Construction of a theoretical measure of educational output is attempted in this paper. The effort begins by specifying the purpose for which this measure is intended: centralized allocation, according to the equimarginal principle, of limited resources to alternative units producing education. It then specifies a simple and narrow goal that education is to meet: satisfaction of an individual student's preferences for market goods, on-the-job consumption, and the direct benefits of education. Educational output is defined as a compensating variation, or lump-sum dollar amount. The measure depends upon the particular set of preferences chosen, and as a result there will, in general, be as many distinct and equally correct measures as there are students. Consideration of the preferences of faculty and other relevant individuals will only add to the multiplicity of measures. It is concluded that centralized allocation of resources to alternative education units according to the equimarginal principle cannot be accomplished if the preferences of each individual student are to be taken into account.

(Author/MSE)
This paper is part of the ERIC Collection of 1979-1980. It is recommended for inclusion by the AIR Forum Publications Review Committee.

Robert N. Fenske
Arizona State University
(Editor, AIR Forum Publications)

"Permission to reproduce this material has been granted by

AER

United States Department of Health, Education & Welfare
National Institute of Education

The views expressed in this document do not necessarily represent official National Institute of Education position or policy."
A Theoretical Framework for Educational Output Measurement

William S. Reece
Research Coordinator
Management Information Division
University of Minnesota
412 Morrill Hall
Minneapolis, MN 55455

612-376-7452

April, 1978
This paper presents an attempt to construct a theoretical measure of educational output. We begin by specifying the purpose for which this measure is intended: centralized allocation, according to the equimarginal principle, of limited resources to alternative units producing education. We then specify a simple and narrow goal which education is to meet: satisfaction of an individual student's preferences for market goods, on-the-job consumption and the direct benefits of education. We then define educational output as a compensating variation, or lump-sum dollar amount. The measure depends upon the particular set of preferences chosen, and as a result there will, in general, be as many distinct and equally correct measures as there are students. Consideration of the preferences of faculty and other relevant individuals will only add to the multiplicity of measures. We conclude that centralized allocation of resources to alternative education units according to the equimarginal principle cannot be accomplished if the preferences of each individual student are to be taken into account.
Introduction

Optimal allocation of resources to alternative productive units by centralized administrators requires measures of the outputs of those units. Specifically, administrators must allocate resources according to the equimarginal principle in order to achieve the optimal production levels. This principle states that the marginal output per unit of resources must be equal in all productive units. If this condition were not satisfied, resources could be shifted from those units with relatively low marginal outputs to those with relatively high marginal outputs, and the result would be a net gain in output.

This principle can, at least in concept, be applied to the allocation of resources to the production of education in particular. Rational university administrators would like to use the equimarginal principle (explicitly or implicitly) to allocate university budget resources among schools and departments within the university. Administrators of multi-campus systems would like to use the principle to allocate their budgets among campuses. However, the use of this principle in education has been hampered by the lack of suitable measures of the output of education institutions.

In this paper we will examine how one might proceed to measure educational output. We begin by trying to rigorously define the concept to be measured. Only then can practical measurement methods be considered. This process will make clear why educational output measurement is so difficult. In particular, we will see that output measurement depends upon what goal the output is intended to satisfy and that there is no way to settle upon a unique goal or a set of goals which lead to tractable aggregation. The result is definition of an immense number of distinct and equally correct measures, which is
All measurement is ultimately based upon the numerical representation of ordering relations.\(^3\) This idea has been loosely expressed in some of the better literature on educational output measurement by the notion that the measurement of the output of an educational institution depends upon the specification of an objective function.\(^4\) Restated, a fundamental measure is a rule for assigning numbers to objects (physical or conceptual) which retains all the information needed to compare these objects with respect to some ordering relationship.

To make this idea more clear, consider a concrete example. Suppose we had a set of straight rods. Measuring their lengths would consist of assigning numbers to the rods so as to preserve the relationships to be found by laying the rods side by side, adjusting them so that they coincide at one end, and observing at the other end which rods extend beyond other rods or appear to coincide. Thus, "rod a is longer than rod b" means "rod a extends beyond rod b." The notion "extending beyond" is an ordering relation which is independent of prior measurement. The "length" numbers assigned to each rod carry all the information needed to make all the comparisons of the kind "rod a is longer than rod b" without actually physically observing that rod a extends beyond rod b. (Additional structure can also be added so that we can give precise meaning to the phrase "rod a is twice as long as rod b." See Krantz, Luce, Suppes, and Tversky (1971).)

To proceed with our analysis of educational output measurement we need an ordering relationship on the output of educational units. One simple and relevant ordering relation can be found in the preferences of a
chosen, potential student. In the next section we will relate educational institutions to the preferences of the individual student.

