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AUTHOR Bright, George W.; And Others

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ABSTRACT

In this document, available literature is identified and reviewed in order to determine whether there are substantiated cognitive effects associated with the use of games in the mathematics classroom. Three criteria were used for selection of the literature: the treatment was identified as a game; cognitive measures were obtained; and the dependent variables included mathematical variables. The paper observes that the study of games as an instruction-learning process in mathematics has so far been neither thorough nor systematic and no definite conclusions can be made about the use of games in teaching mathematics. (MP)

COGNITIVE EFFECTS OF GAMES ON MATHEMATICS LEARNING

BY

George W. Bright

Department of Mathematical Sciences
Northern Illinois University
DeKalb, Illinois 60115

John G. Harvey

Department of Mathematics
University of Wisconsin
Madison, Wisconsin 53706

Margariete Montague Wheeler

Department of Mathematical Sciences
Northern Illinois University
DeKalb, Illinois 60115

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COGNITIVE EFFECTS OF GAMES ON MATHEMATICS LEARNING

Games are appropriate instructional activities. This premise, introduced to American education as early as 1798 by John Wallis in Arithmetical Pastime, was strengthened when John Dewey urged that games be considered an integral part of the school curriculum and as more than relief from the strain and tedium of regular school work (Dewey, 1928). It was not until the late 1950's and early 1960's, however, that games were accepted as important components of the curriculum from preschool through graduate school.

Games have been used for a variety of educational purposes: refining complex military strategies, clarifying decision-making, disguising arithmetic practice, developing the skills of athletes, measuring achievement, and engendering favorable attitudes toward mathematics. Bruner (1960) aptly described the prevailing attitude about the instructional uses of games when he asserted that

> Games go a long way toward getting children involved in understanding language, social organization, and the rest; they also introduce, as we have already noted, the idea of a theory of these phenomena. We do not know to what extent these games will be successful, but we shall give them a careful try. They provide a superb means of getting children to participate actively in the process of learning --

> as players rather than spectators. (p. 95)

In mathematics, teachers have been told that

> Games are fun. They are intended to be. They are also tremendously useful devices for developing skill in mathematics. Practice in computation skills is just as effective and much more palatable when disguised in a game context. (Biggs & MacLean, 1969, p. 50)

> Games may be an effective means of making the practice of computational skills palatable. (Johnson & Rising, 1972, p. 224)

> Most mathematical structures can be learned by playing skillfully contrived and excitingly motivating games of a mathematical nature. (Dienes, 1972, p. 64)

> Many mathematical games can be used to lead students to formulation and testing of hypotheses as they strive to discover a winning strategy. (Sobel & Maletsky, 1975, p. 49)

Perhaps in response to such encouragement, mathematics teachers in the United States and Canada began to use an almost bewildering array of games in their classrooms. In 1975, for example, the National Council of Teachers of Mathematics published a collection of papers from the Arithmetic Teacher which describe more than one hundred games and puzzles (Smith & Backman, 1975). The expressed goals of such games are to provide drill and practice in mathematics and to develop better attitudes toward mathematics.

Are these games effective in attaining these cognitive goals? No one knows, for there has never been a review of the literature related to this topic. In fact there has never been any good indication even of the amount of literature that is available to review. The purposes of this paper, therefore, are to identify the literature that is available, to provide a review of that literature in order to determine whether there are substantiated cognitive effects associated with the use of games in the mathematics classroom, and to summarize the results in a way that will foster future research.

Before proceeding, it is germane to discuss the term game. At present it is impossible to define the concept of game with precision or parsimony; that is, to give a mathematical definition of it. Inbar and Stoll (1970) note that many scholars have proposed definitions of this term. Characteristics of games commonly included in these definitions are

1) a game is freely engaged in;
2) a game is a challenge against a task or an opponent;
3) a game is governed by a definite set of rules;
4) psychologically, a game is an arbitrary situation clearly delimited in time and space from real-life activity; and
5) socially, the events of the game situation are considered in and of themselves to be of minimal importance.

Additional characteristics of games that probably ought to be appended to this list are

6) a game has a finite state-space[1]; the exact states reached during play of the game are not known prior to the beginning of play; and

7) a game ends after a finite number of moves (states).

Obviously, there are activities which have all of these characteristics and which are not commonly considered games; for example, putting a jigsaw puzzle together. It is clear, therefore, that the concept is not yet precisely defined and that the seven characteristics can only serve as guidelines. Gaming is the term used in this paper to refer to the activities of the game players in the course of playing the game. An instructional game is a game which has all of the characteristics given above except that it is, usually, initiated by a teacher rather than the student. That is, it may not be freely engaged in. (It should be noted that although an instructional game may not be freely engaged in, it is usually played with considerable enthusiasm.) An instructional game is a game typically perceived as instructional by both the teacher and the students and for which the teacher has determined a possible set of instructional outcomes. Only instructional games are discussed in this paper.

Prior to the discussion of the literature the criteria for selection of the literature should be made explicit. There were three criteria.

1) The treatment was described by the experimenter as being a game.

2) The dependent variables included cognitive measures, as opposed to being strictly affective or psychomotor.

3) The cognitive dependent variables included mathematical variables; for example, logical reasoning, learning of a mathematical structure, acquisition of a mathematical concept, and practice of a mathematical skill.

Early in the examination of the games literature it became quite evident that instructional game was defined in a variety of ways. Hence the need for the first criterion arose and the literature was "self-selected". One goal was the extraction of a useful definition from this literature. That goal was not attained. Rather, the best approximation to a definition seems to be the seven criteria previously identified.

One of the unfortunate consequences of the first criterion is that it forces inclusion of research treatments which the authors of this paper are uncomfortable labeling as "games". Typically in these situations, however, not only the experimenter but also the subjects viewed the treatments as games (Goldin & Luger (Note 13) is an example of this situation). In order to review the literature fairly, however, the first criterion was consistently applied.

Review of the Literature

Simulation Games

Simulation games have been widely used in schools, especially in teaching the social sciences. A simulation game is one derived from a real-world situation. The elements and relationships of real-world situations frequently are so numerous or change so rapidly or so slowly that it is difficult to study them. A simulation game focuses on only some of the elements and relationships and highlights the interplay between those chosen. Because only some of the elements and relationships are selected the simulation game is, by necessity, simpler than the real-world situation from which it is derived.

A careful search of the available research literature on simulation games did not produce any studies in which the goal of the game was primarily a mathematical one. The search, therefore, was extended to other content areas in order to identify any empirically substantiated cognitive effects of simulation games. The conclusions reached by the authors of this paper agree with those reached by Reiser and Gerlach (Note 1) who reviewed substantially the same literature. Reiser and Gerlach analyzed simulation games literature on five dimensions; two of these, knowledge and intellectual skills, are cognitive. Regarding knowledge they concluded that "the findings regarding the effects of simulation games on the acquisition of knowledge have not been impressive; ... student knowledge is not significantly affected by participation in a simulation game" (p. 6). For intellectual skills, that is, an individual's ability to apply knowledge to a new problem, they concluded that the results are ambiguous and state that "it appears that participation in a simulation game has a significant effect on intellectual skills, but this is not a frequent occurrence. The skill most likely to be affected by game participation is the ability to play the game". (p. 7)

Teams-Games-Tournament Procedure

The Model. The teams-games-tournament (TGT) model, developed at the Center for Social Organization of Schools at Johns Hopkins University, is a classroom organizational scheme in which any instructional game can be used as long as at the end of play the players can be rank ordered on their success in play. The structure of the TGT model will be outlined for a class of 24 students; the adaptation of that structure for more or fewer students is reasonably obvious.

