A paradigm for the study of the diffusion of complex innovations through a society is presented in this paper; the paradigm is useful for studying sociocultural change as innovations diffuse. The model is designed to account for change within social systems rather than in individuals, although it would also be consistent with information processing and attitude change models. Setting is defined by complex interactions between cultural objects and between these elements and the innovations. The associational structure of the model makes possible increased predictability of future changes because it takes into account all situation-specific relations affecting the actions of a society. Longitudinal multidimensional scaling is the preferred method for providing accurate measures of alterations. The model is useful for future research into diffusion of innovation phenomena, requiring a continual measurement from the start of the process; this is necessary if the mathematical model is to be tested realistically. Graphs and references are included. (DP)
AN ASSOCIATIONAL MODEL FOR THE DIFFUSION OF COMPLEX INNOVATIONS

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AN ASSOCIATIONAL MODEL FOR THE DIFFUSION OF COMPLEX INNOVATIONS

The process by which innovations diffuse through a society has been one of the more thoroughly investigated areas of the social sciences. Rogers and Shoemaker (1971) report over 1,200 studies describing various aspects of the process. As might be expected, results often are contradictory and provide a less than clear picture of how individuals decide to adopt a new product, practice, or idea. These studies tell us even less about how the process operates at the social level. One reason for this confusion is the lack of a common theoretical perspective which integrates the individual innovation with other cultural aspects of the adopting society.

This paper provides such a paradigm. It begins by making a distinction between simple and complex innovations (those which alter the associative structure of a culture). Complex innovations are the prime concern here. Also, associationism is reviewed to show that innovative or creative thinking can be described as an associative process. This notion is extended to describe the diffusion of innovations. Then, the paper reviews the history of attempts to describe mathematically the S-shaped curve traditionally identified in diffusion of innovation research. A specific method—longitudinal multidimensional scaling—is suggested as the best method for providing accurate measures of alterations in a society's associative structure as they are produced by an innovation. The paper concludes by predicting the mathematical model which most accurately describes the adoption of innovations.
Theory of Complex Innovations

The diffusion of innovation process can be divided into two general types: (1) those which are readily assimilated by a society and (2) those which lead to major cultural changes. The integration of some innovations (whether products, practices, or ideas) causes only ripples of minor significance in a given society; the diffusion of others produces global alterations in a society's cultural patterns.

The first type can be called the diffusion of simple innovations. In this category are innovations which only modify existing practices or objects. Examples include hybrid seed, the change from a carburetor to a fuel injection system in automobiles or a new additive to gasoline or detergent. Change occurs within a single component or subsystem rather than among the relationships of cultural objects. Simple innovations do not substantially alter the normative patterns or the communication structure of society.

The second category can be called the diffusion of complex innovations. Examples here include a new religion, a new drug (such as birth control pills or marijuana), or a new invention (such as the automobile or the telephone). These are innovations which have had global consequences in terms of altering a society's norms. Also, they can modify that society's communication structure and transform the definitions of other social objects in addition to those of the actual innovation.

Both types of diffusion are discussed extensively in theoretical literature. The anthropological approach to diffusion (Linton, 1936; Sharp, 1952) stresses the consequences of an innovation for the culture in which the object is diffused. As Linton writes:
It has been observed that while elements of culture may be diffused alone, they are more likely to travel in groups of elements which are functionally related (332...). Every cultural trait, even the simplest object or manufacturing technique, is really a complex of elements including various associations and ideas as to how it should be used (347).

Chapin (1928) distinguishes between the effects of the sulky plow (a complex innovation) and modifications to that plow (simple innovations) upon a culture.

While the theoretical literature in diffusion describes complex innovations, the empirical literature virtually has ignored the perplexities of cultural change. One of the primary reasons for this is the lack of methodological tools for measuring cultural change along several different dimensions simultaneously. Typically, the researcher selects a single simple innovation and observes its adoption pattern as the function of a single or limited number of independent variables. These may include an adopter's position in the social structure, level of education, pattern of media use, or communications with previous adopters. No one has explored the innovation as an independent variable which alters these and other cultural patterns.

