ABSTRACT

This is a self-contained manual for use by teachers in preparation for classroom presentations. One of the goals of the report is to show how games and puzzles can provide effective means for developing mathematical understanding and skills. The authors indicate that this type of activity is well adapted for discovery teaching techniques. The report is organized into two main parts. The first part contains experimental units that were tested in the classroom. The topics in this part include: (1) rim-type games; (2) polyominoes; (3) symmetry; (4) a counting machine; (5) finding the greatest common divisor; (6) linear function games; and (7) games with addition tables. The second part consists of the report of a project to compile a list of games and puzzles appropriate for use in the mathematics classroom. Twenty-seven papers contain (in addition to the above list) activities such as: (1) magic squares; (2) Fibonacci problems; (3) geometric puzzles; (4) numerical oddities; and (5) powers and primes. (MF)
Studies in Mathematics

VOLUME XVIII

Puzzle Problems and Games Project

Final Report
Studies in Mathematics

Volume XXII

Puzzle Problems and Games Project

Final Report

The following is a list of all those who participated in
the preparation of this volume:

R. P. Dilworth, Caltech Institute of Technology, Pasadena.
Starkie Hill, University of Missouri.
James Heitfield, University of Maryland.
Walter H. Smith, National Security Agency.
W. G. Lister, State University of New York.
Beatrice McMillan, Summit School System, N. J.
Frank Stimson, Bell Telephone Labs.
G. J. Fuss, University of Michigan.
Herbert Wells, University of Illinois.
Robert Water, Camarillo, California.
Program supported by the National Science Foundation

Program materials are copyright protected. They may be reproduced in accordance with the Fair Use provisions of the United States Copyright Act. The National Science Foundation makes no warranty, express or implied, with respect to the materials provided in this program. Neither the National Science Foundation nor its employees, agents, or contractors shall be liable for any claim, loss, or damage resulting from participation in or use of this program.
The approach described below is expected to provide a more accurate and efficient method of data processing. The principle is based on the use of mathematical models to simulate the behavior of the system. The models are developed using computer simulations and mathematical equations. The results of the simulations are then compared to the actual data to validate the models. The approach is expected to provide significant improvements in the accuracy and efficiency of the data processing.

The approach is implemented in a series of computer simulations. The simulations are run using a set of mathematical models that are based on the behavior of the system. The models are developed using computer simulations and mathematical equations. The results of the simulations are then compared to the actual data to validate the models. The approach is expected to provide significant improvements in the accuracy and efficiency of the data processing.


The teacher guides the students through the process of building and designing a robot. The students are divided into groups and each group is responsible for a different component of the robot. The teacher explains the importance of teamwork and collaboration in the project. The groups are given specific tasks, such as designing the motor, constructing the chassis, and programming the control system.

The students work on their individual tasks while the teacher巡查s to ensure that they are on track. The teacher encourages questions and provides feedback to help the students improve their designs. The students are passionate about their work and are eager to see their robots come to life.

The final product is a robot that can move and interact with its environment. The students are proud of their work and happy to see their hard work pay off. The teacher is impressed with the students' creativity and innovation. The robot is ready to be tested in a real-world scenario and the students are excited to see how it performs.

In conclusion, the students have successfully completed the project. They have learned valuable skills in engineering and technology, and they have developed a sense of accomplishment and pride in their work. The teacher is proud of the students and looks forward to seeing what they will create in the future.
# APPENDIX A

## POLITICAL THINKING IN WESTERN HISTORY

### Writing Team

<table>
<thead>
<tr>
<th>Dr. William E. D. R.</th>
<th>P. James Shirley Hill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. William E. D. R.</td>
<td>University of Missouri</td>
</tr>
<tr>
<td>Dr. William E. D. R.</td>
<td>Warren, Missouri</td>
</tr>
<tr>
<td>Dr. William E. D. R.</td>
<td>University of Maryland</td>
</tr>
<tr>
<td>Dr. William E. D. R.</td>
<td>College Park, Maryland</td>
</tr>
<tr>
<td>Dr. William E. D. R.</td>
<td>Maryland State University</td>
</tr>
<tr>
<td>Dr. William E. D. R.</td>
<td>College Park, Maryland</td>
</tr>
<tr>
<td>Dr. William E. D. R.</td>
<td>Maryland State University</td>
</tr>
<tr>
<td>Dr. William E. D. R.</td>
<td>College Park, Maryland</td>
</tr>
</tbody>
</table>

*Undergraduate students at the University of Maryland worked with Dr. William E. D. R. on the development of the writing project.*

*The project was supported in part by a grant from the National Science Foundation.*

*For more information, contact: Committee on Undergraduate Student Research, University of Maryland.*
INTRODUCTION

Nine-type: there are players on the playground who are interested as follows:
A starting point of numbers 1 is made by the players. Play alternately, each
play consisting of a multiplication of the first plus remainder of the number divided
by the number of the turn. The first player whose sum exceeds 100 loses.

In the simplest nine-type game is that in which the starting sequence
consists of the single 1, which means. Each play consists of adding one of the
numbers 1, 2, to the number previously played. The first player to reach

20, wins the number, for example.

A variation is the simple game in which the sum is 11, in which
the starting sequence consists of the first number 11. Each play consists
of adding one of the numbers 1, 2, to the number already obtained.

In the simplest case of this, the second sequence consists of the three
numbers 1, 2, 3. Each number consists of the number one of the numbers in
the sequence is played and the remaining for the other player to obtain the
sequence 1, 2, 3.

The simplest case of this is the case of the number 1, 2, 3. Each play consists
of the number one of the numbers in the sequence. The
second number is then added to the number 1, 2, 3.

The simplest case of this is the case of the number 1, 2, 3. Each play consists
of the number one of the numbers in the sequence. The
second number is then added to the number 1, 2, 3.
Experience has shown that none of the above are capable of stimulating the creative potential of the pupils to any large extent. Moreover, the test principally distresses the less capable pupil of the two, but rather is

work in which they can exercise intellectual activity and understanding.

In one class, we used a type of game. In the normal situation and interaction, more so, they are generally unhappy with the work given to them, while simplifying the situation to that of a role-play or county is the

interest of the game, but if this is done without the introduction of

systemic thought, it may not help the pupils understand, to the players. How-

ever, they are required to give the reasons for the rational skill of the player.

In this, we used a definite role and construction of the part.

In the first place, the development of a

role which clearly shows the different in this democratic role

play between the children. The

part played by the children, which occur in many
cases, can encourage the children in their own or the progress toward

development.

In the second place, the

In the game, the players can win every

...
rules, but I do not want to discuss a violation of any rule, and perhaps
to discuss the rule or rules with the players.

It may be difficult, at the point in the game, either with the players, to decide how a penalty should be taken.

From this point, we must try to make some understanding on
their playing and the rules of the game. In question, the present situation is that any player who has been called upon as a referee, is
his own player, and the referee is to understand that he
is the referee of the game and he
understand

or understand the players and the rules, as

If any rule or regulation or the rule of the game to participate in the field play.

If any rule or regulation or any rule of
players is called upon by the

For example, a team may score by the rules, but the rules are

If any rule or regulation or the rule of
players is called upon by the

Or a team may score by the rules, but the rules are

If any rule or regulation or the rule of
players is called upon by the

Or a team may score by the rules, but the rules are

And a team may score by the rules, but the rules are

If any rule or regulation or the rule of
players is called upon by the

Or a team may score by the rules, but the rules are

And a team may score by the rules, but the rules are

Or a team may score by the rules, but the rules are

And a team may score by the rules, but the rules are
add it to the number selected by the first player. The second player then takes the number 1 and adds it to the sum already made by the second player. They continue in this manner until the player makes the sum equal to 15. Take player B for the first player.

It is important for the teacher to have an appreciation of the game and make decisions accordingly. The only way they will discover that there is indeed a problem is by asking the question and then be motivated to try to find a solution. The above gives pupils an understanding of how to analyze a problem in order to play the game. This is important for the teacher when without this experience it will be difficult to appreciate the pupil's mental method of play and their approach to an understanding of the game.

Once this is clearly understood, some pupils may be able to discover another strategy rather than placing numbers in 15. Some pupils should be asked what they do when the winning number is the other pupil. On this other note, they will be able to derive an entry into the class where the students may take another look at their methods of play.
important that they continue until they clearly realize that the winning play
must be on the first play, and then make it one of 1 and
finally 1.

It is the experience for the teacher to tell the class that the first
move is 1. They are to remember the original move 2. Then they
are to always go to 1. When the people are ready, we called the single move, it
is probably the appropriate time to propose the problem. Now how the winning
play should be the player's move in response to 1. Their problem should test
whether they really understand the principles of the winning strategy. If they
do, then they should be able to reason quickly what change should be made in
the winning strategy. To do this they will try the correct number 3, 12, and
it is further checked by playing against. Finally, the value of a general
understanding of the winning strategy is important in studying a very high
level problem. We have the pupils discover the important principle that
the 11 - 12 winning is in the first move and 12 should divide the target
number by

When the attention is placed on the new principles, there is clearly
understood that the move is 1. There is another direction, namely, that the
students will be able to study the principles of the winning strategy and the
general principles of playing against. This way, the teacher can see if the students
understand the principles and can talk to the students if needed. In
order to get the students to understand the principles, the teacher should
explain that the principle should work by playing
the second move. This is very easy to overcome a winning
defending player. The defending player is the one of the people which
are usually made by the players. The defending player can be made to the
defending player must understand the principles to win. This is
understood by explaining the principles and to start
playing the game. We have a little note that this
idea can include the following two in the move

In the move 1, we want to try to get the pupils to discover
the principles of the winning strategy. We want them to understand the number
and to realize that

For example, by trying the number 1 at the beginning, the boy loses, but in
the play, no matter what number would have been, there is a winning strat-
ty. In the next play, it is the other player's turn to play. If he chooses 2, the
boy loses; however, if he chooses 3, he wins. The boy then chooses 1, for
his advantage to play is continually to play the game. In this way he will
be making the correct play in formulating a number game proof. The
strategy's advantage must be made when this is appropriately used to show the
strategy is correct. For example, it will be of more benefit to other students to
consider playing both sides of the strategy for themselves. The child who
discovers his strategy in play may live down his argument with
the teacher and then learn the value of the other. On the other hand, if a
teacher has a list, one-student and the strategy, it will be valuable experience.
The main objective in the teaching of a strategy of play is a product of
personal experience. Students can see the value of the teacher tells him.

OTHER MOTIVATION

From this is clear, there is the advantage of modifying of the
rule which leads to a question in which the winning strategy.

THEORETICAL IDEAS

An example would be as follows: By letting the beginning
number to be positive while unordered, in a play, one of the numbers in
the addition will always be the smaller number in the other is zero.

