Ways in which the emerging field of cognitive science can have a major impact on practical education are demonstrated by an overview of two projects: the work on Socratic tutoring by Collins and Stevens, and the diagnosing students' misconceptions in arithmetic by Brown and Burton. These two projects are cited as examples of how the techniques of cognitive science make it possible to analyze teaching and learning in genuinely novel ways.

(Author/CMV)
EXPLICATING THE TACIT KNOWLEDGE IN TEACHING AND LEARNING

Allan Collins

Bolt Beranek and Newman Inc.
Cambridge, Massachusetts 02138

Expiration Date: September 30, 1978
Total Amount of Contract - $337,000
Principal Investigator, Allan M. Collins (617) 491-1850

Sponsored by:
Office of Naval Research
Contract Authority No. NR 154-379
Scientific Officers: Dr. Marshall Farr and Dr. Henry Halff

and

Advanced Research Projects Agency
ARPA Order No. 2284, Amendment 4
Program Code No. 61101E

The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Advanced Research Projects Agency, the Office of Naval Research, or the U.S. Government.

Approved for public release; distribution unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government.
Explicating the Tacit Knowledge in Teaching and Learning

Also supported by ARPA.
Presented at AERA symposium on "Modern cognitive science and the challenges of practical instruction" Toronto, March 1978

The paper attempts to show how the emerging field of Cognitive Science can have a major impact on practical education. In particular it provides an overview of two projects at Bolt, Beranek & Newman: the work on Socratic tutoring by Collins and Stevens and the work on diagnosing student's misconceptions in arithmetic by Brown and Burton. These two projects are cited as examples of how the techniques of Cognitive Science make it possible to analyze teaching and learning in genuinely novel ways.
ABSTRACT

The paper attempts to show how the emerging field of Cognitive Science can have a major impact on practical education. In particular it provides an overview of two projects at Bolt, Beranek & Newman: the work on Socratic tutoring by Collins and Stevens and the work on diagnosing student's misconceptions in arithmetic by Brown and Burton. These two projects are cited as examples of how the techniques of Cognitive Science make it possible to analyze teaching and learning in genuinely novel ways.
The question I decided to address is whether Cognitive Science has anything new to say about education, given that some of the world's best minds have been thinking about education for thousands of years. The answer I think is yes; Cognitive Science provides a set of theoretical formalisms and analysis techniques that can be used to study teaching and learning of topics like math and sciences in a genuinely novel way. My plan is to show how this is possible with two case studies: first I will outline some work Stevens and I (Collins, 1977; Stevens & Collins, 1977; Stevens, Goldin & Collins, in press) have done on tutoring by the Socratic method, and second I will describe work of Brown & Burton (1978) on diagnosing students' procedural errors in arithmetic. These are only examples of the kind of analyses possible.

In our work on Socratic tutoring, we analyzed dialogues by a number of tutors teaching different subjects (medicine, geography, etc.) using the Socratic method. The Socratic method is made up of a variety of individual strategies. These strategies involve entrapping the student into mistakes of different kinds; confronting him with counterexamples, forcing him to make predictions, etc. Generally the method requires the student to derive general principles from specific cases, and learn to use these principles to make predictions about new cases.
Using the production rule formalism developed by Newell & Simon (1972), it was possible to characterize the different strategies tutors use in terms of rules of the form "when in situation X, do Y." We can give a few examples to show what these rules look like:

**Rule 1:** Form a general rule for an insufficient factor.

If

1. the student gives as an explanation one or more factors that are not sufficient,

then

2. formulate a general rule asserting that the factor given is sufficient and ask the student if the rule is true.

Example: If the student gives water as the reason they grow rice in China, ask him "Do you think any place with enough water can grow rice?"

**Rule 2:** Pick a counterexample for an insufficient factor.

If

1. the student gives as an explanation one or more factors that are not sufficient, or

2. agrees to the general rule in Rule 1,

then

3. pick a counterexample that has the right value of the factor(s) given, but the wrong value of the dependent variable, and

4. ask what the value of the dependent variable is for that case, or

5. ask why the casual dependence does not hold for that case.
Example: If a student gives water as the reason they grow rice in China or agrees that any place with enough water can grow rice, pick a place like Ireland where there is enough water and ask, "Do they grow rice in Ireland?" or "Why don't they grow rice in Ireland?"

Rule 3: Question a prediction made without enough information.

If
(1) a student makes a prediction as to the value of the dependent variable on the basis of some set of factors, and
(2) there is another value consistent with that set of factors, then
(3) ask the student why not the other value.

Example: If the student predicts they grow wheat in Nigeria because it is fertile and warm, ask him why not rice.

While in one sense the Socratic method is a single approach that involves teaching the student to reason from cases, in another sense it is made up of a variety of these specific strategies that good teachers hit upon in the course of their teaching. Some hit upon one set, some upon another, though there is usually some overlap. There is little need for teachers to verbalize these strategies, since their application only depends on an intuitive feel as to how to use them. If they are taught, they are usually taught by example. So there is no very specific body of knowledge about the Socratic method, and hence there is no theory to be extended and refined. In fact until computers provided us with formalisms for expressing "process models,"
it is unlikely that anyone would have thought of constructing a specific theory about such a thing as the Socratic method.

