Three experiments concerned with methods of teaching mathematical concepts to 2- and 3-year-olds were carried out. The first experiment, in which 12 children were taught the concepts "fat" and "skinny," showed that (1) explicit verbal representation of the concept was a more effective instructional technique than formulation in terms of an extrinsic reinforcer, and (2) a variety of concrete examples has an adverse effect on learning when the number of exemplars exceeds the minimum necessary for complete logical definition of a relational concept. The second study, in which 16 children were taught the concept "round," investigated the effects of different amounts of teacher verbalization and different forms of child action: none of these experimental variables significantly facilitated learning. The third study, in which 16 children were taught the concept "square," compared the effectiveness of different forms of verbal representation when combined with visual and proprioceptive feedback. The two instructional variables were number of verbal contexts and use of contrastive items in the correction procedure. For children who had already learned the meaning of "round," round-square confusions were reduced when the confusions were contrasted, a contrastive term did not help a child who did not already know its meaning. Confusions between square and shapes other than round were reduced when the teacher presented the term "square" in a variety of verbal contexts and the child responded by using a square object in a number of action contexts. (Author/SS)
Representing Mathematical Concepts to Two- and Three-Year-Olds through Action, Image, and Word: An Experimental Comparison of Teaching Methods

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Three experiments concerned with teaching mathematical concepts to 2- and 3-year-olds were carried out. The first experiment, in which children were taught the concepts "fat" and "skinny," showed that an explicit verbal representation of the intrinsic conceptual problem was a much more powerful instruction technique than formulation in terms of an extrinsic reinforcer ("Find the raisin"), as in a classic discrimination learning problem. The results also demonstrated that a variety of concrete examples has an adverse effect on learning when the number of exemplars exceeds the minimum necessary for complete logical definition of a relational concept.

The second study, on teaching the concept "round," employed some verbal rules in all instructional conditions. It investigated the question of how much verbalization on the part of the teacher and what forms of action on the part of the child are most conducive to learning; none of the experimental variables facilitated learning to a statistically significant degree.

The third study, on methods of teaching 2- and 3-year-olds the concept "square," compared the effectiveness of different forms of verbal representation when combined with visual and proprioceptive feedback. The two experimental instructional variables were number of verbal contexts and use of contrastive terms in the correction procedure. For children who had already learned the meaning of "round," round-square confusions were reduced when the teacher contrasted the terms "round" and "square" in the correction procedure. A contrastive term was of no use to a child who did not already know its meaning. Confusions between square and shapes other than round were reduced when the teacher presented the term "square" in a variety of verbal contexts and the child responded by using a square object in a number of action contexts.
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The concepts, "fat" and "skinny," "round" and "square," were vehicles in a search for ways to teach mathematical concepts to 2- and 3-year-old children. Originally, this search was directed by some ideas about modes of representation proper for instructing children at particular developmental levels. These ideas were based on Bruner's (1966) theory in which he describes three forms that cognitive maps of the world may assume—enactive, ikonic, and symbolic. The enactive mode encodes events in terms of action, the ikonic mode in terms of images, and the symbolic mode in terms of arbitrary systems such as language. Bruner (1966) has advanced the notion that concepts in a new area are best learned if instruction follows the developmental order of the three modes: enactive, ikonic, and symbolic. Applied to teaching extremely young children whose semantic systems are still fairly rudimentary (McNeill, 1970), this theoretical position could imply, first, that enactive and ikonic representation makes possible instruction in more advanced concepts than would otherwise be possible at a given age and, second, that once language has developed to a sufficient degree, verbal concepts may benefit from prior training in their enactive or ikonic forms.

**Teaching "Fat" and "Skinny"

The first study tested this hypothesis through an attempt to develop an instructional sequence for teaching two concepts of intensive quantification: "fat" and "skinny." In the initial phase, which took the form of a classical discrimination learning experiment, twelve 3-year-old children attending the Children's Center (described in more detail later) were required to discover under which of two cans or boxes a raisin had been placed. When the concept
to be learned was "fat," the raisin was always under the container with the larger diameter; when "skinny" was the concept, the raisin was to be found under the container with the smaller diameter. To find the raisin consistently, the child had to induce the quantitative concepts from iconic and enactive information. Seeing the cans allowed a visual image of the concepts to be formed, while the act of lifting the cans to look for the raisins provided the material for enactive representation. In the initial instructional sequence, the two cans were exactly alike save for their diameters. Thus, the materials were maximally abstract or noise-free informationally, and the child presumably had to do a minimum amount of mental abstraction to learn the conceptual principle.

Half the children were taught using the minimum number of examples to define each concept unambiguously. The other half of the children were presented with an extra example of the concept as well. What this meant in practice was that one group received two examples of the concept (Fig. 1a, 1b) while the other group received three (Fig. 1a, 1b, 1c). A single example, as in Fig. 1a, leaves the principle ambiguous: the left one could be correct either because of relative or absolute size. A pair of examples, as in Fig. 1a and 1b, resolves the ambiguity in favor of relative diameter. Apparently, complete visual definition of the concept is critical to the teaching of children and adults who lack a verbal definition of the concept in this situation. Thus, all experiments in which children and adults have successfully transposed relational size concepts to a complete range of new examples without being able to verbalize the principle, have involved two different exemplars at the

Additional examples, like Fig. 1c, offer no additional information from a logical point of view. The question was whether they would do so from a psychological point of view. The examples were intermixed and the order was basically a random one. The day after the presentation of a given concept was complete, the same concept was presented in a verbal form. Visual information was still present—the child could see the pairs of stimuli—but now the child was told: "Put this block in the fat can" or "Put this block in the skinny can." The conceptual problem is now a deductive one: to learn to apply the generalization "fat" or "skinny" to concrete situations. The enactive element is gone, for the child no longer manipulates the stimuli. The sequence of problems in verbal form was to serve as a posttest to assess the effects of the nonverbal training procedure.