1. All students are rank ordered on previous achievement which is related to the knowledge used in the game.
2. Based on these rankings, six teams of four students are formed so that intrateam variability with respect to achievement is maximized and interteam variability is minimized.
3. On the first day of the tournament students play the game by threes, in order of ranking. Each set of three is called a table, with table

one denoted the highest table and table eight, the lowest table.

4. At the end of each day of the tournament a high and low scorer is identified for each table. Each high scorer plays at the next higher table on the following day and each low scorer plays at the next lower table. The high scorer at table one and the low scorer at table eight do not move. This procedure enhances achievement, at least as measured by game success.
5. At the close of each day of the tournament, the high, middle, and low scorer at each table receive predetermined numbers of points. These points can be accumulated on an individual or team basis. Team standings for the day and individual or team standings for the tournament can be maintained and publicized.
6. Periodically, team practice sessions are scheduled so that peer tutoring can take place. Since team members rarely play against each other, cooperation among team members by sharing knowledge increases the potential for accumulating points for the team.
7. The tournament is terminated after the specified number of days.

It is important to note that the TGT model is independent of the grade level, the level and range of abilities and achievement of the students, the experience of the teacher, and the structure and content of the game used. The tournament can be run by students with very little guidance from the teacher. Even though the TGT model can be described as an independent entity, it may interact with several of these extraneous variables in affecting learning. Careful consideration of these variables, therefore, is demanded.

In the ensuing discussion the use of the TGT model is considered in studies of both mathematics and related skills and concepts and in studies which attempt to generalize the resulting conclusions across content and grade levels.

The Research. Edwards, DeVries, and Snyder (1972) randomly assigned four intact seventh-grade classes (two average ability and two low ability, $N = 96$) by ability level to two treatments, traditional instruction and TGT with Equations. The game was played twice a week for nine weeks and periodic practice sessions were scheduled. The three pre- and post-measures of achievement and the results were

1) on the Stanford Achievement Test, computations subtest, the TGT groups gained more than did the control groups ($p < .04$).
2) for the computation subtest items relevant to the content of the game, the TGT groups gained more ($p < .03$).
3) on the experimenter constructed divergent solutions test, the TGT groups gained more ($p < .04$) and ability × treatment interaction was significant ($p < .05$) due to the large gain by the low ability TGT group.
4) on all three measures the subjects learned ($p < .01$) and the effect of ability was significant ($p < .01$ for (1) and (2) and $p < .05$ for (3)).

Edwards and DeVries (Note 2) used stratified random sampling (with respect to ability) to assign 117 seventh graders to four treatments:

(games vs. quizzes) × (team vs. individual reward). The game played was Equations. Rewards were given by publishing team or individual scores, daily and cumulatively, in newsletters. The treatments were used two days per week, one period per day, for four weeks. A third day each week was set aside for team practice sessions. On a computation test only ability produced a significant result. On a divergent solutions test, the TGT groups performed better $(p < .05)$, and there was a significant task × ability interaction $(p < .05)$. The TGT groups had more positive attitudes $(p < .05)$ on four questionnaire scales.

Edwards and DeVries (Note 3) also examined the effects of a scoring variation and a non-competitive form of the TGT model. The scoring variation involved weighting the contributions of the better players more than those of the poorer players. In the non-competitive treatment, the weighted scoring procedure was used without competition among teams. Subjects were 128 seventh graders. The game was Equations. The treatment lasted 12 weeks. Each week two days were devoted to practice sessions, and one day, to game playing. The games groups performed better than the control group on the divergent solutions test $(p < .01)$. There was no significant difference on a computation test, and the TGT variations seemed to have no effects. In fact, the two variations produced less, though not significantly less, learning than the TGT alone. Two of eight scales of attitude and classroom climate favored the TGT groups $(p < .05)$.

Hulten (Note 4) randomly assigned eight intact classes of seventh graders in a ten-week TGT experiment with the Puzzle Problem Game. He reported that the use of team rewards rather than individual rewards promoted greater learning ($p < .05$). The degrees of freedom reported, however, seem to be in error. There was no differential effect of the mode of practice (individual versus team). Slavin, DeVries, and Hulten (Note 5) using eight intact seventh-grade classes (232 students) also studied the effects of team versus individual practice and team versus individual reward in a TGT setting, with newsletter reports of standings distributed to the students. The game was a modification of TUF. The treatment lasted ten weeks, with practice on one day per week and game playing on one day per week. Dependent variables were game success as measured by movement among tournament tables and changes in sociometric status. Team reward seemed to sustain a greater relationship between the two dependent variables than did individual reward ($p < .05$).

DeVries and Edwards (1973) reported that the game task with team reward produced more helping relationships, game groups reported greater satisfaction and less difficulty, and team reward groups showed more mutual concern. Subjects were 110 seventh-graders who used Equations, but no achievement variables were measured. DeVries and Edwards (1974) reported that "team rewards to heterogeneous (on both sex and race) groups ... reduce race and sex barriers inhibiting student interactions" (p. 746). Main and Jakubowski (Note 6) in a TGT study lasting several weeks with four classes ($N = 104$) of seventh-graders reported that students viewed their place in the tournament hierarchy, that is, their table numbers, as an accurate reflection of their ability, that students expressed a willingness to meet stiffer competition, and that students felt winning was under their control.

Several studies have tried to generalize the findings of the mathematics related studies and are briefly reported here to provide a context for interpreting the findings. DeVries, Edwards, and Wells (Note 7) used six high school history classes and teacher-made study questions in a TGT format. No significant difference was observed for a standardized test, but a marginal effect ($p < .10$) was observed for a content specific test favoring the TGT groups. DeVries and Mescon (Note 8) used 60 third-grade students in an initial test of the TGT model with a language arts curriculum. The TGT groups scored significantly better ($p < .05$) on the achievement test. DeVries, Mescon, and Schackman (Note 9) used 22 simple, teacher-designed language arts games with 54 third-grade students. Three of six subscales from a standardized test and three of four content specific tests significantly favored the TGT groups ($p < .05$ and $p < .01$, respectively).

DeVries (1976) in a summary of eight reports of TGT experiments conducted at the Center for Social Organization of Schools indicated that, in seven of nine cases, significant learning effects ($p < .05$) are attributable to the TGT model. It is precisely this consistency in results which makes the model an important focus of research. DeVries' strong claim must be viewed with considerable caution, however. In at least two of the seven supportive cases the learning effect was measured only by a game specific measure of behavior which the non-TGT groups had not had a chance to rehearse. Thus, it is not quite clear whether participating in the TGT or merely playing the game is the more important factor. It is possible that the TGT model causes positive effects because the model incorporates two uses of small groups in which instruction may occur: the teams practice together and games are played at tables. Bright (Note 10) also has reported data that indicate that the movement of students among tables in the TGT tournament does not completely accommodate for student learning as had been previously claimed by DeVries and Edwards (1973). If his results are replicable, then all of the assumptions underlying the use of the TGT model may need to be reexamined.

