Five regional conferences designed for elementary supervisors and elementary mathematics educators were held in Ohio. The purposes of the conferences were: (1) to provide direction on the effective use of the calculator in the elementary school classroom; (2) to re-emphasize the importance of problem solving as a major curricular outcome; (3) to explore the interaction of the two areas and their relationship to the current emphasis on the basics; and (4) to establish links between supervisors and mathematics educators in each region for continuing curriculum development and improving instructional practice. Contents of this report include: (1) announcement and application forms; (2) sample schedules of the conferences; (3) conference evaluation forms and data from the evaluation; and (4) resource packets including papers presented or discussed at the conferences, sample materials, and transparency masters. (Author/MP)
Ohio Regional Conferences on Mathematics Education

supported by funds from the National Science Foundation
Information Dissemination for Science Education (IDSE)
Grant No. SER 77-20594
9/15/77 - 12/31/78

Conference Development Team:
Marilyn N. Suydam, The Ohio State University (Director)
Kenneth Cummins, Kent State University
Thomas C. Gibney, University of Toledo
Johnny Hill, Miami University
Steven P. Meiring, Ohio Department of Education
Len Pikaart, Ohio University
Ohio Regional Conferences
on Mathematics Education

Purposes of the Conferences

Based on the need to prepare for current and continuing improvement in curriculum and instruction for elementary school mathematics, six regional conferences were held. The purposes of the Ohio Regional Conferences on Mathematics Education were:

a. To provide direction on the use of the hand-held calculator in the elementary school classroom

b. To stimulate a realistic, effective approach to problem solving
c. To explore the interaction of problem-solving goals and the expanded use of calculators
d. To establish linkages between supervisory personnel and mathematics educators in the State of Ohio
e. To evaluate the organizational structure as a model for disseminating information to elementary school teachers.

One focus of the conferences was on the hand-held calculator, a technological tool with potentially vast implications for redirecting the thrust of elementary school mathematics. Strategies, applications, and guidelines for the use of calculators were explored. Included in this exploration were ways to improve instruction on basic skills through the use of calculators.

The second focus was on the principal goal of mathematics programs, problem solving. Too often it is considered a distant goal, addressable only after computational proficiency has been achieved. Problem solving should be considered as an on-going objective: how to teach children to attack real problems whose solution requires mathematical thinking. The interaction of problem solving and calculator use was discussed specifically. The conferences considered practices and instructional materials for use with calculators and for teaching problem solving. "What is" and "what should be" served as thrusts.

Participants

Five conferences were held during Spring 1978 and a sixth conference during Fall 1978. The dates and locations were:

<table>
<thead>
<tr>
<th>Date</th>
<th>Location</th>
<th>Venue</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-17 March</td>
<td>Chillicothe</td>
<td>Holiday Inn</td>
</tr>
<tr>
<td>22-24 March</td>
<td>Toledo</td>
<td>Quality Inn</td>
</tr>
<tr>
<td>25-27 April</td>
<td>Kent</td>
<td>Kent State University</td>
</tr>
<tr>
<td>9-11 May</td>
<td>Middletown</td>
<td>Miami University, Middletown Campus</td>
</tr>
<tr>
<td>17-19 May</td>
<td>New Philadelphia</td>
<td>Holiday Inn</td>
</tr>
<tr>
<td>1-2 December</td>
<td>Columbus</td>
<td>Ramada Inn</td>
</tr>
</tbody>
</table>
A brochure announcing the first five conferences (see Appendix A) was mailed in late November 1977 to all county superintendents and supervisors, to LEAs in exempted villages and cities, and to colleges with mathematics educators. The brochure for the sixth conference was mailed in October 1978. Application forms were included and appropriate persons encouraged to submit applications. Follow-up letters were sent, and phone calls were made to all school districts in two regions. The phone calls, however, generated few applications at much expense of Staff time, so they were not made (systematically) for the other four conferences.

While it had been anticipated that participants would be selected on the basis of (1) category (supervisor or mathematics educator), (2) region or district, and (3) interest, in practice only sufficient numbers of persons applied so that all could be accepted. [Unfortunately, December 1977 and January-February 1978 were poor months for Ohio schools. For instance, some schools were open only five or six days during January due to weather conditions. (Every Staff meeting scheduled for January was snowed out.) In addition, monetary limitations precluded some districts from participation. Federal funds provided lunch and materials; districts, however, were expected to provide transportation costs. For many Ohio schools, tight budgets precluded such expenditure.]

The first five conferences were held in five diverse geographical regions of Ohio, determined on the basis of school population distribution and so that they were within commuting range for all participants. The sixth conference was held in the central (Columbus) area, so that it might be accessible to anyone in the state.

To reach the vast number of teachers in Ohio's 600-plus districts directly was deemed impossible; even selecting one teacher from each district would virtually eliminate a participatory conference. We therefore focused on supervisory personnel who work directly with teachers, in order to gain a "multiplier effect". Many districts sent principals or teachers, as the persons responsible for conducting in-service activities with teachers. To provide these supervisors and teachers, who frequently do not have a mathematics background, with linkages with resource personnel, a group of mathematics educators was also selected.

For each conference, attendance consisted of:

<table>
<thead>
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<th>Anticipated</th>
<th>Actuality</th>
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</thead>
<tbody>
<tr>
<td>Approximately 40 supervisors or other leaders of in-service activities in schools</td>
<td>Chillicothe 17</td>
</tr>
<tr>
<td></td>
<td>Toledo 25</td>
</tr>
<tr>
<td></td>
<td>Kent 32</td>
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<td>Middletown 25</td>
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<tr>
<td></td>
<td>New Philadelphia 26</td>
</tr>
<tr>
<td></td>
<td>Columbus 44</td>
</tr>
</tbody>
</table>

| Approximately 10 mathematics educators from colleges and universities | Chillicothe 5 |
| | Toledo 6 |
| | Kent 5 |
| | Middletown 10 |
| | New Philadelphia 5 |
| | Columbus 2 |
Staff

The Staff consisted of:

Kenneth Cummins, Professor of Mathematics, Kent State University
Thomas C. Gibney, Director, Division of Curriculum and Instruction, University of Toledo
Johnny Hill, Campus School Teacher and Professor, Miami University
Steven P. Meiring, Mathematics Consultant, Ohio Department of Education
Len Pikaart, Robert L. Morton Professor of Mathematics Education, Ohio University
Marilyn N. Suydam, Professor of Mathematics Education, Ohio State University (Director)

Planning meetings were held in October, December, and February. Preceding each conference day, the Staff met to discuss evaluative comments and to plan for the following day.

Schedules

The conference schedules were planned to include a variety of active and more passive activities, in large and small groups. The Staff used a team-teaching approach. A sample schedule is found in Appendix B. The first day of each of the five Spring 1978 conferences was planned for mathematics educators from colleges and universities, to discuss questions related to their functioning as linkage personnel both in the conference and in the activities to follow in individual schools and districts, and to provide a setting in which mathematically-related questions beyond the concerns and background of the supervisors could be discussed. The following two days were planned for all participants, involving them in a somewhat prototypic in-service session. The sixth conference consisted of only the two-day plan.

Resource Packets

Resource packets were compiled by the Staff, and provided for each participant for use in the conference and for use in work directly with elementary school teachers. Materials were drawn from a wide variety of sources, including projects funded by the National Science Foundation and other non-commercial and commercial sources. In addition, many documents were prepared by the Staff to meet particular needs.

The packet is included in Appendix D. As it is included here, it is not fully complete, for it includes only references to some printed materials which were included in the packet but could not be placed in this report due to copyright restrictions. In addition, videotapes were prepared; these were shown and discussed at the conferences, and made available for participants to use when conducting in-service activities.
Evaluation was conducted in two phases, immediately following each conference and in a follow-up questionnaire sent in Fall 1978. The immediate evaluations were helpful in planning for the following conference; the follow-up form gives some indication of the extent to which the dissemination goal was met.

Immediate Evaluation. Evaluation data are reported in Appendix C. For the multiple-choice form, some data analyzed by computer are included, indicating responses for each of the questions by type of participant by conference. For the free-response questions, a synthesis of participants' comments is given.

As can be noted, the response was positive, with many helpful comments.

Follow-up Questionnaire. This was an open-ended form, asking participants to specify:

1. Have you had, or are you having a meeting, workshop, or other type of in-service session? If so, when? Where? Number of participants? Teaching level of participants?

2. Have you done any newsletters, news items, or other forms of dissemination? If so, what?

3. How have you used the resource packets?

4. Have you additional suggestions for the ORC Staff?

Sixty-six participants responded. Of these,

- 51 held meetings, workshops, or other in-service activities involving 2000 participants
- 33 used written forms of communication or dissemination
- 64 indicated that they had used the resource packets
Appendix A:

Announcement and Application Forms
The National Science Foundation has funded five Regional Conferences in Elementary Mathematics Education for the State of Ohio. The major topics of these conferences are the roles of the hand-held calculator and problem solving in the elementary mathematics curriculum. The purpose of the conferences is to instruct representatives from Ohio school districts to use resource packets on these two areas for in-service work with local teachers, and to help them to form links with mathematics educators in their region.

Enclosed are conference brochures and application forms. We hope that you will identify the appropriate persons in mathematics education in your institution and forward these materials to them.

We appreciate your cooperation.

Sincerely,

Marilyn N. Suydam
Project Director
Professor of Mathematics Education

Steven P. Meiring
Mathematics Consultant
Ohio Department of Education

MNS/lcs
SELECTION OF PARTICIPANTS

For each conference, approximately 50 participants will be selected:

- 11 elementary supervisors from each region (county, city, or exempted village district)
- 11 elementary mathematics educators from public and universities in each region

The Project Staff will select the participants on the basis of:

- geometry - elementary supervisor (or other district representative), or mathematics educator
- region or district - attempting to provide wide representation within geographical area
- interest - expressed commitment to elementary education from the conference
- support - a letter of support from superintendent or college administrator

An APPLICATION FORM is enclosed.

The application should be returned by January 20

to: Marilyn A. Suydam, Director
The Ohio State University
1986 Worthington Rd.
Columbus, Ohio 43212

Participants will be notified of their acceptance by January 31, 1978.

Late applications may be accepted if vacancies occur.

PARTICIPANT SUPPORT

Lunch and "coffee break" refreshments will be supplied for all participants each day. However, neither overnight lodging nor all-expense-paid costs can be reimbursed. Each conference is within commuting distance for participants; locations were selected in terms of accessibility.

The school districts of Ohio have been divided into five geographical areas. Participants are expected to attend the conference in their geographical area (see map on back of application form).

In the operation of the project and in selecting participants, no discrimination will be practiced in terms of sex, race, creed, color, or national origin.

One unit of university credit may be available for participants. If interested, please indicate on the application.

Ohio Elementary Supervisors
and Elementary Mathematics Educators

Chillicothe - March 15, 16-17
Toledo - March 22, 23-24
Kent - April 25, 26-27
Middletown - May 9, 10-11
New Philadelphia - May 17, 18-19

Mathematical Directions

Solving

Uses
of
Calculators

Problem

Sponsored by

The National Science Foundation and
The Ohio State University
here is the role of the calculator in the elementary school classroom.

- to recognize the importance of teaching as a major curriculum outcome
- to use the interplay of the calculator and their relationship to the general emphasis on the basics
- to establish links between superintendents and mathematics educators in each region for continuing curriculum development and improving instructional practice.

Why?

Elementary mathematics education is currently in a state of flux, prompting:

- outside forces resulting from national influence of recent mathematics programs to meet learner and societal needs
- internal examination of the role that mathematics instruction should play in the present and future
- the role of the calculator and problem solving in current elementary mathematics programs

Teachers in this need to be aware of the major issues, innovations, and alternatives in elementary school mathematics as they plan for the future.

The focus of the conferences will be on the handheld calculator, a technological tool with potentially wide implications for addressing the thrusts of elementary mathematics: structure, applications, and guidance for the use of calculators will be explored. Included in this exploration will be ways to improve instruction on basic skills through the use of calculators.

The other focus will be on the principle goal of mathematics program, problem solving. Too often it is considered a distant goal, achievable only after computational proficiency has been achieved. Problem solving should be considered as an ongoing objective how to teach children to attack real problems whose solution requires mathematical thinking.

The conferences will consider practices and instructional materials for use with calculators and for teaching problem solving. "What is" and "what should be" will serve as thrusts.

PROJECT STAFF

Kenneth Cummings, Kent State University
Thomas Gibney, University of Toledo
Johnny Hill, Miami University
Steven Meiring, Ohio Department of Education
Len Pikaart, Ohio University
Marilyn Suydam, The Ohio State University (Director)

H O W ?

Each conference will begin with a one-day session for elementary mathematics educators in the region. Mathematical tools and procedures for aiding in the development of effective in-service education programs will be explored.

The second and third days of each conference will involve elementary supervisors with the mathematics educators. The conferences will feature:

- review of current practices and research on problem solving and calculator use
- small-group sessions for work with resource materials (which will include transparency masters and copier-ready materials for use by participants in conducting workshops with teachers)
- videotapes and films which examine effective strategies for each topic
- hands-on experiences with calculators (which will be provided for participants)
- discussion of strategies for in-service meetings
- consideration of varied instructional materials and strategies
APPLICATION FORM

Ohio Regional Conferences in Elementary Mathematics Education
sponsored by The National Science Foundation and The Ohio State University

Name ____________________________________________

Title/Position ____________________________________________

School/Office Address ____________________________ Street
Ohio State Zip

City State Zip

Home Address ____________________________ Street
Ohio State Zip

City State Zip

I would like to attend the Regional Conference in: (select one; see map on back for counties in each region)

____ Chillicothe - March 15, 16-17 FIRST DAY OF EACH CONFERENCE includes MATHEMATICS EDUCATORS only; SECOND AND THIRD DAYS include SUPERVISORS and MATHEMATICS EDUCATORS

____ Toledo - March 22, 23-24, 1978

____ Kent - April 25, 26-27, 1978

____ Middletown - May 9, 10-11, 1978

____ New Philadelphia - May 17, 18-19, 1978

What type of in-service experiences have you had with elementary teachers?

________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________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It is with pleasure that we accept your application to attend the National Science Foundation-sponsored Ohio Regional Conference on Elementary Mathematics Education at New Philadelphia on May 17, 18, and 19. The meeting will be held at the Holiday Inn. A map is enclosed pinpointing the location.

The conference will start at 9 a.m. each day, and will end by 4 p.m. at the latest. Lunch will be provided for you. (As you realize, no other costs can be reimbursed.) You will also receive a Resource Packet on problem solving and on calculators, which we hope will be useful to you as you conduct in-service work on mathematics education.

If you cannot attend, please contact me at the above address. If I don't hear from you, I'll assume that I'll see you in New Philadelphia.

I apologize for the delay in sending this acceptance: I know that you realize the problems that the weather has been giving everyone.

Sincerely,

Marilyn N. Suydam
Director
Ohio Regional Conferences
Appendix B:

Sample Schedules
OHIO REGIONAL CONFERENCE
ON ELEMENTARY MATHEMATICS EDUCATION:
New Philadelphia

Wednesday, 17 May 1978

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30 - 9:00</td>
<td>Coffee</td>
</tr>
<tr>
<td>9:00 - 9:30</td>
<td>Introductions</td>
</tr>
<tr>
<td></td>
<td>Rationale and purpose of the conference</td>
</tr>
<tr>
<td>9:30 - 9:50</td>
<td>Your needs and perceptions</td>
</tr>
<tr>
<td>9:50 - 10:05</td>
<td>Review of problem solving packet</td>
</tr>
<tr>
<td>10:05 - 10:20</td>
<td>Review of calculator packet</td>
</tr>
<tr>
<td>10:20 - 10:30</td>
<td>Coffee</td>
</tr>
<tr>
<td>10:30 - 11:45</td>
<td>Issues concerning problem solving</td>
</tr>
<tr>
<td>11:45 - 12:00</td>
<td>Browsing + readying for lunch</td>
</tr>
<tr>
<td>12:00 - 1:00</td>
<td>Lunch</td>
</tr>
<tr>
<td>1:00 - 2:15</td>
<td>Issues concerning calculators</td>
</tr>
<tr>
<td>2:15 - 3:15</td>
<td>Possibilities for preservice education</td>
</tr>
<tr>
<td>3:15 - 4:00</td>
<td>Opportunities for using this project's outcomes</td>
</tr>
</tbody>
</table>
OHIO REGIONAL CONFERENCE
ON ELEMENTARY MATHEMATICS EDUCATION:
New Philadelphia

Thursday, 18 May 1978

8:30 - 9:00 Coffee
9:00 - 9:45 Introductions: to the conference and to each other...
   Why problem solving? Why calculators?
   Contents of the Resource Packets

9:45 - 10:30 Goals of problem solving in the elementary school
10:30 - 10:45 Coffee/browsing
10:45 - 11:45 Problem solving activities and strategies
11:45 - 12:15 The evidence on problem solving
12:15 - 1:15 Lunch
1:15 - 2:00 Problem solving with children
2:00 - 2:30 Problem solving in the primary grades
2:30 - 2:45 Coffee/browsing
2:45 - 3:30 Teaching strategies for problem solving
3:30 - 4:00 Planning workshops for teachers
Friday, 19 May 1978

8:30 - 9:00 Coffee
9:00 - 9:15 Reviewing yesterday and starting today
9:15 - 10:15 The use of calculators in grades K-8
10:15 - 10:30 Coffee/browsing
10:30 - 11:00 The effects of calculator use
11:00 - 12:00 Activities with calculators
12:00 - 1:00 Lunch
1:00 - 1:45 Children using calculators: some techniques
1:45 - 2:30 Using the calculator in problem solving
2:30 - 2:45 Evaluation of the conference
2:45 - 3:00 Coffee/browsing
3:00 - 3:45 Planning workshops for teachers
3:45 - 4:00 In conclusion...
Appendix C: Evaluation Data
1. Participant position— Are you:

A. Elementary School Supervisor or Consultant.
B. Supervisor of Grades K-12.
C. Elementary School Principal.
D. Mathematics Education (College Level).
E. Other.

<table>
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<th>COUNT</th>
<th>TOT PCT</th>
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<th>COL PCT</th>
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<td>4</td>
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</table>

20
2. How clear were the objectives or purposes of this conference?

   The objectives and purposes:

   A. Were clearly outlined from the beginning.
   B. Became clear as the conference developed.
   C. Became somewhat clear as the conference progressed.
   D. Were referred to only indirectly.
   E. Were never made clear.

<table>
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<tr>
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<th>Became Clear</th>
<th>Somewhat Clear</th>
<th>Contrast</th>
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<tr>
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3. The agreement between the announced purpose of the conference and what was actually presented was:

A. Superior.
B. Above average.
C. Average.
D. Below average.
E. Poor.

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COLUMN TOTAL | 95 | 66 | 8 | 1 | 1 | 171 |
4. How well was the conference organized?

A. The conference was extremely well organized and integrated.
B. The conference was adequately organized.
C. The conference had less organization than would seem desirable.
D. The conference had no apparent organization.
E. The conference was too tightly organized, there was not enough flexibility to meet participant needs and desires.

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5. Concerning the mixture of participants, do you think:

A. There should have been more supervisors than there were.
B. There should have been more college teachers.
C. The mixture was about right.
D. The groups should have met separately at the conference.
E. The groups should have separate conferences.

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Notes:
- Column 1: More Supt.
- Column 2: More Cols.
- Column 3: Mixture Right
- Column 4: Should Meet Separately
- Column 5: Separate Conferences
6. How well did this conference contribute to your professional needs?

A. Made a very important contribution.
B. Was valuable, but not essential.
C. Was moderately helpful.
D. Made a minor contribution.
E. Made no significant contribution.

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7. How well would you rate the usefulness of the Resource Packet on Problem Solving?

A. Extremely valuable.
B. Very useful.
C. Useful.
D. May be of use.
E. Useless.

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9. How clearly were your responsibilities in this conference defined?

A. I always knew what was expected of me.
B. I usually knew what was expected of me.
C. I usually had a general idea of what was expected of me.
D. I was often in doubt about what was expected of me.
E. I seldom knew what was expected of me.

\[\begin{array}{|l|c|c|c|c|}
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 & 20.9 & 14.3 & 26.0 & 0.0 & & \\
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 & 14.3 & 28.6 & 14.3 & 33.3 & & \\
 & 7.6 & 10.5 & 1.2 & 0.6 & & \\
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\text{New Philadelphia} & 15 & 8 & 1 & 0 & 24 \\
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 & 53.7 & 36.8 & 8.2 & 1.8 & & 100.9 \\
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\end{array}\]
10. How would you rate the conference effectiveness relative to your investment of time and effort?

A. Very high value for my effort.
B. High value for my effort.
C. Moderate value for my effort.
D. Low value for my effort.
E. No value for my effort.

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11. Do you feel that the presenters were willing to give personal help in this conference?

A. I felt welcome to seek personal help as often as I needed it.
B. I felt free to seek personal help.
C. I felt he or she would give personal help if asked.
D. I felt hesitant to seek personal help.
E. I felt that he or she was unsympathetic and uninterested in participant problems.

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40
12. Freedom of participation in conference meetings: questions and comments were:

A. Almost always sought.
B. Often sought.
C. Usually allowed.
D. Seldom allowed.
E. Usually inhibited.

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13. Did the conference help prepare you to lead in-service activities on problem solving and calculators?

A. Definitely.
B. It was a help on both.
C. On one of the topics.
D. It was little help.
E. It was no help.

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32
14. Would you recommend this conference to a good friend whose interests and background are similar to yours?

A. Recommend highly.
B. Generally recommend.
C. Recommend with reservations.
D. Definitely not.

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15. How would you rate your understanding of **Problem Solving** as a result of this conference?

A. I learned a lot.
B. My understanding improved.
C. A few ideas were new to me.
D. I learned very little.
E. I learned almost nothing.

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| COLUMN TOTAL | 58.9 | 97.6 | 8.8 | 0.6 | 100.0 |
16. How would you rate your understanding of the use of Calculators in schools as a result of this conference?

A. I learned a lot.
B. My understanding improved.
C. A few ideas were new to me.
D. I learned very little.
E. I learned almost nothing.

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The presenters seemed:

A. Always prepared.
B. Almost always prepared.
C. Usually prepared.
D. Frequently not prepared.
E. Never prepared.

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35
18. How would you rate the presenters' sensitivity to what you consider to be the important problems in elementary school mathematics?

A. They were well aware of the important problems.
B. They were aware of these problems.
C. They had a general idea of the problems.
D. They had a vague knowledge of some problems.
E. They did not seem informed of significant problems.

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37
19. How would you rate the presentations, in general?

A. Outstanding and stimulating.
B. Very good.
C. Good.
D. Adequate, but not stimulating.
E. Poor and inadequate.

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20. Would you attend conferences on other (like these) topics in this geographic area?

A. Definitely.
B. Yes, but in a bigger city.
C. It would be a good idea.
D. Probably not.
E. Definitely not.

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Column Total: 143 | 3.5 | 20 | 2 | 171 | 100.0
21. How would you rate the use of instructional media in this conference?

A. The uses of media were almost always effective.
B. The uses of media were usually effective.
C. The uses of media were sometimes effective.
D. The uses of media were seldom effective.
E. The uses of media were never effective.

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COLUMN TOTAL 171

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22. Do you believe that the conference helped establish (or improve) positive linkages between school system personnel and college mathematics educators?

A. Definitely.
B. Somewhat.
C. Very little improvement.
D. No improvement.
E. The linkages should not be established.

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Following the 22 multiple-choice questions were five open-ended questions. Responses to the first four of these follow; the fifth question asked for participants' plans for conducting in-service activities.

The responses on the following pages have been roughly grouped. The number in parentheses indicates the number of persons giving that answer or one very similar in wording.

It is hoped that other persons may obtain some clues about conducting in-service conferences from these reactions.
1. **Best features of the conference were:**

The resource packets — most valuable/excellent [35]

Abundance of materials for in-service [8]
Including both background literature and classroom activities. [2]
Ready-to-use ideas; many items that are useful in the classroom. [5]
The fact that all the materials used plus the talks are included for future reference. [3]
The presentations were correlation with the packets. [3]

Organization, integration and presentation of concepts and ideas in relation to goals. Nice blend between theoretical and practical. [1]

Discussion of issues and research relative to rationales. [5]

Enjoyed each aspect — gained much through lectures, discussion and activities. [5]

Broad scope covered, with up-to-date, concise information: excellent overview. [6]

Continuity of subject matter as well as interesting topics; openness and ability to voice an opinion; also stressing that there is not just one correct answer. [1]

Experiences with problems and problem-solving strategies. [24]

Not being afraid to tackle problem solving! [1]
The problem-solving examples and presentations were outstanding. Both days moved quickly, with an idea shown, practiced, and wrapped up. [2]
Helped me to realize the importance of problem solving being presented by teachers with the emphasis on a variety of solutions and not just one solution as too often presented in a textbook. [2]
The problems worked, the educational philosophy — EVERYTHING! [2]

Experiences with calculators. [14]

Just a good conference overall. [2]

Openness and willingness of high active involvement of participants. [13]
Selection of manipulative devices and materials. [1]

Mix of participants. [6]

Exchange of ideas/interaction with others from different areas, background. [15]
Illustrations of ideas with examples and other participants who had input to share. [2]
Opportunity to participate. [3]
Regional (and thus easy to attend). [3]
Evidence of good planning, clear objectives. [8]
High degree of organization. [9]
Time schedule observed and everyone moved quickly and efficiently. [8]
  Presentations kept on target. [3]
Entire conference had good change of pace. [4]
It was organized and kept moving. I had little feeling of "when
will this be over", even though such an all-day session should be
tiring. [1]
The conference seemed to relate to actual classroom problems and situations.
[2]
We all seemed to be working to the same end — worthwhile in-service. [1]
Display of materials. [4]
Research information. [3]
Videotapes of students. [8]
Food, meeting place. [9]
Good variety of presenters and presentations. [10]
  Stimulating presentations. [4]
  Presenters all very well organized and prepared/well-informed. [15]
  Wide scope of opinions and knowledge presented. [3]
  The obvious preparation done by the staff — and the staff cohesiveness — seeing a need and trying to meet it. [3]
  The preparation and concern the team exhibited for the conference. [2]
Excellent/outstanding staff. [5]
  Dynamic personalities of presenters (good blend of humor and academic presentation). [4]
  The expertise of those conducting the workshop. They were marvelously prepared. There was such good humor throughout — a great working atmosphere. [2]
Teamwork: the coordination, preparedness, and delivery techniques of the presenters. The sessions were lively and stimulating, moved at a
good pace, and with a beautiful example of team teaching at its best. [2]
  The delightful personalities of some of the presenters and the friendly banter which added to the accepting climate. [1]
Excellent conference staff -- blended well -- represented good cross-section of mathematics education -- appreciated the input from the wide background of experience. [2]

It was GREAT! The "turn-over" of presenters kept the conference progressing at a motivating and stimulating pace. [4]

Enthusiasm of presenters. [11]

Everyone was so friendly and enthusiastic -- each presenter communicated the idea that they believed in what they were doing. [1]

Friendliness and willingness to help. [10]

Openness to ideas -- sharing of ideas and everyone working together on problems and ideas and seeing the many different ways of solving them. [1]

Enthusiastic presenter/presentation which exemplified method of presentation in structuring an in-service and for teachers in classroom situations. [1]

Rapport/compatibility between presenters. [4]

The interaction between the participants and the project team.

It was great! [6]

Friendly, relaxed atmosphere -- pleasant, informal, fun-filled, non-threatening. [13]

Warmth and humor/jokes. [2]

Variety of people -- realistic in their attitudes to day-to-day experiences (not in "ivory tower" of what "ought" to be instead of what "is"). [2]

Points of disagreement between presenters were most instructive. They provided balance and a realistic attitude seldom found in such conferences. [2]

Ken Cummins. [13]
2. Worst aspects of the conference were:

Too rushed — a great deal to accomplish in a limited time — couldn't digest all the information. [15]

The feeling of being rushed when exploring some problems. [7]

Too time-conscious — we were always clock-watching. [2]

Not sufficient time for interchange with presenters and other participants. [8]

Needed more time for reflection and discussion. Another day covering only the same amount of material may be useful. [3]

Two days could be devoted to a single topic. [1]

I did feel hurried — wish this had been a four-day conference. [1]

Too short a time — a week in summer and we could have taken this in creative directions. [1]

Length of time together with driving time for three days. [1]

Too much repetition of the first day on the other two days. [1]

It didn't last longer! [1]

The first part on philosophy and issues. [1]

Occasionally getting enmeshed in unimportant (to me) disagreements. [3]

Repetition of same concepts. [1]

One seemingly poor preparation. [1]

Sometimes conferees were confused as to what was taking place. [1]

Packet organization. [3]

Overwhelming number of handouts. [1]

Did not go through packets and discuss individual inclusions. [1]

Didn't particularly care for research results. [2]

Introduction to calculator use was fuzzy. [1]

Little help in setting up in-service programs, particularly for those new to the game. [1]

Insufficient room to work at the tables. [2]

Seating not positioned for comfortable viewing of presenters. [1]

Sitting time. [7]

Large group. [1]

Nametags are good, but a get-acquainted activity (not just introductions, more than the rope problem) would have been effective. [2]
There should be representations of administrators -- in particular, elementary principals, decision makers in instruction, school boards. [2]

Not including more classroom teachers. [1]

Unfortunately, some of the districts that need help didn't send representatives. [1]

Communicating information about the conference to the schools. Since I did not see the notice, the purpose was different from what I thought it would be. [1]

Could more emphasis be given to help supervisors with weak math backgrounds? [6]

Assumption that all participants were expert mathematicians, especially in the use of the calculator. [2]

Some problems presented were far beyond some elementary students. More needed in rudiments for those with slower learning rates. [3]

Quality of videotapes (noise, equipment quality). [4]

Film on problem solving. [2]

Many transparencies could have been clearer, more readable. [2]

None -- it was the best I've ever attended/I have no complaints. [4]

That it's over! [1]

[There were also comments on the room temperature, meals, and dates.]
3. I would suggest the following:

Either fewer topics or more time. [4]

- Shorten the conference (less time per day or only two days). [3]
- More time to consult with presenters. [2]
- More interaction in small groups. [5]
- Some demonstrations by participants. [1]
- Take a wee bit more time to enable participants to do -- and therefore cut back slightly on amount in presentations. [2]
- Occasionally too much time was spent on individual activities. You might tailor materials to fit time a little better. [1]
- So that everyone would have more time to work with one another, expand the conference to three or four days a week. [2]

Some people seemed to feel rushed. I appreciated the effort to keep on schedule and to "pack in" as much as possible. [2]

Follow-up on these two topics in the near future. [2]

- More conferences of this nature throughout the years. [3]
- Additional workshops with smaller doses of information, with time between for classroom use. [1]
- Have this type of conference as an annual event to deal with "timely" topics. [1]
- Present a similar conference to persons working with supervision of high school math. [2]
- Hold a follow-up session with a college person from our region. [2]

Include more classroom teachers. [6]

- Get teachers and principals to attend. [3]
- Communicate to administrators the need to send people to such conferences. [1]
- Involve more elementary specialists in your planning. [3]

Involve legislators in sessions! [1]

Continued communication between presenters and us -- on new thinking, research, meetings coming up, good books and products, etc. [3]

Follow-up by coordinating this into development of courses of study and curriculum guides. [1]

More time to look at materials -- plan presentation of them with evaluative comments. [4]

More detail on the actual organization and management of an in-service workshop. [5]
Better organization of handout materials -- index or table of contents, number pages, notebook binding. [8]

Perhaps a more selected group of handouts. [1]

More materials on lower elementary level. [5]

Give a variety of problems with different levels of difficulty so participants can be more comfortable. [3]

Since many could not handle the material presented, it would be appropriate to mention to participants to remember how students can also easily feel "lost" in math. [1]

I would have enjoyed a little background on the development of the calculator and its operation and use. [3]

Provide a collection of calculator activities coordinated with a typical K-8 mathematics program. [1]

Description and differentiation of calculators appropriate for various grade levels. [4]

Fewer problems with more alternatives generated per problem. [3]

Answers to the problems. [2]

Define the types of problems and grade levels better. [1]

Perhaps create problem centers, and let participants move to each center rather than passing out problems. [1]

The problem-solving film should be specifically scheduled. [1]

Little less use of videotape. [1]

More videotapes of classroom situations. [1]

More emphasis on individual inclusions from packet. [1]

More philosophical discussion (debates) on the two issues. [2]

More practical problems, leave most theory out. [1]

Have participants give their expectations/reasons for coming. [1]

Tell how the project came about, who people are, what is expected of participants. [1]

Objective stated for each session was not always clear. [2]

Temper the missionary zeal just a little. [1]

Keep up the good work. It's great to really go away anxious to get back into the classroom full of enthusiasm. [1]

No changes, the three days were great! [4]
4. Were there materials on display that you would like to see included in the Resource Packets?

Yes, but cost would have been prohibitive. [9]

Perhaps some books could be available for purchase. [4]

Sample pages from several books and curriculum guides. [2]

Illustrations of problem-solving strategies and techniques. [2]

I'd like to own the booklets from CIC! I'm especially happy to have the sheets I can duplicate for my kids to do, and to share at home. [1]

Cited specifically: books or activities from books by Burns [3]
in-service handbook from NCTM [3]
Immerzeel's cards [4]
the two NCTM yearbooks (1975, 1978) [2]
SRA kit (Judd) [3]
Western Springs booklets [1]
Creative Publications kits [2]
locking cubes [1]
commercial catalogs [1]
NCTM membership blanks [1]

A comprehensive list of materials on display, with sources and prices, as well as film sources. [11]

A critique sheet or evaluation of the (commercial) material available. [2]

Latest updates on materials from free sources. [1]

Uses and special features of calculators. [1]

More information on calculator research. [1]

More materials and information on in-service presentations. [2]

Packet is fine as it is -- seems to be sufficiently representative of the materials available. [2]

I'll have enough problems plowing through what I have. [2]

No. [17 explicit, plus many blanks]

I have not had a chance to look closely at the materials on display. [11]

You've done enough: let us do something for ourselves if we're really interested. [1]
Appendix D:

Resource Packet
OHIO REGIONAL CONFERENCES ON MATHEMATICS EDUCATION

Problem Solving Packet

Contents

National Council of Supervisors of Mathematics Position Paper on Basic Mathematical Skills

Problem Solving in the Classroom

Roles and Goals of Problem Solving

An Overview of Problem Solving in Elementary School Mathematics

Problem Solving in the Primary Grades

Abstracts of Selected Current Articles on Problem Solving in *The Arithmetic Teacher*

Application for Free NCTM Materials

Student Strategies for Solving Problems

Research on Problem Solving at the Elementary School Level

Selected Abstracts from Resources in Education (ERIC) on Problem Solving

Unified Science and Mathematics for Elementary Schools (USMES): Sample Materials

Oregon Mathematics Resource Project (MRP): Sample Materials

Mathematics Problem Solving Project (MPSP): Sample Materials, Calculators and Problem Solving

A Variety of Problems

Excerpts from Berea City Schools Scope and Sequence

Transparency Masters

In-Service Education
INTRODUCTION

The currently popular slogan "Back to the Basics" has become a rallying cry of many who perceive a need for certain changes in education. The result is a trend that has gained considerable momentum and has initiated demands for programs and evaluations which emphasize narrowly defined skills.

Mathematics educators find themselves under considerable pressure from boards of education, legislatures, and citizens' groups who are demanding instructional programs which will guarantee acquisition of computational skills. Leaders in mathematics education have expressed a need for clarifying what are the basic skills needed by students who hope to participate successfully in adult society.

The narrow definition of basic skills which equates mathematical competence with computational ability has evolved as a result of several forces:
1. Declining scores on standardized achievement tests and college entrance examinations;
2. Reactions to the results of the National Assessment of Educational Progress;
3. Rising costs of education and increasing demands for accountability;
4. Shifting emphasis in mathematics education from curriculum content to instructional methods and alternatives;
5. Increased awareness of the need to provide remedial and compensatory programs;
6. The widespread publicity given to each of the above by the media.

This widespread publicity, in particular, has generated a call for action from governmental agencies, educational organizations, and community groups. In response to these calls, the National Institute of Education adopted the area of basic skills as a major priority. This resulted in a Conference on Basic Mathematical Skills and Learning, held in Euclid, Ohio, in October, 1975.

The National Council of Supervisors of Mathematics (NCSM), during the 1976 Annual Meeting in Atlanta, Georgia, met in a special session to discuss the Euclid Conference Report. More than 100 members participating in that session expressed the need for a unified position on basic mathematical skills which would enable them to provide more effective leadership within their respective school systems, to give adequate rationale and direction in their tasks of implementing basic mathematics programs, and to appropriately expand the definition of basic skills. Hence, by an overwhelming majority, they mandated the NCSM to establish a task force to formulate a position on basic mathematical skills. This statement is the result of that effort.

RATIONALE FOR THE EXPANDED DEFINITION

There are many reasons why basic skills must include more than computation. The present technological society requires daily use of such skills as estimating, problem solving, interpreting data, organizing data, measuring, predicting, and applying mathematics to everyday situations. The changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators demand a redefining of the priorities for basic mathematics skills. In recognition of the inadequacy of computation alone, NCSM is going on record as providing both a general list of basic mathematical skills and a clarification of the need for such an expanded definition of basic skills.

Any list of basic skills must include computation. However, the role of computational skills in mathematics must be seen in the light of the contributions they make to one's ability to use mathematics in everyday living. In isolation, computational skills contribute little to one's ability to participate in mainstream society. Combined effectively with the other skill areas, they provide the learner with the basic mathematical ability needed by adults.

DEFINING BASIC SKILLS

The NCSM views basic mathematical skills as falling under ten vital areas. The ten skill areas are interrelated and many overlap with each other and with other disciplines. All are basic to pupils’ development of the ability to reason effectively in varied situations.

This expanded list is presented with the conviction that mathematics education must not emphasize computational skills to the neglect of other critical areas of mathematics. The ten components of basic mathematical skills are listed below, but the order of their listing should not be interpreted as indicating either a priority of importance or a sequence for teaching and learning.

Furthermore, as society changes our ideas about which skills are basic also change. For example, today our students should learn to measure in both the customary and metric systems, but in the future the significance of the customary system will be mostly historical. There will also be increasing emphasis on when and how to use hand-held calculators and other electronic devices in mathematics.
Problem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unafraid of arriving at tentative conclusions and they must be willing to subject these conclusions to scrutiny.

Applying Mathematics to Everyday Situations

The use of mathematics is interrelated with all computation activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in light of the initial situation.

Alertness to the Reasonableness of Results

Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential.

Estimation and Approximation

Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand.

Appropriate Computational Skills

Students should gain facility with addition, subtraction, multiplication, and division with whole numbers and decimals. Today it must be recognized that long, complicated computations will usually be done with a calculator. Knowledge of single-digit number facts is essential and mental arithmetic is a valuable skill. Moreover, there are everyday situations which recognition of, and simple computation with, fractions.

Because consumers continually deal with many situations that involve percentage, the ability to recognize and use percents should be developed and maintained.

Geometry

Students should learn the geometric concepts they will need to function effectively in the 3-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties which relate to measurement and problem-solving skills. They also must be able to recognize similarities and differences among objects.

Measurement

As a minimum skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools.

Reading, Interpreting, and Constructing Tables, Charts, and Graphs

Students should know how to read and draw conclusions from simple tables, maps, charts, and graphs. They should be able to condense numerical information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Using Mathematics to Predict

Students should learn how elementary notions of probability are used to determine the likelihood of future events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election forecasts.

Computer Literacy

It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching/learning, financial transactions, and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations.
BASIC SKILLS AND THE STUDENT’S FUTURE

Anyone adopting a definition of basic skills should consider the “door-opening/door-closing” implications of the list. The following diagram illustrates expected outcomes associated with various amounts of skill development.

<table>
<thead>
<tr>
<th>Scope of Skill Development</th>
<th>Expected Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXPANDED SKILLS</td>
<td>POTENTIAL LEADERS</td>
</tr>
<tr>
<td>Mathematical skills beyond those described here plus a desire to learn more.</td>
<td>Employment and educational opportunities will continue to increase as mathematical skills continue to grow.</td>
</tr>
<tr>
<td>basic mathematical concepts. teachers should utilize the full range of activities and materials available, including objects the students can actually handle.</td>
<td></td>
</tr>
<tr>
<td>BASIC SKILLS</td>
<td>EMPLOYMENT VERY LIKELY</td>
</tr>
<tr>
<td>The skills described here.</td>
<td>Employment opportunities are predictable. Doors to further education opportunities are open.</td>
</tr>
<tr>
<td>MINIMAL SKILLS</td>
<td>LIMITED OPPORTUNITIES</td>
</tr>
<tr>
<td>Limited skills, primarily computation. Little exposure to the other skill areas described here.</td>
<td>Unemployment likely. Potential generally limited to low-level jobs.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Any systematic attempt to develop basic skills must necessarily be concerned with evaluating and reporting pupil progress.

In evaluation, test results are used to judge the effectiveness of the instructional process and to make needed adjustments in the curriculum and instruction for the individual student. In general, both educators and the public have accepted and emphasized an overuse of and overconfidence in the results of standardized tests. Standardized tests yield comparisons between students and can provide a rank ordering of individuals, schools, or districts. However, standardized tests have several limitations including the following:

a. Items are not necessarily generated to measure a specific objective or instructional aim.

b. The tests measure only a sample of the content that makes up a program; certain outcomes are not measured at all.

Because they do not supply sufficient information about how much mathematics a student knows, standardized tests are not the best instruments available for reporting individual pupil growth. Other alternatives such as criterion tests or competency tests must be considered. In criterion tests, items are generated which measure the specific objectives of the program and which establish the student’s level of mastery of these objectives. Competency tests are designed to determine if the individual has mastered the skills necessary for a certain purpose such as entry into the job market. There is also need for open-ended assessments such as observations, interviews, and manipulative tasks to assess skills which paper and pencil tests do not measure adequately.