The "fat-skinny" study has been described in detail just sufficient to present the results that are relevant to the studies which follow in this paper. The main outcome was that the basic nonverbal training procedure was a complete failure; only one child out of the twelve mastered the first and simplest learning sequence to the criterion of eight correct in a row within the first 24 trials, or 12 in a row thereafter.

Despite this failure, it was decided to give the verbal posttest anyway to see what could be learned from it. Applying the same criterion of eight correct in a row to the 18 items of the posttest, ten children out of 12 met the criterion for one or both of the concepts "fat" and "skinny."

As far as the extra example was concerned, it seemed to have a negative effect on learning. Those children who were given an extra instance of the
two concepts fared worse in their ability to apply the corresponding verbal labels "fat" and "skinny" on the posttest. According to an analysis of variance, this finding attained the .05 level of statistical significance.

If verbal concepts are the ultimate pedagogical goal, then purely nonverbal means appear, at best, very inefficient, according to these results. Many studies have shown that verbal labels aid in the discovery of solutions to nonverbal concept attainment problems, and a large number of others have indicated that verbalization makes conceptual principles more transferable to new situations. Few studies involving nonverbal responses, however, have addressed themselves to demonstrating the value of formulating the conceptual problem itself in verbal terms.

Reese's (1966) study of intermediate size discrimination and transposition is directly to the point here. One group of children from 34 to 63 months of age was told during training that the reward was under the "medium" stimulus; another group was told nothing about where to find the reward. The first group learned the initial discrimination faster and transposed the principle to more new examples at varying distances from the original pair, even though the concept "medium" was no longer labeled for either group during the transposition tests. Disambiguation of size discrimination problems by verbally defining the solution as relational after nonverbal training was found to lead to relational choices on transposition tests for 5- to 7-year-olds. Similarly, verbal definition of the solution as absolute led to absolute choices on transposition tests (Zeiler, 1967).

Emphasis should be placed on the function of the label in expressing the true goal of the problem. Under these conditions, a label gives the child information about why he is wrong if he fails to make a correct choice. But verbal labels may also be introduced under conditions where they do not
represent the goal of the problem and therefore cannot give informative feedback to the child. Under these conditions, verbal labels do not help transposition learning (Tighe & Tighe, 1969).

As far as the number of instances optimal for learning a concept is concerned, these results are in accord with those from other studies indicating that additional concrete examples of a concept more often impede
learning than help it. The literature in this area could perhaps best be
synthesized by saying that examples must be learned well to be of use, and
that a balance must be maintained between memory load and complete definition
of the concept (Amster and Marascuilo, 1965; Marascuilo and Amster, 1966).

These considerations led to the design of instructional procedures that
would use a concept's name while trying to teach its semantic content. The
adverse influence of the additional example of "fat" and "skinny" cans probably
stemmed, at least in part, from the fact that all examples were presented
before any single one was mastered. For this reason it was decided that in
future teaching procedures mastery in applying the concept within a particular
concrete situation would be requisite to going on to a new situation. Examples
beyond those required for complete conceptual definition were also to be
avoided.

Based on these results, the next study compared methods for teaching
the geometric concept "round."

Teaching "Round"

Granted that at least some verbal rules are called for when working
with 3-year-olds, one question still to be answered was how much verbalization
to introduce and at what stage in the learning process. A second question
involved the forms of action that are most productive for conceptual learning.
The study on teaching "fat" and "skinny" demonstrated that mere manipulation
of objects exemplifying conceptual relationships is not sufficient. The
Montessori method emphasizes a certain type of action in the so-called
"self-teaching" materials (Montessori, 1966). This type of action consists
of manipulation with corrective feedback; an example is the action involved
in fitting puzzle pieces together.
Three teaching methods were compared; all involved the same sequence of stimulus displays. In each display, the child was asked to pick out the "round" one. Round sometimes meant a piece of wood, a hole, or a cup. The displays involved two or three stimuli, and "round" was contrasted with triangular, square, kidney-shaped, and hexagonal. Sometimes shape was the only variable; in other displays, color was introduced as an irrelevant variable. The displays were arranged from easy to hard on an a priori basis.

In all conditions the child was told whether he was right or wrong and induced to correct an incorrect response; he was also rewarded with a raisin when he was right and lost one when he was wrong. This reinforcement or feedback procedure was designed to maximize learning for all children, and it was based on the results of a number of different studies (Eisenberg, 1966; Gollin, 1966; Meyer and Seidman, 1960; and Zigler and Delabry, 1962).

Under the first instructional condition, the teacher used the label "round" and the child manipulated the stimuli and received the corrective feedback of the Montessori type. The second method was identical except that the child manipulated the stimuli without receiving any corrective feedback for his action. In practice, this meant that whereas in the first condition the round stimulus was the only one that could fit somewhere (for example into a round hole), in this condition any stimulus would fit into the large oval hole where the child had been asked to place the "round" one. The third condition was like the first, except that the label "round" was not used until the beginning of the second half of the training. Up to that point the child was asked merely to find the one that "fit," that is, a piece to fit a hole. There was also a control group who received no training.
There were 16 children in the study ranging from 2 to 3 years of age. All the children were given a test before and after training. Half the test displays consisted of new or transfer stimuli not used in the training, while the other half were included in the training. Half the transfer items and half the familiar items used the verbal concept "round" in the instructions but provided no action feedback, whereas the other half provided only action feedback. The items on the test were randomly ordered, and the children were in no way informed as to whether they had given a correct answer.

Although all three of the training groups showed more positive change from pre- to posttest than did the untrained control group, the effect of training on the total test or any of its component parts was not statistically significant according to an analysis of variance. It is interesting to note, however, that the largest difference between the control and training groups occurred with respect to improvement on the verbal transfer items which, it will be remembered, provided no action feedback. Similarly, the differences between the changes produced by the various training methods were not large enough to be statistically significant, either for the total test or for any of its parts. Again, though, the largest gain occurred on the verbal transfer items and was made by the group whose training had consisted of manipulation with corrective action feedback, as well as labeling throughout.

