriculum but in a more extended manner or from a novel point of view. Others deal with topics not usually found at all in the standard curriculum.

This particular series of pamphlets, the Reprint Series, makes available expository articles which appeared in a variety of mathematical periodicals. Even if the periodicals were available to all schools, there is convenience in having articles on one topic collected and reprinted as is done here.

This series was prepared for the Panel on Supplementary Publications by Professor William L. Schaaf. His judgment, background, bibliographic skills, and editorial efficiency were major factors in the design and successful completion of the pamphlets.

PREFACE

Modern mathematics, as distinguished from mathematics in ancient and medieval times, may be said to have begun about the middle of the seventeenth century with the invention of analytic geometry by Descartes and the calculus by Newton. The link between Renaissance mathematics and modern mathematics was effected chiefly by Kepler, Galileo, Desargues, Pascal and Fermat. The calculus of Newton was foreshadowed by the contributions of Cavalieri, Wallis, Barrow, and Roberval.

The calculus as we know it today, or modern analysis, developed from the contributions of Leibniz rather than from those of Newton, largely because of the former's more fortunate notation, although both men are to be regarded as having arrived independently and almost simultaneously at the same basic concepts.

Among the more distinguished successors of Newton who continued to develop the calculus must be mentioned the English mathematicians Gregory, Halley, Taylor, Cotes, and Maclaurin. On the continent, the calculus flourished even more dramatically under the stimulating contributions of John Bernoulli, Euler, Laplace, Lagrange and Legendre, to name the outstanding mathematicians who created the mathematics of the eighteenth century.

Nineteenth century mathematics must be thought of as encompassing two rather distinct eras. The first half of the century was ushered in by the brilliant Laplace, noted among many others things for his classic five-volume treatise, the *Mécanique céleste,* an analytical, mathematical discussion of the solar system. This was an era which saw the elaboration and exploitation of the germinal creations of the hundred and fifty preceding years. The capstone of this development was reached in the contributions of Gauss, often considered to be one of the three greatest mathematicians of all time, the others being Archimedes and Newton.

Toward the midcentury mark, at the end of Gauss' fruitful career, a new era was about to dawn. When Bolyai and Lobachevski introduced non-Euclidean geometry, the spirit of mathematics inevitably changed. Henceforth the outlook was to be entirely different. Modern mathematics, in a narrower sense, began roughly about this time. Increased rigor, more intense generalization, the search for structure, and preoccupation with the logical foundations became the leit-motifs of modern mathematics. Many mathematicians contributed to this vast expansion—to name only a few, we recall the work of Cauchy, Abel, Galois, Jacobi, Lobachevski, Hamiltcn, Sylvester, Weierstrass, Hermite, Kronecker, Riemann, Kummer, Poincaré, Cantor, and Hilbert.

It would appear that contemporary mathematics is the culmination of this development. Lacking adequate historical perspective, however, it is palpably too early to assess the long-range contributions of philosophically-minded mathematicians such as Whitehead, Russell, and Brouwer, logicians such as Quine, Hempel, and Tarski, and generally, creative mathematicians such as Polya, Weyl, von Neumann, Wiener, and a host of others.

The present pamphlet begins with sketches of Laplace and Gauss, respectively. The epoch-making influence of the invention of non-Euclidean geometry is then reflected by the essay on W. and J. Bolyai, together with a note on Lobachevski. Something of the flavor of late nineteenth century mathematics is conveyed, admittedly inadequately, by the essays on Galois and Gibbs. After all, the major purpose of this pamphlet is to familiarize the reader with but a few of the many memorable personalities of these times rather than with their work.

If the reader is intrigued by these few sketches of prominent men who made mathematics, he will doubtless find the following books of interest.

Bell, E. T. *Men of Mathematics.* McGraw-Hill, 1940.

Hooper, Alfred. *Makers of Mathematics.* Random House, 1948; Faber & Faber (London), 1949.

Muir, Jane. *Of Men and Numbers.* Dodd Mead and Company, 1961.

Turnbull, H. W. *The Great Mathematicians.* New York University, 1961.

—William L. Schaaf

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THE AMERICAN MATHEMATICAL MONTHLY

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SCRIPTA MATHEMATICA

E. T. Bell, "*Father and Son, Wolfgang and Johann Bolyai*," vol. 5 (1938), pp. 37–44, 95–100.
I. Malkin, "*On the 150th Anniversary of the Birth Date of an Immortal in Mathematics*," vol. 24 (1959), pp. 79–81.

THE SCIENTIFIC AMERICAN

J. R. Newman, "*Laplace*," vol. 190 (June 1954), pp. 76–81.

The editor also wishes to acknowledge his indebtedness to Mr. T. D. Korm·ichuk's article "A Great Russian Mathematician — Pioneer in Education" (*Soviet Education*, 1963, No. 1, pp. 41–47) for observations on Lobachevski as a teacher.

FOREWORD

To enable the reader to see Laplace in proper perspective to his contemporaries and to the evolution of mathematical thought, it should be pointed out that the leading mathematicians of this period (c. 1730–1830) were Euler, Lagrange, Laplace and Legendre. Euler (1707–1783) rounded out, so to speak, the achievements of his predecessors. Lagrange (1736–1813) extended the calculus and enriched theoretical mechanics. Legendre (1752–1833) developed elliptic integrals and contributed to the theory of numbers. Among the major contributions of Laplace (1749–1827) we must include his work in the calculus, his applications of the calculus to the theory of universal gravitation and planetary motion, and the creation of a calculus of probabilities.

Upon the death of Laplace we may say that mathematics entered a new era, championed by the celebrated Gauss, a worthy successor of Laplace and the type of mathematics which by that time had been elaborated to its utmost heights — the mathematics of Descartes, Newton and Leibniz. It was to be succeeded by a new type of mathematics — new ideas and new methods were about to be created — and Gauss was to bring them to fruition.

LAPLACE

He did great work in mathematics, physics and astronomy but was not an entirely admirable man. While pursuing his scientific career he served three French governments: republic, empire and monarchy.

James R. Newman

Historians of science have rightly called the Marquis de Laplace the Newton of France. He earned the title for his immense work on celestial mechanics, which capped the labors of three generations of mathematical astronomers and produced a universal principle that has been applied to almost every field of physics. Biographers have found Laplace no less interesting — though less impressive — as a person than as a scientist. He was a man of curiously mixed qualities: ambitious but not unamiable, brilliant but not above stealing ideas shamelessly from others, supple enough to be by turns a republican and a royalist in the tempestuous time in which he lived — the era of the French Revolution.

Pierre Simon de Laplace was born at Beaumont-en-Auge, a Normandy village in sight of the English Channel, on March 23, 1749. The facts of his life, of the earlier years especially, are both sparse and in dispute. Most of the original documents essential to an accurate account were burned in a fire which in 1925 destroyed the château of his great great grandson the Comte de Colbert-Laplace; others were lost during World War II in the bombardment of Caen. Many errors about Laplace's life have gained currency: that his father was a poor peasant, that he owed his education to the generosity of prosperous neighbors, that after he became famous he sought to conceal his "humble origins." Recent researches by the mathematician Sir Edmund Whittaker seem to show that whatever Laplace's reasons for reticence about his childhood, poverty of his parents was not among them. His father owned a small estate and was a syndic of the parish; his family belonged to the "good bourgeoisie of the land." One of Laplace's uncles was a surgeon, another a priest. The latter, a member of the teaching staff of the Benedictine Priory at Beaumont, where Laplace had his first schooling, is said to have awakened the boy's interest in mathematics. For a time it was thought that Laplace would follow his uncle's profession as a priest, but at the

University of Caen, which he entered at the age of 16, he soon demonstrated his mathematical inclinations. He wrote a paper on the calculus of finite differences which was published in a journal edited by Joseph Louis Lagrange, the great mathematician, 13 years Laplace's senior, with whom he was later to collaborate.

When Laplace was 18, he set out for Paris. He carried enthusiastic letters of recommendation to Jean le Rond d'Alembert, the most prominent mathematician of France. D'Alembert ignored them; Laplace, not an easy fellow to put off, thereupon wrote him a letter on the general principles of mechanics which made so strong an impression that d'Alembert at once sent for the precocious young man and said: "Monsieur, as you see, I pay little enough attention to recommendations; you had no need of them. You made your worth known; that is enough for me; my support is your due." A short while later d'Alembert procured for him an appointment as professor of mathematics in the *Ecole Militaire* of Paris.

Laplace's rise was rapid and brilliant. He submitted to the Academy of Sciences one memoir after another applying his formidable mathematical capabilities to the outstanding questions of planetary theory. "We have never seen," said a spokesman for the usually imperturbable savants of the Academy, "a man so young present in so short a time so many important memoirs on such diverse and difficult problems."

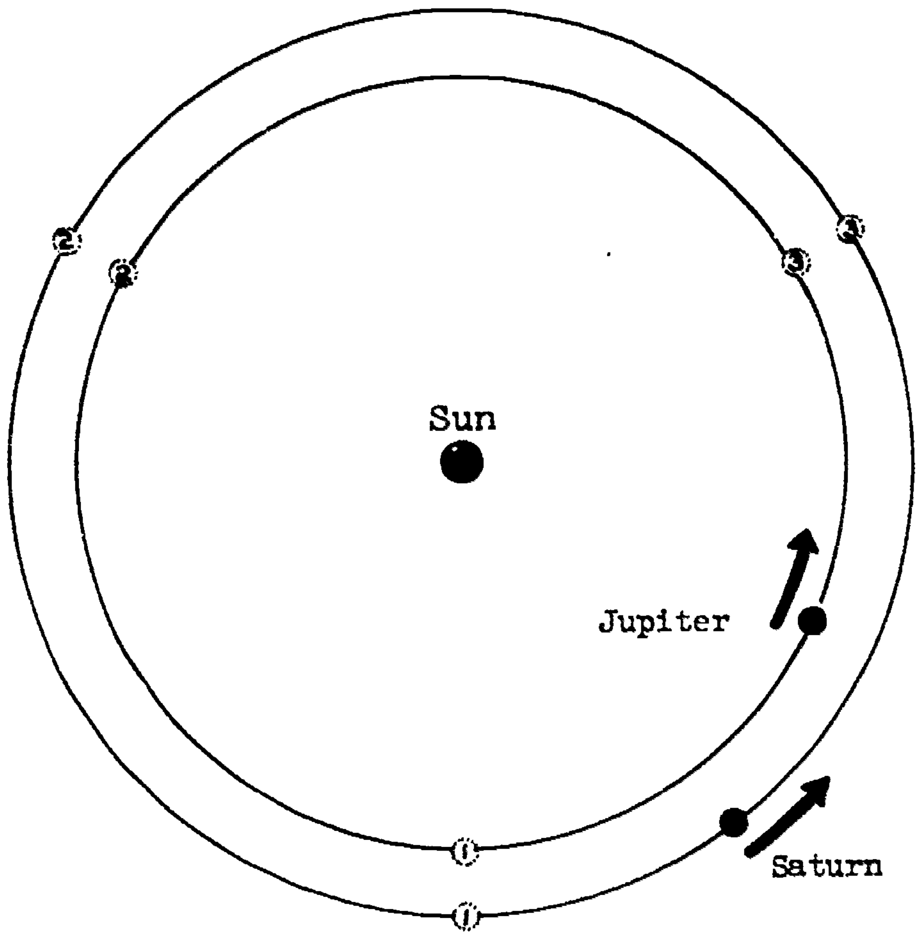
One of the main problems Laplace ventured to attack was the perturbations of the planets. The anomalies of their motion had long been known; the English astronomer Edmund Halley had noted, for instance, that Jupiter and Saturn over the centuries alternately lagged behind and were accelerated ahead of their expected places in a peculiar kind of orbital horse race. The application of Newton's theory of gravitation to the behavior of the planets and their satellites entailed fearful difficulties. The famous three-body problem (how three bodies behave when attracting one another under the inverse square law) is not completely solved today; Laplace tackled the much more complex problem of all the planets cross-pulling on one another and on the sun.

Newton had feared that the planetary melee would in time derange the solar system and the God's help would be needed to restore order. Laplace decided to look elsewhere for reassurance. In a memoir described as "the most remarkable ever presented to a scientific society," he demonstrated that the perturbations of the planets were not cumulative but periodic. He then set out to establish a comprehensive rule concerning these oscillations and the inclination of the planetary orbits. This work bore on the fate of the entire solar system. If it could be shown that disturbances in the machinery were gradually overcome and the status quo restored — a kind of self-healing and self-preserving process analogous to the physiological principle which Walter Cannon has called homeostasis — the future of the cosmic machine, and of its accidental passenger, man, was reasonably secure. If, however, the disturbances tended to accumulate, and each oscillation simply paved the way for a wilder successor, catastrophe was the inevitable end. Laplace worked out a theoretical solution which seemed to fit observation, showing that the outcome would be happy, that the changes of the solar system merely "repeat themselves at regular intervals, and never exceed a certain moderate amount." The period itself is of course tremendously long; the oscillations are those of "a great pendulum of eternity which beats ages as our pendulums beat seconds."