Reports of pupil progress will surely he made. But while standardized tests will probably continue to dominate the testing scene for several years, there is an urgent need to begin reporting pupil progress in other terms, such as criterion tests and competency.
measures. This will also demand an immediate and extensive program of inservice education to instruct the general public on the meaning and interpretation of such data and to enable teachers to use testing as a vital part of the instructional process.

Large scale testing, whether involving all students or a random sample, can result in interpretations which have great influence on curriculum revisions and development. Test results can indicate, for example, that a particular mathematical topic is being taught at the wrong time in the student's development and that it might better be introduced later or earlier in the curriculum. Or, the results might indicate that students are confused about some topic as a result of inappropriate teaching procedures. In any case, test results should be carefully examined by educators with special skills in the area of curriculum development.

CONCLUSION

The present paper represents a preliminary attempt by the National Council of Supervisors of Mathematics to clarify and communicate its position on basic mathematical skills. The NCSM position establishes a framework within which decisions on program planning and implementation can be made. It also sets forth the underlying rationale for identifying and developing basic skills and for evaluating pupils' acquisition of these competencies. The NCSM position underscores the fundamental belief of the National Council of Supervisors of Mathematics that any effective program of basic mathematical skills must be directed not "back" but forward to the essential needs of adults in the present and future.
Improving one's effectiveness in teaching problem solving skills requires consideration of many components that affect the success and richness of such learning experiences. Grouping of students, mode of presentation, external and internal variables, realistic modeling, and avoiding pitfalls are some of the important elements that will be discussed in this paper.

Grouping

Three kinds of basic grouping situations can be used effectively in teaching problem solving: large group instruction, small group instruction, and individual learning. Each has advantages, disadvantages, and a role in helping students develop broad problem solving skills.

Large Group. Large group activities are effective for introducing and practicing the application of a new problem solving strategy like "construct a table"; for examining a variety of different methods of solution of the same problem; and for discussing general aspects of problem solving such as initiating strategies and looking back strategies.

Large group instruction has the drawback that individual problem solving growth may be difficult to foster directly for any but the quickest students because individuals respond at different rates to problems and in different ways (favor individual strategies). Therefore, a quick student may solve a problem posed to the whole class before other students have had a chance to consider it thoroughly. Moreover, the wide range of problem solving abilities may make a problem appear trivial to some students and impossibly difficult to others. For these reasons, discussions about problem solving are possible with large groups but the process of solving problems may be practiced by very few of the group.

Small Group. Small group instruction makes it possible to group students according to problem solving ability and interest. This makes the task of selecting problems of an appropriate degree of difficulty much easier than with large groups. If the group is not too large, students also have the opportunity to engage in group problem solving efforts. In such small groups, students can generally solve more problems than those who work alone, but the groups may take a longer time on each problem than for pupils working alone. Group discussion in order to reach agreement on how to proceed has been shown to produce significantly better achievement than being told how to solve the problem.

Disadvantages of small group problem solving include the need for a large list of problems of varying degrees of difficulty, classroom management procedures, and difficulties in determining individual problem solving growth.

Individual Learning. Problem solving by individuals has the greatest potential for developing problem solving skills that can be both easily ascertained and measured. The individual, left to his own resources, can progress at his rate, use strategies that are comfortable to him, and experience the "aha" feeling wholly on his own. And problems can be selected
for individuals according to their needs, interests, and abilities.

Here again, however, monitoring progress and providing individual assistance is limited even more dramatically. Furthermore, one must be more alert to motivation needs, stress concerns, and incorrect procedures than in small or large group efforts.

Mode of Presentation

Real life problems come in a variety of formats and under a melange of circumstances. Consequently, teaching efforts should be directed toward varying the mode in which problems are presented for solution. Such variety will improve student capabilities to transfer school learned skills to practical problem situations.

Teacher Directed. Teacher led problem solving in which a particular strategy (or computational process) is applied to a group of very similar problems is the most common form of school problem solving activity. Although the customary procedure is to use this approach to practice and refine computational techniques through word problems, variations of this procedure can be used to focus more directly on the problem solving process.

Rather than working problems through to completion, a class discussion of a problem set can be restricted to a thorough talking through of each problem to assure an understanding of the given information, constraints, and problem task. Students can be asked to restate a problem in their own words in order to determine whether they fully comprehend the situation posed by the problem. This activity will help students develop the skill of clear identification of the problem task and help them differentiate between relevant and irrelevant information.

Asking students to describe the problem situation without reference to specific numbers can also help them to generalize the problem situation and processes involved for solution. In this way, student attention will be focused on the problem process, away from the specific answer, enabling an appreciation of the wider application possibilities of the solution process.

Another teacher directed technique with a whole class is to use a problem approach to motivate the need for further mathematical knowledge. By posing a relevant and realistic problem situation that students cannot solve (or solve only with great difficulty), a teacher can motivate the need for new mathematical techniques by answering before it is asked the question, "Why do we need to know this stuff?" And frequently, the new process can be skillfully developed within the context of solving the motivation problem.

Teacher directed problem solving efforts with small groups or individuals call for less direct involvement by the teacher as the problem solving process shifts to the student. After posing a problem of suitable difficulty, the teacher assumes the role of "silent partner," monitoring progress carefully, and reluctantly asking directive questions only as a student exhausts his resources or wears thin his patience. This is the most critical point of the instructional process. Acting too soon will encourage the learner to be dependent upon the teacher. Acting too late will produce anxiety and defeat—the precursors of mathophobia.

Student Generated. A most overlooked opportunity for problem solving instruction arises out of situations that come from students. Using situations that are student generated or revolve around student activities can help bridge the chasm between school practice and actual life applications.
For example, suppose a student observes the phenomenon that standing atop an object and looking down appears greatly different than standing below and looking up at the object. He asks whether there is any physical explanation for this common observation.

A great opportunity for problem solving is at hand! By throwing this question back at the class and asking them what kinds of strategies they might use (listing them for review) to resolve this problem, a very rich albeit unplanned activity can result. Students will see a reason for what they have been learning and discover how to go about applying their skills to a question that intrigues them from real life.

Similar opportunities can be drawn from student activities. Many examples of relevant student situations are found in textbooks. Abstracting these situations from a book format and casting them in a familiar locale with real people will not only enhance the interest level of students. They will also generally find such situations easier to comprehend and be able to transfer the solution process more easily to related problems.

Indirect. Solving a problem is perceived by most students as a challenge. This permits a teacher a third mode for problem solving—indirect presentation. Having established a "place" where interesting and challenging problems are located, students can either elect to use some of their undirected class time to use the "problem place" or be encouraged to that resource by the teacher.

Such "places" might be an attractively decorated bulletin board with The Problem of the Week; Sleuths Corner (or table) where a problem resource is located; stations where problems and equipment or materials useful to solution are gathered; or simply a resource file, cards, or other collection of problems in a selected place (including texts or other resource books.)

An advantage of indirect problem presentation is that a student has flexibility in being able to choose problems which he finds interesting or otherwise appropriate. Another is that he can experience the full problem solving process at his own pace without the threat of failure.

Problems posed for the teacher by the indirect presentation method include establishing a reward (reinforcement) system, student monitoring and evaluation, and matching student ability with problem difficulty. These concerns can be resolved, however, through means such as indexing problems with point values related to their difficulty or use of class problem progress charts that identify the problem, solver, and strategy used.

External and Internal Variables

Associated with problem solving instruction are several factors that affect the learner's ability to solve problems. Some of these factors are external to the learner and are easily controlled through the teaching situation. Others are internal to the learner and are therefore less easily influenced by the teacher.
External variables. Time is a primary consideration in teaching problem solving. The student must have the necessary opportunity to mull a problem over thoroughly, time to understand fully the task and conditions, time for incubation and illumination, and time to think about the solution. An important task for the teacher is to encourage students to extend the amount of time they are willing to work on a problem before "giving up."

All of these factors clearly demand more time from the curriculum; teaching for problem solving is not as efficient as teaching by rote. Therefore, rearrangement of some teaching priorities may be necessary. Or opportunities to teach other subject matter through problem solving may need to be utilized.

Learning style is another important external consideration. Learners respond differently to different stimuli. Some students may need tactile aids to understand the posing of a problem. Others may not. Some students work well independently. Others need more structure and direction. Some students may find certain problem solving strategies more productive than other strategies. Other students may disagree with this list.

Teachers also vary in their uses of instructional styles. There are usually some problem solving approaches that a teacher feels comfortable with and others that he feels less skillful in using. Generally, a teacher is well advised to use methods that utilize one's strengths while giving time to instructional approaches that one feels less expert with. This will widen the range of problem solving experiences for students, capitalize on a teacher's known strong areas, and give the teacher the opportunity to expand his expertise.

Motivation is both an internal and external variable. Most teachers are quite adept at being able to present learning situations in a highly interesting way. But in problem solving the motivation problem centers more around the need for continuing motivation that is sustained through teacher and peer recognition and reinforcement of accomplishments. One goal of problem solving instruction is to encourage students to work more independently and to expand their ability to deal with frustration. Therefore, it becomes very important for the teacher to recognize and reward efforts toward this goal even though a successful problem solution may not be immediately forthcoming.

In internal variables. A problem is not necessarily solved when a correct solution has been found. It is not truly solved until the learner understands precisely what he did and why it was appropriate.

This is one example of what we mean by an internal consideration—a variable that describes the extent of understanding within the learner. Generally we can describe certain elements that are related to this process.

1. Being able to recognize important features of a problem and associating these features with promising solution steps is an important problem solving skill.

2. The process of understanding a problem is related to the ability to restate the problem in an appropriate representation. For example, many problems can be classified as subtraction problems because subtraction is the necessary process required for solution. However, the problems may pose different structures within which that subtraction will occur. Choice of an appropriate numerical sentence to describe the structure can aid in the understanding of the problem. One might choose from the forms

\[ c - b = \] \[ c - ? = a \] \[ a + ? = c \]

each of which leads to a subtraction solution.
3. For a learning situation to be appropriate for problem solving development, the techniques necessary for its solution must be well within the student's range of capabilities and experience.

4. Meaningful problem solving experience should not be restricted to repetitious practice with the same technique applied to similar problems. Such experiences should consist of some practice of different approaches applied to similar problems and the same technique used to solve very different problems.

5. Protection from errors is not desirable in problem solving. Students should be encouraged to be sensitive to reasonability of results, to detect errors, and to explain where a mistake lies.

6. Skills and understandings are most useful in problem solving when they have been developed through problem solving.

7. Excluding lack of knowledge and limited mental capacity, there are known factors that cause some problems to be difficult to solve:
   a. misleading incorrect solutions - the learner halts his problem solving efforts without realizing his solution is wrong;
   b. difficulty in selecting from given alternatives - the learner is not able to systematically reduce the number of possibilities for problem solution;
   c. response having low priority in one's range of experience - the learner's background causes him to assign a low probability to productive problem approaches;
   d. requirement for generating an unusual response - the problem solution deviates markedly from the solver's past similar experiences;
   e. moving too quickly from idea-getting to idea-evaluation - the learner spends too little time generating solution possibilities before trying one;
   f. difficulty in identifying surmountable obstacles - the learner fails to distinguish obstacles to problem solution that can be overcome from those that cannot;
   g. motivation factors
   h. degree of stress

Realistic Modeling

The goal of problem solving instruction is to develop the capability of students to apply successfully school-learned skills to problems arising in real life. However, past experience indicates that transfer of school skills to life situations is not an easy task. The degree to which classroom activities can realistically model real life problem solving is related to how successfully students can make this transfer. Therefore, one of the teacher's tasks is to be aware of realistic problem characteristics.

1. In a real situation, the task to be accomplished or the problem to be resolved is usually well understood. The problem may need to be recast in an appropriate representation, but generally there are no difficulties with interpreting the problem task from given information as is the case in school story problems.

2. Real problems frequently deal with tactile materials and/or real people and situations. Being able to see the problem in terms of concrete materials or to associate it with specific persons or situations makes the problem easier to understand.
3. Real problems have a built-in motivation factor. The successful solution of the problem accomplishes something for the solver. He can immediately appreciate his success or be penalized for his failure until the problem is dealt with. He is therefore willing to wrestle with a problem for a considerable time period. This contrasts to a school situation where a student either moves on from an unsuccessful problem attempt after a brief effort or seeks help from the teacher.

4. There may not be "the" correct solution to a real problem. Many solutions may be acceptable in terms of the specific needs of the solver and the given solution. Rather than requiring an exact answer, it may be sufficient to be "close enough." For this reason, estimation and approximation skills are relatively high in importance in real problem solving.

5. There is no preferred strategy to the successful solution of a real problem. The sole criteria is that the problem be successfully solved in an efficient manner to the problem solver. Therefore, trial and error might be judged just as satisfactory a method as a careful, reasoned problem attack if it solves the problem.

6. Real problem situations frequently reoccur for the solver. This gives the problem solver the motivation to find a solution strategy that will accomplish the repeated task most efficiently. He therefore has a reason to look for more than one solution process to the problem.

7. Real problems are usually cluttered with a lot of irrelevant information or may even be missing some necessary data. One of the solvers first tasks is to distinguish between pertinent and nonpertinent information and to decide whether he has enough information to generate a solution.

8. Few real problems are purely mathematical. The problem may consist of many nonmathematical elements. The task of the solver may be to restate and simplify the problem to mathematical terms. Or a mathematical solution may only be one component to the resolution of a larger problem. Affective considerations frequently play a role in real problem solving.

Obviously, it is not possible nor desirable to simulate each of these real life problem characteristics in classroom problem solving. However, an awareness of them should enable the teacher to occasionally model such situations, making the transfer between school and life skills much easier for students.

Avoiding Pitfalls

In problem solving, as for most areas of instruction, there are a meshing of more than one set of objectives. That is, we pursue problem solving not solely as an end unto itself but in conjunction with other goals and objectives such as refinement and maintenance of computational skills. In trying to accomplish more than one goal through an activity, it is easy to lose sight of the main objective or to occasionally get instructional goals at cross purposes. The following listing contains frequent pitfalls involving problem solving that might arise in this way and that should be avoided.

a. Overemphasis on verbal cues such as equating the word "of" with "times" should be avoided. Such a practice may lead students to misinterpretations for given circumstances as well as focusing attention to early on the parts of the problem before he gains a grasp for the situation wholly.

b. Do not insist that a particular procedure be used in the solution of a set of problems. If the intent is to practice a particular procedure, then the objective may well be computational rather than problem solving.

c. Try not to imply to students that one strategy of many which can
be used to solve a problem is the "best." The intent is to increase the number of such strategies that a student has command over. And what may be best for one person may not be best for another.

d. In working with large groups, avoid instructional practices that involve a designated learner—that is, a class representative that does the talking, thinking, and actual problem solving for the class with no measure of how many other students understood or participated in what occurred.

e. Consider problem solving as an activity suitable for all students—not just the more capable ones.

f. Problem solving should not be withheld from any group of students until they master a certain set of basics. It can and should be an instrumental activity in the development of basic skills.

In summary, the task of teaching for problem solving development can appear to be an awesome task. But like most complex activities, the goal is well worth pursuing. However, in this case, the task is not as difficult as it might first seem. Most of the components for good problem solving instruction are ingredients for good teaching of any mathematical topic. And many of the aspects discussed in this discourse on problem solving are common tools of the good teacher.
Roles and Goals of Problem-Solving in the Elementary School

I. Introduction

John Dewey (1909) and his followers were perhaps the first in American education to attempt to cite problem-solving as an important goal in the curriculum as suggested in Dewey's *How We Think*. In 1918 Kilpatrick urged the "purposeful act" as the major item of curricular concern. In 1925 Collings reported work on using situations and activities in the lives of boys and girls as a basis for topics in their "content subjects." Kilpatrick commented:

Our highly artificial study of arithmetic, geography and physics has too often meant that the child lived but meagerly in and through the school studies (1:109).

Kilpatrick taught that the "wider problem of method"—that is, problem-solving—is synonymous with living or similar to the "moral problem of life itself." (1:3). Learning, he urged, should help engage students in living now and not be looked on solely as preparation for later life.

In 1932 came the Eight-Year Study (thirty secondary schools) in which the curriculum was planned around problems arising from various aspects of present-day (1932-1940) living. This was followed by the development of core curricula by Albery, et al in the 1950's. Still later in the 1950's, with concern for problem-solving still in mind, there arose the "essentialists" as Bestor and Rickover with the philosophy that different disciplines have different techniques of problem-solving—that the inquiry process differs from subject to subject. Indeed, some thrusts went so far as to encourage, it seems, that the best preparation for life was to take on the style of the researcher and, in the sense that students should explore and make conjectures there may be some point to this suggestion—it is wrong, though, if it means early and hurried abstractions before the student is ready. In the fifty years of thought and development many had forgotten the early tenets of Kilpatrick that "education be considered as life itself and not a mere preparation for later living" (2).

Although points of view apparently changed from 1909 to the present there always seemed to be concern with "problem-solving" no matter what the name. That this strand is of present-day concern and that educators see the continuing need for a problem-solving curriculum is stated by David Ost as follows:

Problem-solving and related skills have long been of concern to education planners... it is becoming increasingly clear that problem-solving abilities are essential for an evolving culture. As a result, increased effort is being made to incorporate problem-solving situations into the educational process (3).

But while we have traced the attention to problem-solving as an important curricular strand in the past years, we may yet ask, "What is
problem-solving?" Some answers:

We first think, "Solving problems!"

"Problem-solving is a search for alternatives (4)"

Problem-solving is a way to employ or apply cognitive skills to answer a felt question about a situation

Problem-solving is a process

Problem-solving is creative and reflective thinking brought to bear on a situation

Elsewhere we shall no doubt perceive differences between exercises and problems and so we will not dwell on these here.

II. Roles of Problem-Solving

By "role" we shall mean "what part should it play?" --"what part in the curriculum and in the classroom?" It seems to the writer that the spirit of "solving problems" or the atmosphere of eagerness and challenge to try to use various strategies of problem-solving should always be present in the classroom. Of course there must be time for skill development (perhaps per se in various modes), for diagnosis and remedial work, for practice on the ordinary text problems and exercises and/or problems on developing skills from "real problems" or variations of them previously done. But in nearly every class the students should be in exploratory and conjecture-making stages on some problem which arises either from the ongoing development of the material of from sources connected with the life of the student.

Problem-solving in all of its aspects is one of the main "reasons" for mathematics. Mathematics "came to be" as a "problem-solver" and, in its infancy, it took on problems of business, commerce, navigation, engineering and many other facets in the lives of people and it is still doing so as it continues to reign as the "queen of the sciences!" In one of his talks Morris Kline suggests that "the beauty of mathematics does not justify mathematics (instruction)" and neither does "the intellectual challenge" but what mathematics does and can do justifies it. Hence problem-solving should play a big role!

Specifically problem-solving can be an effective device for motivation and it can give valuable and continuing experience in the "art of investigation" and, while applying strategies, mathematical skills and concepts of various kinds are developed and created which enable us to attack more sophisticated problems.

III. Goals of Problem-Solving

Although the various strategies of problem-solving will be discussed by others one immediate goal of problem-solving is to acquire skill in the use of
various approaches and to perceive which method might be more applicable. These skills are of "higher order" than merely those of using the basic operations in mathematics but, of course, the basic operations and much more are used. We will call "acquiring skill" goal (a).

Other goals, just as important, are:

(b) to be able to sense a problem. Bruner suggests that "sensing a problem is the most natural intellectual activity. It is an awareness that something needs to be different, improved, modified. (5) Teachers can encourage the "sensing of problems" in the classroom by an exciting handling of subject matter and by helping the student recognize problems which arise in the ongoing development or the subject itself. Not all "problems" need come from the "outside."

(c) to be able to determine the scope of the problem and to delimit it to a level which can be handled by the student with skills, materials and information readily available---this does not mean that one will not attempt to develop new skills. "Delimiting" is sometimes urgent and most desirable for sometimes the "gravity of a problem produces inertia" (Morris Kline).

(d) to develop flexibility to entertain the possibility of attack by several different strategies

(e) to tend to try the simplest strategy first --- try common sense.

(f) to develop the ability to suggest variations on a problem

(g) to live in anticipation of other paths or turns a problem might take

(h) to be looking for problems and "eyes open" for relationships --- sensing sources of problems in our surroundings: social, environmental

(i) to have a spirit of adventure, creativity, anticipation, excitement and a (humble) feeling of CONFIDENCE. No suggested strategy should be an object of scorn or belittling or embarrassment. "Nothing breeds success like success itself."

IV. Beginnings on Achieving Goals

---START NOW with ongoing development in the classroom--- helping the subject grow by sensing problems

---RAISE QUESTIONS in a casual way-- questions which give rise to or introduce considerations from which grow new concepts
--INTRODUCTION-- of interesting situations which come from problem sources
(as the packets assembled for this conference) and which employ "problem-
solving strategies."

--BEGIN WITH SIMPLE THINGS-- simple ideas perhaps also in physical
settings as $\frac{1}{3} \times \frac{1}{4}$ by means of fraction cards (6). Big ideas
often come from little questions and little situations.

Dwight L. Moody has said, "When God wants to move a mountain,
he does not take a bar of iron, but he takes a little worm."

References

1. William H. Kilpatrick. Foundations of Method: Informal Talks on

2. William H. Kilpatrick. "The Project Method." Teachers College Record,
19-4 (September, 1918), 6-7.

Bakersfield, California: California State College, 1974. (Mimeographed)

4. Leland F. Webb and David H. Ost. "Real Comprehensive Problem Solving
As It Relates to Mathematics Teaching in the Secondary Schools,"

5. Ibid.

many examples of student-centered approaches to teaching which help in
the ongoing development of the subject.
SOME GEMS FOR TEACHERS

"Begriffe ohne Anschauung sind leer." (Kant)

"...mathematics 'in statu nascendi'--in the process of being invented--has never before been presented in quite this manner to the student, or to the teacher himself, or to the general public." (Polya)

"The investigator himself...does not work in a rigorous deductive fashion. On the contrary he makes use of phantasy and proceeds inductively, aided by heuristic expedients." (Felix Klein)

"The premise here is that education has a great teaching facility which as yet is unused--the student." (Ohio State University Educational Research Bulletin, XXXIX: 6, September 14, 1960).

"Learning can be deepened and be made more genuinely human as well as beneficial as a human act if it appropriates the procedures of both the creative artist and critic." (Meland)

"...it is first necessary to arouse his (the student's) interest and then let him think about the subject in his own way." (J. W. Young)

"Nothing is more important than to see the sources of invention which are, in my opinion, more interesting than the inventions themselves." (Leibnitz)

"The most natural methods of advance is a series of successive approximations to logical rigor, and, in fact, this is the way in which the subject has actually grown up." (Saxelby)

"...the teacher should lead up to an important theorem gradually in such a way that the precise meaning of the statement in question...is fully appreciated ...and furthermore, the importance of the theory, and indeed the desire for formal proof is awakened, before the formal proof itself is developed. Indeed, much of the proof (of the theorem) should be secured by the research of the students themselves." (E. H. Moore)

"In our mathematics classes we ought to concentrate less on covering a certain body of knowledge and more on thinking about what we have done, how that can be generalized and applied to other problems, how it can be changed into new problems, and how to go about finding general principles. (Willoughby's "Discovery." The Mathematics Teacher, January, 1963).

"No matter how modern the mathematics program, it will suffer a miserable death if there is no creativity or imagination in the classroom. On the other hand, the most traditional program can become most exciting if somehow the student, if in a small part only, relives the discovery of mathematical ideas and the clashing of minds which have been a part of its development."

"'Covering pages' may deaden, 'uncovering ideas' enlivens the classroom."

"Non verba, sed res." (Comenius)
"Quod non fuerit priusquam in sensu, non est in intellectu." (Comenius)

"The mathematical experience of a student is incomplete if he has never had the opportunity to solve a problem invented himself." (Polya)

"This intuitional direct vision method is intended, not to take the place of, but to prepare the way for a more rigorous analytical study of the subject." (Saxelby)

The student "in each new advance is to begin with the concrete object, something which he can see and handle and perhaps make, and go on to abstractions only for the sake of realized advantages." (Durell)

"Most of the footprints in the sands of time have been made by workshoes."

"What is most important for teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization or more conceptually modes of thought." (Bruner)

"...concepts are formed out of experiences, and hence the classroom should be arranged so that the children learn mathematics in much the same way as they learn most of the things they know--by manipulating actual objects." (Dienes mentioned in Arith. Teach. Nov., 1968, p. 594.)

"The typical mathematics lesson...no physical apparatus for the children to play with, and a lesson involving merely talking, reading and writing--may eventually be outmoded." (Arith. Teach. Nov., 1968, p. 594.)

"The teacher must attempt to rescue the low achiever from discouragement and despair and make it possible for him to succeed at his own ability level"...while keeping in mind that not all types of slowness are permanent...it may take only interest on the part of the student to increase his achievement." (Mathematics Teacher, March 1967, "Mathematics for the Low-Achiever in High School.")

"There is nothing in the intellect that was not first in the senses..." (Morris Kline, AMM, March, 1970, p. 263).

"Clearly the intuitive approach can lead to error, but committing errors and learning to check ones' results are part of the learning process." (Ibid., p. 266).
"Learning to solve problems is the principal reason for studying mathematics." (National Council of Supervisors of Mathematics (1978), p. 147). Knowledge, skills, and understandings are important elements of mathematical learning but it is in problem solving that the student synthesizes these components for the purpose of achieving a goal, answering a question, or reaching a decision.

Though most educators agree on the importance of problem solving, nearly any discussion about its role in the elementary school curriculum raises issues in need of further clarification. Four questions which usually emerge are: (1) What is the relationship of problem solving to basic skills? (2) Exactly what is meant by problem solving? (3) How should problem solving be taught? (4) Whose responsibility is it to help students develop problem solving abilities? That is, when should problem solving be introduced in the curriculum?

The purpose of this paper is to present brief answers to these four questions and thus orient you to the problem solving approach taken in these materials.

Is Problem Solving a Basic Skill?

Computational proficiency has been long recognized as a basic skill goal of the elementary mathematics curriculum. The role and importance of problem solving in the elementary program has been generally less well defined. Traditionally, problem solving has been pursued with less immediacy and emphasis than computational skill development and, consequently, has not been considered by many as a basic goal. However, current interest
in redefining learning outcomes considered as basic—that is, learning areas important for mastery by all individuals—suggest the need for an expanded definition of basic skills.

Assessment results have highlighted a fact known informally to us all: computational skill proficiency is not synonymous with the ability to apply these skills in meaningful situations. Basic skills by today's standards are judged in terms of competencies. That is, we are concerned with not only what an individual knows but with how well he can use this knowledge. Problem solving constitutes this additional dimension of the basic skill definition in the area of mathematics.

"I think problem solving is the basic skill in mathematics. By problem solving I mean more than knowing what to do, in the sense of having knowledge. I mean having a kind of commitment to problem solving—a willingness to tackle a problem even when one doesn't know right away what to do, and to keep plugging away at the problem until one finds a reasonable solution." This quote comes not from a mathematics educator but from a Georgia state legislator.

While it may be surprising to some to think of problem solving as a basic skill, its development marks the difference between mathematically literate and minimally functioning individuals. A learner with uncultivated problem solving capabilities is a passive and limited individual dependent upon others for his growth and needs. But a learner with well developed problem solving skills is an active, confident, and flexible individual capable of reshaping his learning environment and satisfying his own needs. Education can no longer afford a narrow definition of basic skills. Development of active learners must be the first priority. Consequently, problem solving must be considered a basic mathematical skill.
What is a Problem?

The word problem is derived from the Greek problema, which translated literally means "something thrown forward" (from ballein, "to throw").
Less inspiring but more common is the standard definition, "a question raised for inquiry, consideration or solution . . . a source of perplexity . . ." (Webster's New Collegiate Dictionary, 1975).

For our uses then, a problem is a perplexing question or situation. It is important to note that a problem is not simply a question or situation—it must be perplexing.

A question or a situation can be judged perplexing, and thus be a problem, only in relation to a person and a time. What is a problem for one student now may not be a problem for that student in another month or year, or it may not be a problem for another student now. Hence teachers must select questions and situations which will likely be problems for their students.

Another implication of our definition is the idea that a question or situation must be accepted by the student as a problem. "Perplexing" implies that the question or situation is of some interest and that the student will accept it.

The characteristics of a problem for a student, then, are that:

- It is a question or a situation.
- It is accepted by the student.
- At the time it is presented to the student, there is some blockage or challenge so that the solution is not immediate.

Problem solving is the ability to solve problems. We are interested in problem solving in mathematics—the ability to solve problems which use mathematics. However, ultimately problem solving for tomorrow's adults is an important facet of life—be it mathematical or not. Questions such as, "Should I buy a new car?" or "How can I finish painting this chair?" may or may not be mathematical, but they can be real problems to people. Our aim is to equip students with problem solving strategies and our hope is that...
these students, tomorrow's adults, will be able to use these strategies to solve the problems they must encounter.

How to Teach Problem Solving

In the past, problem solving has been most often taught by presenting specific types of problems. Textbooks are arranged so that every so often there are one or two pages of "word problems," all very similar in setting and in solution technique. In upper elementary, junior high, and secondary school you find distance-rate-time problems, work problems, mixture problems, or age problems. In elementary school problem types are often associated with mathematical skills--multiplication problems, fraction problems, or merely problems. Actually, such "problems" are really exercises to practice very specific skills. Results are that the student must continually practice the skills to maintain the ability and that he or she is equipped to solve only those problems which are very similar to the exercises.

What we suggest is that the students learn very general strategies for attacking problems. (One set of these strategies will be described with examples of their use in another paper, but for now, three are mentioned to indicate the sort of activity we mean--guess and check, construct a table, or work backwards). There are four important points associated with using a strategies approach to teaching problem solving:

1. Students can be taught each strategy. The strategies themselves are general skills or abilities.
2. Each strategy can be used in a great variety of problems.
3. Often a student will use from two to four strategies in a single problem (as opposed to acquiring one technique to solve one type of problem.)
4. When faced with a new problem a student armed with a set of attack strategies has a way to begin to work. (Often the most difficult step in problem solving is getting started).

In general, to teach a strategies approach to problem solving a teacher will follow this procedure:
It is clear that to use this approach teachers need (1) a clear idea of all strategies, (2) a large resource of good problems, (3) time in the teaching schedule (mathematics period) to teach problem solving. Most of the strategies appropriate for the elementary program are familiar to teachers and little time is needed to understand them. Developing a resource of good problems is a continuing process. Supplementary materials provide one rich source. Also, teachers can use their textbooks as a resource. But even more fruitful, teachers can make up a great variety of problems themselves (and share them with each other). Finding time for the approach involves, (1) reducing the current classroom time devoted to teaching problem solving and (2) organizing the learning sequence so that part of the time for practice of computational skills is included in the
problem solving activities--during problem solving, students are using computational skills and thus practicing them.

The big pay-off in the strategies approach to problem solving is that the students will experience a variety of problems--the strategies are so general that they can be used in all types of problems. The student acquires experience in solving problems which are new to him or to her. Thus when receiving an entirely new problem, one we cannot even anticipate now, the student has a set of strategies to employ which hold promise of helping to find a solution. In this way, we hope to prepare students to solve problems in the future--even problems we can't think of or know about today.

When Should Problem Solving Instruction Begin?

Problem solving begins as soon as we become aware of our environment, develop some need which we determine to be within our capability of addressing, and go about trying to satisfy this need. Soon after birth, we begin this kind of problem solving and continue it through the rest of our lives.

Insofar as learning is involved, we use new information and knowledge to extend and expand our problem solving capabilities. But we also use problem solving to acquire additional knowledge and skills. Thus, problem solving and learning are inextricably interwoven in the way we naturally discover and react within our environment.

How then do educational objectives relate to problem solving? A broad statement of purpose of the function of education might be: (1) to provide activities that encourage the development of skills and information that are known to be valuable for participation within one's society, and (2) to examine formally processes which are productive for acquiring further information.
and satisfying one's needs.

This definition brings us back to a comparison between computational skill development and problem solving goals in mathematics education. Quite accurately, computational skills can be judged as fulfilling the first aspect of the function of education and problem solving the second. However, many curriculum designers and other educators use this artificial separation of these two basic goals of mathematics as the basis for curriculum separation and sequential treatment that places computation development before problem solving.

The unfortunate consequences of this decision are threefold. First, the natural manner in which we learn is suddenly interrupted and replaced by a learning model that concentrates on only one aspect of mathematical learning at a time. Secondly, formal consideration of problem solving is withheld from many students until some level of computational proficiency is attained. For some students this means that they will never be exposed to formal problem solving processes. And it means for other students the adjustment to a sharp change in philosophy and emphasis as the focus shifts from computation to problem solving. Thirdly, problem solving as a process too frequently becomes an incidental by-product in problem situations where the primary emphasis continues to dwell on skill refinement and maintenance, application practice, and further computational development. Sadly, this equates to a sizeable number of students never gaining the opportunity to become skillful problem solvers.

To avoid this undesirable and relatively ineffective procedure for learning mathematics, we advocate that the natural mode of learning be continued from nonformal early childhood activities to later formal school learning experiences with an appropriate balance maintained for both aspects of mathematics education. This implies that problem solving should begin in kindergarten at an appropriate level of sophistication. And its development should progress from a natural learning tool to a formal activity in which the focus is on the
process as a reproducible strategy that can be utilized to solve similar and dissimilar problems. Continuing responsibility for improving this basic skill is shared by every teacher throughout the rest of the mathematics curriculum. Only with this kind of emphasis can we expect to realize the goal of confident and flexible problem solvers who can assume an active role in modern society.
Problem Solving in the Primary Grades

- Many of the best problem-solving situations in the primary grades come from everyday situations: "How many more chairs will we need if we're having five visitors and two children are absent?" "How many cookies will we need if everyone has two?"

- In many textbooks, problem solving in grades 1 and 2 is ignored. In most textbooks, there is no clear problem solving program (in which children are taught strategies or alternative ways to solve problems). In almost no textbooks are creative, open-ended problems used.

- Unfortunately, problem solving in primary school mathematics has been limited to finding the answers to word problems in textbooks. Getting answers to such problems may involve problem solving, but it may not. If the problems are so easy that children know the answers automatically, there is really no problem at all. Problem solving is the reason behind teaching mathematics: to help the child resolve difficulties which he or she wants to resolve.

- Problems should be used throughout lessons, from introduction through reinforcement. But it is important that children be asked to solve real problems. Word problems in textbooks and most problems used in the physical representations of number ideas and operations are generally only situations from which modeling (such as $4 + 5 = 9$) can be derived, to help the child relate a real-life occurrence to a mathematical representation of that occurrence.

- Problems should be interesting to the child: the child must want to solve the problems.

- Real objects should be used (or available) for solving problems. But children in the primary grades are increasingly able to make estimates, keep records of their observations, make mathematical statements to fit events or situations, and make decisions on the basis of what may happen. They need to be faced with a variety of problems and record-keeping procedures.

- Children should believe that they can solve a problem, and should know when they have a solution for it. Confidence in their ability to solve problems must be developed; the teacher must create an atmosphere in which they feel both free and secure.

  - Let children use their own language to express the problem.
  
  - Let children work together: they need to discuss problems, share ideas, debate alternatives, and verify solutions.

  - Develop problems appropriate for different ability levels: the problems should allow for different levels of solution.

  - Provide problems that have no answer or that have many (equally correct) answers. Often, instructional materials leave children with the impression that every mathematical problem has exactly one answer.

  - Give children problems for which they must collect information or data. The typical textbook problem gives all the necessary facts. For everyday problems involving mathematics, people often must seek out the necessary
data, or at least select from what is available those facts which are needed.

- Similarly, provide experiences in which the formulation of the problem to be solved is required. People have to ask questions before they begin to solve many everyday problems.

- Help children to sense what the problem is about, to form a picture of the relationships and patterns, to systematically determine procedures and alternatives.

- Discuss problems, plausible answers and estimates, and varied procedures for finding solutions.

- Select problems which provide for maximum pupil involvement and minimum teacher guidance.

- Break complicated problems into manageable parts when frustration approaches.

- Problem solving is not dependent on reading ability. Children should be given problems orally both before and after they learn to read. Remember that many problems faced in real life do not come neatly packaged in words: children must learn to interpret most oral and non-verbal cues that comprise the problem or are related to the solution of the problem.

- The child needs to begin to develop a variety of strategies for solving problems. The problems which follow are illustrative for some of the strategies that primary-level children need to learn. (In many cases, problems could also be used with another strategy, such as naming only the operation needed for solution.) Many of these problems were adapted from those appearing in recent textbooks. In addition, a variety of more creative problems is included following page 6.
Using drawings and diagrams

A bus had 10 rows of seats.
There were 4 seats in each row.
How many seats in all?

The children at Lincoln School are packing boxes of books to send overseas.
How many boxes will they need if they have 24 books and put 6 books in a box?

The pupils found that 2 out of each 5 books were in poor condition.
If one room collected 25 books, how many of these were in poor condition?

You are riding on an elevator.
Enter on the main floor.
Go up 6 floors.
Go down 3 floors.
Go up 9 floors.
Go down 7 floors.
Go up 8 floors.
Go down 2 floors.
Go down 5 more floors.
Get off the elevator.
On what floor are you?

It is two miles from Dr. Jones' house to her office.
She walks to her office five days a week.
How far did she walk back and forth to her work every week?

Dramatizing or acting out problems

Six children were standing at the teacher's desk. Five children join them.
How many children were at the teacher's desk then?

Nine children stood at the back of the room. Seven went to their desks. How many children were left?

Selecting needed information (or deleting unneeded information)

There were twenty-eight children in Mrs. Black's third-grade classroom.
On Friday the whole class made kites in art class. Fourteen of the children decided to fly their kites that afternoon. Only one of the children were able to get their kite up in the air. How many kites did not fly?

Eight people talked to a reporter.
Twenty-three people watched the fire.
Six more people talked to the reporter.
How many people in all talked to the reporter?

Kate got a new paint set for her birthday. It has 36 colors and 5 brushes. She painted 18 pictures, each with a different color. How many colors are still unused?

Pete mowed Mr. Wynn's lawn for $3.50. He bought 2 cards for 75¢. The clerk gave him a gift of 3 packs of gum. How much money did Pete have left?
Problems without numbers

George had some money.
He gave a customer some change.
How much money does he have now?

Sally counted the pages in her order book. She wrote a dozen orders. How many pages were left?

If you know how many times your heart beats each minute, how can you find out how many times it beats in 24 hours?

Jane knows how many yards of ribbon she needs and the cost of each yard. How can she find how much the ribbon will cost?

Collecting information

How much will it cost each person in our class if we share equally the expenses of taking a field trip to the state park and having a picnic lunch while we're there?

The PTA has given $45.00 to each class in the school to be used for magazine and newspaper subscriptions. How should we spend the money?

How far can you walk in 5 seconds?
How far can you walk in 1 second?
How far can you walk in 1 minute?

How long does it take a ball to stop bouncing?
Try a big ball. Try a football. Try a little ball.

Get a stack of 15 cards.
How long does it take you to turn them over, one at a time, using your right hand only? Your left hand only? Both hands?

How long does it take to turn over 30 cards?

Making up problems

Make up a story using 9, 5, 14, tigers, lions, circus, wagons, left.

Have students find interesting pictures (with or without data) in newspapers and magazines. Have them make up problems to fit the picture.

Have students put problems in a file box for other students to solve.

Write problems based on a real experience.

Give students a table or other set of data (e.g., baseball statistics). Have them make up problems using the data.
Using materials

Have students use manipulative materials or flannelboard materials to verify the solution to a problem.

When decorating the room (or for some other measuring situation), let the children determine and measure the amount of crepe paper, twine, etc.

Making up an easier problem

Jen saved $3.56.  
Jeff saved $5.27.  
How much more money has Jeff saved?  
Jen has 36c.  
Jeff has 5c.  
How much more does Jeff have?

Reasonable answers

Jim weighs 70 lbs. standing on one foot, so he must weigh 140 lbs. standing on 2 feet.

When Henry was 12 years old, his mother was 3 times as old as he. Since Henry is now 30 years old, his mother must be 90 years old.

Matching mathematical sentence with drawing

Write number sentences on board.
Draw picture to match one sentence.
I had three cups. I broke one. How many cups are left?
Have child match sentence and picture to go with the problem.
You might read the problem and have pupils find the picture that illustrates it, then write the mathematical sentence.

Noting missing information

If you have enough information to answer a question, answer it.  
If you do not have enough information, write NM for need more information.

Every mouse in Europe eats 73 cheeses each year.  
Every mouse in Africa eats 86 cheeses each year.  
Most of the cheese is Swiss cheese.
1. How many mice are in Africa?  
2. Which mice eat more cheeses each year: those in Europe or those in Africa?
3. Which of the two places has more cheeses?
4. How many more cheeses does a mouse living in Africa eat in a year than a mouse in Europe?

Using maps, tables, etc.

Give pupils a modified train or bus schedule. Ask questions about arrival, departure, and traveling times.

Give pupils a simplified map. Have them locate specified points, and answer questions about distances.
Using maps, tables, etc. (continued)

Have pupils fill in a table for each child in the class, noting names and distance traveled to school from home. See how many problems or questions they can make using the facts given in the table.

Make a chart or graph which shows, for instance, how many cars pass through the traffic lights at the corner. (Children will, of course, collect the data.)

Show the children a time line, such as this one which shows the order in which some chipmunks stopped chomping branches. (They were having a contest to see who could chop the longest time.)

Ask: Which chipmunk was still chomping after Betty quit? Which chipmunk finished before Charlie? How many gave up chomping between Alice and Zeb? Which chipmunk came in second in the contest? Which chipmunk stopped chomping first?

In how many different ways can a bus driver get from City A to City B if the driver always moves toward B?