Treatments Embodying Games

Games have sometimes embodied treatments or parts of treatments in studies designed primarily to investigate mathematics learning. In all studies of this kind examined, the reason for choosing this instruction-learning process is not given. Games may have been chosen because they would facilitate instruction or learning, because they were a convenient vehicle for the treatment, or because the researcher had a disposition toward the use of them.

These studies fall into three categories. In the first, games were used as advance and post organizers; in the second, they were used to investigate the acquisition of problem solving strategies and mathematical structure; and in the third, they were used to investigate the effect of activity upon mathematical learning. Each category will be dealt with in turn.

Organizers and games. Scandura and Wells (1967) compared the effect of historical advance organizers (H) and game advance organizers (O) when used before instruction on either abstract mathematical groups (G) or combinatorial topology (T). One hundred four elementary education majors (100 females, 4 males) were randomly assigned to four treatment groups (HT, HG, OT, OG) of equal size. In one class period each treatment group studied their advance organizer, the material to be learned and took a nine-item test to measure learning and transfer. It was concluded that (a) the O-groups outperformed the H-groups ($p < .05$) and that (b) the OT-group outperformed the HT-group ($p < .005$). The means on the posttest for the OG-group were higher than those of the HG-group, but the difference was not statistically significant.

Lesh and Johnson (1976) examined four independent variables: (a) type of organizer (advance vs. post), (b) type of example (model vs. application), (c) number of examples (one vs. several), and (d) size of instructional group (individual vs. small group). The two application organizers consisted of a game or series of games, while the models organizers were not embodied in games. The subjects, 240 fourth-grade children and 240 seventh-grade children, were randomly assigned to the sixteen treatment groups. Instruction, a 30-minute self-instruction unit on motion geometry, was followed by a 15-minute posttest. Regarding the effect of games this study concluded that for fourth graders the non-game-embedded models organizers were more effective than the game-embedded applications organizers ($p < .05$).

Problem solving strategies and mathematical structure. Dienes and Jeeves (1965, 1970) used games which embodied the structure of a mathematical group. They employed a mechanism which had three panels: "State", "Play", and "Predict". The elements of the group, represented by colored geometric shapes, were displayed in each panel of the mechanism. Each subject was told that a game would be played against the mechanism and that the subject had won the game when he/she could accurately predict the next state of the mechanism after choosing the next "Play" shape and looking at the "State" shape but before pressing the "GO" button on the front of the mechanism. When the "GO" button was pressed, the chosen "Play" element of the group and the then showing "State" element were combined using the group operation, and the "State" panel changed to show the group element which resulted from that combination. The mechanism recorded all of this information as well as whether the prediction was correct.

In the initial study (Dienes & Jeeves, 1965) university students and eleven-year-old children each played two games. Approximately half of the adult subjects (15 males, 14 females) were designated as the selection subjects; the machine was operated by these subjects. The remaining adult subjects (15 males, 17 females) were designated as the reception subjects; the machine was operated for these subjects by the experimenter. Half of each of these groups played a 2-group game followed by a Klein 4-group game; the other half of each group played the same games in the reverse order. Thirteen of the eleven-year-old boys and 12 of the girls played a 2-group game and a cyclic 4-group game; 12 of the boys and 12 of the girls played a 2-group game and a Klein 4-group game. Within each grouping half of the subjects played the 2-group game followed by the 4-group game; the other half played these games in reverse order. Each game was played by each subject until every prediction was correct when all combinations were shown to the subject.

The second study (Dienes & Jeeves, 1970) was designed to investigate what would happen when the tasks were made so complex that short-term memory alone was not sufficient to cope with the tasks. The games were based on the Klein 4-group, the 3-group, the 5-group, the abelian 6-group, the 7-group, the cyclic 9-group, and the abelian, noncyclic 9-group. There were six treatments each consisting of four games. Play of each game was limited to 140 moves. At the end of the play of each game, subjects were asked to make predictions when both the "State" and the "Play" elements were given by the experimenter. All possible combinations of group elements were checked at this time.

Each treatment was given to five adult males, five adult females, five eleven-year old females, and five eleven-year old males. Treatments were administered on four successive days. The dependent measures of each game were: (a) the number of incorrect predictions during the play of each game (A-operation score), (b) the number of correct responses (corrected for guessing) to questions of the form: What would you play with X to get state Y in the window next? (These questions were asked at the completion of the play of each game and gave the Equation score), and (c) the number of incorrect predictions when both the state and play were given (B-operation score). Other, rather complex, measures were used, but they play no role in the conclusions relevant to the questions at hand. In addition, at the end of each game each subject was asked what he/she thought the game had been about.

Using the results of both studies a large number of conclusions were reported by Jeeves (Note 11), among them are the following:

1. Subjects' descriptions of the purpose of the game fell into three recognizable categories: the operator evaluation, the pattern evaluation, and the memory evaluation. With each of these evaluations, there was an associated strategy. In the first study, the mean numbers of moves for these categories were 101, 120, and 151, respectively. The first of these means is significantly different from the last ($p < .025$). By assigning

the categories scores of one, two, and three, respectively, a product moment correlation between scores and numbers of moves was 0.5 ($p < .005$). "It seems therefore that subjects do tackle the task more efficiently if they give 'higher order' evaluations." (Jeeves, Note 11, p. 15)

2. The A-operation mean scores, the Equation mean scores, and the B-operation mean scores support the hypothesis that "children consistently found it easier to particularize than to generalize, and more difficult to generalize than adults". (Jeeves, Note 11, p. 15)

3. The A-operation mean scores, the Equation mean scores, and the B-operation mean scores support the hypothesis that both children and adults found generalization by a factor to be more difficult than simple generalization.

4. Evidence from three different directions could be interpreted to support the view that when load on short term storage is reduced the rate at which a mathematical structure is learned improves. (Jeeves, Note 11, p. 25).

Branca and Kilpatrick (1972), Ackerman (1972), Chilewski (Note 12) and Pereira (Note 13) have also conducted studies using games based on the mathematical structure of the 4-groups. Branca and Kilpatrick used two Klein 4-group games and a network game to study the performance of 51 junior high school and 49 senior high school girls. At approximate two-week intervals each subject played all three games until she thought she knew all of the rules; she was then asked to give the rules and was encouraged to continue playing if some of them had been overlooked or forgotten. They reported results similar to those recounted by Dienes and Jeeves (1965, 1970); they also reported that subjects who were successful on the 4-group games had high strategy scores on the network game while those who were unsuccessful on the 4-group games had low strategy scores. Ackerman used the same Klein 4-group games as did Branca and Kilpatrick. His subjects were 45 fourth- and 45 sixth-grade boys with low (below the 50th percentile) mathematics achievement scores; these boys were assigned to one of three treatment groups which played one or both of the games. Ackerman concluded that "it appears that boys who are generally low mathematics achievers have not had the successful experiences with patterns and structure necessary for success with group structure games". (p. 1501A)

Using both 4-groups and two modes of presentation (static vs. transformation) Chilewski (1974) designed four games treatments. Fifty-two fourth-grade children were randomly assigned to one of the four two-week treatments. This experimenter found no significant differences across treatments for the strategy choices of children; he did find a significant training effect $(p < .025)$.