Since the results of this study were largely negative, its main use was as a pilot to the next study to be described.

Teaching "Square"

The "round" study suggested that if correct referential use of the descriptive term "round," especially in new situations, was the ultimate
goal of training, then "manipulation with corrective feedback plus labeling" might be the best method. Consequently, the techniques for teaching "square" were all based on this method, although the sequencing was improved to reflect the order of difficulty that had emerged from the earlier data.

The principle question which this study asked was: Exactly what should the teacher say about the concept to be taught, granted that some sort of verbal representation was critical.

Psychologists generally stress the relation between word and referent in their treatment of the development of word meanings. Linguistic discussions of word meaning, in contrast, revolve around the relation of words to other words (Olson, 1968). The experiment on "fat" and "skinny," just described, indicated that increasing the number of referential relations of the word in question can have, under some circumstances, an adverse effect upon learning the concept. But what about increasing the number of verbal relations? How does variation in verbal context affect conceptual learning?

The particular way in which this question was applied to teaching the concept "square" to 2- and 3-year-olds was derived from some Russian work on the effect of a variety of verbal and action contexts on very early vocabulary learning. In one study by Kol'tsova (Razran, 1961), 20-month-old children were presented with a doll 1500 times. To half the children, the doll was presented in three contexts: "Here is a doll," "Take the doll," and "Give me the doll." The other half were exposed to 30 different contexts: the same three plus others like "Rock the doll," "Feed the doll," and so forth. The test was to "pick a doll" out of an array including the experimental doll, other dolls, and other toys; the children were to learn the range of reference
of the word "doll." Those children who had had the experience of 30 different contexts were more successful in discriminating dolls from non-dolls and also responded to the task more quickly. Thus, the variety of verbal and action contexts in which a word is placed has a positive effect on learning its referential meaning, independent of the variety of concrete referents with which the word is associated.

In another experiment, Koltsova (Razran, 1961), using 18-month-old children, showed that placing a word in a number of different contexts is more effective in teaching the meaning of that word than exposing the child to an equal number of referents or concrete examples.

Although the evidence points to the value of a variety of linguistic contexts on the stimulus side and a variety of action contexts on the response side in teaching concepts of all sorts, no one has investigated this variable in relation to learning analytic or attribute concepts. The study to be described assessed the effect of variety of verbal context on learning the meaning of the geometric term "square."

The role of verbal contrast in fostering conceptual learning was the other issue taken up by this study. Brown (1958), Werner (1968), and Vygotsky (1962) suggest that "true" concepts are a synthesis of discrimination and generalization. In terms of the stimuli or examples of a concept given in a learning situation, this principle is tacitly assumed: the learner is exposed to both positive and negative instances of the concept in question. Within linguistics, the role of contrast is more central and more explicit in its relation to language learning. On the phonological level, for example, the functional sound units--phonemes--are traditionally defined by oppositional
pairs. Clearly, the contrast case defines the boundary of the phoneme concept and thus is as crucial to its precise definition as the positive instance is. From a logical point of view, this is true of any concept. In perception, we know from the work of Garner (1966) that the array of contrasting items in which a particular item is found is critical to its identification. There are indications from work in West Africa (Greenfield, Reich & Olver, 1966; Price-Williams, 1962) that the nature of the contrast cases can be critical in determining whether or not a certain classificatory principle is brought into play at all, and that this is more the case the less various classificatory principles are integrated into a single unified system. Similarly, it is clear from the work of Lantz and Steffire (1964) that the value of a particular linguistic encoding for identifying or remembering a stimulus depends on the array in which the stimulus is found or more accurately, on whether it differentiates the referent from the other stimuli in the array. Extending the role of contrast, Olson (1970) has recently advanced a general view of semantic encoding as the partitioning of alternatives. Is it also the case, then, that the ease with which a conceptual label can be learned and applied to concrete examples depends on the existence of contrasting terms to denote the alternative stimuli in the array? This question has never been asked experimentally, although Bereiter and Engelmann (Osborn, 1969) have exploited the idea of a system of contrasting terms in teaching disadvantaged preschool children; they present attribute labels from the outset in the company of their polar opposites. Clepareede (Olsen, 1968) proved experimentally that dissimilarity leads to awareness. Thus, one might hypothesize that contrasting terms would be of value not only in promoting the bare learning of a particular concept, but also in fostering conscious control over conceptual knowledge.
The study to be described incorporated contrastive terms into the correction procedure and tested the value of doing this for teaching the meaning of "square" to 2- and 3-year-olds. The particular contrastive terms used were therefore dependent on a particular child's errors and on the stimulus array. The way in which this worked in practice will become clear when the teaching sequence and procedure is described in detail.

The Children

All the children in the study were attending the Children's Center in Syracuse, New York. The Center, organized and run at this time by Dr. Bettye M. Caldwell, is a school for children from the age of 6 months to kindergarten (Caldwell and Richmond, 1968).

The children ranged from 2 years, 6 months to 3 years, 10 months. Because of the small numbers available there was in fact no sampling. All the children available for the duration of the study were used unless they had a sibling in the study or already knew the concept "square" as evidenced by a perfect score on the pre-test. Four children attained perfect scores; their average age was 3 years, 7 months. The average age for the 18 children in the study was 3 years, 1 month. This slight contrast in age is not statistically significant.

As for the social makeup of the groups, three of the four who had the concept at the outset were middle-class white children. The fourth was a lower-class black child. In the group who failed to achieve a perfect score, only three out of 18 were middle-class, two white and one black. Thus, class background made a big and statistically significant difference in initial level of knowledge of the concept (Fisher Test, p. < .05).
Design of the Study

Eighteen children who did not demonstrate perfect knowledge of the meaning of "square" on the pre-test participated in the study. The total group of 18 children was divided into three age levels. Within a given level, children were randomly assigned to one of the five groups comprising the study. One-third of these children were in a control group; they were not taught the concept "square," but they were given the "square" test twice—at the same interval as the 12 children who were exposed to one of four types of experimental teaching.