Thus Laplace's theorems gave assurance of the reliability of the stellar clockwork of the universe; its peculiar wobbles and other irregularities were seen to be minor, self-correcting blemishes which in no sense threatened the revolutions of the engine as a whole. Indeed, Laplace regarded the anomalies as a boon to astronomers. He wrote in the *Mécanique céleste:* "The irregularities of the two planets appeared formerly to be inexplicable by the law of universal gravitation; they now form one of its most striking proofs. Such has been the fate of this brilliant discovery, that each difficulty which has arisen has become for it a new subject of triumph — a circumstance which is the surest characteristic of the true system of nature."

Two reservations about this work have to be noted. Laplace's solution did not completely prove the stability of the solar system. His solution would be valid for an idealized solar system undisturbed by tidal friction or other force; but the earth is now known, as it was not in Laplace's day, to be a non-rigid body subject to deformation by tidal friction, which thus acts as a brake on its motion. The effect is very small but acts always in one direction. Consequently we cannot conclude, as Laplace did, that nature arranged the operations of the celestial machine "for an eternal duration, upon the same principles as those which prevail so admirably upon the Earth, for the preservation of individuals and for the perpetuity of the species."

The second point concerns Laplace's failure to mention his indebtedness to Lagrange. Almost everything that Laplace accomplished in physical astronomy owes a debt to Lagrange's profound mathematical discoveries. It is impossible in many instances to separate their contributions. Lagrange was the greater mathematician; Laplace, for whom mathematics was only a means to an objective, was primarily a mathematical physicist and astronomer. Others have severely censured Laplace for his lack of acknowledgment of his collaborator's contributions, but Lagrange, obviously a saintly soul, did not; the two always remained on the best of terms.



PERTURBATIONS of the planets were mathematically described by Laplace and Lagrange. When Jupiter and Saturn are in conjunction (on a line with the sun), one planet speeds up and the other slows down. Numbers indicate they are in conjunction three times in every five circuits of Jupiter.

Laplace's *Mécanique céleste* appeared in five immense volumes between 1799 and 1825. He described its scope as follows:

"We have given, in the first part of this work, the general principles of the equilibrium and motion of bodies. The application of these principles to the motions of the heavenly bodies has conducted us, by geometrical reasoning, without any hypothesis, to the law of universal attraction; the action of gravity, and the motion of projectiles, being particular cases of this law. We have then taken into consideration a system of bodies subjected to this great law of nature; and have obtained, by a singular analysis, the general expressions of their motions, of their figures, and of the oscillations of the fluids which cover them. From these expressions we have deduced all the known phenomena of the flow and ebb of the tide; the variations of the degrees, and of the force of gravity at the surface of the earth; the precession of the equinoxes; the libration of the moon; and the figure and rotation of Saturn's rings. We have also pointed out the cause why these rings remain permanently in the plane of the equator of Saturn. Moreover, we have deduced, from the same theory of gravity, the principal equations of the motions of the planets; particularly those of Jupiter and Saturn, whose great inequalities have a period of above 900 years."

Napoleon, on receiving a copy of the *Mécanique céleste*, protested to Laplace that in all its vast expanse God was not mentioned. The author replied that he had no need of this hypothesis. Napoleon, much amused, repeated the reply to Lagrange, who is said to have exclaimed: "Ah, but it is a beautiful hypothesis: it explains many things."

To mathematicians the work is especially memorable. The Irish mathematician William Rowan Hamilton is said to have begun his mathematical career by discovering a mistake in the *Mécanique céleste*. George Green, the English mathematician, derived from it a mathematical theory of electricity. Perhaps the greatest single contribution of the work was the famous Laplace equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Laplace's expression is a field equation, which is to say it can be used to describe what is happening at every instant of time at every point in a field produced by a gravitational mass, an electric charge, fluid flow and so on. Another way of saying this is that the equation deals with the value of a physical quantity, the potential, throughout a continuum. The potential function u, introduced in the first instance as a purely mathematical quantity, later acquired a physical meaning. The difference between the values of the potential function at two different points of a field measures the amount of work required to move a unit of matter from one of these points to the other; the rate of change of potential in any direction measures the force in that direction.

By giving u different meanings (*e.g.*, temperature, velocity potential and so on) the equation is found to have an enormous range of applications in the theories of electrostatics, gravitation, hydrodynamics, magnetism, sound, light, conduction of heat. In hydrodynamics, where u is the velocity potential (distance squared divided by time), the rate of change of potential is the measure of the velocity of the fluid. The equation applies to a fluid which is incompressible and indestructible; if as much fluid flows out of any tiny element of volume as flows in, the potential function satisfies Laplace's equation. A rough explanation of why this equation serves as an almost universal solvent of physical problems is that it describes a characteristic economy of natural behavior—"a general tendency toward uniformity so that local inequalities tend to be smoothed out." Thus a metal rod heated at one end tends to become of uniform temperature throughout; a solute in a liquid tends to distribute itself evenly.

The *Mécanique céleste* is a book whose difficulties are proportional to its bulk. Laplace made no concession to the reader. He bridged great gaps in the argument with the infuriating phrase "it is easy to see." The U. S. mathematician and astrologer Nathaniel Bowditch, who translated four of the volumes into English, said he never came across this expression "without feeling sure that I have hours of hard work before me to fill up the chasm." Laplace himself, when required to reconstruct some of his reasoning, confessed he found it not at all *"aisé à voir"* how his conclusions had been reached. Nor is it a modest or entirely honorable writing. "Theorems and formulae," wrote Agnes Mary Clerke, the noted historian of astronomy, "are appropriated wholesale without acknowledgment, and a production which may be described as the organized result of a century of patient toil presents itself to the world as the offspring of a single brain." The biographer Eric Temple Bell has remarked that it was Laplace's practice to "steal outrageously, right and left, wherever he could lay his hands on anything of his contemporaries and predecessors which he could use."

For those unable to follow the formidable abstractions of the *Méca-*

nique Laplace wrote in 1796 the *Exposition du système du monde,* one of the most charming and lucid popular treatises on astronomy ever published. In this masterpiece Laplace put forward his famous nebular hypothesis (which had been anticipated by Immanuel Kant in 1755). Its gist is that the solar system evolved from a rotating mass of gas, which condensed to form the sun and later threw off a series of gaseous rings that became the planets. While still in the gaseous state the planets threw off rings which became satellites. The hypothesis has had its ups and downs since Kant and Laplace advanced it. In Laplace's theory revolution in a retrograde direction by a member of the solar system was impossible; yet before Laplace died Sir William Herschel found that the satellites of Uranus misbehaved in this way, and others have since been discovered. Yet the theory was an intellectual landmark, and much of its basic reasoning is still accepted by some cosmologists as valid for astronomical aggregates larger than the solar system.

Another subject upon which Laplace bestowed his attention, both as a mathematician and as a popularizer, is the theory of probability. His comprehensive treatise *Théorie analytique des probabilités* described a useful calculus for assigning a "degree of rational" belief to propositions about chance events. Its framework was the science of permutations and combinations, which might be called the mathematics of possibility.

The theory of probability, said Laplace, is at bottom nothing more than common sense reduced to calculation. But his treatise seemed to indicate that the arithmetic of common sense is even more intricate than that of the planets. No less a mathematician than Augustus De Morgan described it as "by very much the most difficult mathematical work we have ever met with," exceeding in complexity the *Mécanique céleste.*

Laplace's contributions to probability are perhaps unequaled by any other single investigator; nevertheless the *Théorie analytique,* like the *Mécanique,* failed to acknowledge the labors of other mathematicians, on which many of its conclusions depended. De Morgan said of Laplace: "There is enough originating from himself to make any reader wonder that one who could so well afford to state what he had taken from others, should have set an example so dangerous to his own claims."

In a companion work, the *Essai philosophique sur les probabilités,* presenting a nontechnical introduction to the laws of chance, Laplace wrote a passage which is regarded as the most perfect statement of the deterministic interpretation of the universe, a symbol of that happy and confident age which supposed that the past could be described and the future predicted from a single snapshot of the present:

"We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes. The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of this intelligence. Its discoveries in mechanics and geometry, added to that of universal gravity, have enabled it to comprehend in the same analytical expressions the past and future states of the system of the world. Applying the same method to some other objects of its knowledge, it has succeeded in referring to general laws observed phenomena and in foreseeing those which given circumstances ought to produce. All these efforts in the search for truth tend to lead it back continually to the vast intelligence which we have just mentioned, but from which it will always remain infinitely removed. This tendency, peculiar to the human race, is that which renders it superior to animals; and their progress in this respect distinguishes nations and ages and constitutes their true glory."

Together with the great chemist Antoine Lavoisier, Laplace engaged in experiments to determine the specific heats of a number of substances. They designed the instrument known as Laplace's ice calorimeter, which measures heat by the amount of ice melted, a method employed earlier by the Scottish chemist Joseph Black and the German Johann Karl Wilke.

Laplace prospered financially and politically; Lavoisier died on the guillotine. In 1784 Laplace was appointed "examiner to the royal artillery," a lucrative post and one in which he had the good fortune to examine a promising 16-year-old candidate named Napoleon Bonaparte. The relationship was to blossom forth 20 years later, much to Laplace's advantage. With Lagrange, Laplace taught mathematics at the *Ecole Normale*, became a member and then president of the Bureau of Longitudes, aided in the introduction of the decimal system and suggested, in keeping with the reform spirit of the Revolution, the adoption of a new calendar based on certain astronomical calculations.

There is some reason to believe that for a brief period during the Revolution Laplace fell under suspicion; he was removed from the commission of weights and measures. But he managed not only to hold on to his head but to win new honors. He had a knack for riding the waves of his turbulent era. Under the Republic he was an ardent Republican and declared his "inextinguishable hatred to royalty." The day following the 18th Brumaire (November 9, 1799), when Napoleon siezed power, he shed his Republicanism and formed an ardent attachment for the first consul, whom he had helped earlier to form a Commission for Egypt. Almost immediately Napoleon rewarded Laplace with the portfolio of the Interior. The evening of his appointment the new ministry demanded a pension of 2,000 francs for the widow of the noted scholar Jean Bailly, executed during the Terror, and early the next morning Madame Laplace herself brought the first half-year's income to "this victim of the passions of the epoch." It was a "noble beginning," as Laplace's protégé François Arago wrote, but it is hard to discover any other noble accomplishment gracing Laplace's ministerial career. His tenure of office was brief – six weeks. Napoleon wrote tartly of Laplace's shortcomings in his St. Helena memoirs: "He was a worse than mediocre administrator who searched everywhere for subtleties, and brought into the affairs of government the spirit of the infinitely small." But to soothe the hurt of his dismissal the deposed minister was given a seat in the Senate and in 1803 became its Chancellor.

Historians have amused themselves describing Laplace's skill in running with the hare and hunting with the hounds. The neatest evidence appears in his introductions to successive editions of his books. He inscribed the first edition of the *Système du monde* in 1796 to the Council of Five Hundred, and in 1802 prefixed the third volume of the *Mécanique céleste* with a worshipful paean to Napoleon, who had dispersed the Council. Laplace dedicated the 1812 edition of the *Théorie analytique des probabilités* to "Napoleon the Great"; in the 1814 edition he suppressed this dedication and wrote "that the fall of empires which aspired to universal dominion could be predicted with very high probability by one versed in the calculus of chances." Napoleon had made Laplace a count; this gave him the opportunity to join in the 1814 decree of forfeiture banishing the man who had made him a count. When the Bourbons returned, Laplace was one of the first to fall at their feet; for this genuflection he received a marquisate.

Laplace was not an evil or a malicious man. He gave a hand up to many younger scientists. At his country home in Arcueil he surrounded himself with "adopted children of his thought": Arago, an astronomer and physicist; the physicist Jean Biot, noted for his investigations of the polarization of light; Baron Alexander von Humboldt, the celebrated German naturalist and traveler; Joseph Gay-Lussac, the great chemist and physicist; Siméon Poisson, the brilliant mathematician. Biot related that after he had read a paper on the theory of equations, Laplace took him aside and showed him "under a strict pledge of secrecy papers yellow with age in which he had long before obtained the same results." Having soothed his ego, Laplace told the young man to say nothing about the earlier work and to publish his own.

The almost universal admiration for Laplace's scientific genius did not mitigate the widespread distrust inspired by his political adaptability. The more tolerantly cynical of his contemporaries referred to his "suppleness." The stock appraisal is to compare him to the Vicar of Bray. The Vicar, an accommodating man who was twice a Papist and twice a Protestant, is said to have defended the charge of being a time-server by replying: "Not so, neither, for if I changed my religions, I am sure I kept true to my principle, which is to live and die the Vicar of Bray." Laplace could have made similar answer.

About his family life and personal habits there is a strange lack of information. Laplace's marriage with Charlotte de Courty de Romanges, contracted in 1788, was apparently a happy one. They had a daughter and a son, Emile, who rose to the rank of general in the artillery. In later years Laplace passed much of his time at Arcueil, where he had a house next to the chemist Count de Berthollet. There in his study, where the portrait of Racine, his favorite author, hung opposite that of Newton, he pursued his studies with "unabated ardor" and received "distinguished visitors from all parts of the world." He died on March 5, 1827 a few days before his 78th birthday. Illustrious men are required to say deathless things on their deathbeds. Laplace is said to have departed after expressing the reasonable opinion, "What we know is very slight; what we don't know is immense." De Morgan, observing that "this looks like a parody on Newton's pebbles," claimed to have learned on close authority that Laplace's very last words were: "Man follows only phantoms."