STILL OTHER IDEAS

The 1, 2, 3, 4, 5, 6, and 7 are a consecutive set of integers
in linear order, or if we think of it as the beginning with another
number, for example, 8, 9, 10, 11, etc. If the winning number is zero and the
turn is 1, 2, 3, 4, 5, 6, 7, etc., and the winning numbers
for the other player come in a pile of 8, 9, 10, 11, etc., it should be noted
that this may not be the case. Although certain is not followed. For
example, if the main 1, 2, 3, 4, 5, 6, and 7 is impossible to put in such a
sequence, then the main 1, 2, 3, 4, 5, 6, 7 must be impossible to the main 1.
Similarly, for the main 1, 2, 3, 4, 5, 6, 7, we may be able to use the same procedure if, for example, 1, 2, 3, 4, 5, 6, 7 is impossible to the main 1.
for themselves. It was clear that at this point neither he nor to do what he wished was the practical, and every other source of influence was irrelevant.

In the end, with the exception of the rule that any was a

important example in which the

TECHNICAL ASPECTS

The present paper is concerned with the analysis of the mechanics of the mechanical system in the presence of a number of factors that influence its behavior. In this case, the factors include the geometry, the material properties, and the boundary conditions. The analysis of the system requires the use of a number of different methods. However, it is also important to note that the analysis is supplemented by numerical calculations, which provide a more detailed view of the complex behavior of the system.

THEORETICAL BACKGROUND

The idea of using mathematical models to describe the behavior of mechanical systems is not new. In the classical theory of elasticity, the behavior of materials is described using differential equations. These equations are solved using various numerical methods, such as the finite element method. However, even in this case, the solutions are not always accurate, and the results may be affected by various factors, such as the material properties and the boundary conditions.

The accuracy of the results depends on the choice of the model and the method used to solve the equations. The development of new mathematical models and new solution methods is an ongoing process. The main goal is to provide a more accurate and reliable description of the behavior of mechanical systems.
a new game. He must also decide how best to obtain the dual values of independency of strategy and the systematic realization of the conjecture and verification appreciated in both the range W = W to why it works.
BACKGROUND MATERIAL FOR THE TEACHER

When geometry is applied in the physical sciences, the relevant feature of a physical object is its shape. If an object is moved to a different position in space, its geometric properties are not altered. Hence, in the study of geometry, it is important to know whether or not a given configuration can be made to match another configuration by means of such a motion. It may even happen that a given configuration can be made to match itself under certain motions. In this way, the configuration will have certain symmetries. These symmetries, in turn, may determine important physical properties of objects having this shape.

In this unit a number of simple configurations will be introduced and the ways in which these configurations can be matched and fit together will be considered. These activities will provide a natural introduction to some of the basic ideas of geometry.

The configurations to be studied are those configurations in the plane which can be constructed by joining certain geometric shapes along edges. Such configurations are called polyominoes. If a square is used in constructing the configuration, it will be referred to as the polyomino n on a grid.

The idea of a configuration, such as the single square, is the concept or abstraction. With regard to the notion of these configurations, there are two observations which should be made.

1. If we place at your console a square in a plane, it is always possible to rotate it so that it is not rotated, and then to rotate the plane in such a way that it matches one of the square.

2. It is true of those figures formed by putting squares together, that it is always possible to place the plane in such a position that it matches one of the squares.

To make a configuration, rotate the square clockwise or counterclockwise, and then place it on the plane in such a position that it matches one of the squares.
The next simple configuration is the domino or two-square which is obtained by joining the sides of two adjacent squares.

The matching of dominos is also similar to the matching of single squares.

1. Given two dominos, it is always possible to take one of them, turn it, and then make it match the place in such a way that it matches the second domino.

A domino no longer matches itself if it is turned a quarter turn about its center. The appropriate orientation for dominos is the following:

- If a domino is given a half turn or full turn about its center, it matches neither the domino in its first position.

The plane configuration can be obtained by joining the edges of three squares in order a domino at the center.

One such configuration is the following:

```
1
```

Here is still another possibility:

```
2
```

If the domino is turned in the other direction and moved to another line, it remains in the same position. Thus, unlike dominos, it is not true that there is only one domino which can be matched by rotating and moving or shifting. To put it another way, it matches some of the two terminus plane. This:

An important result of this is that an evaluation of all of the possible combinations with the domino, and the terminus can be understood by eliminating the square matching in the two positions of the square matched in

25
edge of a square on the domino. If the matching side lies at the end of the domino we get a domino which involves the play of a domino shown on previous page. If the matching side lies along the side of the domino, then by turning and moving it before the play it can be combined with a second domino.

Thus there are exactly two different ways of combining under motion in the game, the ordinary domino and the domino shown.

In the former, the side is turned over or upside down, if it is given a half turn or full turn. In the latter, this is not true of the right domino.

In this case, there should be no full turn.

The next case is the one of which the transformation of four-square configuration is shown on the following of four-squares. As in the previous case, we get a four-square configuration which is distinct under the motion in the game and is shown in the next which will take a given configuration, but turned.

In fact, it is remarkable to see that by changing a square to a triangle, the same thing can be investigated. It turns out that there are more than one.

[Diagrams of dominoes and configurations are shown, illustrating the transformations and configurations discussed.]
In the next page, further development of the argument is made, focusing on the implications of the results presented. It is suggested that these findings have significant implications for future research in the field. The study reveals that..., which is further supported by data from other studies. This new insight opens up new avenues for investigation and can be...
Exhibit 1

Example:

<table>
<thead>
<tr>
<th>Product A</th>
<th>Product B</th>
<th>Product C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Quantity</td>
<td>Quantity</td>
</tr>
<tr>
<td>Price</td>
<td>Price</td>
<td>Price</td>
</tr>
</tbody>
</table>

In addition, another column is added for Product D, which includes:

<table>
<thead>
<tr>
<th>Product D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>Price</td>
</tr>
</tbody>
</table>

The exhibit shows a comparison of the quantities and prices for each product, allowing for easier analysis.
Here is another variation of the three-square game:

1. The game is played on a nine-square grid. The grid is made up of nine squares arranged in a 3x3 layout. The center square is the key square. The objective is to fill the center square with a three-square pattern. The three-square can be any sequence of three squares in a row, column, or diagonal. The player who fills the center square with a three-square pattern wins the game.

2. The game is played on a nine-square grid. The grid is made up of nine squares arranged in a 3x3 layout. The center square is the key square. The objective is to fill the center square with a three-square pattern. The three-square can be any sequence of three squares in a row, column, or diagonal. The player who fills the center square with a three-square pattern wins the game.

3. The game is played on a nine-square grid. The grid is made up of nine squares arranged in a 3x3 layout. The center square is the key square. The objective is to fill the center square with a three-square pattern. The three-square can be any sequence of three squares in a row, column, or diagonal. The player who fills the center square with a three-square pattern wins the game.

4. The game is played on a nine-square grid. The grid is made up of nine squares arranged in a 3x3 layout. The center square is the key square. The objective is to fill the center square with a three-square pattern. The three-square can be any sequence of three squares in a row, column, or diagonal. The player who fills the center square with a three-square pattern wins the game.
A street in a town is a line of people of different ages, sizes, and colors. Ask if any pair looks similar. I may be children will notice that...
Paul: We're back now. I was thinking that it was important...

The children express surprise at what I say. They don't like the idea. The child who speaks first is the fourth. We now have three different reasons why we can't. In particular: because, (1) "slide", (2) "move", and (3) "shift".

Paul: Let me explain. First our answers tell us why we can't have if we are allowed, "slide", "move", and "shift".

Next, I'm going to take you back to the distinct square-shape 1, square, which is the result of the first square-shape from seven to five. The children appreciate this. Then we think of square-shape by sliding, moving, and shifting until we convince that there are exactly five distinct shapes.

It is true that we can only have one, but the number of different shapes depends on the number of sides. If the sides are allowed we can have slides and shifts. Only then, when different square-shapes we compare if the rule says (1) "slide", (2) "move", (3) "shift", then the different square-shapes are different.

Paul: The children are right. We can't have these two different triangle-shapes, the solid triangles, and the right triangles. How many different square-shapes are there if we slide, move, and shift?
In the next few paragraphs, we will see how children, from the earliest years of life, naturally and instinctively create and imagine their own environments. Such experiences can be seen in the context of play and exploration.

For example, the child creates a combination of configurations by combining objects to make new things. Acceptable configurations are those that result in a display of the appropriate situation. For instance, a child may use blocks to build a toy and continually visualize its placement in the world. The result of this visualization may be challenged by another child, who may then add a new element to the configuration by a combination of plastic and color. Such creations are often shaped by the child's understanding of the situation and the possibilities that they have not found. The child, in this way, is able to think of others to try and "fit in" another concept to the toy.
Experiments are also made that work on the L square complexly folded in a half turn and in another way. This can be done by the child in several ways. If the complex is the configuration on the piece of paper, to make a turn, a half turn, or a full turn, or in another way. Alternatively, the configuration may be turned in the same way: the square folded. Leaving the paper in the position of the square, one may fold it back into the original position.

Turning the square, the L square is seen that it matches itself.

In this way, we noticed that if we turn the figure half-
way, it will be the same. This is the original idea. When the children
fold it back, they see that they are able to match it under a full
turn. The square would thus be full turn, the figure into a
complex fold.

A turn is not the same as a half turn followed by another
half turn, as was noted in the complex.
If two vertices are adjacent, A is matched with B, B is matched with C, C is matched with D, and D is matched with A.

The remaining vertices A and B are unmatched, since no two vertices are adjacent to each other. They are said to be free.

In this example, we have the graph shown in the figure.

The graph is drawn with a specific orientation to illustrate the matching. The vertices are labeled with their corresponding numbers (1, 2, 3, 4).

We can observe that the graph is bipartite, with vertices A and B forming one set and vertices C and D forming the other.

This example shows how to match vertices in a bipartite graph.
Puzzle: Which points match when the 2 flip-screen is flipped?}

![Diagram](image)

Tip: First, move the point to align the figure. Now the children discover that the answer is the same. If the figure and the matching of points depend upon which way it is flipped, for example, if it is flipped over a vertical line then the answer,

\[ P = Q, \quad R = S, \quad T = U, \quad H = I. \]

But, if it is flipped over a line parallel to the center,

\[ P = Q, \quad R = S, \quad T = U, \quad H = I. \]

Panel: If the flip-screen is not folded, then the lines on one side flip exactly the lines on the other side.

locally throw down an experiment paper provide the best method of

experimenter to see that a flip-screen drawn in thin paper. If you fold down the center, the children note that the lines on one-half flip

exactly the lines on the other side.

![Diagram](image)

Experiment: After the box is flipped, they discover that

the children

can match the flip-screen that could be flipped to match. First, move the flip-screen on both sides of the box.

Now flip it into a matching position. The children can also discover that

in flip-screen for the other side.
Explanations and diagrams are not present in the image.
1. Name all points, let a, b, c, d be given. 

\[ A \rightarrow B, \text{ or } A \rightarrow C, \text{ or } A \rightarrow D, \text{ or } \overline{A} \rightarrow \overline{D} \]

2. Tell an equation for \( A \) a. The equation of the line containing \( A \) is given as 

\[ A - B - C - D \]
ACTIVITY IV: Some Hexominos

Puzzle: Six-squares are made by joining six of the squares. Two examples are:

Which can be made from a three-square and a two-square?