And a few more . . .

Write true, false, or NM for each:

In one day Jake hopped 11 times and skipped 14 times. He jumped less than he hopped.
1. He hopped more times than he skipped.
2. He jumped 0 times.
3. He ran 9 times.
4. He skipped more than he hopped.

You left the house at 4:15. You had $2.00 to buy 1 pound of hamburger. You got back from the store with the hamburger at 4:45. You have $ .21 change. How long were you gone? How much money did you spend?

You bought a comb for 29c. You had 50c. Can you also buy a book for 21c?

The sum of two numbers is 15. Their difference is 3. Name the two numbers.


Abstracts of Selected Current Articles on Problem Solving in The Arithmetic Teacher


Krulik provides a definition of a problem which excludes exercises and points out that it must be accepted by the student. Many "textbook problems" may not satisfy the definition. The major thrust of the article is a discussion, with sample problems, of seven suggestions for teachers.

1. The problem solver must carefully "digest" the problem.
2. Encourage your students to make many suggestions toward solution of the problem and to analyze why they reacted as they did.
3. Help students to examine data in a meaningful way.
4. Organize the data carefully.
5. Allow time for the problem solver to think.
6. Encourage alternate solutions.
7. Look for patterns within the data of the problem.

Following is one of the sample problems:

Two logs are found in a woodpile, and are identical in every way. Using a power saw, it takes 9 seconds to cut the first log into 4 pieces, how long should it take to cut the second log into 5 pieces?


A pattern is provided to cut a square into smaller squares of 3 different sizes and triangles of 2 different sizes—thus, "Five easy pieces." A great many questions and explorations for students are provided, grouped into the following study areas: basic relationships, puzzles, patterns, logic, areas, and costs. The activities are most appropriate for grades 1-4.

Lester attempts to identify some of the useful ideas for elementary school problem solving found in the writings of three prominent psychologists. At the risk of over summarizing a summary, we list some key implications for teachers.

From Herbert A. Simon:
"An important component of problem-solving skill lies in being able to recognize salient problem features rapidly and to associate (them) with promising solution steps."

"The processes of understanding (a problem) include the processes of constructing representations of problem situations."

From Norman R.F. Maier
"Many problems are solved incorrectly because the problem solver gets a wrong solution and stops without realizing that it is incorrect."

"A problem can be made difficult if it requires a response that deviates from past experience."

"Efficient problem solving...is both a matter of perceiving obstacles that can be readily surmounted and of ingenuity in dealing with a particular obstacle."

"Finding a final solution to a problem involves two stages, idea-getting and idea-evaluation."

...the degree to which the problem solver will respond to a challenge, the length of time the individual will stick with a problem..., and the person's tolerance of ambiguity are some of the motivation-related factors that should be considered."

"An individual's performance during problem solving varies depending on the types of pressures involved and the person's frustration threshold."

From William A. Brownell
"...the relationships necessary to (solve a problem) should be well within the child's understanding and identifiable by him..."

"To the limits desirable and possible, solutions to problems should be summarized clearly, stated verbally, and generalized."

"...practice in problem solving should not consist of repeated experiences in solving the same problems with the same techniques..."

"A problem is not necessarily 'solved' because the correct response has been made."
"Instead of 'protected' from error, the child should many times be exposed to error and be encouraged to detect and to demonstrate what is wrong, and why."

"... meaning and understandings are most useful when they have themselves been acquired through the solving of problems."


"Teaching problem solving is a problem, but like most other problems it can be solved." This is an excellent teaching oriented article which demonstrates how teachers can help students through the following stages of solving problems, with typical textbook problems and with process problems (which help demonstrate the stages):

1. Understanding the problem.
2. Planning to solve the problem.
3. Solving the problem.
4. Reviewing the problem and the solution.

Specific questions and comments are exemplified for teachers to assist students and sensitize them to the problem elements. Two problems are explored and the use of tables, diagrams, and lists are demonstrated.

"Advice such as 'Think' and 'Read the problem again' does not help the child..."


"The importance of problem solving as an aim in mathematics instruction is not just that the students be able to solve the problems in the book; the ultimate aim is that they be able to solve problems whenever the need arises. This may be in the grocery store, or it may be in some other school subject... Not all of the possible applications can be illustrated in the mathematics program; there are just too many... The implication is that there is a need for problems that expose the potential for application."

The article explores a great variety of problems, which are problems that: (1) explore the "domain of definition", (2) require reflective thinking, (3) exploit the notion of a function, (4) emphasize relations in general, (5) require deduction and (6) involve transformation.

Two activities, with a tear-out center section, are presented to provide students with experience in writing and solving "story" problems.


Research has shown that the different hemispheres of a brain are used for different purposes. In a usual right-handed individual the left hemisphere "excels in performing routine sequential tasks, logical reasoning, and analysis of stimulus components. Language is processed in the left hemisphere." "Rule application is characteristic of left-hemisphere processing."

"It appears that problem solving will be enhanced by greater use of the right hemisphere ... Activities that encourage right-hemisphere use are puzzles, particularly of a spatial nature. Problem-solving activities with tessellations, pentominoes, tangrams, and soma cubes require imagery for solution."


Ames describes the potential of exploring problem solving situations in the mechanics of a bicycle. Through questions, such as "How do pedal revolutions and wheel revolutions compare?", a great deal of mechanics and mathematics can be explained. Both single-gear and multigeared bicycles are explored.


A review of several research studies leads to the following suggestions:

I. Use aids.
II. Require the use of manipulatives in all introductory work.
III. Stress accurate modelling behaviors.
IV. Expect a wide performance range.
V. Encourage and allow children to use manipulative and pictorial aids all year.
VI. Teach all types of addition and subtraction, including comparison and additive problem solving.
These recommendations are discussed with examples of their implementation.


West considers three blockages to solving simple arithmetic verbal problems: (1) comprehending the problem (2) translating the data into a computational format and (3) carrying out the computation required. This article appears to emphasize "getting the right answer" as the goal of problem solving and examples are fairly mundane arithmetic exercises.
STUDENT STRATEGIES FOR SOLVING PROBLEMS

We believe that the best way to teach students to solve problems is to teach them to use general strategies which they can apply in a wide variety of problem settings. There are a great many ways to organize and list problem solving strategies. Those listed here are provided to show one good list which students can learn to help solve problems. In any given problem a student might employ any one strategy, two strategies, or six strategies. Not all will always help, but by becoming familiar with all of them, a student acquires a repertoire that he or she can draw on to start to attack a problem.

Following is a list of 17 strategies and some examples of problems in which they could be used.

1. Select appropriate notation.

Examples:

(a) If I give my friend 6 pieces of candy from a bag and I have 7 pieces left, how many pieces did I start with? (Grade 1)
Notation might be a picture of a bag. By writing an open sentence (strategy 5) the student may be able to solve, \( 8 - 6 = 7 \)

(b) How many rectangles can you find which have integral sides and an area of 36 cm\(^2\)? (Grade 4) Combined with making a drawing (strategy 2) and writing an open sentence (strategy 5), the student may find several or all solutions (other strategies can be used, e.g., 6, 7, 8, 9, and 11.)

2. Make a drawing, figure, or graph

(a) If there are 2 roads from Albany to Bakers and 3 other roads from Baker to Centerville, how many different ways can we travel from albany to Centerville? (Grade 5)

(b) How much carpet would we need to cover our classroom floor? (Grade 4)

(c) Sarah put 15 brownies on a dish that has 3 sections. Each section is to hold \( \frac{1}{3} \) of the brownies. How many brownies did Sarah put in each section? (Grade 3)

3. Identify wanted, given and needed information

(a) If it is 2 km from home to school, how far do I travel between home and school each day (I eat lunch at school). (Grade 2)

Wanted: How far do I travel?
Given: It is 2 km from home to school.
Needed: How many trips do I make?

(b) If you have a 3 liter container and a 5 liter container, how could you pour 4 liters into a large tub? (Grade 5)

Wanted: A way to measure out 4 liters.
Given: 3 liter container, a 5 liter container and a large tub.
Needed: ?
(c) Run a 25m course as fast as you can. What is your average speed? (Grade 6)

Wanted: Average speed.
Given: 25m course.
Needed: Time to run the course.

Restate the problem

(a) Find 3 different integers such that the sum of their reciprocals is an integer.
(Grade 6) Using A, B, C, and D as a notation for integers (strategy 1) and writing an open sentence (strategy 5), the restatement could be:

Find integers A, B, C, and D such that:
1/A + 1/B + 1/C = D

(b) Which is the best buy, 2 two-pound packages of cookies for 45 cents each or 4 one-pound packages which cost 21 cents each? (Grade 4)
Which is less money? 2 x .45 or 4 x .21?
Or which is less? One pound or 45/2 or 21 cents?

5. Write an open sentence
(Some examples have already been mentioned.)

(a) What numbers (whole numbers) are greater than 11 but less than 20? (Grade 2)
Find numbers which will make both of the following true:
11 < □□ and □□ < 20
or simply, 11 < □□ < 20

(b) If you put 24 cans of a drink in each case, how many cases can you fill with 470 cans of the drink? (Grade 4)
24 x □□ ≤ 470 and 24 x (□□ + 1) > 470

6. Draw from your cognitive background
(Problem solving very often involves synthesizing previous learning)

(a) About how many revolutions will a tire make in going a mile? (Grade 5)
Recall that C = π x d or C = 2πr

(b) If the perimeter of a square is 16 cm, what is its area? (Grade 4)
Use p = 4s and A = s²

7. Construct a table
(a) If parking tickets are $2.00, how much will 3 tickets, 5 tickets, or 10 tickets cost? (Grade 3)

<table>
<thead>
<tr>
<th>Tickets</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>
(b) How many line segments can you draw connecting 6 points on a circle? (Grade 4)
See also strategy 10, make a simpler problem.

<table>
<thead>
<tr>
<th>points in circle</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>line segments</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Guess and check

We emphasize "and check". Guessing is a good strategy and children should be encouraged to use it, but random guessing is not often productive. If a check is made, the student may acquire an insight into the problem.

(a) Can you find two numbers such that their sum is 15 and their product is 36? (Grade 3)

\[
\begin{align*}
\square + \Delta &= 15 \\
\square \times \Delta &= 36
\end{align*}
\]

Using open sentences (strategy 5) and a table (strategy 7) helps.

\[
\begin{array}{c|c|c|c|c|c|c}
 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\square + \Delta & 15 & 15 & 15 & 15 & 15 & 15 \\
\hline
\Delta & 15 & 14 & 13 & & & \\
\hline
\square \times \Delta & 0 & 14 & 26 & & & \\
\end{array}
\]

(b) If each letter is a code for a digit (0, 1, ..., 9), what is the following addition problem? (Grade 5)

\[
\begin{align*}
\text{SEND} + \text{MORE} &= \text{MONEY} \\
\text{Is } D &= 0? \\
\end{align*}
\]

9. Systematize

Making a table is one system, already mentioned.

(a) What counting numbers less than 20 can be written in only one way as a product of exactly two counting numbers (except for order)? (Grade 3)

\[
\begin{align*}
\sqrt{1} &= 1 \times 1 \\
\sqrt{2} &= 2 \times 1 \\
\sqrt{3} &= 3 \times 1 \\
\sqrt{4} &= 4 \times 1 \\
\sqrt{5} &= 5 \times 1 \\
\sqrt{6} &= 6 \times 1 \\
\sqrt{7} &= 7 \times 1 \\
\sqrt{8} &= 8 \times 1 \\
\sqrt{9} &= 9 \times 1 \\
\end{align*}
\]
(b) Here is a map of Cindy's paper route. The dots represent her customers. What route should Cindy follow, starting and finishing at her home? Each block is 100m. Is there more than one best route? (Grade 4)

10. Make a simpler problem

See 7(b).

(a) How long would it take for 9,000 people to hear the good news if each person who hears it tells 4 new people in 10 minutes, but then tells no one else? (Grade 5)

Try to find how long it would take 10, 50, 300 people to hear the good news.

(b) If the perimeter of a rectangle is 108cm and one side is 36cm, what is the length of the other side? (Grade 3) Could you solve the problem if the perimeter were 10cm and one side is 3cm?

11. Construct a physical model

(a) A baseball player has 8 baseballs. Seven of them weigh exactly the same, but one is heavier. Using a balance scale, how can you find the heaviest ball in just 2 weighings? (Grade 6)

A physical model could be made with 8 baseballs, marking one as the heaviest, and a balance scale. However, students could simply use 8 pieces of paper as models of the baseballs and simulate weighing them.

(b) If 4 people in a room each shake hands with everyone else, how many handshakes will there be? (Grade 4)

The model could be four students carrying out the handshakes.

12. Work backwards

(a) If two whole numbers have a sum of 18 and a product of 45, what are the numbers? (Grade 4) A student could list all the pairwise addends of 18 and all the pairwise factors of 45 to find the pair in both lists.

(b) Sue baked some cookies. She put one-half of them away for the next day. Then she divided the remaining cookies evenly among her three sisters so that each received 4. How many cookies did Sue bake? (Grade 3)

Working backwards: each of the sisters received \( \frac{1}{3} \) cookies so Sue had divided \( 3 \times 4 = 12 \) cookies among them. But those 12 cookies were half of the total. Thus Sue baked \( 2 \times 12 = 24 \) cookies.
Note: The following are often called "looking back strategies" because the student uses them after he or she has a solution to the problem. These are probably the hardest to teach because students often believe that they are finished as soon as they find an answer. However, if students learn to use these strategies they will be using a lot of mathematical thinking, they will often discover better ways to solve the problems, and they will discover many new ideas, such as the solutions to other problems. Teachers can provide an environment which encourages students to try the "looking back strategies."

13. Generalize
(a) A generalization of problem 2(a) is that if there are \( m \) roads from Albany to Bakers and \( n \) other roads from Bakers to Centerville, then there are \( m \times n \) different routes from Albany to Centerville.

(b) A generalization of problem 7(a) is that the cost of \( N \) parking tickets is \( 2 \times N \) dollars.

14. Check the solution
(a) If a student finds 3 and 15 as the two numbers sought in problem 12(a), he or she could check the answers (or prove they are the correct answers) by noting that:

\[
\begin{align*}
3 + 15 &= 18 \\
3 \times 15 &= 45
\end{align*}
\]

(b) A student with an answer of 24 for problem 12(b) could check it by noting that:
1/2 of 24 is 12. Thus, Sue saved 12 cookies and divided 12 among 3 sisters, \( 12 \div 3 = 4 \). Yes, this agrees with the information that each sister received 4 cookies.

What would happen if a student mistakenly found an answer of 36 cookies?

15. Find another way to solve the problem
(a) In problem 2(a) a student might try giving names to the roads and listing all the different routes from Albany to Centerville.

(b) In looking back at problem 6(b) a student might generalize (strategy 13) that if the perimeter of the square is \( p \) cm, the area is given by:

\[
\left( \frac{p}{4} \right)^2 \text{ cm}^2
\]
16. Find another result

(a) One result of solving problem 4(a) is that a solution is 2, 3, 6. Another result is that there are the only integers that will work.

(b) In looking back at problem 11(a) a student might discover that he could solve the problem even if there were 9 baseballs (and one is known to be heavier than the others.) This is a different result. However, a student might generalize also that for 3 weighings he could solve the problem for 27 baseballs or that with n weighings he could solve the problem for $3^n$ baseballs.

17. Study the solution process

This strategy can help students see more clearly how they are using all the other strategies and how the use of strategies help them find solutions. Teachers can ask students what they thought of while they worked on a given problem to help emphasize this review of the solution process. As a student describes the process, the teacher can point out the different strategies employed. Other students could be asked if they proceeded differently and the teacher might be able to show that different students used different strategies but reached the same solution.
Teachers and pupils both know that verbal problems cause problems. Because of this, and because it is considered to be an ultimate goal of mathematics instruction, researchers have devoted much attention to problem solving over the years. This research has focused on characteristics of problems, characteristics of those who are successful or unsuccessful at solving problems, and teaching strategies that may help children to be more successful. Recently attention has begun to be focused on the heart of the problem -- the strategies which children use in solving problems, the process of problem solving.

From research we have learned about a variety of points connected with problem solving. Instead of giving many details about each study, we have summarized the main findings very briefly. You might want to see how many of these points agree with the conclusions you have reached on the basis of your experiences with children. You might also want to note those things you have learned that do not appear on this list; we are aware that the list is not totally comprehensive.

1 A revised version of this article appeared in the November 1977 issue of the Arithmetic Teacher.

2 For additional information on the studies or for references to the research reports, you could refer to:

Suydam, Marilyn N. and Weaver, J. F. Using Research: A Key to Elementary School Mathematics. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1975. (Also available from NCTM.)


The annual listing of research in the Journal for Research in Mathematics Education.
Children are probably a little more successful at solving problems with familiar settings, but problems with unfamiliar settings do not seem to cause undue difficulty.

At least one study with black children from a lower socioeconomic environment indicated that there was no significant difference in achievement between problems from a textbook and problems written by children, using familiar settings and people.

Generally, it has been concluded by many researchers that children like a variety of problem settings. And it seems important that children be interested in the problems, as well as in solving them.

In problems, the operation which appears to be easiest is addition, followed by subtraction, multiplication, and division.

A problem which involves only one of the four operations is generally less difficult than a problem which involves two operations.

When the data in a multi-step problem are in the order required for solution, higher scores can be expected than when the data are not in the order in which they will be used.

The time needed to solve a problem is less when the question is placed at the beginning of the problem rather than at the end; however, achievement will probably not differ significantly for either positioning of the question.
When pupils work on sets of verbal problems in small groups of 2 or 4 pupils, they can solve more problems than those who work alone, but the groups might take a longer time on each problem than pupils working alone.

Group solutions to problems may be no better than the independent solutions of the most able member of the group, if he or she is perceived by the group to be most able.

There is some evidence that group discussion in order to reach agreement on how to proceed results in significantly better achievement than being told how to solve the problem.

Systematic teaching of a variety of problem-solving procedures aids children in developing problem solving strategies.

Giving pupils many opportunities to solve problems has frequently been suggested as being of great importance.

Encouraging children to solve problems in a variety of ways appears to aid children in becoming better problem solvers.
Having pupils write an equation or mathematical sentence for a problem can be helpful. Writing equations which fit the problem situation (expressing the real or imagined actions in the problem) and using equations which emphasize the operations by which the problem may be solved directly each appear to have some advantages.

Emphasis on isolated word cues (such as "left" or "in all") can be misleading, for attention is directed away from recognition of the relationships inherent in the problem which may be crucial to its solution. Some discussion and illustrations of how word cues may be misleading could help, however.

Problems with extra, irrelevant data are more difficult than problems without extra data.

Problems with materials, diagrams, or some other type of visual aid are generally easier than those without such aids.

Instruction on what process to use and on why that process is appropriate will generally result in higher scores than merely solving problems without discussion. Emphasize what needs to be done and why it needs to be done rather than just obtaining an answer.
Many other specific techniques have been reported by researchers to be helpful; among those suggested are:

(1) Provide a differentiated program, with problems at appropriate levels of difficulty.

(2) Provide many and varied situations that give children opportunities for structuring and analyzing situations that really constitute a problem and not just a computational exercise.

(3) Have pupils dramatize problem situations and their solutions.

(4) Have pupils make drawings and diagrams, using them to solve problems or to verify solutions to problems.

(5) Have pupils write their own problems, formulating them for given conditions.

(6) Present problems orally.

(7) Use problems without numbers.

(8) Have pupils designate the processes or operations to be used.

(9) Have pupils note the absence of essential data or the presence of unnecessary data.

(10) Have pupils test the reasonableness of their answers.

(11) Use a tape recorder to aid poor readers.

(12) Present some problems in separate sentences rather than in the usual paragraph format.

Opportunities should be provided for children to determine the question to be answered, select specific facts necessary to solution, and choose the appropriate process. However, rigid adherence to a formal analysis procedure (that is, requiring pupils to answer a specific set of questions in a specified order) does not appear to be effective.
Researchers generally conclude that:

(1) IQ is significantly related to problem solving ability;
(2) sex differences do not appear to exist in the ability to solve problems; and
(3) socioeconomic status alone does not appear to be a significant factor.

Computational difficulties appear to be a major deterrent to finding correct answers when solving problems, with reading a secondary cause of difficulty.

Higher levels of problem solving ability are often associated with higher levels of computational and reading ability, but much of this apparent relationship may be the result of the correlation of these abilities with IQ.

Among the reasons commonly found for why children make mistakes as they solve problems are:

(1) errors in reasoning
(2) ignorance of mathematical principles, rules, or processes
(3) insufficient mastery of computational skills
(4) inadequate understanding of vocabulary
(5) failure to read to note details

Many researchers have proceeded on the assumption that if we can ascertain what problem solvers who are successful have in common, we may be able to help those who do not do as well. Among the
many factors in addition to skill in computation, reading comprehension, and higher IQ scores which may characterize those good at solving problems are:

(1) ability to estimate and analyze
(2) ability to visualize and interpret quantitative facts and relationships
(3) understanding of mathematical terms and concepts
(4) ability to note likenesses, differences, and analogies
(5) ability to select correct procedures and data
(6) skill in noting irrelevant detail
(7) ability to generalize on the basis of few examples
(8) ability to switch methods readily
(9) higher scores for self-esteem and lower scores for test anxiety

- More impulsive students are often poor problem solvers, while more reflective students are likely to be good problem solvers.

What reading skills will help?

- Good and poor achievers in problem solving differ on many aspects of reading.

- Activities stressing certain reading skills, such as selecting main ideas, making inferences, constructing sequences, and following directions, may improve problem solving achievement.

- Specific instruction on quantitative vocabulary may be helpful for some pupils.
Creative or divergent thinking is a successful strategy, but is used by relatively few pupils. Blind guessing and trial-and-error are considered to be the most unsuccessful strategies.

Having a pupil think aloud as he or she solves a problem may help you to diagnose the particular reason why a pupil is having difficulty.

A new factor has entered many classrooms recently, the hand-held calculator. Problems in current elementary school curricula are often included merely to provide practice on particular computational skills. With the use of the calculator, however, there can be more focus on problem solving "for problem solving's sake." The focus can be on strategies and process when the calculator is used, with less emphasis on computation within the problem solving context. More real problems can be used and the range of problems extended. Research has not yet considered the effect of the calculator on problem solving. There is only some preliminary evidence from schools in which the calculator is used during mathematics instruction that problem solving achievement on standardized tests may increase. We need to refocus the curriculum and see what the advantages of the calculator can be for helping children to learn how to solve problems of all types.

Some characteristic differences between high and low achievers in problem solving were analyzed. (grade 7)


Pictures elicited more ideas and more fluency in problems than did written stimuli. (grades 4, 5)


The ability to express problem relationships as number sentences was highly related to problem solving ability, while knowledge of specified vocabulary was important but may not have been critical. (grade 6)


Problems with extraneous data took more time to solve. No significant effects were found for question placement. (grade 4)


General reading ability had an effect on problem solving, but was highly related to IQ. Computation ability also had a significant effect on problem solving ability. (grade 6)


Both problem setting and degree of attention to distractions had a significant effect on performance and behavior in solving division problems. (grades 1-3)


Five criteria for developing problem situations to promote strategies of thought and problem solving were cited. (grades k-12)

Problems which contained immaterial data were most difficult, followed by those with irrelevant data, at each of three ability levels. (grade 8)


Cognitive structuring techniques increased problem-solving success for field-dependent children. (grade 4)


No significant differences were found for position of the question or use of like elements in sets, but problems presented without a visual aid were more difficult than those with a visual aid. (grade 1)


The physical structure of apparatus used as vehicles for problems influenced problem difficulty. Partitive division appeared to be more difficult than measurement division. (ages 3-8)


Incorrect responses were used as alternative answers on diagnostic test items, with twelve error categories forming the basis for a matrix which was found to diagnose "satisfactorily". (grade 5)


Accuracy of computation was not affected by the problem setting; however, it seemed to take more time to solve problems with unfamiliar settings. Least skilled pupils were most affected by unfamiliar settings. (grade 5)


Pupils tended to score higher on the test which did not require them to go through steps of formal analysis. (grades 4-6)

Burns, Paul C. and Yonally, James L. Does the Order of Presentation of Numerical Data in Multi-Steps Affect Their Difficulty? School Science and Mathematics 64: 267-270; April 1964.

Pupils were less successful when numerical data were presented in an order unlike the way in which the data were used to solve the problem. (grade 5)

Percentages correct for 9-year-olds on four problems ranged from 22 to 46. Many did not make any response. (age 9)


The ability to compute, to note details in reading, and knowledge of the fundamental concepts of arithmetic predicted problem-solving ability essentially the same as a combination of the 15 variables studied. (grade 6)


Pupils achieved higher scores in determining what the problem asked them to find than they did in steps involving the number processes used. None of the steps distinguished between good and poor problem-solvers. (grade 6)


Children’s approaches to problem solving were analyzed, and considered in relation to models of cognitive processes. (grades 3, 4)


In grades 3-4, word problems presented in separate sentences resulted in higher achievement than did work with regular problem format. In grades 5-7, work with problems in any of three formats was more effective on a word problem test than was only computation drill. (grades 3-7)


No significant differences in the improvement of peer relations, attitude toward mathematics, or mathematical achievement between pupils working in small groups or independently were found. (grades 4-6)


Pupils performed better in selecting correct process for solving verbal problems with word clues. (grade 6)

Extraneous information and extra numbers made problems more difficult; telling children that there were extra numbers reduced errors. (Elementary)


Generalizations from research on problems were grouped in terms of teaching techniques, student methods, student skills and abilities, and other factors. (Elementary)


The better estimators were also better problem solvers. No significant difference in problem solving ability was found between students given or not given estimation instruction, but those having instruction were significantly better in estimating. (Grade 5)


It was concluded that increased emphasis should be given to those skills in reading which were shown to be closely related to problem solving. (Grade 6)


An expository and an inquiry method were equally effective for general problem-solving instruction at all IQ levels. (Grade 6)


Both treatments improved children's ability to solve problems, with no significant difference between the two procedures. Specific reading abilities did not appear to be more essential than general reading ability or computational ability. (Grade 4)


Successful problem solvers appeared to be able to comprehend mathematical relationships expressed in the problem, use abstract analytical reasoning, use insightful reasoning, and use a minimum number of steps to solve the problem. Ability to note accurately the information given or not given did not appear helpful. (Grade 6)
Houtz, John Charles. Problem-Solving Ability of Advantaged and Disadvantaged Elementary School Children with Concrete and Abstract Item Representations. (Purdue University, 1973.) Dissertation Abstracts International 34A: 5717; March 1974.

Models, slides, and picture-book forms of problem items resulted in higher performance than did the abstract form. (grades 2, 4)


Group solutions to problems were not better than the independent solutions by the most able member of the group if perceived to be most able. (grades 5-8)


No major change in error patterns in problems was noted in the 48 years since John's (1927) study. (grade 6)


Children who spent ten percent of instructional time in verbal generalizations made significantly greater average growth in problem solving and in computation than children in the control group. (grade 4)


For both partitive and quotitive types, sharing problems were significantly easier than sharing-implied or non-sharing problems. (grade 2)


Problems with extra data were more difficult than problems without extra data. Routine computation was easier than either type of problem. (grade 6)


No significant differences were found between groups using a general problem-solving program, a wanted-given program, or the regular textbook. (grade 5)


Analysis of the work of 60 pupils revealed 40 types of errors. (grades 4-6)

Partition problems were more difficult than measurement problems. Problems with whole number quotients were easier than those with fractional number quotients. (grade 6)


A summary of some of the writing on problem solving and a critical review of the procedures and findings of studies is presented in terms of method of solving problems, comparison of types of problems, and the relation of practice exercises to success. (elementary)


No significant difference in achievement was found between use of problems written by children and textbook problems. (grade 5)


Students who wrote and solved problems of their own scored significantly higher than students having textbook problems. (grade 6)


For first graders, problems with sets with three different names were more difficult than problems with sets having the same name. For second graders, use of a visual aid affected difficulty level. (grades 1, 2)


Measures of quantitative ability, mathematics achievement, word fluency, general reasoning, and a reflective conceptual tempo were positively correlated with using equations in solving word problems. (grade 8)


Bibliographies and reviews are noted, and studies on problem-solving ability, tasks, processes, instructional programs, and teacher influences are discussed. (elementary, secondary)
Klugman, Samuel F. Cooperative Versus Individual Efficiency in Problem Solving. *Journal of Educational Psychology* 35: 91-100; February 1944.

Children took less time to work 20 problems individually than when working in pairs, but achieved higher scores when working in pairs. (grades 4-6)


Poor reading was not a factor in 52 percent of the standardized achievement test problems solved incorrectly; computational factors were also clearly a deterrent to success on the remaining problems. (grade 6)


Children at low levels of conservation and the low I.Q. group were more dependent on aids and transformations in solving subtraction problems than were higher-level pupils. (grade 1)


Pupils who studied a structured equation approach were better able to program problem-solving situations than those who used a traditional approach, but the groups did not differ on processing ability. (grade 5)


Problem-solving ability was found to increase with age, but certain aspects of proof could be taught in upper elementary grades. (grade 6)


Syntactic structure and vocabulary levels were both found to be determiners of difficulty in problems, with vocabulary level perhaps the more crucial. Those with high ability and high reading achievement met greater success in problem solving. (grade 4)


Four variables were identified which significantly affected the difficulty of problems: number of operations, sequence of problems, complexity, and conversions. Verbal clues, order of operations, and number of steps had little effect on difficulty. (grade 6)
Trends pre- and post-Sputnik were traced. (grades 3-6)

High correlations among reading, ability, and computation scores were found, indicating a complex interaction and the cruciality of all to problem-solving skill. (grades 4, 8)

Students using individualized problem-solving assignments made significantly greater score gains than those using regular mathematics textbook materials. (grade 5)

Children who completed diagrams to solve problems scored significantly higher than those who did not use diagrams. (grade 3)

Diagrams aided pupils in solving problems.

The multi-experience approach to problem solving was more effective than the verbal approach. (grade 4)

The group having systematic discussion made statistically significant gains. Both groups made gains on some types of problem solving. (grade 4)

Students trained with either a conceptual or a behavioral strategy scored significantly higher than a control group on acquisition and transfer tests of problem-solving, but no differences were found between strategies or patterns of decision-making. (grade 6)
Portis, Theodore Roosevelt. An Analysis of the Performances of Fourth, Fifth and Sixth Grade Students on Problems Involving Proportions, Three Levels of Aids and Three I.Q. Levels. (Indiana University, 1972.) 

Performance on tests using physical and pictorial aids was significantly higher than when only symbolic aids were used. (grades 4-6)

Possien, Wilma Martens. A Comparison of the Effects of Three Teaching Methodologies on the Development of the Problem-Solving Skills of Sixth Grade Children. (University of Alabama, 1964.) 

Students trained in the use of inductive procedures exhibit some characteristics of effective problem-solving behavior more frequently than pupils taught by the deductive method. (grade 6)

Post, Thomas Robert. The Effects of the Presentation of a Structure of the Problem-Solving Process upon Problem-Solving Ability in Seventh Grade Mathematics. (Indiana University, 1967.) 

Special instruction in structure of problem solving appeared not to improve problem-solving ability significantly. Intelligence was a significant factor. (grade 7)


Groups taught specific problem-solving procedures on two levels of difficulty achieved significantly more than those who followed typical textbook procedures. (grade 6)


Seventeen suggestions for instruction, based on research findings, were listed. (elementary)

Robinson, Mary L. An Investigation of Problem Solving Behavior and Cognitive and Affective Characteristics of Good and Poor Problem Solvers in Sixth Grade Mathematics. (State University of New York at Buffalo, 1973.) 

Good problem-solvers had significantly higher scores on IQ, reading comprehension, arithmetic concepts and problem-solving, and self-esteem measures, and were less test-anxious. More impulsive pupils were poor problem-solvers, while more reflective pupils were good problem-solvers. (grade 6)


Problems in which events were mentioned out of chronological order and problems with the starting set unknown were more difficult to solve. (grade 3)
Scott, Ralph and Lighthall, Frederick F. Relationship Between Content, Sex, Grade, and Degree of Disadvantage in Arithmetic Problem Solving. Journal of School Psychology 6: 61-67; Fall 1967.

No statistically significant relationship was found between "need content" of problems and degree of disadvantage of pupils. (grades 3, 4)


Difficulty level of a problem was affected by the operations needed to solve it; an ordering by difficulty level was reported. (grade 7)


The students used identifiable problem-solving strategies to solve each of five varied problems. (grade 4)


The composite readability scores for sixth-grade textbooks ranged from 5.0 to 5.8. Analysis of selections indicated a range of below grade 4 to grade 8. Tests ranged from below grade 4 to grade 6. (grade 6)


Excellent prediction of relative success in solving addition problems and learning addition facts for children entering first grade was found using tests of numerosness. (grade 1)


Described action did not differentially affect problem-solving performance on the four basic problem structures studied. Mean scores for problems of the type \[ a + b = c \] were higher than for other problem types. (grade 1)


Instruction consisting of equal amounts of content dealing with computational structure and verbal problem solving may have a more favorable effect on pupils' immediate problem-solving performances than does computational structure alone, and at least as favorable an effect as emphasis upon problem solving alone. (grade 4)

For usual textbook verbal problems, those who were directed to draw a picture "telling the entire story" did as well as those who were taught to write equations to solve the problems. (grade 4)


Questions about problem solving are answered in terms of research findings.


The main findings from research on problem solving at the elementary school level are briefly summarized. (elementary)


Factors which could be used to classify high and low achievers in problem solving with up to 95 percent accuracy were identified. (grade 6)


Good achievers in problem solving were significantly superior to poor achievers in all 15 reading skills studied. It was concluded that reading in problem solving must be considered a composite of specific skills rather than a generalized ability. (grade 7)


Pupils who studied quantitative vocabulary achieved significantly higher scores on tests of problem solving and of concepts than pupils who did not have such instruction. (grade 5)


Reflective pupils were significantly better than impulsive pupils at selecting the correct operation to solve a problem; differences on the estimation test were not significant. (grade 4)

Conclusions for the tests given by the Committee of Seven were reported, including: (1) children apparently had no difficulty with one-step problems; (2) formal analysis appeared to have practically no relation to ability to solve problems; (3) and unfamiliar situations did not affect problem-solving achievement as much as was supposed. (grades 3-7)


Reliabilities of the test were found to be .79 and .84, with the degree of agreement between intended and judged classifications at .78. (elementary)


Both pupils and teachers had favorable attitudes toward problem solving. "Rather stable" and significant positive correlations were found between attitudes and achievement. (grade 4)


A double operation (addition and subtraction) was more difficult than a single operation. Not-checking of answers significantly affected achievement, and four interactions were found to be significant. (grades 2, 3)

Williams, Mary Heard and McCreight, Russell W. Shall We Move the Question? Arithmetic Teacher 12: 418-421; October 1965.

Little difference was found in the two placements of the question in problems in terms of achievement, but time was less when the question was shown first. (grades 5, 6)


Students significantly improved in problem-solving ability when taught by discovery approaches. (elementary)


The wanted-given treatment was found to be superior to either practice-only or action-sequence on all dependent variables studied. (grade 4)


Pupils were significantly better at solving problems when they contained word clues than when they did not. (grade 5)
Selected Abstracts from Resources in Education (ERIC) on Problem Solving

ED 073 926  SE 015 809
Beardale, Edward C., Jerman, Max E.
Linguistic and Numerical Variables in Verbal Arithmetic Problems.
Pub Date [73] Note—26p.
EDRS Price MF-$0.65 HC-$1.29

This paper describes a study in which 14 linguistic variables were used to determine which variables would account for a significant amount of the observed variance in the error rate in verbal arithmetic problems. Three forms of verbal problems were used in which the number of words in the problem statement were systematically varied. The 14 variables entered in the first three steps of the multiple regression in the first analysis were significant in the presence of the new variables. OP)

ED 095 008  SE 017 943
Beardale, Edward C., Jerman, Max E.
Pub Date Apr 74
EDRS Price MF-$0.70 HC-$2.06 Plus Postage.

Five structural, four linguistic and twelve topic variables were used in regression analyses on results of a 50-item achievement test. The test items were related to 12 topics from the third-grade mathematics curriculum. The test items reflect one of two cases of the structural variable, cognitive level; the two levels are characterized, inductive (generalization) or algorithmic thinking. Fourth-grade and fifth-grade students (N=120) served as subjects. Three regression analyses were carried out. The first used the 12 topic variables. Geometry was the only significant topic variable entered in the final step (p greater than .05, R^2 = .5347). The second analysis added Cognitive Level to the topic variables. Cognitive Level entered first and was significant (p greater than .01) along with Geometry (p greater than .05) at the last step. At the sixth step Cognitive Level, Geometry and Prediction entered the second step and accounted for 39 percent of the variances in test scores. This was a 13 percent increase over step two in the first analysis. In the last analysis, use of those entered in the first three steps of either of the previous analyses plus the linguistic variables, both Cognitive Level and Geometry continued to be significant in the presence of the new variables. (JP)

ED 141 087  SE 022 547
Blumberg, Phyllis
Chaining in Problem Solving: A Critique and Reanalysis.
Pub Date Apr 77
Note—27p. Paper presented at the annual meeting of the American Educational Research Association (New York, New York, April 4-8, 1977); Contains occasional light type
EDRS Price MF-$0.83 HC-$2.06 Plus Postage.

Identifiers—Research Reports

This study investigated the question of whether young children can form response chains in problem solving. After reviewing the literature relating chaining as a component of problem solving, the author argues that a test of chaining should be free of requirements to recall previously learned material, to remember general information, or apply abstract principles. The current study used a task in which subjects were required to execute a sequence of trades. Subjects were drawn from kindergarten, third grade, sixth grade, and college populations, and were individually tested. Results indicated that college and sixth-grade students were able to solve all problems without hints. Younger students were not able to solve the problems after a few trials. The author concluded that children were capable of forming chains at young ages. (SD)

ED 076 414  SE 016 057
Bright, George W.
Geometric Problem Solving Abilities of Children in the Primary Grades.
Pub Date Apr 73
Note—27p. Paper presented at the annual meeting of the National Council of Teachers of Mathematics (Dallas, Texas, April 1973)
EDRS Price MF-$0.65 HC-$1.29

Identifiers—Research Reports

The problem investigated was the analysis of a complex figure by identifying simpler figures embedded in it. The primary goal was to determine the level of sophistication of analysis employed by children in the primary grades; a secondary goal was to determine if students could be led to expand their analyses. Drawings and problems were prepared by the experimenter. Fifteen students were randomly selected from second-to-sixth year old third-grade students, average age was 8.1 years. Each student was interviewed individually. Results showed that subjects almost universally failed to identify overlapping figures, that the subjects seemed to employ search techniques and that those techniques were most frequently used by older students. By chance there was a lack of statistical relationship between age and problem-solving ability, and that limited instruction was somewhat effective. (DT)

ED 128 179  SE 020 954
Cohen, Harvey A.
The Art of Searing Drusms, Artificial Intelligence and Education.
Pub Date [77] Note—60p.; Not available in hard copy due to marginal legibility throughout original document

Available from—The Artificial Intelligence Laboratory, 545 Technology Square, Cambridge, MA 02139 ($1.70)
EDRS Price MF-$0.83 Plus Postage. HC Not Available from EDRS.

Several models for problem solving are discussed, and the idea of a heuristic frame is developed. This concept provides a description of the evolution of problem-solving skills in terms of the growth of the numbers available and increased sophistication in their use. The heuristic frame model is applied to two sets of physical problems to illustrate the components involved. Several teaching strategies related to the heuristics and to promoting students' self awareness of their developing problem solving ability are discussed. In an appendix, the problem-solving model is related to the Piagetian idea of conservation. (SD)

ED 129 631  SE 021 501
Domina, Suzanne K.
Problem Solving: Polya's Heuristic Applied to Psychological Research.
Pub Date [76] Note—40p.
EDRS Price MF-$0.58 HC-$2.06 Plus Postage.

Identifiers—*Polya (George)

Using the "How to Solve It" list developed by Polya as a vehicle of comparison, research findings and key concepts from the psychological study of problem solving are applied to mathematical problem solving. Hypotheses concerning the interpretation of psychological phenomena for mathematical problem situations are explored. Several areas of needed research with respect to the solution of mathematical problems are discussed. Three elements of Polya's list are identified as having primary importance in the solution process. It is argued that psychological research does not support the usefulness of "deriving a plan," but rather implies that problem solution is facilitated by the restructuring of data. (Author)
Teaching Children How to Think: Synthesis. In about teaching methods or techniques which can be readily adapted by the teacher to any grade level and subject area are presented. Specific directions on how to get a project started in the classroom are given. Also presented are guidelines for developing learning modules or packages for teaching creative thinking and problem solving. The teacher is also outlined and described the procedures for organizing a workshop on creativity and problem solving. The appendix contains the descriptions of published teaching materials for specific grade levels and subject areas.

Authors: Ahti


Note: For the teachers edition of this document see ED 016 723.

EDRS Price MF-50.83 HC-522.00 Plus Postage

Note—408p. For the teachers edition of this book as a resource tool. The book also offers problem solving. In the hook there are also out directions on how to get a project started in the classroom. Information about teaching methods or techniques which can be readily adapted by the teacher to any grade level and subject area are presented. Specific directions on how to get a project started in the classroom are given. Also presented are guidelines for developing learning modules or packages for teaching creative thinking and problem solving. The teacher is also outlined and described the procedures for organizing a workshop on creativity and problem solving. The appendix contains the descriptions of published teaching materials for specific grade levels and subject areas.