In this paper the concern is with the diffusion of complex innovations and their effects on altering the cultural definitions of society— or, the normative associative structure of its social system. Associationalist models long have been proposed as explanations for the innovations process (that is, how various combinations of thought patterns result
In new ideas, practices, or objects). While these notions have been applied to innovation processes, they have not been used to describe diffusion within a social system. Such a model is proposed here.

Associationism

While associationist models of cognition can be traced back to Aristotle, the basic ideas of modern associationism were first advanced by Locke (1690) and other British associationist philosophers (Hobbes, Berkeley, Hume, Hartley, both Hills, and Bain [Anderson and Bower, 1973]). Basic tenets of this school of thought is that the human mind begins tabula rasa (as a blank slate), and that the structure of the mind at any specific time is contingent on the individual's past experience (as opposed to each thought having an innate and proper locus in the mind).

In Locke's (1690, I i 15) words:

The senses at first let in particular ideas, and furnish this yet empty cabinet, and the mind by degrees growing familiar with some of them, they are lodged in the memory, and names got to them. Afterwards the mind, proceeding further abstracts them, and by degrees learns the use of general names.

Another canon—atomism—holds that all knowledge can be derived from discrete simple ideas which, though the associative process, are combined into complex ones. Simple ideas are assumed to be so elemental that they are unanalyzable. Hume argues that complex ideas do not necessarily resemble simple ones because they evolve from combinations of ideas which somehow produce new patterns.

Thus, the associationists view the human mind from a holistic perspective—with each simple element interacting, through its relations.
with other elements, to organize the mind. Cognition and the process of organizing simple ideas into novel and complex ones are assumed to be governed by three principles: (1) similarity, (2) contiguity, and (3) contrast.

(1) The **Principle of similarity** holds that objects which are conceptually similar are associated in an individual's mind. For example, thinking of lemons can easily lead to thoughts of limes. The two objects perceptually are similar; they share specific traits (including shape, texture, and internal physical structure). The principle of similarity involves this kind of synchronous association which structures simultaneous ideas into more complex ones. It is a structural concept because it relates through patterns to a single coherent series of relations, or structures.

Word-substitution provides a good illustration. If two words are synonyms (that is, semantically identical), then one can replace the other without any alteration of the interrelationships among the symbols. If the words are semantically different, they cannot be interchanged without altering the relational structure. And the greater the dissimilarity, the greater the interrelationships among the symbols will change.

(2) The **principle of contiguity**, as argued by Deese (1965:12), is: "Two psychological processes occurring together in time or in immediate succession increase the probability that an associative connection between them will develop." This assertion is to time what the principle of similarity is to space and, as such, can be considered a special case of similarity (as when thoughts of limes immediately follow thoughts of lemons).
The principle of contrast advances the notion that associative links are formed between objects which are conceptual opposites. For example, thoughts of black often lead to thoughts of white, large to small, good to bad, and so on. This, then, is but another special case of the similarity principle. Given a pair of maximally dissimilar opposites—"hot" and "cold"—it is not hard to think of the terms "warm" and "cool," which are moderately dissimilar to both the extremas. This leads to the idea of a temperature dimension, with "hot" and "cold" at the extremes. Bipolars specify a single attribute which, in this case, is temperature. Terms become similar because a single shared attribute was chosen from among all possible points of comparison. Similarity between the bipolars relates to definition in terms of an identical attribute, even though they represent different values of that attribute.

Locke, Berkeley, Hume, and Bain considered similarity to be the irreducible law of association. Hartley and James Mill, however, viewed such relationships as special cases of the principle of contiguity and as tautology (that is, things are similar because they are similar).

In my work, I consistently have taken the position that all words and simple concepts are related in the mind according to their degree of similarity (Barnett, 1976). This supports Locke's opinion that ideas are structured as similar because past experiences specify such a relationship between objects of thought. Objects are not similar because they are similar. And the organization of ideas need not be determined by the perceptual process. Many metaphysical concepts have no perceptual referent, yet they are associated with other ideas.