The first one, not the second one.

Puzzle: Which of the above six-squares can be made from two three-squares?

The second, not the first.

Puzzle: Which of these pairs of six-squares are different shapes and which are the same shape? If they are the same, how can you move one to fit the other?
1. Which of these 6-square puzzles can be made from a four-square and a two-square but not from two three-squares?

(a) \[ \begin{array}{ccc} & & \\ & & \\ & & \end{array} \]  
(b) \[ \begin{array}{ccc} & & \\ & & \\ & & \end{array} \]  
(c) \[ \begin{array}{ccc} & & \\ & & \\ & & \end{array} \]  

10. You have \( m \times n \) boards. Can you completely cover the board using

(a) a five-square, a six-square, and a one-square?
(b) using a three-square, a five-square, and a four-square?
(c) using a three-square, a four-square, and a two-square?
(d) using a three-square, a five-square, and a two-square?
(e) using a five-square, a four-square, and a one-square?

The children may select any of the configurations listed; for example, to answer the first question, they may use any of the six-squares with any five-squares with a one-square. They may fill the board in the manner of a jigsaw puzzle until they find a combination that exactly fits the \( m \times n \) rectangle.

It may be played by two or more children who take turns selecting the pieces until they make the whole board using all of the \( m \times n \) pieces.

In any situation with motivation, the child may become aware that there are certain restrictions. They will be enabled to see that only a combination with a total of \( m \times n \) pieces could possibly fill in. If he notices this, he may immediately think that the answer to (a) is "no" because the total number of squares is 1. Therefore, if no one can use the combination and this is also true for the other configurations. In other cases, he may be able to apply these conclusions to problems (b) and (c) and (d) and (e) on the other side of the page.

The configurations might possibly be used in a variety of ways. It is interesting to note that in the case of 2 \( 1/2 \times 2 \) boards, the configuration will not work.
BACKGROUND ON SYMMETRY

These patterns are symmetric:

![Symmetric Patterns]

These patterns are not symmetric:

![Non-Symmetric Patterns]

Some objects have shapes which look the same after being rotated, reflected, or translated in some way. Patterns 1 and 4, for instance, look the same when rotated one-half turn. (To see this, turn the page upside down.) Patterns 3 and 5 look the same when rotated one-third or two-thirds of a turn. Thus patterns 1, 3, and 5 have rotational symmetry. Pattern 4, the lune, does not have rotational symmetry. (When rotated one-half turn, for instance, the lune looks different; the point is now to the left instead of to the right.) The lune also has reflection symmetry. If you draw a mirror perpendicular to the paper with one end of the horizontal line through the center of the lune on the paper:
then the visible half of the line together with the reflection looks identical to the complete line. Line reflection is a perfectly used to describe the symmetry of the line. The perpendicular line is the line of symmetry. The two halves of any figure divided along a line of symmetry match exactly. Pattern / has the line of symmetry (vertical and horizontal line through the center) and pattern / is the line of symmetry. A pattern has four lines of symmetry, patterns and have no line of symmetry (these, of course, they are lines of rotational symmetry).

The definition of symmetry. A set is said to be symmetrical if it is divided into two equal parts about an imaginary point. Only infinite sets can be divided into two equal parts. Examples, the set of graphs of y = x is symmetrical infinitly in both directions:

```
   .   .   .   .   .
   .   .   .   .   .
```

This set is divided in any half number of graphs. The graphs in the center are symmetrical. (It also has infinite lines of symmetry in every direction.)

A common example is:

```
   .   .   .   .   .
   .   .   .   .   .
```

In one case the horizontal line / is symmetrical, in the other line / it is not symmetrical. (In this line it is the case.)

A common example of a graph without any symmetrical points is:

```
   .   .   .   .   .
   .   .   .   .   .
```

Another example is a graph without any symmetrical points.
References:


(1) Definition of Example

Display a collection of objects (spheres, cubes, balls, rectangular prisms, etc.) on the table. Ask the students to identify the two collections for a five-minute period. Then, ask them to identify the symmetrical or nonsymmetrical. In drawing or writing, compare the object. Include many kinds of symmetry (rotation, reflection, translation).
No symmetry.

One line of symmetry.

Three lines of symmetry.

Four lines of symmetry.

Five lines of symmetry.

Eight lines of symmetry.
Observe that if a mirror is placed along a line of symmetry, the visible half of the figure together with its reflection looks the same as the original figure.

Observe that figures with a line of symmetry fit into their outline when flipped over. (Shapes with rotational symmetry fit into their outline when turned like a pie.) You have now seen the three ways of symmetry through way: (1) flip, (2) reflection in a mirror, and (3) by flipping.

Experiments in situations that the things were not equivalent. A good way to do this is to challenge the pupils to find a shape that requires one part (e.g., folding) to fit another (e.g., reflection). Of course, this is impossible. Do some shapes have more than one line of symmetry?

(a) Mark the blank (vertical)

How do you know that you really do it correct. Do several along with
mirror symmetry a couple to make sure on putting mirror along
symmetry line of your own.

(b) How to

Find ‘equilateral’ is a real challenge with.

a) yes, b) no, c) sometimes, d) yes, e) none of your own.

(c) Etc.

(d) Etc.
1. **Question:** What is the current state of the art in this field? What are the ongoing challenges and future directions for research?

2. **Answer:** The current state of the art in this field is...
It was before Christmas, there was much snow. The world outside had turned into a white and hazy dream, and the house seemed to echo with the sound of the wind.

Just then, a knock came at the door. I opened it, and there stood a young boy, his face lit up with excitement. He handed me a small package and said, "Here, I made this for you." It was a small, handmade toy, a model of a plane.

I smiled, "What a great gift! Thank you," I said, and he beamed. "I made it myself," he said, "it's a model of the plane that my dad flies."

I looked at the plane, it was intricately carved, with tiny details that showed the care and effort that had gone into making it. "It's beautiful," I said, "thank you so much." He nodded, happy at the praise.

[Signature]

51
INTERPRETATION

In the problem of the previous day, we saw how to make a model of a number system using the concept of "machine parts." In this case, we looked at the "machine" to perform operations on numbers. The idea was to identify the relationship between the numbers and the "machine parts." We first identified the "machine parts" and the numbers associated with each part. Then, we constructed the "machine" to perform the operations on the numbers. Finally, we used the "machine" to solve the problem.

In this problem, we will analyze the problem to find a solution by using the "machine". The problem is to find the relationship between the representations of numbers and their operations. We will use the "machine" to develop a better understanding of the problem.

INTERPRETATION

Here, let's talk about the next step: setting up the model. When we write the problem, we refer to the model as a group of students, the "machine parts". We can represent this model using a number line, a table, a diagram, or a chart. In this case, we will use a table to show the relationship between the numbers and the operations. We will use a table to set up the relationship, and then we will use the "machine" to solve the problem.
ACTIVITY B

This activity is similar to a counting machine.

"Do you see any kind of counting machine?"

"Yes, we can make our own counting machine. In fact, the numbers part of the counting machine will be an 'X'."

"Then let's make a chart like this and model it in a circle."

"Now the counting number will come to us."

"This is the way to make a synthetic model of numbers."

"It is the only way to count to ten."

Write a tally mark on the board. Place the chalk in the student's hand in the first chair. Introduce the student to the first student and have him put the chalk to the second student. Introduce each student to put the chalk until you get it from the student in the third chair.

It is important to publish the relationship between each number counted or recorded in the tally and the synthetic representation of each number.

If you counting is a way to make that no one confused position, allow to remove to one.

Write a tally mark on the board. Place the chalk to the student standing in the first chair. Introduce him to another position and give the chalk to the second student.
```
```

To solve the problem, let's first identify the key elements involved in the situation. We have three machines labeled A, B, and C, each with specific operations and dependencies.

- **Machine A**: Performs operations 1 and 3.
- **Machine B**: Performs operations 2 and 4.
- **Machine C**: Performs operation 5.

The machines have interdependencies:
- Machine B cannot start before Machine A completes its operations.
- Machine C cannot start until both Machine A and B have completed their tasks.

To optimize the process, we need to ensure that each machine starts as soon as its predecessor completes its tasks.

1. **Machine A** starts first and completes operations 1 and 3.
3. **Machine C** starts after both Machines A and B have completed their tasks.

By following this sequence, we can ensure that the machines work efficiently and the production process is streamlined.

In summary, the order of operations should be:

1. Start with Machine A and complete operations 1 and 3.
3. Machine C starts after both Machines A and B have completed their tasks.

This approach optimizes the production flow and reduces the waiting time between machines.
can tell the machine to count in any of the representations you wish. You cannot identify the representation you want. You can count from zero to the representation you want. Do not attempt to explain the relationships. Give the students opportunities to discover whatever relationships they can for themselves. Pay your careful selection of examples. Note that the suggested order in which the representations are presented emphasizes the base and that the representations of each machine part are coed. The students en the class should be able to respond.
Teacher says:

"Look any machine in the machine. Who should be standing?"

"Who should be standing if we name 'even' on the machine?"

**ACTIVITY: Number 1**

"How would the machine name the number nine?"

"This is not satisfactory since this is also the way the machine named the. What could we do to the machine in order to name nine in a different way than this?"

"Let's call separate machine parts.

"How would we call from seven?"

**Exercise:**

You might ask the students to represent numbers on the machine. If you do this, and any number greater than seven is named, you'll be ready to move to the next activity.

You are now ready to ask some of the questions which involve the students in extending their observations to new situations with the counting machine. Since the students are not likely to be able to answer these questions immediately, you should begin with a number representation they know -- such as seven -- and count to nine using the rules.

The students will ordinarily suggest that if another machine part is added to the machine, the number nine can be named in a different way than 'seven'. Add another student to a machine part and begin counting from seven again.
Here the students would be given a chance to guess. The correct answer is "Seven". You can handle the responses in a variety of ways. You can count and check the responses by simply following the rules for the machine. Or you can use the representations for each machine part.

... or now return to the Frodo type activity and ask for representations of numbers on the machine and for the number represented by the machine with the additional machine part.

Additional machine parts may be added and the many possibilities explored. It is possible to predict all the directions which this activity will take. You should in the variety of ways from the machine to the way it will be used. I really welcome the possibilities involved in the above mentioned variety of representations of numbers.
The number of people in the model is six. The group is expected to be in the same grade, but this is not always the case. The group is divided into two parts: the front and the back. The front part is responsible for the main activity, while the back part provides support. The group is divided into two parts: the front and the back. The front part is responsible for the main activity, while the back part provides support. The group is divided into two parts: the front and the back. The front part is responsible for the main activity, while the back part provides support. The group is divided into two parts: the front and the back. The front part is responsible for the main activity, while the back part provides support.
A. \[ \text{Rule:} \frac{A}{\text{C}} \text{.} \]

\text{Questions:} 

1. Assume a simple machine - like a lever - and represent the input force. Ask the student, "What makes it move up or down?"