All four groups who were exposed to some form of experimental teaching received the same sequence of instructional materials (Fig. 2) presented in different ways. Basically, all these children were given practice in picking out the square stimulus from arrays which became increasingly difficult as the instructional sequence progressed. The word "square" was used in every instructional condition, and every child handled the square stimuli with corrective feedback at least some of the time. In other words, all four teaching methods were variants on what the "round" experiment had shown to be the most promising approach. Test conditions contrasted with teaching conditions in that there was no corrective feedback for manipulatory action.

The first experimental variable was variety of context. The word "square" in a single verbal and action context was presented to six of the 12 children. For example, if the task was to find a piece to fit a square hole, the child would be told "The square piece just fits in here (teacher pointing to hole). Can you find it and put it in?" The other six children were presented the
word "square" in multiple verbal contexts and consequently manipulated the square stimulus in a variety of ways. For these children, eight of the 23 instructional arrays, shown in Fig. 2, were presented using the type of instructional sentence just described. But two other types of verbal contexts were also introduced: "Put the square piece in this can" (eight arrays shown in Fig. 2) and "Give me the square piece." (seven arrays shown in Fig. 2). The corresponding action contexts are clear, but it should be noted that they involved no corrective feedback; any piece in the array would "fit" equally well into the can or the teacher's hand.

Each of these two instructional groups was also divided in half according to the type of correction procedure used. For half of each group the correction procedure involved labeling the shape of the stimulus that had been wrongly picked. This was the "contrastive terms" group. The other half of the group was corrected the same way except that the contrastive label was omitted. For example, a child is presented with an array consisting of a square piece, a round piece, and a square hole, and is told "The square piece just fits in this hole. Can you find it and put it in?" He responds by choosing the round piece. If he is in the "contrastive terms" group, he is told: "No, this one (teacher points to it) is round. This (teacher points to is) is the square one. Can you put it in?" If the child is not in the "contrastive terms" group, he is simply told "No, this (teacher points to it) is the square one. Can you put it in?"

Thus, four instructional groups are generated. They can be described as follows:

Group 1: the word "square" is presented in a variety of verbal contexts, and errors are corrected by using a term contrastive to "square."
Group 2: the word "square" is presented in a variety of verbal contexts, and errors are corrected without introducing any contrastive terms.

Group 3: the word "square" is presented in a single verbal context and errors are corrected by using a term contrastive to "square."

Group 4: the word "square" is presented in single verbal context and errors are corrected without introducing any contrastive terms.

Procedure and Instructional Sequence

Testing the concept "square." All the children except one (who was added to the study later because two children stopped coming to school) were given the "square" pre-test individually by one of the regular teachers in the school whom the children all knew. The one exception was tested by the author. The same form was administered as a posttest by the author. Eight of the 12 children who were taught the meaning of "square" were retested the day after their training was completed. The maximum interval from teaching to posttest was eight days. The control group was retested at the same time.

Following an introduction to the test situation, the child was given a warm-up task in which he was given practice in taking the test without having to discriminate shapes. The test proper, which followed, comprised 17 items arranged in a random order (Fig. 3). In each item, the task was to pick out the square element from an array and place it in an oval from which was large enough so that all items fit equally well. Some of the displays (numbers 4, 11, 12, 13, 16, and 17) later appeared in the teaching materials, while all the rest (numbers 1-3, 5-10, 14-15) were to be found only on the test.

As to feedback, the tester could give general encouragement but no specific information about whether a response was right or wrong.
Teaching the concept "square." The teaching was carried out by the author about five weeks after the testing. (The only exception was the one child added to the study late who was trained two days after being tested.) Nine of the children mastered the instructional materials in one session, two required two sessions, and one child took three sessions to master the sequence. None failed.

The teacher introduced the instructional session by saying, "We are going to play a game. Every time you are right, I will put a raisin in your cup. When you make a mistake, I will take a raisin away and put it in my cup. O.K.?” The child was also praised when right and corrected when wrong in one of the two ways demanded by the experimental design.

The sequence of materials, which was the same for all four experimental groups, comprised five sections, each of which had to be mastered before proceeding to the next, theoretically more difficult, one (Fig. 2). (The brackets next to sections B and C indicate how they were further subdivided.) Each section was made up of a set of conceptually related stimulus arrays from which the square item had to be selected. The order of difficulty of the various sections was based on the results of the earlier study on teaching the concept "round." In many cases, this order substantiated earlier research on stimulus factors complicating the attainment of concepts (e.g., Meyer, 1968; Osler, 1966; and Piaget, 1952).

Analysis of Data

The main analytic tool was an analysis of variance and of covariance (Steel and Torrie, 1960) in which the dependent variable was the change in test scores from pre-test to posttest. Initial level of knowledge of the concept "square," as displayed on the pre-test, served as the covariable. It
was thought that the use of initial score as a covariable would increase accuracy in determining the effectiveness of the various instructional methods by taking into account the "law of initial values": the more initial errors, the more room for improvement. This turned out to be the case. There was a correlation of .27 between number of initial errors and amount of improvement. A comparison suggested by Steel and Torrie (1960) of the treatment variance before and after adjustment for initial level indicated that the introduction of the covariable into the analysis yielded a 9% gain in precision.

Although the variable of social class was supposed to be randomly distributed among the groups, the small number of middle-class children made even distribution impossible. In point of fact, two of the three middle-class children ended up in Group 1 and the remaining one in Group 4. Although this study was not concerned with the effect of social class, it did seem necessary to show, where applicable, that group differences are not entirely attributable to differences in social class composition. Auxiliary analyses were used to this purpose.