FOREWORD

To give in a few words, an adequate impression of Gauss's place in the development of mathematics is frankly impossible. Carl Friedrich Gauss, astronomer and mathematician, was a prolific creator of new mathematical ideas, some of which he did not exploit for one reason or another. Thus he anticipated the invention of the non-Euclidean geometry of Bolyai and Lobachevski, but lacked the moral courage to publish it, fearful of adverse criticism. Not unlike Newton, Gauss wished only to avoid controversy and detested acrimonious quarrels.

We are content to point out that his greatest contributions were in the fields of number theory and analysis, although there was scarcely an area of mathematics that he did not enrich. His name is forever associated with congruences and modular arithmetic; with proofs of the fundamental theorem of algebra; with the constructibility of the regular 17-sided polygon; with the theory of complex numbers; and with the method of least squares, to say nothing of his innovations in astronomy, geodetic surveying, and electromagnetic theory. It is no exaggeration to say that the "Prince of Mathematicians" was in some way connected with nearly every aspect of early 19th century mathematics.

The definitive biography of Gauss is the substantial and scholarly volume by G. Waldo Dunnington titled *Carl Friedrich Gauss, Titan of Science.*

CARL FRIEDRICH GAUSS

B. F. Finkel

Tn versatile and prolific mathematician, Carl Friedrich Gauss, was born at Brunswick, Germany, April 30*, 1777, and died at Göttingen on February 23, 1855. His father was a bricklayer and was desirous of profiting by the wages of his son as a laborer, but young Gauss's talents attracted the attention of Bartels, afterwards professor of mathematics at Dorpat, who brought him to the notice of Charles William, Duke of Brunswick. The duke undertook to educate the boy and sent him to the Caroline college in 1792. By 1795 it was admitted alike by professors and pupils that he knew all that the professors could teach him. It was while at this school that he investigated the method of least squares, and proved by induction the Law of Quadratic Reciprocity. He gave the first rigorous proof of this law and succeeded in discovering eight different demonstrations of it.† While at Caroline college, Gauss manifested as great an aptitude for language as for mathematics, a very general characteristic of eminent mathematicians.

In 1795 Gauss went to Göttingen, as yet undecided whether to pursue philology or mathematics. While at Göttingen he studied mathematics under Abraham Gotthelf Kästner, who was not a very inspiring teacher and who is now chiefly remembered for his *History of Mathematics*, 1796, and by the fact that he was a teacher of the illustrious Gauss. In 1796 he discovered a method of inscribing in a circle a polygon of seventeen sides, and it was this discovery that encouraged him to pursue mathematics rather than philology—a rather insignificant incident to be fraught with such stupendous consequences—consequences materially affecting our present progress in mental and material development.

A detailed construction of this problem by elementary geometry was first made by Pauker and Erchinger.

Gauss worked quite independently of his teachers at Göttingen, and it was while he was there as a student that he made many of his greatest

*Cf. *Britannica Encyclopedia* and *Century Dictionary*.
†For Gauss's third proof as modified by Dirichlet, see Mathews's *Theory of Numbers*, pages 38–41.

discoveries in the theory of numbers, his favorite subject of investigation. Among his small circle of intimate friends was Wolfgang Bolyai, the discoverer of non-Euclidean geometry.

In 1798 Gauss returned to Brunswick, where he earned a livelihood by private tuition. Later in the year he repaired to the University of Helmstadt to consult the library, and it was while here that he made the acquaintance of Pfaff, a mathematician of great power. Laplace, when asked who was the greatest mathematician in Germany, replied, Pfaff. When the questioner said he should have thought Gauss was, Laplace replied: "Pfaff is the greatest mathematician in Germany; but Gauss is the greatest in all Europe."*

In 1799 Gauss published his demonstration that every algebraical equation with integral coefficients has a root of the form $a+bi$, a theorem of which he gave three distinct proofs. In 1801, he published *Disquisitiones Arithmeticae,* a work which revolutionized the whole theory of numbers. "The greater part of this most important work was sent to the French Academy the preceding year, and had been rejected with a sneer which, even if the work had been as worthless as the referees believed, would have been unjustifiable."† Gauss had written far in advance of the judges of his work, and so the recognition of its merits had to wait until the mathematical world came in sight of this splendid creation. Gauss was deeply hurt because of this unfortunate incident, and its was partly due to it that he was so reluctant to publish his subsequent investigations.

The next important discovery of Gauss was in a totally different department of mathematics. The absence of a planet between Mars and and Jupiter, where Bode's Law would have led observers to expect one, had long been remarked, but not until 1801 was any of the numerous groups of minor planets which occupy that space observed. On the first of January, 1801, Piazzi of Palermo discovered the first of these planets, which he called Ceres, after the tutelary goddess of Sicily.‡ While the announcement of this discovery created no great surprise, yet it was very interesting, since it occurred simultaneously with a publication by the philosopher Hegel, in which he severely criticised astronomers for not paying more attention to philosophy, a science, said he, which would have shown them at once that there could not possibly be more than seven planets, and a study of which would have prevented, therefore, an absurd waste of time in looking for what in the nature of things could

*Cajori's *A History of Mathematics.*
†Ball's *A Short History of Mathematics.*
‡Young's *General Astronomy*, edition of 1898, page 368.

not be found. This is only one instance of the many refutations of dogmatic statements of philosophers who presage nature's laws without confirming them by actual observations.

However, the new planet was seen under conditions so unfavorable as to render it almost impossible to forecast its orbit. Fortunately the observations of the planet were communicated to Gauss. Gauss made use of the fact that six quantities known as elements completely determine the motion of a planet unaffected by perturbations. Since each observation of a planet gives two of these, e.g., the right ascension and declination, therefore three observations are sufficient to determine the six quantities and therefore to completely determine the planet's motion. Gauss applied this method and that of least squares and his analysis proved a complete success,* the planet being rediscovered at the end of the year in nearly the position indicated by his calculations. This success proved him to be the greatest of astronomers as well as the greatest of "arithmeticians."

The attention excited by these investigations procured for him in 1807 the offer, from the Emperor of Russia, of a chair in the Academy of St. Petersburg. But Gauss, having a marked objection to a mathematical chair, by the advice of the astronomer Olbers, who desired to secure him as director of a proposed new observatory at Göttingen, declined the offer of the emperor and accepted the position at Göttingen. He preferred this position because it afforded him an opportunity to devote all his time to science. He spent his life in Göttingen in the midst of continuous work and after his appointment never slept away from his observatory except on one occasion when he accepted an invitation from Humbolt† and attended a scientific congress at Berlin, in 1828. The only other time that he was absent from Göttingen was in 1854, when a railroad was opened between Göttingen and Hanover.‡

For some years after 1807 his time was almost wholly occupied by work connected with his observatory. In 1809 he published at Hamburg his *Theoria Motus Corporum Coelestium*, a treatise which contributed largely to the improvement of practical astronomy, and introduced the principle of curvilinear triangulation. In this treatise are found four formulae in spherical trigonometry, commonly called "Gauss's Analogies," but which were published somewhat earlier by Karl Brandon Mollweide of Leipzig, 1774–1825, and still earlier by Jean Baptiste Joseph Delambre (1749–1822). §On observations in general (1812–1826) we

*Berry's *A Short History of Astronomy*.
†*Britannica Encyclopedia*, 9th edition, Vol. X, page 104.
‡Cajori's *A History of Mathematics*.
§Cajori's *A History of Mathematics*.

have his memoir, *Theoria Combinationis Observationum Erroribus Minimis Obnoxia,* with a second part and supplement.

A little later he took up the subject of geodesy and from 1821 to 1848 acted as scientific adviser to the Danish and Hanoverian governments for the survey then in progress. His papers of 1843 and 1866, *Ueber Gegenstände der höhern Geodäsie,* contain his researches on the subject.

Gauss's researches on *Electricity and Magnetism* date from about the year 1830. In 1833 he published his first memoir on the theory of magnetism, the title of which is *Intensitas vis Magneticae Terrestris ad Mensuram Absolutam Revocata.* A few months afterward he, together with Weber, invented the declination instrument and bifilar magnetometer. The same year they erected at Göttingen a magnetic observatory free from iron (as Humbolt and Arago had previously done on a smaller scale), where they made magnetic observations and showed in particular that it was possible and practical to send telegraphic signals, having sent telegraphic signals to neighboring towns. At this observatory he founded an association called the *Magnetische Verein,* composed at first almost entirely of Germans, whose continuous observations at fixed times extended from Holland to Sicily. The volumes of their publications, *Resultate aus der Beobachtungen des Magnetischen Vereins,* extend from 1833 to 1839. In these volumes for 1838 and 1839 are contained two important memoirs by Gauss, one on the general theory of earth-magnetism, the other on the theory of forces attracting according to the inverse squares of the distance. Like Poisson, he treated the phenomena in electrostatics as due to attractions and repulsions between imponderable particles. In electro-dynamics he arrived, in 1835, at a result equivalent to that given by W. E. Weber in 1846, viz: that the attraction between two electrified particles, e and e', whose distance apart is r, depends on their relative motion and position according to the formula

$$ee' r^{-2}[1 + (rd^2r - \tfrac{1}{2}dr^2)^2 c^{-2}].$$

Gauss, however, held that no hypothesis was satisfactory which rested on a formula and was not a consequence of physical conjecture, and as he could not form a plausible physical conjecture he abandoned the subject. Such conjectures were proposed by Riemann in 1858, and by C. Neumann and E. Betti in 1868, but Helmholtz in 1870, 1873 and 1874 showed that these conjectures were untenable.

In 1833, in a memoir on capillary attraction, he solved a problem in the Calculus of Variation, involving the variation of a certain double integral, the limits of integration also being variable; it is the earliest example of the solution of such a problem.

In 1846 was published his *Dioptrische Untersuchungen*, researches on optics, including system of lenses.

As has already been observed, Gauss's most celebrated work in pure mathematics is the *Disquisitiones Arithmeticae*, and a new epoch in the theory of numbers dates from the time of its publication. This treatise, Legendre's *Théorie des nombres* and Dirichlet's *Vorlesungen über Zahlentheorie* are the standards on the Number Theory.

In this work Gauss has discussed the solution of binomial equations of the form $x^n = 1$, which involves the celebrated theorem that the only regular polygons which can be constructed by elementary geometry are those of which the number of sides is $2^m(2^n + 1)$, where m and n are integers and $2^n + 1$ is a prime. These equations are called *cyclotomic equations*, when n is prime and when they are satisfied by a primitive nth root of unity.

Gauss developed the theory of ternary quadratic forms involving two indeterminates, and also investigated the theory of determinants on whose results Jacobi based his researches on this subject.

The theory of Functions of Double Periodicity had its origin in the discoveries of Abel and Jacobi. Both arrived at the Theta Functions which play so large a part in the Theory of Double Periodic Functions. But Gauss had independently and at a far earlier date discovered these functions and their chief properties, having been led to them by certain integrals which occurred in the *Determinatio Attractionis*, to evaluate which he invented the transformation now associated with the name of Jacobi. In the memoir, *Determinatio Attractionis*, it is shown that the secular variations, which the elements of the orbit of a planet experience from the attraction of another planet which disturbs it, are the same as if the mass of the disturbing planet were distributed over its orbit into an elliptic ring in such a manner that equal masses of the ring would correspond to arcs of the orbit described in equal times.

Gauss's collected works have been published by the Royal Society of Göttingen, in seven 4-to volumes, 1863–1871, under the editorship of E. J. Schering. They are as follows: (1) The *Disquisitiones Arithmeticae*, (2) *Theory of Numbers*, (3) *Analysis*, (4) *Geometry and Method of Least Squares*, (5) *Mathematical Physics*, (6) *Astronomy*, and (7) *Theoria Motus Corporium Coelestium*. These include besides his various works and memoirs, notices by him or many of these, and of works of other authors in the *Göttingen gelehrte Anzeigen*, and a considerable amount of previously unpublished matter, *Nachlass*. Of the memoirs in pure mathematics, comprised for the most part in volumes ii, iii and iv (but to these must be added those on *Attraction* in volume v), there is not one which has not signally contributed to the branch of mathematics to which it belongs, or which would not require to be carefully analyzed in a history of the subject.

His collected works show that this wonderful mind had touched hidden laws in Mathematics, Physics and Astronomy, and every one of the subjects which he investigated was greatly extended and enriched thereby. He was also well versed in general literature and the chief languages of modern Europe, and was a member of nearly all the leading scientific societies in Europe.

He was the last of the great mathematicians whose interests were nearly universal. Since his time, the literature of most branches of mathematics has grown so rapidly that mathematicians have been forced to specialize in some particular department or departments.

