An analysis is presented of the basic instructional productivity formula, \( PR = TL \times CS \), where \( PR \) is the index of productivity, \( TL \) is the index reflecting teaching load, and \( CS \) is the class-size index. Objectives of the analysis are to: extend the basic instructional productivity formula to include recognition of different types of instruction; indicate that the formula may be applied at any level of organizational or temporal aggregation; specify alternate measures of faculty-count data for use on the formula; specify how the course data of the formula may be credit, hour, course, or section measures; suggest other factors and options for consideration in developing the course measures of the formula; and suggest additional extensions and applications of the resulting algebra. (SW)
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SPECIFICATIONS FOR THE INSTRUCTIONAL PRODUCTIVITY
TEACHING LOAD - CLASS SIZE FORMULA

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Abstract

The familiar instructional productivity relationship, \( PR = TA \times CS \), involves measures of amount of instruction, \( S \), number of classes, \( C \), and number of faculty, \( F \). These measures are used to express productivity, \( PR = S/F \), average teaching load, \( TL = C/F \), and average class size, \( CS = S/C \). This paper extends the basic formula to recognize differing types of instruction, e.g., individual instruction, and provides general specifications for the many alternate measures of \( S \), \( C \), and \( F \) which may be utilized in the instructional productivity analysis. The specifications and alternatives should assist analysts in designing analyses and in assisting consumers of the analyses in achieving the purposes for which the analyses are carried out.
SPECIFICATIONS FOR THE INSTRUCTIONAL PRODUCTIVITY -
TEACHING LOAD - CLASS SIZE FORMULA

The relationships among class size, teaching load, instructional productivity, and unit costs in higher education are well known. Fifty years ago, Reeves and Russell (1929) noted that high unit costs may result from high faculty salaries, low teaching loads, and small classes. Certainly this was not a startling revelation even then. About twenty-five years ago empirical relationships among the several variables were examined in The California and Western Conference Cost and Statistical Study. The reciprocal relationship between teaching-salary (unit) costs and weekly student-class-hours per full-time-equivalent teaching-staff member was specifically identified (California and Western Conference, 1956).

Algebraic statements of the relationships have also been given in varying contexts and for varying purposes. Gulko (1972) provided fundamental teaching load and class size algebra as a basis for discussing the student-faculty ratio. Elements of the algebra are incorporated in the Resource Requirements Prediction Model (Clark, Huff, Haight, and Collard, 1973). Perhaps the most comprehensive treatment of the instructional productivity algebra is incorporated in the Sheehan and Gulko (1976) presentation of the instructional cost index.

The purposes of this paper are (1) to extend the basic instructional productivity formula to include recognition of different types of instruction, (2) to indicate that the formula may be applied at any level of organizational or temporal aggregation, (3) to specify alternate measures of faculty-count data for use on the formula, (4) to specify how the course data of the formula may be credited hour, course, or section measures, (5) to suggest other factors and options for consideration in developing the course measures of the formula.
and (6) to suggest additional extensions and applications of the resulting algebra. The principal contribution of the paper is not expected to be entirely new insights and information, but rather the comprehensive treatment of the topic in a single place.

The Basic Formula

The basic and familiar instructional productivity formula is written as

$$PR = TL \times CS$$

(1)

where $PR$ is the index of productivity, $TL$ is the index reflecting teaching load, and $CS$ is the class size index. The values of the three terms are defined to be

$$PR = S/F,$$

$$TL = C/F,$$ and

$$CS = S/C,$$

where $S$, $F$, and $C$ are counts or aggregates reflecting amount of instruction provided based upon numbers of students, $S$, number of faculty, $F$, and number of classes taught, $C$. Generally, of course, each of the three terms is an "average" and it is important to recognize this, but the "A" is omitted from the descriptors for the sake of simplicity.