Innovation as an Associative Process

According to H.G. Barnett (1953), the innovation process takes
place on a mental plane, with every innovation seen as a combination of associative elements. Such ideas are defined socially by the innovator's cultural setting through the society's symbol system. When an innovation occurs, there is a linkage or fusion of two or more elements that have not been combined previously. The result is a qualitatively distinct whole. In other words, the associative structure of the innovator is altered.

As criterion for novelty, Barnett emphasizes reorganization of mental configurations rather than qualitative variation: "Innovation does not result from the addition or subtraction of parts. It takes place only when there is a recombination of them (1953:9)." There are three distinct processes by which concepts can be reorganized. They are identification, substitution, and discrimination. Each process provides specific motion in the configuration which alters the spatial relations binding the elements. The impression which the individual holds is a function of the distance among the elements in his or her psychological system.

Creative solution and novel mental configuration stem from individuals who are placed in states which tend to bring the required associative elements into ideational contiguity. Mednick (1962) suggests three ways of achieving a creative solution: (1) serendipity, (2) similarity, and (3) mediation. The first is self-explanatory; in the second, the required elements for change result when stimuli which are alike elicit the associative elements; in the third, a mediating process is used on common elements to evoke the requisite associative elements in contiguity. He claims that the degree of creativity is a function of how mutually
remote elements of the new configuration are. Similar associational models of innovation have been presented by Golvin (1936) and Lin and Zaltman (1973).

Stein (1963) considers creativity to be the result of social processes. He claims that the process occurs within an individual as a result of the process of social transactions during which information is made available to the potential innovator. And—in order to be labelled an "innovation"—the novel product which results from this process must be accepted as tenable or useful by the social system. Indeed, it is this final criterion which determines the ability of the creative product to diffuse throughout a society.

Kasperson (1976) provides a comprehensive review of empirical literature on the innovation process. He concludes that innovativeness is a function of the variety and scope of the information made available to an individual. One person’s radius of exposure may be determined to some extent by his or her environment or the variety of published materials available. Environment can be the organizational structure or climate, membership in an invisible college, or some other set of interpersonal relations. Exposure to novel information allows the requisite stimuli to reorganize the associational structure of the individual's cognition. Innovations are the result.

To summarize this section, an associationist model of innovation suggests that the associational structure of an individual’s mind at any point is contingent on the individual’s past experiences (or information). And it is this information about single elements or symbols and their degree of similarity that determines the way in which they can be combined. Such combinations lead to innovations. The process takes place at a
social or cultural level because an individual must receive inputs in order to recombine disparate elements into novel patterns and because the creative product must be judged worthwhile by the larger social organization.

Measurement of the Associational Model

The associational model of the innovation process has implications for the study of their diffusion. It suggests that an associationist model can increase predictability and explain more about the adoption of new ideas, practices, or products.

The diffusion of innovations and the acceptance (or rejection) of a new idea is also a mental process involving the reorganization of elements in associational configurations by members of a culture. Most often, it involves the addition of new concepts into a culture's meaning system. As an innovation spreads throughout a society, the configuration shared by members of that society is modified to provide an accurate representation of the innovation and the cultural changes produced.

The degree of reorganization of the associative structure is a function of the amount of information members of the social system receive about the innovation. Communication scientists long have been interested in the effect of message variables on the adoption process. Indeed, it is these messages which must alter the existing associations and form new ones. The associations formed with the innovation must indicate compatibility with "...existing values, past experiences and the needs of the receiver (Rogers & Shoemaker, 1971:145)." The Rogers and Shoemaker book is full of examples of innovations which failed to be adopted by a society because compatible associational links were not formed. The more compatible an innovation is with existing associations,
as perceived by members of a social system, the faster its rate of adoption (1971:1352). From an associational perspective, compatibility is the cognitive introduction of a novel element which minimizes the configuration's change at the cultural level.

The associative model demands a measurement scheme for the study of the diffusion of innovations that meets the following requirements:

1. Associational links among a set of elements are measured. This relationship is the similarity among the set of items. It must be capable of relating existing practice and the innovation to that constellation of items used to define the new idea.