Gauss was a contemporary of Lagrange and Laplace, and these three, of which he was the youngest, were the great masters of modern Analysis. In Gauss that abundant fertility of invention which was marvelously displayed by the mathematicians of the preceding period, is combined with an absolute rigorousness in demonstration which is too often wanting in their writings. Lagrange was almost faultless both in form and matter, he was careful to explain his procedure, and, though his arguments are general, they are easy to follow. Laplace, on the other hand, explained nothing, was absolutely indifferent to style, and, if satisfied that his results were correct, was content to leave them either without a proof or even a faulty one. Many long and abstruse arguments were passed by with the remark, "it is obvious." This led Dr. Bowditch, of Harvard University, while translating Laplace's *Mécanique Céleste,* to say that whenever he came to Laplace's "it's obvious," he expected to put in about three weeks of hard work in order to see the obviousness. Gauss, in his writings, was as exact and elegant as Lagrange, but even more difficult to follow than Laplace, for he removed every trace of the analysis by which he reached his results, and even studied to give a proof which, while rigorous, should be as concise and synthetical as possible. He said: "Mathematics is the queen of the sciences, and arithmetic is the queen of mathematics," and his *Disquisitions* confirms the statement.

Gauss had a strong will, and his character showed a curious mixture of self-conscious dignity and child-like simplicity. He was little communicative, and at times morose.

He possessed a remarkable power of attention and concentration.

and in this power lies the secret of his wonderful achievements. As a proof of this power of attention we quote from Carpenter's *Mental Physiology*. Gauss, while engaged in one of his most profound investigations, was interrupted by a servant who told him that his wife (to whom he was known to be deeply attached, and who was suffering from a severe illness) was worse. "He seemed to *hear* what was said, but either did not comprehend it or immediately forgot it, and went on with his work. After some little time, the servant came again to say that his mistress was much worse, and to beg that he would come to her at once; to which he replied: 'I will come presently'. Again he lapsed into his previous train of thought, entirely forgetting the intention he had expressed, most probably without having distinctly realized to himself the import either of the communication or of his answer to it. For not long afterwards when the servant came again and assured him that his mistress was dying and that if he did not come immediately he would probably not find her alive, he lifted up his head and calmly replied, 'Tell her to wait until I come', — a message he had doubtless often before sent when pressed by his wife's request for his presence while he was similarly engaged."

In bringing this imperfect sketch to a close, we wish to call attention to the fact that it has been conclusively shown that Gauss was not the first to give a satisfactory representation of complex numbers in a plane, this having been first satisfactorily done by Casper Wessel in 1797, though Wallis had made some use of graphic representation of complex numbers as early as 1785. Gauss needs no undue credit to make him famous — the writing alone of any one of the seven of his collected works being sufficient to rank him among the great mathematicians of his day. However, it was Gauss who in 1831, "by means of his great reputation, made the representation of imaginary quantities in the 'Gaussian plane' the common property of all mathematicians." He brought also into general use the sign i for $\sqrt{-1}$, though it was first suggested by Euler. He called $a + bi$ a *complex number* and called $a^2 + b^2$ the *norm*.

FOREWORD

Perhaps no other mathematical creation has had such a profound effect upon the subsequent development of mathematics as the invention of non-Euclidean geometry. This breakthrough was to the history of mathematics what the advent of Copernicus' theory was to the history of astronomy—a truly revolutionary event.

Three names will always be associated with the creation of non-Euclidean geometry: Gauss, Bolyai, and Lobachevski. All three arrived at the same general results independently. Gauss, lacking courage to announce his conclusions, presumably anticipated the other two; he never published his ideas on the subject, nor made any public claim to the invention of non-Euclidean geometry. Nevertheless he encouraged others to study the question of constructing a consistent non-Euclidean system.

Roughly speaking, not only did Lobachevski and Bolyai achieve this momentous result independently of each other, but almost simultaneously. Sometimes Lobachevski is referred to as the creater of non-Euclidean geometry rather than Bolyai; the only justification would be the fact that Lobachevski announced his findings (with some misgivings) in 1826, while Bolyai published his version in 1833.

More important than the matter of priority is the profound and far-reaching impact of their contribution upon mathematics. It is not too much to say that, together with the abstract algebra developed by Boole and others about the same time, the acceptance of non-Euclidean geometry changed the very foundations upon which mathematics was to rest. The contemporary spirit of mathematics, known as axiomatics, is the direct outcome of this bold revolutionary step in thought.

Inasmuch as no essay on Lobachevski has been included in the present booklet, it is desirable to record a few observations about the brilliant Russian who was ahead of his time in other areas as well as in mathematics. Lobachevski spent some twenty years working out his hyperbolic geometry, or, as he called it, *pangeometry*. Although published in 1826–1829, it attracted little or no attention at first. Gauss did not hear of it until 1840. Ironically, some of Lobachevski's contemporaries (e.g., Ostrogradskii) did not comprehend the new geometry nor did they appreciate its significance. Even his algebra and geometry textbooks were rejected. But he never lost heart and was firmly convinced of the value of the geometry he had created.

Not so well known is the fact that Lobachevski was also a dedicated educator and a gifted teacher. For years he devoted his energies to improving curricula and teaching methods in the schools and university in the Kazan district. His lectures were distinguished for their depth and logical organization. He could be profound or captivating, depending upon the subject. He lectured slowly and clearly, writing formulas meticulously on the blackboard, offering his own ideas rather than repeating the words of others, inviting students to acquire first hand acquaintance with mathematical ideas. Drawing upon a vast background of erudition, his colorful lectures on physics and astronomy were deservedly popular. Never irritated, always friendly, he was a warm and sensitive tutor, with a gift for discovering talent and nurturing it where found.

FATHER AND SON
WOLFGANG AND JOHANN BOLYAI

E. T. Bell

The independent invention by Newton and Leibniz of the calculus is but one of several spectacular coincidences in the history of mathematics, where if the facts were not definitely known it would be not at all unreasonable to impute plagiarism to one or other of two practically simultaneous discoverers.

When three men starting from different points arrive at substantially the same goal in the middle of a desert without being aware that they were not alone in their explorations, the coincidence of their meeting takes on the aspect of a miracle. Yet that is what happened in the creation of non-Euclidean geometry by Gauss, Lobachevski, and Johann Bolyai. Gauss, had he cared to hasten his steps a little, might have arrived years before either of his rivals; Lobachevski probably was putting the finishing touches (in 1825) to his work at the same time that Bolyai first clearly saw his way through to the end. Lobachevski and Bolyai were totally unaware of one another's existence, and Gauss had no suspicion that either of the others had completed the task which he had begun before he was twenty.

Wolfgang Bolyai, the father of Johann, appears to have been one of the very few men for whom Gauss had a real affection. Although many of Gauss' letters to his regular correspondents belie the legend that he was the austere prince of mathematicians and nothing more, in only those to the elder Bolyai does he let himself go—occasionally. For this reason, if no other, Wolfgang Bolyai is a figure of interest to mathematicians. With the exception of his work on the foundations of geometry, his somewhat labored mathematical writings are barren of inspiration and have survived only because the appendix of twenty-six pages—"the most wonderful twenty-six pages in the history of mathematics," they have been called—written by his son Johann, was bound up with the rest. But it is unfair to say, as is sometimes said, that the elder Bolyai deemed his son's work only worthy of an afterthought in his own voluminous writings; the contrary is the fact. The father was immensely proud of his son and fully appreciated the magnificence of his work. They fell out, but their estrangement was not due to any lack of understanding on the father's part. This will appear as we proceed. We shall first try to give something of the flavor of Wolfgang Bolyai's friendship with Gauss by quoting from their correspondence.

The Hungarian mathematician, poet, and dramatist, Wolfgang Bolyai, was born on February 5, 1775, at Szekler-Land, Transylvania, and died at Maros-Vásárhely at the age of eighty-one. Feeling himself gifted as a youth, he had some difficulty in choosing a career, but finally, at the age of twenty-one, he accompanied his friend and patron, Baron Kemény, to Germany where both studied for a short time at the University of Jena.

"It was then [1796]," Bolyai writes, "while walking along the banks of the Saale that I, with my limited and haphazard knowledge, took the path I find myself still following in my old age. . . . In the autumn of 1796 we went on to Göttingen, where we were received by Kästner and Lichtenberg, and there I became acquainted with Gauss [who had been studying at Göttingen since the autumn of 1795], whose friend I still am today; but how far I am from being able to compare myself with him. He was very modest and very reserved; one would have had to live with him, not three days, as with Plato, but three years, to realize how great he was. What a misfortune it was for me that I did not know how to open this silent book that flaunted no title; I had no idea of the extent of his knowledge, and he, seeing my tastes, esteemed me much without knowing how inconsiderable I was. What united us was our common passion (which did not show itself openly) for Mathematics and our similar temperaments; so that often, each occupied with his own thoughts, we would walk together for hours without saying a word."

Bolyai outlived his friend a year Writing in 1856, the year of his own death, he recalls his first meeting with Gauss, sixty years before, and tells how their friendship ripened.

". . . I left then [1796] for Vienna . . . something made me go first to Jena. . . . I did not follow the mathematical course but, walking alone along the banks of the Saale recalling my scattered memories, I began, without books, to speculate on the principles of Mathematics, and as I hoped to do something on my own account, it was on the banks of the Saale that first took root those ideas which I later sought to refine and extend. From Jena I went to Göttingen; and it was there, at the house of the kindly professor Seyffer, that I saw Gauss *for the first time*, and I, with my little knowledge, had the audacity to deliver him a lecture (resounding like an empty keg) on the shallowness of the treatment of the principles of Mathematics, as concerns multiplication, division, raising to powers, . . ., the straight line, the plane, equations from their various points of view, and so on. Later, we used to encounter one another on the ramparts, both of us alone; we walked together, we made appointments, and presently we enlisted together, as brothers, under the banner of truth. After that it was to me that he most frequently confided his profound works; he never spoke of them in advance, or even when they were finished; only once did I perceive in him a restrained satisfaction, and that was when he gave me as a souvenir a little tablet on which he had inscribed the calculations for page 662 of the *Disquisitiones Arithmeticae* concerning the polygon of 17 sides.

"We walked together to Brunswick to see his parents; his mother asked me if her son would ever be anything worth while; on hearing my reply: *the first mathematician of Europe!* she burst into tears."

Gauss at the time was nineteen, Wolfgang Bolyai twenty-one. From his self-portrait it is evident that Bolyai was not afflicted with conceit. The best thing he ever did was his son Johann, and he was fully aware of the fact. For himself, he asked to be buried without a marker on his grave. An apple tree would do, he said, to remind him in his long sleep of the three apples of history: the apple which Eve foisted off on Adam; the apple which Paris awarded Helen of Troy as the fairest of the fair, and the apple whose fall inspired Newton to his law of universal gravitation. The first two, he remarked, made earth a hell; the third restored the earth to its dignity among the heavenly bodies. His wish to have no monument has been respected. Today father and son lie in the same grave, their differences forgotten; the pyramid which marks the grave bears only the son's name, Bolyai János, 1802–1860.

After leaving Göttingen on June 9, 1799, Wolfgang Bolyai returned to Hungary, where from 1804 to 1852 he taught in the Reformed Evangelical College at Moras-Vásárhely, holding the professorship of mathematics, physics, and chemistry. His memories of Göttingen and the walks with Gauss had "impassioned" him for rigorous mathematics. They had often discussed the foundations of geometry. Bolyai evidently made more of an impression on Gauss than he modestly admits: "Bolyai," Gauss declared, "is the only one who ever entered completely into my metaphysical ideas concerning mathematics." On reaching home, Bolyai wrote to Gauss (September 11, 1799) announcing his safe arrival. Three months later Gauss replied, expressing his great regret that he had not taken advantage of their recent proximity to go more thoroughly into his friend's work on the first principles of geometry. "Let me know about your work soon," he concludes.

This was in September 1799, almost at the close of the Eighteenth Century. The date is rather important in the history of mathematics because it fixes definitely the epoch at which Gauss saw the light dawn on non-Euclidean geometry. He had at least begun to suspect that Euclid's parallel postulate is incapable of proof, and he had taken the first steps, at the age of 21 or less (actually at sixteen), toward constructing a non-Euclidean geometry. The elder Bolyai was still in the dark. The work of which he had spoken to Gauss was a supposed proof of the unprovable parallel postulate. This came out almost exactly five years later when (1804) Bolyai sent Gauss his manuscript and asked for a frank opinion of its merits.

In the meantime, however, he had sent Gauss the news of something more important, the birth of his son Johann on December 15, 1802. His letter glows with paternal pride, as is customary over the birth of an heir, but he probably had no idea that the infant before so many years had passed was to succeed easily, splendidly, where he himself was foredoomed to life-long failure. The problem obsessed him. "As I was not satisfied with my attempts to prove the parallel postulate," he writes, "and as the fruitless efforts of so many years had destroyed my peace of mind, my fire for mathematics was quenched and I turned to poetry." But poetry was not his only solace. As he later confessed to his more fortunate son, "If long ago I had succeeded in reaching some conclusion in the matter of the parallel postulate, I would never have busied myself with the construction of stoves nor the art of poesy, and I would have been a better man and a better father of a family." The marriage of poetry with stoves must be the most singular misalliance in the history of the arts.