By making the tacit knowledge about Socratic teaching explicit, it becomes possible to educate teachers about the Socratic method. We can teach them the strategies that many different teachers have developed and the situations where they apply. If they were to evolve the method on their own or from watching other good teachers, they would at best develop only a subset of these strategies.

More generally the method of analysis can be applied to analyze the specific strategies used by the most effective teachers in the country. Such an analysis can be derived from videotapes of actual classes. The analysis would determine the teacher's systematic patterns of response to particular teaching situations. It is a kind of formal analysis that simply was not possible in terms of earlier formalisms in psychology, such as S-R bonds or stage models. Simply put we can now construct a formal theory of the strategies used by our most successful teachers, and we can thereby make their accumulated tacit knowledge available to every potential teacher.

My second example concerns modelling the learner to diagnose his misunderstandings. In a system called BUGGY, Brown & Burton (1978) developed a representation called a procedural network in which they represent all the procedures necessary to carry out addition and subtraction in explicit data. In BUGGY, procedural errors are represented as perturbations of the correct procedures. In this way the program can simulate any
consistent procedure that students follow even if it is a wrong procedure, as in the example below.

Consider five "snap shots" of a student's performance doing addition as might be seen on a homework assignment. Before proceeding, discover the student's bug.

Sample of the student's work:

\[
\begin{array}{cccccc}
41 & 328 & 989 & 66 & 216 \\
+9 & +917 & +52 & +887 & +13 \\
\hline
50 & 1345 & 1141 & 1053 & 229 \\
\end{array}
\]

Once you have discovered the bug, try testing your hypothesis by "simulating" the buggy student so as to predict his results on the following two test problems.

\[
\begin{array}{cc}
446 & 201 \\
+815 & +399 \\
\hline
1361 & 700 \\
\end{array}
\]

The bug is really quite simple. In computer terms, the student, after determining the carry, forgets to reset the "carry register" to zero and hence the amount carried is accumulated across the columns. For example, in the student's second problem, \((328 + 917 = 1345)\) he proceeds as follows:

\(8 + 7 = 15\) so he writes \(5\) and carries \(1\), \(2 + 1 = 3\) plus the one carry is \(4\), lastly \(3 + 9 = 12\) but that one carry from the first column is still there -- it hasn't been reset -- so adding it in to this column gives \(13\). If this is the bug, then the answers to the test problems will be \(1361\) and \(700\). This bug is not so absurd when one considers that a child might use his fingers to remember the carry and forget to bend back his fingers, or counters, after each carry is added.
It turns out that it is very difficult for experts to diagnose systematic errors that BUGGY makes, especially when the errors are in low level procedures that are called by different high level procedures, or when there is more than one bug in different procedures. To a teacher, the students' errors are likely to look random.

One way the BUGGY program can be used is to train teachers and students how to diagnose bugs. The program selects a bug, and gives a few examples of the bug as manifested in working problems. The user then selects problems for BUGGY to work until he thinks he has figured out the bug. Then BUGGY presents him with five test cases where he must simulate the bug. If the user cannot produce the same answer as BUGGY on any of the test cases he then goes back to trying to diagnose the bug. In this way teachers can be taught how to be expert diagnosticians.

More importantly Brown & Burton have used BUGGY to analyze automatically a subtraction test taken by 1300 students in the 4th to 6th grades. They found that about 40% of the students used essentially correct procedures, about 20% have what appear to be random errors to BUGGY, and about 40% have systematic procedural errors. From their data they have compiled a list of the 20 most frequent errors in subtraction and their frequencies among grade school students. The three most common bugs are the following:

1. **Borrow from zero** (e.g., \(103 - 45 = 158\)) Frequency 107/1300
   When borrowing from a column whose top digit is 0, the student writes 9, but does not continue borrowing from the column to the left of the 0.
2. Smaller from larger (e.g., 253-118 = 145) Frequency = 54/1300
   The student subtracts the smaller digit in a column from the larger digit regardless of which one is on top.

3. Difference of 0-N = N and jump over zero in borrowing (e.g., 204-25 = 129) Frequency = 34/1300
   When ever the top digit in a column is 0, the student writes the bottom digit in the answer; i.e., 0-N = N. When the student needs to borrow on a column whose top digit is 0, he skips that column and borrows from the next one.

The implications of BUGGY for testing are profound. No longer need a test be a means to assign a score to a student. A score is an arbitrary measure that tells very little about how well the student is doing or what his real problem is. For example, a single bug in a low level procedure will produce many more wrong answers than several bugs in higher level procedures. What a test becomes, if BUGGY is taken seriously, is a method for diagnosing the student's underlying errors. The outcome is a statement of whatever specific procedural errors the student is making. Then teaching can be directed explicitly at the bugs the test has diagnosed.

Surely before BUGGY arithmetic had been studied very thoroughly. But by applying a Cognitive Science analysis, Brown & Burton could find new insights into the problems children are having. Other basic skills (higher mathematics, reading, writing, etc.) are more difficult to analyze in this way, but the lesson should be obvious. There is enormous leverage in this kind of analysis and the potential implications for practical education are profound.
REFERENCES