With a small sample, effects must be substantively large and consistent among subjects to achieve statistical significance. Thus, it is well to bear in mind that statistical significance in the present study indicates substantive significance as well. In addition, an attempt has been made to account for the behavior of every individual child through subsidiary analyses.

Results

Age was not a significant factor in determining either initial score or improvement. Thus, the objective of comparing various instructional techniques in terms of their effect for a single age group could be pursued, and age was not taken into account in the remaining analyses. It must be remembered that the children spanned only the range from 2 years, 3 months, to 3 years, 10 months
and, in fact, the expected trend for older children to start out with higher scores did appear.

The basic fact about the instructional procedures was that they were successful. The average pre-test score for the entire group of 18 children was about 50% correct or 8.2 out of a total of 17 items. The control group showed, on the average, no change on the posttest in their knowledge of the concept "square." In contrast, the four instructional groups eliminated an average of 5.3 errors or 56% of all pre-test errors. Using data from all five groups, analysis of covariance, summarized in Table 1, indicated that the various groups did reliably differ from each other in amount of change from pre- to posttest, controlling for initial score. This overall effect attained the .005 level of statistical significance. Even without statistical control of initial level, however, the groups demonstrated reliably different amounts of improvement. The analysis of variance of the change scores yielded an overall difference that was statistically significant at the .01 level ($F = 5.68; df = 4.13$).

Dunnott's test for making a series of comparisons with the mean of a single control group showed that each instructional group except the "single context-contrastive terms" group was reliably different from the control group at the .01 level of statistical significance. As in the remaining results to be reported, this comparison relates to change scores corrected for initial level of competence.

Let us now compare the four instructional groups with each other, rather than with the control group, in order to see precisely how the two instructional variables were operating.
Variety of context exerted a strong positive influence on learning the concept. A one-tailed t-test indicated that the effect of this factor was statistically significant at the .025 level ($t = 2.4$). On the average, scores (corrected for initial level) improved 7.25 points more for the six children exposed to "square" in a variety of contexts than for the six children exposed to the concept in a single context. (One point is given for each item correct.)

Although the level of improvement was greater for the three middle-class children, they did not deviate from the overall pattern. Table 2 makes this point clear.

The use of contrastive terms did not, in itself, affect learning the concept. There was, however, a sizeable and unanticipated interaction between the two factors such that the use of contrastive terms in the correction procedure did facilitate learning when it was used in combination with a variety of contexts. A two-tailed t-test indicated that this interaction effect achieved statistical significance at the .05 level ($t = 2.4; df = 7$). One may conclude that contrastive terms are of use under some conditions but not others. An analysis of pre-test errors shows how the pre-existing semantic system of conceptually related terms determines the effect of contrastive terms in learning a concept.

The analysis of errors on the "round" test in the earlier study supported the idea that identifying an example of the concept became more difficult when the number of items in the array got larger and when it was necessary to abstract the two-dimensional shape from a three-dimensional object. An analysis of errors on the "square" pre-test indicated that these factors were much less
powerful. Instead, a tendency to confuse "round" and "square" emerged as the greatest source of error. Table 3 uses the difference between the observed and

Table 3 about here

the chance rates of success for a given item as a measure of relative difficulty and shows that the inclusion of a round stimulus increased the difficulty of an array to a statistically significant degree (p = .01 by a two-tailed median test).

What is the significance of this tendency to confuse round and square? Is it simply attributable to the fact that certain children (randomly distributed among groups) were exposed to the concept "round" in the earlier study? This is not the case, for the "inexperienced" group confuses round and square just as often as the group who participated in the "round" study. The earlier experience with "round" does not affect total errors either; the inexperienced group averages 9.2 pre-test errors out of 17 items while the inexperienced group averages 8.5. Thus, it appears that, somehow, "round" is the more basic, perhaps the unmarked, concept in this particular situation.

The application of the unmarked-marked distinction originated by Jakobson (1969) would lead to the following hypothesis: the unmarked geometrical shape "round" will be the first to be recognized as such by the child and he may take for granted that all geometric terms refer to this category. This initial hypothesis must then be corrected through appropriate feedback. Indeed, there is evidence of the marking process in the semantic development of quantitative terms (Donaldson and Balfour, 1968; Donaldson and Wales, 1970). The results of Donaldson and Balfour indicate that "more" is unmarked relative to "less" for 3-year-olds interpret both "more" and "less" as "more."

The situation is analogous to the present one in which both "round" and "square" are interpreted as "round."
This argument is based on the notion of a predetermined or natural order of concept acquisition. The only other possible explanation of the "round"-"square" confusion would be in terms of earlier geometrical teaching. Since 12 of the 18 children in this study had been in a previous systematic study of geometrical discrimination conducted by Henning and Hayweiser (1968), we have some evidence that bears upon this point. In that study, the children were to learn a circle-triangle discrimination. First, they had to learn to choose the arm of a T-maze containing a large triangular stimulus. Then the rewarded stimulus was reversed and the children had to learn to choose the "round arm" of the maze. Since all the children had more experience with the triangle as the positive stimulus than with the circle, and since only three out of the 12 ever mastered the concept "round" in the reversal situation, this nonverbal experience, if anything, should have worked in the direction of diminishing the tendency to choose a round stimulus. Therefore, these experimental results tend to discredit specific experiences as the critical factor in generating the tendency to round-square confusions. Henning and Hayweiser (Henning, personal communication, 1969) found, moreover, that there was a definite tendency for the children to start out in their learning procedure by choosing the arm of the maze containing the round stimulus, even though it appeared on the opposite side from the child's previously determined preferred position. Thus, the shape "round" seemed perceptually dominant for a substantial group of these very same children even a year or more before the present study was begun.