II. Student Demand for Education

We assume students demand higher education for three reasons: 1) the process of higher education yields direct and immediate satisfaction, 2) the results of higher education give direct satisfaction later in life; and 3) higher education produces changes in the value of the individual's labor services. This last factor has, of course, been widely treated in the "human capital" literature. These studies have focused primarily on highly abstract models which yield conditions for optimal educational investment behavior in the face of a one-dimensional rate of return to education.

Our model is more complex in that we specify a particular labor market mechanism which determines different "returns" to different dimensions of education. This makes explicit the means by which different institutions yield different quantities of educational output. Our labor market model is one in which workers' characteristics determine their productivities. As a result, employers are willing to offer different wages to workers with different characteristics. Secondly, jobs also have characteristics (in addition to wage rates) associated with them. We assume potential workers have preferences defined over these job characteristics, so that a given individual would have to be paid a premium to induce him to accept a job with relatively unpleasant characteristics (or would be willing to sacrifice some money income for relatively good job characteristics). To summarize, this is a labor market in which heterogeneous workers are employed in heterogeneous jobs. This is, roughly, the kind of market analysed by Rosen (1974)
and generalized to the labor market by Lucas (1977). If we assume an equilibrium is achieved in the market, the wage rates received by the workers will be seen to depend upon their own characteristics and the characteristics of the jobs they fill. The relation between wage rates and the characteristics of jobs and individuals is known as a "hedonic wage equation." If we assume an equilibrium is established in this market, we will observe a hedonic tuition function relating tuitions to school characteristics and student initial characteristics.

III. Definition of Educational Output

We can now precisely define educational output from the point of view
of a particular individual student's preferences for market goods, on-the-job consumption and the direct benefits of education. We will measure educational output with a "compensating variation," which is the quantity of money required to make the individual in question indifferent between two alternative sets of constraints on his behavior. We define the quantity of output of higher education institution h to be the dollar amount $Q_h$ which would make the individual indifferent between receiving the lump sum payment $Q_h$ along with no higher education and attendance at institution $h$ but with no lump sum payment. That is, first we determine the highest indifference curve to be attained by the individual given that he attends institution $h$ (at no tuition cost). Then we determine what lump-sum payment must be made to the individual to bring him to the same indifference curve given that he receives no higher education and thus enters the labor market with his initial endowment of characteristics.

Let us begin with a very simple model to see more precisely how the measurement process works. Suppose the individual in question does not derive any direct benefits from education and there is no on-the-job consumption. In this simple case $Q_h$ is merely the difference between his labor market earnings to be received if he were to attend institution $h$ and his earnings to be received if he received no higher education. This is precisely the view which underlies Welch's innovative empirical study of educational quality measurement. (Welch (1966)).

If we relax the assumptions that the individual does not derive direct benefits from education and that there is no on-the-job consumption, then the compensating variation $Q_h$ not only compensates the individual for lost money income but also for the loss of the direct benefits of education, and
possibly for lost on-the-job consumption.

By construction, the compensating variation has the properties required to be an ordinal measure of the individual's preferences for the characteristics of alternative institutions. That is, by construction $Q_a > Q_b$ if and only if the highest indifference curve to be attained by attending institution $a$ is farther from the origin than that to be attained by attending institution $b$. Further the compensating variation we have defined has the required cardinal properties to be useful in implementing the equimarginal principle. That is, it makes sense for us to say institution $a$ produces twice the output of institution $b$. Also, it makes sense to say the difference in output between institutions $a$ and $b$ is the same as that between institutions $c$ and $d$. (This is true because the compensating variation is a distance in a preference map and is entirely independent of the choice of ordinal utility indicator.)

IV. Conceptual Problems with the Compensating Variation Measure of Educational Output

We have in the previous section constructed a well-defined measure. However, it is based only upon student preferences, and further, only upon the preferences of a single student. Had we chosen a different student we would, in general, expect to get a different set of output measurements for the feasible set of schools. Thus, there are, in general, as many equally well-defined output measures as there are students. We also could, presumably, construct distinct and well-defined compensating variation output measures based on the preferences of faculty, administrators, state legislators, and others. Thus, there is almost no limit to the number of equally well-defined output measures which can be constructed within this
framework.

Is there any way to avoid this multiplicity of measures? I don't believe there is any even remotely reasonable way to avoid the problem of distinct measures for the broad classes of students, faculty, etc. However, there are two possible approaches one might try to get a single measure of educational output from the point of view of the student. The first of these is the well known approach of forming an aggregate preference ordering. The conditions needed for existence of a well-defined aggregate preference ordering, or, stated in another way, for the existence of a "representative individual," are very restrictive and are very unlikely to be satisfied in practice.