2. It is holistic. That is, it must be capable of measuring, simultaneously, along all integrating dimensions to produce a total description of the complex innovation rather than just describing separate aspects of the relationship. Such attributes must not be imposed by the researcher but must emerge from measurements on the adopting society.

3. To describe the cognitive state of a social system, measurement must take place on a societal or cultural level. It must involve consensual measures. Only in this way, prediction about the degree of adaptation can be made.

4. Finally, the measurement scheme must be capable of measuring changing conceptions in a culture's associational structure, over time, as the members of the social system become exposed to information about the innovation. It must describe the adoption process. This means that ratio measures must be used to make possible descriptive calculations of the rate (or velocity) of cultural change. Given multiple time periods, accelerations also can be calculated. Such velocities and accelerations
are necessary for any discussion of process (Arundale, 1971, 1973).

Theoretically, the scale must be infinite; actually, it must only be long enough to examine the phenomenon in question and it must be infinitely dense (that is, capable of measuring the most miniscule changes in the configuration).

**Longitudinal Multidimensional Scaling**

One measurement system which satisfies these demands of the associative model is longitudinal multidimensional scaling (or MDS) as proposed by Woelfel (1972, 1973, 1974), Woelfel and Barnett (1974), Barnett (1976), Wigand and Barnett (1976), and Barnett, Serota, and Taylor (1974, 1976). The fundamental approach to MDS is: the associational structure for any set of concepts can be represented on an N x N distance matrix; each vector of this matrix describes a concept's relationship with all other concepts; in diffusion of innovation studies, these concepts are (a) the innovation itself, (b) previous products or ideas which the innovation may displace, and (c) a series of cultural objects which have stable and well-established relations with previous practices and the innovation.

Data on these concepts can be gathered through a series of direct paired comparisons elicited with questions phrased this way: "If X and Y are U units apart, how far apart are a and b?" Such wording demands dissimilarity judgments from a respondent, but specifies that such judgments be made in terms of a standard distance provided by the experimenter.

Dissimilarity matrices formed from measures taken in this way provide static pictures of the interrelationships among concepts held by individuals. The average distance matrix generated from all members of
a social system (or a representative sample of that population) represents the collective consciousness -- that aggregate psychological configuration which constitutes culture. In successive matrices, process is recorded at known time intervals. Changes between the matrices are calculated. Such a procedure minimizes measurement unreliability. Although data for a given individual may be unreliable (or, inversely proportional to the difficulty of the judgment task) application of the Central Limit Theorem and Law of Large Numbers forces the arithmetic mean of all responses--for any cell of the matrix--to converge on the true population mean, as the sample size increases. Reliabilities in the .85 to .90 range have been reported with as few as 50 cases (Barnett, 1972; Danes and Woelfel, 1975).

Mean distance matrices are transformed further to scalar-products matrices which are double-centered (Torgerson, 1958) to establish origins at distribution centroids. Such matrices subsequently are factored to achieve coordinate matrices whose columns are orthogonal axes and whose rows are projections of the concept location on each of the dimensions. This space simultaneously represents average distance judgments for all possible pairs. Also, the multidimensional space is constructed from the unstandardized distance vectors. Thus, all variance in the sample population ordinarily is accounted for by the N-1 dimensional space (although under some conditions it may be less) (Barnett & Woelfel, 1976).

This procedure is repeated over time. Provided that no additional information affects the relative stability of concepts, spaces are rotated about the centroid to a least-squares best fit. From the resultant cross-time coordinate matrices, one can fit motion trajectories which describe relational changes for the set. When additional information is
present -- such as knowledge of the relative inertial masses (amount of prior information [Saltiel & Woelfel, 1975]) of the concepts-- alternative rotational algorithms exist (Woelfel et al., 1975; Serota et al., 1977).

Least-squares rotation has the effect of over-estimating some changes while underestimating others. This can lead to erroneous conclusions. As an alternative to this procedure, Woelfel et al., (1975) have proposed a method which makes use of theoretical or "extra" information to provide a rotation which yields a simpler apparent motion. Such information concerns location of the concepts in space and is independent of coordinate values. Because of this, it can be treated as invariant under rotation and when coordinates are translated.

