Computer-based courseware for the intermediate grades developed by the PLATO Elementary Mathematics Project was tested for a three year period in the public schools of Champaign and Urbana, Illinois. This brief report describes the project in terms of the student session, curriculum, educational effectiveness, and data feedback to teachers. Descriptions of 19 lessons on a variety of topics include objectives and purpose, as well as a few selected screen displays for most of them. A sample of student work shows how ten students responded to similar problems, and sources from which the lessons were taken are cited. (CMV)
ELEMENATARY MATHEMATICS WITH PLATO

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CONTENTS

The PLATO Elementary Mathematics Project ........ 1
The Student Session .................................. 1
The Curriculum ....................................... 2
A Sample of Student Work ............................. 23
Educational Effectiveness ............................. 25
Data Feedback to Teachers ............................. 28
References ........................................... 28
The PLATO Elementary Mathematics Project

For the past five years a team at the Computer-based Education Research Laboratory at the University of Illinois, under the direction of Robert B. Davis, has been developing, implementing, and testing computer-based mathematics courseware for the intermediate grades.* During a three year trial in cooperation with the public schools in Champaign and Urbana, Illinois, this courseware has been used in eighteen classrooms by approximately 700 fourth, fifth, and sixth grade students for a total of about 43,000 hours of instruction delivered. Each participating classroom is equipped with four PLATO terminals. Each teacher has worked out a schedule for use of the terminals so that every student can use PLATO daily.

The Student Session

Each student uses a PLATO terminal for about thirty minutes each day. (The teacher specifies how long each student's sessions will be—usually between 20 and 35 minutes.) During this time, PLATO offers the student a variety of appropriate lessons to work on. A typical session might proceed as follows:

First, a few minutes of quick review exercises, probably related to the lessons the student will work on later in the session.

Second, a list of several appropriate instructional lessons for the student to choose from. These lessons are the main material that the student is working on. As the student masters skills and concepts presented in these lessons, new lessons are added to the choice list and the completed lessons may be dropped.

Last, a list of several general experience lessons for the student to choose from. Some of these lessons are games that reinforce recently learned mathematical skills. Others may encourage exploration of new and different ideas and strategies or provide practice in previously learned skills.

*Funding for this project has been provided by the National Science Foundation and the State of Illinois.
The Curriculum

The currently available curriculum includes about 250 PLATO lessons and covers three major areas:

1. Whole number arithmetic, including
   - meaning of operations
   - computation techniques and practice
   - algorithms
   - place value
   - renaming and symbols
   - word problems

2. Fractions, mixed numbers, and decimals, including
   - meanings of fractions and mixed numbers
   - equivalent fractions
   - addition, subtraction, and multiplication of fractions and mixed numbers
   - the meaning of decimal numerals

3. Graphs, functions, and variables, including
   - signed numbers
   - variables and open sentences
   - exponents
   - graphing equations
   - functions

The teacher can choose what topic(s) each student will study on PLATO. Many of the topics have companion booklets and worksheets to help correlate the PLATO lessons with classroom activities.

The following pages describe some of the elementary mathematics lessons. With each description there are a few selected screen displays. These lessons are representative of the elementary mathematics courseware available on PLATO.
Speedway: A Race to Improve Your Speed with Number Facts

Type: game
Exposure: multiple

Objective: Student quickly and accurately answers one-digit addition, subtraction, multiplication, and division problems.

Purpose: Provides a game situation for drill, practice, and remediation in the basic number facts involving the four basic operations.

Description: Student works 10 problems at a time, attempting to beat his previous speed (so winning is based on self-improvement). Missed problems are presented again; when he has mastered the problems at one difficulty level he advances to more difficult problems.

Graphs and charts showing the student's performance are presented before and after the game. A student can use this information to decide where he needs practice. However, if he repeatedly practices at one operation to the exclusion of areas in which he needs work, PLATO limits his choices.
Subtraction with Sticks -- Part 2

How to borrow using sticks

\[
\begin{array}{c}
43 \\
-25
\end{array}
\]

Here is a problem. We need to take away 5 leaves but we only have 3. We can get more leaves by opening a bundle. Put a to open a bundle.

How many leaves are there now?

How to borrow using sticks

\[
\begin{array}{c}
48 \\
-25
\end{array}
\]

Type: instructional, practice

Exposure: two or three segments (based on performance)

last segment repeats for practice.