Bolyai had put high hopes on his letter of 1804 to Gauss. The Austrians at the time were bullying the Hungarians; to circumvent the censorship of the Black Chamber of the Austrian Government, Bolyai forwarded his manuscript in several small packages—the whole might have excited suspicion. He begins by saying that circumstances had interrupted his researches on the parallel postulate for three years, but that he has now resumed them with a view to teaching his pupils what he had found. "I can find no mistakes in it," he says; "check the correctness of it and write me as soon as possible; tell me whether I have expressed myself badly or whether it is too condensed. . . . If you deem this little work worth the trouble, send it to some Academy capable of judging and giving it the seal of its approval. I am prepared to hear an unfavorable verdict. I am not disturbed because I have not yet published this [so that I might live at peace among my numerous critics].

You know, don't you, what Hamlet said: 'The spurns that patient merit of th' unworthy takes.' "

Ten weeks later Gauss replied. He lets his friend down kindly. "I have read your memoir with the greatest interest and attention and I was highly delighted at the profound perspicacity you evince. But it is not worthless encomiums you wish; such, up to a certain point, might also seem partial, for the course of your ideas is very similar to that which I myself formerly followed in seeking the solution of this Gordian knot, a quest which is still in vain. It is only my sincere judgment, without subterfuge, that you want. Here it is: your method does not satisfy me *at all*."

Gauss then goes on to point out the fatal flaw in Bolyai's ingenious attempt to prove the parallel postulate. "You asked me for my honest opinion," he concludes; "I have given it you, and I repeat again that it would afford me the keenest pleasure to see you surmount these difficulties."

But the elder Bolyai was never to overcome the obstacles to a proof of the parallel postulate; they were insurmountable, and Bolyai lacked the imagination to see that a consistent geometry could be constructed independently of Euclid's assumption. This triumph, as has been said, was reserved for Wolfgang's son Johann. As an interesting coincidence we note that Abel and Johann Bolyai were born in the same year, 1802, and that both, in their youthful impatience, fell foul of Gauss and both misjudged him.

Johann's early maturity as a mathematician is even more astonishing than Pascal's. A mystic might be tempted to think that the son "remembered" all his father had ever known and had solved in his prenatal sleep the problems which had tormented his father and were to baffle him to the end of his long life. The boy showed a marked talent for music, which the father encouraged, and soon became an accomplished violinist. But of mathematics he knew absolutely nothing — not even common addition — till he was nearly ten. Then his father began teaching him the rudiments. By the age of thirteen Johann had mastered the calculus, differential and integral, and was so proficient in analytical mechanics that he astonished the examiners who tried in vain to stump him. The father nearly burst with pride. Describing his son's rapid advance, Wolfgang declared that Johann's "progress in mathematics was like a streak of lightning. He did not wait to be shown the proofs of theorems but gave them himself first. He leapt at me like a devil and begged me to go ahead faster."

Moved by paternal pride and the certain knowledge that Johann was a mathematical genius of the first rank — an estimate in which Gauss later concurred privately, so that it could do the young talent no good — Bolyai wrote to the friend of his youth, begging Gauss to take the fourteen-year-old boy into his own household for two years, in order that the young recruit might have a training worthy of his promise such as only the veteran of a hundred successful campaigns could give. Gauss never replied to this letter. After all it was a pretty steep favor for even an old friend to ask, and Gauss' own sons had caused him trouble enough — there is one distracted letter in which he pours out his anxiety over Eugene whose whereabouts he has been seeking vainly to discover for weeks. So far as is known the correspondence between Gauss and his old friend was not resumed till 1831, when Bolyai again wrote, sending Gauss tangible evidence that Johann, then a dashing army officer of twenty-nine, had amply fulfilled his early promise and was indeed a great mathematician.

Johann himself has left a record of how he became interested in the riddle of parallels. "My father pointed out to me the great gaps in the theory of parallels. He made me see that although he had done far better than his predecessors, he had nevertheless found nothing satisfying or suitable, in the sense that none of his new postulates, each of which moreover sufficed to prove Euclid's parallel postulate rigorously, possessed the necessary degree of geometrical obviousness, however admissible and justified it may have appeared at the first glance. He declared, but without demonstration, that it is impossible to prove Euclid's postulate, and fearing that I might fritter away my whole life vainly and fruitlessly over it, he strove by all possible means to deflect me from the continuation of my researches and to inspire me with a horror of them."

Hearing nothing from Gauss in response to his urgent plea that the Prince of Mathematicians accept Johann as a pupil, the elder Bolyai did the next best thing and sent his son to the Royal Engineering College at Vienna. Johann entered the College in 1817 at the age of fifteen and left in 1822. During his course the young geometer, far from being turned from his destiny by the horrible example of his father and the latter's anxious warnings, thought incessantly about parallels and communicated some of his enthusiasm to his friend Carl Szász (1798–1853), who in his turn made ingenious suggestions to Johann. Szász left Vienna in 1821 to teach law at the college of Nagy-Engel, Hungary; Johann continued his researches alone, corresponding with his father about the difficulties and pitfalls of his undertaking, some of which were painfully familiar to the old man himself. In 1823, at the age of twenty-one, Johann found the right road: he admits that his supposed proof is fallacious, and he will now show that Euclid's assumption is *not necessary* by creating what he calls the *Absolute Science of Space* in which the parallel postulate is ignored. Flushed by his first and brilliant successes he shares his exultation with his father.

"I am firmly determined," he writes, "to publish a work on the theory of parallels as soon as I can get the materials in order and circumstances permit. I have not done so *yet*, but the path I have followed has, so to say, almost reached the goal; the end itself is not attained, but I have discovered such beautiful things that I have been dazzled; it would be a tragedy if they were lost. When you see them you will agree. In the meantime I can say only this: *I have created a new universe out of nothingness.* All that I have communicated to you up to now is only a house of cards compared to this tower. I am as fully convinced that this will bring me honor as if I had already done it."

This letter is dated November 3, 1823. Wolfgang Bolyai was then engaged in writing up his own *Tentamen*. With a prophetic insight which might have dismayed him could he have foreseen how bitterly right he was to prove, he counsels Johann to hasten. "If you have really succeeded, no time must be lost in publishing, for two reasons: first, ideas are transmitted easily from one man to another, who can anticipate publication; second, there is a certain truth in the belief that many things have their own epoch, in which they are discovered simultaneously in many places, just as the violets bloom everywhere in spring. Again, every scientific struggle is no less than a fierce war, in which it is impossible to say when peace shall ensue. Thus we should win while we can, since the advantage always rests with the man who gets there first."

As a matter of historical fact Gauss had already arrived in 1825 — indeed long before — the year in which the sanguine Johann finished his masterpiece at the age of twenty-three. It would take us too far afield to enter in detail into what Gauss had been keeping locked up in his desk or in his head, but a few extracts from his correspondence may suffice to indicate that his claim — never made publicly — that he had anticipated the young son of his old friend was well founded.

Writing in 1831 to Schumacher, Gauss observes that his "meditations" on parallels were already forty years old, but that he has never found or taken the opportunity to write them out. For some weeks he has been putting them into shape for publication. On another occasion he declares his intention of having his researches on non-Euclidean geometry (he invented the term) come out only after his death. Few persons, he said, have any conception of the nature or difficulties of questions concerning the foundations of geometry, and this holds for amateurs and professional mathematicians alike. He himself had begun to reflect on the matter before he was sixteen. His correspondence contains many propositions of non-Euclidean geometry; but all to whom he communicated his discoveries were pledged not to divulge his geometrical heresies. Gauss, like Descartes, desired only tranquillity and repose in in his life; the barbaric and ignorant yawpings of orthodox professorlets, should his work become public, rang in the ears of his imagination and he could not force himself to face the racket—"the clamor of the Boeotians," he called it.

Among Gauss' correspondents were two who deserve mention, Ferdinand Karl Schweikart (1780–1859), a distinguished professor of law at the University of Königsberg, who had studied mathematics as an undergraduate at Marburg, and Schweikart's nephew, Franz Adolph Taurinus (1794–1874), who had been induced in 1820 by his uncle to devote his fine talents to what was in effect non-Euclidean geometry—"Astral Geometry," according to Schweikart. Both of these men were on the right track and far along; the work of Taurinus in particular evoked Gauss' admiration and praise. But these two, like the rest, were begged not to divulge the hints Gauss threw their way as to his own work. For some obscure reason Taurinus in later life destroyed all copies of his own work on which he could lay his hands. The elder Bolyai had good cause, although he was unaware of any grounds for his premonition, to counsel his son to hasten.

At last Johann got his work into print in 1831. He had already communicated it to others, notably to one of the professors at the Royal Engineering College in 1825, who, Johann says, presumably kept the original draft of the completed exposition. At his father's request, Johann prepared a Latin translation for inclusion in the *Tentamen* as an appendix. This was printed in 1831. Realizing the magnitude of the modest Appendix of twenty-six pages, Wolfgang had it printed first; the rest of the *Tentamen*—his own work—could wait. Johann was impatient for Gauss' verdict; Wolfgang also wanted to hear what the Prince thought of the Appendix. Owing to the cholera epidemic then raging, Wolfgang took extraordinary precautions that his son's work should reach Gauss. He might have spared his pains. Commending the work to Gauss' attention, Wolfgang writes as follows from Maros-Vásárhely on June 20, 1831.

". . . My son is already high up in the Engineering Corps, and will soon be Captain; he is a fine fellow, a virtuoso on the violin, a strong fencer and courageous, but he is always dueling and is perhaps too hot-headed a soldier, but also a gallant man: out of light comes darkness, and out of darkness, light. He is passionately devoted to mathematics, for which he has a rarely gifted mind. At present he is in the garrison at Lemberg. He has the greatest veneration for you and is capable of understanding and appreciating you. It is at his request that I send you this little work of his; have the goodness to judge it with your perspicacious eyes, and, in the reply which I await impatiently, write me your verdict without consideration. . . ." He calls Gauss' attention (as an afterthought written on the inside of the envelope) to one miraculous theorem in Johann's non-Euclidean geometry: it is possible in this geometry to square the circle. Unlike some of the classical miracles this one is true. Again, on January 16, 1832, to make assurance doubly sure in view of the cholera epidemic, Wolfgang transmits a copy of the Appendix with corrections of typographical errors. "My son was away when his work was printed," he writes; "he has corrected the mistakes (in the *Errata* at the end). I have corrected most of them by pen, to save you this bother. He writes me from Lemberg that he has since made several places simpler and more elegant, and that he has proved the impossibility of determining *a priori* whether Euclid's parallel postulate is true or not."

It seems rather a pity that a mathematical genius of Johann Bolyai's caliber should have squandered himself on the army—although all armies have desperately needed geniuses, everywhere and in all times, God knows. Nevertheless Johann seemed to enjoy the stupid life, and his contumacious brawling (dueling, according to his doting father) took his mind off the overwhelming disappointment he was presently to suffer.

While waiting for six weeks to elapse during which Gauss concocts his kindly crushing reply, we may regale ourselves with the famous incident of that Gargantuan duel of Johann's which is always retailed in connection with his misspent life. It sounds like a thundering lie, but is well attested.

During his vegetation in the garrison at Temesvar, Johann, the young officer of engineers, fell out with the Austrian cavalry officers for two necessary and sufficient reasons: he was a Hungarian, they were Austrians; he was a skilled builder of military bridges and such things, they were more or less honest fellows whose loyal duty it was to wait hand, nose, and foot on a bunch of smelly horses and see that their charges did not gorge themselves into a state of chronic bellyache. All in all they were too much, in their jolly, bluff way for the fastidious Johann. One day he challenged no less than thirteen of them to a duel on condition that after each combat he be allowed to play an air on his violin to refresh himself. One after another he laid them all out cold.

This is only one duel of many in mathematical history. If the astronomer Tycho Brahe rates as an applied mathematician, he also deserves a medal, for in his duel he got his nose sliced off. At present mathematicians favor purely verbal weapons, and it is not unusual, and quite refreshing, occasionally, to hear one eminent mathematician calling another a damned fool — behind his back. Mathematicians have been compared (maliciously) to artists; certainly sometimes they act like artists.

Gauss has now had time to answer Wolfgang's urgent letter. His reply is dated March 16, 1832.

". . . Let us now speak a little about the work of your son. If I begin by saying that *I cannot praise this work* you may well recoil in momentary astonishment; but I can say nothing else; to praise it would be to praise myself; for indeed the entire content of the work, the trail your son has blazed, the results at which he has arrived, coincide almost wholly with my own meditations which have partly occupied my mind now for thirty or thirty-five years. Thus I was completely dumfounded. As for my own personal work, of which indeed I have so far confided but little to paper, my intention was to let nothing be published during my lifetime. For the majority of men lack an open mind on questions of this sort, and I have found only a very few who take a special interest in what I have communicated to them on the subject. To be capable of taking this interest it is necessary to have realized vividly what is lacking, and on these matters most are in utter darkness. On the other hand it was my intention, given the time, to put all this in writing so that it should not perish with me.

"Thus I am agreeably surprised to see that this trouble is now spared me, and I am overjoyed that it is none other than the son of my old friend who has anticipated me in so remarkable a manner."