Finally, Pereira (Note 12) investigated the relationship between overt verbalization and mathematics learning using a Klein 4-group game. Forty eleven- and twelve-year-old girls were randomly assigned to one of four treatments in which they were instructed to verbalize or to be silent during the four learning tasks or during a subsequent evaluation period. While time was not controlled it is reported that each subject completed the four learning tasks and the evaluation task in approximately one hour. On the basis of the terminal test scores it was concluded that (a) subjects who played the game silently outperformed those who verbalized during play ($p < .05$) and that (b) there was a significant interaction ($p < .05$) between performance on the terminal test and these same independent variables; an analysis of the retention test scores yielded the same conclusions. Pereira also concluded that there was a relation between the number and nature of the rules discovered and the scores on both the terminal and retention tests ($p < .05$).

Goldin and Luger (Note 13) proposed a different approach to the study of mathematical structure and problem solving strategies. Their model of problem solving incorporates two ideas: (a) there is a fundamental correspondence between Piagetian conservation operations and automorphic transformations and (b) subjects decompose problems into subgoals and subproblems. The state-space representation (Nilsson, 1971) was used to formally describe problem structures and to generate five hypotheses about the paths generated by human problem solvers in the state-space of a problem. These hypotheses were tested by Luger (Note 14) using the 4-ring Tower-of-Hanoi problem. Forty-five adult subjects were asked to play a game in which the goal was to solve this problem in as few steps as possible. Each subject worked until he/she gave up or succeeded. Subjects' moves were mapped to the state-space representation. Luger concluded that (a) the subjects paths were not random and were goal-directed,

(b) subgoal states are important in problem solving, (c) the majority of subjects executed minimal solutions to more than 50 percent of all of the isomorphs of an n-ring subproblem prior to executing minimal solutions to more than 50 percent of the isomorphs of the (n+1)-ring subproblem, (d) there was not an expected congruence of nonminimal solution paths across isomorphic subproblems, and (e) 50 percent of subjects' paths started towards the goal state while the other 50 percent did not.

Game and activity learning. Humphrey (1966) administered a 78-item number concepts pretest to 35 first-grade boys and girls. On the basis of equal pretest scores ten pairs of boys and girls were taught and played eight active games dealing with number concepts. An active game was defined to be one which involved physical movement. At the conclusion of the ten-day treatment the pretest instrument was administered as a posttest. A comparison of the pre- and posttest scores revealed significant gains ($p < .001$). No significant differences were observed between girls and boys.

From a pool of 319 third-grade classes Crist (1969) randomly selected 42 classes. These classes were randomly assigned to one of three ten-day instructional treatments on telling time; the treatment approaches were developmental-meaningful, drill, and active games. Two parallel forms of a 74-item instrument which tested time concepts were developed and administered as pre- and posttests. Crist reports no significant differences between the posttest scores of the treatment groups. In their discussion of this experiment Ashlock and Humphrey (1976) report "it was found that all groups learned from pretest to posttest" (p. 60); however they do not specify the probability level for this result.

Wright (cited in Ashlock & Humphrey, 1976) divided 60 five- and six-year-old children into three instructional groups of equal size on the basis of their scores on 80-item verbal pretest of eight mathematical concepts; the way in which these scores were used to form the groups is not described. One group was assigned to a traditional treatment of number concepts; a second was assigned to motor learning activities; and the third group observed but did not participate in the treatment given to the second group. At the conclusion of the eight-day treatments the posttest scores revealed no significant differences between the treatment groups (Ashlock & Humphrey, 1976, p. 63). However there were significant differences ($p < .01$) between the pre- and posttest scores of the motor activities treatment group (Schoedler, 1976, p. 35).

Droter (cited in Ashlock and Humphrey, 1976) compared traditional, passive games, and active games treatments. Sixty kindergarten children were randomly assigned to one of these three treatments and, for an unspecified period of time, were taught arithmetic readiness skills. Both pre- and posttests of these skills were administered to all children; the active games treatment group had the highest mean gain although it was not significantly different from the mean gains of the other two treatment groups (Ashlock & Humphrey, 1976, p. 62).

Schoedler (1976) chose concepts from geometry and measurement in order to compare a traditional academic approach to an active games learning approach. Boys and girls were rank ordered separately on the basis of their scores on the Delaware Assessment Test given at the end of first-grade. Strata of six children were created using these rank orderings. Within each stratum children were randomly assigned to one of six second-grade classes. For fifteen class periods these six classes were assigned to one of three treatment groups: a control group, the academic group, and an active games learning group. Data from content specific pre-, post-, and retention tests indicated that there were no significant differences.

Games Used in Mathematics Instruction

The studies discussed below are different from those previously discussed in that the experimental settings are probably more typical of the ways in which classroom teachers use games in their everyday mathematics instruction. No coordinated chain of inquiry is represented by this collection of studies either with respect to the ages of the students or with respect to the instructional setting.

The instructional setting, often a neglected aspect of experimentation, may be important in understanding the cognitive effects of games since it related to the conceptual development of the student. The instructional setting can be described as follows:

(1) Pre-instructional. Games used in a pre-instructional setting serve to introduce a concept or skill. The game may be used as

an advance organizer (e.g., Scandura & Wells, 1967) or may serve to prepare for cognition.

(2) Co-instructional. In a co-instructional setting games are used, alone or in conjunction with other instruction-learning processes, at the point at which mastery is anticipated. When games are used in this sense it is assumed that the concept has been introduced but that it has not been mastered.

(3) Post-instructional. In a post-instructional setting, games are used to promote retention or to facilitate retraining or retrieval. Games used in a post-instructional setting may be used to disguise practice, to inconspicuously diagnose student mastery, or to provide opportunities for enrichment.

The discussion that follows is structured primarily according to the instructional setting and secondarily according to the ages of the students.

Games in a pre-instructional setting. Wynroth (1970) used two kindergarten and three first-grade intact classes in a study of learning about the natural number system. One class at each grade was designated as experimental and the other three, as control. In the experimental kindergarten class the children played oral games for one-half of a 45-minute period twice each week during the fall and three times each week during the spring. In the experimental first-grade class the children played the games for one-half of a 50-55 minute period each day all school year. In both experimental classes, the remainder of the mathematics instruction time was spent working in workbooks. The amount of mathematics instruction in the control kindergarten class was not reported; the two control first-grade classes spent about 30 minutes per day on mathematics. For eight computation tests all but one comparison of experimental and control classes at each grade level favored the experimental classes ($p < .05$); that one comparison was not significant. It appears that the statistical procedures employed were not appropriate and that the individual student was incorrectly used as the unit of analysis.

COUNT-O, a flashcard, bingo-type game designed to teach number recognition 1-100, was used by Wheeler and Wheeler (1940) in an examination of the efficiency of a game to teach recognition and reading of numbers. Prior to the treatment subjects received some instruction on number concepts and on reading the numbers 1-10. Individual pretesting eliminated those first-grade children who knew 90 or more of the first 100 numbers. For the 114 first-grade children who played COUNT-O for twenty minutes a day for five consecutive school days in November, the median score rose from $6.29 \pm .83$ on the pretest to 60.00 ± 4.24 on the posttest (range 0-87 and 2-100, respectively). It was concluded that the use of a game had considerable value for primary children and that the curriculum placement of these number skills could not be determined solely by the mental age of the children.