Objective: Student works vertical 2-digit subtraction problems from right to left, using "borrowing" or "breaking up a ten into ones" where appropriate.

Purpose: Relates the sticks model to the standard algorithm.

Description: PLATO pictorially shows the sticks meaning of each digit in a vertical subtraction problem and of each step in the algorithm. Student works problems with and without the sticks pictures, including problems of his own making.

Practice sessions are more frequent for those who make frequent errors or need the help sequence.

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How the West Was One \times Three + Four

Type: game
Exposure: multiple
Objective: Given 3 numbers, student constructs a mathematical expression to put his playing piece in a good position on the game board and correctly evaluates his expression.
Purpose: Provides a game-type situation in which computational skills, ability to make combinations, and strategy win.
Description: Two players take turns constructing mathematical expressions with 3 random numbers from spinners. If the student can correctly evaluate his "move", his playing piece moves on the board accordingly. Advantageous moves include: making the maximum result \([a \times (b + c)]\), landing on a town or a shortcut; or bumping the opponent.
Options include: playing against a friend or PLATO setting the quality of PLATO's play (very good or just competitive); and having signed numbers or natural numbers on the spinner.
**Rubber Stamp**

Type: direct teaching
Slot: instructional
Exposure: two segments

**Objective:**
Student writes a repeated addition number sentence for a given picture.

**Purpose:**
1. Introduces model to later relate repeated addition to multiplication.
2. Emphasizes the correspondence between a picture and a number sentence.
3. Shows how an addition number sentence can be interpreted in words.

**Description:**
With his finger as a rubber stamp and PLATO's screen as a piece of paper, student touches the screen and "stamps" the paper repeatedly. He writes a repeated addition number sentence to correspond with the stamped paper. PLATO responds by interpreting the number sentence in words and asking the student to judge the appropriateness of his number sentence.
Claim Game

Type: game
Slot: choice
Exposure: multiple

Purpose: To give students practice in recognizing factors of numbers up to 36.

Description: This is a two person game. There is a list of the numbers from 2 to 36. Players take turns removing numbers from the list and putting them on their own list. After a number is chosen, the player who chose it can also "claim" any remaining factor of the number he chose for his own list. If he overlooks a factor, it can be chosen by his opponent at the start of his turn.

A player's score is the sum of the numbers he has chosen. The player with the highest score wins.

Students can play against a friend or PLATO. The PLATO playing algorithm plays two moves ahead, except in the case that the player is more than ten points behind, in which case PLATO chooses the highest number remaining on the list.

The game was programmed from a description in the Arithmetic Teacher.

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Introduction to the Multiplication Algorithm

Type: instruction and practice
Exposure: 3 - 5 segments, repeats
Objective: The student understands the standard multiplication algorithm and can use it to work multiplication problems.
Purpose: To teach the multiplication algorithm in a way that relates it to previously taught techniques.
Description: The student first learns how to work a multiplication problem by breaking it up into a sum of multiplications by numbers have one non-zero digit. Then he learns how the algorithm allows him to do that quickly without having to do much writing of auxiliary problems. In each case the student must practice using the method to demonstrate his understanding.

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Purpose: Practice constructing and identifying fractions of a set of objects.

Description: The student turns on fractions of a set of light bulbs. After some success, PLATO turns on some lights and the student identifies the fraction represented. For this lesson the denominator of the fraction is always the number of lights in the set. The student is expected to move quickly to the next lesson ("Lights: Equivalent Fractions") where the denominator is different from the number of lights. Difficulty and length of the problem sequence adjust to the student's performance.

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Paintings Library (touch)

Purpose:  1) Encourage painting a fraction of a box different ways.
         2) Initiate the idea of re-arranging the painted areas without changing the fraction painted.
         3) Provide opportunities to notice equivalent fractions.
         4) Let students share their work with others.

Description: The student can paint a box to keep in the library or look at paintings done by classmates. Due to space limitations, each student can keep only two paintings. A student can replace his or her paintings with new ones.

Reprinted with permission from The Fractions Curriculum of the PLATO Elementary Mathematics Project by Sharon Dugdale and David Kibbey, © 1975, Board of Trustees of the University of Illinois.
Make-a-Monster (touch)

Purpose: 1) Re-emphasize that \( \frac{3}{4} \) is 3 of a fourth.
2) Provide experience with a new unit — the cup.
3) Familiarize the student with a model that will be useful later for equivalent fractions and common denominators.