Gauss then goes on to praise Johann's work and to suggest a few improvements in the nomenclature and presentation. His observations in this letter alone are sufficient to convince any impartial judge that Gauss had indeed reflected long on the foundations of geometry and that he had, as he claimed, anticipated Johann Bolyai in his whole great project — that of the creation of a self-consistent geometry independent of Euclid's assumption. But Johann, naturally, was not an impartial judge. The young man was infuriated, not only with Gauss, but with his generous and patient father, whom he accused of having betrayed him to the 'old grabber' of Göttingen. Johann never forgave Gauss, although he was forced to admit years later that the Prince of Mathematicians had not exaggerated his claims to priority.

In this connection it should be emphasized that Gauss never made any public assertion that could be even remotely construed as a claim of priority; all that he said on the matter was in private letters to friends, and neither he nor they had any thought that the correspondence would some day be made public. In none of Gauss's letters is there a trace of anything resembling envy or chagrin that Johann Bolyai and Lobachevski had anticipated him, and we may believe that the second paragraph in the above quotation is a true expression of his sentiments in the matter. Yet terrific logomachies have been waged over this meaningless dispute, leaving all parties to it soreheaded and blindly unreasonable. So much for mathematics (or any other intellectual discipline) as a "training" in objectivity of judgment. Outsiders — those who do not make their living at mathematics — may be inclined to smile indulgently at the whole ridiculous uproar; but to the protagonists in this tragi-comedy it was as serious as are the rows today over free trade, salvation by grace, or the inspired sanctity of some particular form of government. In such arguments we reason wholly with our viscera and usually succeed only in getting ourselves into a mess.

Another disappointment was in store for Johann. Seventeen years after the publication of his own work he first learned that Lobachevski had anticipated him. Determined to find a flaw in the Russian's work he began an intensive study of Lobachevski's *Geometrical Researches* in 1848. Hoping to prove himself the better man he planned a comprehensive work on the foundations of mathematics, but was unable to complete the project successfully. It is said that he left over a thousand pages of manuscript, none of which has been published. But for his unfortunate attitude toward Gauss and his jealousy of all who had written on the principles of geometry, Johann might have capitalized his talent even more gloriously than he did. Lobachevski of course was unassailable. To judge by the comparative futility and frustration of Johann's career after the publication of his immortal twenty-six pages, the fatal problem of parallels may, after all, have raised the havoc with his life which his father had predicted.

Johann's seed also fell practically by the wayside, just as had Lobachevski's. Although greatly impressed himself by the famous Appendix, Gauss did not go out of his way to advertise it to the scientific world. Nor, apparently, did he bother to tell his old friend anything about the work of Lobachevski when he became acquainted with it. To his narrow circle of intimates Gauss was enthusiastic enough over Johann's work. Thus, writing to Gerling on February 14, 1832, he says, "I remark that today I have received a short paper on non-Euclidean geometry wherein I rediscover all my own ideas and RESULTS developed with great elegance, although for anyone unacquainted with the matter it is in a form somewhat difficult to follow, owing to the conciseness. The author is a very young Austrian officer, the son of a friend of my youth with whom I often discussed the subject in 1798, though at that time my ideas were far from being developed and immature. . . . I consider this young geometer Von Bolyai a genius of the first magnitude." High praise, indeed the highest possible, when we consider the source, and well merited according to those who should know. What a pity it is that Johann could not have read the last sentence of Gauss' comments over Gerling's shoulder. His touchy pride might have been soothed and instead of eating his heart out in futile jealousy he might well have capped his masterpiece with another.

In a letter of December 21, 1843, Gerling gives Gauss news of his old friend Wolfgang. "You may picture him [Wolfgang Bolyai] as a rather stout, robust and friendly old man who seems to live out his old age in his garden. . . . His son (who, you tell me, has written a good book) . . . is pensioned and lives, I believe, with his father." Gerling asks Gauss for the title of the "good book," piqued, no doubt, by the praise of the Prince who was not addicted to flattery. Gauss replies on February 4, 1844, giving the desired information and telling Gerling of Lobachevski's work.

"Moreover, in the past decade, a Russian, Lobachevski, has broken a similar way." Gauss goes on to say that he thinks he possesses all of Lobachevski's works on the subject, "but their thorough reading has not yet been undertaken;" Gauss is waiting until he resumes the subject in his own fashion and also until he can read Russian with ease. He then makes an observation which may cheer up some young and neglected genius who thinks he has the world by the tail but who cannot convince the critics that he has caught anything livelier or more impressive than a dead cat. "I read a very disparaging review of Lobachevski's work, which [the review] for any well-informed reader leaves the impression of having been written by a totally ignorant man. Since I have had the opportunity to go into this little book [Lobachevski's] itself, I have formed a very favorable judgment of it."

The work in question is one of the shorter essays; Gauss likens the more extensive treatise to a forest in which it is impossible for the traveler to find his way until he knows all the trees individually. It is in another letter than Gauss makes the assertion that "I find nothing new for me in Lobachevski's work, but the development is carried out in another way from that which I myself followed, and indeed by Lobachevski in masterly fashion in the true geometric spirit." In the same letter he refers to Schweikart, for whom he had a high regard. For Taurinus, Schweikart's nephew, Gauss had only praise and encouragement. So we may discount to zero the lying legend that Gauss was a selfish, jealous, and secretive old man who kept his secretiveness up his sleeve as a knife to stick in the backs of enthusiastic young mathematicians poaching on his preserves. He was serene, not jealous, and judicial, but perhaps a shade too cautious for his own good and for that of mathematics.

In one of his letters, after having heard of Lobachevski, Gerling remarks that "the steppes of Russia seem to be a suitable soil for these speculations [on non-Euclidean geometry], as Schweikart, now in Königsberg, thought out his 'Astral Geometry' while in Charkow."

The creation of non-Euclidean geometry was a turning point in human thought. Three outstanding figures share equally and independently in that epoch-making advance: Lobachevski, Johann Bolyai, and Gauss.

Johann died in 1860 at the age of fifty-eight. To one of his devoted countrymen, the architect Schmidt, is due the rescue of his personality from oblivion.

FOREWORD

The tragic and all too brief life of Galois is one of the most poignant episodes in the history of mathematics. The creation of the theory of groups — the work of a few fleeting years — was eventually to put the name of Galois in the gallery of immortals. Not the least significant of this man's facets was his extraordinary capacity for thinking through mathematical investigations almost entirely in his head. In this respect he would appear to have resembled J. von Neumann. This power of concentration was to Galois both an asset and a liability. When dealing with mathematicians of lesser stature, this gift put him at a disadvantage; they, for their part, would often fail to recognize sheer mathematical creativity and power when they encountered it. All concerned might profit from this lesson today.

The interested reader will find Leopold Infeld's fictionalized biography *Whom the Gods Love* a fascinating revelation of the man Galois.

ÉVARISTE GALOIS

ON THE 150TH ANNIVERSARY OF THE BIRTH DATE OF AN IMMORTAL IN MATHEMATICS

I. Malkin

The following lines are a humble tribute to the memory of a great French mathematician of unexcelled brilliance and of unsurpassed tragedy of personal destiny and scholastic career, a mathematician who is now considered one of the greatest geniuses the human race has ever produced. To achieve immortality in pure mathematics virtually as a teenager, as he did, is a case without parallel even in the annals of French mathematics of the Napoleonic epoch.

Two princes were born to France in the year of 1811. They were born in the same year, and they died both in the same year of 1832. Both were victims of a cruel fate—though in different ways. The one was the son of the great Emperor of the French; the other was a prince of Science. The one, "Le Roi de Rome" at birth and "L'Aiglon" of France posthumously, owes his fame to the heroic glory of his father; the other, a boy of comparatively obscure origin, contributed himself immensely to the scientific glory of his country by the miraculous might of the intellectual powers, with which he was endowed by the grace of God.

Évariste Galois was born on October 25, 1811, at Bourg-la-Reine near Paris. His father, a man of pleasant and lovable character, was owner of a boarding house for young people; his mother, daughter of a city official, was a young person of unusually high education. At the age of twelve young Évariste went to Paris, where he entered the Collège Louis le Grand. At the age of fifteen his extraordinary powers in mathematics became evident. Standard texts in algebra do not satisfy him and he turns to the original works of the great master Joseph Louis Lagrange (1736–1813). At the age of seventeen he was already the creator of highly important results in the theory of algebraic equations. His first paper was published in Gergonne's *Annales de Mathématiques* of 1828–1829, while he was still a pupil of the Collège.

But right at this point we have to start with an account of his unique misfortunes.

In some respects Galois was not an easy character. He did not care to keep in conformity with the curriculum of the school, preferring to choose his own ways in his mathematical studies. He recognized very early and very well the order of magnitude of his titanic abilities, and he became full of understandable pride and more or less isolated from his surroundings. He felt himself far superior to his teachers. All this was only the source of the first blows and disappointments in a short and tortured life.

Twice did he try to enter the celebrated École Polytechnique, the prototype of this country's West Point Academy, and he failed both times. Once he simply refused to answer the examiner's question, which he considered ridiculous. While he was accustomed to work out his mathematics in his head, he was being asked to develop the solutions on the blackboard, which was again much against his liking. Around this time his father, whom he loved affectionately, committed suicide, which contributed disastrously to the unhappiness of the young man.

Galois' first outstanding results, preceding the publication just mentioned, were submitted to Cauchy, and the latter obliged himself to present the important paper at one of the earliest sessions of the Academy of Sciences. Augustin Louis Cauchy (1789–1857), one of the greatest figures in the history of mathematics of all times and nations, the rival of the "Princeps Mathematicorum" Carl Friedrich Gauss (1777–1855) of Germany, was unfortunately not very careful about the publications of young colleagues. Another immortal of pure mathematics, Niels Hendrik Abel (1802–1829), a Norwegian counterpart of Galois, submitted to Cauchy a paper of historical importance in 1826; it was published only in 1841, twelve years after Abel's death, and an insistent intervention by the Government of Norway was necessary to have that paper published. Galois was even much less lucky: his paper was simply lost on Cauchy's desk — and that is all there is to it.

In 1830 Galois published three papers on the theory of equations and right afterwards he wrote a résumé of the results of his investigations: this résumé was handed over to another leading French mathematician, Jean Baptiste Joseph Fourier (1768–1830). It was to be submitted to the Academy of Sciences for competition for the Grand Prix de Mathématiques of the Academy. Fourier took the manuscript home for examination. The Academy did not get it, because Fourier died before he could take care of Galois' work. The manuscript could not be found among Fourier's papers.

The "coup de grâce" was given Galois, though unvoluntarily, by S. D. Poisson (1781–1840), member of the same Academy of Sciences, a prolific author of books and other publications on various branches of Applied Mathematics, a man whose name is familiar to every engineer. Poisson was animated by the sincere desire to help Galois. He persuaded the young man to reproduce the manuscript previously lost among the papers of Fourier, with the intention of presenting it to the Academy. Poisson kept the new manuscript for four months and then he returned it to the author with the laconic remark: "Incompréhensible."

The exasperation of Galois knew no bounds: "If a cadaver were necessary to lead the people to a revolt, I would give them mine." He joined the most embittered opponents of the regime of King Louis-Philippe. The question what had politics to do with mathematics at those times, finds a concise answer in a book of a prominent contemporary British engineer-mathematician, N. W. McLachlan. In the historical introduction to his *Bessel Functions for the Engineer* (Oxford University Press) he tells us the following little story about Fourier's (see above) classical *Théorie Analytique de la Chaleur*: "The year 1822 is of singular importance in the history of mathematics owing to the publication of J. B. Fourier's treatise on the Analytical Theory of Heat. This epoch making analysis had adorned the archives of the Paris Academy of Sciences for about twelve years. Its publication was delayed for fear that it should adversely affect the prestige of the powers that were." Things of the kind just alluded to may have been responsible for the political radicalization of Galois. He became too loud and the consequence was confinement to the prison of Sainte-Pélagie, where he was kept for five months. He was discharged in March 1832, a month before his term expired, because of the poor state of his health. This act of clemency led to his undoing. Soon after his release he met a woman of rather base characteristics and became entangled in the nets of a heartless coquette. The result was a duel with an unknown individual. Galois was physically a small and weak and nearsighted young man; to challenge him to a duel was virtually an abominable crime and murder. Galois had no illusions about the outcome, but there was no alternative to accepting the challenge. On the morning of May 30, 1832, he was mortally wounded by a bullet in his abdomen. In the evening peritonitis developed. His younger brother visited him in the hospital and started crying. To him Galois said: "Ne pleure pas, j'ai besoin de tout mon courage pour mourir à vingt ans." ("Don't cry, I need all my courage to die at the age of twenty.")

A few hours later, at 2 a.m. on May 31, 1832, ended this short tortured great life.

When Poisson had written down his "Incompréhensible" (see above), he made a statement, for which we cannot blame him unrestrictedly, as will be seen presently.

Almost entirely certain of the coming catastrophe, Galois wrote on the eve of the duel a letter to his friend Auguste Chevalier, in which he gave a short summary of his research work including his nonpublished manuscripts. In this summary Galois states that, while he is absolutely certain of the correctness and validity of his theories, he did not care to develop their proofs; he himself calls his work "a mess" in this respect. Here is a famous paragraph of his letter to Chevalier: "You will publicly ask Jacobi and Gauss to give their opinion, not on the truth, but on the importance of the theorems. After this there will be, I hope, some people who will find it to their advantage to decipher this mess" ("ce gâchis"); Carl G. J. Jacobi (1804–1851), a man of most astonishing productivity, was the first in the series of great German-Jewish mathematicians. It is important to note that according to available information Galois' letter had never been submitted either to Gauss, or to Jacobi. The work of Galois continued for a long time to be a "mess." It required a great deal of concentrated deep studies to be understood thoroughly. Poisson was an outstanding authority in various branches of Applied Mathematics, but the work of Galois constitutes one of the most difficult branches of Pure Mathematics.