Bowen (1970) hypothesized that students who used the WFF'N PROOF game would demonstrate a significantly higher degree of proficiency in their learning of logical principles than their peers who were given a structured, textbook approach. Students in two classes of fourth-grade honor students (IQ range: 131-159; N = 40) were paired on the basis of sex and IQ; members of each pair were randomly assigned to one of the two instructional treatments. On the basis of two parallel tests, one for each instructional treatment, the structured, textbook treatment was more effective than the games treatment ($p < .0005$).

Karlin (1972) had some success in designing a card game to teach prime number factorization skills, recognition of the Fundamental Theorem of Arithmetic, and multiplication facts to fifth-grade children. In this three-day methods comparison study, groups using the card game were contrasted with groups using the traditional textbook-oriented methods. Eight classes from two schools were assigned in equal numbers to the two groups. The game approach proved significantly more effective than the text approach in fostering recognition of the Fundamental Theorem of Arithmetic on the posttest ($p < .05$) but not on the retention test given three weeks later. No significant differences were found for treatment main effects with respect to prime factorization skills, multiplication skills, or attitude toward arithmetic. Introverts as classified by their teacher learned better with text material, whereas extroverts learned better through gaming. Neither difference was statistically significant.

Games in a co-instructional setting. Games at the co-instructional level are presented alone or concurrently with a more formal presentation of the mathematics to be learned. In an early study of games, Steinway (1918) addressed three issues: the extent to which number games are of interest to first-grade children, the extent to which games could be used to supplement formal work in arithmetic, and the extent to which games promoted knowledge of number concepts. The children in two intact first-grade classes were the subjects for this study. The 21 children in one class received the games-only treatment for six weeks. In this treatment group the children could elect to play or not to play games during daily instructional periods of approximately 45 minutes. The games emphasized counting, reading, writing, and recognizing numbers and simple addition; every fifth day a new number game was introduced. The children in the other first-grade class received formal work in arithmetic and played games; the number of children, the ratio of formal work to game playing, and the length of the instructional periods was not specified. The investigator concluded that number games were of interest to first-grade children (average interest span, games-only group: 17 minutes), that a need for arithmetic knowledge was displayed by children during the play of the games, and that achievement in arithmetic skills, as measured by a 25-item test, increased. The pre- and posttest means for the games-only treatment groups were 17.1 and 20.5, respectively; for the formal work and game treatment group, they were 17.1 and 19.0, respectively. Statistical comparisons were not reported.

Kennedy and Newman (1976) used a daily 45-minute instructional period to emphasize reasoning and analytic thinking to first-grade children. All children were instructed in the experimental curricula for approximately one-third of each period. Experimental children used the remainder of the period to play games whereas comparison children used the remaining time for social studies. Games available to the experimental group, selected to complement the skills developed in the lessons, included chance games (e.g., <u>Hi Ho Cherry O</u>), strategy games (e.g., <u>Checkers</u>) and chance/strategy games (e.g., <u>Parcheesi</u>). Two of eight scales of reasoning and thinking reflecting predetermined cells from Guilford's Structure of the Intellect Model favored the games group ($p < .01$). No significant differences were found for the other six measures.

While concurrently teaching the 100 basic addition facts, Goforth (1938) used ADD-O, a flashcard game with a gameboard similar to that used in bingo. Although a single game of ADD-O could be played in two to four minutes, play continued for 20 minutes each day for thirty consecutive school days. The subjects, 31 second-grade children, were tested every ten days; both vertical and horizontal formats of the addition facts were used. On the average, the children learned 1.65 addition facts for each day of game play. Learning of the addition combinations was not significantly affected by the format.

Wheeler (1939) also used ADD-O with 125 second-grade children for 20 consecutive school days for 20 minutes each day in order to examine the relative difficulty of the 100 basic addition facts. Pretest results from the repeated measures design showed that these subjects knew 10-12 combinations. Ten-day results showed 35-45 combinations mastered, and posttest results showed 55-61 combinations mastered. Posttest results further showed that 25 percent of the children had mastered at least 80 of the combinations. For each of the three measures, the median score was higher when the basic facts were presented in horizontal format than when they were presented in vertical format. The investigator concluded that addition combinations could be successfully taught through informal methods inherent in such games. Statistical comparisons were not reported.

Kincaid (1977) investigated the effects on second-grade children's attitude and mathematics achievement resulting from parent-child use of mathematics games in the home. Parents in the experimental group attended ten weekly meetings where they learned to construct and play mathematical games before introducing them at home. The treatment did not have a significant effect on achievement but did have a positive effect on attitude of the experimental students ($p < .045$).

Group games have been used to teach basic number concepts (including rote and rational counting, one-to-one correspondence, time concepts, money, and informal geometry) to educable mentally retarded children (Ross, 1970). Twenty children (mean Stanford-Binet IQ: 66.23; C.A.: 7.9 years) participated 100 minutes each week for nine months in a games program while 20 matched subjects used a traditional mathematics program. The games program was designed so that intentional learning was general games skills and incidental learning was basic number concepts. Five kinds of games were used: table search games, board games, card games, guessing games, and active racing games. The experimental groups scored higher on both mid-experimental ($p < .0003$) and post-experimental ($p < .0003$) number measures and higher on post-experimental ($p < .002$) spontaneous use of quantitative terms measures.

Motivated drill in a game format for each of the four fundamental operations was used by Hoover (1921) in 30 third-grade classes. The sample classes were divided into two groups of equal size and comparable mental attainments on the basis of the superintendent's judgment. The children in the 15 classes in the games group played four domino-like games, one for each operation, stressing speed and accuracy of computation, for 10 minutes a day, three days a week for three months. As nearly as possible, time was partitioned equally between the four operations. The control group, the other 15 classes, spent the same amount of time in arithmetic instruction. Over the experimental period, class mean accuracy scores increased 17.8 percent for the games group and 14.1 percent for the control group on a 30-second speed test; the content of the speed test was not described. Statistical comparisons were not reported.

Bright, Harvey, and Wheeler (Note 15) examined the question of whether learning is inhibited or enhanced when the instructional objectives of a mathematics game are incorporated directly into the rules of the game. In particular, the study attempted to determine if there were differences in the learning of basic multiplication facts between two versions of the game MULTIG (Romberg, Harvey, Moser, & Montgomery, 1974, 1975, 1976).

In the standard version, the number of points scored in a given turn was determined by counting a specified set (cardinality less than nine). In the modified version, the number of points scored in a given turn was determined by counting two sets (cardinality less than nine) and multiplying the two numbers. The treatments were conducted in a TGT setting over ten consecutive school days during the third quarter of the school year with 12 intact third- and fourth-grade classes (6 selected classes at each grade level, 3 randomly assigned to each version of the game). For both power and speed posttests, grade level was a stronger main effect than treatment. Neither main effect, however, was statistically significant. For both posttests, the third-grade classes performed better after playing the modified game, while the fourth-grade classes performed better after playing the standard game. This effect did not appear to be stable.

Hart (1977) examined the effect of the use of leisure time material on computational achievement and on the number of computational items attempted. Six intact classrooms of nine/ten-year-old students sporadically used leisure time materials in a school setting for five weeks. The experimental group used materials which emphasized mathematical games and puzzles whereas the control group used materials emphasizing reading and word puzzles. Hart concluded that the attitude of the child significantly affected the amount of computation attempted ($r = .36$), although it did not affect the number of games and puzzles attempted.