Description: The student uses measuring cups to mix monster formulas in an eccentric scientist's laboratory.

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Darts

Purpose: Experience locating fractions and mixed numbers on the number line.

Description: The student writes numbers or expressions to shoot darts at balloons tied to a number line. Difficulty adjusts to the student's performance. Higher levels have fewer integers on the line and/or smaller balloons to shoot at.

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Sort Equivalent Numbers (touch)

Purpose: Provide practice identifying equivalent whole numbers, fractions, and mixed numbers. This is for students who already have a basic understanding of equivalent fractions and mixed numbers.

Description: Several numbers (whole numbers, fractions, and mixed numbers) are scattered on the screen along with 2, 3, or 4 loops. The student sorts the numbers into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance. This is a practice lesson which will appear one page at a time.

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Purpose: Practice estimating fractional distances on a number line.

Description: This is an interterminal number line game. Two students compete. One controls a boat, the other a submarine. The object is to move to a point above (or below) the opponent's piece and shoot it. The number line varies from game to game.
Purpose: Provide experience adding and subtracting fractions and mixed numbers, simplifying answers and "borrowing" when necessary.

Description: The student is stepped through subtraction problems involving "borrowing." Difficulty adjusts to the student's performance. Higher levels of difficulty give less help and include addition problems as well as subtraction problems. Most, but not all, of the subtraction problems involve "borrowing."

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High Wire

Purpose: Show addition of fractions on the number line with visual feedback about common denominators.

Description: A monkey swings along under a high wire (number line) making an addition problem (e.g., $2/3 + 1/2$ is 2 swings of length $1/3$ and 1 swing of length $1/2$). The student writes a fraction to hop a feather along above the number line. If the student is right, the feather lands where it can tickle the monkey; and the monkey falls off the line laughing. This is a number line illustration of common denominator; the monkey's swings and feather's hops match up to show equivalences. Difficulty adjusts to the student's performance.

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TRUE, FALSE AND OPEN SENTENCES

There are 6 parts to this lesson:

1. The student is asked which of these statements (5) is true/false.
2. The student is asked to make up two true and two false statements.
3. The student is asked to make this true, then false,
   \[ 3 + [\ ] = 5 \]
   and is told that is is called an open sentence.
4. The student is asked which of the five listed sentences are true, false, or open.
5. This part is like part 3 above but has open sentences like
   \[ (3 \times [\ ]) + 7 = 13 \]
   \[ [\ ] + (\ ) = 14 \]
   and \[ (4 \times [\ ]) + 3 = (\ ) \]
   On the last two the student makes it true and false with two pairs of numbers.
6. The student is asked to make up and write true, false, and open sentences (three of each).

Reprinted with permission from Description of Graphing Strand Lessons by Donald Cohen and Gerald Glynn, © 1974 Board of Trustees of the University of Illinois.
There are six parts to this lesson:

1. The student is told the rule: if you have an open sentence with more than one of the same shape, the same numbers must go in the same shapes. Three examples are given:
   \[ 5 + 3 = 8 \]  
   this is an illegal substitution
   and  
   \[ 3 + 3 = 8 \]  
   legal substitutions

2. PLATO gives an open sentence with numbers substituted, the student has to tell if the substitution is legal (2) or illegal (3).
   example \[ 5 + 3 = 14 \]

3. The student is given the open sentence
   \[ (2 \times \square) + \square = 21 \]
   and is asked to make a legal substitution three times.

4. PLATO substitutes in the open sentence five times
   \[ \square + \bigcirc + \square = 27 \]
   the student is asked to tell if the substitution is legal or illegal.

5. The student is asked to make a legal substitution (twice) in
   \[ \square + \bigcirc = 18 \]

6. The student is asked to make a substitution in the following
   \[ \square + \bigcirc + \square = 20 \]
   so that it is legal and false, illegal and true, illegal and false, and finally legal and true (two of each).
INTRODUCTION TO LINEAR GRAPHING

The student is asked to find a □ -number and a ○ -number which will make this sentence true

$$(2 \times □) + 3 = ○$$

He does this, PLATO plots the point which corresponds to this pair of numbers. He does three of these, then is asked to predict three more points that work for this open sentence by looking at the graph. If he picks a point that doesn't work it plots an x instead of an 0, and shows the pair of numbers in the open sentence labeling it "false." Then he can try again. He leaves upon finding three correct points.