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Authors: Ahti


Note: For the teachers edition of this document see ED 016 723.
An Analysis of the Structural Variables that Determine Problem-Solving Difficulty on a Computer-Based Test

Dyatlov, Elizabeth Jane 

Abstractive Complexity Level 

Computer Oriented Programs, Disadvantaged Youth, Problems, Problem Solving 

The minimum number of different operations required to reach the correct solution is large when it is of a different type than a problem preceding it, when the index complexity of its most complex solution is great, when there are a large number of words in the problem, and when a conversion of unanswerable ones from day to day is required. These variables of problem difficulty were determined to be significant in an experiment using disadvantaged and non-disadvantaged students, who were given access to a computer. Variables that did not make a significant contribution to test regression analysis were the "verb-phrase" variable, the "passive" variable, and the "steps" variable (MF).

ED 691 225 

Moyer, Richard E. 

Learning to Solve Problems: Role of Instructional Method and Learner Activity 

Michigan Univ., Ann Arbor. Human Performance Center 

Sparks Agency—National Science Foundation, Washington, D.C. 

Post Date 1/24 

EDRS Price MF-50.75 HC-54.20 PLUS POSTAGE 


A programed sequence for teaching students to solve word problems was developed using a combination of the information processing and structural variables approaches. Students using the sequence proceeded individually through mastery of a sequence of objectives. In order to evaluate the program, fourth and fifth graders were randomly selected from classes; the remaining students in these classes served as controls. All students were given the appropriate level of the Stanford Achievement Test as a pretest. During the 11 weeks that the experimental students completed the Word Problem Program, control subjects received regular mathematics instruction. The experimental sections of the Stanford Achievement Test served as posttests. Both fourth- and fifth-grade experimental groups scored higher on their respective applications problems than the comparable control groups. (SD)
ED 115 491
Romer, Thomas A. Glove, Richard
Note—22p.; Report from the Project on Conditions on School Learning and Instructional Strategies; A discussion of the results and appendices appear in part 2, SE 019 855
EDRS Price MF-50.76 HC-54.43 Plus Postage
The purpose of this study was to determine whether a process model could be constructed using data identified from flow charts which accounted for somewhat more variance in predicting the difficulty of two-digit multiplication problems than did a process model developed by Cromer. Cromer's data and variables were used as a starting point. Ten new variables were identified from multiplication and addition flow charts. Seven basic models. 4 reduced models. 10 factor models, 24 one-variable models, and a set of systematic restricted models were examined. Multiple regression analysis was used to predict difficulty. The overall results indicate that the flow chart variables do produce somewhat better models. This volume presents the first two parts of this report and includes the problem statement and results. (Author/SD)

ED 115 492
Romer, Thomas A. Glove, Richard
Note—47p.; Report from the Project on Conditions of School Learning and Instructional Strategies; A discussion of the results and appendices appear in part 1, SE 019 855
EDRS Price MF-50.76 HC-54.43 Plus Postage
The purpose of this study was to determine whether a process model could be constructed using data identified from flow charts which accounted for somewhat more variance in predicting the difficulty of two-digit multiplication problems than did a process model developed by Cromer. Cromer's data and variables were used as a starting point. Ten new variables were identified from multiplication and addition flow charts. Seven basic models. 4 reduced models. 10 factor models, 24 one-variable models, and a set of systematic restricted models were examined. Multiple regression analysis was used to predict difficulty. The overall results indicate that the flow chart variables do produce somewhat better models. This volume presents the first two parts of this report and includes the problem statement and results. (Author/SD)

ED 049 909
Rosenhal, Daniel J. A. Resnick, Lauren B.
The Sequence of Information in Arithmetic Word Problems. Project Report No. 71
EDRS Price MF-50.65 HC-53.29
The effects of three variables on the difficulty of verbal arithmetic problems were examined. Variables included problem form, sequence of information, and problem verb. A total of 32 problems was generated, four in each of four problem forms and two sequences of information. Verbal answers were not above second-grade levels, and numbers used ranged from 2 through 9 with no borrowing or carrying required. Two groups of elementary-grade subjects (63 in all) solved all of the problems. Analysis of variance performed on the data indicated that problem form was significant (p<.001) but that the problem verb was not. Reverse sequence problems were most difficult to solve and became more difficult as the problem form became more difficult. It was concluded that subjects need to distinguish sequence of information from sequence even when a WRD (p<.001) and that reverse sequence causes the greatest difficulty in problem solving. Tables and references are included. (MS)

ED 061 533
Sear, Carolyn Ingrid
EDRS Price MF-50.65 HC-53.29
The primary objective of this study was to compare two operations performance among formal operational, transitional, and concrete operational individuals with the effect of relative field dependence taken into account within each of these three cognitive developmental levels. Secondly, the study explored whether a developmental relationship exists between logical thought and field independence. Eight male and eight female subjects per grade were randomly selected from class lists for sixth, seventh, and eighth grades and classified according to cognitive developmental level. All critical problems (to be solved) are fully described. Sex and age differences are discussed. In general, the study concludes that Piagetian developmental levels does provide an overall theoretical framework in which to understand and interpret differences in complex, deductive problem solving performance. but, in the problems used, field independence does not appear to clarify individual differences in a meaningful way. (TL)

ED 064 150
Searl, Barbara H. and others
NOTE—TR-213
EDRS Price MF-50.65 HC-53.29
Using the capability of the computer, the authors have designed an instructional program that emphasizes students' problem solving skills instead of their computational skills and that allows the collection of a large and detailed data base. This report describes the procedures used to (1) identify variables that affect students' performance on arithmetic word problems presented at a computer terminal, (2) identify variables that affect the computer-based problem-solving process, (3) assess the usefulness of the identified variables as predictors of student performance on the newly structured curriculum, and (4) develop a model for sixth-grade students who were from one to three years below average in arithmetic computation skills who were subjected to developing and testing the program. Three groups of participants approximately two-thirds were black students from an economically depressed area, and the remainder came from a middle-class area. Results of several analysis of variance were significant. It was possible to account for a substantial portion of the variance in students' performance on the newly structured curriculum. The report is addressed to those concerned with the development and assessment of new educational technology. It is the work on new instrument development for evaluating other curricula and new measurement instruments. (Author)

ED 135 807
Shaw, Mary H.
Spons Agency—National Science Foundation, Washington, D.C.
Pub Date Jun 76
NOTE—265p.; Material has been removed from the appendixes due to copyright restrictions
EDRS Price MF-50.65 HC-53.29
This report details the development of a paper-and-pencil test of problem solving, the development of PROFILES, a new instrument for testing the instrument, and the technical and information on item analysis, reliability and validity. The chapter on PROFILES includes information on its rationale, development, and the development of a scoring protocol. (RC)
Analysis of Kindergarten and First Grade Children's Addition and Subtraction Problem Solving Modeling and Accuracy
Pub Date 76

EDRS Price MF-090-83 IC-$2.06 Plus Postage
Descriptors—Addition, Cognitive Development, Mathematics Education, Number Concepts, Problem Solving, Research, Subtraction, Identifiers—Research Reports

ED 047 932
Sutphen, Marilyn N.; Weaver, J. Fred
Research: A Key to Elementary School Mathematics
Pennsylvania State Univ., University Park Center for Cooperative Research with Schools.
Pub Date [70]
Note—psp. EDRS Price MF-$0.65 HC-$2.29
Descriptors—Elementary School Mathematics, Instruction, Mathematics Education, Problem Solving, Research Reviews (Publications), Verbal Aids, Identifiers—Center for Cooperative Research with Schools

ED 079 697
Talman, Carolyn Flatman
An Investigation of Selected Mental, Mathematical, and Personality Assessments as Predictors of High Achievement in Sixth Grade Mathematical Verbal Problem Solving.
Pub Date 73
Note—p. 44, D.Ed Dissertation, Northwestern State University of Louisiana

EDRS Price MF-$0.65 HC-$6.58
Descriptors—Elementary Education, Grade 6, Intelligence Tests, Mathematical Concepts, Mathematics, Mathematics Instruction, Personality Assessment, Problem Solving, Reading Ability, Reading Research, Reading Skills, Reading Tests, Verbal Ability

The purpose of this study was to determine if selected mental, mathematical, and personality assessments of sixth grade pupils could predict high achievement in mathematical verbal problem solving for the subjects. The subjects were 112 sixth graders, 56 classified as high achievement in mathematical verbal problem solving and 56 classified as low achievement in verbal problem solving scores available in cumulative school records. Thirty-eight mental, mathematical, reading, and personality test scores, the correlation among scores were analyzed, and four combinations of variables were analyzed, and four combinations of variables resulted: (1) to construct a multiple variable and interest span, motivation and interest, and performance and patience. Research in each of these areas as related to problem solving is cited. A list of 135 references is included. (DT)

ED 100 718
Zarowlski, Donald L
An Exploratory Study to Compare Two Performance Measures in an Interview-Coding Scheme of Mathematical Problem Solving and a Written Test, Part 1: Test Reliability Report No. 306.
Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning
Spans Agency—National Inst. of Education (DEW), Washington, D.C. Cooperative Research Program
Pub Date Mar 70
Contract—OECB (1081)

EDRS Price MF-$0.25 IC-$1.35
Descriptors—Ability Grouping, Arithmetic, Conservation (Concepts), Factor Analysis, Grade 1, Intelligence Quotient, Manipulative Materials, Measurement Instruments, Performance Factors, Problem Solving, Task Performance

This study examined differential performances among groups of first grade children when solving eight different types of arithmetic word problems under two distinct experimental conditions. The categories of children were actually 4 ability groups: (1) low quantitative comparison, (2) high quantitative comparison; (3) (Lorge-Thorndike) 10 tests, (2) low quantitative comparison scores and high IQ, (3) high quantitative comparison scores and low IQ, and (4) high quantitative comparison scores and high IQ. The 112 children who filled these categories were given a 44-item problem solving test, with six problems from each of the eight types previously used in a randomized sequence. Half of the children in each ability group were randomly assigned to the condition of no manipulatable objects, while the other half were provided with manipulatable objects referred to in the problems and were allowed to use them any way they wanted to help solve the problems. Analysis of the data revealed that IQ was not a significant factor, that Problem Condition was significant, that there was a significant interaction due to Quantitative Comparisons and Problem Conditions for one problem type, and that there were significant main effects due to Problem Conditions for the remaining seven problem types. There was also a significant main effect due to Quantitative Comparisons for one of the remaining seven problem types. (MH)
In this study the researcher investigated the feasibility of constructing a valid paper-and-pencil measure of problem solving ability. (Rationale and design of the study are discussed in Part 1.)

The principal feasibility criterion, correlation of at least .71 with scores on taped and coded individual "thinking aloud" problem-solving versions, was not met, however, the obtained correlation (.68) for one test suggested to the researcher that more reliable tests might achieve the criterion. Rank ordering of subjects on the "thinking aloud" procedure and written tests were highly correlated. The use of the "thinking aloud" procedure to establish concurrent validity was evaluated and questions about the validity of this procedure with seventhgrade students were raised. Investigations of the functional differences of this procedure with seventhgrade students were planned.

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LUNCH LINES

Challenge: Recommend and try to have changes made which would improve the service in your lunchroom.

Most children have gripes about their school cafeteria: they have to wait too long, it's too crowded, they don't like the food, too much food is wasted, etc. A general discussion will enable the teacher to learn what observations the children have already made and what principally concerns them. The children may then decide to work in small groups measuring the flow of traffic at the lunch counter and through the aisles, recording the dimensions of the room and the arrangement of furniture, writing and distributing questionnaires of student attitudes and preferences. The data they have collected may be represented by tables, graphs, scale drawings and scale models.

As the children collect their data, draw their conclusions and recommend improvements, they may see the need for more data or data of a different type. Other activities may include the investigation of the cost of suggested improvements such as rearrangement of counters, new tables, more doors, etc. A single solution to the problem is not necessarily sought, but each group of students documents as thoroughly as possible its suggested improvements. In some cases, they receive permission to try out their ideas so that the unit is able to culminate in some action such as the acceptance of some of their recommendations by the principal or school board. At other times, the unit activities may result in a proposal for changes.

The lunch lines problem may lead naturally to follow-up activities related to other school problems such as Play Area Design and Use of Classroom Design.

SOFT DRINK DESIGN

Challenge: Invent a new soft drink that would be popular and produced at a low cost.

Children may quickly become involved in this challenge if they are asked what makes a soft drink popular. Alternatively, a future school party might need a beverage which the children could be asked to concoct.

Frequently a class will separate into small groups to plan their own methods of discovering the important characteristics of a popular drink. One group may devise a questionnaire and conduct a sample survey of various age groups to define the flavor, degree of carbonation and price preferences of different consumer groups. Results of the surveys are tallied and then depicted on histograms and bar or line graph forms. The children might devise a blindfold test to see whether a consumer can actually distinguish between certain drinks. Right and wrong guesses are tallied on a confusion matrix.

Another group may prepare several punches by mixing flavors and sugar in varying proportions. Preferences are determined by blindfold testing. Subsequently, important factors are identified by rating all possible pairs of punches according to similarity. Results are tallied on a two- or three-dimensional map.

Using a similarity map, it is possible to then distinguish the factors in each drink which contribute to discernable test differences between the drinks sampled. If the preference results are then correlated with this factor analysis, the children can figure out which factors should be included in a popular beverage.
CONSUMER RESEARCH—PRODUCT TESTING

Challenge: Determine which brand of a product is the best buy for a certain purpose.

The challenge might arise from a class debate over the quality of some product bought by the children, their parents or the school store or over the claims of TV commercials. The children decide to test several brands of the products which they use most frequently.

The class as a whole, or individual groups, formulate test objectives and procedures. The children may purchase the several brands themselves to make price comparisons and per unit cost calculations. Quantitative data is obtained by testing the properties of the products, such as the lifetime of pens or soap pads, or the strength of batteries or plastic wraps. Often the children construct test apparatus to insure uniform, objective measures. Using the data acquired, the children calculate areas, volumes, lifetimes or strengths. Information is depicted on bar or line graphs. Test procedures and results are periodically reviewed by the class to appraise constructed as well as the suitability and validity of the tests. Finally, using the test results, the children determine which brand is best for their specific purpose, considering also quality and cost.

Having evaluated the several brands of a product, the children decide to inform their schoolmates, parents or the product manufacturers of their test procedures and findings by means of a school magazine or newsletter. A permanent consumer information/product testing group might be formed.

As a follow-up activity, the class may decide to manufacture a new consumer product; they consequently explore production and marketing techniques. This may lead to both Manufacturing and Advertising.

PEDESTRIAN CROSSINGS

Challenge: Recommend and try to have a change made which would improve the safety and convenience of a pedestrian crossing near your school.

Children are aware from an early age that some pedestrian crossings are more dangerous than others. They are urged to cross streets going to and from school at locations where traffic police are on duty for short periods of time before and after school or where there are "Walk" lights.

The challenge might originate from a general discussion about coming to school. If crossing certain streets is a problem, the teacher will learn what observations the children have already made. The children may then decide to work in small groups counting cars to determine traffic-flow patterns, calculating speed of cars, timing pedestrian crossing times and car gap times, measuring length and width of street and height of buildings and trees, interviewing pedestrians about the safety of the crossing and possible improvements. The data they have collected may be represented by tables, graphs, scale drawings and scale models. Classroom simulations based on real data may be used to try out some of the children's suggestions.

As children collect the data, draw their conclusions and recommend certain improvements, they may see the need for more data or a different type of data such as sight distances and car-braking distances. A single solution to the problem should not necessarily be sought, but each group of students should document as thoroughly as possible its suggested improvement. It is hoped that the unit will culminate in some positive action like submitting a report of recommendations to the proper authorities.
The following activities, reproduced with permission, are samples of materials from the Mathematics Resource Project, developed at the University of Oregon and supported by a grant from the National Science Foundation. Project materials are now commercially available from Creative Publications.

Mathematics Resource Project materials consist of five resources, each containing worksheets, calculator activities, games, puzzles, bulletin board suggestions, project ideas and teaching didactics. The resources have been designed and created for teachers in grades 5 through 9.

Resource activities are highly motivational materials designed to provide student practice with all the basic skills including problem solving, mental computation, estimation, and measurement. Individual resource packets are organized under the following titles:

- Number Sense and Arithmetic Skills - 832 pages
- Ratio, Proportion and Scaling - 516 pages
- Geometry and Visualization - 830 pages
- Mathematics in Science and Society - 464 pages
- Statistics and Information Organization - 850 pages

The sample materials are identified by the letters MRP on the top right hand corner of the page.

For more information about Mathematics Resource Project materials, contact:

Creative Publications, Inc.
P.O. Box 10328
Palo Alto, CA 94303
The following materials from the Mathematics Resource Project were included in the packet, but cannot be reproduced here due to copyright restrictions:

Do You Know That
Rumors
Change for a quarter
Wordless Problems
Omar's Dilemma
Balancing Bees
What's My Line
Strictly Squaresville
The Painted Cube
A Startling Discovery
The Parthenon
Impossible? or Improbable?
Maximum Volume
A Sheepish Problem
Let's Not Get A Head
Make a Dip Stick
Forest Fires Are a Real Burn
Operation Please!
Picture Problems 2
Great-Great-Great-...Grandparents
More Fun at the Fair
Area Problems to Attack
Staircases
Going Around in Circles
Petite Proportions 2
A Variety of Volume Vexations
The Wide Open Spaces
Mathematics Problem Solving Project (MPSP)

The following pages, copied with permission, are samples of problem materials from the Mathematics Problem Solving Project (MPSP), a project of the Mathematics Education Development Center at Indiana University funded by a grant from the National Science Foundation. The project was cooperatively developed by staff from the following:

University of Northern Iowa
Cedar Falls, Iowa

Oakland Schools
Pontiac, Michigan

Indiana University
Bloomington, Indiana

Project materials consist of three booklets of lesson outlines and three associated sets of problems organized under the following headings:

- Using Lists
- Using Tables
- Using Guesses

The sample problem materials are identified by the letters MPSP on the top right hand corner of the page.

Inquiries about the project or the materials may be addressed to:

Professor John F. LeBlanc, Director
Mathematics Education Development Center
Education Building
Indiana University
Bloomington, Indiana 47401
The following materials from the Mathematics Problem Solving Project were included in the packet, but cannot be reproduced here:

| BO 1 | 13 BV 6 |
| BO 2 | 14 BM 6 |
| BO 15 | 17 BM 6 |
| BO 16 | 18 BM 6 |
| WO 13 | 19 BM 6 |
| WO 14 | 20 BM 6 |
| GO 1 | 9 YV 6 |
| GO 2 | 10 YM 6 |
| GO 3 | 13 YM 6 |
| GO 4 | 14 YV 6 |
| GO 19 | 21 YM 6 |
| GO 20 | 22 YC 6 |
| 3 BC 1 | 23 YD 6 |
| 4 BC 1 | 24 YC 6 |
| 29 BS 1 | 9 WC 6 |
| 30 BC 1 | 10 WD 6 |
| 6 YM 1 | 5 RV 6 |
| 6 YB 1 | 6 RM 6 |
| 33 YC 1 | 15 RD 6 |
| 34 YC 1 | 16 RC 6 |
| 17 WB 1 | 21 RD 6 |
| 18 WS 1 | 22 RM 6 |
| 7 RB 1 | 9 GD 6 |
| 8 RB 1 | 10 GV 6 |
Egyptians used fractions with some special rules. First, they used only unit fractions, fractions with 1 in the numerator like $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$. Second, they expressed fractions like $\frac{3}{4}$ by using only unit fractions. Also, they never used the same unit fraction in the same expression more than once. One way used to express $\frac{3}{4}$ was $\frac{1}{2} + \frac{1}{4}$. Think like an Egyptian and provide a name for the following fractions using only unit fractions. Use the number of unit fractions you are told to use.

1. $\frac{1}{2}$ - use 2 unit fractions
2. $\frac{5}{8}$ - use 2 unit fractions
3. $\frac{13}{16}$ - use 3 unit fractions
4. $\frac{7}{8}$ - use 3 unit fractions
5. $\frac{9}{16}$ - use 2 unit fractions
6. $\frac{3}{7}$ - use 3 unit fractions
7. $\frac{1}{6}$ - use 2 unit fractions

What is the next number in the sequence?

14, 17, 50,

Start with a number. Form a sequence such that each term is the sum of the squares of the digits of the preceding term. At some point in the sequence each term will be 1 or the sequence will repeat.

16, 37, 58,
Cars have space inside so persons can sit in them. Sometimes we put other things in a car. Estimate how many basketballs could be put into your car.

10 basketballs
45 basketballs
115 basketballs

Write other guesses

Write several large and small amounts of money.

$1,000 2¢
$5 Write more

You are buying something for 43¢. Write several amounts of money which "are small" compared to 43¢.

Write several amounts which are small compared to $417.
Trace the length of the segment with your finger.

\[ \text{__________} \]

Is it as wide as your finger? Your hand?

List several things in your school which you estimate to be as long as the segment.

- Get a box of paper clips.
- Pick up your pencil. Does it weigh as much as 10 paper clips? Put the clips in one hand and the pencil in the other. Make estimates smaller and larger than 10 for the weight of the pencil in paper clips. Circle the best guess.

<table>
<thead>
<tr>
<th>Object</th>
<th>Weight Guesses</th>
<th>Number of Clips</th>
<th>Best Guess</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) pencil</td>
<td>10, 20, 7, 50, 18</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
Find a picture in a magazine or book that has many things in it. List the ideas about space and number which you estimate or see in the picture.

Examples: a road longer than a house
three boys
a plant smaller than a boy

Name smaller things that fit into larger things

Examples: a fire in a house
car in a garage
grass on a lawn

List several things which are found in packages or groups of 100 or more items.

Examples: people at a basketball game
popcorn kernels in a bag
cars in a salesman's lot
Suppose you leave a tip for a waitress at a cafe. Some people leave 15% of the cost of the meal. If your meal cost $6.30, estimate ways that you could use to find 15% of $6.30. List some ideas about 15% and about $6.30. Write some guesses about what 15% of $6.30 might be. Circle your best guess. You do not need to find the exact amount.

Suppose you have 7 coins in your pocket that add up to $1.00. What are the coins?
List the first 2 prime numbers that are in ascending order and the first 2 prime numbers that are in descending order.

A baseball player has 8 baseballs. Seven of them weigh exactly the same, but one is heavier. Using a balance scale, how can you find the heaviest ball in just 2 weighings?

First weighing – put 3 balls on each side of the scale. If one side is heavy, we know the heavy ball is one of the three on that side. If they balance we can weigh the 2 remaining balls in another weighing. Take the 3 balls and put one on each side. If they balance the 3rd ball is the heaviest.
Suppose that a lady bug wants to crawl up to the window where there is a hole leading outside. The lady bug is at the tip of your shoe now. Give the lady bug a map and directions for the shortest route from your shoe to the window.

**CRYTARITHMETIC - DIVISION**

\[
\begin{array}{c}
T D X ) E D E I T X \\
X T X T
\end{array}
\]

\[
\begin{array}{c}
E S X \\
S S I \\
T D X \\
E X T \\
E S X \\
S N X \\
T D X \\
T S I
\end{array}
\]

\[
\begin{array}{c}
1 4 5 ) 7 4 7 0 1 5 \\
5 1 5 1 \\
7 2 5 \\
2 2 0 \\
1 4 5 \\
7 5 1 \\
7 2 5 \\
2 6 5 \\
1 4 5 \\
1 2 0
\end{array}
\]

I = 0 Identity

X = 5

T = 1 Identity

D = 4

E = 7

S = 2

N = 6
POSITION STATEMENT

PROBLEM SOLVING

Learning to solve problems and apply mathematical skills is the major reason for studying mathematics. Therefore the most valuable and lasting strand of the mathematics curriculum is Problem Solving. It is the skill and strategy gained in this area that the learner will use all his life, for it is here that the focus of all skill areas come into use. The knowledge of skills combined with the thinking necessary to find solutions to the unknown is the highest level of mental processing.

Problem solving should involve purposeful and perplexing questions of interest to the student. A true problem is a situation that is novel for the person trying to solve it.

Because this area is so highly dependent on cognitive abilities, it is very important to consider its relationship to child development. Problem solving activities are of a highly individualized nature. If one were to compare a monochromatic development of color from white to black, one could imagine the infinitesimal shades of gray between the black and white. All these shades represent the multilevel skills in problem solving.

There is some commonality in both child development levels and problem solving strategies. According to George Polya, problem solving strategies can be taught like any other skill if the proper stage of development and prerequisite skills have been achieved. He likens it to swimming in that if you want to be a swimmer you must get into the water.

The mathematics Scope and Sequence has been based on the commonalities evident in child development and allows the professional teacher to make adjustments to provide for unique needs and abilities.

There are three stages of child development which are essential for consideration in problem solving approaches. These are defined by Piaget in the following manner:

pre-operational 2-7 years - labeling activities, refining sensory motor activities, simple concept level
concrete-operational 7-11 - operates with concrete objects, begins organizational thinking
formal operations 11+ years - operates with ideas, makes inferences

Problem solving strategies most useful for young children are the following in sequential development:

1. Activities which involve comparing objects on common attributes and which involve much manipulative processing.
2. Activities which represent ideas pictorially rather than concretely.
3. Activities which transfer concrete ideas into abstract symbols like number sentences or equations.
4. Activities which verify a solution by testing it against concrete manipulations.

As a young person begins to think abstractly these skills can be a useful tool to him/her. They should be introduced easily and comfortably with visual confirmation. Difficulty should be increased in small sequential steps.
Many of the above strategies carry over from the young child to the intermediate, junior, and senior high levels. They are dependent on the student's background of experience.

Further extensions of problem solving are prevalent as the child's experiences and needs are broadened. Skills are needed in other content areas and are demanded by interests in hobbies and activities within the child's life.

Students at this level have developed some abstract thinking and many processes can be combined to solve a problem. Some problem solving techniques available to the student at this level are:

1. Estimating
2. Patterning
3. Using a model/design
4. Working backward

The teacher must demonstrate the technique and then allow the student to practice using the technique.

Problem solving is more a part of the life of the secondary student as he/she becomes ready to enter his/her role as an adult in society. Life is not a series of segmented activities but an interplay of all skills, knowledge, and abilities. With this integration in mind, the problem solving strategies at this level are highly complex and multifaceted. They involve, however, many components of those strategies taught at an earlier level and are highly dependent on them. Some strategies used at a highly abstract level of development are the following:

1. Selecting appropriate notation
2. Identifying wanted, given and needed information
3. Systematizing
4. Restating the problem

The development from the concrete to the abstract approach of problem solving must be taught and not be left to chance. It is the responsibility of every educator to provide this essential segment of a child's education and also to help him learn to apply and transfer his math skills to other areas in which they are needed. The Problem Solving strand of the Scope and Sequence supplies the foundations and structure for instruction to take place.
The K-8 mathematics scope and sequence is divided into seven strands:

1. Structural Number Concepts
2. Computation
   a. Whole Numbers and Integers
   b. Decimals
   c. Fractions
3. Geometry
4. Measurement
5. Interpreting Quantitative Data
   a. Ratio and Proportion
   b. Graphing
   c. Probability
   d. Statistics
6. Applications
7. Problem Solving

Structural Number Concepts involves numeration and the theory of real numbers. Students will become aware of the underlying concepts which serve as the foundation for mathematics and its uses.

Computation of whole numbers, decimals, and fractions provides for an understanding of the algorithms used in computation. Students will become aware of the process of computation and be encouraged to check answers or solutions as to accuracy and reasonableness.

Geometry fulfills the individual's need to understand the physical world. From recognition of basic shapes to finding the volume of "pyramids" students will gain insight into the spatial relationships which exist in their world.

Measurement deals primarily with metric measure and incorporates length, area, mass, volume and temperature. Knowledge of measurement terms is essential in one's life, as are the actual skills of estimating and measuring. Students will gain a solid background of knowledge and skills through this strand.

Interpreting Quantitative Data contains ratio, proportion, graphing, statistics, and probability. There is much evidence to support the inclusion of all these topics as each one plays an important role in the individual's participation in the real world. Students will be provided many opportunities to gain practical skills in this strand.

Applications provides many opportunities for students to put to use the knowledge and skills learned in the other strands.

Problem Solving is yet another way to apply cognitive skills but goes far beyond application. Students will learn the processes of problem solving. This strand is the most important of all the strands as it is the goal toward which we are leading our students. If students can approach a problem situation and apply the skills of this strand, they will reach satisfying solutions.

The concepts in the scope and sequence have been coded as spiraling, core, spiraling core, and enrichment. The spiraling concepts are those concepts introduced at a previous grade level which may need to be reviewed. Core items are those concepts considered to be appropriate major concepts for students at a grade level but extended to some degree when reviewed the following year are coded as spiraling core. Finally, the enrichment ideas are those concepts at a grade level for the more capable student.
Numbers in front of objectives indicate grade levels when the objective can be accomplished.

All the objectives are considered important. The experiences of each child will depend upon the development of the child, his/her native ability, and his/her attitude.

It is suggested that teachers maintain an environment where these objectives can be accomplished to some degree by all students.

K, 1, 2, 3 1. Draw from his/her cognitive background.

3 2. Identify useful or extraneous information given in the description of a problem.

1, 2, 3 3. Recognize simple questioning techniques for placing a given situation in a mathematical context. Example: What operation should be used? What units? What comes first?

1, 2, 3 4. Draw pictures to develop understanding of a problem.

K, 1, 2, 3, 5. Manipulate physical objects to develop understanding of a given problem.

1, 2, 3 6. Use tables, drawings or diagrams to develop understanding of a problem.

3 7. Identify the object of a problem, the given information, and the unknowns.

3 8. State a given problem in simpler terms.

1, 2, 3 9. Construct models of problems. (physical, simple number sentences, tables, graphs, geometric shapes)

1, 2, 3 10. Conduct simple physical experiments and collect data to find solutions.

K, 1, 2, 3 11. Check solutions to problems.

1, 2; 3 12. Explore patterns to reason through a problem.
A Note to Conference Participants:

On the following pages are listed the 17 problem solving strategies presented during the conference. You might use these as master sheets to make transparencies for your in-service activity. If you chose, you can type in example problems to illustrate each strategy. (See "Student Strategies for Solving Problems" in an earlier section of this packet.) You may prefer to distribute the "Student Strategies . . ." paper to your teachers and make transparencies from the following masters. Then you could work and discuss the problems in the paper, working on the overhead transparencies.
PROBLEM SOLVING STRATEGIES

1. SELECT APPROPRIATE NOTATION.

2. MAKE A DRAWING, FIGURE, OR GRAPH.

3. IDENTIFY WANTED, GIVEN, AND NEEDED INFO.
4. RESTATE THE PROBLEM.

5. WRITE AN OPEN SENTENCE.

6. DRAW FROM YOUR BACKGROUND.
7. CONSTRUCT A TABLE.

8. GUESS AND CHECK.

9. SYSTEMIZE.
10. MAKE A SIMPLER PROBLEM.

11. CONSTRUCT A PHYSICAL MODEL.

12. WORK BACKWARDS.
13. GENERALIZE.

14. CHECK THE SOLUTION.

15. FIND ANOTHER WAY TO SOLVE IT.
16. FIND ANOTHER RESULT.

17. STUDY THE SOLUTION PROCESS.
The success of any inservice experience depends greatly upon the harmonious interaction of a large number of variables. Some of these variables like timing, group size, location, length, involvement, and assistance, may be within your control. But other factors equally important to the inservice success may be less directly under your influence—components such as participant interest (see Appendix A), attitudes, background knowledge, administrative cooperation, and equipment performance or availability.

This makes the task of inservice planning both difficult and critically important. One tries, of course, to optimize all conditions affecting the inservice. But, if one were to select a single component that must be addressed to assure chances for success (after participants are warm, comfortable, and filled with coffee), that variable is that participants must be convinced of the need for the inservice.

In the case of problem solving and calculators, this inservice element becomes imperative! Nearly every teacher will come to any such inservice experience with very strong attitudes and beliefs. Problem solving responsibility of the high school—my kids have enough trouble learning their multiplication facts, or calculators would make it impossible to motivate children to learn their basic facts—kids, like adults, would only get lazy and become dependent upon calculators.

Therefore, it is extremely important early in your inservice activities to discuss the issue: Why should more attention be focused in the elementary mathematics curriculum on problem solving (calculators)?

The answers to such questions are likely to be fraught with high emotion. One possible strategy to avoid early problems, to defuse some of this emotion, and to capitalize on the intense feelings, is to conduct an open and highly participative discussion using one of these questions as the topic. Allowing participants to verbalize their points of view, hear others, and have certain misgivings answered by the group will get you off to a smooth start with high participant involvement while avoiding your being pushed too early into a defensive posture.

After a period of free wheeling discussion, it will become necessary for you to summarize the group discussion (theirs, not yours, even if it isn't what you might have liked) and to move on to a more substantive study of the topic. A transition like the following might be helpful. Now that we have had a chance to air our feelings about calculators (problem solving), let's take a look at what research and some actual classroom experiences have to say about some of the issues we have discussed.

At this point, a relatively short but convincing presentation will probably be necessary to convince skeptics and to reinforce supporters of the importance of the topic. The following materials may provide you with some solid ammunition for this presentation.
Why should we give more emphasis to problem solving?

Back to basics, minimum competencies, and high classroom priorities on basic fact development are likely to be topics that will cloud the problem solving issue. Traditional strong emphasis on basic fact development (and consequent low priority on problem solving) has been a benchmark of the elementary program. Coupled with the superficial inferences by some back to basics and minimum competency advocates that children are not learning their "basics" as well as previous generations, arguments to place more emphasis on problem solving are likely to meet with some resistance.

Many of the critics of current mathematics education base their concerns about the lack of success of the mathematics program on the performance of graduates, the finished product of our educational efforts. But close listening and questioning of critical concerns yield interesting results. When they say, Today's graduates don't know their basics; most critics actually mean: Today's graduates can't apply their basic computational skills in reasonable problem situations.

As the accompanying table (see Appendix B) of Ohio twelfth grade assessment results for mathematics shows, computational skills are reasonably high and relatively close to the desired performance as determined by a panel of Ohio mathematics educators. However, problem solving capabilities decline very rapidly with increasing complexity of the situation and produced marked discrepancies between actual and desired performances. The sample items also illustrate that the problem situations are quite relevant to adult needs and, with one exception, are quite within the level of difficulty of the elementary program.

National assessment data reflect precisely the same trends as this Ohio information. Problem solving skills are not well developed for a significant number of students. Basics and minimum competencies may continue to be a concern. However, for most students, the concern should not be restricted to development of computational skills, but rather extended to how to use them.

Two key issues that remain to be answered to convince a group of the importance of problem solving in service are: 1) Is teaching problem solving my responsibility? and 2) What is there to teaching problem solving that I am not doing now? Other packet information can be used to help answer these questions. An excellent additional resource is the November, 1977 issue of The Arithmetic Teacher. A somewhat more sophisticated reference is the March, 1978 issue of School Science and Mathematics.

Why Should We Consider the Use of the Calculator in the Elementary Program?

The hand calculator is the second most successful electronic instrument (after television) ever used in the consumer world. They are becoming as common place as pencils and paper. The small size, lack of noise, durability, and enormous price drops of 95% have made the idea of a personal hand calculator for every pupil and adult a real possibility.

Hand calculators can make the performance of complicated computations less tedious and more accurate but their use does not lessen the need for pupils to understand which operations are needed to solve a particular problem, to make sensible estimates, and to analyze their results. If the hand calculator is used properly in the classroom it should help alleviate tedious hand calculations and enhance a pupil's mathematical understanding and computational skills.
The use of the hand calculator by teachers will be a new experience and will provide them with new insights in evaluating mathematics instruction by themselves and their pupils.

OTHER CONSIDERATIONS FOR INSERVICE PLANNING

In designing an effective inservice program, there are many other factors that should be considered after plans for answering the need issue and for organizing the general content have been made. Some of these factors involve general considerations of what is known about successful inservice. Others concern specific considerations appertaining to mathematics inservice.

Ten Considerations for the Planning of Successful Staff Development*

Significant changes in teacher behavior will occur when activities are directed toward specific outcomes. However, it should not be concluded that most procedures will be effective for most objectives. Differences in materials, approaches, and formats are related to differences in the effectiveness of professional growth programs. Research supports the following characteristics of successful inservice education.

1. Inservice programs that are school-based have more influence on teacher attitudes than those which are college-based.

2. Self-instructional inservice materials designed for teachers to use in the school setting have proven to be effective.

3. School-based programs where teachers participate as planners and provide resources to one another are more successful in achieving their objectives than those which are conducted by college or other outside personnel without the help of teachers.

4. Staff development activities that are individualized are more likely to accomplish their objectives than those which offer the same program for all participants.

5. Professional growth programs that place the teacher in an active role constructing and generating are more likely to realize their objectives than those where the teachers are in a receptive role.

6. Inservice activities that emphasize demonstrations, supervised trials, and feedback are more likely to reach their goals than are those where teachers are asked to store up prescriptive ideas for a future time.

7. Inservice education programs which are organized in such a way that teachers can share with and help one another are more likely to achieve their objectives than are those where teachers work separately.

8. Teachers benefit more from staff development activities that are ongoing in nature and a part of the school's total program than those which occur only as isolated episodes.

9. Teacher selection of goals and related activities are more beneficial to participants compared to those programs, where goals and activities are preplanned.

10. Self-initiated and self-directed inservice activities are associated with successful achievement of program goals.

Implications for Mathematics Inservice Education

A. Parental Concern: Three factors seem to affect both pupil attitude and pupil performance in mathematics. 1) parental expectation of pupil achievement in mathematics, 2) parental encouragement regarding mathematics, and 3) parent's attitude toward mathematics.

B. Affective: Wholesome attitude toward mathematics is promoted when the pupil feels: 1) mathematics is useful, 2) success is possible, and 3) teacher is enthused about mathematics.

C. Elementary school teachers prefer inservice that combines content and methods of instruction.

D. Some elementary school teachers may lack confidence in their ability to learn mathematics.

E. The first inservice session should be closely related to classroom work.

F. Inservice sessions should include problem solving and hand calculator exercises and methods that can be adapted for use in teachers' own classrooms.

G. Teachers who have taught the same grade for many years may need a review of the K-6 mathematics program goals.

Some Themes Around Which an Inservice Might Be Organized

How problem solving (calculator use) relate to the total curriculum; to the mathematics curriculum

Teacher attitudes about calculator use (problem solving) compared to research and assessment findings

How to teach for problem solving: strategies approach; types of problem solving

How to organize your classroom and instruction for problem solving

Problems, pitfalls, and other considerations in simulating real problem solving in the classroom

Some Questions as You Plan for an Inservice

How can you best provide inservice to groups? Do K-2 teachers have different perceptions and needs than 3-6 teachers?

What size group is most desirable for inservice?
How do your teachers feel about mathematics? about their abilities to teach it?

Is a needs assessment desirable? When should it be administered—before or after the first session?

Should members of the junior high or senior high school staffs be involved in the inservice? in what capacity?

Is problem solving a strand in your curriculum guide? Is enough importance assigned to it? What kinds of other resources for teaching problem solving do you have in your district?

Are calculators readily available in any of the buildings of your district? Could they be made available? through what means?

What kinds of attitudes do your teachers, parents, and school board members have about basic skills? minimum competencies? problem solving? calculators? What needs to be done to overcome any resistance? to point out the potential or importance?

How much time will you have for inservice? What themes should you build your inservice periods around?

Should you inservice at the district level or building level? Should you work with total staffs or representatives (who may or may not multiply their experiences)? What role do administrators have in your plans? parents?

What can you do about establishing a resource center, corner, file, etc? How many per building or district are needed? What can you do to make teachers aware of resources from this conference? from other sources?

Can direct assistance to teachers be provided by you or someone else in the classroom?

How can you utilize other expertise available to you within the district? outside the district? How might you use the college educators you met at this conference?

Can you benefit from planning together with people from other districts? How could you do this effectively?

What additional assistance do you need that has not been provided here? Where can you obtain it?

What implications do issues raised relative to problem solving and calculator use have for preservice training?

What is the nature of your commitment to these inservice topics? Will you do one inservice and then move on to the next "hot topic?" or will you pursue this topic over a long period of time?

What kind of support can you expect from your administration at the district level? at the building level? What kinds of support can parents provide?
What kind of resources, financial and other, do you need to do proper inservice?

Should you attempt to begin two big inservice thrusts at the same time? Do the topics need to be treated separately? Could you work on one topic with teachers from some grade levels and on the other topic with other teachers? If so, which would initiate with which group?

Should you seek outside funding sources for a comprehensive program within your district? If so, what sources are available?

Should you make problem solving an interdisciplinary effort in your inservice plans? What kinds of additional considerations might need to be made? What resources (materials and personnel) are available for such an approach?

Should you set up model classrooms working with only a few teachers before launching more extensive inservice efforts? How could you use the model classrooms?

What kinds of plans should you consider for parent and public awareness?

How long should individual inservice sessions be? How far apart should sessions be? What kinds of provision for feedback and input need to be considered?

What kinds of curriculum changes or teaching emphases might be required to place more stress on problem solving (calculator use)?

In what ways may your evaluation program be affected?

Would a school wide activity (problem solving olympics) or theme be productive in encouraging a total school involvement to help reach your inservice goals? (Another activity might be a contest to design a safe way to drop an egg from the top of the school building without breaking the egg.)

Inservice education is too important to be left entirely to university and college professors, P.T.A.'s, curriculum committees, curriculum directors, county or state supervisors, bargaining units, or school board members. The professor or supervisor may instigate the planning at each building by teachers under the leadership of the administrators but the topics must come from the teachers. Districts should develop an inservice program building by building where a needs assessment is developed and administered. Then appropriate inservice should be planned and implemented on a long term basis with continual feedback.

GOOD LUCK!
APPENDIX A

In order to plan inservice, a professor or supervisor should obtain data concerning the needs of the proposed population. The following summary of a needs assessment obtained from 153 teachers and administrators in 17 elementary schools in the Toledo Metropolitan area is listed as an example of data that would be useful during the planning stage of an inservice program.