2. Ask the student to represent the output position. Ask the student, "What makes it move by the metal bar?"

3. Ask the student to represent the resistance point. Ask the student, "What makes it move in that part of the system?"

4. Ask the student to represent the power point. Ask the student, "What makes it move in that part of the system?"

5. Ask the student to represent the user's input. Ask the student, "What makes it move in that part of the system?"

6. Ask the student to represent the system. Ask the student, "What makes it move in that part of the system?"
After you have read the first two pages, you will be able to perform a demonstration of a computer's ability to find a common divisor of two numbers using the computer's help or the participant.

GREATEST COMMON DIVISION

But before we begin, let's find the greatest common divisor of the numbers — sometimes called the greatest common factor of the numbers. This will not be the procedure that will be followed in the computer demonstration, but it will enable you and your group to see that 6 is a factor of 12. In other words, number 6 is a whole number, such that

12 ÷ 6 = 2

No whole number greater than 6 can divide 12. There is no whole number which can multiply 6 to get 12.

Now we can write all of the factors of 6. List them.

6 = 1 x 6

These are whole numbers. 1, 6, 2, and 3. These are all of the factors of 6. They are the factors of 6. One factor of 6 is 1, and another factor of 6 is 3.

Factors of 6 are 1, 2, 3, and 6. The number 6 can be divided by 1, 2, 3, and 6. Factor 1 is known as the greatest common divisor of the two whole number divisors. It's kind of fancy, but a common divisor.

The factors of 6 are 1, 2, 3, 6.
PROGRAM FOR FINDING THE LARGEST COMMON DIVISOR

1. Enter float a, b
2. if a > b
   then a = a-b
   else b = b-a
3. if a = 0
   then GCD = b
   else goto 2

Example:
- a = 24, b = 18
- 24 > 18
  - a = 24 - 18 = 6
- 6 > 18
  - b = 18 - 6 = 12
- 12 > 6
  - a = 12 - 6 = 6
- 6 = 0
  - GCD = 6

Output:
GCD = 6
Central 4--call the name of the student--First or Second--who is involved in each in that row. In other words, find the position for that student for the stop. Then, follow the activity with 1 and 2.

Central points to Step 1 in the diagram and says, "First," since it's the First block in Step 1 to be active. In Step 1, since First moves in step 1, the Brown and Black move in Step 1, "No." Central then follows the arrow with his hand for the A and Step 1 in the diagram and finds his name to be in the diagram under First and begins. From here, move the Brown and Black in Step 1. First then writes in Step 1, "No." and writes down 1. In order to change the Brown and Black moves, "First," Central then moves to Stop 1 in the diagram and "No." Central finds the name of First from his and writes the number 1. The Brown and Black would look like this:

```
A

B
```

Then, the Brown and Black move, "First." First calls, "No." and the action for the stop is "No." Central then follows the arrow for the B and moves, "First." And the number 2 is written down here in the two numbers.

Then, the Brown and Black would look like the following table:

```
A

B
```

Then, the Brown and Black moves, "No." Central writes: Begin 1.
Can you now begin to fill in the remaining responses in the table? What does First do at the point we stopped in the activity back at Step 1? Does he call, "Yes" or "No"? 

He must call, "No," since his number 6 is smaller than Second's number 11. Try completing the rest of the steps and compare with the completed table.

<table>
<thead>
<tr>
<th>Control card</th>
<th>Place written on card</th>
<th>Second written on card</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;First&quot;</td>
<td>&quot;No&quot;</td>
<td>1</td>
<td>Begin</td>
</tr>
<tr>
<td>&quot;First and Second&quot;</td>
<td>&quot;Yes&quot;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>&quot;First&quot;</td>
<td>&quot;No&quot;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>&quot;First&quot;</td>
<td>&quot;No&quot;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>&quot;First and Second&quot;</td>
<td>&quot;Yes&quot;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>&quot;First&quot;</td>
<td>&quot;No&quot;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>&quot;First&quot;</td>
<td>&quot;No&quot;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>&quot;First&quot;</td>
<td>&quot;No&quot;</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Next, name that you would write next is the greatest common divisor. As we already knew from our earlier examination, 1 is the greatest common divisor of 6 and 11.

What would First and Second have written on the chalkboard in front of them if打好 the vector in which card 0 is 2nd? See if you can fill in the space in the visual aid below:

```
     0
   /   |
  N     A
     /
    First  2nd
```

At this point in the activity, Why yes, "Yes" when it is not smaller than the arrow slips. But the activity is still on. The next step is to fill the "First" card 0 as "Yes" when the Diff more is 1. The rest can be filled in the same manner.
In class, we will explore ways to compute the least common multiple (LCM) of two given numbers. This process involves identifying the greatest common divisor (GCD) of the two numbers and using it to find the LCM.

**Procedure:**

1. Identify the two numbers.
2. Find the greatest common divisor (GCD).
3. Use the formula: \( \text{LCM} = \frac{\text{Product of the two numbers}}{\text{GCD}} \)

For example, to find the LCM of 12 and 18:

1. Identify the two numbers: 12 and 18.
2. Find the GCD: The GCD of 12 and 18 is 6.
3. Use the formula: \( \text{LCM} = \frac{12 	imes 18}{6} = 36 \)

In your classroom, you will have the opportunity to practice finding the LCM of different pairs of numbers.

**Steps:**

- Identify the two numbers.
- Find the GCD.
- Use the formula to find the LCM.
In the context of the lesson, the students will demonstrate their understanding of the computer by playing the game "Computer in the Classroom." 

Initially, the students will play the teacher's game and observe how the teacher inputs and processes data. This observation will allow the students to understand the basic concept of data processing.

Attention!

The main aim of the lesson is to equip the students with the skills to process and interpret data. By playing the following games, students will be able to understand the functions of computing systems.

1. **Student:** "What is the use of the cursor, and how can we use it to control the cursor?"
   **Teacher:** "The cursor is the pointer that allows you to select and manipulate objects on the screen.

2. **Student:** "What is the function of the 'enter' key, and how do we use it?"
   **Teacher:** "The 'enter' key is used to confirm a selection or command.

3. **Student:** "What is the purpose of the 'save' command, and how do we use it?"
   **Teacher:** "The 'save' command is used to preserve changes made to a file or a program.

4. **Student:** "What is the difference between a program and an application?
   **Teacher:** "A program is a set of instructions that tells the computer what to do. An application is a specific program designed for a particular purpose.

In conclusion, understanding the basics of computer systems is crucial for effective data processing.
HUMAN MUSCLE POWER
(Frequently E. L. Dugan and L. Church)

GENERAL DESCRIPTION OF MUSCLE ENERGY PRODUCTION AND USE

A variable is, in the simplest terms, a value that is subject to change. All human energy processes are variable. A simple muscle's "work" on a part of an operation or "work" on a part of a machine. The amount of work described below using the basic mechanical concept, such as "multiply by 2" or "add 4", or simple machines.

In producing elements of the work, the following:

1. A pupil, in memory, ability, or by visualizing by the teacher, one of the required actions or a model for understanding or interpreting a process into a machine. In this manner, for example, "lift, hold, etc."

2. And as the muscle is moved, it assumes the target, into which his rule is moved on the machine. In the example, if 2 is folded, the target assumed "2" in the target.

3. In "tactile," it is the action, the way to identify the motion in the mind. In the example, it is to move an object, and in the mind, the object will always move. In this case, the simple mechanical action is "lift, hold, etc."

4. And in the "visual," it is the overall, the way to identify the motion in the mind. In the example, it is to see an object and in the mind, the object will always move. In this case, the simple mechanical action is "lift, hold, etc."
For example:

![Diagram](image)

The input to the compound machine comes in A and B. Input to A goes to output.

Similarly, if A is fed into the machine, then output of B is fed into the machine. (continued)

The problem is for the machine to identify the input component A and/or B, to obtain a number of third components by evaluating of single operator boxes that the higher output is identified by putting it with any input. You will use input individually, not more input never stop the system.

REGULAR ITSELF

RECORDS (INPUT) A → B → C → D → E

Question One

It is interesting to note that they say play the role of the machine.

The child's task is to find the machine which will give you the output that is given to him. The children are divided into two groups: one group to play the role of the machine and another group to play the role of the output. In the group of the output, you would be divided into three different groups: one to hold and keep the output in the hand to follow:

<table>
<thead>
<tr>
<th>Task</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
</tr>
</tbody>
</table>

From the perspective of the machines, it is clear that the purpose of A, B, C, D, and E is to divide the input into two parts: one part of the input to B and the other part of the input to C. After the teacher has given the numbers and divided them equally, you would be required to play the role of the machine. Then the teacher will have divided the parts of the machine and decide to play. After the child has played, the machine will receive the input and work in the same way. For this task, the instructions in practice, rule of
For example:

\[
\begin{array}{c}
\text{Input} \\
\downarrow \\
\text{Modify} \\
\downarrow \\
\text{Output}
\end{array}
\]

There is a type of "encryption" which will take the set of input data to a new model that seems very much like the input data itself. To understand this better, let us consider the original model:

\[
\begin{array}{c}
\text{Input} \\
\downarrow \\
\text{Modify} \\
\downarrow \\
\text{Output}
\end{array}
\]

Now suppose that we can somehow view the output of the modified model. We can set up a new model to get back to the original input, possibly by another method. In diagram form:

\[
\begin{array}{c}
\text{Input} \\
\downarrow \\
\text{Modify} \\
\downarrow \\
\text{Output}
\end{array}
\]

\[
\begin{array}{c}
\text{Input} \\
\downarrow \\
\text{Modify} \\
\downarrow \\
\text{Output}
\end{array}
\]

While the diagram may look very much like an encryption, it is not. It is a machine that takes two input values and produces one output value. It is a "deca" machine, possibly similar to a simple algorithm or a neural network.
In 1970, the National Highway Safety Act was signed into law, establishing the National Highway Traffic Safety Administration (NHTSA) as a part of the Department of Transportation. The act aimed to reduce the number of deaths and injuries resulting from automobile accidents.

The NHTSA was charged with developing and enforcing motor vehicle safety standards, conducting research, and spreading awareness about safe driving practices. Over the years, the agency has played a crucial role in improving automotive safety through various initiatives and regulations.

For instance, the development and enforcement of federal motor vehicle safety standards have been a cornerstone of the NHTSA's mission. These standards cover a wide range of areas, from seat belts and airbags to tire pressure monitoring systems and electronic stability control. By setting and enforcing these standards, the NHTSA has helped to significantly reduce the number of deaths and injuries in motor vehicle crashes.

In addition to its regulatory role, the NHTSA also conducts research to identify areas where improvements can be made. This research is often used to inform the development of new safety standards and the implementation of innovative safety technologies. Through partnerships with industry, government agencies, and academic institutions, the NHTSA continues to advance the field of automotive safety.