There is a geometric pattern which is related to the open sentence and the pairs of numbers.

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PROBLEMS WITH LINEAR GRAPHS

The student is asked to choose one of seven different problems, such as write an open sentence whose graph goes through the point (2,6). The student is given the form \((A \times \square) + B = \bigcirc\) and asked to fill in \(A\) and \(B\). If incorrect, PLATO plots his graph, lists it under "incorrect rules" and he is asked to try again. If the student is correct he is asked to find another way to do it (if that is possible). After three attempts (right plus wrong) he is given another point like (4,9). The student can come back to this lesson seven times.

The seven problems are:

1. Make up an open sentence whose graph goes through the point (2,6)
2. Make up an open sentence whose graph is flatter than the one we give you
3. Make up an open sentence whose graph has the same over-up pattern as the one we give you
4. Make up an open sentence whose graph meets the one we give you at the point (1,5)
5. Make up an open sentence whose graph goes through both points \((0,0)\) and \((2,2)\)
6. Make up an open sentence which will be at right angles \((90^\circ)\) to the one we give you
7. Make up an open sentence which will go through these three points: \((2,5), (3,7),\) and \((5,11)\)

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In problem 1, the student is given the form

\[(A \cdot x \ [\square]) + B = \bigcirc\]

and is asked to put in an A and B. PLATO then plots the graph which corresponds to this open sentence. If it goes through the point (2,6), the student is asked to do it another way. If the graph doesn't go through the point (2,6) the student is asked to try again, listing the incorrect open sentence on the side of the screen. After three incorrect tries the correct open sentence is given.

The other problems work in a similar manner.
TOWER PUZZLE (GAME)

This is a game used later to derive an empirically-determined function.

The object of the game is to move the pile of discs from one peg -- l(left), m(middle), or r(right) -- to another peg.

The rules are:

1. you can only move one disc at a time
2. you can't place a larger disc on top of a smaller one.

The student is shown how this is done for five discs, then can choose the number of discs he wants to play with after choosing his starting and ending peg. The student plays one game, then can choose to play again at another session.
A Sample of Student Work

Some of the lessons on PLATO are "library" lessons. In these lessons the student is able to save his work so others may see it. He can also look at work saved by other students. One such lesson is the Paintings Library. Here the student must paint a particular fraction of a box. If the painting is correct, it can be saved in the library. Here are a few of the paintings students have placed in the library.

This is how Reg I painted 1/5 of the box.

This is how Keiran B painted 2/3 of the box.

This is how Cheryl S painted 1/4 of the box.

This is how Lawton T painted 5/9 of the box.
This is how Shannon painted 2/7 of the box.

This is how Lisa painted 3/7 of the box.

This is how Fred painted 1/2 of the box.

This is how Frederic painted 1/2 of the box.

This is how Joshua painted 5/9 of the box.

This is how Jane painted 1/4 of the box.
Educational Effectiveness

In 1974-75, and again in 1975-76, PLATO Elementary Mathematics materials were used in about a dozen 4th, 5th, and 6th grade classes. In each year, data were also collected from about a dozen roughly comparable non-PLATO classes. In 1976-77 PLATO Elementary Mathematics materials were used in eight 4th, 5th, and 6th grade classes.

Analyses performed at CERL have been oriented toward determining strengths and weaknesses in the curriculum, rather than toward producing an overall "proof of effectiveness". Nevertheless, the available data indicate that having PLATO present in elementary classrooms can have a positive effect on the mathematics achievement of the pupils in those classrooms.

For example, in 1975-76 the Level 2 (Form R) of the Comprehensive Test of Basic Skills, a standardized test widely used in public schools across the country, was administered to most of the PLATO and non-PLATO classes studied* (a different test was used with a few high 6th grade classes). Pairs of PLATO and non-PLATO pupils were found who were essentially identical in entering math level as measured by the Fall (pretest) administration of the CTBS **; these pairs were drawn from classes that were matched by grade level and roughly comparable as to children's ability and neighborhood background. The CTBS was administered again in the Spring as a posttest measure. Results for the matched pair sample are given in the following table. (Units are Grade Equivalents (G. E.).)

<table>
<thead>
<tr>
<th>Fall</th>
<th>Spring</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>PLATO</td>
<td>129</td>
<td>5.0</td>
</tr>
<tr>
<td>non-PLATO</td>
<td>129</td>
<td>5.0</td>
</tr>
</tbody>
</table>

As shown in the table, the average PLATO pupil gained 1.4 G. E. in math during the year, while the average non-PLATO pupil gained 1.1 G. E. There is less than one chance in a hundred that a difference this large could have occurred by accident (matched pair t(128) = 2.807, p = .0058).