Needs Assessment Summary
153 responses

<table>
<thead>
<tr>
<th>Need Description</th>
<th>No</th>
<th>Some</th>
<th>Much</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Effective Classroom Management Techniques</td>
<td>17</td>
<td>72</td>
<td>49</td>
</tr>
<tr>
<td>2. Exploring Alternative in Reporting to Parents</td>
<td>32</td>
<td>79</td>
<td>31</td>
</tr>
<tr>
<td>3. Techniques for &quot;turning on&quot; the &quot;turned off&quot; students</td>
<td>13</td>
<td>39</td>
<td>83</td>
</tr>
<tr>
<td>4. Diagnosis of Learning Problems in Students and Developing Strategies for Overcoming Them</td>
<td>8</td>
<td>53</td>
<td>75</td>
</tr>
<tr>
<td>5. Effective Teaching Strategies for Reaching the Below Average Student in the Classroom</td>
<td>13</td>
<td>54</td>
<td>71</td>
</tr>
<tr>
<td>6. Utilizing Television as a Tool for Improving Instruction</td>
<td>59</td>
<td>69</td>
<td>11</td>
</tr>
<tr>
<td>7. Identifying and Utilizing Area Resources for the Extension of Classroom Learning Experiences</td>
<td>28</td>
<td>66</td>
<td>44</td>
</tr>
<tr>
<td>8. Maintaining Efficient, Effective, and Legally Correct Student Records</td>
<td>56</td>
<td>57</td>
<td>29</td>
</tr>
<tr>
<td>9. Developing Learning Resource Centers in the Classroom</td>
<td>25</td>
<td>56</td>
<td>59</td>
</tr>
<tr>
<td>10. Gearing up for the Mainstreaming of Handicapped Children in the Regular School Program</td>
<td>36</td>
<td>66</td>
<td>30</td>
</tr>
<tr>
<td>11. Preparation of an Instructional Unit</td>
<td>72</td>
<td>56</td>
<td>15</td>
</tr>
<tr>
<td>12. Planning Strategies for a week, quarter, year</td>
<td>59</td>
<td>60</td>
<td>18</td>
</tr>
<tr>
<td>13. Subject Emphasis—How is a particular subject, such as mathematics, taught Today in Elementary School</td>
<td>55</td>
<td>59</td>
<td>21</td>
</tr>
<tr>
<td>14. How to deal with prejudice in the classroom</td>
<td>58</td>
<td>66</td>
<td>13</td>
</tr>
</tbody>
</table>

Each respondent was asked to indicate the degree of no, some, or much, relative to the need for inservice work for each item. As the summary implies the respondent's immediate needs centered around topics 3, 4, and 5 while they expressed little need for topics 11, 12, 13, and 8. It is with this type of information that a professor or supervisor can accurately plan inservice sessions that are needed by a specific group of teachers.
### APPENDIX B
**SOME PERFORMANCES ON THE**
**TWELFTH GRADE OHIO MATHEMATICS ASSESSMENT (1975)**

<table>
<thead>
<tr>
<th>Number of Items</th>
<th>Problem Type</th>
<th>Actual Performance</th>
<th>Desired Performance</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>whole number computation</td>
<td>91.8%</td>
<td>96.7%</td>
<td>- 4.9%</td>
</tr>
<tr>
<td>17</td>
<td>one step application</td>
<td>85.1%</td>
<td>91.7%</td>
<td>- 6.6%</td>
</tr>
<tr>
<td>18</td>
<td>two step application</td>
<td>67.6%</td>
<td>88.4%</td>
<td>- 20.8%</td>
</tr>
<tr>
<td>3</td>
<td>difficult application</td>
<td>48.3%</td>
<td>87.2%</td>
<td>- 38.9%</td>
</tr>
<tr>
<td>16</td>
<td>interpretation/inference</td>
<td>79.8%</td>
<td>90.7%</td>
<td>- 10.9%</td>
</tr>
</tbody>
</table>

### SAMPLE ITEMS

#### One Step Application

Your club earned $250. You want to spend the money on tickets for the fair. Fair tickets cost $1.25 each. How many can you buy?

- A. 11 (2%)
- B. 20 (14%)
- C. 22 (5%)
- ✫ D. 200 (73%)
- E. I don't know. (4%)

**desired performance** - 88.8%

#### Two Step Application

How many feet of fencing is needed to go around a rectangular swimming pool 60 feet by 120 feet?

- A. 180 feet (9%)
- ✫ B. 360 feet (51%)
- C. 720 feet (6%)
- D. 7,200 feet (29%)
- E. I don't know. (4%)

**desired performance** - 90.6%

#### Difficult Application

The cost of frozen orange juice is 6 ounces for 34¢, 12 ounces for 52¢, 16 ounces for 74¢, or 24 ounces for $1.07. Which is the best buy?

- A. 6 ounces for 34¢ (5%)
- B. 12 ounces for 52¢ (38%)
- C. 16 ounces for 74¢ (13%)
- ✫ D. 24 ounces for $1.07 (36%)
- E. I don’t know. (6%)

**desired performance** - 86.2%

#### Interpretation/Inference

What is the weight of ✿ if □ is one pound?

- A. 3 pounds (5%)
- B. 2 pounds (5%)
- C. ½ pound (10%)
- ✫ D. ⅓ pound (68%)
- E. I don’t know. (11%)

**desired performance** - 77.1%
OHIO REGIONAL CONFERENCES ON MATHEMATICS EDUCATION

Calculator Packet

Contents

Use of Hand Calculators in Grades K-8
Hand Calculators: Home and School
Excerpts From Articles on Calculators
Using Calculators: How--Not Should
Where Can Use of the Calculator Lead
State-of-the Art Review on Calculators
Calculator Information Center Bulletins
Minicalculators in Mathematics Classes
Teacher Notes for Estimation and Your Calculator
A Variety of Calculator Activities and Games

Oregon Mathematics Resource Project (MRP):
  Sample Materials, Calculator

Oregon Mathematics Resource Project (MRP):
  Sample Materials, Calculators and Problem Solving

Mathematics Problem Solving Project (MPSP):
  Sample Materials, Calculators and Problem Solving

MCTM Monograph
A Selection of Transparency Masters
Workshop Evaluation
Conference Evaluation Form
Teacher In-Service Evaluation Form
Introduction

Pocket calculators are ruining our kids by turning them into a generation of mathematical illiterates, says Kansas State University Physics Professor Dr. William Paske. He states that years of reliance on the ubiquitous machines leads to a significant diminishment of a pupil's ability to make basic calculations without help. "If we continue to encourage the use of hand calculators at the primary grades we're going to create a large mass of mathematical illiterates," Paske asserts.

Dr. Paske's fears are seconded by John Renner, professor of science education at the University of Oklahoma. "The calculator will get you the right answer without your understanding the basics of mathematics," Renner says. "That's my fear. The pupils will say there's no need to learn because this little black box will do it for them."

The question is not whether to use hand calculators, but when, says Dr. Jesse A. Rudnick, professor of mathematics education at Temple. His research findings with 600 seventh grade students indicated that using the calculator didn't affect a pupil's ability to (mentally) calculate at all.

Hand calculators are becoming as commonplace as pencils and chalkboards. There are even models out for pupils as young as five years old. Some educators feel that calculators may inhibit a pupil's learning basic mathematics skills if they're introduced too early by becoming a crutch. Other educators state that we are living in the age of the calculator, so let pupils have them as young as possible to be used as learning tools. Most educators agree that the hand calculator should not replace the merits of drilling and home assignments but used rather to help pupils increase their effectiveness with these endeavors.

Approximately 36 percent of Florida's eleventh grades who took the state's new functional literacy test in the fall of 1977, failed the mathematics section. The test was a measure of a high school graduate's minimal skills, including their ability to determine a monthly electric bill, calculate sales tax on different items, and compute interest on a savings account. Florida is the first state to actually administer a test students must pass to earn a diploma. Many states are planning to follow Florida's testing and we can expect similar disappointing results from today's mathematics instruction regarding the use of mathematics in the "real" world. The hand calculator is a teaching aide that may help provide the high school graduates with the mathematics skills they need to operate and survive in the world of work.

We will not engage ourselves in a discussion of whether or not the hand calculator should be used in grades 1-8. The hand calculator is here and readily available to students. This conference's role will be to provide suggestions for classroom use that are in accord with approved classroom methods and learning objectives. The hand calculator should be regarded as another teaching tool available to the teacher, much like flash cards, blackboards, overhead projectors, flannel boards and trigonometric tables. How you actually use the hand calculator will be up to you to decide.
Hand Calculators

Stores are selling hand calculators with many different characteristics. For classroom use in the elementary school the following minimal characteristics are recommended.

1. A display of at least eight digits
2. A display that is easy to read by more than one person
3. Four basic functions - addition, subtraction, multiplication, and division
4. Floating decimal, decimal is placed in each answer automatically
5. No dual purpose keys
6. AC adapter
7. Have all the calculators in your classroom the same.

The need for similar hand calculators in high school decreases. Additional features needed by high school pupils are:

1. Scientific notation
2. Square root function
3. Reciprocal function
4. Logarithmic function
5. Trigonometric functions

Data from hand calculator manufacturers suggest that sales of hand calculators have increased from 7.4 million dollars in 1973 to 72.6 million dollars in 1977. Sales in over half the instances are being made to housewives and pupils for home and school use. As one visits schools and talks with teachers it is clear that the number of pupils using hand calculators is increasing. The predicted retail price in 1978 for the basic hand calculator is $5. The predicted retail price for scientific calculators is $25. Given the price, the number sold, and the fact that teacher educators, state and city supervisors, and textbook publishers agree on the desirability of the use of the hand calculator in schools, there should be widespread use of calculators in schools in 1978-1979.

Basic Assumptions on Hand Calculator Use

1. The use of hand calculators should not replace instruction in skills and concepts.
2. The hand calculator can be a useful teaching-learning device in an elementary mathematics classroom.
3. In courses and in mathematics topics where computational skill is not a major objective, the hand calculator may be used to decrease the pupil's computational load.
4. Use of the hand calculator should be taught to all pupils before they complete their formal mathematics instruction.
5. Hand calculators exist. They are here to stay in the "real world," so we cannot ignore them.
6. The hand calculator will help stimulate pupil interest in solving mathematical problems.
7. Decimals and metric computations will be introduced early in the elementary school. Hand calculators will facilitate early continuous experiences with this new set of numbers.

8. The amount of mathematics used by the common person (MR. T. C. M.I.T.S.) will increase drastically.

9. The principle purpose of a hand calculator is to make calculations easy.

NCTM Statement

The National Council of Teachers of Mathematics Board of Directors have identified nine ways in which the hand calculator can be used in the classroom:

1. To encourage pupils to be inquisitive and creative as they experiment with mathematical ideas.
2. To assist the pupil to become a wise consumer.
3. To reinforce the learning of the basic number facts in addition, subtraction, multiplication, and division.
4. To develop understanding of computational algorithms by repeated operations.
5. To serve as a flexible "answer key" to verify the results of computation.
6. To promote student independence in problem solving.
7. To solve problems that previously have been too time-consuming or impractical to be done with paper and pencil.
8. To formulate generalizations from patterns of numbers that are displayed.
9. To decrease the time needed to solve difficult computations.

The impact of widespread use of hand-calculator will produce:

1. Less emphasis on paper-and-pencil algorithms.
2. More significant and interesting mathematics in the school.
3. Consumers and adults much better prepared to deal with mathematics in the world today.

How Hand Calculators are now used in Schools

Presently hand calculators are being used for small scale exploratory activities, remedial mathematics in Title I classes, advanced secondary Science and Mathematics classes, low high school achievers, and in various NSF funded project. General recommendations presently being suggested for a district's use of hand calculators are:

1. Primary level: incidental use, especially in an interest corner or learning center.
2. Intermediate level: available in the school of class sets for occasional use.
3. Junior High level: available of class sets for each teacher.
4. Senior High level: a hand calculator for every student, available anytime.
The hand calculator is being used to save time, to reinforce learning, to develop concepts, to motivate the learner, and to apply mathematics in everyday situations. Hand calculators are quiet enough to be used in a library or classroom and versatile enough to be used wherever they can be carried.

The hand calculator reduces computational time and provides immediate feedback to reinforce learning. It also provides immediate reinforcement of definitions, functions, and basic properties. As an answer key it provides immediate feedback for checking exercises. Pupils are becoming more inquisitive, creative, and independent as they use hand calculators. Number-systems concepts and an understanding of computational algorithms are being strengthened by the use of hand calculators. Estimation and error-identification skills are being developed as a result of using hand calculators, whether the need for such skills is caused by human or machine errors. Problem solving is more realistic since the number no longer have to be simple. With the help of the hand calculator the relationships between mathematics, the sciences, social studies, economics, and geography, as well as physical education and vocational education, are being reinforced by the integration of mathematical applications within these subjects. Consumer economics information, tax computations, stock market analyses, and sports are examples of data that regularly appear in newspapers and that can be used with hand calculators in problem solving situations.

Adults as well as elementary pupils now have a tool that can complete effectively with the marketing techniques of the business community. Pupils can help their parents save money by comparing unit prices, verifying bills and cash register tapes, and figuring discounts. The hand calculator is also being used to balance checkbooks and family budgets, to calculate income tax, and for computing gasoline mileage.

Although research at this time is inconclusive projects tend to state that pupils who use hand calculators with their regular mathematics instruction gained in reasoning ability, computational ability, and interest in mathematics at a faster rate than pupils without the use of hand calculators.

Management

1. Most hand calculators work and are tough enough for direct use by pupils. Rough handling by pupils is rare and calculators are seldom hurt if they are dropped.
2. A school should plan for a 10%-20% per year replacement (or repair) rate.
3. Hand calculators should be numbered and checked in daily to reduce the risk of theft. A pocket or space should be provided for each hand calculator so anyone can easily check on their location.
4. Some rechargeable batteries and adapters will be needed for school owned hand calculators.
5. Pupils should be taught how to use a hand calculator.
6. Teachers and pupils must determine if hand calculators may be used for local and national tests.
Suggested Uses of the Hand Calculator

Grades K-3
1. Count sets of objects, one to twenty, and display the numerals on the hand calculator. (Counting)
2. Given a set of number cards, one through twenty, have one pupil point to a card and the students then show that number on their hand calculators. (Number Recognition)
3. Respond to verbal number names by showing one, two, and three digit numbers on the hand calculator. (Number Production)
4. Given an oral name, a written name, or a set of objects, the pupil produces the correct numeral on the hand calculator. (Number Production)
5. Display a number on the hand calculator that comes before or after a given number, in the middle of two given numbers. (Number Sequence)
6. Show the number that is four (4) tens and two (2) ones. (Place Value)
7. Counting on the hand calculator (punch 1, punch +, punch 1, punch +, punch 1, punch +, etc.). Count by 2s, 5s, 10s, etc. (Counting Forward and Reverse)
8. Use the hand calculator to help pupils fill in the 9x9 basic addition and subtraction chart. (Discovering Patterns)
9. To find or verify missing addends or sums for addition and subtraction exercises in vertical or horizontal form. (Operations)
10. To discover the role of zero in addition and subtraction algorithms. (Zero Property)

Grades 4-6
1. Give an oral or written name the pupil shows the correct numeral on the hand calculator. Four thousands, 7 hundreds, 3 tens, and 6 ones or four thousand two hundred six. (Number Production)
2. Use the hand calculator to find the number that is 1000 less than 6271 or 8000. Show the number that is 100 greater than 4821 or 4935. (Place Value)
3. Use the hand calculator to verify column addition, subtraction particularly where regrouping or borrowing was necessary, and multiplication and division exercises. (Operations)
4. To help recognize the relationship between addition and multiplication. $6 \times 4 = 24$, $4+4+4+4+4+4 = 24$. Similar relationship between division and repeated subtraction. $21+7 = 3$, $21-7 = 14$, $14-7 = 7$, $7-7 = 0$ (Operations)
5. Entering a large number (6-8 digits) and having it read by a partner. (Place Value)
6. Verify answers to multiplication and division exercises that involve 10, 100, and 1000. (Operations)
7. Use the hand calculator to determine if the following number are even or odd. Thirty-eight, 653, 1,692, 29, 8,769, 728. (Number Theory)
8. Check this exercise by using your hand calculator

$$\frac{37 \ R \ 2}{9 \ ) 335}$$ (Operations)
9. First estimate the answers and then use your hand calculator to check your estimations. (Estimation)

<table>
<thead>
<tr>
<th>Estimation</th>
<th>Hand Calculator Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 5812 + 4406</td>
<td></td>
</tr>
<tr>
<td>b. 846 - 231</td>
<td></td>
</tr>
<tr>
<td>c. 479 x 24</td>
<td></td>
</tr>
<tr>
<td>d. 8153 ÷ 41</td>
<td></td>
</tr>
</tbody>
</table>

10. Select the largest number in each row. Use your hand calculator to check your results. (Order relations)

| a. 19 x 31 | 32 x 19 |
| 889 - 496  | 889 - 496 |
| 380 - 20   | 400 - 19  |
| 2 x 20 x 40| 2 x 19 x 30|

* Grades 7-8 *

1. Find the value of $53^2$ or $46^2$. (Exponents)

2. Multiply $1 \times 2 \times 3 \times 4 \times 5 \ldots 9 = \boxed{}$. This product is called 9 factorial and is written $9!$. (Factorial)

3. Insert the correct symbol $>$, $<$, or $=$ to make the statement true. Then check the results with your hand calculator. (Number properties)

   - $45 \times 31 \quad \square \quad 35 \times 41$
   - $178 - 6 \quad \square \quad 178 - 7$
   - $127 \times 428 \quad \square \quad 428 \times 127$
   - $32 \times (41 + 82) \quad \square \quad (32 \times 41) + (32 \times 82)$

4. Use the hand calculator to find the largest whole number that makes each sentence true. (Division)

   - a. $N \times 6 \quad \leq \quad 493$
   - b. $9 \times N \quad \leq \quad 329$
   - c. $N \times 27 \quad \leq \quad 1746$

5. Find the missing numbers with your hand calculator. (Number properties)

   - $573 + \boxed{} = 573$
   - $17 \times 86 = \boxed{} \times 17$
   - $(49 + 24) + 29 = 49 + (24 + \boxed{})$
   - $617 \times \boxed{} = 617$
   - $9 \times (36 \times 74) = (9 \times \boxed{}) + (9 \times 74)$
6. Which of the following numbers are factors of the first number? (Factoring)

18 → 1, 2, 3, 4, 5, 6, 8, 9, 12, 18
40 → 1, 2, 3, 4, 5, 6, 7, 8, 11, 14, 17, 20

7. Complete these number sequences by using your hand calculator. (Patterns)

a. 7, 14, 21, 28, __, __, __, __, __, 63
b. 4, 0, -4, -8, __, __, __, __, -28
c. 1, 0.5, 0.25, 0.125, __, __, __, __, 0.015625
d. 1, 4, 9, 16, __, __, __, __, __, __, 64

8. Use your hand calculator to determine if the following fractions are equivalent. (Cross-multiplication)

a. \( \frac{5}{9} \div \frac{25}{36} \)  
   \( \frac{17}{31} \div \frac{23}{37} \)

9. Place the decimal point in your answer. Check your answer with your hand calculator. (Decimals)

a. \( 2.1 + 3.2 + 4.1 = 9.40 \)
   b. \( 5.49 \times 3.2 = 17.5680 \)
   c. \( 239.5 \div 0.19 = 1260.526 \)

10. Find the square root of 46 to three decimal places. (Square root)

Summary

The hand calculator is a teaching device that should be found in the elementary mathematics classroom. Its increased use by pupils, teachers, and parents, coupled with the eventual adoption of the Metric System will bring new changes in the content and sequence of mathematics education.

Students should have to demonstrate their knowledge of basic computational skills before, while, and after they work with hand calculators. Pupils must understand what the hand calculator is doing for them.

Finally, you should have a hand calculator too. It will increase your personal use of numbers and also give you ideas on how to use it in your classroom.
HAND CALCULATORS: Home and School

The hand-calculator is the second most successful electronic instrument (behind color television) ever used in the consumer world. The small size, lack of noise, easy maintenance, and enormous price drops of 95% have made the idea of a personal hand-calculator for every person a real possibility. It is now estimated that over 40 million hand-calculators will be sold all over the world each year. The National Council of Teachers of Mathematics has pronounced the hand-calculators as valuable instructional tools and predicted that they are here to stay.

There is no attempt to suggest that arithmetic should not be done in the head. As teachers and parents gain experience using hand-calculators, they will find many ways to help pupils use the hand-calculators to learn and strengthen their knowledge of basic arithmetic facts and skills. The hand-calculator is having a good impact on slow learners. The glowing numerals and push buttons are exerting a powerful appeal that strengthens a pupil's interest in arithmetic. High achievers are also experiencing mathematical gains as they are able to explore advanced application work with their surplus class time.

The hand calculator is a teaching aide that can help a teacher do a better job of teaching mathematics concepts. It can help a pupil focus his attention on mathematical ideas. Teachers are discovering that the hand calculator can help pupils perform the routine calculations that often deflect them from the more meaningful processes that must be mastered for a basic understanding of mathematics. The use of the hand-calculator will never replace the need for understanding mathematical concepts, learning basic skills, or the teaching of mathematics but it will increase the teacher's chances for providing insights into an additional depth and breadth of
mathematical topics.

Because of hand-calculators many adults are actually enjoying mathematics. Although it has long been socially acceptable to be poor in mathematics, adults are now beginning to discuss computational games. The negative public attitude toward mathematics is changing as more adults use hand-calculators. The day may come soon when it will be socially acceptable in our culture to enjoy mathematics.

Using a hand-calculator at home and in the classroom will be a new experience. Listed below are some suggested activities to be used at home. Try some of these with your children and let the hand-calculator open up new avenues to problem solving.

**Uses Around the Home**

1) Retotaling - Routine checking of charges, invoices, and other bills that come every month

2) Monthly electric bill - Total cost for one month ÷ kilowatt hours = cost per kilowatt

3) Cost to operate an appliance - Amperes x 120 = watts
   Watts ÷ 1,000 x cost per kilowatt = cost /hr.

4) Reading your meter - keep a yearly record to find the monthly amounts

5) Real estate taxes - Market value x rate of assessment = assessed valuation
   Assessed valuation ÷ 1,000 x mill rate = annual real estate tax

6) Fire insurance - Coverage ÷ 100 x cost per 100 = annual premium

7) Filling a swimming pool (Length x width x depth x 7.5 = gallons)
   Pool gallons ÷ gallons per minute ÷ 60 = hours to fill or drain

8) Others:
   - Board feet of lumber
   - Changing a recipe
   - Balancing your checkbook
   - Price comparisons for best buy
   - Discount
   - Stock dividends
   - Compound interest
   - Installment carrying charge
   - Monthly loan payments
   - Foreign currency conversion
   - Metric conversions
   - Changing a fraction to a decimal
   - Changing a fraction to a percent
   - Baseball averages
   - Calorie chart - calculate anticipated weight loss

George Immerzeel compares the use of a hand calculator in the elementary classroom to a story about a small boy and a wise old man. "According to the story, the wise man came to the village once a year to share his wisdom. The small boy, as small boys will, decided to show the village that the wise man wasn't always right. He captured a small bird and held it in his hands. He planned to ask the wise man if the bird was alive. If the wise man said, 'Yes,' he would squeeze the bird and the wise man would be wrong. If the wise man said 'No,' he would open his hands and the bird would fly away.

"When the boy finally got to the front of the line and asked the wise man if the bird was alive, the wise man responded, 'It's in your hands.'"


Comments from the parents of seventh-grade pupils regarding their children's use of the calculator were:

"It's all right to introduce the calculator in the higher grades, after the students learn their basic skills."
"Let's go back to teaching the basics, not teach our children to be dependent upon a machine."
"Under no circumstances should the taxpayers' money be spent on this."
"The calculators are too easily stolen."
"It could serve as an incentive to the child and add interest to what otherwise might be a dull subject."
"Stop experimenting with our kids; you have already lost one generation to modern math."
"No way our kids should use the machines. Teach them basics."
"It's a good idea! But what will teachers do with the time left over?"


"Educators and laymen alike have shown a great resistance to the use of hand calculators in schools, particularly in the lower grades. Educators have been willing to admit—or to consider the admittance of—hand calculators only in a case where the calculator adds a 'new dimension' of learning to the experience of the child; only in a case where a calculator allows a child to do something, or learn something, that he could not before. I submit that this resistance to the use of hand calculators is irrational and I propose that we make fullest possible use of calculators in all grades of our schools...

"The real problem involved in the use of this new instrument of calculation—and here the reactionaries are speaking—is that the use of the
proposed instrument of calculation does not carry with it an understanding of the basic operations of arithmetic as does the use of the present instruments—pencil and paper . . .

"After hand calculators have been introduced into the schools in all grades, I am sure that teachers will find a myriad of things to do with them and with the time that is released by the use of calculators. We should not try to prevent the introduction of hand calculators into schools, or fear it; we should, I think, accept it as inevitable and begin a study of ways to make it as fruitful as possible."


Mr. Munson believes that each school district should develop a policy on the use of hand calculators in school and in doing assignments at home. He raises the following questions for each district to consider:

1. Should calculators be used for computation or merely to check on computations done by hand?
2. Should slow learners be given access to calculators in the belief that students will gain mathematical facility with their use?
3. At what grade level should calculators be introduced if they are to be used as a part of an instructional unit on calculator use?
4. Is there need for an instructional unit on calculator use?
5. Will the use of calculators make students mathematically lazy?
6. Should restrictions be placed on the use of calculators in subjects other than mathematics?
7. If calculators are used for daily work, may they be used in tests?
8. Should special allowances be made for the use of calculators in achievement test batteries?
9. Does the school have a responsibility to furnish calculators for student use?
10. Should calculators replace slide rules in advanced mathematics and science classes?


Bell's article addresses three topics: student reactions to calculators, some pedagogical questions that may need further investigation, and some practical and management questions.

**Student Reactions To Calculators**

1. Is explicit instruction in use of calculators necessary?
2. Are children interested in using calculators, and does the interest last?
3. Do children "naturally" detect errors and reject unreasonable results?
4. Can calculators help in diagnosing gaps in conceptual understanding?
5. Do children become curious about unfamiliar functions on the calculator?
Some Pedagogical Issues
1. Do children become dependent on calculators? Does it matter?
2. Are there pedagogical consequences from choice of machine configuration?

Some Comments on Management Problems
1. Are calculators durable enough for classroom use?
2. What about losses from thefts?
3. What source of power is best?

VI: Rogers, Joy J. The Electronic Calculator--Another Teaching Aid?

In commenting on teaching aids that have been relegated to school storage rooms, the author proposes four features that seem to separate enduring teaching aids from the others. These were: inexpensive and durable, controllable by learner, does something learner wants done, and flexible usage.

Ms. Rogers summarizes her position by suggesting that the hand calculator has the potential, as assessed against these four criteria, to be a teaching aid of enduring value.
CALCULATORS: THEIR USE IN THE CLASSROOM*

Gregory Aidala

1) The purchase of all calculators should include a one year warranty to replace or repair any malfunctioning machine.

2) Distinct and permanent identification is necessary for all calculators and adaptors.

3) The authors highly recommend the use of electrical adaptors as opposed to any type of recharging device. Adaptors will provide uninterrupted and longer lasting service in the utilization of calculators.

4) A locking cabinet must be provided to enhance the easy distribution, collection, and protection of all calculators and adaptors.

5) Designated calculators should be assigned to students so that a particular machine is utilized by the same pair of students on a continuous basis.

6) Rules and regulations involving the use of calculators must be clearly stated and enforced so that students will exercise care in the operation of each calculator.

7) A trustworthy student should assist the teacher in the distribution and collection of calculators during a class period.

8) At least two full class periods of instruction should be provided to all students vis-a-vis methods of operating a calculator.

9) Although educators should be urged to explore all avenues of incorporating calculator usage into daily lessons, we highly recommend that the utilization of calculators not exceed one experience per week. The novelty of calculators in a classroom environment can easily be eroded by overuse that more importantly basic computational skills might eventually become weaker.

*Taken from School Science and Mathematics, April 1978, Volume LXXVIII, No. 4, Whole 686, pages 307-311.
Suppose you were faced with the question, "What are the three most compelling reasons for using hand-held calculators in the classroom?" Or, "What are the three most compelling reasons for banning hand-held calculators from the classroom?" What would your answers be?

Those are two of the questions that were asked in a survey conducted last spring for a study sponsored by the National Science Foundation.* The aim of the study was to provide a critical analysis of the role of hand-held calculators in pre-college education. Data on the availability of calculators, statements of opinion and practice, published articles, and research results were collected. A primary purpose was to identify the positions of teachers and other educators regarding the use of hand-held calculators in elementary and secondary schools.

Questionnaires were sent to samples of teachers and other school personnel, state supervisors of mathematics, and mathematics educators in colleges and universities in every state. In addition to the questions above, answers were sought to such questions as:

--How should hand-held calculators be used?
--What uses are most important at various levels?
--How should the curriculum be modified if calculators are readily available to students at all times?
--What would you recommend to elementary and secondary school personnel considering the selection and use of calculators?

The most frequently given reasons for using hand-held calculators in the schools are that they:

1. Aid in computation; they are practical, convenient, and efficient.
2. Facilitate understanding and concept development.
3. Lessen the need for memorization, especially as they reinforce basic facts and concepts with immediate feedback and as they encourage estimation, approximation, and verification.
5. Motivate: curiosity, positive attitudes, and independence are encouraged.
6. Aid in exploring, understanding, and learning algorithmic processes.
7. Encourage discovery, exploration, and creativity.
8. Exist: they are here to stay in the "real world."

The last reason -- the pragmatic fact that they exist and that they are appearing in the hands of increasing numbers of students -- is perhaps the most compelling. How they can be used to facilitate the growth of mathematical skill and understanding is a concern that each teacher must attack. Research is being conducted that will provide some indication of how well the other beliefs in the benefits of using a calculator are supported: this research can be supplemented by teachers testing calculator applications in a wide variety of mathematics classes.

The most frequently cited reasons for not using calculators in the schools are that:

1. They could be used as substitutes for developing computational skill: students may not be motivated to master basic facts and algorithms.
2. Are not available to all students.
3. May give a false impression of what mathematics is: mathematics may be equated to computation, performed without thinking.
4. Are faddish.
5. Lead to maintenance and security problems.

* "Using Hand Calculators: The Implications for Pre-College Education," National Science Foundation Grant No. EFP 75-16157.
The first concern — that students will not learn basic mathematical skills — is the one expressed most frequently by parents and by other members of the lay public, as reflected (and created) by newspaper articles. But it builds a strawman, for few educators believe that children should use calculators in place of learning basic mathematical skills. Rather, there is a strong belief that calculators can help children to develop and learn more mathematical skills and ideas than is possible without the use of calculators. Teachers and other educators need to give serious attention to proving that this belief can be implemented and become fact.

To aid in implementing the use of calculators, a variety of recommendations was made by the teachers and other educators surveyed. These range from the general to those specific to the curriculum, and include:

1. EXPERIMENT AND PLAN — don't wait for "the word":
   a) Learn to use calculators yourself first, finding meaningful ways to use them.
   b) Use them with students only after considerable thought as to how, when, and why.
   c) Develop a school-wide policy and guidelines.
   d) Develop ways to incorporate calculators into the existing curriculum, and develop new curriculum as necessary.
   e) Plan a reasonable inservice program, evaluation, and research.
   f) Use in early grades with care, if at all.

2. Survey available calculator models carefully and buy good equipment, commensurate with student needs. Make sure that all students have access to a calculator.

3. Change teaching emphases to concept development, algorithmic processes, when to apply various operations, and problem solving.

4. Do not ignore the development of computational skill.

5. Think of calculators as a tool to extend mathematical understanding and learning by making traditional work easier. The focus can be on process because the product is assured.

6. Place more emphasis on problem-solving strategies. Use practical, realistic, significant problems, and more applications.

7. Spend less time on computational drill, more time on concepts and the meaning of operations. Use more laboratory activities where computation is involved but the emphasis is on learning mathematical concepts. Decrease the use of tedious, complicated algorithms; emphasize algorithmic learning, including student development of algorithms.

8. De-emphasize fractions, and emphasize decimals, introducing them earlier.

9. Emphasize estimation and approximation (including mental computation skills), checking and feedback, exploration and discovery.

10. Do more and/or earlier work with such ideas as place value, the decimal system, number theory, number patterns, sequences, limits, functions, iteration, statistics, probability, flow charting, computer literacy, large numbers, negative numbers, scientific notation, data generation, and formula testing.

There is one thing you should be wary of: purchasing materials related to using calculators. Look over such materials carefully; make sure they will really be useful to you. Don't buy "sight unseen." A few useful materials are available — and a lot that appear to be helpful mostly in providing dollars for the sellers.

A great deal of exploration needs to be done to determine how the calculator can best be used. But that it will be used in schools appears certain — so start exploring in your classroom!
Where Can the Use of the Calculator Lead?

The hand-held calculator can be regarded as the GREAT EQUALIZER in the sense that now ALL can calculate.

Reluctance to use the hand-held calculator may well be analogous to that of using any of the "new" calculators or algorithms as they were introduced throughout history. Calculations were in earliest times with the aid of pebbles (stones, "calculi" (Latin)) in grooves in the ground or in a tray of sand or soil. Later came the various kinds of abaci and then algorithms on numerals. Even algorithms have changed through the years, for there were times during which $34 \times 76$ was done in the ways shown below.

The one at the left is called the "scratchout method" and the one at the right the "gelosia" or "lattice method." The latter led to the development of "Napier's Rods" in 1617. Later came the slide rule and the computer and now the calculator. Would it be natural that there was some resistance to some of the -- yet each offered an improvement in some way over the preceding -- and our minds and skills did not cease to grow! Indeed, through their use came new opportunities in mathematics and in problem-solving.

Problem: How can the calculator be used effectively and not compromise with understanding and competency.

Where can the calculator lead?

a) to increased facility in the use of numbers. Problems need not now be made "to order" -- more "real life" problems now!

Students can also be encouraged to "set up" entire problems and to make simplifications before using the calculator -- e.g. the value of the fluid in the tank at $63\varepsilon/gallon$ is given by

$$8.5 \times 8.5 \times 3.1416 \times 20 \times 15/2 \times 63/100$$

Note that the expression can be simplified and then calculated.
An example of a "real life" problem whose solution is made easier now is to study the problem of how far to cut in at each corner of a rectangle in order to make a box of maximum volume --- the use of a table and a calculator!

b) to better understanding of different kinds of numbers-- the rationals as repeating decimals (perhaps an invitation to study periodicity of the repeating decimal); the idea of square root and cube root but here to explore: \(5 \times 5 = 25\). \(\sqrt{25} = 5\) but what two numbers (alike) multiply to give 26? -- problems from our environment too!

c) to greater efficiency in estimation -- if used in this direction

12.6 \(\times\) 40 equals about what? Let us see!
.13 \(\times\) 457 equals about what? Let us see!
What number \(\times\) 25 lies between 290 and 310? / Estimate!
Check!

d) to gain early ideas on ratio and the concept of (later to be called) trigonometric ratio.

A right triangle drawn as shown and with the calculator one can divide each height by the corresponding base --- ratio always about the same! Useful --- or that can arise from attempting to find the height of a pole and the previous activity become a part of the problem-solving process! Students can make their own "ratio tables" for different angles.

e) to become more proficient in mental arithmetic. Why not "pit one's self" against the calculator!
E.g.

\[
\begin{align*}
5 & \times 6438 = 32190 & 45 x 45 & = 2025 & (13)^2 \\
20 & \times -734 = -14680 & 35 x 35 & = 1225 & (18)^2 \\
98 + & 34 = 132 & 72 x 78 & = 5568 & (14)^2 \\
1328 & - 98 = 1230 & 94 x 96 & = 9024
\end{align*}
\]

We need to encourage mental arithmetic. "Contesting" against the calculator can help us.

f) to explorations which lead to concepts in algebra or which can be substantiated by algebra.

\[
\begin{align*}
45 & \times 45 & = 2025 & (13)^2 \\
35 & \times 35 & = 1225 & (18)^2 \\
72 & \times 78 & = 5568 & (14)^2 \\
81 & \times 89 & = 7249 & (15)^2 \\
94 & \times 96 & = 9024 & (18)^2 \\
93 & \times 97 & = 8981 & (31)^2
\end{align*}
\]

Is there a "quick way" to get the answers just above? Calculator answers are correct but can we devise a way (algorithm) to perform
on the digits to get the answer quicker?

Another interesting exercise is to ask the student to calculate simultaneously \((13)^2\) and \(14 \times 12\); \((15)^2\) and \(14 \times 16\); \((17)^2\) and \(16 \times 18\); \((16)^2\) and \(15 \times 17\). Hence how can we quickly calculate \(23 \times 25\)? \(29 \times 31\)? et cetera.

Later, can the above "quick methods" be substantiated by Algebra?

g) to facilitate the estimation of roots as \(\sqrt[3]{1632}\)

What whole number cubed seems to be just less than 1632? just greater? Hence we have a lower bound and we have an upper bound. Can we find lower and upper bounds in tenths? then in hundredths? Indeed here are some important mathematical terms growing from simple things: upper bound, lower bound, sequence, limit-- and students gain a "feel" for these terms. Calculators can help us learn good mathematics vocabulary.

h) to deepen ideas of number-theoretic concepts: prime number, factor, factorization. There are examples of this in various sources of suggested uses of the calculator.

i) to lead to the idea of solving equations.

What number "works" in this expression -- that is, what number makes it valid (or makes up the solution set!)?

\[
3 \Box^2 + 4 \Box = 39
\]

j) to employ "algebra" in a new setting.

Although the discussion should perhaps concern the elementary school, it might be pointed out here that the attempt to solve

\[
x^2 + 6x - 43 = 0
\]

by the use of the calculator is made easier by rewriting the above as

\[
x(x + 6) - 43 = 0
\]

--"easier" in the sense that the number of steps is reduced.

Although the calculator is the GREAT EQUALIZER one should use it also in school to develop more mental powers in arithmetic. Indeed one use of the calculator in the classroom can well be to help devise ways and skills so we use it less!

Computers may be of great value in problem-solving, but apparently the human brain alone is able to tackle the subtler aspects of creating an effective correspondence between the mathematical world and the world of experiment and observation.
The one hand, mathematics teaching should be permeated with concrete examples which give an impression of how widely and diversely mathematical ideas penetrate into human problems generally, including everyday, technical and scientific matters. On the other hand, it is necessary to tell at least one lengthy connected story of the application of mathematics in real depth. This will amongst other things communicate the message that no-one can expect to solve the whole of any problem mathematically. There must be an integration of experiment and theory; there must be a combination of mathematical investigation with inferences from observation and experiment and from non-mathematical modes of reasoning. The best primary-school teaching is a good reminder of how effectively such integration can be carried out, and can be an inspiration to those of us attempting the same at other levels of education.

State-of-the-Art Review on Calculators: Their Use in Education

Background

At approximately one-tenth the price they were four years ago, handheld calculators are a bargain. They have progressed rapidly from being considered a status symbol to the point where, for many adults, they are considered a necessity. While not every household has a calculator, marketing figures indicate that over 80 million calculators have been sold in this country.

Increasingly, these data reflect sales not only to individual parents, who may let their children use the tool, but also sales to schools. Not surprisingly, the calculator was readily accepted at the college level -- as a tool in mathematics, engineering, science, and other courses, for all levels of students from remedial to advanced. At the secondary school level, there has also been a high degree of acceptance. The calculator was recognized as a tool which could help to save time spent on hand calculation and thus allow more time to be spent on mathematical ideas and on more interesting content and problems. Use of calculators is by no means incorporated into instruction by every secondary school mathematics teacher, but their use is widely allowed. The main question has been, "Should or shouldn't they be used on tests?", and even this is fading as an issue; teachers are using tests where calculators, available to all, are neither an aid nor a hindrance in terms of the goals being tested.

From the junior high school years downward, hesitancy about using calculators increases. Especially in classes for low achievers in the junior
high, there are many teachers who still hold firmly to the belief that students must master computational facts and procedures before they use calculators. On the other hand, an increasing number of teachers say, in effect, "Why should these students go on struggling to master what they've obviously had trouble mastering for the past six or seven or eight years? Why not let them use the calculator so they can go on to learn some real mathematics -- and maybe attain a different viewpoint about mathematics?"

In the elementary school, use of calculators is greater at the intermediate level (grades 4 through 6) than at the primary level. The most obvious reason for this is the widespread belief, held by both parents and teachers, that children should master the basic facts and the procedures for addition, subtraction, multiplication, and division before they use calculators. Associated with the tendency to use calculators may be the teacher's level of mathematical background: the greater the teacher's knowledge and confidence about mathematics, the more "comfortable" or secure he or she may feel with a tool that can process numbers so quickly. Another factor may be the firmer belief held by primary-level teachers in the role of manipulative materials in developing children's understanding of and competency with mathematical ideas and processes, as evidenced by the fact that the use of such materials is high in the primary grades but has tapered off by the fourth-grade level. Thus intermediate-level teachers may be more ready philosophically to use a tool which required no physical manipulation beyond key-pushing. (It might also be noted that fear of audiovisual equipment in general decreases as grade level increases.)

The "back-to-the-basics" bandwagon has also undoubtedly played a part in suppressing use of calculators at the elementary school level. As concern has been expressed by parents and school boards, teachers have re-emphasized the stress placed on work with computation. Extended practice exercises and
drill work have been viewed as the way to meet the demands for a more "traditional" type of arithmetic program. Energy that might have gone into exploring the use of calculator applications has been deflected to the development of drill-and-practice materials; the open-mindedness needed to incorporate instructional applications of calculators has been tamped by their "newness" in an era when "old" values are being given priority by a vocal segment of the population.

**Extent and Type of Use in Schools**

No data have thus far been cited about the extent to which calculators are being used in schools. The reason may be obvious: such data are not widely available. We do not know exactly how many students are using calculators in schools; we do not know exactly how many teachers are incorporating calculator use in the instructional program. We have only the results of a few relatively small-scale surveys, plus the perceptions of those who work with and observe school programs.