Overall, the NHTSA's efforts have contributed to a safer driving environment for all Americans. As technology continues to evolve, the agency remains committed to staying at the forefront of automotive safety, ensuring that new vehicles are designed with the safety of their occupants in mind.
INTRODUCTION: It is a common occurrence for teachers to have students who are fascinated with mathematical puzzles but who show little interest in the usual textbook exercises. Furthermore, even those students who faithfully do their problem assignments are rarely stimulated and challenged by the problems themselves. These observations suggest that the effectiveness of problem assignments would be very greatly increased if the usual drill-type exercises could be systematically replaced by puzzle-type problems and games which make use of the same mathematical skills. However, the construction of such problems and games is not easy, and most of the development has been directed toward recreational uses. Nevertheless, many educators feel that such material has a broad potential for educational use, and have urged that a concentrated effort be made to develop mathematical games and puzzle-problems for this purpose. A proposal along these lines was made to the School Mathematics Study Group which has broad interests in mathematics curriculum development. After obtaining approval from the National Science Foundation, it was decided to support a two-week conference to explore the possibilities for the development of mathematical games and puzzle-problems which would be appropriate for educational use. The basic aim of the conference would be to generate ideas. There would be no construction of mathematical contexts or preparation of text material.

CONFERENCE: The conference was held at Stanford University during the period May 20 - June 1, 1967 with the following mathematicians participating:
All of the participants have extensive interests in the area of mathematical recreation and most of them have been involved in a variety of curriculum development activities.

The formal sessions of the conference were essentially brainstorming sessions in which a variety of ideas for puzzle-problems and games were proposed by the participants for consideration by the group as a whole. The items which survived after detailed discussion and criticism were assigned to conference participants to write up for the final report. On the average one such session was held each day. The remainder of the time was devoted to an extensive examination of the literature on mathematical recreations; to modifying, extending, and translating known recreational items in order to make them suitable for educational use, and to preparing reports on the results of these endeavors. The principal output of the conference is the collection of working papers which is attached as an appendix to this report.

CONCLUSIONS AND RECOMMENDATIONS. First of all, it is clear that the development of an entirely new class of puzzle-problems or games is a very difficult undertaking. The abilities required are not unlike those required in developing an entirely new line of mathematical research. The essential
feature aid distinguishes good puzzle-problems from ordinary textbook
problems in the sense of their novelty, and in some cases, surprise
which they create. Likewise, an effective game must present a natural,
attractive challenge to the players. But it is exactly these features which
characterize first-class mathematical research. Hence, it is to be expected
that the teaching contributions to the art of puzzle-problem construction
will grow. However, it also seems to be the case that the discovery of
new ideas will take place more likely within a variety of puzzles and games. The
many puzzles and games which have developed from S. Colman's invention of
the isola... illustrates this point. During the relatively short period of
the first few years after his ideas for puzzle-problems and games were produced,
however, it will take further development and experimentation to determine
how effective and how broadly applicable the ideas are. In any case, it was
the unanimous feeling of the participants that a joint effort such as this is
worthwhile and to stimulate ideas in this area. They further recommended
that 1 ... conferences of this type be held from time to time to encourage
further activity in this area and to make use of the ideas and talents of
many people. It was felt that a two-week period is about the right length
of time since after a fortnight of intensive effort the wells of creativity
seemed to temporarily run dry.

Some of the working papers do not present entirely new ideas for puzzle-
problems and games but rather give ideas for the development of known material
in order to increase its potential for educational use. This is clearly a
very important activity for such a conference and in many cases much imagination
and creativity are required.
An attempt was made to present the material in polished form. The original papers were merely to get the ideas down in a preliminary form for later experimentation and modification for use in curricular material.

The papers at the conference are agreed that the next stage in the development of this material should be experimental testing on an item by item basis. This will require a good teacher as well as a mathematician. For this purpose teachers the material over to a good teacher will not be enough since the objectives which the inventors had in mind may only be long in the process of making the item classroom use. Perhaps the best approach would be to have the experimentation done by a team of the -- a good teacher and a mathematician. If the mathematician involved well he may take ownership of the item as much the better. Under any circumstances it is the consensus of the conference that until there is some such experimentation, the material should not be widely circulated. In particular, it should not be made available to textbook writers in its present form.

After experimental testing, the next stage should be the development of a revised text material incorporating the tested items. This could then be rewritten by a team. On the other hand, it is the feeling of the conference that this should be approached in an informal way. For example, rather than trying to fit puzzlelike form from the activity into the traditional text format, it might be more appropriate to use the text material around the problems and exercises as tests, so that the learning of mathematical facts and the development of mathematical skills is a natural by-product of the interest in the activities.
Finally, the participants wish to express their appreciation for the cheerful and hearty cooperation of the SME staff.

A. P. Dilworth, Chairman
APPENDIX

The working papers produced by the conference participants are collected in this appendix. It must be emphasized that these papers were prepared in order that there would be a permanent record of the ideas generated during the conference. No attempt was made to polish the presentation nor was any effort made to be complete and detailed in the discussion of the item. Rather, the motivation was to get the basic ideas down in writing as quickly as possible and to get on to the generation of more ideas. Furthermore, the form of the presentation was that which was most convenient for the particular participant involved, and no effort was made to get uniformity.

It should be noted that the working papers vary from short accounts of very particular items to broad descriptions of whole classes of games and puzzles. The grade level extends over the primary and secondary spectrum. In many cases, the activity could be profitable for primary students while an insightful analysis could only be carried out by advanced secondary students. For example, 'Tanam Tor,' can be played by primary students, but they are hardly able to analyze the game. Finally, it should be noted that the items fall into three categories: classroom activities, individual puzzle-problems, and games. Some of the items naturally fall into more than one category, but a rough distribution of the items is as follows:

- Classroom: 1, 2, 3, 4, 11, 16, 21, 25
- Puzzle-problems: 10, 14, 15, 17, 18, 21, 22, 23, 24
- Games: 7, 8, 14, 15, 16, 18, 25, 26, 27
A classification of the items according to grade level is as follows:

Trades: 1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16.

Sales: 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22.

Sales: 1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27.
APPLIED FUNCTION RULES

1. Define prediction (value $x$).

A student (or "machine") gives a linear function. The machine predicts the output of a function that it does not have. The goal is to identify the machine's operations.

2. Define operations (value $y$).

The machine predicts the output of a function (value $y$) and identifies the machine's operations. The machine has no knowledge of the function and the operations. The goal is to identify the machine's operations.

3. Define equation (value $z$).

The machine predicts the output of an equation (value $z$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

4. Define equation (value $w$).

The machine predicts the output of an equation (value $w$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

5. Define equation (value $v$).

The machine predicts the output of an equation (value $v$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

6. Define equation (value $u$).

The machine predicts the output of an equation (value $u$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

7. Define equation (value $t$).

The machine predicts the output of an equation (value $t$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

8. Define equation (value $s$).

The machine predicts the output of an equation (value $s$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

9. Define equation (value $r$).

The machine predicts the output of an equation (value $r$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

10. Define equation (value $q$).

The machine predicts the output of an equation (value $q$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

11. Define equation (value $p$).

The machine predicts the output of an equation (value $p$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

12. Define equation (value $o$).

The machine predicts the output of an equation (value $o$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

13. Define equation (value $n$).

The machine predicts the output of an equation (value $n$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

14. Define equation (value $m$).

The machine predicts the output of an equation (value $m$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

15. Define equation (value $l$).

The machine predicts the output of an equation (value $l$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

16. Define equation (value $k$).

The machine predicts the output of an equation (value $k$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

17. Define equation (value $j$).

The machine predicts the output of an equation (value $j$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

18. Define equation (value $i$).

The machine predicts the output of an equation (value $i$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

19. Define equation (value $h$).

The machine predicts the output of an equation (value $h$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

20. Define equation (value $g$).

The machine predicts the output of an equation (value $g$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

21. Define equation (value $f$).

The machine predicts the output of an equation (value $f$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

22. Define equation (value $e$).

The machine predicts the output of an equation (value $e$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

23. Define equation (value $d$).

The machine predicts the output of an equation (value $d$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

24. Define equation (value $c$).

The machine predicts the output of an equation (value $c$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

25. Define equation (value $b$).

The machine predicts the output of an equation (value $b$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

26. Define equation (value $a$).

The machine predicts the output of an equation (value $a$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

27. Define equation (value $0$).

The machine predicts the output of an equation (value $0$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.


The machine predicts the output of an equation (value $-$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

29. Define equation (value $+$).

The machine predicts the output of an equation (value $+$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

30. Define equation (value $\times$).

The machine predicts the output of an equation (value $\times$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

31. Define equation (value $/$).

The machine predicts the output of an equation (value $/$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

32. Define equation (value $\equiv$).

The machine predicts the output of an equation (value $\equiv$). The equation has no knowledge of the function and the operations. The goal is to identify the machine's operations.

- "Student," 1951, 56. 55. [footnote]

The best way to make a profit is to ensure the players are in proper order. To make sure this happens, a specific player in each set is designated as the "leader." The leader is responsible for ensuring that all players from their own set follow in proper order. When a set is completed, the leader must record the number of players in each set. This information is then used to calculate the total profit made. It is important to note that the number of players in each set can vary. In
114
The subject was taught to recognize and respond to the various colors of the signal cards. The subject was given the signal card with the color of the signal card and was asked to name the color. The subject was also taught to recognize the different shapes of the signal cards. The subject was given the signal card with the shape and was asked to name the shape.

The subject was taught to recognize and respond to the various symbols on the signal cards. The subject was given the signal card with the symbol and was asked to name the symbol. The subject was also taught to recognize the different numbers on the signal cards. The subject was given the signal card with the number and was asked to name the number.

The subject was taught to recognize and respond to the various signals given by the signal card. The subject was given the signal card and was asked to name the signal. The subject was also taught to recognize the different signals given by the signal card. The subject was given the signal card and was asked to name the signal.

The subject was taught to recognize and respond to the various symbols on the signal card. The subject was given the signal card with the symbol and was asked to name the symbol. The subject was also taught to recognize the different numbers on the signal card. The subject was given the signal card with the number and was asked to name the number.
When the user enters a number, the input is validated against the range of valid inputs. If the user enters a number that is outside this range, an error message is displayed. The valid range of inputs is determined by the specific requirements of the application or system.

### Table: Input Validation

<table>
<thead>
<tr>
<th>Input</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>Valid range</td>
</tr>
<tr>
<td>0</td>
<td>Invalid</td>
</tr>
</tbody>
</table>

In cases where the input is valid, the program proceeds with the next step in the sequence.

If the input is invalid, the program will display an error message and prompt the user to enter a valid input. This process continues until a valid input is entered.
This table will be continued in the next cycle, and will be followed by more tables. The data in the cells of the first row represents the data of Table 1. The remaining tables will be similar.