It should be noted that beginning with the basic productivity formula, the paper does not explicitly recognize the unit cost implications of the algebra. Rather, what is dealt with here may be viewed as the central factors which underlie unit cost figures. It can be argued that it is more important for many purposes to develop and understand indices of these underlying variables than it is to calculate unit costs (see Adams, Hankins, and Schroeder, 1978, p. 126). The extension of the points of this paper to formulas for unit costs should be straightforward.

Aggregation Levels

The basic formula (1) may be applied to an individual member of the
faculty, to all faculty of a department, to a school, college, or division, to a campus or institution, and to even higher levels of aggregation. Similarly, the formula may be used to describe attributes of instructional productivity for an academic term, an academic year, or a fiscal year. The only requirement is that the course data, S and C, appropriately match the faculty measure, P, in the application. This feature of the basic formula is noted by the addition of a superscript, \( a \), to the symbols of the algebra,

\[
P^a, S^a, C^a, FR^a, TL^a, \text{ and } CS^a.
\]

The superscript may be read as a variable which has values denoting the several possible levels of aggregation, organizational and temporal. Similarly, the presence of the superscript in the symbolism should serve as a reminder to define, carefully, and consistently the level or levels at which the indices are being calculated.

Matching Course and Faculty Data

If productivity, teaching load, and class size measures for individual faculty members or for special categories of faculty; e.g., categories defined by academic ranks, are desired, the course data aggregates must be developed from records of the specific teaching assignments of individual faculty members. Generally this matching requirement is easily achieved. A difficulty arises when two or more faculty members perhaps with graduate teaching assistants, share the responsibility of or assignment to a single course section. In such cases it is necessary to allocate the course measures, S and C, which derive from the section among the responsible instructors. The concept of "proportion of shared responsibility" is suggested as a basis for the required allocations.

If productivity indices for a department or higher levels of aggregation, only, are desired, then normally it will be possible to avoid associating
course data with individual instructors. The required matching is achieved by independently aggregating the course and faculty data to obtain totals for the department. The assumption is that the faculty appointed to a department teach only the courses of the department and that no other persons participate in the teaching of these courses. Adjustments to the departmental totals may be made on the basis of the individual cases in which this assumption is violated. Specifically, the course data or the faculty data may be "moved" in order to achieve the desired match. Depending upon the ultimate use to be made of the data and the degree to which it is known the assumption is violated, the "benefits" of making such adjustments to the data may not justify the "costs" of making them.

Definition of Faculty

The faculty-count measure, F, may be defined in more than one way. For non-complex colleges F may be a headcount of faculty, perhaps modified only by adjustments for part-time faculty. For complex universities where split appointments (abound and) are used to recognize the assignment of faculty to differing types of activities F may be defined as the number of full-time-equivalent (fte) faculty appointments or assignments to engage in instruction and related activities. The institution's up-to-date budget is generally the preferred source for such fte counts. (The argument that the budget does not sufficiently reflect the actual assignments or activities of the faculty to be used as a source for teaching load data raises questions about the integrity of the budget and of the use of data from it for any other purpose.)

Definitions of F as headcount or budgeted fte lead to productivity and teaching load measures in which instruction and instruction-related activities are included. As a third alternative F may be defined as the fte assigned or fte of activity devoted to instruction and determined from a faculty
assignment or activity analysis instrument. The intent of definitions of this type is to avoid charging departmental research, committee work, and the like to the production of "direct" instruction. (The alternative of defining \( F \) in terms of a standard teaching load, e.g., twelve weekly class hours equals one fte, is intentionally not described, because such a formulation contaminates the basic productivity formula.)