*The CTBS data from the 1975-76 trial were gathered under the aegis of Educational Testing Service, Princeton, New Jersey, under an NSF contract. The analysis described was a by-product of formative evaluation work performed at CERL. We thank ETS for sharing the raw data with us.

**Although the criteria used in selecting these pairs were very stringent, the number of pairs obtained was 75% of the maximum number of pairs that could have been obtained if comparable standing on the CTBS had not been required. Hence, the results for the matched pairs may confidently be taken as representative of results for the total sample.
Data analysis for 1976-77 is still in progress, but preliminary results show a mean gain in PLATO classes of two full G. E. Detailed analysis may attenuate this figure somewhat, but the gain by PLATO classes in 1976-77 is clearly greater than the corresponding gain shown in the table above for 1975-76.

There are several reasons why the data discussed above may be taken seriously. (a) Each trial involved hundreds of pupils, not the one or two classes of many educational experiments. (b) Each trial lasted the better part of a school year, not a few hours, days or weeks. (c) There have now been three successive trials in each of which PLATO classes have done better than non-PLATO classes. Furthermore, the superiority of PLATO group performance has appeared to be greater each year than it was the year before.

**Attitudes**

One could imagine an educational treatment that, while effective, might be disliked by pupils as a result of being too demanding or boring. All evidence shows that PLATO is NOT regarded in this way. For example, math pupils seek extra work sessions on PLATO far more often than they try to avoid assigned sessions. Results of a written attitude questionnaire given in 1974-75** show that pupils not only were enthusiastic about PLATO math materials, but that their attitudes toward math itself, and toward their own ability to cope with math, clearly improved with experience on PLATO. Teachers' observations support these findings, even to the extent that access to PLATO is sometimes used as an incentive to improved scholastic and social behavior unrelated to PLATO or to math. Thus, it would appear that the PLATO Mathematics materials, besides being effective, are intrinsically motivating as well. The pupils learn a lot, and they like it.

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*For further information on the change from 1974-75 to 1975-76, see Section 3.5 of the Final Report: Demonstration of the PLATO IV Computer-Based Education System, A Report made under the NSF Contract C-723, Computer-Based Education Research Laboratory, January 1, 1972—June 30, 1976. (March, 1977)

**We thank ETS for sharing with us summarized results of this attitude questionnaire.
Fig. 1. Changes in arithmetic standing (in grade equivalents) of individuals in PLATO and non-PLATO 4th grade classes. Each vector represents a specific student. The tail of the vector shows the student's grade equivalent on the pretest; the head of the vector shows the student's grade equivalent on the posttest. The shaded area on each graph shows the "expected" growth range of a 4th grade student, from 4.0 to 5.0. The bottom row of graphs corresponds to the 3 non-PLATO classes tested in 1975-76. The middle and top rows are the PLATO classes tested in 1975-76 and 1976-77 respectively. The non-PLATO classes were chosen as comparison classes for the 1975-76 PLATO classes. (No non-PLATO classes were tested in 1976-77.) Teacher "f" taught a PLATO class in each of the two years. PLATO vs. non-PLATO differences have usually shown up most clearly at the 4th grade level. The test was the Comprehensive Test of Basic Skills, Level 2, Form R, 1968-69 edition.
Data Feedback to Teachers

PLATO offers the teacher much information about the students' work. The following types of feedback have proved most valuable:

1. An over-all view, grade-book style, of students' progress and performance in selected key lessons.

2. A list of students who appear to be needing human help in important lessons. The student's name is listed with the name of the problem lesson and a short explanation of the difficulty.

Although this information has been available to the teacher through the same terminal that the students use, teachers have appreciated the extra convenience of having a hard-copy of these two features mailed to them once each week. They have used the terminal to see up-to-the-minute information and to see more detailed data. For example, many lessons report diagnostic data to help teachers pinpoint the student's difficulties.

References

More information about the PLATO elementary mathematics curriculum is available in the following publications:


Dugdale, Sharon and David Kibbey, Programs from the Skywriting and Spider Web Library: A Sample of Student Work, October 1975, CERL Publications, University of Illinois, Urbana, Illinois.