Paris (1971) adapted experimental materials for cooperative and competitive game situations from Equations to examine the relation of a personality trait (assertive-obedient) to mathematics achievement and to attitude toward mathematics games. The experimental materials for the game situations consisted of one set of playing cards that presented 25 game problems and another set that presented the solutions. The Childrens' Personality Questionnaire was administered to 302 suburban fifth-grade students in order to classify them as assertive or obedient. Each student was randomly assigned to a no-treatment, competitive-win, competitive-lose, or cooperative game situation. In the competitive treatments subjects were paired as opponents and used scoring rules. In the cooperative treatment subjects were paired as collaborative teams and did not use scoring rules. Two measures for this posttest-only experiment were designed by the investigator. The achievement test was based on problems from Equations and on examples from fifth-grade arithmetic texts. The attitudinal measure was adapted from the Blum and Dutton Scale. The investigator reported, without supporting statistics, the following conclusions: (1) subjects in the cooperative game setting scored higher in achievement than subjects in the competitive settings; (2) assertive subjects scored higher in achievement than obedient subjects; and (3) attitudinal differences in general were not found for the assertive-obedient factor or for the cooperative-competitive game settings.

Achievement in division was the focus of Fishell's (1975) investigation. Eight intact fifth-grade classes were randomly assigned to the treatment group or to the control group. The classes comprising the treatment group played a math trading game and were shown a division process which used the principles of this game. Those in the control group used no manipulatives or games but studied division. Both the treatment group and the control group received instruction in division for fourteen consecutive class periods. Pre-, post-, and retention tests were given. Fishell concluded that the use of the math trading game did not significantly improve division achievement.

Using two variants of NIM, Peele (1972) examined learning patterns of 49 volunteer fifth- and sixth-grade students. The games, Last-One-Loses (LOL) and Even-Wins (EW), were played by subjects against a computer which had been programmed to display characteristics of intelligence during play of the games. Some randomly selected subjects played with an "executive option" in which, after the play of any game, they could adjust the level of the computer program to "play easier" or to "play harder". All subjects played LOL on one day and EW on a subsequent day; they were permitted to play each of the games for a maximum of one hour but could stop at any time during that hour. The average playing times were not reported. Statistical differences were not found on two measures of understanding of the strategy for playing these variants of NIM.

Weusi-Puryear (1975) integrated a 40-minute game treatment into one-day field trips for summer school classes of eight- to eleven-year-old students. The questions addressed were whether the game elements of computerized treatments could produce significantly greater achievement than the tutorial elements of those treatments. The subjects ($N = 258$) were randomly assigned in equal numbers to three treatments: computerized tutorial, computerized tutorial interwoven with a simulated Tic-Tac-Toe game (GAMBO), and control. For the two tutorial treatments the content was addition for students aged eight and nine and multiplication for students aged 10 and 11. All subjects received the same pre- and posttests, two of which were mathematical: a computational inventory (Wilson, Cahen, & Begle, 1968, p. 25) and an experimenter-designed addition/multiplication test (5 items per operation). These instruments correlated significantly ($p < .05$). The students in the games/tutorial treatment group were allowed to take a turn only after responding correctly to randomized addition or multiplication exercises (depending on the subject's age). These subjects achieved more ($p < .05$ for addition; $p < .01$ for multiplication) than the subjects in the tutorial-only group even though the game-playing subjects did fewer exercises. Correlations between a Tic-Tac-Toe strategy rating and the pre- and posttest scores were not significant.

Bonner (1975) used an interactive classroom-based computer terminal with 60 seventh-grade students over an eight week period with a median subject participation of seven days. Three treatments, drill, puzzle, and game, had the common objective of providing practice on signed numbers. Achievement gains were assessed with two-way multivariate analysis of variance of treatment (drill, puzzle, game) X ability (high, low). Overall significance ($p < 0.047$) was found between treatments on the achievement measures, with students in the game and drill treatments achieving more ($p < 0.035$) when measured immediately and two weeks later.

WFF'N PROOF, a series of 21 games of mathematical logic of increasing difficulty, was used with junior high school students by Allen, Allen, and Miller (1966). Twenty-three students (average IQ: 114.5), who enrolled in a summer session program, progressed at their own rate through the WFF'N PROOF series for 45 minutes per day, 5 days per week, for 29 days. The remaining part of the daily two-hour instructional period was spent reading and discussing the rules and underlying concepts of the games. Students in the non-comparable control group ($N = 22$, average IQ: 104.5) were enrolled in the regular fall mathematics classes in the same school building. The California Test of Mental Maturity (Junior High Level, 1957 S-Form) was used as the pre- and posttest. The dependent variable, the change in non-language IQ score over the two administrations, was considered by the researchers to be a measure of problem solving skills. The comparison between the experimental and control groups favored the experimental group ($p = .02$). When initial IQ score was used as a covariate, the significance increased ($p < .01$).

The regular use of teacher-made mathematics games to teach mathematics to low achieving students in grades 7 through 12 (median grade level ninth-grade; mean age 14.5 years) was investigated by Burgess (1970). Twenty-four classes from nine secondary schools were randomly assigned in equal numbers to experimental and control groups. For eight weeks each class received its usual instruction in computation with fractions during one-half of the daily mathematics period and either content related game activities (experimental treatment) or activity sheets (control treatment) during the other half of the period. Multivariate analysis of covariance showed the control group scored significantly better ($p < .03$) than the experimental group on multiplication and division of fractions, but there was no significant difference on addition and subtraction of fractions. There was a significant difference ($p < .003$) favoring the females, and a significant difference ($p < .03$) favoring the younger students.

During his feasibility study Jones (1968) used a modified programmed lecture and mathematical game treatment with 38 low achieving students enrolled in a ninth-grade general mathematics program scheduled during a nine-week summer session. The games used and the instructional objectives of those games were not specified. The Cooperative Arithmetic Test was administered as a pretest and a posttest; a significant improvement ($p < .05$) in students' scores was reported.

Janke (Note 16) looked for differential effects between teacher-made games and published games in mathematics instruction in a TGT setting with behavioral disabled adolescents. Two of the eight dependent variables were mathematics achievement variables. Subjects were 90 urban males randomly selected from three nongraded, nonresidential schools for the emotionally impaired. Three mathematics classes of 10 students each were formed in each of the three schools by random assignment of subjects. One class in each school was randomly assigned to each of the three treatments: control, published games (TUF and Equations), or teacher-made games (mathematical variations of Sorry and Monopoly). The TGT instructional organization structured the classrooms during the 18-week study. Game playing subjects played three times each week for one-half hour per session. The investigator concluded that published mathematics games appeared to be as effective as teacher made games ($F(2, 87) < 1$) and that the TGT model was effective for improving selected dimensions of student behavior ($p < .01$) but not for improving mathematics achievement ($F(2, 87) < 1$).

Allen, Allen, and Ross (1970) compared 43 junior and senior high school students who played WFF'N PROOF with 34 students who studied prealgebra concepts. Games subjects studied and played WFF'N PROOF four hours per day, five days per week for three weeks; the amount of time which the prealgebra subjects engaged in that treatment is not reported. There is a 21 day interval between the time of the pre- and posttests for the prealgebra subjects. For these noncomparable treatment groups, the differences in mean scores on the non-language part of the California Test of Mental Maturity was significant ($p < .01$). The pre- and posttest change scores for boys and girls in the games treatment group were not significantly different.