To account for the options in defining \( F \) the superscript, \( d \), is added to the symbolism which becomes

\[
P_{ad}, S^a, C^a, PR_{ad}, TL_{ad}, \text{ and } CS^a. \tag{3}
\]

Faculty Category

The productivity formula may be applied to subsets or individual categories of the faculty of a department or an institution. Indices can be calculated for the individual academic ranks; for men and for women; for tenured, untenured-on-track, and for untenured-not-on-track faculty; for groups defined by age; and so forth. To account for this possibility a superscript, \( f \), is added to the affected symbols. One value of \( f \) might, of course, be "all". The symbolism now becomes

\[
P_{adf}, S^a, C^a, PR_{adf}, TL_{adf}, \text{ and } CS^a. \tag{4}
\]

It should be noted that the superscript for faculty category is added only to those terms directly affected. It could have been added to the course-data terms, because the requirement that faculty and course data match will cause these terms to be functions of the selected faculty category.

Basis of Course Data Measurement

As Sheehan and Gulko (1976, p. 65) point out, the course data of the productivity formula may be measured on the basis of credits, hours, or courses. To this list may be added the section as a basis for developing measures of \( S \) and \( C \). Normally the same measurement basis will be used for both variables in a single analysis. The bases of measurement can, however, be combined
by deriving ratios of measures from differing bases. For example, an estimate of $C$ based on credits may be taken as $k \times C_h$, where $C_h$ is the hour-based measure and $k$ is a previously determined ratio between the credit-based and hour-based measure.

The general nature of the measures of $S$ and $C$ derived on each of the four bases may be described as follows.

Credit Basis. Aggregates of $S$ are student credits, commonly called student credit hours, and $C$ is an aggregate of section credits. Section credits are measured at the level of the individual course section where they may be defined to be the number of academic credits a student earns by enrolling in (and passing) the section. Course schedule and registration records normally do not include section credit values for the sections of courses taught by means of different section types, e.g., a lecture section and several laboratory sections. Algorithms for calculating section credit values in such instances, that is, for allocating the course credit value among the two or more section types, can be specified on the basis of the course credit value and section weekly meeting hours.

Student credits, $S$, at the section level is the product of section credits and section enrollment. At the course level it is the product of the course credit value and the (unduplicated) number of students enrolled in the course or simply the aggregate number of credits for which the students in the course are enrolled.

It should be noted that this exposition does not provide specifications in all possible detail. The treatments to be accorded "zero-credit" courses, "variable-credit" courses and the specification of what constitutes a section, for example, are left as exercises for the user.

Hour Basis. Aggregates of $S$ are (weekly) student hours, often referred
to as student class hours or student contact hours, and \( C \) is expressed in (weekly) section hours, which may be called faculty (class or contact) hours.

The section hours, \( C \), of an organized class section is the number of hours per week the section is scheduled to meet. Class meeting days and begin and end times are often converted to section hours using rules (such as fifty minutes equals one section hour), which force section hours to be the same as credit values when this is the intended relationship. Special rules may need to be developed for "sections" other than those taught as organized classes.

The variable of type of instruction is considered below.

The student hours, \( S \), measure is defined in a fashion parallel to the definition of student credits. In general, it is a product of a section (or course) hour measure and the corresponding number of students.

**Course Basis.** Aggregates of \( S \) are course enrollment counts and \( C \) is the number of different courses taught.

**Section Basis.** Aggregates of \( S \) are section enrollment counts and \( C \) is the number of different sections taught. Note that in the lecture-laboratory course the section-based value of \( S \) exceeds the course-based value, because students are double-counted in the former, but not in the latter.

To account for the options in the measurement of course data a subscript, \( b \), is added to the symbolism of the basic formula. The symbols now become

\[
\begin{align*}
\text{adf} & \quad \text{b}^a \quad \text{CS}^a
\end{align*}
\]

It should be noted that the course-based teaching load, \( TL \), index has a different meaning from the indices based upon credit, hour, or section measures which have in common the section as the basic unit of analysis.