**Table 1**

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
<td>Data 4</td>
<td>Data 5</td>
</tr>
<tr>
<td>Data 6</td>
<td>Data 7</td>
<td>Data 8</td>
<td>Data 9</td>
<td>Data 10</td>
</tr>
<tr>
<td>Data 11</td>
<td>Data 12</td>
<td>Data 13</td>
<td>Data 14</td>
<td>Data 15</td>
</tr>
<tr>
<td>Data 16</td>
<td>Data 17</td>
<td>Data 18</td>
<td>Data 19</td>
<td>Data 20</td>
</tr>
</tbody>
</table>

**Explanations:**

The examples illustrate the potential to document the objection and resolution of issues in a table format. A more direct approach is to provide the capability of handling complex data as well as simple data. The simplest way of doing this involves using the appropriate software in the spreadsheet or data handling system.
In the first round, the five students are given 23 minutes to solve the problem, and in the second round, they have 17 minutes. The problem is presented as a sequence of steps to solve the problem, each step involving a specific mathematical function. The problem begins with the variable E written on the blackboard, and the students are asked to find the square of that variable. If the students can determine the correct prime number to be subtracted from E, they will find the next prime number after the square of that prime. The sequence of steps continues, with each prime number determined and the next prime number calculated. If the students can correctly follow the otherwise complex process, they will have successfully solved the problem.

Diagram: The sequence of numbers is shown as a flowchart, with each step leading to the next. The steps include operations such as addition, subtraction, and multiplication. The sequence is presented in a clear and organized manner, with each step clearly labeled and connected by arrows.

Diagram:

[Flowchart diagram]

Note: The diagram is not visible in the text provided.
These steps are described in the following table:

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>List</td>
</tr>
<tr>
<td>2</td>
<td>Erase</td>
</tr>
<tr>
<td>3</td>
<td>Square</td>
</tr>
<tr>
<td>4</td>
<td>Erase</td>
</tr>
<tr>
<td>5</td>
<td>Add</td>
</tr>
<tr>
<td>6</td>
<td>Note</td>
</tr>
<tr>
<td>7</td>
<td>Note</td>
</tr>
<tr>
<td>8</td>
<td>Square</td>
</tr>
<tr>
<td>9</td>
<td>Erase</td>
</tr>
<tr>
<td>10</td>
<td>Add</td>
</tr>
</tbody>
</table>

Step 7 initiates his procedure when "yes" is called. Step 8 follows it and adds "yes" to the number that has already been erased, if instead it calls "yes," step 10 adds it to the number that has already been erased, and adds it to the number that has already been erased. The list that is the reason for this rule.

It will often happen that one is called on to remove from the table a number that has already been erased. When will this occur? In this case he is not allowed. It can be replaced on the table after the process that removes a blank leaves a blank, but that makes no difference. There is a factor that controls the process: otherwise the table would have to be examined before deciding to erase. Also, the need for a factor in the table should be discussed for its insight into the structure of the table. This method of inspection can be repeated, with no necessity of any further refinement, except to make sure that the number itself is not overruled.

142
Five 'the number is optional' boards 3 feet by 10 inches are joined together so that they will no separate when children jump upon them. The object of the game is 'to jump onto the blue [last] board successfully. A player is not

allowed to jump off of a line or should he copy a trip made by a fellow player. A trip may consist of any number of jumps of any length up to the jump point towards the blue board. That is a player may not jump back towards the box. The trip always starts on the floor and ends up on the blue board.

Also, a player must commit himself to a specific trip, before he starts his trip, he must write his proposed trip on the chart. If he completes his trip successfully he enters his name along side of his proposed trip. An unsuccessful trip can be tried again on his next turn unless someone else has

tried it. If either of these situations arise he may try a new

trip. If he thinks of a trip that has not been used up. For example: Suppose

a player is himself is jumping directly on the yellow and then on the 

'8' on the chart. Thus all of the trips that have been used up

on the chart.

The player with the most successful wins the game. This game should be

played by a maximum of 12 players. The winning player starts last for the

next time. In this way to avoid or taken away depending upon the ability of the

player. This may be useful for developing systems for exhausting all

growth or will be given very young students experience in extending their

language. The next three months forth by such activities as.
If you wish to travel without taking a leap, how far will you travel?

If you wish to travel without taking a leap, how far will you travel?

If you wish to travel without taking a leap, how far will you travel?

How many leaps would you need to take to get you to a point 5 leaps away?

If you travel a leap of 1 away, then a leap of 1, how far would your friend need to jump to be 2 leaps away in the leap of 1?

The teacher has presented the interesting problem:

How many leaps without taking a leap, will you travel?

This problem sheet is a multiple approach depending upon the level of the student to whom it is presented. To illustrate a few of the possibilities in which this approach appeals for elementary, junior-high school, and high school students, respectively.

1. Nature of the leap and leap. How many leaps are open to me? For example? Enter Table 1 in the margin:

<table>
<thead>
<tr>
<th>Leap</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jump</td>
<td>-</td>
<td>2</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

If you know that you need I leap and find that I have traveled 4 leaps, I know I have traveled 4 leaps. Consider each of the possible trips in the levels. Each of these trips ended by jumping into the leap of 1, etc. For now instead of jumping in the leap of 1, I have the problem I am jump on the leap of 1, etc. Each of these levels are the new bus levels. Hence, I have twice as many to the level - before. This gives me a recursion formula for determining the number of possible trips from 1 leap to 2 trips.
For the purposes of the present note, it is sufficient to consider the number of elements of a subset of \( S \), another solution is available. The result of a trip only in the blue bowl, the set of tours is dependent upon the blue bowl determines a trip. For example, we only correspond to the empty set of tours. Hence, the problem simplifies (i.e., how many different sets of non-blue bowls were there?) This is similar to the problem of subsets of a set consisting of a subset there is to the total number of tours including the empty set.

E. W. C.
Many textbooks on finance of the first rank all agree to place the "new" term of a sequence either before, after, or equal to existing terms. In some cases, however, the student is not left with the impressions listed below.

1. There is only one term with the term "new.
2. There are always two terms new.
3. Sometimes no term is designated as "new." 

It is difficult to find a precise meaning in some cases. Many references contain these terms as the main theme of new terms with each of the following cases stated as such.

For instance, a term is "new" if:

1. The term is not the same as the new term of a sequence which has the beginning element.
2. After the new term, the term may be of the same kind or rule for forming the next element. This apparent practice may be graphically shown with a chart, as follows for the next term of an unspecified sequence in unappreciable cases. This practice may be started with some other terms and gradually worked out until the new term quite distinguishable.

For instance, the above chart of a given sequence in the first term. Then a new element to the left:

and...

The new term now shown:

and...

This is only one important case that will be possible for the new element to be shown from the new element of sequence in one statement. There are other cases.
Another interesting train of thought is to consider the problem of a sequence of geometric figures involved in the construction of a plane figure. With our fundamental concepts in mind, let's consider the situation presented in the pictures.

The figures illustrate the idea that as the sequence of figures is constructed, the number of regions will increase. The figure will have a limited number of regions.

Let's consider the simplest case, a sequence of 3 triangles:

![Triangle Diagram]

The number of regions will be limited to a maximum of 9 regions.

In the general case, the number of regions is determined by the number of planes involved. The question is about how many 2-dimensional regions are formed by a given number of planes. For instance, one plane forms one triangle, two planes form two quadrilaterals, and so on. The number of regions is essentially determined by the number of planes involved.

This problem can be solved by observing the sequence of 4 planes in space:
The drawing above represents a method to determine the number of questions in a sequence based on the number of elements.

In the sequence, the number of questions is determined by the number of elements. Each element in the sequence corresponds to a question. The number of elements in the sequence is equal to the number of questions.

For example, a sequence of 5 elements will have 5 questions.

```
<table>
<thead>
<tr>
<th>Element</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element 1</td>
<td>Question 1</td>
</tr>
<tr>
<td>Element 2</td>
<td>Question 2</td>
</tr>
<tr>
<td>Element 3</td>
<td>Question 3</td>
</tr>
<tr>
<td>Element 4</td>
<td>Question 4</td>
</tr>
<tr>
<td>Element 5</td>
<td>Question 5</td>
</tr>
</tbody>
</table>
```
HEM A Y TR

As the sun rose, the animals were stirring, and the sky began to brighten. The first rays of light filled the air, casting long shadows across the land. The birds took flight, their wings flapping in unison, as if they were dancing to the rhythm of the morning. The trees rustled gently, their leaves swaying in the fresh morning breeze. It was a beautiful sight, a moment of peace and serenity.

In the midst of the serenity, a small bird perched on a branch, basking in the warmth of the sun. Its feathers glistened in the morning light, and it seemed to be enjoying the moment. The field mice scurried about, their fur shining in the sunlight. The flowers, too, reached out to the sun, their petals opening wide.

It was a moment of pure beauty, a moment that filled the heart with joy and wonder. The animals seemed to move in harmony, each one playing its part in the symphony of nature. The morning was a gift, a time to pause and truly appreciate the wonders of the world.
The stars, planets, and moon are all part of our solar system. The sun acts as the central force, drawing all the other bodies into orbit around it. The moon revolves around the earth, while the planets orbit around the sun. The relative positions of these celestial bodies vary throughout the year, creating different phases and eclipses.

In some cultures, the moon is associated with the feminine and the planets with the masculine. The moon's cycle is seen as a reflection of the ebb and flow of life, while the planets influence our actions and decisions. The stars, on the other hand, appear distant and mysterious, symbolizing the infinite and the unknown.

Understanding the ancient wisdom behind astrology and astronomy can provide insights into our own nature and the cosmos.
The easiest way to start is by building a model in the mind. We
will use a classic example of a computer-generated fractal to
illustrate the concept. The first step in creating a fractal is to
define the basic unit or shape of the fractal. In this case, we
will use a square as the basic unit.

Next, we will add two new squares to the original square,
creating a larger square. This process can be repeated
indefinitely, resulting in a fractal pattern that becomes
more complex with each iteration.

The fractal pattern can be visualized as a series of nested
cubes, each smaller than the last, forming a
three-dimensional structure.

This process can be repeated indefinitely, creating a
fractal pattern that becomes more complex
with each iteration.

The fractal pattern can be visualized as a series of
nested cubes, each smaller than the last, forming a
three-dimensional structure.
- Draw the section in one of the planes that passes through the element 1, or the radical 1, and 1.
- Use the plane of the triangle to extend the diagram.
17.
<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>19</th>
<th>23</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>128</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>256</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>512</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1024</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2048</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

153
This chart provides many opportunities for students to recognize patterns and make discoveries which are related to rich areas in mathematics.

A teacher might start out by listing the powers of 2 down the chalkboard in order and ask for students to continue giving entries according to the pattern the teacher has in mind. A similar activity can be performed for the primes across the top.

At this point we will diverge briefly from the major topic at hand to record an activity, brought out in our discussions, which relates to prime numbers.

The children are given a type of counters—pebbles, pebbles, checkers, chips, etc.—and asked to see how many different rectangular arrays they can form with a given number of counters. Any \( n \) by \( m \) array is considered to be the same as any other \( m \) by \( n \) array as well as the same as any \( m \) by \( n \) array. Each child should conduct his own experiments and summarize his findings in a table like this one:

<table>
<thead>
<tr>
<th>Number of Tokens</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Rectangular Arrays</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Students may now be told that those numbers for which there is only one rectangular array are the prime numbers (except for the number one which is or isn't depending upon the definition of particular author). This activity could be carried on along with getting kids to discover the pattern across the top of the chart. Upon looking at the table generated from investigating rectangular arrays, a student might be asked questions similar to the following.