What is interesting is the way in which previous experience with the term "round" determines the effect of using contrastive terms in the correction procedure. In practice, the introduction of contrastive terms meant primarily...
contrasting "square" with "round," for 50% of all errors during the teaching phase consisted of mistaking round for square. It turns out that contrastive terms do effectively reduce confusion between "round" and "square" if the child has been systematically exposed to the semantics of "round" in the earlier study in this project. It seems as though refraining from mention of the term "round" is as effective for the "inexperienced" child as is the "round"-"square" contrast for the "experienced" child. From these data emerges the idea of appropriate and inappropriate introduction of contrastive terms. Contrastive terms really means the single term "round" here. "Appropriate" applies both to the case where the term "round" is not introduced because of lack of requisite semantic experience and to the case where the term is introduced because the relevant experience has been acquired through participating in the earlier experiment.

Because the contrastive-terms variable seemed to have a specific effect on round-square confusions, it was thought that the number-of-contexts variable might have a specific effect on other types of confusion. Thus, multiple verbal and action contexts might operate to help the child distinguish square from other shapes perceptually less distinctive. This formulation led to the prediction that those children appropriately introduced to contrastive terms would show the greatest relative reduction in round-square errors, and those children exposed to "square" in multiple contexts would show the greatest relative reduction in other types of error. Thus, every child was predicted to fall above or below the median in relative reduction of round-square errors, and above or below the median in relative reduction of other confusions. The probability of correctly predicting both aspects of a child's behavior by chance is .25. Eight out of 12 children conformed to the prediction for both
aspects of their behavior. (Two out of three middle-class children and six out of nine lower-class children conformed to the prediction.) The probability of this occurring by chance is .0016 according to a two-tailed binomial test. Thus, the following pattern of effects seems to be a reliable one. For children who have already learned the meaning of "round," round-square confusions are reduced when the teacher contrasts the meaning of "round" and "square" during the correction procedure. For children who have not already learned the meaning of "round," round-square confusions are reduced when the teacher demonstrates the meaning of "square" but does not use the contrastive term "round" during the correction procedure. Finally, multiple contexts help all children distinguish square from other less distinctive shapes.

Discussion

The overall success of the instructional methods are relevant to the question of what modes of representation are most effective in teaching 2- and 3-year-old children. In all cases, the children being taught the meaning of "square" were presented during the teaching phase with a representation in three modes simultaneously--the enactive, the ikonic, and the symbolic. The enactive mode entered when the child picked up the square object and did something with it. For some children, the particular act varied from array to array. The stimulus arrays themselves furnished an ikonic representation both of the class of square things and of contrasting classes of geometric shapes. More important was the fact that the solution to the problem was represented in all three modes. The word "square" inserted into a command provided a symbolic representation of the solution. There were, moreover, perceptual cues and kinesthetic feedback on at least one-third of the training items in the form of something with which the square thing would
fit. This additional square stimulus thus provided an ikonic and enactive representation of the solution to the problem of discovering the referent of the term "square."

The simultaneous representation of the problem in three modes during teaching resulted in an ability to deal with the concept later in the posttest when primarily different exemplars were presented in a new action context with no ikonic or enactive cues to the solution. During testing, the only representation of the solution was the symbolic term "square." These results suggest that the ideal sequence of representational modes for learning symbolic concepts can be imagined as a gradual peeling away of enactive and ikonic support from an originally trimodal representational structure.

Experiments by Carlson (1969), Peterson (1969), and Svenstrom (1966) comparing methods for teaching conservation of quantity, as well as Kohnstamm's (1967) work on classification, suggest that efficient learning depends on initial access to a multimodal representation of the problem.

The contrast between this experiment where the child is faced with a trimodal representation contrasts with the first study of teaching "fat" and "skinny." There, although the child has visual and kinesthetic contact with exemplars, the solution (that is, the concept) is really not represented in any mode at all. The "fat" and "skinny" posttest, in which the solution was represented by a label, demonstrated the effectiveness of a verbal representation. Bem (1970) has found verbal representation of the goal state to facilitate the solution of much more difficult problems at later ages. The absence of any representation of the solution or true goal is what is usually meant by the term "discovery" learning. In the literature, it is opposed to directed
teaching or to "guided discovery." Evidence seems to be accumulating that pure discovery is the least successful method for teaching specific concepts and that it does not improve the transferability of a concept (Kersh and Wittrock, 1967). Discovery methods seem to be useful when the aim is to teach techniques of discovery, per se. If, however, the task is so difficult that the learner does not succeed in discovering the concept, then the discovery technique will not be reinforced either (Kersh and Wittrock, 1967). The impossibility of discovering the concepts "fat" and "skinny" in the first experiment suggests that other analytic concepts, indeed, most concepts upon which school focuses, may be too difficult at young ages to be "discovered" successfully. If so, then the practice of requiring discovery, as in the methods used to teach "fat" and "skinny," not only fails to teach the concept but may also discourage the development of the discovery process itself.

A newly translated Soviet article (Pushkina, 1970) on the development of the relational concept "smaller" indicates that "discovery" of the solution to a nonverbal discrimination problem can also be most effectively promoted by an initial trimodal representation of the concept. In a series of training experiments with 1- to 5-year-olds, Pushkina found that a combination of visual, verbal, and tactile activity with feedback was better than two other methods. In one of the less successful training experiments, children were asked to compare verbally the size of a number of different object pairs; in the other one children manipulated pairs of objects in a way that required perceptual size comparisons. Again, effective sequencing goes from trimodal representation to representation in a single mode and from highly informative feedback to less informative feedback.
The methods used to teach "fat" and "skinny" contrasted with those used to teach the concept "square" in a second respect: two or three examples of the concept had to be mastered simultaneously. The results indicated that the extra example actually hindered learning. In the methods used to teach the meaning of "square," a given type of discrimination was always mastered before the child was exposed to new examples. A third difference from the earlier study was the absence of prolonged repetition of stimulus displays. This time it was possible to face each stimulus display only once, whereas in the earlier study the minimum number of repetitions in the learning sequences was four under two of the conditions. Repetition had had such a negative effect on the children in the first study that it was almost impossible to complete that experiment. This was one of the motives for making variety an explicit variable in the final study.