The game Equations was used by Allen and Ross (Note 17) to assess ability to solve computation and reasoning problems; the sample consisted of 10 to 14 eighth-grade mathematics classes. Each of five treatment groups used the game in some way; the effects of the various experimental conditions were measured by two different forms of an investigator-designed inventory. Even though three of the five treatment groups had played Equations for two years, at the end of this two-week study Allen and Ross concluded that skills in applying mathematical ideas can be significantly improved ($p < .0001$) by learning procedures which were rich in opportunities for application at appropriate levels of complexity; the student was incorrectly used as the unit of analysis.

In a multifacted, observational, quasi-experimental investigation, Freitag (1974) described five case studies each of which used a different mathematics game. Each study was one week in duration, and each used different teachers and students. The games used included a function game with fourth-grade students ($N = 6$; one 50-minute playing period), a form of bingo which simulated keeping an accurate savings account with sixth-grade students (2 classes, $N = 63$; one 40-minute game playing period), a card game, Bridget (Johnson, 1958) for the evaluation of algebraic fractions with above-average eight-grade students (2 classes, $N = 30$; 35 minutes over two days), a variation of Euclid with tenth-grade, Algebra II students ($N = 28$; 75 minutes over two days), a game for ordering and operating with fractions with senior high school students in a consumer mathematics class ($N = 18$; one 50-minute game session). The videotaped game sessions were preceded by a pretest specific to the mathematical content of the game played; the pretest was administered the day before the game sessions in each study. Different posttests, again specific to the mathematical content of the game, were administered as soon after the game playing session as possible. For each study, the average posttest score was greater than the average pretest score. No analyses of the data were reported even though the investigator states "significant gains in learning were accomplished between the pre- and posttests" (Freitag, 1974, p. 100).

Equations and Tac-Tickle were used by Henry (1974) to determine if mathematics games would affect quantitative or nonverbal cognitive abilities. Three intact seventh-grade classes at three different junior high schools participated in the experiment; one class played Tac-Tickle, one played Equations and the third served as a control group. In the two experimental classrooms students played games for half the class period approximately every other day for a period of six weeks; conventional instruction alternated with playing the games in these classrooms. On the basis of pretest and posttest scores Henry reported no significant results.

Carter (1975) combined mathematical games and team competition in an out-of-class setting to examine the effects of a mathematics club upon the achievement of seventh-grade inner city students. A card-type game was introduced at the first of two weekly sessions for each of eight weeks. During the first session, the team members practiced together, and during the second one, the teams competed. A control group of comparable size was randomly selected from the same population. Parallel forms of a standardized achievement test were used to collect pre- and post-treatment data. Achievement was not significantly affected by the treatment, and sex did not have a significant effect upon achievement either within or across groups.

In a methods comparison study, Page (1971) compared the effectiveness of two instructional procedures on the ability of high school students to generalize the strategy of a simple counting game. Subjects were paired on IQ scores, and members of each pair were randomly assigned to a Discovery Group (N = 23) or an Instruction Group (N = 24). Subjects in the Discovery Group had opportunity to learn the strategy only by playing the game, whereas subjects in the Instruction Group had opportunity to learn the strategy by reading an explanation of the strategy. Every subject "played other forms of the counting game until each form had been played an arbitrary number of times" (p. 4628A). Evidence that a student had generalized the winning strategy was the "learning of the perfect strategy for two consecutive forms of the game" (p. 4628A). On the average, students in the Instruction Group played significantly more perfect games than students in the Discovery Group; no probability level is reported. However, there was no significant difference between the two groups in their ability to generalize the winning strategy. For Instruction Group subjects who learned the perfect strategy for at least one form of the game, there was a strong positive correlation between IQ and the ability to generalize the winning strategy for the game.

Addleman (1972) used four intact classes of undergraduate students who were prospective school teachers. Three classes were randomly assigned to one of three treatments which lasted for eight weekly 50-minute sessions. One of the treatments consisted of playing Equations six times, NIM once, and a die game once. The other two experimental treatments did not involve games. On the basis of gain scores, each experimental group performed better than the control group ($p < .05$), but they did not differ significantly from each other on a measure of numerical achievement. When initial numerical achievement was used as a covariate, the result was no longer significant. There were no significant differences for a measure of problem solving. In all cases subjects were incorrectly used as the unit of analysis.

Freitag (1974) randomly selected 14 of 30 college students from an intact class to play a game which involved set properties and operations. These 14 students played the researcher-designed game in a single 90 minute setting. For similar, treatment-specific pre- and posttests, their average posttest score was higher than their average pretest score (13.8 versus 8.4 on 24 items). No statistical significance was reported.

Games in a post-instructional setting. Bright, Harvey, and Wheeler (Note 18) investigated the effects of games on the retraining of multiplication skills in two studies. The treatments were conducted in a TGT setting during the first ten instructional days of two school years with 14 classes of students from grades four, five and six and 10 classes from grades five and six, respectively. The students had previously been instructed in the 100 basi multiplication facts. The researchers concluded that the drill-and-practice games MULTIG and DIVTIG (Romberg, Harvey, Moser, & Montgomery, 1974, 1975, 1976) were effective in retraining of skill with basic multiplication facts both for all of the basic facts

($p < .025$ and $P < .001$, respectively) and for the 36 facts (4×4 through 9×9) specific to the games ($p < .01$ and $p < .001$, respectively).

McCann (1977) used all pairs of three treatments (two games and conventional drill) with two problem types (powers of ten and formula solving) in a remedial mathematics review with naval trainees. The drill and game treatments utilized PLATO IV. Random assignment of 48 subjects to the six combinations of instructional modes yielded four subjects per treatment in each counterbalanced task grouping. Depending on the instructional mode, the time-on-task (treatment time) varied between 17 and 31 minutes. No significant differences in performance or time-on-task were found between the three training methods.

Conceptual Framework and Implications for Future Research

It was stated at the beginning of this paper that the research on the cognitive effects of games on mathematics learning is extremely fragmented. One possible cause of this fragmentation is that the majority of the research did not focus upon the cognitive effects of games; for example, the research on the teams-games-tournament (TGT) model emphasized and studied the use of that model rather than the effects of the games played.

Four dimensions of the problem seem to be particularly relevant

1) characteristics of the game,
2) instructional objectives of gaming
3) learner-game interactions, and
4) learner-learner interactions.

Each of these will be discussed in turn.

1. Characteristics of the game

One important collection of games is simulation games. A simulation game simplifies a real-life situation, and since mathematics is not a simplification but an abstraction of the real world, it is unlikely that there are simulation games which are also primarily instructional games in mathematics. This is not to say that simulation gaming does not use mathematical skills and concepts; for example, a knowledge of simple arithmetic is necessary and a knowledge of probability may help if one is to be successful in playing Monopoly. However, the use of simulation games in mathematics teaching is at best marginal.

In mathematics a more important collection of games is nonsimulation games. These games can be examined by focusing on stimulus and response characteristics. The stimulus characteristics include the format and the constraints imposed upon the players. Important groups of questions regarding the stimulus characteristics are

1a) What effect does the format of the game have upon learning? Are there differential effects if the game is a board game, with or without a path? a card game? if no visual stimulus is a part of the game? The conclusions reached by Levin, Divine-Hawkins, and Kerst (Note 19) indicate that the inclusion of visual stimuli, for example, a gameboard, may assist children in storing and recalling information.