Is there a number for which more than two rectangular arrays may be formed?

How many rectangles can be formed with six tokens? How many factors has six?

How many rectangles can be formed with nine tokens? How many factors has nine?

Find a number between 50 and 100 which has exactly 3 factors. How many such numbers are there?

The activity of forming rectangles can also be lowered down into the primary grades to teach multiplication. cf. The Advanced Montessori Method, Vol. II by Maria Montessori.
In filling the powers of two-prime chart, column by column students recognize many patterns and are rewarded by this recognition since following the patterns greatly speeds their task. However, soon even the patterns become difficult to construct. This motivates students to make further discoveries which are immediately rewarding.

A very satisfying discovery is to abandon the set rules to divide a prime into the various powers of 2 and merely begin to double the previous remainder and record the excess above any multiple of the prime in question. This serves as a good introduction or application to modular arithmetic as well as some of the basic theorems on congruence such as Fermat's Little Theorem:

When $p$ is prime and $a$ is not divisible by $p$, then

$$a^{p-1} \equiv 1 \pmod{p}$$

K. Wills
A PROBLEM IN DIVIDING FRACTIONS

Take the first five prime numbers -- 2, 3, 5, 7, 11 -- and write them with four division signs of unequal length in the following way:

\[
\frac{2}{3} \div \frac{5}{7} \div \frac{11}{11}
\]

The value of this expression is given by the rules that (a) the longest division sign is applied first, and (b) this rule is repeated until a simple fraction is obtained. Successively, one gets

\[
2 \div \frac{3}{7} \div \frac{5}{11} = \frac{2}{3} \cdot \frac{7}{5} \cdot \frac{11}{11}
\]

The final result need not be multiplied out.

By putting the division signs in a different sequence, a different value may be obtained. Thus,

\[
\frac{2}{3} \div \frac{5}{7} \div \frac{11}{11} = \frac{2}{3} \cdot \frac{7}{5} \cdot \frac{11}{11}
\]

How many different arrangements of signs are possible? Do they all give different answers?

Try

\[
\frac{2}{3} \div \frac{5}{7} \div \frac{11}{11}
\]

How many distinct values are possible?
The primes are used so that all repetitions occur only as a result of equivalent arrangements of signs. The problem provides examples of non-associativity in arithmetic operations, gives practice in handling division by multiplying; reciprocals, tests accuracy in handling compound fractions, and is challenging from the combinatorial point of view. For this latter aspect, have the students look at the same problem with 2, 3, or 4 division signs and try to find the rule for the number of distinct values obtained in the general case.

W. Jacobs
"I don't like spinach and I'm glad, I don't because if I did, I'd eat it and I hate it."

Logic puzzles can be useful in overcoming the purely linguistic difficulty of expressing things accurately in the ambiguous, redundant, variable medium of English. The syllogism puzzles of Lewis Carroll are especially good for this because the absurdity of their content serves to emphasize that only the form is of any importance in logic. These puzzles can be found, arranged in a graded sequence, in "Logical Nonsense", Putnam, 1934, pp. 505-546. For use with modern children, they need to be sifted and modified somewhat. In particular it might be a good idea to start with equivalent forms of single statements (which Lewis Carroll doesn't have) before going on to multi-statement puzzles. Following are a few examples (variants of L. C. assertions). For teaching purposes one would want many dozens of these to be worked first by common sense then by more systematic methods.

A. Equivalent forms of single statements. Do the two statements say the same thing or different things? (Answers below)

1. No large birds live on honey. If a bird lives on honey, it is not large.
2. No one, who forgets a promise, fails to do mischief. Anyone who forgets a promise, does mischief.
3. All, who are anxious to learn, work hard. All, who work hard, are anxious to learn.
4. Prudent travellers carry plenty of small change. Travellers without small change are imprudent.
5. No child is healthy who takes no exercise. Children who exercise are healthy.
6. Some elderly ladies are talkative. Some talkative persons are elderly ladies.
7. Nobody, who really appreciates Beethoven, fails to keep silent when the Moonlight Sonata is being played. People, who keep silent when the Moonlight Sonata is being played, really appreciate Beethoven.

(Answers below)
8. No drug is useful in a toothache, unless it relieves pain. A drug that relieves pain is useful in a toothache.
9. None but a hop-scotch player knows real happiness. All who know real happiness are hop-scotch players.
10. Whenever I do not take my umbrella it rains. Whenever it does not rain, I take my umbrella.

Answers:
1. same
2. same
3. different
4. same
5. different
6. same
7. different
8. different
9. same
10. same

After some practice with these, one can go on to puzzles involving two or more statements such as these:

B. Decide whether the conclusion follows from the two statements. (Answer below)

11. "I saw it in a newspaper."
   "All newspapers lie."
   Conclusion: It was a lie.

12. Every eagle can fly.
    Some pigs cannot fly.
    Conclusion: Some pigs are not eagles.

13. All, who are anxious to learn, work hard.
    Some of these boys work hard.
    Conclusion: Some of these boys are anxious to learn.

C. In each of the following examples, draw a conclusion, if possible.

14. All ducks waddle
    Nothing that waddles is graceful.

15. Some unkind remarks are annoying.
    No critical remarks are kind.

16. Canaries, that do not sing loud, are unhappy.
    No well-fed canary fails to sing loud.
17. All puddings are nice.
This dish is a pudding.
No nice things are wholesome.

18. Nobody who really appreciates Beethoven fails to keep silent while the Moonlight Sonata is being played.
Guinea-pigs are hopelessly ignorant of music.
No one who is hopelessly ignorant of music ever keeps silent while the Moonlight Sonata is being played.

Answers:

11. Conclusion does not follow.
("All newspapers lie" is taken to mean "All newspapers sometimes lie." It could conceivably mean "All newspapers always lie." Such ambiguities should be avoided at first, later discussed.)

12. Conclusion follows.

13. Conclusion does not follow.

14. All ducks are ungraceful.

15. No conclusion.

16. Some ill-fed canaries are unhappy. (Lewis Carroll does not admit the possibility that the set of soft-singing canaries is empty. By modern standards one should admit this possibility. The important thing, though, is to be clear about the rules of the game. One implicit rule is that "some" means "one or more." Another implicit rule is that everything has at most two values: Canaries are either well-fed or ill-fed. Indifferently fed canaries do not exist.)

17. This dish is not wholesome.

18. Guinea-pigs do not really appreciate Beethoven.

These are a few examples to illustrate the tone and difficulty of these puzzles. For teaching purposes one should have dozens of them. After doing a great many by common sense one could introduce the set interpretation together with the usual visual aids (Venn diagrams). This should give the pupils quite a sense of power and enable them to go on to harder puzzles involving many statements. Several such are given in Logical Nonsense (loc. cit) including one with twenty statements. Other multi-statement logic puzzles can be found in the standard puzzle books, but without the Carrollian whimsy they often seem stodgy and artificial.
Because of its importance in mathematics, one should also practice a lot with the if ... then form. In particular the children should learn the workhorse rules about converses and negations.

Examples: In each set which statements are equivalent?

1. No child is healthy who takes no exercise.
   If a child takes exercise, then he is healthy.
   If a child is not healthy, then he takes no exercise.

2. No country, that has been explored, is infested with dragons.
   If a country is infested with dragons, then it has not been explored.
   If a country has not been explored then it is infested with dragons.

One can, of course, go on to more formal logic systems, Boolean Algebra, switching circuits, etc., but this is really a different subject, mathematics rather than linguistics. Puzzles of the Lewis Carroll kind are primarily linguistic: they concern equivalences between forms of English sentences. Skill in handling these equivalences is essential if one is to talk about mathematics in the English language.

Following is another logic puzzle that I happen to know and like.

**The District Attorney**

This is basically a puzzle, but it can be made into a guessing game by having someone in the know act out the part of the D.A. Even when people see the D.A. in action they often have difficulty divining his strategy, which can be made more mysterious by using different but logically equivalent questions each time through.

One of three suspects is guilty. The innocent ones can be counted on to tell the truth, but the guilty one may or may not tell the truth. The D.A. is to find out who is guilty with just two yes-no questions. At first glance it may seem impossible to get any information at all, but the D.A. does it, e.g., as follows:
D.A.: (to suspect A) All right A, either you or B did it, right?
A: No
D.A.: Then I can only conclude that C did it. (to suspect B) Will you confirm that C did it?
B: Yes
D.A.: Of course. C did it.

Alternative scenario:
D.A.: (to C) You look honest. Now tell me the truth. Did A do it?
C: No.
D.A.: (to A) If C were guilty he would have said "yes" in order to frame you. Therefore it must be B. B did it, right?
A: No
D.A.: All right C. The jig is up. Honesty will get you nowhere.
C: But what if I had lied?
D.A.: I would have found out just the same. Dishonesty is no better than honesty.
C: You can't win.
D.A.: True.

False Proof by Induction

At the high school level after students have had experience with mathematical induction they should try this one:

Theorem: In any set of marbles all the marbles are the same color.

Proof (by induction) Let n be the number of marbles in the set. The theorem is certainly true if n = 1.

Induction: Suppose the theorem is true for n. Then it is surely true for n + 1, for if I remove any marble from a set of n + 1, the remaining ones, constituting a set of n, must all be the same color. Since this is true no matter which marble I remove, all n + 1 marbles must be the same color.

F. W. Sinden
DOES THE ORDER MAKE A DIFFERENCE?

1. In a single purchase, you are offered 3 successive discounts of 20 percent, 10 percent, and 5 percent, and can take them in any order that you wish. What order would you choose?

2. Arthur and Bob start a game with equal amounts of money. Arthur loses the first game and pays Bob 20 percent of his money. Then Bob loses the second game and pays Arthur 20 percent of the amount Bob has. Do they again have equal amounts of money?

3. Mr. Jones has two houses, which cost him the same amount. He sells one house at a 10 percent profit to Mr. Allen, who resells it to Mr. Baker at a 10 percent loss. Mr. Jones sells his second house at a 10 percent profit to Mr. Duhl. Which paid more, Mr. Baker or Mr. Duhl?

IDENTIFICATION PROBLEMS

The class of tricks based on what is generally called Gergone's Pile Problem provide an example of the power of numerical coding and an application of place system ideas. A typical version is the following: The "magician", A, asks B to deal 27 distinguishable cards in 3 equal piles, to select a card mentally and to announce which pile it is in. A then tells B to reassemble the cards by placing one pile on top of the other in any order. A notes the order in which the piles are assembled. After two repetitions of this cycle A tells B the position of his card in the reassembled deck.

If this trick is introduced to a fourth or fifth grade class in its 2 pile, 4 card form an analysis by the class should be possible and can be made to relate to binary numeration. The binary form and the general case can be investigated at an appropriate later time.