The following graph presents data from one such survey, conducted with over 22,000 students in the Shawnee Mission (Kansas) Public Schools.
It is one of the few studies in which data collected recently (1977) were compared with data collected earlier (1975). Terry Parks reported on both ownership and accessibility: substantial increases were found at each of the three school levels. The data reflect a pattern of increasing availability of calculators to students.

These data are paralleled in several other reports. It must be noted, however, that they may not be applicable to districts that have not collected data: the mere lack of collection of data may indicate less interest and less availability.

Just as data on the extent of the use of calculators are limited, so are data on the types of uses being made of calculators. But we do know that, at the elementary school level, four types of uses are predominant:

1. Checking computational work done with paper and pencil.
2. Games, which may or may not have much to do with furthering the mathematical content, but do provide motivation.
3. Calculation: when numbers must be operated with, the calculator is used with the regular textbook or program.
4. Exploratory activities, leading to the development of calculator-specific activities where the calculator is used to teach mathematical ideas.

At the secondary school level, the emphasis varies:

1. Calculation, used whenever numbers must be operated with.
2. Recreations and games.
3. Exploration: because secondary school mathematics teachers' backgrounds are generally good, there is much more of this type of activity than at the elementary school level. In addition, the students who continue in higher-level courses are often intrigued to explore.
4. Use of calculator-specific materials. There is at least one text integrating the use of calculators, with several others being field-tested.

Anna Graeber and several others at Research for Better Schools conducted a survey in 1977 of 1,343 teachers in Delaware, New Jersey, and Pennsylvania.
in grades 1, 3, 5, and 7.

In the first grade, calculators were used most frequently for drill; the next three most frequent usages were for checking, motivation, and remediation. Use of the calculator for drill decreased with grade level. Above first grade the most frequent usage was for checking. Motivation and word problems were the next most frequently reported uses for calculators at the higher grade levels.

Between 15 and 30 percent of the teachers indicated that they were using "instructional materials specifically designed for use with the calculator", although the nature of those materials is not noted.

Reasons For and Against Using Calculators

In a survey reported to the National Science Foundation in 1976 by Marilyn Suydam, reasons cited by educators and the authors of published articles for using or not using calculators in schools were listed. Literature published since then has affirmed the continuing acceptance of the reasons for using calculators:

(1) They aid in computation.

(2) They facilitate understanding and concept development.

(3) They lessen the need for memorization.

(4) They help in problem solving.

(5) They motivate.

(6) They aid in exploring, understanding, and learning algorithmic processes.

(7) They encourage discovery, exploration, and creativity.

(8) They exist: this pragmatic fact is perhaps the most compelling, as they appear in the hands of increasing numbers of students.

The reasons for not using calculators also continue to have pertinence:

(1) They could be used as substitutes for developing computational skills.

(2) They are not available to all.

(3) They may give a false impression of mathematics -- that it involves only computation and is largely mechanical.
There is recent research on their effects.

They lead to maintenance and security problems. The first concern is one expressed most frequently by parents and other members of the public. They fear that students will become lazy and will not "make use of their brains -- a wonderful calculator if it is cultivated properly." But few educators believe that children should use calculators in place of learning basic computational skills. Rather, they express a strong belief that calculators can help children to develop and learn more mathematical skills and ideas than is possible without the use of calculators.

As one example of this, a survey of parents in West Chester, Pennsylvania in 1975 indicated that about half of the parents feared that calculators would hinder students' performance on basic skills -- but at least as many thought calculators would improve their children's attitudes toward mathematics. In another survey, parents were asked if calculators should be in elementary schools; about three-fourths of them said "no". But when the question was changed to ask if calculators should be used along with paper-and-pencil computational work, over three-fourths said "yes".

Teachers' opinions about calculators have changed in recent years. In the Shawnee Mission survey already cited, teachers were asked, "Should calculators be used in schools by students?". In 1975, 65.2% said "yes"; in 1977, 71.9% said "yes". Analysis of reasons for responses indicated their awareness of how to use calculators as a tool to assist in teaching computational skills had increased. In 1975, teachers were concerned about the effect of calculators; by 1977, as ideas and guidelines had developed, concerns decreased. In the RBS study, however, the percentage of teachers who had used calculators at each level was far lower than might be anticipated: 3.9% at grade 1, 8.4% at grade 3, 19.4% at grade 5, and only 25.6% at grade 7. Obviously there is much variance in the use of calculators.
at different locations. The effect of leaders who are actively interested in helping teachers learn how to use calculators as an instructional tool seems evident.

Research on Calculator Effects

Most educators believe that the use of calculators should not replace instruction on skills and concepts; rather, calculators are a useful teaching-learning device. Evidence from the research to date supports this contention. In most of the studies at the elementary school level, the data were collected to provide an answer (to parents and school boards, as well as to teachers) to the question, "Will the use of calculators hurt mathematical achievement?" The answer appears to be "No": in all but a few studies, achievement is as high or higher when calculators are used for mathematics instruction (but not used on tests) than when they are not used. But there is variability in the findings, depending in part on the test used: scores may not be as high for problem solving or for concept sections of a test. However, considering the fact that the curriculum was not changed to use the calculator to promote problem solving or concept development specifically, this may not be surprising. Unfortunately, it is unclear in the reports from such evaluations just how the calculator was used, so that specific ways in which the calculator might have been used to enhance problem solving or concept scores remains unknown. What we do know is that the calculator, in general, facilitates mathematical achievement across a wide variety of topics, and this finding is verified at both the elementary and secondary school levels.

In addition, there are a few studies which indicate that children learn basic facts and skills with the use of calculators, and they learn mathematical ideas (such as understanding of mathematical properties) with the use of calculators. Such research in the United States is supported by evidence from other countries, such as Britain and West Germany. There is also evidence
that children do not tend to use the calculator when they realize that it is unnecessary. For example, one researcher cited the example of $79 + 23 - 79 = ?$; children did not use the calculator to find the result — as many adults might have.

At the Wisconsin Center for Cognitive Learning, exploratory work has been underway for the past four years. Summarizing his investigations for the Center in 1976-77, Fred Weaver stated:

At the second-grade level, teachers were given explicit suggestions regarding use of calculators in connection with their on-going mathematics programs, particularly when working with basic addition and subtraction facts and algorithms. At the third-grade level, an emphasis was placed upon the use of calculators in connection with mathematical properties and their applications with particular attention to doing-undoing ideas. At the seventh-grade level, emphasis was placed upon calculator algorithms for whole number situations.

Generally, calculators facilitated instruction, making certain approaches to content more feasible than otherwise would have been the case. However, at each grade level some difficulty was observed in recording calculator algorithms.

His work has been concerned less with developing materials than with exploring the effect of the calculator on promoting mathematical learning.

Several other projects sponsored by the National Institute of Education or the National Science Foundation have the same focus on learning rather than materials.

As the research is surveyed, it becomes evident that there is a need for many more studies to provide knowledge of how calculators can be used to facilitate learning.

**Curriculum Development**

Monies from federally funded programs, including Title IV of the Elementary and Secondary Education Act, as well as from NIE and NSF, are currently being devoted to the exploration of what uses of calculators are feasible and to the development of materials for children and teachers. For
example, the Columbus (Ohio) Public Schools have a grant under Title IV to develop materials for grades 4, 5, and 6, and are presently field-testing modules for a range of topics including place value, decimal computations, rounding, estimation, basic facts, and applications in measurement and money.

Materials are being developed by both individuals and groups. For instance, a group of teachers in the Minneapolis Public Schools produced a set of worksheets complete with objectives and teaching suggestions, designed for students in grades 9 and 10. The topics range from the decimal system to applications such as finding the cost of an oil change or charting population growth. The calculator is used as a tool to help children learn mathematical ideas, and as a computational device to help them to understand ideas and applications that they might not otherwise have been able to.

Several of the state mathematics councils have similarly involved members in developing materials. The Michigan Council of Teachers of Mathematics monograph provides a variety of activities for grades K-3, 4-8, and 9-12, with the mathematical objectives clearly specified. Unfortunately, in many other current publications, the calculator itself is being taught, not mathematics. Students learn some interesting things to do with a calculator, but instructional objectives may not be furthered in the process.

Other states are at the stage of incorporating recommendations on calculators in their curriculum guides. For instance, Indiana's 1977 publication states:

Calculators certainly will have an impact on mathematics curricula. They may change not only the kinds of computational skills which are taught but the manner in which they are taught. It is our feeling that mathematics teachers and curriculum planners must incorporate calculators into regular classwork rather than ignore or banish them. Teachers must find effective uses at all levels from primary grades to calculus.

The guide then suggests ways to use calculators: to reinforce computational
skills, to improve estimation skills, to aid in teaching place value, to develop number concepts, to solve problems with factual data, and to extend textbook problems using more realistic numbers are among the points cited.

A teacher in Fairview Park, Ohio, provides a typical illustration of what an individual teacher may do. When her school received 12 calculators, she put together a unit for her sixth-grade class. She wanted the children not only to become familiar with the functions of the calculator, but also to use the calculator to solve everyday problems and to learn more about number patterns. She used the overhead projector to teach the children how to use the calculator, and made posters, worksheets, task cards, games, and other materials. She found student interest high, with many students gaining confidence in their problem-solving and estimating skills.

The majority of the materials being published contain activities for using the calculator to promote existing curricular ideas. Some of the recommendations of the 1976 NIE/NSF Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics concerned curriculum development for the long-range future. Although little has been published that stretches the curriculum to new bounds, NIE is supporting the development of some future-oriented prototypic curricula that integrate calculator use.

Next Steps

We need to know much more: not just what calculators can do, but what it is possible for them to do given specified curricular and instructional options. We need to know how learning is affected by the use of calculators and how mathematics can be taught differently because of the existence of a new tool. As one respondent to a survey at the 19th Annual Meeting of the National Council of Teachers of Mathematics noted:

The calculators' relationship to problem-solving ability is a question of vital concern. Although the research reported
in Suydam's 1976 report for the NSF shows conflicting reports about calculator effects on problem solving, all of the research ... had the common element that the calculator was an adjunct to units in problem solving -- it was not incorporated into a specific problem solving strategy. This appears to be the best hope for meaningful use of the calculator -- by incorporating it into a specific strategy.

Summary

The use of calculators in education is increasing, although there is some concern and resistance at all levels. The fact that they have become more widely available and that children will use them in their daily lives throughout life makes their use in schools seem imperative to many people. Others fear that growing dependence on calculators will be harmful. However, there is initial evidence that calculators can be used to further the development of mathematical ideas and skills. The efforts of both individuals and groups are focused on studying the effects of calculator use and on developing needed materials. The calculator is not and will not be ignored as a useful learning tool.

Prepared by Marilyn N. Suydam, Calculator Information Center.

This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy.
References


Suydam, Marilyn N. *Electronic Hand Calculators: The Implications for Pre-College Education.* Final Report, Grant No. EPP 75-16157, National Science Foundation, February 1976. ERIC: ED 127 205, ED 127 206.


Additional references are found in the bulletins distributed by the Calculator Information Center, 1200 Chambers Road, Columbus, Ohio 43212.
Looking for Information about Calculators?
So is the Calculator Information Center!

The Calculator Information Center has been established by the National Institute of Education to collect information about the use of calculators in elementary and secondary schools -- and to provide you with information. As recommended in the Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics*, the information collection and dissemination process is important in gathering appropriate development and use of calculator materials by coordinating research and development efforts, aiding needless duplication, and providing a source of knowledge and assistance ...

Thus, the Center will

(1) Develop an information data base . . . [so that] information on calculator activities in such places as local school systems, State agencies, universities, and industry will be routinely routed to the Center.

(2) Develop an easy way to gain access to the information . . . (p. 45)

To help to establish the information base, you can send information to the Center:

- instructional applications
- studies on the effect of using calculators

(Materials will not be released or entered into the ERIC system without specific permission.)

From the Center you will be able to obtain:

- annotated bibliographies:
  - of curricular and instructional applications
  - with background information pertinent to educators
  - on research
- information bulletins on such topics as:
  - available commercial instructional materials
  - available non-commercial instructional materials
  - schools in which calculators are being used and which have indicated willingness to be contacted directly by those with specific questions
  - summaries of characteristics of various calculators
  - points to consider when selecting a calculator
  - aspects to consider when designing school-based studies
  - other topics as requests make a need evident

If you have information to share, or if you wish to learn what others are doing with calculators, contact:

M. N. Suydam, Director
Calculator Information Center
1200 Chambers Road
Columbus, Ohio 43212

Or phone: 614-422-8509 between 9 and 5 (Eastern time zone)

*Copies of the report can be obtained from E. Esty, Mail Stop 7, NIE, 1200 19th St., NW, Washington, D.C. 20208.
Please complete and return to the above address!

If you would like to have your name placed on the Center's mailing list, please check here: ☐

The following reference bulletins are currently available. If you would like any of them, please put a check in the box:

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Please indicate the type of material in which you are interested, so that you can be sent additional materials as they are prepared:

Circle level

☐ References to instructional applications  ☐ References to research  ☐ References to commercial materials  ☐ References on general concerns

Circle level

elementary  secondary  college

elementary  secondary  college

elementary  secondary  college

elementary  secondary  college

Please print your name and address:

Name: ____________________________

Address: ____________________________

Zip Code __________________________

If you know of other persons interested in calculators, please make copies of this page for them (or send us their names and addresses).
Instruction with Hand-Held Calculators, K-12

Bulletin No. 7
August 1977

Articles


This is the first in a serialization of a "self-teaching" student workbook about simple four-function calculators and elementary computer programming. Basic addition, subtraction, and multiplication are covered, some computer-like notation for writing programs is introduced, and directions are given for playing NIM.


A method of finding logarithmic values using a simple matrix and a four-function calculator is provided.


Student reactions to calculators, some pedagogical questions that may need further investigation, and some classroom management suggestions are discussed.


Using the calculator for interdisciplinary projects is discussed, with a list of 15 topics and activities in which calculators can be used. Six interdisciplinary activities are described in detail: for primary grades -- car counting, planning and planting a garden; for intermediate grades -- population growth, organizing a recycling program; for upper grades -- school election prediction, pollution analysis.


Computations yielding results which can be read as words or phrases are presented.


The "passing" of slide rules is humorously decried.


Activities are suggested which focus on developing skills for using the calculator, exploring basic arithmetic operations, understanding algorithms, mental calculation and estimation, and problem solving.


Ways of using the calculator with the blind are discussed.

A method of finding large primes using calculators or computers is presented.


An algorithm is given for using a calculator to compute the cube root of any real number.

Elder, Mary C. Mini-Calculators in the Classroom. *Contemporary Education* 47: 42-43; Fall 1975.

Student discoveries about numbers and operations, stemming from the use of a calculator, are described.


The calculator's effect on sales of slide rules is briefly discussed.


A number-word puzzle to be solved on the calculator is presented.


A cryptarithm, activities involving limits, a keyboard game, and several number tricks to be done on a calculator are described and explained.


Use of the calculator is considered from a functional view and from a pedagogical view.


Ways in which the calculator can be used are discussed.


Activities are described in which the calculator is used in a discovery learning situation for exploring number patterns in multiplication exercises.

Gregory, John W. Use the Calculator for Drill. *Instructor* 86: 104-105; April 1977.

Activities are described which feature the use of a calculator's constant key in drills on basic facts.


Steps in organizing a calculator tournament among schools are presented, with sample problems given.

A mathematical inquiry method (making and organizing observations, generalizing, specializing, inventing symbolism, and proving conjectures) and the calculator's role as an important tool for this process are discussed. An example is given in which the concept of area of a circle can be investigated, with a worksheet.

Hoffman, Ruth I. Don't Knock the Small Calculator -- Use It! Instructor 85: 149-150; August/September 1975.

Several examples of using calculators to explore mathematical ideas are given.


A proposal that the fullest possible use of calculators be made at all grade levels is discussed.


Interesting things to do with a calculator are presented.


Activities and games for the calculator are presented.


Topics discussed include algorithms, calculator problems, homework, problem solving, mental arithmetic, and designing tests for use with calculators. Specific problems for primary, middle, and upper elementary grades are also included.


Teachers are reminded of their responsibility for deciding how best to use the calculator in their classrooms.


Computing numbers approaching a limit is discussed, with a convergence theorem and several examples.


As calculators are increasingly used, drill will be de-emphasized in favor of problem-solving activities. Considerations for selecting calculators and selected games for use with calculators are described.


Seven games are presented, with the mathematical concepts involved, the objective, the number of players needed, the rules, and variations for different grade levels.

A method for using the calculator to determine the day of the week on which any given date falls is explained.


A rationale for the use of calculators with trainable adolescents is presented, with a description of a pilot study assessing an approach to teaching addition and subtraction to six moderately retarded students. A detailed presentation of the program used is included.


An addition activity based on the calculator keyboard is described.


Suggestions are given for using calculators for computations too trivial for computer processing but too time-consuming for hand calculation. Several examples from arithmetic, algebra, and calculus are included.


Details are given concerning the inner workings of the electronic calculator.


Several examples of elementary technical problems in which students can use calculators are given: finding average monthly sales, dealing with lengths and with circuits, figuring percents, and converting from English to metric units.


The development of calculating machines is traced.


Using the calculator as an aid in problem-solving situations is discussed, with suggestions for the classroom. Goals for teaching about calculators are stated, including mechanical aspects, capabilities, limitations, errors, problem solving, and when to use calculators.


Activities and games for computation and estimation, measurement and geometry, functions, and problem solving and applications are described.

Directions for two calculator games are given. One can be played on either a four-function or a programmable calculator, the other only on a programmable calculator. Flowcharts and programming steps are provided.


The significance of the simple type of calculator as a tool in the classroom is discussed, as well as the contribution of the more sophisticated type.


How to program an HP-25 calculator to serve as a clock/timer with display in hours, minutes, and seconds is described.


A program for a programmable calculator is provided which simulates the problems of rocket propulsion, hovering, and soft landing.


The content and teaching of secondary-school mathematics in the calculator era are discussed.


A method to check addition and subtraction in any number base less than ten, using a simple adding machine or a calculator, is given.

Rogers, James T. 10 Games You Can Play with a Pocket Calculator. Science Digest 77: 42-45; May 1975. (See also: Rogers, James T. Seven Pocket Calculator Games. Creative Computing 2: 19; January-February 1976.)

Brief directions for games and "tricks" are presented, with the mathematical rationale behind the "tricks" not explained.


Seven advantages of using a calculator in teaching mathematics to handicapped learners are identified. The use of the calculator as a tool for discovery, for drill and practice, and for motivation is discussed.


Several applications of calculators at various elementary grade levels are suggested, with some punch/display sequences illustrated.

Classroom use of calculators is described, including a brief discussion of ways in which the use of the calculator can enrich, supplement, support, and motivate the regular program.


Some uses of calculators in biology classes are described; for example, charting exponential population curves, evolution by natural selection, and random genetic drift.


Six mathematics units for middle and upper elementary grades are described: banking, transportation, taxes, budgeting, shopping, and pricing and advertising. For each unit, mathematical skills and practical applications are identified, vocabulary is listed, and a series of student learning activities, some using calculators, is suggested.


Worksheets are provided for an activity involving the use of the calculator in discovering patterns when a number is divided by 9, 99, and 999.


A two-person game to be played on a pocket calculator is described and the winning strategy discussed.


A collection of six games for the programmable calculator are presented: Battle the Dive Bomber, Football, Blackjack, Space Flight, Biorhythm Forecast, and Test Your ESP. Goals and rules are described, with programs (for the HP-25).


This report from the NCTM Instructional Affairs Committee presents nine justifications for using the hand-held calculator in classrooms, with some specific examples of curricular applications.
Non-Commercial Publications


The role of the calculator in the elementary-school classroom is briefly discussed. Suggestions for calculator activities are given, with informal evaluations of some.


An overview is provided for each of four reports dealing in whole or in part with calculators (the NACOME report, the Euclid Conference report, the status study report to the NSF on calculators in pre-college education, and the report on the Conference on Needed Research and Development on Hand-Held Calculators). Activities in several school systems are also cited.


Uses and implications of use, research, selection guidelines, and activities are discussed, with the activities keyed to the various functions of the calculator.

Suydam, Marilyn N. Electronic Hand Calculators: The Implications for Pre-College Education. Final Report, Grant No. EPP 75-16157, National Science Foundation, February 1976. [Available from ERIC Document Reproduction Service, Box 190, Arlington, VA 22210 -- 50-page body, ED 127 206 (microfiche, $0.83; hardcopy, $2.06, plus postage); full report, ED 127 205 (microfiche, $0.83; hardcopy, $20.75, plus postage)

The range of beliefs and opinions about the impact of the calculator on pre-college educational practice is reported, with information derived from a literature search and from a surveys of several groups. Appendices in the full report discuss plausible instances with which to use calculators, criteria for redesigning the curriculum, needed research, and perspectives for curriculum revision.


This document will contain 20 articles on different ways a calculator can be used in a classroom.


Worksheets were designed for use with low-achieving ninth and tenth graders; they also seem appropriate for seventh and eighth graders. Activities are included on: introduction to the calculator, games, exploring algorithms, pattern search, estimation and reinforcement of basic computation, consumer applications, and societal applications. Notes for teachers are included with each lesson.

Activities appropriate for the middle school are presented, most as worksheets.


Calculator activities are organized under the following categories: algebra, advanced algebra, general mathematics, geometry, junior high, probability, trigonometry, and miscellaneous.


This free resource list cites articles, books, newsletters, and media, plus information on calculator models appropriate for school use.


Results of the conference, intended to provide a well-defined framework for future research and development efforts, are reported. Twenty-one recommendations covering the development of an information base, curriculum development, research and evaluation, teacher education, and dissemination are discussed.


Part I considers desirable features of calculators for classroom use, basic assumptions, and curriculum concerns. Part II is a collection of activities for grades K-3, 4-8, and 9-12, with a bibliography.


Calculator activities are presented on place value, rounding off numbers an estimating answers, whole numbers, decimal fractions, common fractions, number patterns, powers and roots, algebra, geometry, and advanced topics. Other sections discuss calculator selection, types of use, the keyboard, displays and problems.

This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy.

Books on Calculator Applications


Immerzeel, George; and Ockenga, Earl. *Calculator Activities--Book 1 and Book 2.* Creative Publications and Omron Corporation of America.


Jamele, P. R. *How to Select and Use a Calculator—or Getting the Most from Your 4-Function Calculator.* Los Angeles: Crescent Publications, 1975.


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Introduction to Research on Hand-Held Calculators: K-12

For several years, research efforts have been directed at obtaining information on the effects of using hand-held calculators in elementary and secondary schools. This bulletin has been prepared to acquaint readers with these research efforts and to provide summary information regarding their major findings. In addition to an annotated bibliography listing the studies alphabetically, a table is provided for quick reference. For each study, the table lists the author(s) or major researcher(s), the date of the research report, the grade or age level(s) of the subjects involved in the study, the number (N) of subjects or classes involved, the length or duration of the study, the subject matter area, the type of research, and the major findings.

Most of the studies involved comparisons of Calculator and Non-calculator groups; that is, groups in which the calculator was used or was not used for instruction. Some of the studies had several sub-analyses of the data, so that 40 findings were noted. In 19 cases the Calculator group achieved significantly higher on paper-and-pencil tests (with which the calculator was not used). No significant differences were found in 18 instances; In only three instances was achievement significantly higher for the Non-calculator group.

Such gross tabulations provide some support for the belief that calculators can be used to promote achievement. At the same time, awareness needs to be maintained about the variety of focus, the limitations of research designs, the lack of sufficient descriptions to make a study replicable, and similar factors pertaining in some of the studies to date. Moreover, most of the research efforts were short-term, and dealt with fitting calculator applications into the existing curriculum. Attention must be directed to the long-term effects of the use of calculators, and to ways of using calculators to promote the learning of mathematics. Attitudes also need additional study.
<table>
<thead>
<tr>
<th>Author</th>
<th>Date</th>
<th>Grade/Level</th>
<th>N</th>
<th>Length</th>
<th>Type of Research</th>
<th>Topic</th>
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<td>Allen</td>
<td>1976</td>
<td>6</td>
<td>6</td>
<td>25 days</td>
<td>experimental</td>
<td>decimals, metric</td>
<td>posttest, NSD; retention, N &gt; C</td>
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<td>Anderson</td>
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<td>7</td>
<td>12</td>
<td>20 wks.</td>
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<td>general</td>
<td>C improved attitudes; NSD, achievement (concepts, computation); C &gt; N, problem solving</td>
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<td>Borden</td>
<td>1977</td>
<td>6</td>
<td>4</td>
<td>4 wks.</td>
<td>experimental</td>
<td>*decimals</td>
<td>Both groups achieved significant gains; N had significant negative change in attitude</td>
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<tr>
<td>Campbell/</td>
<td>1976a</td>
<td>5-6</td>
<td>1</td>
<td>school</td>
<td>experimental</td>
<td>checking</td>
<td>NSD, computation; C &gt; N on concepts, problem solving in grade 5</td>
</tr>
<tr>
<td>Virgin</td>
<td></td>
<td></td>
<td></td>
<td>7 mos.</td>
<td></td>
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<tr>
<td>Fischman</td>
<td>1976</td>
<td>9-10</td>
<td>6</td>
<td>1 sem.</td>
<td>experimental</td>
<td>business mathematics</td>
<td>NSD, attitude, concepts; C &gt; N, skills</td>
</tr>
<tr>
<td>Hutton</td>
<td>1977</td>
<td>9</td>
<td>2</td>
<td>1 year</td>
<td>action</td>
<td>general</td>
<td>C &gt; N, computation, concepts; N &gt; C, problem solving</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>powers, roots, radicals</td>
<td>NSD</td>
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<tr>
<td>Jamski</td>
<td>1977</td>
<td>7</td>
<td>6</td>
<td>4 wks.</td>
<td>experimental</td>
<td>rational numbers, percents,</td>
<td>Significant differences on posttest; NSD, retention</td>
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<tr>
<td>Jones</td>
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<td>6</td>
<td>171</td>
<td>9 wks.</td>
<td>experimental</td>
<td>general</td>
<td>C &gt; N; NSD, attitudes</td>
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<td>Lenhard</td>
<td>1977</td>
<td>7-12</td>
<td>8</td>
<td>1 sem.</td>
<td>experimental</td>
<td>general</td>
<td>NSD, achievement, attitudes</td>
</tr>
</tbody>
</table>

* c = classes; p = pupils; t = teachers
** C = Calculator group; N = Non-calculator group; NSD = no significant differences
<table>
<thead>
<tr>
<th>Author</th>
<th>Date</th>
<th>Grade Level</th>
<th>N</th>
<th>Length</th>
<th>Type of Research</th>
<th>Topic</th>
<th>Finding</th>
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</thead>
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<tr>
<td>Luxton/Spungin</td>
<td>1976</td>
<td>ages 15-20</td>
<td>15 p</td>
<td>4 wks.</td>
<td>action</td>
<td></td>
<td>Blind and partially sighted were able to use cassette manuals to learn to use three calculators.</td>
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<td>Miller</td>
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<td>5</td>
<td>4 grps.</td>
<td>12 days</td>
<td>experimental</td>
<td>division</td>
<td>C &gt; N, lower-ability groups, on skills, division; NSD between higher-ability groups</td>
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<tr>
<td>Muzeroll</td>
<td>1976</td>
<td>7</td>
<td>207 p</td>
<td>60 days</td>
<td>experimental</td>
<td>general</td>
<td>NSD, achievement, attitudes</td>
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<tr>
<td>Nelson; Bitter/Nelson</td>
<td>1976</td>
<td>4-7</td>
<td>196 p</td>
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<td>Quinn</td>
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<td>8-9</td>
<td>184 p</td>
<td>8 mos.</td>
<td>experimental</td>
<td>algebra</td>
<td>NSD, achievement; C had less anxiety, better self-concept</td>
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<td>1976</td>
<td>7</td>
<td>600 p</td>
<td>1 year</td>
<td>experimental</td>
<td>general</td>
<td>NSD, achievement, basic skills</td>
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<tr>
<td>Scandura et al.</td>
<td>1976</td>
<td>k-2, 3-4</td>
<td>1 summer</td>
<td>preliminary</td>
<td>general</td>
<td>Parental attitudes indicate reservations about calculator use found to be source of motivation</td>
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<td>Schafer et al.</td>
<td>1975</td>
<td>5</td>
<td>5 c.</td>
<td>2 days</td>
<td>preliminary</td>
<td>general</td>
<td>C &gt; N on calculator examples; NSD on non-calculator examples</td>
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<tr>
<td>Schnur/Lang</td>
<td>1976</td>
<td>ages 9-14</td>
<td>60 p</td>
<td>2 mos.</td>
<td>experimental</td>
<td>general</td>
<td>C &gt; N</td>
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<tr>
<td>Shirey</td>
<td>1976</td>
<td>10-12</td>
<td>9 days</td>
<td>experimental</td>
<td>consumer, business</td>
<td>math.</td>
<td>C &gt; N, inquiry</td>
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<tr>
<td>Spencer</td>
<td>1975</td>
<td>5-6</td>
<td>84 p</td>
<td>8 wks.</td>
<td>experimental</td>
<td>general</td>
<td>C &gt; N on grade 5 reasoning, grade 6 computation, total test</td>
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<td>Sutherlin</td>
<td>1977</td>
<td>5-6</td>
<td>8 c</td>
<td>8 wks.</td>
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<td>decimals, estimation</td>
<td>NSD, estimation</td>
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<td>Vaughn</td>
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<td>9</td>
<td>8 c</td>
<td>8 wks.</td>
<td>experimental</td>
<td>decimals, percents retention</td>
<td>C &gt; N, achievement; NSD, attitudes, retention</td>
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<td>Wajeeh</td>
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<td>9</td>
<td>13 c</td>
<td>15 wks.</td>
<td>experimental</td>
<td>general math.</td>
<td>C &gt; N, achievement; NSD, attitudes</td>
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<tr>
<td>Author</td>
<td>Date</td>
<td>Grade</td>
<td>N</td>
<td>Type of Research</td>
<td>Topic</td>
<td>Finding</td>
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<td>Weaver</td>
<td>1976a</td>
<td>2,3,5</td>
<td>7 c</td>
<td>exploratory</td>
<td>number sentences</td>
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<tr>
<td></td>
<td>1976b</td>
<td>3</td>
<td>2 c</td>
<td></td>
<td>addition, subtraction sentences</td>
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<td>Whitaker</td>
<td>1977</td>
<td>1</td>
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<td>general</td>
<td>C &gt; N, non-timed computation and verbal problem solving; N &gt; C, concepts; NSD, timed computation, total achievement, attitude</td>
<td></td>
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<tr>
<td>Zepp</td>
<td>1976</td>
<td>9, 179 p</td>
<td></td>
<td>experimental</td>
<td>proportions NSD</td>
<td></td>
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<tr>
<td>MT</td>
<td>1974</td>
<td>teachers and others</td>
<td></td>
<td>survey</td>
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</table>
Research on Hand-Held Calculators, K-12


During a 25-day unit on decimal algorithms and on the metric system, four intact sixth-grade classes used calculators for all computations while two classes used only paper and pencil. No significant differences between the two groups were found on the posttest, but on the retention test the group using paper and pencil only scored significantly higher on both the decimal and the metric tests than the calculator group.


Three seventh-grade mathematics classes taught by the same teacher were randomly selected at each of four schools for the 20-week study. One class in each school was permitted restricted use of calculators (checking paper and pencil computations and as an aid in problem solving), a second class was permitted unrestricted calculator use, and the third class was not permitted to use calculators. Pupils using calculators showed improved attitudes toward mathematics but no change in achievement, understanding mathematical concepts, or computational skill. On an untimed problem solving test, pupils using calculators solved problems correctly at almost twice the rate of pupils not using calculators.


A unit on decimals constructed to precede a study of common fractions was taught to four classes of sixth graders, with two of the classes using calculators while the other two classes did not. Both groups showed a significant gain in mathematics achievement of certain concepts and skills in decimals. It was not reported if any differences in achievement of attitude were found when the calculator group was compared to the non-calculator group. The non-calculator group showed a significant negative change in attitude toward mathematics.


For a seven-month period, fifth and sixth graders in one school had calculators available in their classrooms for checking their work, while at a second school no calculators were permitted. A standardized mathematics achievement test
and an attitude questionnaire were given as pre- and posttests. On the computation subtest there were no significant differences in the gain scores between the two schools. On both the mathematics concepts and the problem-solving subtests, fifth graders in the calculator group scored significantly higher than fifth graders in the non-calculator group.


Investigator-constructed questionnaires were distributed to fourth-, fifth-, and sixth-grade teachers and to elementary school principals to determine their attitudes toward the use of calculators in the classroom. Results showed that just over half of the 183 teachers responding did not think that the use of a calculator would help them realize their teaching objectives for mathematics. Almost half of the teachers felt that the calculator could be introduced between grades 4 and 6, while 44% indicated a preference for after grade 6. Teachers and principals were not unlike in their attitudes toward the use of the calculator in the classroom and were consistent in identifying similar advantages and disadvantages of using calculators.


Three business arithmetic classes of ninth and tenth graders used calculators while three other business arithmetic classes did not. All classes were taught the same material. No significant differences in attitudes toward business mathematics or in understanding of concepts were found between the calculator and the non-calculator group. However, the calculator group scored significantly higher on a test of arithmetic skills than did the non-calculator group.


This is a report on the study by Barrett and Keefe, involving two sixth-grade classes. A posttest at the end of the year indicated that the students using calculators scored higher on tests of concepts and computation than a non-calculator group, but not as high on problem-solving tests.


A 4-week unit on powers, roots, and radicals was studied by one group of ninth-grade algebra students who had traditional instruction with no calculators, a second group who had traditional instruction but could use calculators during class, and a third group who had special calculator instruction plus access to calculators during class. No significant differences were found when groups were compared on achievement or attitude.

Three classes of students at the seventh-grade level used calculators during a 4-week unit on finding equivalent forms for fractions, decimals, and percents, while three other classes did not use calculators. On an immediate posttest, a significant difference was identified between groups on items involving conversion from a simplified fraction to a decimal. No differences were found between groups when a retention test was given.


One hundred thirteen sixth-grade students used calculators for about an hour daily for 9 weeks, while another group of 58 sixth-graders used paper and pencil only. Results showed that students using calculators made significant gains in total achievement, computation, and concept scores; no differences were found in attitude or self-concept. Girls in the experimental group scored significantly higher on concepts than did boys.


Analysis of at least eight tests taken by a total of 125 secondary students in grades 7 through 12 showed no differences in performance between those using and not using calculators during the test on test scores, concept and computation errors, attitudes, time, and rank.


During this four-week study, 15 blind and partially sighted subjects ages 15 to 21 used and evaluated the instructional manuals for three calculators: the TSI Speech Plus, the Master Specialty Audio Response, and the APB Braille Calculator. Data indicated that the students were able to use the cassette manuals to learn to use each calculator. Several suggested improvements for future development of materials are given.

Miller, Donald Peter. Effectiveness of Using Minicalculators as an Instructional Aid in Developing the Concept and Skill of Long Division at the Fifth Grade Level. (The Florida State University; 1976.) Dissertation Abstracts International 37A: 6327; April 1977.

Two fifth-grade classes were separated into a low group and a high group on the basis of a prerequisite skills test of multiplication, subtraction, and division by one-digit divisors. One class was chosen to use calculators during a 12-day unit on division while the other class was permitted the use of multiplication tables. The low groups were taught by an elementary education major. The low calculator group scored significantly higher than the low control group on posttests of prerequisite skills and division.

One group of students was allowed to select activities from seven resource areas (one of which involved calculators), while the comparison group was taught under a no-choice option. A total of 207 seventh-graders participated. Results showed no significant differences in mathematics attitude or achievement between the two groups. Overall, there was a significant decline in students' attitudes toward mathematics from the end of grade 6 through the end of grade 7 for both groups of students.


A total of 196 summer school students in grades 4 through 7 were placed in one of four different curricular programs: the regular mathematics program, the regular mathematics program plus calculators, a commercial calculator-involved curriculum, or a diagnosis-remediation calculator program. Findings showed that gains in basic computational skills and attitudes of students toward mathematics were significantly improved when students used hand calculators.

Quinn, Donald Ray. The Effect of the Usage of a Programmable Calculator upon Achievement and Attitude of Eighth and Ninth Grade Algebra Students. (Saint Louis University, 1975.) Dissertation Abstracts International 36A: 4234-4235; January 1975.

The programmable calculator was used in eighth- and ninth-grade algebra classes for evaluating algebraic expressions and for solving linear, quadratic, and systems of equations. Findings showed no significant differences in achievement when performance of students in the calculator classes was compared to performance of those in non-calculator algebra classes. However, students in the calculator classes showed less "anxiety toward mathematics," and had better "self-concept in mathematics" than students in non-calculator classes.


The effect of the availability and use of a calculator on seventh-graders' mathematics achievement was investigated. Preliminary findings on parental attitude toward the use of calculators and on student achievement are discussed.


Four K-2 studies and one for grades 3-4 considered which mathematical topics could be taught most effectively with a hand-held calculator, which new mathematical topics could be successfully introduced when using such calculators, and what implications there might be for problem solving.

Students in the calculator group scored significantly higher on calculator examples, while no differences were found on non-calculator examples between calculator and non-calculator groups.


Groups using calculators gained significantly more whole number computational ability than control groups not using the calculator. Sex of student and calculator usage interaction was not significant, nor was the interaction between ethnic/economic background and gain in computational ability.


Tenth, eleventh, and twelfth graders in consumer and business mathematics classes were randomly assigned to receive computer-augmented instruction or a low-cost alternative using tables and calculators to complete inquiry exercises. The instructional unit covered nine days. Results showed that the calculator group did significantly more inquiry beyond the minimum required than did the computer group.


For an eight-week study, 40 fifth graders and 44 sixth graders were randomly assigned to either a calculator group or a non-calculator group. Both groups worked with computation sheets prepared by the experimenter. The calculator group scored significantly higher than the non-calculator group on the reasoning test in grade 5 and on the computation test and total arithmetic test in grade 6.


As they studied a unit on decimal operations and on estimation techniques, four fifth- and sixth-grade classes used calculators while four other classes did not. No significant differences in estimation skills were found.


Four ninth-grade general mathematics classes used calculators as they studied decimals and percents for 8 weeks in a specially-designed curriculum, while four other classes received traditional instruction with no calculators.
Results showed that the calculator group scored significantly higher than the non-calculator group on an achievement test, but no differences between groups were found with respect to attitude or retention of mathematical skills.


The group of ninth-grade general mathematics students using an investigator-developed unit plus calculators for 15 weeks scored significantly higher on a standardized computation test than the group using only the developed unit, but there were no significant differences in attitudes.


Explorations involving the use of calculators in connection with mathematics instruction were conducted with two fifth-grade classes, two second-grade classes, and three third-grade classes. The data suggested that pupils encountered no consequential problems with the mechanics of using simple four-function, algebraic-logic calculators in routine contexts, and that pupils elected not to use calculators in situations where their use is unnecessary or of no particular advantage. While elementary-school mathematics programs usually emphasize binary operations, project explorations have moved increasingly toward content interpretations in terms of unary operations.


Limited systematic instruction was provided for two third-grade classes on a calculator-assisted approach to solving selected types of simple open addition and subtraction sentences involving three-place whole-number addends, moving from a guess-and-test procedure to more direct and efficient sentential transformations. Pupils exhibited a relatively high level of computational accuracy in their use of calculators (94%) but substantially lower levels of proficiency in providing mathematically correct solutions for "taught" and related "untaught" open-sentence types (ranging from 66% to 23%). Serious questions are raised regarding the appropriateness for young children of certain instructional approaches to the solution of simple open addition and subtraction sentences.


The 30-day study examined the effect of calculator use with first graders upon achievement, attitudes, and interests related to mathematics. Each group completed daily worksheets, with one group checking their results on a calculator and the other group relying upon their teachers to check results. Findings indicated
that students using calculators were aided in non-timed computation and in solution to verbal problems but they displayed smaller gains in mathematics conceptualization. No differences were found for the variables of timed computations, total achievement gain, and attitude gain.


One hundred seventy ninth graders and 198 college freshmen were classified as having high, middle, or low ability in solving proportions. Half the students in each ability group were given calculators to use while working on a programmed unit in linear interpolation, while the rest of the students could only use paper and pencil for their computations. Results showed no significant differences between performances of students using calculators compared to those not using calculators, nor was there any significant interaction of use of calculators with ability to solve proportions. The hypothesis that students could understand a proportional train of thought better if the barrier of computation were removed was not borne out.


A survey of teachers, mathematicians, and laymen is reported, with seven questions and percentage of responses noted.
References on Desk Calculators, K-12

Research


Eighteen students (ages 12 to 15) used four calculators for six months to check mathematics problems. Comparisons of pre- and posttest data indicated significant increases in student interest and positive attitudes toward mathematics, while disruptive behaviors decreased.


Eighty-three middle school remedial mathematics students used calculators as they worked through lessons in the Computational Skills Development Kit on an individualized basis for 4 weeks, while 90 students did all the required calculation with paper and pencil. Results on a standardized arithmetic skills posttest showed that the non-calculator group scored significantly higher ($p < .001$) than the calculator group. No significant differences between groups were found when compared by grade level.


Fourth-, fifth-, and sixth-grade classes used Monroe Educator calculators: these calculators perform the basic operations in much the same way they are done with paper and pencil. Although complete results were unavailable at the time of writing, several observations were made. The calculators could be operated by the students; when used as a regular classroom tool, they tended to motivate and reinforce understanding and achievement in basic skills. Children seemed to enjoy using the calculators, and to exhibit better work habits. Place-value concepts were reinforced.


The effect of practice with a calculating machine on the pupil's problem-solving techniques and computational skills was studied. Thirteen pupils in the second half of sixth grade completed the year's work in the six-week treatment period. Gain scores from four tests were compared, with improvement found in each case. Pupils were able to analyze more problems in the time available than they usually did.