One feature that the two studies had in common was the use of both material and verbal corrective feedback for right and wrong responses, and self-correction for wrong responses. The results of the two experiments clearly show that such theoretically optimal feedback is not a sufficient condition for learning a given concept, although it may be effective when the proper representational conditions are met.

While it is interesting to compare the methods used in the two studies, the difference in the success of their teaching techniques could be due to any or all of these three reasons and to some others besides. Nevertheless, it seems useful both for future research and for practical application to make these differences explicit.

Similarly, a comparison of the techniques used to teach "round" with those used to teach "square" suggests some interesting possibilities. The
variable in the "round" study dealt with the **forms of action representation** and the **timing of verbal representation**. Neither seemed to matter very much. In the "square" study, the variables all involved the **forms of verbal representation**, and the differences were much larger.

The untaught control group showed some improvement in the "round" study whereas they did not in the "square" one. Perhaps it was natural to attach a label to the most perceptually dominant category of stimuli, the round ones, whereas it demanded more specific experience to label the less dominant "square" category. This result, therefore, is another piece of evidence in favor of the hypothesis that "round" is the unmarked or more basic geometric concept on which the square category is superposed. Research on infant perception also points to round as the naturally more primitive geometric concept: infants between ten and thirteen weeks of age will reliably follow a moving two-dimensional round shape in preference to a similar square one (Graefe, 1963; described in Kessen, Haith, and Salapatek, 1970). It is interesting that the natural dominance of certain perceptual categories should influence the referential properties of words in semantic development. After the completion of the experiments described here, Heider (1970, 1971) reported related evidence in the domain of color as well as shape. Most relevant in the present context is the finding that shape categories have natural focal points and that these are related to the use and learning of shape terminology. Her data support, moreover, the notion that round is a more basic or primitive concept than square.

The experiment on teaching "square" actually varied the form of the verbal context in which the term "square" was placed while holding all other factors constant. Psychologists have tended to devote a great deal of energy to the question of whether verbal representation enters into concept formation at all and have generally neglected the problems of what form this representation should take. It is, therefore, interesting that variables relating to form of
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verbal representation should turn out to be such powerful ones. This point is especially interesting since the children were so young. Clearly, the original idea that pure action might be most suited to this developmental level was exploded by the results of several studies. The experiment on teaching "square" seems to indicate that what is said about a given concept label is as important to the development of the child's definition as whether the concept is labeled at all. The main effect of a variety of verbal contexts on the stimulus side seems to be in making exemplars of a given concept distinguishable from less perceptually dominant stimuli. When a more dominant category exists along a given dimension, in this case "round," the category tends to be over-generalized to other stimuli in that dimension and the form of correcting this overgeneralization becomes crucial. Labeling the category to be learned is an ideal way to mark it if the child has not yet labeled the overgeneralized or unmarked contrast class. If he has, however, then effective teaching seems to require explicit contrast between the two labels and their referents in the correction procedure. This finding adds to the general principle demonstrated by Cole (1968) with much older children and with adults that instructional effects are enhanced if the conceptual principle being taught is explicitly contrasted with other developmentally prior principles relating to the same domain.

Probably, contrastive labels do not work with children who do not already understand the label fairly well for the same reason that an extra exemplar did not work in the "fat"-"skinny" study: there are more things to learn at once, an added cognitive burden.

Note that multiple verbal contexts on the part of the experimenter involved multiple action contexts on the part of the child. This latter may
be crucial to the effectiveness of a variety of contexts, although speech and action factors cannot be disentangled in this experiment. We may have found a more precise way of understanding the frequent observation that children do not learn what is irrelevant and that they sometimes do not display their "true" abilities for want of motivation. If one thinks of the action context as providing a goal, then the meaning of a particular word included in the directions becomes a means to this goal. One could say, at a molecular level, that the concept has "relevance" in a broader task structure and that the child now has a "reason" to learn its meaning. Recent studies of nonverbal skill development suggest that, indeed, the goal precedes the means ontogenetically, and that when a successful means first comes into being it is indissoluble from the goal which motivated its existence. Only gradually does it become a separate entity able to be applied to a variety of ends (Bruner, 1968). Miyamoto's (1969) study of concept formation among the Zinacanteco Indians indicates that this principle applies to verbal concepts as well. He found that the broader the variety of action contexts into which objects fitted in Zinacanteco culture, the more likely they were to be called similar on the basis of common class membership and the less likely they were to be called similar on the basis of association in a common action. In other words, multiple action contexts seemed conducive to the development of intensive definitions closely related to the extensive definition at which the "round" and "square" training was focused. The implication is that the broader the variety of ends for which a given concept can be used, then the more likely it is to achieve an independent life of its own. Surely in everyday life those concepts most important to action are the ones placed in the greatest variety
of contexts and therefore learned the most thoroughly. The role of familiarity then becomes that of providing a higher order structure into which the unfamiliar can be fitted as a necessary component.

There are several types of variety in this study, and variety probably has several different effects. First of all, there is the variety or nonrepetitiveness of the stimulus displays, a quality which contrasts with the approach of the first study. Certainly this difference must be partly responsible for the much more enthusiastic response of the children to the procedures used to teach the concept "square." At the same time, we see that sheer variety can have adverse effects on learning if it is associated with increasing the number of things to be mastered simultaneously. One way to deal with this potential conflict is to ensure mastery of each new element as it arises. This was one strategy in the "square" study. Another way is to provide a variety of familiar elements. The verbal and action contexts in which "square" was embedded were familiar and served this function.

All these types of variety—stimulus arrays, words, and actions—could be operating to improve learning through enhancing attentional processes. Certainly there is a great deal of evidence that variety has this effect (Fiske and Maddi, 1961). Enhanced attention does not explain, however, why a variety of verbal and action contexts has a positive effect on learning that is much larger than that produced by a variety of referents.