**Type of Instruction (or Section)**

To this point the options specified for the basic instructional productivity formula have been most clearly applicable to instruction provided in
organized classes. The inclusion of data for individual instruction "sections" in the basic formula detracts from the meaningfulness and interpretability of the teaching load and class size indices which are produced. This difficulty is removed by expanding the basic formula in a way which recognizes different types of instruction. The subscript, \( t \), is used. A value of \( t \) is assigned to each section and indicates the instructional method (intended to be) used for the section. As described here, type of instruction is an attribute of a section; that a large class is called a "lecture" section but seldom experiences a lecture does not alter the designation.

In the simplest case, \( t \) has two values—organized class and individual instruction. In the general case, \( t \) has several values for organized classes, e.g., lecture, discussion, seminar, and laboratory; and several for individual instruction, e.g., individual lesson, field/clinical, independent study, and research.

The basic formula is expanded to account for type of instruction as follows. First, the course data measures are disaggregated by type of instruction:

\[
C = \sum C_t \quad \text{and} \quad S = \sum S_t.
\]

Then, the original productivity index may be expressed as a sum of the indices for the several types of instruction,

\[
PR = \sum PR_t = \sum (S_t/F).
\]

Similarly, the teaching load index may be expressed as a sum,

\[
TL = \sum TL_t = \sum (C_t/F).
\]

Finally, the class size index is stated for each instruction type,

\[
CS_t = S_t/C_t.
\]

There are two characteristics of this expansion that bear noting. First, the productivity and teaching load indices for single types of instruction cannot be interpreted in isolation. Each is a ratio of \( S \) or \( C \) for one
type of instruction, to the total faculty, \( F \). (The next step in expansion would recognize that \( F \) may be defined and measured for each type of instruction and in practice this is done. Some analysts would argue that the disaggregation of \( F \) leads to indices which are difficult, at best, to relate to the reality of providing instruction to students.)

Second, comparisons of TL, CS, and to some degree PR, indices for different types of instruction generally should not be attempted. They are calculated because their values are expected to differ. In particular, the measure of \( C \) for one type of instruction may not be very comparable with that for another. As a matter of fact, in some applications \( C \) for individual instruction types may not be defined and TL and CS calculated for organized classes only.

The addition of the subscript, \( t \), for type of instruction to the basic symbolism produces

\[
F_{\text{adj}}, b_{t}^{a}, b_{t}^{c}, b_{t}^{T}, b_{t}^{L_{\text{adj}}}, \text{ and } b_{t}^{C_{a}}. \quad (6)
\]

Responsible Faculty

Some purposes may be served by relating one category of faculty, e.g., those with instructor to professor ranks, to \( S \) and \( C \) for all of the courses at a given level of aggregation, e.g., a department. For other purposes, interest may focus on the relationship of a category of faculty to \( S \) and \( C \) for the course sections the members of the category actually teach or for which they are responsible. Thus, a subscript, \( r \), indicating faculty category as an attribute of the course data is introduced and the symbolism becomes

\[
F_{\text{adj}}, b_{r}^{a}, b_{r}^{a}, b_{r}^{T}, b_{r}^{L_{\text{adj}}}, \text{ and } b_{r}^{C_{a}}. \quad (7)
\]

The development of course data in which faculty category is recognized requires that the teaching assignments of individual faculty members be known and, unless the responsible faculty are in the same category, that cases of shared responsibilities be resolved. Note that superscript \( f \) and subscript
Both designate faculty category. One, \( f \), identifies the category of faculty included in \( F \) and the other, \( r \), identifies the category of faculty responsible for teaching the sections which produce \( S \) and \( C \). The values of \( t \) and \( r \) may be the same, \( f \) may be differentiated and ignored, i.e., have the value "all", but it would be unusual for \( r \) to be differentiated and \( f \) ignored.