A program for low achievers in grades 7-9 from disadvantaged areas which emphasized real-world applications and use of flow charts, calculators, and other materials, resulted in significant achievement gain. Sixty per cent of the students who had participated in the program continued to take mathematics courses, compared with 40 per cent in a control group.


Only 13 per cent of the schools reported (in 1967-68) having calculators in the mathematics department, with 2 percent of these having computer features. Five per cent of the schools had computer facilities which were used by mathematics classes.


See also: Cech, Joseph P. *The Effect of the Use of Desk Calculators on Attitude and Achievement with Low-Achieving Ninth Graders*. Mathematics Teacher 65: 183-186; February 1972.

The two main reasons for using calculators with low achievers in mathematics classes are motivation and achievement. This study of calculator effectiveness involved two teachers each teaching a calculator section and a regular section of general mathematics for seven weeks. Students in the experimental group were encouraged, but not forced, to check answers with the calculators. All classes were given pre- and posttests of attitude and achievement. Results did not support the hypothesis that students using calculators would show positive gains in attitude toward mathematics or increased paper-and-pencil computational skill. Students could compute better with the calculator than without it, however.


From grades 6-8 in a single school, 35 pairs of students were matched according to IQ and grade placement in arithmetic. One from each pair was then selected to use the calculator. Analysis of data from the nine-week study indicated that in computation, reasoning, and concepts, the calculator had no effect except in the area of reasoning in grade 7.


An experimental and a control class were administered pre- and posttests to check the effects of calculator use on the achievement, attitude, and academic motivation of low achievers. The use of printing calculators did not produce a statistically significant change in mathematics achievement. More favorable attitudes and weaker academic motivation were recorded for both groups at the end of the experiment.
Fehr, Howard F.; McMeen, George; and Sobel, Max. Using Hand-Operated Computing Machines in Learning Arithmetic. *Arithmetic Teacher* 3: 145-150; October 1956.

A controlled experiment on learning multiplication by using a two-digit multiplier was conducted for a two-week period. No significant difference was found in the performance of students in experimental and control groups. However, the experimenters felt that longer use of the devices might have produced an effect, and therefore conducted a half-year experiment using the Monroe Educator model hand-operated calculator. Students using this machine made significant gains in both computation and reasoning. Although their gains were greater than those of a control group, these differences were not statistically significant. Both students and teachers using calculators had a very positive attitude toward calculator use in the mathematics classroom.


The group using the traditional textbook and calculators for a full year gained significantly more than the group using the traditional textbook alone or the modern textbook with calculators, but only on arithmetic fundamentals achievement.


Use of units in which fractional numbers were converted to decimals and examples then solved on a calculator was found to be a "viable alternative" to use of conventional textbooks (including fractions) with or without a calculator, for low-ability or low-achieving students.


The effectiveness of an electronic calculator, programmed as an immediate feedback device, was compared with the effectiveness of pencil-and-paper exercises without immediate feedback for the learning of the 100 basic multiplication combinations. Twelve students in each of seven fifth-grade classes were identified as low achievers and randomly assigned to treatment. Significant differences favored the electronic calculator practice group over the pencil-and-paper practice group on both acquisition and short-term retention, but not on long-term retention (one month or three-and-one-half months retention periods).
Johnson, Randall Erland. 'The Effect of Activity Oriented Lessons on the Achievement and Attitudes of Seventh Grade Students in Mathematics.' (University of Minnesota, 1979.) Dissertation Abstracts International 32A: 305; July 1971.

Activity-oriented instruction, including one treatment in which calculators were used, did not appear to be more effective than instruction with little or no emphasis on activities, for units in number theory, geometry and measurement, and rational numbers.


The group using calculators achieved significantly more on a standardized test than did a group not using them.


Two hundred one low achievers were randomly scheduled into one of five control sections or one of five experimental sections. All groups followed the same lesson sequence, with control groups using only paper and pencil for all calculations and experimental sections using electronic calculators. Significant differences were found on both attitude and achievement tests from pre- to post-treatment for both groups, but no significant differences in posttest mean scores were found between groups.


This study was designed to test the use of calculators with two groups of ninth graders and one group of fifth graders. The findings were equivocal, concerning the effect of calculators on students' performance, self-confidence, and attitudes toward mathematics. Teacher enthusiasm for calculator use was unrelated to student performance. Teacher enthusiasm was highest in classes of low-average IQ. While some teachers felt calculators interfered with their daily operations, others felt that the productivity of students increased, especially among those previously incapable of producing. Classroom behavior problems were eased.


The calculator, when used as a teaching aid with slow learners in mathematics in the seventh and eighth grades, did not significantly improve attitude, increase mathematical achievement, or increase non-calculator computational skill, mastery of mathematical concepts, or ability to solve mathematical problems. However, the students did at least as well in all areas as those students not using calculators.

A study with students in grades 4 through 9 is reported; the groups using calculators achieved higher than groups not using calculators.


The group having calculator instruction had significantly higher scores than a group not using calculators, on computation but not other tests or an attitude measure.

Stocks, Sister Tina Marie. The Development of an Instructional System Which Incorporates the Use of an Electric Desk Calculator as an Aid to Teaching the Concept of Long Division to Educable Mentally Retarded Adolescents. (Columbia University, 1972,) *Dissertation Abstracts International* 33A: 1049-1050; September 1972.

All students demonstrated an improvement in scores between pre- and posttest; however, no tests of significance were made. A positive change in attitude was also found.

Other References


Various types of computing devices are described, and their usefulness in the classroom discussed. Scant attention is devoted to calculators in general, and none to the hand-held calculator as a distinct instructional aid. The authors state that electronic calculators are "far more powerful problem solving tools than conventional machines."


A summary of key features to compare when deciding on either a programmable calculator or a minicomputer are listed. Consideration is given to the kinds of work to be done, flexibility needed, experience of users, operating features, and price.


Advantages of printing calculators are discussed. Consumer concerns (such as power source, types of display, noise factors, special keys, memory) are identified. Four printing calculator models are described in detail.

A game in which teams of students compete with each other using an adding machine can be used to practice and enhance basic skills.


A kit was designed to introduce gifted students to basic computer activities. The kit included an abacus, slide rule, desk calculator, punch-card equipment, and an electronic computer, as well as books. A series of objectives and activities is outlined.


Many problems cannot be done by the pupil alone, but can be handled by the pupil-plus-computer combination. Two such problems involve exponents and the Pythagorean theorem. The facility to do many computations enables students to get a better feel for rational and irrational numbers and for the definition of a logarithm.


This series of four text-workbooks was designed for tenth-grade mathematics students who have exhibited lack of problem-solving skills. Electric desk calculators are to be used with the text. In the first five chapters of the series, students learn how to use the machine while reviewing basic operations with whole-numbers, decimals, fractions, and percents. The rest of the chapters present word problems in simple consumer mathematics, business activities, installment buying, banking, stocks and bonds, insurance, taxes, and utilities. A chapter on the use of formulas is included.

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Introduction to
Pros and Cons of Using Hand-Held Calculators

Should the use of calculators be permitted in elementary and secondary schools? Many of us concerned with education are facing this question and its possible implications for curricula in the elementary and secondary school. This bulletin has been prepared to provide summary information regarding the pros and cons of using hand-held calculators in schools. In the first section, frequently cited reasons for using calculators in schools are listed. This list is followed by another which gives frequently cited reasons for not using calculators in schools. Each of these lists was taken from Electronic Hand Calculators: The Implications for Pre-College Education, a report prepared for National Science Foundation by Suydam (1976). A check of more recently published literature indicated that these reasons, derived from a survey sent in the spring of 1975, are still the ones most frequently cited. The third and final section of this bulletin provides an alphabetized list of references which contain information relevant to the pros-and-cons issue of the calculator controversy. It is hoped that this bulletin will assist you in forming your own position on this important issue.

Reasons for Using Calculators

(1) **They aid in computation.** They are practical, convenient, and efficient. They remove drudgery and save time on tedious calculation. They are less frustrating, especially for low achievers. They encourage speed and accuracy.

(2) **They facilitate understanding and concept development.**

(3) **They lessen the need for memorization, especially as they reinforce basic facts and concepts with immediate feedback.**

(4) **They help in problem solving.** Problems can be more realistic and the scope of problem solving can be enlarged.

(5) **They motivate.** They encourage curiosity, positive attitudes, and independence.

(6) **They aid in exploring, understanding, and learning algorithmic processes.**

(7) **They encourage discovery, exploration, and creativity.**

(8) **They exist.** They are here to stay in the "real world," so we cannot ignore them.

The last reason — the pragmatic fact that they exist and that they are appearing in the hands of increasing numbers of students — is perhaps the most compelling. How they can be used to facilitate each of the other seven beliefs is therefore a question that must be attacked.
Reasons for Not Using Calculators

(1) They could be used as substitutes for developing computational 
skills: students may not be motivated to master basic facts and 
algorithms.

(2) They are not available to all students. Because they cannot 
afford a calculator, some students are at a disadvantage.

(3) They may give a false impression of what mathematics is. Mathe-
ematics may be equated to computation, performed without thinking. 
Emphasis is on the product rather than on the process; structure 
is deemphasized. Mental laziness and too much dependence are 
encouraged; lack of understanding is promoted. Some students and 
teachers will misuse them.

(4) They are faddish. There is little planning or research.

(5) They lead to maintenance and security problems.

[Note: The security problem appears to be almost non-existent, according 
to reports from those actually working with calculators.]

The first concern—that students will not learn basic mathematical skills—is 
one expressed most frequently by parents and by other members of the lay public, 
as reflected (and created) by newspaper articles. But it builds a strawman, for 
few educators believe that children should use calculators in place of learning 
basic mathematical skills. Rather, there is a strong belief that calculators can 
help children to develop and learn more mathematical skills and ideas than is 
possible without the use of calculators. Much serious attention must be given 
by teachers and others to proving that this belief can be implemented and become 
fact.
Pros and Cons of Using Hand-Held Calculators Bulletin No. 11 August 1977


Denman, Theresa. Calculators in Class. *Instructor* 83: 56-57; February 1974.


Quinn, Donald R. Yes or No? Calculators in the Classroom. NASSP Bulletin 60: 77-80; January 1976.


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Selecting a Calculator

See also: Calculators—They Just Keep Multiplying. Administrative Management 33: 68; August 1972.

Descriptions of calculator models introduced up to early 1972 are provided.


Features to look for, how much to spend, and consideration of one's particular needs are discussed. Both desk and hand-held calculators are considered.


Features to look for and what models have those features are indicated in this guide to buying a hand-held calculator.


A checklist of features to consider when choosing a hand-held scientific calculator are listed. Features are described, with a summary of the characteristics of 17 engineering calculators.

See also: Selecting a Minicalculator. Arithmetic Teacher 23: 547-550; November 1976; Mathematics Teacher 70: 360-363; April 1977.

Checklists are provided which are designed to help educators select an appropriate calculator for particular situations.


The positive contributions of the calculator to basic education are explored. The introduction discusses questions educators ask about the use of the calculator. The first section briefly describes uses of the calculator in the classroom (as a time saver, for reinforcement, for motivation, as an aid to conceptualizing, and for applications), discusses research on calculators, describes NCTM involvement, and presents the NACOME recommendations concerning
calculators. The second section covers the implications of the use of the calculator in terms of curriculum, teacher in-service education, classroom management, instruction, and testing and evaluation. The third section gives guidelines for selecting and using calculators. The final section includes classroom activities keyed to the various functions of the calculator.


Advantages of printing calculators are discussed. Consumer concerns (such as power source, types of display, noise factors, special keys, memory) are identified. Four printing calculator models are described in detail.


Features of calculators such as precision and constant keys are briefly discussed.


Features to consider when buying calculators for business or business education purposes include output type, decimal control, automatic rounding, portability, and programmability. It is suggested that buyers know terminology pertaining to the machine and test machines with the types of problems to be used in class.


Features and functions for various models are tabulated.


Three groups of programmable calculators are identified: key programmable (volatile memory), card programmable, and key programmable (non-volatile memory). (Volatile memories are erased and lose the program when the power is shut off.) Eleven calculators are compared on the following features: program steps, branching, addressable memories, logic, stack registers (reverse Polish notation), parenthesis levels, pending operations (algebraic operating system), and price. Two programs are given.


Some common-sense things to look for when buying an electronic calculator are given. Some algorithms are also presented, for use with the less expensive calculators which do not have all capabilities built in.


Who will use the calculator and for what purpose, how much mathematics the user has and/or will study, and how much the buyer wants to pay should be considered when purchasing a calculator.

Features to consider when selecting a calculator are noted.

Jamele, P.R. *How to Select and Use a Calculator--or Getting the Most from Your 4-Function Calculator.* Los Angeles: Crescent Publication, 1975.

Special features of different calculators are explained. Instructions for solving some special problems with a four-function calculator are given; algebraic equations, exponents, higher roots, geometric problems, annuities, linear interpolation, series evaluation, and use of the calculator when answers exceed the eight-digit capacity are covered.


This is the first part of a two-part article surveying calculators of interest to the chemist. The range of electronic calculators currently available and their capabilities are surveyed.


This is the second part of a two-part article surveying calculators of interest to the chemist. Desk top calculators now on the market are described, including manufacturers, prices, and functions performed. A table of specifications for programmable desk calculators is provided.

LaBar, Martin; Wilcox, Floyd; and Rickman, Claude M. Programmable Calculators as Teaching Aids and Alternatives to Computers. *School Science and Mathematics* 74: 647-650; December 1974.

The authors provide a list of calculators which have a capacity for handling programs, and a list of programs for such calculators which are available at cost. They argue that the use of these materials at many levels of mathematics instruction enhances both motivation and understanding.

Mims, Forrest M. Here are the New Programmable Calculators! *Popular Electronics* 9: 29-35; May 1976.

Reverse Polish notation and algebraic methods of entry, and branching and conditional-comparison capabilities of programmable calculators are discussed. A shopper's guide to four elementary programmable models is provided. Sample programs for determining volume of a cylinder, for incrementing a number, for squaring consecutive integers with a display of each result, and for a Hi-Lo game area included.


Computer-like functions which can now be done by electronic desk calculators are described, and areas where calculators are deficient are listed.
Keyboard, number display, batteries, logic systems, and warranties are briefly discussed.

Features, prices, and ratings are included in these reports.

Basic characteristics, prices, and ratings for calculators are listed.

Features of various types of calculators are listed.

Some features to look for when buying a hand-held calculator include floating decimal, negative function, clear key, and power source.
Mathematics Teacher. New Products, Programs, Publications.

In this monthly feature of the NCTM journal, calculators and materials for use with calculators in the classroom are frequently reviewed.


General types of advanced calculators, calculator logic, special features and functions, programmable calculators, and calculator construction are topics discussed in this booklet.

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References on Calculators, Post-Secondary Level


For an eight-week period, each of four intact introductory statistics classes (n = 172) was randomly assigned to one of four treatment conditions corresponding to two instructional modes (emphasizes on situational problem solving vs. no such emphasis) and two computational methods (access to electronic calculating instruments vs. no such access). Significant main effects of computational achievement on statistics content achievement and of instructional mode on both attitude toward mathematics and view of statistics were found.


This study focused on faculty and student attitudes toward the use of calculators in college accounting and business mathematics courses. Two different surveys were used; one was administered to 35 full-time and part-time faculty in the accounting and business mathematics areas at one college, while a second survey was administered to a random sample of 244 students. Responses from both the faculty and students indicated fairly consistent attitudes. A seemingly larger portion of students felt that calculators should be allowed unconditionally, while the faculty appeared to have some reservations on the use of calculators, especially in classes of business mathematics.


The author designed an instructional package including slides and tape cassettes for individual use by students learning to use a hand calculator to perform computations. Students (n = 17) using the package were given pretests and posttests of ability and attitude. On the three sets of cognitive objectives, mastery was achieved by 96 percent, 76 percent, and 76 percent of the students, respectively. All students reported favorable attitudes to the unit. The author discusses planned revisions of the program, and relationships among the variables.
Two uses for the programmable calculator in the laboratory are suggested: as a means of determining whether a student's raw data from a laboratory experiment fall within acceptable tolerance limits, and as a means of checking the reliability of unknowns and grading on quantitative experiments.


The problems that arise in test situations when some, but not all, students use calculators are discussed, and some solutions to these problems are suggested.


Three general types of applied problems in structural geology are discussed and trigonometric solutions are indicated. In addition, a five-example problem set is included.


A solution is given to the problem, "Given a calculator with no memory, but with a squaring key, find the smallest power of n that cannot be computed without entering the value of n at least k times in the keyboard."


The use of a Hewlett-Packard 9810A programmable desk calculator with plotter for drawing ball-and-line stereopairs as well as three-dimensional structural formulas which are useful for teaching stereochemical principles and molecular structure is described.


Among new technological developments affecting business education courses is the calculator. A course in business applications of the calculator at one university is noted.


Using calculators in college courses is discussed.

The usefulness of card-programmable hand-held calculators in the management curricula of the Naval Postgraduate School and in the fleet were investigated, using manufacturer-provided information, NPS classroom experimentation, "hands-on" programming, interviews, and other literature. All aspects of calculator functions, programming, and programmability were surveyed with particular emphases on educational and practical applications. It was concluded that calculators provide significant advantages in teaching or learning mathematical concepts and that they are potentially important management and tactical support tools navy-wide. In addition, "thinking process transmutation", discovered in this study, is concluded to be an inevitable and important by-product of calculator programming which significantly improves the user's overall analytic capacity.


A college-level remedial mathematics course is described in which students were required to use calculators as a part of their course work.


The possible subtle influence of calculators in the routine evaluation of the arithmetical proficiency of students is noted. Results of a study with a class of 71 first-year professional pharmacy students taking a two-credit pharmaceutical calculations course are interpreted.


Questions about the use of calculators in the engineering classroom are discussed, a freshman-year course in engineering computation emphasizing calculators is described, and features needed for calculators used in engineering are listed.


Directions are given for gaining access to the clock function in the Hewlett-Packard 45 calculator. A method of using the function for timing and storing elapsed times of up to nine separate events is described. The accuracy of the HP-45 as a timer is discussed.


Reasons for and against the use of calculators in an accounting class are given, and a compromise solution is suggested.
No significant differences were found when achievement and attitude of college basic mathematics students in classes using the calculator were compared to those of students in non-calculator basic mathematics classes. Among students using calculators, those having higher aptitudes in mathematics showed significantly higher achievement and attitude scores than students having lower aptitudes.

An experiment in which a minicomputer was used as an instructional aid in a calculus classroom and as a laboratory device for students is described.

In a pilot study (October 1976), 48 college students were randomly assigned to manual (no calculator), basic, or advanced calculation conditions to work statistical problems. Calculator groups took less time and made fewer errors than the manual group. In a second study, 60 college students worked easy and hard statistical problems under one of the three calculation conditions. Results indicated that calculator usage reduced working time and errors, especially on hard statistical problems. Subjects in the calculator groups also expressed more positive attitudes about themselves and the problem solving tasks.

A procedure is described for presenting routine practice problems on a programmable calculator with attached teletype. The program uses a random number generator to write problems, gives feedback, and assigns grades according to the procedures outlined and charted by the author.

Two programs for performing Fourier analysis and synthesis with a Hewlett-Packard (HP-25) calculator are described.

The use of a programmable calculator for student experiments, grading of laboratory reports, and assigning accuracy and precision scores is described.

Two classes of community college business mathematics students used calculators in the classroom, while two other business mathematics classes at the same school followed the same curriculum without using calculators. There were no significant differences between groups on standardized tests of arithmetic achievement, mathematical reasoning, and critical thinking abilities.


An experiment is presented in which a programmable calculator is employed as a data-generating-system for simulated laboratory experiments. The example used is a simulated conductometric titration of an aqueous solution of HCl with an aqueous solution of NaOH.


Presented is a brief description of a study done to assess the impact of the use of calculators during examinations, showing that the use of the calculators did play a major role in chemistry grade determination.


Nine principles to help the consumer and user of sophisticated pocket calculators are identified.


Uses of calculators and computers in the college classroom are described, a philosophy about their use is discussed, and several problems (especially amenable to use of a computer with plotting facility) are presented.


This statistics text includes a review of the four basic mathematics operations, square roots, and algebraic equations, and points out how the calculator can be used to assist the statistics student.


One hundred seventy ninth graders and 198 college freshmen were classified as having high, middle, or low ability in solving proportions. Half the students in
each ability group were given calculators to use while working on a programmer unit in linear interpolation, while the rest of the students could only use paper and pencil for their computations. No significant differences were found between performances of students using calculators compared to those not using calculators, nor was there any significant interaction of use of calculators with ability to solve proportions. The hypothesis that students could understand a proportional train of thought better if the barrier of computation were removed was not borne out.


Included among papers on the use of computers and electronic equipment in instruction is one paper discussing the use of programmable, hand-held calculators for calculus instruction.
Instruction with Hand-Held Calculators, K-12

Update: December 1977

Articles


Shortcuts for converting base ten integers to or from other lower-base integers are discussed in terms of the key strokes used on a four-function calculator. A student worksheet with accompanying answer sheet is included.


This is the second article in a series about simple, four-function calculators and elementary concepts in computer programming. This article covers multiplication, powers, and a multiplication game. Student exercises and answers are provided.


Predictions involving the production of calculators in 1976 are verified, and new predictions for 1977 are made.


Features, listed in order of preference, are discussed: natural-order arithmetic, floating point, underflow, constant key to operate on all four operations, eight-digit display, fingertip-size keys, rechargeable batteries with alternative plug-in operation, and clear-entry key.


Four examples are given to show how children can be involved in intellectual experiences in mathematics. The fourth example describes an activity which uses number properties to extend the range of a calculator beyond its eight-digit display capabilities.


Ten problem-solving activities are presented which involve large numbers and real data, written for use with a calculator.

School systems are urged to formulate a system-wide policy that will govern the use of calculators in the classroom. Ten questions to consider when developing such a policy are given.


The words for this crossword puzzle are found by working computational problems on a calculator and reading the inverted display.


Rules to play the game "Frogs" on an SR-52 programmable calculator are given. Flowcharts, a program listing, and a sample game are included.


A set of steps for introducing calculators to elementary school children is suggested.


Advantages of allowing children to use calculators in school are discussed briefly. Features to look for in selecting a calculator are described.


Procedures for evaluating three algebraic expressions using a four-function calculator are given.


Four games which can be played by both children and adults are given, for use with a four-function calculator.


Simpson's rule for calculating volume is discussed and applied to finding the volume of several solids, with four student worksheets provided. The calculator is mentioned as a computational aid.

Eighty college-preparatory CHEM Study chemistry students were randomly assigned to either a calculator group or a non-calculator group for this one-semester study. The calculator group used calculators during class sessions to perform calculations for homework, laboratory exercises, and chapter tests and quizzes. No significant differences in chemistry achievement were found between the two groups on the posttest measures.


Over a period of 19 weeks, a group of 51 twelfth-grade consumer mathematics students used calculators to perform all computations while a group of 43 students used traditional paper-and-pencil methods. No significant differences in mathematical problem-solving achievement or in attitude toward mathematics were found between the two groups. Strong positive attitudes toward the use of calculators in the classroom were found.


The electronic calculator industry was studied with respect to international trade and transfer of technology.


This is the report by two "outside evaluators" of the kindergarten and grade 1 program developed by Texas Instruments.


A model was developed to estimate the probabilities of purchase associated with various products within a category. The technique was piloted using the product category of hand-held calculators.
Books


This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy. December 1977: Additions to Bulletins issued in August 1977.
Instruction with Hand-Held Calculators, K-12


The use of 15 calculators to promote problem-solving skills in an eighth-grade class is described. Suggestions and recommendations are included.


The use of calculators in an eighth-grade class is described, with an exercise on volume cited.


This is the fourth installment of a "teach yourself" style workbook; it introduces mixed operations.


The change sign key is introduced in this series of workpages.


A calculator version of the game Krypto is presented.


Operating characteristics and features of programmable calculators are compared.


The effects of calculators on education, including discussion of potential strengths and weaknesses, are described. Specific suggestions for needed curriculum development are presented.


This discussion on the role of computers in mathematics classes points out
that "many of the teaching and learning activities that were enhanced by computers in schools during the 1970s can now be carried out with relatively inexpensive calculators."


A two-hour module used in a Calculator Usage in Elementary Schools class for teachers is presented; it is appropriate for use in grades 7 and 8. Using worksheets, students convert fractions to decimals, do the prime factorization of the denominator, and find patterns formed by the denominator of the fractions that are terminating decimals (using any four-function calculator).


This year-long course for Mathematics 9-10 is designed to assist students who have had difficulty in mathematics as well as those who are unmotivated. Computational skills are applied in practical situations. Number, operation, space, symbolism, relation, proof, and approximation concepts are included as well as skills in computation with whole numbers, fractions, and decimals.


Calculators may be used in solving pattern-recognition problems.


A worksheet specifically for use with calculators is included in the presentation of the problem.


Two calculators using long-life batteries (2000 hours) were rated.


Two solar-powered calculators were rated.


Ten calculators are rated, included two printing calculators. Display characteristics and types of calculators are also briefly described.


Five calculators are rated—1 four-function, 1 specialized, and 3 scientific calculators.

This project, which can be carried out in any junior or senior mathematics class with at least one calculator with an exponential function, is described in terms of the elementary function aspect, the statistical aspect, data, fitting the curve, and a summary.


A reason for the popularity of calculator games is discussed: "a calculator game, then, is a simple but challenging vehicle by which a quantitative mind can play directly with the objects of its affection--numbers--regardless of the amount of mathematical skill."


Using calculators to check work is discussed.


Examples of notational problems are cited, to point out the need for "a complete and consistent set of explicit operators."


This report lists arguments and attitudes resisting and favoring the use of calculators in the classroom, curriculum comments from articles, and the author's reactions.


A summary of uses of calculators and sources of information and materials are given.


Students are to use calculators to demonstrate their knowledge of the correct order of operations for a given mathematical expression in doing a worksheet and a crossword puzzle.


Some historical approaches to finding the area of a circle are presented, with calculators suggested for use in solving some of the problems.
The uses of programmable calculators in secondary school classes are discussed, including grading, laboratory, exercises, computing T-scores, and a quantitative approach to chemical equilibrium.


The method of inquiry is applied to find the area of a circle using the calculator.


An algorithm for generating on any calculator as many digits as desired in the decimal representation of the rational number N/D is given in detail.


This bimonthly column presents some problems, some hints on how to use a calculator, and some comments on particular calculators.


A programmable calculator program to solve the equations in the Tin Can problem (Clyde, 1978) is presented in this letter.


Three programs for exercises in Scott (1978) are given in this letter.


A calculator is used as a counter in activities for levels 4-8 for determining students' paces per kilometer and relating the metric system to students' own lives.


Reasons for exploring $0^0$ are discussed, and then the role of calculators in examining the problem is presented.


Characteristics of 20 printing calculators are presented and discussed.

A variation of Bingo is given, with directions for the calculator activity and extensions.


Calculators in school need not cause decline in either practical skill or mathematical understanding. With the right curriculum materials, they can help bring about major improvements in mathematics education.


This article, a portion of Chapter 1 from *Calculator Calculus*, presents a repetitive method for approximating square roots using the calculator.


The importance of functions is discussed and specific illustrations of the calculator as a function generator are presented.


The investigation of decimal equivalents (Billstein and Lott, 1978) is extended with three problems.


Activities using four-function calculators are classified by purpose: to explore number patterns, to discover relationships and develop concepts, to practice mental estimation, to reinforce inverse relationships, to solve application problems, to develop the "guess-then-check" technique, and for individual exploration and enrichment.


The calculator number system, underflow and overflow, calculator arithmetic, integer arithmetic, working with fractions, use of memory, and other memory-like features are discussed, with 11 exercises included.


The calculator makes it feasible for students to find the sum of all n-digit whole numbers that may be formed from an n-digit whole number.
Four worksheets are given in which the calculator can be used to provide practice on estimation.


This preliminary set of materials, field-tested during 1977-78, incorporates the use of programmable calculators in the standard mathematics curriculum in grades 11 and 12. The first chapter makes students familiar with different kinds of calculators and teaches algebraic, AOS, RPN, and arithmetic calculator logics, as well as simple programming. The second chapter (thus far available) is on exponents and logarithms.


A unit is presented which specifies how the calculator can be used for two problem-solving techniques, trial-and-correction and make a table.


An opportunity for students to discover numerical patterns is provided by three worksheets.


Illustrations are given of how doing certain algebraic manipulations prior to doing calculations can eliminate unnecessary storage of data, reduce the number of steps required to obtain an answer, and avoid data overflow.


The "repeat" capability of most four-function calculators is used in an algorithm to find any integral root of any number.


Determining repeating decimals is examined in this article, with 17ths, 19ths, 23rds, 31sts, and 41sts suggested for exploration.


Three experiments used with seventh graders are presented: finding quotient and remainder, listing 10 non-zero multiples of a given number, and stating which of the numbers 2 to 20 are factors of a given number.

That 'a crutch is a bad thing is questioned; use of the calculator as a useful crutch is illustrated.


A method is given for approximating the nth root of any positive number with a four-function calculator with square root key and repeat multiplication capability.


A calculator program for finding a Klingon spaceship is given.


Answers for Wavrik (January 1978) are given with comments.


Advantages of the calculator over the computer for certain uses are discussed.


A rationale is given for the consideration of unary operations which can be facilitated by using calculators. Suggestions and illustrations of some ways to accomplish this at the pre-algebra level are given.


This module is excerpted from material prepared for ongoing calculator explorations with a class of accelerated seventh-grade students. Greater attention is given to unary or monadic operations. Relationships and properties to be identified, suggestions for the teacher, calculator algorithms, and record sheets are provided.


This is the first of a two-part article; it considers the pros and cons of the use of calculators in the classroom, attempting to differentiate between the facts and assumptions about the problem. Important questions for research are identified.
This is the second of a two-part article on the controversy over the use of calculators in the classroom. The author points out the possible effective uses for the calculator and suggests guidelines for those who wish to incorporate it into their classroom approaches.


The 1974 NCTM Board of Directors' position statement on calculators is reprinted.
Books on Calculator Applications


Ways to use a calculator as an integral part of mathematics instruction in existing curricula are provided. Activities are proposed that will help children think about mathematics rather than merely push buttons.


This text, developed to help teachers learn a diagnostic/prescriptive approach, includes cross-references to pages in Calculator Power (Bitter, 1977.)


Puzzles, pattern searches, problems, and a variety of other activities are included in this book.


This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy.

Copies of Calculator Information Center bulletins may be made for distribution.
Research on Calculator Uses, K-12


This "informal study" investigated the effectiveness of using calculators as an immediate feedback device in learning some basic multiplication facts. One class of 26 second graders was given a pretest of 20 facts, with about five seconds allowed for each response. Seven days of classroom instruction by the regular teacher involved nine textbook pages plus activities using physical and pictorial models. Students also practiced for 8 to 10 minutes on 6 days, using calculators to check immediately their recall on each combination (but not using other materials). The set of 20 facts was given again. The mean increased from 6.37 to 12.16. The largest gain in recall occurred on those facts practiced on the calculator alone; the smallest gain was on those facts presented by the textbook alone.


In a survey in 1977 of 1,343 teachers in grades 1, 3, 5, and 7, questions on calculator use were included. The percentage of teachers who had used calculators was: 3.9% at grade 1, 8.4% at grade 3, 19.4% at grade 5, and 25.6% at grade 7. In the first grade, calculators were used most frequently for drill; the next three most frequent usages were for checking, motivation, and remediation. Use of the calculator for drill decreased with grade level. Above first grade, the most frequent use was for checking, with motivation and word problems next most frequently reported uses.


This study investigated the effects on achievement and attitudes resulting from use of a calculator-based curriculum and calculators in ninth-grade basic mathematics (in which students were at least two grade levels behind in mathematics achievement). Twelve classes from three schools were randomly assigned to either calculator (n = 83) or non-calculator groups (n = 84). For four weeks, teachers used guides prepared by the investigator to teach estimation, computation, and problem solving using the four operations with whole numbers. One half of each group was randomly selected to take the posttest with calculators available, while the other half did not have calculators. Data were analyzed by analysis of covariance. Students using...
calculators in instruction scored as well in computation and significantly better in problem solving as their peers not using calculators. Attitudes were not significantly different. Students using calculators on the posttest did significantly better in both computation and problem solving than students not using calculators.

Kasnic, Michael James. The Effect of Using Hand-held Calculators on Mathematical Problem-Solving Ability Among Sixth Grade Students. (Oklahoma State University, 1977.) Dissertation Abstracts International 38A: 5311; March 1978. (Order No. 7801276)

Sixth-grade students from four randomly selected schools in a suburban school district were assessed on problem-solving ability and placed in a low, average, or high ability group. Ten students were randomly selected from each group at each school; schools were randomly assigned to one of four treatments: using calculators to practice problem solving; using calculators to practice problems and on the posttest of problem solving; practice on problems with paper and pencil only; or a control group. Nine 50-minute sessions of practicing problem solving comprised the treatment period. The calculator groups did not complete significantly greater numbers of practice problems than the non-calculator group, nor did groups differ on the number of correct responses. No significant differences were found between low and high ability groups.


A survey of leadership personnel in Los Angeles County is reported; although a wide range of reactions was found, on the whole teachers seemed favorable to the idea of calculators in the classroom.


The results of a student attitude survey among junior high school students who had been allowed to use calculators to check mathematics computations are reported. Significant preference for using calculators in the classroom was displayed.


Approximately 700 seventh-grade students in two schools were randomly assigned either to calculator or control groups. Each of six teachers taught two experimental and two control classes, using the regular textbook. Students were "on their own" as to how and when they used calculators; they kept logs of when and for what operations calculators were used. No significant difference in computational skills was found between groups. Attitudes of students in both groups varied little. Parental attitudes, however, changed: while 50 percent opposed the use of calculators at the outset, only 33 percent were opposed at the end. At the start, 49% felt that their children would become highly dependent on the machine, while at the end this number dropped to 22%.
A national survey conducted for the National Science Foundation included questions about the extent of use of calculators in schools. For four grade ranges, the percentage of schools having calculators was: K-3, 28%; 4-6, 36%; 7-9, 49%; 10-12, 77%. Rural schools are as likely as suburban schools to have calculators, and both are significantly more likely to have them than schools in small cities or urban areas. The percentage of mathematics classes using calculators increased with grade level: K-3, 6%; 4-6, 14%; 7-9, 30%; 10-12, 48%. Most K-3 teachers indicated that calculators are not needed; in 4-6, 44% indicated that they were not needed, while 39% needed them but did not have them; for 7-9, the comparable percentages were 42% and 28%; for 10-12, 33% and 18%.


Results from a survey of 415 secondary teachers are briefly summarized. Guidelines were developed and a booklet for teachers prepared. The effect of using calculators and the curriculum supplement has been studied in five schools.


Two-hundred-fifty parents and teachers of grades K-9 replied to a 12-item questionnaire about the use of calculators. They were increasingly more accepting of calculator use as grade level (3-6, 7-8, 9-12) increased, with teachers significantly more favorable than parents at the two lower levels. Both groups were moderately negative about the use of calculators for homework and about whether skills with calculators will be essential to future success. They were accepting of the use of calculators for enrichment, motivation, and games. While moderately negative replies were given about the use of calculators to replace paper-and-pencil skills, they were very accepting of the use of calculators along with paper-and-pencil computation.

This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy.

Copies of Calculator Information Center bulletins may be made for distribution.
References on Calculators, Post-Secondary Level  

Ball, John A.  

Eisberg, Robert M.  

Phillips, R. F.  
Simple Gravitation Using a Programmable Pocket Calculator.  
Physics Education 12: 360-363; September 1977.

Described is the calculation (with a programmable calculator) of the potential of the earth's gravitational field strength and the energies of satellites in orbit around the earth.

Roberts, Dennis M. and Glynn, Shawn M.  

The purpose of this study was to examine the relative magnitude of benefit that accompanies use of calculators compared to hand computation. Forty-eight students in an introductory statistics course were randomly assigned to three treatment groups: (1) Manual—all computation by hand; (2) Basic Machine—use of a four-function calculator without memory or square root key; or (3) Advanced Machines—use of calculators with either two memories or square root. They were given a test with five routine sets of data and problems involving frequency distributions. Large differences were found between either of the machine conditions and the manual condition, but very little difference was found between the two machine conditions. The machine work was more than twice as efficient as calculation by hand.

Roberts, Dennis M. and Glynn, Shawn M.  
Numerical Problem Solution as a Function of Calculation Mode and Task Difficulty.  

This is the second in a series of studies designed to test the hypothesis that calculators facilitate problem solution by saving time and increasing accuracy. Sixty students in an introductory statistics course were involved; computational tests containing both easy and difficult statistical problems were used. Students were randomly assigned to manual, basic machine, or advanced machine conditions. The calculator groups were superior on measures of number correct, time, and efficiency scores and in attitude, with the two types of calculator groups not significantly different. Task difficulty was not significant, although there was a larger difference between performance on easy and difficult problems in the hand condition than in the machine condition.

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This study investigated the effect on computation performance of amount of prepractice and type of calculator, when students were told to work very fast versus very accurately. Sixty students in an introductory statistics course were randomly assigned to four treatments: basic machine, 2 or 5 prepractice trials; advanced machine, 2 or 5 trials. Half of each group worked under "fast" or "accurate" conditions. Amount of prepractice made no difference in number correct or in time required to work the problems. Students using the more advanced calculators needed only about half the time and had more correct answers. Time and accuracy were greater under the "accurate" condition. The advanced calculator group was significantly more efficient, as was the "fast" condition. On four of five attitude clusters, advanced calculator users were significantly more positive.


One-hundred-two students in a beginning statistics course were randomly assigned to six conditions—use of handwork or of same or advanced calculators with either easy or difficult problems. The difficulty of the problems did not have a significant impact on the number of problems answered correctly. Time varied with type of calculator and difficulty level. Efficiency improved as sophistication level of the calculator improved. The group using the most sophisticated calculator expressed more positive attitudes than other groups did.


(Note: A copy of this fifth article in the set was not available for review.)


A problem concerning the computation of hydrogen spectral line wavelengths, appropriate for computations with calculators having eight significant digits, is outlined for a general physics class.


Selected programs of astronomical interest that have been written for calculators are noted. Topic and source are indicated.
Types of Calculators
Bulletin developed by Jon L. Higgins

Perhaps "a rose is a rose is a rose", but it is not true that "a calculator is a calculator is a calculator"! The way one enters information into a calculator, and the way the calculator processes the entered information, varies according to the brand and the model calculator being used. While there is a wide variety of calculator types, there are three basic variations (and a minor combination) that you should be aware of.

The first, and perhaps simplest of these, accepts numbers and operations just as they are written in horizontal mathematical notations. That is, to do the problem 2 + 3 =, one simply keys in 2, 3, and DI. Perhaps the most important key is the DI key, for it actually instructs the calculator to execute the operation which was keyed in previously. This seems natural for problems written in horizontal notation, but it can be confusing for young children if they have only seen a vertical format such as $\frac{1}{2} + \frac{1}{4}$. Not only is there no = sign in a vertical format, $\frac{1}{2} + \frac{1}{4}$, but the horizontal line is important in signaling the end of the problem statement. Of course, there is no horizontal bar on any calculator keyboard. Fortunately, the elementary mathematics curriculum has attempted to make children comfortable with both horizontal and vertical formats for many years now. Widespread adoption of hand-held calculators, however, may finally provide a reason for emphasizing the horizontal format.

When a series of arithmetic operations is entered into calculators of the first type, the calculator processes the operation in the order in which they are entered. Thus the expression

$$2 + 4 \times 5 - 9 \div 3 =$$

would be evaluated as

$$6 \times 5 - 9 \div 3 =$$

then as

$$30 - 9 \div 3 =$$

and finally as

$$21 \div 3 = 7$$

This procedure seems perfectly reasonable until one begins to include fractions in the arithmetic operations. A type 1 calculator would evaluate the expression

$$\frac{1}{2} + \frac{1}{4} =$$

or

$$0.5 + 1 \div 4 =$$

or

$$1.5 + 4 = 0.375$$
This is a disastrous state of affairs, since the correct answer is 0.75! One way to avoid this difficulty is to agree on a new order of operations: evaluate all multiplications and divisions in an expression first, and evaluate the additions and subtractions last. With this agreement, \( \frac{1}{2} \times 2 + 1 \div 4 = \)

becomes \( 0.5 + 0.25 = 0.75 \)

Because of difficulties such as this, a second-type calculator has been constructed which follows this new rule of order. It would evaluate the expression

\[ 2 + 4 \times 5 - 9 \div 3 = \]

as \( 2 + 20 - 3 = 19 \).

Type 1 calculators which process all operations in the order in which they are entered are known as algebraic logic calculators. Type 2 calculators which perform all multiplications and divisions in an expression before evaluating additions and subtractions are known as algebraic operating system calculators.

It is not easy to modify a type 1 calculator to perform like a type 2 calculator unless the calculator has a memory. But it is relatively easy to make a type 2 calculator perform like a type 1 calculator. The secret is simply to have each operation performed before the next operation is keyed in. The simplest way to remember to do this is to press the \( \neq \) key after each operation expression is completed. That is, if you are using a type 2 calculator and want it to evaluate the expression \( 2 + 4 \times 5 - 9 \div 3 = \) in the same way as a type 1 calculator, you should use the following key sequence:

\[ 2, +, 4, \neq, \times, 5, \neq, -, 9, \neq, \div, 3, \neq. \]

(This is the simplest procedure to remember. But it is not the most efficient procedure. Since the type 2 calculator performs multiplications and divisions first, it is really only necessary to press the equal key after additions and subtractions. Thus the sequence \( 2, +, 4, \neq, \times, 5, -, 9, \neq, \div, 3, \neq \) will give the same results. For beginners, however, the longer procedure is more consistent and less confusing.