The variety of verbal and action contexts certainly promotes the relevance of the concept, as has been discussed before. Multiple verbal contexts may also be particularly useful in illustrating the semantics of terms which have no stable relation to extralinguistic contexts, that is, to concrete referents.
Some vocabulary test data indicate that it is just such words which pose the greatest problems for lower-class children (John and Goldstein, 1964). The finding here is consistent with this line of reasoning, for it is fairly well established that lower-class people tend to rely relatively more on extralinguistic context and relatively less on linguistic context in their use of language for communication (Greenfield, in press).

The results of a study by Werner and Kaplan (1950) provide yet another reason why multiple contexts might be more effective than a single context in teaching a concept. They found that younger children do not differentiate a word from its verbal context and may regard a given word as carrying the meaning of the whole or a part of the context. If this is so, then variable contexts may help in establishing the invariant properties of word and referent.

In terms of preschool education, and compensatory education in particular, the results of this series of studies seem to indicate that what is said about concrete experience is more important in the semantic development of analytic terms than the nature of the experience itself. This conclusion seems to support Bereiter and Engelmann's (1966) approach to the education of severely deprived preschool children, for they use a wide variety of statement forms or logical relations in connection with a small variety of concrete referents. On the other hand, if the action context constitutes a goal which gives relevance to the words embedded in a related sentence, then use of a given concept in a wide variety of meaningful situations could be an equally critical factor.

In general, these experiments support the linguistic approach to semantics rather than the psychological: the relation of words to other words appears more crucial in semantic development than the relation of words to things. They also provide experimental evidence against preschool instructional approaches
which emphasize direct experience with materials and leave unspecified the communicative context in which all such experience must take place. The study of ways to teach the meaning of the term "square" to 2- and 3-year-olds that has been reported here suggests that systematic examination of this question will bear fruitful results.
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Table 1

Analysis of Variance and Covariance for Change in Knowledge of "Square" from Test to Retest under One Control and Four Instructional Conditions

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>$\xi^2$</th>
<th>$F$</th>
<th>$\xi_{xy}$</th>
<th>$\xi^2_{X}$</th>
<th>df</th>
<th>$\xi^2_{Adjusted}$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditions</td>
<td>4</td>
<td>177.1</td>
<td>5.7*</td>
<td>17.1</td>
<td>59.8</td>
<td></td>
<td></td>
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<tr>
<td>Error</td>
<td>13</td>
<td>101.3</td>
<td></td>
<td>70.6</td>
<td>252.0</td>
<td>12</td>
<td></td>
<td>81.9</td>
</tr>
<tr>
<td>Total</td>
<td>17</td>
<td>278.4</td>
<td></td>
<td>87.1</td>
<td>311.8</td>
<td>16</td>
<td></td>
<td>254.1</td>
</tr>
<tr>
<td>Conditions Adjusted</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>172.2</td>
<td>8.4**</td>
</tr>
</tbody>
</table>

* $p < .01$

** $p < .005$
Table 2

Average Percentage of Pre-test Errors
Eliminated After Two Types of Instructional Treatment
For Middle-Class and Lower-Class Children, Separately and Combined

<table>
<thead>
<tr>
<th></th>
<th>Multiple Context Treatment</th>
<th>Single Context Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>N</td>
</tr>
<tr>
<td>Lower-class Children</td>
<td>71%</td>
<td>4</td>
</tr>
<tr>
<td>Middle-class Children</td>
<td>100%</td>
<td>2</td>
</tr>
<tr>
<td>Total Group</td>
<td>81%</td>
<td>6</td>
</tr>
</tbody>
</table>
Table 3

Rate of Success on Pre-test for Arrays with and without Round Stimuli

<table>
<thead>
<tr>
<th>High Success Rate (from 28% to 11% above chance)</th>
<th>Arrays With Round Stimuli</th>
<th>Arrays Without Round Stimuli</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Low Success Rate (from 16% below chance to 8% above chance)</th>
<th>Arrays With Round Stimuli</th>
<th>Arrays Without Round Stimuli</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

P < .01, Median Test
Figure Legends

Fig. 1 Some displays to represent concept "fat."

Fig. 2 Sequence of displays used to teach meaning of term "square."

Fig. 3 Displays for testing understanding of term "square."
Fig. 1 Some displays to represent concept "fat."
Fig. 2 Sequence of displays used to teach meaning of term "square."

VERBAL AND ACTION CONTEXTS

- a. Just fits
- b. In can
- c. Give to teacher
Fig. 3 Displays for testing understanding of term "square."
I should like to thank Dr. Bettye M. Caldwell, formerly Dual Professor of Child Development, Syracuse University, for giving me the opportunity to participate in the life of the Children's Center and to carry on research there. Mrs. Melanie Applegate, a teacher at the Center, did the major part of the testing and a significant portion of the teaching in a most skillful and sensitive way. Dr. Paul Sheehy of the Department of Preventive Medicine, Upstate Medical Center, gave invaluable help with the statistical design of the experiments and planned many of the analyses. Mr. Herbert Weisberg of the Department of Statistics, Harvard University, completed the statistical planning, and Miss Alice Galenson of Radcliffe College helped with the computations.

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The twelve children with experience in the Henning-Hayweiser study were randomly distributed among all five groups in the present study. There was no relationship between participation in that study and our results. For example, five out of six members of our control group were trained in
the t-maze; yet the control group showed no improvement from pre- to posttest. There was, moreover, no correlation between participation in our earlier study to teach "round" and participation in the Henning-Hayweiser procedure. Children who had been through the t-maze procedure were as evenly distributed as possible among the four relevant groups: "experienced" verbal contrast, "inexperienced" verbal contrast, "experienced" no contrast, "inexperienced" no contrast.