Course Level

Recognition that \( S \) and \( C \) may be aggregated by course level is provided by the introduction of a subscript, \( c \), to represent the course level variable. Course level analysis is common and generally straightforward. With this addition the symbolism becomes

\[
\text{F}^{\text{adf}}, \text{S}^{\text{ad}}, \text{C}^{\text{ad}}, \text{PR}^{\text{adf}}, \text{TL}^{\text{adf}} \text{ and } \text{CS}^{\text{ad}}
\]

Student Level

Although not as straightforward as aggregation by course level, \( S \) and \( C \) may be aggregated by student level. For some purposes student level, rather than course level, defines the appropriate basis for analysis. Values of \( S \) stated by student level by aggregating for each level over sections or courses, are familiar and meaningful. Values of \( C \) aggregated by student level may be less familiar and have meanings which are less obvious.

The value of \( C \) for a section (or course) may be disaggregated to student levels by calculating \( p \times C \) for each student level, where \( p \) is the proportion of \( S \) for the section (or course) which is counted for the student level. The student level values of \( C \) for section (or courses) may then be summed to the desired level of aggregation.

The interpretation of \( \text{PR} \), \( \text{TL} \), and \( \text{CS} \) indices calculated from measures of \( S \) and \( C \) for student levels is not identical to the interpretation of corresponding course level indices. For example, the average class size for freshman and sophomore students represents the (weighted) average size of the classes in
which freshman and sophomore students are enrolled.

As developed here the symbolism of the instructional productivity formula is completed by the addition of a subscript, s, indicating student level. The final product is

\[ F^{adf}_{s,trcs, b} b^{c}_{trcs, b} trcs, b^{pr}_{trcs, b} trcs, b^{TL}_{trcs, b} trcs, b^{CS}_{trcs} \]  

(9)

Summary

This paper has suggested specifications for the basic instructional productivity, teaching load, and class size formula. Eight specific considerations or options in the application of the formula have been suggested and some remarks on the incorporation of the options in applications of the formula have been offered.

The paper is not intended to be complete in terms of variants of the data that might be used in the basic measures and the nature of the indices that can be produced. Other analysts, if not ultimate consumers of the data, can certainly describe additional "cuts" of the data which will be meaningful and useful. However, what has been included should be sufficient to indicate that (a) the calculation and subsequent interpretation of, say, a ratio of student credit hours to full-time-equivalent faculty is not as straightforward as on the surface it may appear, and (b) the basic productivity formula, \( PR = TL \times CS \), is a flexible tool which enables quantitative descriptions of central variables of instruction along a variety of dimensions and from a variety of perspectives.

As demonstrated elsewhere (e.g., Gulko, 1972; Sheehan and Gulko, 1976) the algebra of the basic productivity formula may be extended to incorporate such constructs as the full-time-equivalent student, average student load, student-faculty ratio, and unit cost. Some or all of the variables considered in the paper are relevant to each of the additional constructs and their quantification. The natural algebraic relationships among the several
descriptors of the instructional program provide an orderliness which is a powerful aid in interpreting the quantitative data and using them to understand the operations of the programs.

The paper contains no suggestions regarding how or for what purposes, if at all, specific analyses of the types described might or should be carried out. The argument that quantitative analyses of academic variables are incapable of revealing academic reality and that such analyses lead to more harm than improvement in the academic enterprise has been avoided. The question of the applicability of the term productivity to academic affairs has been ignored. The premises of the paper are that such analyses have been carried out for years and will continue to be done in the foreseeable future and that the achievement of purposes stated for them can only be enhanced by better understandings of the measures and indices used.

The principal product of the paper is implied by superscripts and subscripts of the basic measures and indices of expression (9). To summarize, in this expression:

- a specifies the organizational and temporal level of aggregation,
- d indicates the definition or type of faculty count,
- f denotes the category of faculty included,
- b indicates the basis of course data measurement,
- t indicates type of instruction,
- r specifies the category of faculty responsible for the courses included,
- c denotes course level, and
- s denotes student level.

These are variables which can or should be considered and specified as the basis for an instructional productivity or related analysis and which by their specification can increase the likelihood that the analysis will serve the purposes for which it is undertaken.


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