A slow posthumous turn in favor of Galois began only fourteen years after his tragical end, when his eminent countryman Joseph Liouville (1809–1882) had published in his *Journal de Mathématiques* for the first time most of Galois' works. Great leaders in mathematics became interested in the "gâchis" and the gigantic work of its deciphering began, as Galois had predicted it: Enrico Betti (1823–1892) of Italy, Camille Jordan (1838–1922) of France, Felix Klein (1849–1925) of Germany, Sophus Lie (1842–1899) of Norway, men of the highest reputation in the history of Mathematics, actually found it "to their advantage" to decipher the great "mess." Their studies have led to the result that Galois' prophecy concerning the truth, validity, and faultlessness of his theory was absolutely correct. That "mess" is nothing less than the celebrated "Galois Theory of Groups," erected upon his investigations in the theory of algebraic equations and considered now one of the most outstanding achievements in the Mathematics of the nineteenth century.

The new developments in exact sciences have led to a downfall of the boundaries between the Pure and the Applied Mathematics, and so we find that the Galois Theory of Groups and its tremendous modern expansion acquire growing importance in the Theoretical Physics as well.

Entirely independently of the question of practical utilization of scientific research we all share the conviction that scientific studies belong to the highest and noblest interests of human existence. Evariste Galois is one of the greatest heroes of scientific achievement and one of the immortals of Mathematics. Therefore we all unite in most profound sympathy, respect and admiration for the memory of the great martyr and most illustrious mathematician — Evariste Galois of France.

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FOREWORD

Occasionally the most profoundly significant contribution to science has been made by a man of whom one scarcely ever hears. A case in point is Josiah Willard Gibbs, the brilliant American mathematical physicist, whose life and work were so nearly one, that his name is hardly known outside the circle of scientific workers. To sum up Gibbs' contributions to theoretical physics by saying that he developed the science of thermodynamics, created the branch of mathematics known as vector analysis, created the science of statistical mechanics, and laid the foundations of physical chemistry, while accurate enough, fails to suggest the deep significance of these achievements. Gibbs' work bridged the gap between classical physics and atomic physics. Indeed, it was Max Planck who proclaimed that the name of Gibbs, "not only in America, but in the whole world will ever be reckoned among the most renowned theoretical physicists of all times."

Interested readers will find excellent accounts of Gibbs' life and work in the following books:

Bernard Jaffe, *Men of Science in America* (Chapter 13). Simon and Schuster, 1944.

Benedict A. Leerburger, Jr., *Josiah W. Gibbs: American Theoretical Physicist*. Franklin Watts, 1963.

Muriel Rukeyser, *Willard Gibbs, American Genius*. Doubleday, 1942.

Lynde Phelps Wheeler, *Josiah Willard Gibbs: The History of a Great Mind*. Yale University Press, 1962.

JOSIAH WILLARD GIBBS*

R. E. Langer

In the great glass of time the grains of sand are even now stirring which by their fall will soon mark the lapse of a century since the birth of Josiah Willard Gibbs, a man who must be placed at once among the greatest and the least generally known of American scientists.

To us, in this scientific group, it is a matter as we will it — the unexalted choice of a perspective — whether the century be regarded as a unit great or small. The geologist, the astronomer, the biological evolutionist, thinks effortlessly in eons by comparison with which the century is an insensible instant, and the scholar of the arts, the science, the philosophy or the mathematics of antiquity reaches aptly back over centuries, be they fifty or more. And yet, as students of modern science we remember that only three centuries or less have rolled by since Galileo paid in discouragement and anxiety the penalty of his heroic thought; since Descartes preached his scientific doctrine and founded modern mathematics; and since the mechanics of the world of today took shape in the great mind of Newton. The life of our national identity is not yet to be measured by two centuries, and even a single century ago the soil we tread was still in all truth that of a new land.

The American scientist of a hundred years ago stood forth, therefore, very much in the rôle of the pioneer. He won out in his achievements over many handicaps; without traditions at his back, with little in the way of schools and libraries or laboratories to his hand, and often in the face of small sympathy or encouragement from his neighbors, who were for the most part engrossed in the busy incidents peculiar to the industrial and commercial and agricultural enterprise of the new nation. The issues which were soon to lead to the test whether the bonds uniting the nation were tender or tough, were already inflaming the passions of men, and were all too well designed to overshadow the inconspicuous scholarly searchings and deductions, in the tiny impulses of which the revolutions of scientific discovery are generally born.

I need not recall to you the pomp and circumstance of a few years ago by which a great American city saw fit to mark the consummation of a century of scientific progress. There were exhibits to swell with pride the hearts of the disciples of science, and there was much that called to all mankind for homage on behalf of those rare men, who, facing the

*An address to the Minnesota section of the Mathematical Association of America, October 28, 1938; also given at Northwestern University on November 30, 1938.

questions which the facts of nature constantly pose to the human mind, were of that superior wisdom that they could see the principles behind the facts, and grasping these place into the hands of their fellows the machineries of modern science for the control and understanding of nature and the consequent amelioration and enrichment of life. Of these rare men of extraordinary intellect was Josiah Willard Gibbs. An ancient proverb notwithstanding, the world-wide scientific authority of today is agreed that in his own field he was one of those uncommon men who could see tomorrow's sun.

Josiah Willard Gibbs was born on the eleventh day of February in the year 1839, in the city of New Haven, Connecticut. His family had made its homes upon American soil from almost the earliest of colonial days, and had been identified over many generations with the finest in American thought and culture. Thus one finds among Gibbs's direct though more remote ancestors a President Willard of Harvard College, and a President Dickenson of the College of New Jersey which later became Princeton College, while Gibbs's father, also named Josiah Willard, was a professor at Yale, and an eminent scholar widely noted for the profundity of his thought in the fields of sacred literature and the classics. From men such as these Gibbs inherited genius and those traits of industry, power of concentration, and intellectual integrity, keenness, and modesty which were to characterize him throughout his life.

Though records of personal incidents or anecdotes of Gibbs's boyhood or youth are scant or utterly missing, it is not hard to reconstruct in essence the atmosphere of the environment in which he lived. New Haven was even then the seat of Yale College, and so was a center of American learning. From early colonial times it had been a community built largely around the church, and possessed of a strongly local and very individual character. Traditionally its code of preference was inclined in all matters toward a general and extreme conservatism, and this attitude, especially in matters of politics and religion, had been unwaveringly maintained. Although as a seaport it had cherished, and in those earlier days of the steamboat perhaps still did cherish, some dreams of commercial greatness, the life and ambitions of the city, as they expressed themselves through its more influential citizens, were focused in large measure upon the College. Yale was the acknowledged center of civic pride and ambition. It requires little in the way of imagination to conceive of how Gibbs, born into the family of a scholar and professor in such surroundings, must during his boyhood have been saturated with the atmosphere of collegiate life, and how he must have had instilled into him a thorough sense of the prime importance of things academic.

During these years the community of New Haven merited with far more justice a description as an overgrown village than as a city. Its connection with New York by railroad was not materialized until the year 1848, and in that same year street lighting by gas lamps was first introduced upon the main streets of the town. Until 1854 there were no graded schools, and for years beyond that fire protection was on a volunteer basis, and policing in the hands of a Department of the Watch. By common custom scavenger duty was delegated to swine and was allocated to the gutters of the city streets, and not until 1861, when Gibbs had already attained his majority, were pig-pens banished from the city, and a law passed abating the nuisance of horses and cows pasturing in the streets. On the main thoroughfares sidewalk paving was an innovation which had been bitterly opposed, and it is recorded that a prominent citizen still strode over this object of extravagance only when necessity compelled him to, and then in haste, and walked by preference in the street, fortified in his conviction that God's soil was still good enough for him.

Gibbs prepared himself for college at the Hopkins Grammar School in his home city. He was a distinguished student who divided his interests between mathematics and the classics. At the age of fifteen he entered Yale, and continued there to live up to his earlier promise. His interests were exclusively cultural, and so following his graduation he remained at Yale for five years more to receive his doctorate at the end of that time, in 1863. Yale had established the degree of doctor of philosophy only three years earlier, and in doing so had taken the lead among American institutions. With his degree Gibbs received an appointment as tutor in the College, and was assigned to teach classes at first in Latin, and then in Mathematics and Science. He found this assignment no easy one, nor one for which he was temperamentally well equipped, or in which expectations of brilliant success could be his. Whatever his genius, it was not that of the college teacher.

In physical aspect Yale presented at that time the picture of a row of old and homely buildings of brick, which were flanked round about by a campus noted for the distinction of its archways of great and stately elms. Along the street this campus was edged by a long rail fence around which much of Yale tradition entwined itself, and which from immemorial time had been the favorite roost of the students. As a barrier to ingress the fence was a negligible matter, and the campus, therefore, assumed much the character of a village green, where tramps, and beggars, and pedlars, and organ-grinders, and in fact all who would, mixed freely in the college life. As at noon the elms lent shade, they at night increased the darkness, and so unwittingly conspired to enhance the lure and convenience of the unlighted campus for all such as were bent upon purposes of academic mischief. The fugitive student could depend here at once upon ready access to the city streets and upon the asylum of many handy refuges and concealments, temptations which, it has been said, rarely failed to convert original saints into undergraduate sinners.

To the common creature comforts of the students the dormitories of the old brick row afforded only the rudest of satisfaction, and of this sort also were the classrooms. The beams sagged, the floors were billowy and the ceilings cracked; and the walls were gouged and furrowed, for it had long been the prime ambition of every student to leave some lasting mark of his upon pillar or doorway. The rooms were illy lighted and seemed, therefore, steeped in permanent gloom. There was a general air of mustiness about and an appearance of roughness, and the sanitation was far from modern. Such details, however, seemed of small effect toward mitigating the intensity of life for the five hundred students of the College, and faculty vigilance enjoyed but rare respite from its appointed task of keeping pace with the students' inventive genius for pranks and insubordination.

With all this it was an academic era noted as one in which professors were, or at least were popularly regarded to be, highly individualized beings. Among them oddities of personality or habit were not regarded the exception but the rule, and even the younger tutors and teachers generally shared in these distinctions. All too commonly custom and their own expanding dignity placed them under the incumbency of assuming an aspect of austerity which made them appear as nothing so much as arch-foes to their charges. It was their first duty to match their discipline vigilantly against their opponent's ever-present urge for fun, and a tutor in actual physical pursuit of a fleeing undergraduate was by no means an uncommon sight.

For this sort of thing Gibbs could hardly have been more poorly endowed by nature. He was shy and retiring, gentlemanly, contemplative and reserved. The subjects he taught, moreover, Latin and Mathematics, were at that time required of all students throughout their first and second years, and, as is the way with requirements generally, these bred neither a desire to spare the tutor, nor a fondness for the subjects. Quite the contrary we may infer, for of the many campus customs at Yale in those times none appears to have been entered into with so much gusto and zest as the annual farcical pageant of the Burial of Euclid, with which the sophomore class was wont to celebrate its mathematical emancipation.

There are many records of this ceremonial in the Yale archives, and though in its details it naturally varied with the genius of the class, it maintained its identity in form over a period of generations. The sophomore class having been summoned to gloat over Euclid's death, assembled in some college hall which was bedecked suitably to the occasion. The scene was dominated by a large and lurid cartoon which bristled in detail with fire and fury, and depicted how in the presence of Jupiter demon stokers were assisting at the consumption of Euclid's remains in a sea of blazing tar. A dismal forest with embattled demons filled the remoter parts of the scene, while in the foreground a student visibly filled with despair lent company to a weeping crocodile. Under this aspect Euclid's volume was perforated with a glowing poker, each man of the class thrusting the iron through in turn to signify that he had gone through Euclid. Following this the book was held for a moment over each man to betoken that he had understood Euclid, and finally each man passed the pages under foot that he might say thereafter that he had gone over Euclid.

These preliminaries accomplished, the funeral cortege was formed, and proceeded lugubriously, with grotesque garb and blazing torchlights to the chosen place of interment. At times Euclid himself was impersonated, dressed in classic raiment and pressing his beloved volume to his breast, and at others the book alone was borne suitably shrouded at the head of the procession. At the pyre the celebration waxed in boisterousness and assumed more the aspects of revelry. There was elaborate mock lamentation, a funeral oration was held, and dirges more or less derisive were sung.

"No more we gaze upon that board
Where oft our knowledge failed.
As we its mystic lines ignored,
On cruel points impaled."

"We're free! Hurrah! We've got him fast
Old Euk is nicely caged at last."

"Black curls the smoke above the pile
And snaps the crackling fire:
The joyful shouts of Merry Sophs
With wails and groans conspire.
May yells more fiendish greet thy ears,
And flames yet hotter glow:
May fiercer torments rack thy soul
In Pluto's realms below."