There is another major type of calculator that is available to students. The type 3 calculator (Reverse Polish Notation) focuses upon the arithmetic operations as functions on ordered pairs of numbers. That is, addition matches the number pair \((15, 10)\) with the number 25. Subtraction matches \((15, 10)\) with 5. Multiplication matches \((15, 10)\) with 150. Division matches \((15, 10)\) with 1.5. The order of the numbers in the pair is important, since not all operations are commutative. For subtraction \((15, 10)\) is matched with 5, but \((10, 15)\) is matched with -5. Because of this focus on number pairs, a type 3 calculator requires that both numbers be entered before the operation is specified. Thus a type 3 calculator has a key on the keyboard just for entering numbers. That key is usually marked \( \text{ent} \). To add \(2+3\), key in \(2, \text{ent}, 3 \) (which establishes the ordered pair \((2, 3)\) and then instruct the calculator which operation function to perform by pressing an operation key. Thus the complete keystroke sequence for adding \(2 + 3\) is:

\[ 2, \text{ent}, 3, +. \]

Pressing the operation key actually performs the operation, so that no \( \neq \) key is necessary. The absence of an \( \neq \) key is the easiest way to identify a type 3
(Reverse Polish Notation) calculator. Less expensive type 3 calculators omit the enter key and use the + key both to enter numbers and to perform the addition operation.

Finally, some calculators act like a combination of type 3 and type 1 calculators. These calculators work with Reverse Polish Notation for addition and subtraction, and with algebraic logic for multiplication and division. They are most easily identified by double marked keys $\text{[C]}$ and $\text{[CE]}$. Many business calculators and printing calculators use this combined system, known as arithmetic logic. Because this combination logic could be confusing, these calculators probably should not be used with young beginning students.

Each of the three major types of calculators has its own advantage. A type 1 calculator operates just as a person with minimal mathematics training would expect that it should. A type 2 calculator operates consistently with conventions made in algebra. A type 3 calculator emphasizes ordered pairs and functions. As we have tried to point out, it is not difficult to switch back and forth between different types of calculators. But it is risky to approach a new calculator and assume that it will work exactly like your old familiar one.

This Information Bulletin was prepared by Jon L. Higgins, The Ohio State University. An expanded version will appear in a forthcoming publication of the National Council of Teachers of Mathematics, A Calculator Handbook for Teachers (being prepared in cooperation with ERIC/SMEAC with Jon L. Higgins as editor).

This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position of policy.
Suggestions for Calculator Selection

This is a synthesis of considerations and appropriate suggestions for selecting a calculator for elementary school use. Most of the considerations are also appropriate for secondary school users, but such factors as the number of functions, the type of logic, and programming capability assume increased importance in upper-level courses. For additional information, check the references in Reference Bulletin 12.

Note that it is important that a calculator be selected in relation to anticipated curricular applications. It is strongly suggested that the way a particular calculator operates should be checked carefully: test the calculator before you buy a classroom set to be sure it will serve your needs.

### Things to Consider

<table>
<thead>
<tr>
<th>Type of logic</th>
<th>Suggestions for Elementary Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic (allows data to be entered as mathematical sentences are usually written; see Information Bulletin 1 for a discussion of types of logic)</td>
<td></td>
</tr>
<tr>
<td>At least +, -, x, ÷</td>
<td></td>
</tr>
<tr>
<td>Floating decimal point; negative sign that immediately precedes a negative number; check the way the calculator rounds numbers</td>
<td></td>
</tr>
<tr>
<td>Clear indication of when display, input, or processing limit is reached, or when &quot;illegal&quot; operation is used</td>
<td></td>
</tr>
<tr>
<td>8-10 digits; easily readable; acceptable viewing angle (depends in part on how many persons are to view—one child or more than one; how calculator must be positioned); note that the vision-impaired child may have difficulty with certain types of displays (see also comments on page 3)</td>
<td></td>
</tr>
<tr>
<td>In general, each key should have only one purpose</td>
<td></td>
</tr>
<tr>
<td>Configuration of keys should facilitate accurate entry; easily accessible on-off switch—check the ease with which it works; adequately sized keys; keys should give some response when pressed (click, beep, or other sense); note the position of the numeral in relation to the keys</td>
<td></td>
</tr>
</tbody>
</table>
Things to Consider

Size and weight

Power source

Special keys
constant (K), change sign (+/-), parentheses, square root, percent, fraction, squaring

Memory: two-key

four-key

Memory indicator on display

Scientific notation

Automatic constant

Suggestions for Elementary Level

Appropriate for the user

Should provide long service, conserve energy. One opinion: "Consider the number of operating hours per battery replacement or charging. Automatic power-down displays and delayed power-off features insure the maximization of battery life. Long-life replaceable batteries seem to be the most cost- and time-efficient. Charging batteries and contending with electrical cords can be tedious." (Caravella, 1976, p. 548)

Analysis of the curriculum in which the calculator is to be used will aid in deciding how important these keys are to the user (for example, the +/- key is important if you want convenient manipulation of integers); generally you will have to "trade" some features for others you consider more desirable. Note how the keys handle the procedures.

Stores (STO) the displayed number for later recall (RCL): a useful feature for users even at early levels

Allows functions, usually addition (M+) and subtraction (M-), to be performed on the content of a memory register, with retention for later recall (MR); includes "memory clear" (MC): could be useful at upper levels

Helpful; make sure that it is easy to interpret the symbol (for example, an "M" is easier than a ".")

Note when and how it works (it may "cover up" repeating decimals)

Allows calculator to count: note for which operations a constant applies, and the position of the number treated as the constant—note also that it may operate differently with different functions.
## Things to Consider

**Printout**

- Not worth the current cost—but could be helpful to some users if cost dropped; note that a printout may take an unexpected form—check how symbols appear

**Durability**

- Check on droppage, malfunctioning incidents, etc., and weigh this in relation to cost

**Cost**

- Within the budget...

**Reliability of manufacturer**

- Adequate (12-month) warranty; repair service

**Reliability of vendor**

- Prompt, responsive service

## Types of Display

Currently two different types of display are available: LED (Light Emitting Diode) and LCD (Liquid Crystal Display). Each has advantages and disadvantages.

### LED
- In use longer
- Less expensive
- Durable (depending on the particular calculator)
- "Flashing" of symbols can be read in dark
- Uses 9-volt battery; relatively short life
- Red numerals or blue/green numerals: higher battery drain for blue/green than for red numerals
- Red numerals not readable from wide angle; blue/green generally readable from wider angle

### LCD
- More recently on market
- More expensive
- Less stable, reportedly (e.g., dropping may cause display to shift or lose part of a symbol)
- "Immediate" display of symbols depends on good reflected room light
- Uses silver oxide battery: hundreds of hours of life
- Black numerals on gray, yellow: low battery drain
- Readable from wide angle
Minicalculators in Mathematics Classes

The advent of inexpensive, yet powerful, “hand-held” calculators must be considered in connection with mathematics curriculum revision. Although many questions about the potential of using calculators in mathematics classrooms remain unanswered, the National Council of Teachers of Mathematics has taken a positive stand and encouraged exploration and research. Preliminary investigations are inconclusive. Only the future can reveal the emphasis and direction that will be followed.

Calculators certainly will have an impact on mathematics curricula. They may change not only the kinds of computational skills which are taught but the manner in which they are taught. It is our feeling that mathematics teachers and curriculum planners must incorporate calculators into regular classwork rather than ignore or banish them. Teachers must find effective uses at all levels from primary grades to calculus.

Standardized achievement tests generally cover three phases of mathematics—computation, mathematical concepts and problem-solving. Minicalculators are computational wizards and children can use them for basic operations with a minimum of instruction. The unanswered question at this stage is, “How can paper and pencil computational skills be preserved if calculators are available?” The countless hours spent on computation drill during the middle grades needs to be reevaluated. Is checking of answers by calculators all that can be done in this area? Many educators believe it is possible to improve computational skills, for example, in estimation and place value determination, by using calculators.

The second area of standardized testing is that of mathematical concepts. Instruction in primeness, factors, odd or even, LCM, and other phases of number theory can be enhanced with use of the calculator. The concept of limits and related theorems can be taught with greater effectiveness by using calculators. There are many other concepts which can be developed or reinforced through the use of calculators and deserve greater exploration.

Finally, standardized tests include a sub-test on problem-solving, sometimes called applications. Here is a prime area where calculators should be used effectively. Too often there is little time spent on problem-solving in mathematics classes due to difficulties with computation. This is true despite the obvious importance of practical applications for the majority of students. Also, many textbook problems are trivial and the numbers are kept artificially simple because of difficult calculations involved. This need be true no longer since the minicalculator can perform realistic computations as easily as the simplified versions found in texts. The student is provided the opportunity of keeping abreast of the problem at hand without getting sidetracked with tedious computations.

The grade level and specific courses will influence potential uses of calculators. In advanced mathematics courses, the value of scientific calculators, including programmable types, is quite evident. They will replace slide rules and books of tables. We believe calculators belong in all advanced mathematics classes as invaluable tools.

Nearly all prerequisite skills to higher mathematics can be strengthened by well-planned experiences with the calculator. Experimentation with the use of calculators in mathematics classrooms at all levels is strongly recommended.

Following are some suggested uses of calculators in the classrooms:

1. Reinforce computational skills.
2. Improve estimation of results.
3. Aid in teaching place value.
4. Develop number concepts.
5. Stimulate interest through games.
6. Solve problems with factual data, e.g., local or national sports statistics, business data, consumer buying, discounts, etc.
7. Check answers to computations.
8. Drill on arithmetic “facts” (alternative to flash cards).
9. Extend problems in the text to use larger or more realistic numbers.

A goal concomitant to learning more useful mathematics is the development of skill and confidence in using the calculator. Many adults are reluctant to use calculators and school is the most effective place for such orientation and practice.

In conclusion, it is clear that calculators are becoming a regular part of daily life for many people. The mathematics classroom is the logical place to prepare future breadwinners for the increasing complexity of modern life—including effective use of calculators. Experimentation with the use of calculators in mathematics classrooms at all levels is strongly recommended.
Teacher Notes for Estimation and Your Calculator

Prerequisite Knowledge:

(1) Round whole numbers to nearest 10, 100

(2) Round decimals to nearest whole number

(3) Estimate sums

Objectives: Given a calculator the child will be able to:

(1) Use estimation skill to decide on reasonableness of answers to addition problems solved on a calculator.

Materials:

(1) A calculator for each student

(2) Student Page 5-1

Even though children will have the ability to solve problems rapidly with the calculator, answers will be wrong if they enter the numbers incorrectly. Children will need to be able to judge if the answers the calculators give them are reasonable. Estimating skills are as important as being able to enter numbers on a calculator. This exercise will give students a chance to practice this estimation skill. Make other sheets like this for subtraction, multiplication and division.
Estimation and Your Calculator

Denny did the following problems on his calculator. He wasn't very careful and incorrectly entered some of the numbers. Look carefully at each problem, estimating each sum. Circle all problems you think are wrong. Go back to each problem and find the correct answer using your calculator.

2.57

2.57

36.9

103.472

143.242

943.06

14.37

1936

2275

773.12

263
Teacher Notes for Multiplying Decimals by 10, 100, 1000

PREREQUISITE KNOWLEDGE:
1. Enter decimals on calculator
2. Multiply on a calculator

OBJECTIVES: Given a calculator a child will be able to:
1. Discover a rule for multiplying a decimal by 10, 100, 1000.

MATERIALS:
1. Calculator for each student
2. Student Pages 3-3, 3-4, 3-5 duplicated for each child.

This activity should follow multiplying whole numbers by 10, 100, 1000. Have the students do page 3-3 alone or in small groups and then compare results in a large group. On page 3-3, they should discover that a quick way to multiply a decimal by 10 is to move the decimal point 1 place to the right. Talk about why that is so.

On page 3-4, children again can work independently but come together as a large group to compare rules. They should discover that to multiply a decimal by 100, move decimal point 2 places to the right; for 1000, move decimal point 3 places to the right. If they see this, try to generalize a rule for $10^n$.

For page 3-5, you are trying to combine rules for multiplying decimals and whole numbers by 10, 100, 1000. Once a final rule is written, make a large sign and post it in the classroom.
Multiplying Decimals by 10, 100, 1000

Use your calculator to find each product.

\[
\begin{align*}
23.6 \times 10 &= \underline{236} \\
.427 \times 10 &= \underline{4.27} \\
9.36 \times 10 &= \underline{93.6}
\end{align*}
\]

\[
\begin{align*}
2.36 \times 10 &= \underline{23.6} \\
4.27 \times 10 &= \underline{42.7} \\
.936 \times 10 &= \underline{9.36}
\end{align*}
\]

Look closely at the first factor in each problem and the answer. What happened to the decimal point?

Now without using your calculator answer each problem.

\[
\begin{align*}
53.6 \times 10 &= \underline{536} \\
.701 \times 10 &= \underline{7.01} \\
5.36 \times 10 &= \underline{53.6} \\
7.01 \times 10 &= \underline{70.1} \\
.536 \times 10 &= \underline{5.36} \\
70.1 \times 10 &= \underline{701.0}
\end{align*}
\]

Check with your calculator. Were you correct?

Write a rule for multiplying decimals by 10.

Will your rule work for other decimal?

<table>
<thead>
<tr>
<th>Guess</th>
<th>Actual Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.472 \times 10 =</td>
<td>\underline{634.72}</td>
</tr>
<tr>
<td>.9372 \times 10 =</td>
<td>\underline{9.372}</td>
</tr>
<tr>
<td>3473.6 \times 10 =</td>
<td>\underline{34736}</td>
</tr>
<tr>
<td>.00785 \times 10 =</td>
<td>\underline{.00785}</td>
</tr>
<tr>
<td>34.56 \times 10 =</td>
<td>\underline{345.6}</td>
</tr>
</tbody>
</table>
Place Value

One number - giver and any number of players each with a calculator.

Object of the Exercise --
To remove one digit from the display without changing any of the other digits.

How to Play --
The number giver picks a number which all players enter into their hand calculators, and says which digit is to be removed.

Example: In the display 876543 wipe out the 7 without changing any other digit. (Answer: 806543)

Example: Wipe out the 8 in .567891. (Answer: .567091)

ADDITION EXERCISE

Two players and a hand calculator

Object of the exercise -- To get 100 on the display

How to Play --
The first player pushes a single digit key (not zero) then pushes the (+) key. The next player takes his turn by pushing a single digit key (again not zero), then pushing the (+) key. Players take turns until a player pushes the (+) key and the display reads 100. The player who pushes (+) and gets the display to show 100 wins. If a player pushes (+) and the display shows a number larger than 100, that player loses.
Circle all the pairs of numbers whose product is \( .24 \). Use your hand calculator.

<table>
<thead>
<tr>
<th>.6</th>
<th>X</th>
<th>.4</th>
<th>X</th>
<th>.3</th>
<th>X</th>
<th>.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>.02</td>
<td>X</td>
<td>12</td>
<td>X</td>
<td>24</td>
<td>X</td>
</tr>
<tr>
<td>.8</td>
<td>X</td>
<td>3</td>
<td>X</td>
<td>6</td>
<td>X</td>
<td>.04</td>
</tr>
<tr>
<td>X</td>
<td>48</td>
<td>X</td>
<td>.005</td>
<td>X</td>
<td>.01</td>
<td>X</td>
</tr>
<tr>
<td>.3</td>
<td>X</td>
<td>.08</td>
<td>X</td>
<td>2</td>
<td>X</td>
<td>.12</td>
</tr>
<tr>
<td>X</td>
<td>30</td>
<td>X</td>
<td>.008</td>
<td>X</td>
<td>.06</td>
<td>X</td>
</tr>
<tr>
<td>.36</td>
<td>X</td>
<td>1.5</td>
<td>X</td>
<td>5</td>
<td>X</td>
<td>.048</td>
</tr>
<tr>
<td>X</td>
<td>.48</td>
<td>X</td>
<td>.5</td>
<td>X</td>
<td>4</td>
<td>X</td>
</tr>
</tbody>
</table>

Your Score.
- 12 or more: Tops
- 9 to 11: Very Good
- 6 to 8: A Good Start
- 3 to 5: Look Again
- Less than 3: You need help

FIND THE MISSING NUMBERS

Make the numbers (horizontal and vertical) add up to the sum in the small square. You can work across and down. Your calculator can help you find missing numbers.
Work each exercise to complete the magic square. The sums in the rows, columns, and diagonals are equal. What is the sum?

1. $591 - 98 = \underline{\hspace{2cm}}$
2. $635 - 237 = \underline{\hspace{2cm}}$
3. $1569 - 1065 = \underline{\hspace{2cm}}$
4. $1303 - 827 = \underline{\hspace{2cm}}$
5. $974 - 509 = \underline{\hspace{2cm}}$
6. $832 - 378 = \underline{\hspace{2cm}}$
7. $1185 - 759 = \underline{\hspace{2cm}}$
8. $1179 - 647 = \underline{\hspace{2cm}}$
9. $3712 - 3275 = \underline{\hspace{2cm}}$

Use a calculator to check these answers. Circle every correct answer.

Use a calculator to add.

What is the sum of your six answers? \underline{\hspace{2cm}}
Object of the Game -- To get your number over 999999.

The Play -- Each player enters a six-digit number in his or her calculator, no two digits of which are the same. A coin toss then decides who goes first. Player A says, "Give me your 5's." (This is an example: a player can ask for any number from 1-9.) Player B, reading his or her calculator says, "You get 500."

The "give and take" of the above number depends on where the 5 occurs in the number of Player B. If the number on the calculator reads 12345, B says, "You get 5"; 12354, B says, "You get 50"; 12534, B says, "You get 500"; 15234, B says, "You get 5000"; et cetera.

The player who "takes" adds the value of the number. The player who "gives" subtracts the same value. Thus, Player A says, "Give me your 5's." Player B says, "Take 50." A adds 50 to his or her number; B subtracts 50.

If one player asks for a number the other player does not have (for example: A asks for 6, and B says, "I have no 6's"), the play continues with B asking A for a number.

Play continues until one player wins by going over 999999. No player can ask for 0. If a player has two or more of the same digits in his or her number, the smaller of the two numbers may be given. (For example: 663790. The player gives 60,000, not 600,000).

Strategy -- Putting a large or small initial number in the calculator can be very risky. As you play, numbers starting with 5 and 6 will look increasingly attractive. Also, players sometimes reveal the number they don't want to give away by asking for it.

Sample Play --

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Number:</td>
<td>Starting Number:</td>
</tr>
<tr>
<td>765432</td>
<td>658792</td>
</tr>
<tr>
<td>Plays:</td>
<td></td>
</tr>
<tr>
<td>A: &quot;Give me 2&quot;</td>
<td>765432</td>
</tr>
<tr>
<td></td>
<td>+ 2</td>
</tr>
<tr>
<td></td>
<td>765434</td>
</tr>
<tr>
<td>B: &quot;Give me 5&quot;</td>
<td>- 5000</td>
</tr>
<tr>
<td></td>
<td>760434</td>
</tr>
<tr>
<td>A: &quot;Give me 6&quot;</td>
<td>+ 60000</td>
</tr>
<tr>
<td></td>
<td>820434</td>
</tr>
<tr>
<td>B: &quot;Give me 8&quot;</td>
<td>- 800000</td>
</tr>
<tr>
<td></td>
<td>20434</td>
</tr>
</tbody>
</table>

B wins
Calculators have also increased the need for estimation skills and mental arithmetic. Even though the student generally learns to visually check each entry in the calculator, it is still possible to make errors. Estimation skills are necessary to catch them.

We often spend class time in playing games using the calculator to increase estimation skills. One of the favorite games is played with partners. The partners agree on a target range, such as 490 to 500. Through a series of multiplications they try to reach a number within that range. For example, the first student enters the start number 15 and pushes \((x)\). The second estimates, chooses to enter 28, and pushes \((x)\). The machine shows 420. The first student estimates 1.3 and pushes \((x)\). The machine displays 546. The second player estimates .9 and multiplies to get an answer of 491.4. This number is in the target area, so he is the winner. We play this game with all four operations and with a variety of targets.
Mathematics Resource Project (MRP)

The following activities, reproduced with permission, are samples of materials from the Mathematics Resource Project, developed at the University of Oregon and supported by a grant from the National Science Foundation. Project materials are now commercially available from Creative Publications.

Mathematics Resource Project materials consist of five resources, each containing worksheets, calculator activities, games, puzzles, bulletin board suggestions, project ideas and teaching didactics. The resources have been designed and created for teachers in grades 5 through 9.

Resource activities are highly motivational materials designed to provide student practice with all the basic skills including problem solving, mental computation, estimation, and measurement. Individual resource packets are organized under the following titles:

- Number Sense and Arithmetic Skills - 832 pages
- Ratio, Proportion and Scaling - 516 pages
- Geometry and Visualization - 830 pages
- Mathematics in Science and Society - 464 pages
- Statistics and Information Organization - 850 pages

The sample materials are identified by the letters MRP on the top right hand corner of the page.

For more information about Mathematics Resource Project materials, contact:

Creative Publications, Inc.
P.O. Box 10328
Palo Alto, CA 94303
The following materials from the Mathematics Resource Project were included in the packet, but cannot be reproduced here due to copyright restrictions:

- Estimation and Approximation
- Fix That Leak
- I Need a Job Like That!
- Closer & Closer
- Four Investigations
- Stretch Your Calculator
- Calculator Capers I
- Calculator Capers II
- Calculated Codes
- 9-Time
- Cheery Sequences
- Palindromes!
- Curiosities
- Persistent Numbers
- That's Just About the Size of It!
- Shopping with a Newspaper
- More Investigations
- 2x3 Good Times
- The Reverse Double-Digit Magic Trick - Act 1
- When You're Hot You're Hot
Mathematics Problem Solving Project (MPSP)

The following pages, copied with permission, are samples of problem materials from the Mathematics Problem Solving Project (MPSP), a project of the Mathematics Education Development Center at Indiana University funded by a grant from the National Science Foundation. The project was cooperatively developed by staff from the following:

University of Northern Iowa
Cedar Falls, Iowa

Oakland Schools
Pontiac, Michigan

Indiana University
Bloomington, Indiana

Project materials consist of three booklets of lesson outlines and three associated sets of problems organized under the following headings:

- Using Lists
- Using Tables
- Using Guesses

The sample problem materials are identified by the letters MPSP on the top right hand corner of the page.

Inquiries about the project or the materials may be addressed to:
Professor John F. LeBlanc, Director
Mathematics Education Development Center
Education Building
Indiana University
Bloomington, Indiana 47401
The following materials from the Mathematics Problem Solving Project were included in the packet, but cannot be reproduced here:

23 YC 1
24 YS 1
11 WC 1
12 WS 1
13 RM 1
14 RB 1
27 RR 1
28 RS 1
13 GB 1
14 GM 1
1 BC 6
2 BC 6
17 YM 6
18 YC 6
1 WM 6
2 WC 6
11 RV 6
12 RV 6
13 RM 6
14 RV 6
1 GM 6
2 GC 6
5 GC 6
6 GV 6
11 GM 6
12 GC 6
19 GM 6
20 GC 6
NCTM  Recommendations

1. TO ENCOURAGE PUPILS TO BE INQUISITIVE AND CREATIVE AS THEY EXPERIMENT WITH MATHEMATICAL IDEAS.
2. TO ASSIST THE PUPIL TO BECOME A WISE CONSUMER.
3. TO REINFORCE THE LEARNING OF THE BASIC NUMBER FACTS IN ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.
4. TO DEVELOP UNDERSTANDING OF COMPUTATIONAL ALGORITHMS BY REPEATED OPERATIONS.
5. TO SERVE AS A FLEXIBLE "ANSWER KEY" TO VERIFY THE RESULTS OF COMPUTATION.
6. TO PROMOTE STUDENT INDEPENDENCE IN PROBLEM SOLVING.
7. TO SOLVE PROBLEMS THAT PREVIOUSLY HAVE BEEN TOO TIME-CONSUMING OR IMPRACTICAL TO BE DONE WITH PAPER AND PENCIL.
8. TO FORMULATE GENERALIZATIONS FROM PATTERNS OF NUMBERS THAT ARE DISPLAYED.
9. TO DECREASE THE TIME NEEDED TO SOLVE DIFFICULT COMPUTATIONS.
1. COUNT SETS OF OBJECTS, ONE TO TWENTY, AND DISPLAY THE NUMERALS ON THE HAND CALCULATOR. (COUNTING)

2. GIVEN A SET OF NUMBER CARDS, ONE THROUGH TWENTY, HAVE ONE PUPIL POINT TO A CARD AND THE STUDENTS THEN SHOW THAT NUMBER ON THEIR HAND CALCULATORS. (NUMBER RECOGNITION)

3. RESPOND TO VERBAL NUMBER NAMES BY SHOWING ONE, TWO, AND THREE DIGIT NUMBERS ON THE HAND CALCULATOR. (NUMBER PRODUCTION)

4. GIVEN AN ORAL NAME, A WRITTEN NAME, OR A SET OF OBJECTS, THE PUPIL PRODUCES THE CORRECT NUMERAL ON THE HAND CALCULATOR. (NUMBER PRODUCTION)

5. DISPLAY A NUMBER ON THE HAND CALCULATOR THAT COMES BEFORE OR AFTER A GIVEN NUMBER, IN THE MIDDLE OF TWO GIVEN NUMBERS. (NUMBER SEQUENCE)

6. SHOW THE NUMBER THAT IS FOUR (4) TENS AND TWO (2) ONES. (PLACE VALUE)

7. COUNTING ON THE HAND CALCULATOR (PUNCH 1, PUNCH +, PUNCH 1, PUNCH +, PUNCH 1, PUNCH +, ETC.). COUNT BY 2s, 5s, 10s, ETC. (COUNTING FORWARD AND REVERSE)

8. USE THE HAND CALCULATOR TO HELP PUPILS FILL IN THE 9 X 9 BASIC ADDITION AND SUBTRACTION CHART. (DISCOVERING PATTERNS)

9. TO FIND OR VERIFY MISSING ADDENDS OR SUMS FOR ADDITION AND SUBTRACTION EXERCISES IN VERTICAL OR HORIZONTAL FORM. (OPERATIONS)

10. TO DISCOVER THE ROLE OF ZERO IN ADDITION AND SUBTRACTION ALGORITHMS. (ZERO PROPERTY)
1. Give an oral or written name the pupil shows the correct numeral on the hand calculator. Four thousands, 7 hundreds, 3 tens, and 6 ones or four thousand two hundred six. (Number production)

2. Use the hand calculator to find the number that is 1000 less than 6271 or 8000. Show the number that is 100 greater than 4821 or 4935. (Place value)

3. Use the hand calculator to verify column addition, subtraction particularly where regrouping or borrowing was necessary, and multiplication and division exercises. (Operations)

4. To help recognize the relationship between addition and multiplication. 
   $6 \times 4 = 24$, $4+4+4+4+4+4 = 24$. Similar relationship between division and repeated subtraction. $21 \div 7 = 3$, $21 \div 7 = 14$, $14 \div 7 = 7$, $7 \div 7 = 0$ (Operations)

5. Entering a large number (6-8 digits) and having it read by a partner. (Place value)

6. Verify answers to multiplication and division exercises that involve 10, 100, and 1000. (Operations)

7. Use the hand calculator to determine if the following number are even or odd. Thirty-eight, 653, 1,692, 29, 8109, 728. (Number theory)

8. Check this exercise by using your hand calculator

   \[
   \begin{array}{c}
   37 \div 2 \\
   9 ) 335
   \end{array}
   \] (Operations)

9. First estimate the answers and then use your hand calculator to check your estimations. (Estimation)

<table>
<thead>
<tr>
<th>ESTIMATION</th>
<th>HAND CALCULATOR ANSWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 5812 + 4406 =</td>
<td></td>
</tr>
<tr>
<td>B. 846 - 231 =</td>
<td></td>
</tr>
<tr>
<td>C. 479 \times 24 =</td>
<td></td>
</tr>
<tr>
<td>D. 8153 \div 41 =</td>
<td></td>
</tr>
</tbody>
</table>

10. Select the largest number in each row. Use your hand calculator to check your results. (Order relations)

   | A. 19 \times 31 | B. 32 \times 19 |
   | 889 - 496 | 889 - 496 |
   | 380 - 20 | 400 - 19 |
   | 2 \times 20 \times 40 | 2 \times 19 \times 30 |
1. Find the value of $5^3$ or $4^6$. (Exponents)

2. Multiply $1 \times 2 \times 3 \times 4 \times 5 \ldots 9 = \Box$. This product is called 9 factorial and is written $9!$. (Factorial)

3. Insert the correct symbol $\geq$, $\leq$, or $=$ to make the statement true. Then check the results with your hand calculator. (Number Properties)
   - $45 \times 31 \Box 35 \times 41$
   - $178 - 6 \Box 178 - 7$
   - $127 \times 428 \Box 428 \times 127$
   - $32 \times (41 + 82) \Box (32 \times 41) + (32 \times 82)$

4. Use the hand calculator to find the largest whole number that makes each sentence true. (Division)
   - A. $N \times 6 \leq 493$
   - B. $9 \times N = 329$
   - C. $N \times 27 = 1746$

5. Find the missing numbers with your hand calculator. (Number Properties)
   - $573 + \underline{\Box} = 573$
   - $17 \times 86 = \underline{\Box} \times 17$
   - $(49 + 24) + 29 = 49 + (24 + \underline{\Box})$
   - $617 \times \underline{\Box} = 617$
   - $9 \times (36 \times 74) = (9 \times \underline{\Box}) + (9 \times 74)$

6. Which of the following numbers are factors of the first number? (Factoring)
   - $18 \underline{1, 2, 3, 4, 5, 6, 8, 9, 12, 18}$
   - $140 \underline{1, 2, 3, 4, 5, 6, 7, 8, 11, 14, 17, 20}$

7. Complete these number sequences by using your hand calculator. (Patterns)
   - A. $7, 14, 21, 28, \underline{\Box}, \underline{\Box}, \underline{\Box}, \underline{\Box}, \underline{\Box}, 63$
   - B. $4, 0, -4, -8, \underline{\Box}, \underline{\Box}, \underline{\Box}, \underline{\Box}, -28$
   - C. $1, 0.5, 0.25, 0.125, \underline{\Box}, \underline{\Box}, \underline{\Box}, 0.015625$
   - D. $1, 4, 9, 16, \underline{\Box}, \underline{\Box}, \underline{\Box}, 64$
8. Use your hand calculator to determine if the following fractions are equivalent. (Cross-multiplication)

A. \( \frac{5}{9} \square \frac{25}{36} \)
B. \( \frac{17}{31} \square \frac{23}{37} \)

9. Place the decimal point in your answer. Check your answer with your hand calculator. (Decimals)

A. \(2.1 + 3.2 + 4.1 = 9.40\)
B. \(5.49 \times 3.2 = 17.5680\)
C. \(239.5 \div 0.19 = 1260.5263\)

10. Find the square root of 46 to three decimal places. (Square root)
WORKSHOP EVALUATION

Ohio Regional Conferences on Mathematics Education

Evaluation of the 5 Conferences

The staff conducting the 5 regional conferences are interested in the participants' reactions to the conferences. Each participant will be asked to complete a questionnaire, (pink form) which includes a set of multiple choice questions and a section for open responses.

The goal of the conferences is to help the participants prepare to conduct workshops or other in-service activities with teachers in their own or nearby districts. We will contact you later to see if the conferences resulted in such workshops. (This is an evaluation of the effectiveness of the Ohio Regional Conferences.) At that time you may have other suggestions for the staff and we hope you will send them.

Evaluation of Follow-up Activities

If you present a workshop or other activity for teachers, you may want to use the evaluation form in this section (white form). Answer sheets are available from:

Len Pikaart
Ohio University
College of Education
McCracken Hall
Athens, Ohio 45701

The answer sheets will be provided free (as long as the supply lasts). We ask that you reproduce the questionnaire itself. Please request your best estimate of the number of answer sheets needed and return unused ones so they will be available for others. We will score the answer sheets and send you a complete analysis if you will permit us to include the data from your program in our summary data.

We emphasize that no individual workshop or activity will be identifiable in any report of analysis we conduct. We would like to examine the reactions to all the programs conducted as a result of the conferences to determine if we should use the same sort of conference format in the future--thus, we want to evaluate us--not you. Positive results would have great implications for the National Science Foundation (supporting this project) and perhaps for the Ohio State Department of Education. (Negative results would have implications too!)

We hope you will decide to use the evaluation form and to share the dates with us.
Ohio Regional Conferences on Mathematics Education

Conference Evaluation Form

Select the single best answer for you to each question and mark all answers on the answer sheet. If the question is not appropriate, leave it blank.

Participant Position

1. Are you:
   A. Elementary School Supervisor or Consultant.
   B. Supervisor of Grades K-12.
   C. Elementary School Principal.
   D. Mathematics Education (College Level).
   E. Other.

Conference Objectives and Purposes

2. How clear were the objectives or purposes of this conference? The objectives and purposes:
   A. Were clearly outlined from the beginning.
   B. Became clear as the conference developed.
   C. Became somewhat clear as the conference progressed.
   D. Were referred to only indirectly.
   E. Were never made clear.

3. The agreement between the announced purpose of the conference and what was actually presented was:
   A. Superior.
   B. Above average.
   C. Average.
   D. Below average.
   E. Poor.

Organization

4. How well was the conference organized?
   A. The conference was extremely well organized and integrated.
   B. The conference was adequately organized.
   C. The conference had less organization than would seem desirable.
   D. The conference had no apparent organization.
   E. The conference was too tightly organized; there was not enough flexibility to meet participant needs and desires.
5. Considering the mixture of participants in the meeting:
A. There should have been more female teachers.
B. There should have been more male teachers.
C. The mixture was about right.
D. The groups should have met separately at the conference.
E. The groups should have separate conferences.

Conference Content

6. How well did this conference contribute to your professional needs?
A. Made a very important contribution.
B. Was valuable, but not essential.
C. Was moderately helpful.
D. Made a minor contribution.
E. Made no significant contribution.

7. How would you rate the usefulness of the Resource Packet on Problem Solving?
A. Extremely valuable.
B. Very useful.
C. Useful.
D. May be of use.
E. Useless.

8. How would you rate the Resource Packet on Calculating?
A. Extremely valuable.
B. Very useful.
C. Useful.
D. May be of use.
E. Useless.

Participant Participation

9. How clearly were your responsibilities in this conference defined?
A. I always knew what was expected of me.
B. I usually knew what was expected of me.
C. I usually had a general idea of what was expected of me.
D. I was often in doubt about what was expected of me.
E. I seldom knew what was expected of me.

10. How would you rate the conference effectiveness relative to your investment of time and effort?
A. Very high value for my effort.
B. High value for my effort.
C. Moderate value for my effort.
D. Low value for my effort.
E. No value for my effort.
Presenter-Participant Relationship

11. Do you feel that the presenters were willing to give personal help in this conference?
   A. I felt welcome to seek personal help as often as I needed it.
   B. I felt free to seek personal help.
   C. I felt he or she would give personal help if asked.
   D. I felt hesitant to seek personal help.
   E. I felt that he or she was unsympathetic and uninterested in participant problems.

12. Freedom of participation in conference meetings: questions and comments were:
   A. Almost always sought.
   B. Often sought.
   C. Usually allowed.
   D. Seldom allowed.
   E. Usually inhibited.

Conference Effectiveness

13. Did the conference help prepare you to lead in-service activities on problem solving and calculators?
   A. Definitely.
   B. It was a help on both.
   C. On one of the topics.
   D. It was little help.
   E. It was no help.

14. Would you recommend this conference to a good friend whose interests and background are similar to yours?
   A. Recommend highly.
   B. Generally recommend.
   C. Recommend with reservations.
   D. Definitely not.

15. How would you rate your understanding of Problem Solving as a result of this conference?
   A. I learned a lot.
   B. My understanding improved.
   C. A few ideas were new to me.
   D. I learned very little.
   E. I learned almost nothing.

16. How would you rate your understanding of the use of Calculators in schools as a result of this conference?
   A. I learned a lot.
   B. My understanding improved.
   C. A few ideas were new to me.
   D. I learned very little.
   E. I learned almost nothing.
17. The presenter seemed:
   A. Always prepared.
   B. Almost always prepared.
   C. Usually prepared.
   D. Frequently not prepared.
   E. Never prepared.

18. How would you rate the presentation's suitability to what you consider to be the important problems in education? [Please check one.
   A. They were well more of the important problems.
   B. They were aware of the problems.
   C. They had a general idea of the problems.
   D. They had a very limited awareness of these problems.
   E. They did not even touch on significant problems.

19. How would you rate the presentation? [Please check one.
   A. Outstanding and stimulating.
   B. Very good.
   C. Good.
   D. Adequate, but not stimulating.
   E. Poor and inadequate.

20. Would you like to attend conferences on other (like these) topics in this geographic area?
   A. Definitely.
   B. Yes, but in a different city.
   C. It would be a good idea.
   D. Probably not.
   E. Definitely not.

21. How would you rate the use of instructional media in this conference?
   A. The use of media were almost always effective.
   B. The use of media were usually effective.
   C. The use of media were sometimes effective.
   D. The use of media were seldom effective.
   E. The use of media were never effective.

22. Do you believe that the conference helped establish for improved positive linkages between school system personnel and college mathematics educators?
   A. Definitely.
   B. Somewhat.
   C. Very little improvement.
   D. No improvement.
   E. The linkages should not be established.
1. Best features of the conference were:

2. Worst aspects of the conference were:

3. I would suggest the following:

4. Were there materials on display that you would like to see included in the Resource Packet? (Which ones?)

5. Do you feel you are prepared to lead in-service activities on problem solving using calculators for teachers? Can you tell us what activities you expect to organize? For how many teachers? When?
TEACHER IN-SERVICE ACTIVITY

Evaluation Form

Select the single best answer for you to each question and mark all answers on the answer sheet. If the question is not appropriate, leave it blank.

1. Are you:
   A. Elementary school teacher, grades K-4.
   B. Elementary school teacher, grades 5-8.
   C. Elementary School Principal.
   D. School system administrator.
   E. Other.

2. This activity focused on:
   A. Problem solving.
   B. Using calculators.
   C. Both problem solving and calculators.
   D. Some other topic but referred to problem solving or to calculators.
   E. Some other topic.

3. The objectives and purposes:
   A. Were clearly outlined from the beginning.
   B. Became clear as the activities developed.
   C. Became somewhat clear as the activities progressed.
   D. Were referred to only indirectly.
   E. Were never made clear.

4. The agreement between the announced purposes of the activity and what was actually presented was:
   A. Superior.
   B. Above average.
   C. Average.
   D. Below Average.
   E. Poor.

5. How well was the activity organized?
   A. It was extremely well organized and integrated.
   B. It was adequately organized.
   C. It had less organization than would seem desirable.
   D. It had no apparent organization.
   E. It was too tightly organized, there was not enough flexibility to meet participant needs and desires.

6. How well did this activity contribute to your professional needs?
   A. Made a very important contribution.
   B. Was valuable, but not essential.
   C. Was moderately helpful.
   D. Made a minor contribution.
   E. Made no significant contribution.

7. How would you rate the usefulness of the materials on Problem Solving?
   A. Extremely valuable.
   B. Very useful.
   C. Useful.
   D. May be of use.
   E. Useless.
8. How would you rate the usefulness of the materials on Calculators?
   A. Extremely valuable.
   B. Very Useful.
   C. Useful.
   D. May be of use.
   E. Useless.

9. How clearly were your responsibilities during this activity defined?
   A. I always knew what was expected of me.
   B. I usually knew what was expected of me.
   C. I usually had a general idea of what was expected of me.
   D. I was often in doubt about what was expected of me.
   E. I seldom knew what was expected of me.

10. Considering the size of the group, do you feel that the leaders were willing to give personal help?
    A. I felt welcome to seek personal help as often as I needed it.
    B. I felt free to seek personal help.
    C. I felt he or she would give personal help if asked.
    D. I felt hesitant to seek personal help.
    E. I felt that he or she was unsympathetic and uninterested in participant problems.

11. Would you recommend this conference to a good friend whose interests and background are similar to yours?
    A. Recommend highly.
    B. Generally recommend.
    C. Recommend with reservations.
    D. Definitely not.

12. How would you rate your understanding of the use of problem solving as a result of this conference?
    A. I learned a lot.
    B. My understanding improved.
    C. A few ideas were new to me.
    D. I learned very little.
    E. I learned almost nothing.

13. How would you rate your understanding of the use of calculators in schools as a result of this conference?
    A. I learned a lot.
    B. My understanding improved.
    C. A few ideas were new to me.
    D. I learned very little.
    E. I learned almost nothing.

14. How would you rate the activity in general?
    A. Outstanding and stimulating.
    B. Very good.
    C. Good.
    D. Adequate, but not stimulating.
    E. Poor and inadequate.
1. Best features of the activity were:

2. Worst aspects of the activity were:

3. I would suggest the following:
Ohio Regional Conferences
on Elementary Mathematics Education

The two films shown during the conference are available for rental.

**Expanding Math Skills with the Minicalculator: Classroom Management**
Aesop Films, 18 min., 1976
Encyclopedia Britannica Educational Corporation
425 North Michigan Avenue
Chicago, Illinois 60611

Rental: $14 for three days

**Solving Verbal Problems**
The Pennsylvania State University, 21 min., 1970
Audio Visual Services
The Pennsylvania State University
Special Services Building
University Park, Pennsylvania 16802

Rental: $12 for first day, $6 for each additional day

Arrangements to obtain either of the films on loan may also be made with:

Marilyn N. Suydam
The Ohio State University
1200 Chambers Road
Columbus, Ohio 43212

Phone: 614-422-6717
Ohio Regional Conferences: Materials on Display


* In addition, Calculator Information Center bulletins should be checked for other displayed materials on calculators.