1b) What is the effect of the constraints imposed upon the players by the game? A number of constraints can be imposed upon the players by games; these include the number of players, the number of plays required, the time allotted or required for each play or for the game, the resources required or available, and the overt behavior required of players. Some games, for example, Solitaire, are played by only one player and are a challenge against a task while others, like WFF'N PROOF and Equations, require or permit opponents or partners. Are there differences when the game is a challenge against a task rather than an opponent? Are there differences if a game is played with a partner or without one?

Next, the number of plays required and the time required to play the game may be important variables. A game may be effective primarily because it focuses a player's attention on a task for longer periods of time than do other instruction-learning processes. Thus, if the time required for one player to complete a single move is too long, the other players may not attend to the task as they should. Or if the time required to complete play of the game is too long, the same possible deleterious effect may occur.

Finally, the overt behavior required of the players may have an effect upon learning. In this area one important concern is the amount and kind of verbalization required of the players. Wynroth (1970) observed that oral games contributed significantly to the learning of natural number properties while Pereira (Note 12) concluded that those who played his game silently outperformed those who verbalized during play. Do these results hold for other types of games when used for different purposes? Does it matter whether players are required or permitted to manipulate objects, to write down descriptions of relationships, or to keep records during play of the game? Finally, what differences are there when each player is permitted or required to make a move at the same time as do other players as opposed to taking turns? Are there differences if different players make or do not make the same number of moves during play of the game? In some games the players do not move in sequence; for example, Double Solitaire; and may not even make the same number of moves.

1c) There are also the response characteristics of a game; that is, what completes a move during play of the game? Quite frequently the response of the player may be unimportant in instructional games; for example, the final positioning of a piece in a board game may not be important except that it indicates that player correctly completed the move. Similarly the score which results from a given move may not be important nor may be the existence of winner(s) or loser(s) at completion of play.

2. Instructional Objectives of Gaming

One of the characteristics of an instructional game is that the teacher predetermines a possible set of instructional outcomes for the game before the start of play. One instructional objective which must be determined is the mathematics content to be taught by the game, for example, basic multiplication facts or the recognition of three-dimensional geometric solids. A second instructional determination is the instructional setting intended. That is, is the instructional level of the game pre-instructional, co-instructional, or post-instructional? A third instructional objective which must be specified for a game is the level at which the content is to be utilized during gaming; for example, one of Bloom's (1956) taxonomic levels could be specified.

3. Learner-Game Interactions

Once the instructional objectives have been specified and the game begins, the interaction of the learner with the game begins. The amount and kind of these interactions would seem to be important, and the following questions should be considered:

3a) What heuristics is the learner using?

3b) What strategies does the learner use? If the game has a strategy, does the learner recognize and use it? If the game has a strategy does the association of the strategy with the instructional objectives of the game enhance or interfere with learning?

3c) What products does the player produce? For example, the player might find the sum of two numbers or challenge the move of another player. Which of these products are in accord with the instructional objectives of the game and which are not?

3d) What is the load on short-term memory? on long-term memory? Does an excessive loading in short-term (long-term) memory inhibit learning? The research of Dienes and Jeeves (1965, 1970) suggests that this may be an important factor.

4. <u>Learner-Learner Interactions</u>

At the time that a given learner is interacting with the game, the player may be interacting with other learners as well. These interactions may have an enhancing or deleterious effect upon learning.

4a) What is the level of competition between players? What is the effect of that competition upon learning? If the conclusions reported by Paris (1971) are valid, then the level of competition may critically affect learning.

4b) Does peer teaching take place? What is the effect of any peer teaching which occurs?

In summary then, the following variables have been identified within each of the four relevant game dimensions. Other variables and indeed other dimensions may arise from future research or from further review of the existing research.

1) characteristics of the game
   a) format
   b) constraints
   c) responses
2) instructional objectives
   a) content of the game
   b) level of instruction
   c) level of utilization
3) learner-game interactions
   a) heuristics
   b) strategies
   c) products
   d) memory load
4) learner-learner interactions
   a) level of competition
   b) amount of peer teaching

In addition to the game characteristics, there are learner characteristics which may also be important in determining the effects of games. At the very least they need to be considered in order to better develop or to complete the global view of the effects of games. These variables include sex, age, socioeconomic level, grade level, cognitive level, prior achievement, cognitive style, problem-solving skills, and a multitude of personality, interest, motivation, and attitude factors. For example, Karlin's (1972) results suggest that extroverted children learn better by playing games than do introverted children.

Synthesis and Projections

It should be clear on the basis of the literature identified in this paper that the study of games as an instruction-learning process in mathematics has so far been neither thorough nor systematic. The extant literature is very fragmented, and most is based on weak or unstated foundations. Most often the "game" involved is viewed wholistically rather than as an entity with distinct and interactive characteristics. The multitude of game dimensions discussed in this paper suggest that the wholistic view cannot provide sufficient information to determine why a game is or is not an effective teaching device.

As a result, at this time no definitive conclusions can be made about the use of games in teaching mathematics. The cognitive effects associated with the classroom use of games are unclear. When the dimensions of games have been identified and investigated, with appropriate attention also paid to learner characteristics, then it seems reasonable to expect that definitive conclusions can be reached. Such information would facilitate identification of which games should be used, when they should be used, how they should be used, and with whom they should be used. That, however, is the last step in a systematic investigation of games. The point of departure seems to be to investigate the characteristics of games singly and in combination in order to document ways in which learning is affected.

It is not unreasonable to expect that studies of dimensions of games will also yield information important to an understanding of the process of learning. A game, because it is structured, more or less independently of environmental influences, allows manipulation of variables in much the same way as does a "laboratory" procedure such as programmed instruction. At the same time, game playing -- and play in general -- is a part of the natural learning environment of western culture. Consequently, the results of a study of games are likely to be generalizable in an important and natural way to the study of learning in a much broader context than just learning via games. Game playing, therefore, holds considerable promise as an avenue of research.

The thrust of the suggested research on the effects of games is to investigate the effects of games, independent of any comparison of relative effects of other instructional procedures. It is essential to understand first, why a game is or is not an effective teaching tool, and second, how the effects of a game can be altered by altering the game dimensions. Only then is it reasonable to compare the effects of a game with the effects of other instruction-learning processes. Such comparisons should be made, however, only in the presence of clearly defined learning outcomes.

In light of the exhortations to use mathematics games and the potential generalizability of games research, it is somewhat surprising that so little attention has been given to the study of mathematics games. Perhaps the deficiency is but another instance of the discrepancy between educational practice and educational theory. The authors of this paper accept the challenge to discover the role of games in mathematics instruction.

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Footnote

[1] The state-space of a problem (including a puzzle or a game) is (a) a description of all of the outcomes -- states -- which are related to the original problem -- the initial state -- (i.e., those which can be derived from it using the permitted problem solving procedures), (b) a description of the states which are solutions of the problem (the goal states), and (c) a description of the way in which each state is related to all other states. If the states are represented as points and the nontrivial state relationships are presented as directed segments, then the state-space can be thought of as a tree graph in which the initial state is the origin of the graph, the branches represent the state relationships, and the goal states are among the terminal points of the branches. (Nilsson, 1971)