As a student at Yale, Gibbs must have participated in such a rite, and during his life he must have witnessed it many times. For him personally, however, Euclid never died in any but a metaphoric sense; for geometry and geometric imagination were the cornerstone and buttress of his genius.

At the expiration of his appointment as tutor Gibbs went to Europe for further study. The Civil War, which had been running its course, ravaging the country and depressing personal incentive, had meanwhile come to its close, and in departing this country Gibbs figuratively became one of a brilliant troop of young intellectuals who were destined to play a decisive role in the cultural development of America. These men went to Europe in search of learning, and they found the fulfillment of their quest at the great German universities. There they found great minds in numbers and under conditions which made those minds accessible to others less mature. They found there also a breadth of academic viewpoint, a freedom of research, and an insistence upon productive scholarship which they had not theretofore known. Upon their return they brought back with them these ideals and perspectives, together with an abundance of enthusiasm; and under the spur of this inspiration they became the institutors of the system of post-graduate instruction and research which is the essence of the modern American university, and became the founders of the many learned societies which are today conspicuous forces in the intellectual and cultural life of the nation.

Gibbs spent three years in Europe, studying for a time at Paris, but principally at Heidelberg, as a student of those great teachers, Helmholtz and Kirchhoff, and at Berlin under the influence of the supreme rigorist genius of the great Weierstrass. Subsequent to his return to America, Gibbs was elected, in 1871, to the Professorship of Mathematical Physics at Yale. He was then thirty-two years old, and was to hold this professorial chair without interruption for precisely that many years again until his death. The distinction which had thus been bestowed upon him was apparently a cheap one for Yale, and one which for Gibbs remained long empty of all but honor. During many years it carried no remuneration at all, and during many more only a fragment of an otherwise customary salary.

Had Gibbs been under the necessity of maintaining himself, his position at Yale would, of course, have been untenable. Fortunately this was not so. His father had bequeathed him a competence which, though small, was matched by the smallness of his need. He was, and always remained unmarried. Throughout his life he retained the occupancy of rooms in the old family house in which he had spent all the boyhood he could remember, and which stood in close proximity to the College campus. This house sheltered now the family of a married sister, and in this family he found his own permanent home. His life was routine and uneventful; for social contacts he felt but little need, and the craving to see and hear, which impels one to travel, was not his. He had few aesthetic needs, was abstemious in his habits, and seemed to find in his work all that he sought from life.

The study which claimed the initial interest of Gibbs as a professor was that of thermodynamics, the science which treats of heat as a form of energy, and concerns itself with the laws governing the transformations of heat into energy in different forms. This was then a new science, and for Gibbs it was an absorbing one. The results of his first two years of research were given out by him in the form of two papers which were respectively entitled "Graphical Methods in the Thermodynamics of Fluids," and "A Method of Geometrical Representation of the Thermodynamic Properties of Substances by Means of Surfaces." The papers were remarkable. By using as mathematical coordinates such physical quantities as volume and pressure, energy, temperature and entropy, they derived, on the one hand, heat diagrams of various types which later became instruments of great importance for the thermal engineer, and, on the other hand, showed how with any physical body there might be associated a graph or so-called thermodynamical surface, from the geometrical configuration of which could be recognized the many relations between volume, energy, temperature, pressure and entropy, the conditions for stability and equilibrium, and the passage from the liquid to the solid or gaseous states. In these papers Gibbs revealed himself at once as possessed of a rare imagination in the domain of abstract geometry, and as a master in its application.

These initial works of Gibbs's genius were followed in 1876 and 1878, that is, in his thirty-seventh and thirty-ninth years, by his greatest memoir, "On the Equilibrium of Heterogeneous Substances." Here Gibbs rose to the pinnacle, and revealed himself to be a true intellectual giant. The work is a monumental one, immense in its scope, and one which shows, as almost no other scientific work does, the sheer power of human thinking. In the science of physics the law of the conservation of energy, under the transformation of mechanical work into heat and vice versa, was even in Gibbs's time a familiar instrument, and one of the most effective, in the hands of the theoretical investigator, to his purpose of deducing from observed phenomena an intelligible picture of nature. In this respect the science of chemistry was far behind, for the relations between the energies of chem l reactions and heat had almost completely eluded all attempts to bring them within the realms of scientific law. It was to the difficulties of this problem that Gibbs had bent his thought, and these difficulties he had at one stroke subjugated with an astounding completeness.

The paper stands as a great model of the role which mathematics rightly plays in its relations with the sciences. Assuming command over a bewildering welter of apparently unrelated facts, it imposes upon them a few fundamental laws, and reduces the whole to rule and order. Gibbs based his authority upon the first two laws of thermodynamics, namely, the law of the conservation of energy and the law that heat will not of itself flow from a colder body to a hotter one. To these laws he adjoined a few experimentally determined primary chemical facts, and from this basis proceeded by mathematical deduction alone, with unbending rigor, to clear his intellectual way step by step, and to uncover again and again a basic principle and intrinsic likenesses between things in which such had superficially seemed remote. Specifically, Gibbs deals in this great work with the statics of chemical substances which are in contact with each other, and derives for them conditions for their co-existence, their equilibrium, or their stability as solids, liquids, vapors, or gases, or as liquid films, gaseous mixtures, solutions, or crystals, and discusses the effects upon them of osmosis or gravity, of electromotive or capillary or catalytic forces. In its reasoning it is a true unfolding of nature's law, and in its results it laid the foundation of a new science, a science of great present-day vitality, the science of Physical Chemistry.

Gibbs's great achievement was slow to attain its deserved and destined influence, and for this there were many reasons. As a scientist he was a thoroughly solitary figure, and the researches of his paper constituted a scientific departure which in its originality had been entirely unforeshadowed by the work of others. He had had no helpers, and as a true innovator he had no rivals. His paper was modestly published in the *Transactions of the Connecticut Academy of Arts and Sciences*, and was, therefore, in large measure obscure, and certainly in Europe almost inaccessible. Finally his paper was of a most forbidding aspect, as Gibbs was certainly no easy writer to read. His style was bare and concise, the cast of his ideas was severe, and his reasoning prompt, unerring and rigorous. Of emphasis he gave little, and a thing once said was done with. Finally, but critically, the great treasures he uncovered were treasures for the chemist, whereas his paper barely mentioned as many as five or six chemicals, all of them simple, and at the same time extended over three hundred printed pages which are covered with some seven hundred mathematical formulas. Such a memoir, one may venture, would be no mean test of mettle for the prospecting chemist of today. Small wonder that it remained largely unexplored some three score years ago.

The past half century has spoken in emphatic term for the brilliance and profundity of Gibbs's achievement. His paper is rated as a pre-eminent document of scientific prophecy, for many phenomena which it predicted, which were at the time unknown and unsuspected, have since been discovered or rediscovered by experimental means. The principles which Gibbs laid down have led to a wealth of original and fruitful researches which even now are apparently far from exhaustion. Emerson might well have said of them, "The creation of a thousand forests is in one acorn."

There is every reason to believe that Gibbs was himself fully aware of the great ultimate importance of his work. Nothing would have been more foreign to him, than to have lent word or action of his own to further its recognition or acclaim. It was a conspicuous characteristic of his nature to mantle his own personality with a distinctive cloak of reserve. In his personal contacts he seems to have been always friendly and considerate, kindly and affable and of a ready and spontaneous if somewhat subdued sense of humor. As an intellect, however, he withdrew into himself. Though his associates and colleagues easily recognized him as having in full measure those traits of character which Francis Bacon held characteristic of the true scholar, "the desire to seek, the patience to doubt, fondness to meditate, slowness to assert, readiness to reconsider, carefulness to dispose and set in order, and repugnance to every kind of imposture," they were permitted to recognize beyond this none of the more intimate things which filled his mind. He loved his work. and was possessed of an impelling enthusiasm for it, but these matters he kept in concealment. He worked without either the stimulus of conversation or that of criticism, and never spoke of his ideas until they were rounded out and in every way ready for publication.

Herein undoubtedly lay Gibbs's greatest failing. By profession he was a teacher, by temperament he lacked entirely the teaching spirit. Silent and reserved men have often been great as teachers, but this was not so with Gibbs. He taught only graduate students, and his students were never more than a few, but even to these he never confided the matters which at any time were at the focal point of his interest. To the few who had the will and the ability to follow him he was inspiring. All found his lectures difficult. They were invariably well prepared, but their thought was heavily concentrated, the progression of ideas often precipitate, and exercises, if such were included at all. were all too often and readily brushed aside.

The repertoire of courses from which Gibbs lectured from year to year seems to have been essentially the following: Vector Analysis, Capillarity, The Wave Theory of Light and Sound, Least Squares, The Theory of Potential, The Mathematical Theory of Electricity and Magnetism, and Multiple Algebra, and later — not until fifteen years after his great memoir — Thermodynamics, Statistical Mechanics, and The Computation of Orbits.

Much of the theory set forth in these lectures was original. This was so, for instance, with the Vector Analysis, a creation of Gibbs's for which almost all scientists of the present day are indebted to him. The Cartesian coordinate geometry, powerful mathematical tool though it be, becomes involved and unwieldy in the extreme when it is applied to elaborate space relations, or to the study of strains, twists, spins, or other common aspects of rotational motion. The desirability of an emancipation from it had been recognized by many mathematicians before Gibbs, and had led the German mathematician Grassmann to his "Ausdehnungslehre," and the Englishman Hamilton to his Theory of Quaternions. This last as a theory is concise, consistent and elegant. As an instrument, however — and though it had been used with masterly effect by Maxwell — it had generally been found unfortunately artificial. The mathematician Cayley speaking to this point, compared Quaternions to a pocket map, which to be used has to be unfolded. The quaternion formula, he felt, had to be retranslated into coordinates to be really understood.

These defects Gibbs sought to overcome through the medium of a sort of fusion of the German and English works. The result was his creation of Vector Analysis, which during the lapse of half a century has established itself as an indispensable tool of the theoretical scientist. With customary modesty Gibbs refused to regard his work here as properly original. He said of it: "The notions are only those which he who reads between the lines will meet on every page of the great masters of analysis, the only difference being that the vector analyst, having regard for the weakness of the human intellect, does as the early painters did who wrote beneath their pictures, 'This is a tree and this a horse.' " He had an account of the Vectors printed privately for the use of his students in 1881. Only in 1901 did he consent to the publication of a book on the subject.

During the years 1882 to 1889 Gibbs published a series of papers on the theory of light, and an important paper for the astronomer, "On the Determination of Elliptic Orbits from Three Observations." Of these the latter has become classical. and has achieved an immense saving in astronomical calculations. The former constitute together what is generally regarded as the simplest and most conclusive argument on purely theoretical grounds for the acceptance of the electromagnetic theory of light. At this day the correctness of his contentions has, of course, been long established, by those experimental results which meant the triumph of Maxwell's theory.

The genius of Gibbs had meanwhile received recognition, not alone in this country but throughout the world. Though to the layman, and even to many of his immediate colleagues in other fields, he remained unknown, and though he has been called an author whose books no one of his generation was ready to read, he had been elected to membership in the National Academy of Sciences, and had been awarded the Rumford Medal of the American Academy of Arts and Sciences at Boston. Before his death he was to hold honorary degrees from universities in three different countries, and to become a corresponding member of fifteen of the world's great learned societies. He was to receive from the Royal Society of London its Copley Medal, the highest distinction for research in any land, and to be elected to a vice-presidency of the American Association for the Advancement of Science.

In his final work, "The Principles of Statistical Mechanics," Gibbs was again to step forth as the innovator, and to open a most fertile field for subsequent extended scientific investigation. The belief had long been common among scientists that heat in a substance was due to motion of the molecules, but simple as this conception was, it had persistently defied all efforts aimed toward its theoretical demonstration. This failure was stigmatized by the Physicist Kelvin as "a cloud upon the history of science in the nineteenth century." To remedy it Gibbs undertook the study of mechanical systems which are composed of vast aggregates of particles. Though these particles individually were to be regarded as obeying the classical Newtonian laws of motion, a consideration of them, in the face of their great number, would have been hopeless. He took, therefore, as cornerstones upon which to build, the laws of averages and of probability, and with none but the simplest of mechanical assumptions as tools proceeded to erect his theory. From it emerged in fine succession all the basic laws of heat as they are embodied in the science of thermodynamics.

Among American men of science Gibbs holds a preeminently high place. His was one of those rare intellects from which the race obtains its pictures of the world as a cosmic universe. His mind was one in which disjointed phenomena were organized, and such generalized statements of scientific law as mark epochs in the advance of exact knowledge were thought out. Such minds are very rare, and their thoughts are incalculable treasures. The mathematician finds peculiar satisfaction in the work of Gibbs, for in it is revealed the quintessential power of mathematics for "spreading its net over the Cosmos and calling forth from it order, abstract form, and the law of science."

Gibbs died suddenly in 1903, in the sixty-fourth year of his life. He left little in the way of notes, for he had always been accustomed to carry his unfinished work only in his head. It was known that he planned an extension of his great work in thermodynamics, but his thought in that connection will never be known. He waited with them too long.

"They do not die who leave their thought
Imprinted on some deathless page,
Themselves may pass: the spell they wrought
Endures on Earth from age to age."