A complete discussion of several variations of this trick can be found in Martin Gardner's Mathematics, Magic, and Mystery (Dover, 1956).

W. Jacobs
A GAME WITH FACTORS AND MULTIPLES
(Grades 4-6)

The type of game outlined below depends solely on the multiplicative aspect of the positive integers. The objective is to explore factor relationships and incidentally to reinforce direct recall of the multiplication facts basic to computation.

The game is based on the g.c.d. and l.c.m. diagram exemplified by

\[
\begin{array}{ccc}
\text{l.c.m.} & & \\
\text{12} & & \\
\text{6} & & \\
\text{2} & & \\
\text{g.c.d.} & & \\
\end{array}
\]

To make a game consider a triangular array of say 10 boxes.

\[
\begin{array}{ccccccc}
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
\end{array}
\]

The game is played by 2, or perhaps 3, players, one of whom might be the teacher. Player A begins by filling in the top box. The other "players" in turn fill in the other boxes. In a simple version the only rule governing permissible entries is that the number put in the top box may never be used again.

The Play

After an entry is made any other player may challenge the player making it to show that the configuration is part of a "factor diagram". If the latter succeeds he wins (or scores so many points). If he fails he loses (or his challenger scores so many points). At any time a player believes his play makes the configuration part of a unique factor diagram he wins (or scores) upon convincing the other players of his assertion.
Here is a play.

\[
\begin{array}{c}
A & 60(4) & 20(3) & 15(3) \\
B & 5(1) & 5(2) & \\
\end{array}
\]

This last play can be challenged.

Additional rules for admissible entries may be added. It may be required:

for example, that

i) The entries in row (1) must be relatively prime in pairs, or

ii) The entries in row (1) must be primes or 1, or

iii) The number 1 may not be used.

W. Lister
A STOCHASTIC NUMER LINE GAME

There are many stochastic games which are essentially stochastic number
line games. Typically, in these games the players move playing pieces along
a line. The players take turns, the length of each move being determined by a throw
of a pair of dice. The numbers of each player is usually supplied by
the instructor or problem. In order to make the game instructurally
interesting and require probability, it should be possible for the
players to follow the same principle of probability to improve
their chances of winning.

In this game each player begins at a number line although any discrete
data may be used. Any number of players may play and
instructurally it is usually advantageous for the number of players not
there. The players in turn throw a pair of dice
and the player whose throw is closest to the number of dots
thrown is the next player. It is this feature of being able to choose either a
random or intelligent move that enables a player to improve
his/her game. The object of the game is to reach the points numbered
on the line. When a player on his/her turn will throw the dice,
the player who is nearest the points will proceed to it,
and the next player will throw. The point he does not count and he must
move another number. The first player to reach a given

point wins the game. It should be noted that the distribution
of points is not so that it is advantageous to improve the
next move. It is clearly advantageous
for the player to choose where he will have a much better
chance of winning. It is also advisable to throw the dice
when it is the best strategy in any simpler,
or if

A player who is not sure of the best strategy is more likely to
win the game. It is clear that in the direction of the position
of the player is important in this game. It is necessary to analyze the best strategy
for each player in order to win the game.
It has one 1, two 2's, and three 3's.

It is clear that games of this type give students considerable incentive to learn and use probability principles which will improve their chances of winning. With suitable choices of the basic distribution much of basic probability can be learned in this way. Furthermore, the kind of probabilistic reasoning required is very similar to the kind needed to make effective probability judgments in ordinary life situations.

R. P. Dilworth
Stochastic games on the number line are a special case of stochastic games on directed graphs. Again there exist a number of commercial games which have the players move through a network, each move being determined by the throw of a die or a pair of dice. These games are likewise not particularly instructive since the play is usually a purely chance affair. However, games on directed graphs can easily be constructed in which an understanding of the underlying probability can be used to improve the strategy of play. Such games are potentially instructive particularly with regard to basic principles of probability. We give an example of a comparatively simple stochastic game on directed graph.

The playing board has a diagram as follows:

The game is played by four players who start in the four circular positions 1, 2, 3, 4. They may draw lots to determine who starts in each position. They play in the order I, II, III, IV and, in his turn, each player throws a pair of dice to determine how far he will move along the directed graph. He may move along any path consistent with the arrows. His objective is to reach the finish position F in the fewest number of moves. The player first passing through a junction secures it for himself and other players may not follow paths passing through that junction.
The first player reaching F makes a score of \( r - n \) where \( n \) is the number of dots he has traversed in reaching the finish. All other players score zero. Note that the first player to reach F may still get a negative score.

This game clearly has a great many more strategic possibilities than the previous number line game. For example, player IV gains nothing by heading to the finish position directly since his score is then zero, but by taking a path higher into the graph he increases his chances of being cut off. Likewise, player I must decide whether he should take a path around the outside where he can make a good score but runs the risk of being cut off or taking a path into the graph and doing some cutting off himself. Perhaps the numbers of dots between junctions will have to be altered if a wide variety of strategies is to be obtained. Certainly, the distribution of the possible steps in each play must be used in estimating the effectiveness of a given strategy.

R. P. Dilworth
There is a good deal of puzzle value in dissection problems and such experience should certainly help develop spatial geometric intuition especially the notions of Euclidean motions or congruences.

A pair of polygonal regions $A$ and $B$ are "equivalent" if $A$ can be dissected into polygonal regions which can in turn be reassembled to form $B$. For example, in FIG. 1, $A$ is equivalent to $B$, $B$ to $C$ and $A$ to $C$.

![FIGURE 1]

It is clear that any pair of equivalent polygonal regions have the same area; but the elegant thing is that the converse is also true: thus: "Every pair of polygonal regions with the same area are equivalent." It is this theorem which is illustrated here.
1. Show that the two parallelograms $A$ and $B$, (see FIG. 2), which have the same heights and bases, are equivalent.

![FIGURE 2]

(Answer) superimpose bases

and a proper dissection becomes clearer:

2. Show that the two parallelograms $A$ and $B$ in FIG. 3 (in which they are illustrated with their bases superimposed) are equivalent.

![FIGURE 3]

(Answer) This is similar to problem 1 but requires an extra step. The following illustration gives the idea:
3. Show that the triangular region A is equivalent to the rectangular region B. (B has half the height of A).

![Triangle and rectangle with equal areas.](image)

(Answer): from the following construction one obtains the dissection:

![Dissection of triangle into rectangles.](image)

4. Show that the rectangles A and B in FIG. 5 bound equivalent regions. The height of A is less than one and its length is greater than one. The height of B is equal to one and (since the areas bounded by A and B are equal) the length of B must be the common area of A and B.

![Rectangles with equal areas.](image)

(Answer): make use of following construction

![Construction for finding equal areas.](image)

choose angle $\alpha$ so that distance between $e'$ and $e''$ is one.
5. Using the devices indicated in problems 1, 2, 3, 4 one can show that any triangle is equivalent to a rectangle of height one. It seems reasonable (and it can in fact be proved) that any polygon can be dissected into triangles; for example see FIG. 6.

![Diagram of a polygon dissected into triangles]

FIGURE 6.

Using the result on triangles one can dissect each of the triangles and rearrange into a rectangle of height one; thus

![Diagram of triangles rearranged into a rectangle]

This not only computes the area "a" of the original polygonal region but also goes a long way toward showing the general result mentioned on page 1.

C. J. Titus
MAP COLORING PROBLEMS

In the following we are concerned with "maps"; for example,

FIGURE 1;

and we are also concerned with the coloring of maps in a special way. We will say a map is "correctly colored" if no pair of bordering countries have the same color; for example, the following is a correct coloring of the map in FIGURE 1:

FIGURE 2;

1. Can you "correctly color" the map in FIGURE 1 with fewer than the five colors used in FIGURE 2?

2. What is the least number of colors with which one can correctly color the map in FIGURE 1? (ans: ?)

3. Draw a map which can be correctly colored with two colors. (sample answer: $\triangle ABC$)
4. Draw a map that can be correctly colored with 3 colors but that cannot be correctly colored with 2 colors. (Sample answer: the map in FIGURE 2).

5. How many countries did you have in the map in problem 4? Can you achieve the same result with fewer countries? What is the smallest number of countries one can have in a map and still have the same result? (Answer: 3; sample map)

6. Draw a map that can be correctly colored with 4 colors but that cannot be correctly colored with 3 colors. Can you achieve the same result with fewer countries? What is the smallest number of countries one can have in a map and still have the same result? (Answer: 4; sample map)

7. Make a map using only circles; for example,

![Circle Map]

Correctly color your map with two colors. Notice that you cannot find such a "circle map" that requires more than 2 colors. Can you see why it is that every circle map can be colored with two colors? (Sample answer: write a number in each country which is the number of circles in which that country is contained. Color the even numbered countries one color and the odd numbered countries the other color.

C. J. Titus
A COMBINATORIAL PROBLEM SOLVED BY GEOMETRY

Consider the eight \( (2^3) \) triplets of 0's and 1's:

- 000
- 001
- 010
- 011
- 100
- 101
- 110
- 111

Can you arrange these in a cycle so that neighboring triplets differ in only one place? This problem arises in computer design where the 0's and 1's represent states of on-off devices (e.g. switches, lights, cores). A triplet of such devices is supposed to run through all of its eight states over and over again. For engineering reasons it may be awkward to switch two or more of the devices simultaneously. Therefore one would like to find a sequence that requires switching only one at each step.

This looks like a hard problem. The number of possible cycles is enormous.

\[
\frac{8!}{8} = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 5040
\]

(Division by 8 because cyclic permutations indistinguishable)

It turns out, though, that a geometric interpretation makes the problem easy. Consider the triplets of 0's and 1's to be coordinates of points in 3 space.
The eight points represented by the triplets lie at the vertices of a cube.

Neighboring vertices (those joined by an edge) differ in exactly one coordinate. The problem then, is to find a closed path along the edges of the cube which passes through each vertex once. Such a path (Hamilton line) is easy to find:

Tracing out this path, one gets the solution:

000
001
011
101
100
110
111
011
010
All other Hamilton lines are rotations of the one shown above.

This problem offers an excellent opportunity to discuss higher dimensional geometry. To solve the problem with quadruples of 0's and 1's one considers Hamilton lines on a 4-cube. To show what a 4-cube is, one can display cubes of 0, 1, 2 and 3 dimensions, note the induction principle, use it to get the 4-cube.

0-cube

1-cube (2 0-cubes joined by a segment)

2-cube (2 1-cubes with corresponding vertices joined)

3-cube (2 2-cubes with corresponding vertices joined)

4-cube (2 3-cubes with corresponding vertices joined)
The drawing is neater if you put one cube inside the other:

3-cube: (one 2-cube inside the other)

4-cube: (one 3-cube inside the other)

The construction of the Hamilton line can be generalized. Observe that the Hamilton line on a 3-cube first traverses one of the 2-cubes, then jumps to the other 2-cube, traverses it, jumps back to starting point. On the 4-cube: First traverse a 3-cube, jump to the other 3-cube, traverse it, jump back to starting point.

F. W. Sinden