Another approach with more general applicability is to use the mean of the distribution of the compensating variations. Since, in concept, we have a value of our output measure for each institution and for each student, we could define the output of each institution to be the average over all students of their output measures for that school. Adopting this approach, we could calculate the compensating variations for a small sample of students and use the sample means and sample standard deviations to form confidence intervals for the true mean output measures or make other probabilistic statements about the distribution of the measures.

The problem with this probabilistic measure is that while it is unique and well-defined, it is not the kind of measure needed to make the equimarginal principle operative. The equimarginal principle is not based on means of distributions of outputs, but rather on the outputs themselves. Thus, while the probabilistic output defined above may be useful for some purposes it will not be the tool which enables central administrators to allocate resources in an optimal manner.
V. Summary and Implications

We have constructed a meaningful and unambiguous measure of educational output based upon the economic concept of the compensating variation. To do this we have had to specify a particular set of preferences over market goods and the direct benefits of education. Since, in general, we cannot aggregate preferences, our framework leads to as many equally well-defined measures as there are students. Further, in constructing our output measure we have not even begun to consider the preferences of faculty and other relevant groups. Consideration of these groups will lead to the construction of even more equally valid measures. If we assume that central administrators cannot ignore the preferences of any student, our framework suggests there is no hope at all of finding the sort of output measures needed to enable centralized resource allocation mechanisms to arrive at the optimal allocation of resources to alternative productive units.

These results suggest the technocratic approach (as espoused by Razin and Campbell (1972) and Jablonski (1978), e.g.) to resource allocation in higher education can never be justified by arguing that such mechanisms lead to the optimal allocation of resources since the measures required to make the approach work do not exist even in concept. Then what are the alternatives? One is to muddle along with the technocratic approach using the measures which are available and abandoning claims to optimality. One problem with this procedure is that outputs will become merely those objective results which are easily measured rather than the results which are of value. Further, if resources were allocated to various units according to their relative production of these objective measures, we would expect to see the units shifting their efforts toward the production of these objective measures to
the detriment of the unmeasured, and perhaps more important activities.

There are (at least) three other alternative allocation mechanisms which do not rely on output measurement. One is the dictatorial approach. By this I mean centralized decisionmaking by a single individual based upon his own preferences. The problem with this mechanism is in choosing the right dictator. With any dictator chosen we can always expect some of the relevant individuals to be dissatisfied with his decisions. And with some possible choices we can expect almost all the relevant individuals will be dissatisfied. Thus, this is a risky mechanism. A second possibility is the political approach. This mechanism relies upon decision-making by a committee made up of a small group of roughly equally powerful individuals who represent various constituencies. A problem with this approach is that personalities and institutional arrangements will be important factors in determining the allocation of resources, and these factors may overwhelm the preferences of the relevant individuals. Finally, there is the decentralized market approach. This mechanism relies on student demanders and faculty suppliers to make their own decisions. With this mechanism, central administration does not need output measures since they have little or no role in allocating resources.
References

Abowd, J. An econometric model of the U.S. market for higher education.


Hanushek, E. Learning by observing the performance of schools. New Directions for Institutional Research, 1975, 2, 17-38.


References (continued)


The author would like to thank Stephen Hoenack and William Weiler for helpful comments on an earlier draft.

1. See Bowles (1967) or Razin and Campbell (1972) for examples of applications the equimarginal principle to the allocation of resources to the production of education.

2. For a more detailed treatment of the ideas presented in this paper see Reece (1978).

3. See, e.g., Hempel (1952), Pfanzagl (1971) or Krantz, et al. (1971) for the foundations of measurement.

4. See, e.g., Hanushek (1975) and Solmon (1973).


6. See Kalachek and Raines (1976) and Lucas (1977) for empirical hedonic wage equation estimates.

7. See Abowd (1977) for an application of Rosen's model of markets for heterogenous goods to the market for higher education.

8. See Hicks (1956) for a detailed treatment of compensating variations.

9. In calculating $Q_h$, we set tuition at institution $h$ equal to zero because we want the quantity of output to be independent of price. That is, in comparing the outputs of two institutions their tuition levels should be irrelevant. See Reece (1978) for a more detailed discussion of this point.

10. Gillingham (1974) has used this approach to interpret the consumer price index and Gillingham and Reece (1978) have suggested a similar approach to quality of life measurement.


12. See Hoenack (1977) for a detailed treatment of the operation of the decentralized market mechanism in higher education.