Another alternative rotation scheme shifts only the theoretically stable concepts to a least-squares best fit and then incorporates dynamic concepts into a new coordinate system. (It is quite similar to procedures used in astronomy when positions of fixed stars are used to measure the motions of other stellar bodies.) With the diffusion of innovations, there are theoretical reasons to suspect that stable relationships exist among the concepts used to define prior practices and objects; dynamic relationships exist among the prior practice or object and the innovation. Thus, one might hold the defining concepts stable and allow the innovation to move in relation to them.

Still a third procedure, when more information is known, is to weigh the concepts according to their inertial masses and rotate them to a weighted solution.
Formulae necessary to perform all of these operations are described in great depth by Woelfel et al., (1975); an empirical example is presented by Serota et al. (1977). A computer program—known as "Galileo™"—with the necessary algorithms is available at several academic institutions.

Once rotations are complete, change in the position of concepts can be calculated by simple subtraction of the coordinates over time. Motion through the space can be expressed as velocities:

\[ v_i = \frac{d_i}{t} = \frac{\sum_{j=1}^{N} (a_{ij} - b_{ij})^2}{t_i - t_0} \]

where \( v_i \) = the velocity of concept \( i \),
\( d_i \) = the distance concept \( i \) has moved across the interval of time \( t \),
\( t = \) time,
\( a_j \) = the coordinate value of concept \( i \) on the \( j \)th factor of the \( t_0 \) space, and
\( b_j \) = the coordinate value of concept \( i \) on the \( j \)th factor of the \( t_1 \) space.

This motion can be decomposed into its components along the orthogonal dimensions on which concepts are differentiated. Velocities and accelerations then can be computed as derivatives of the resultant curves. Partial derivatives are changes on a single dimension. Thus, it is possible to use diffusion messages to determine change in con-
ceptics toward novel ideas in terms of the dimensions:

\[ \frac{d s}{d t} = \frac{d s}{d t} + \frac{d s}{d t} + \cdots + \frac{d s}{d t} = \sum_{i=1}^{N} \frac{d s_i}{d t}. \]

Similarly, accelerations in the space are given by the second derivative:

\[ A_t = \frac{d^2 s}{d t^2} = \sum_{i=1}^{N} \frac{d^2 s_i}{d t^2}. \]

I suspect that this derivative is non-stationary. Research on diffusion of innovations shows that the adoption process can be described with an S-shaped curve—thus, \( \frac{d^2 s}{d t^2} \) is not constant. Also, on the basis of the literature in the field, I predict that acceleration (of the concepts) changes as a function of information supplied to members of the social system. For these reasons, much information on the diffusion process stands to be gained from second-order derivatives.

The Diffusion Curve

So far, I have made no predictions about rates of cultural change over time. It seems clear that there is modification in these rates during the adoption of the innovation. Theoretically, slopes reflecting the velocity of change over time resemble the traditional S-shaped diffusion curves.

For the ideal case, changes in the receiver system are described in this way: initially, the rate is very slow and represented by a small, positive slope; then, the rate increases exponentially until about half of the potential adopters have modified their conceptions of the innovation. At this point, the slope should peak at about 2.0. In the next stage, cultural change continues—but at a decreasing rate. Although still positive, the slope approaches zero and becomes asymptotic with the
number of potential adopters (that is, as associative processes within laggards lead them to change their conceptions of the innovation).

This S-shaped, growth curve first was described by Tarde (1903). Since then, it has been found consistently in the diffusion of innovation literature: Chapin (1928) found it with the diffusion of the sulky plow; Pemberton (1936) reported that postage stamps and state adoption of constitutional or statutory limits upon the taxation rate of municipalities could be described with it; McVoy (1940) found the same curve with the diffusion of city manager plans; Ryan and Gross (1943) and Ryan (1948) identified the S-shaped curve with the spread of hybrid seed corn. Hägerstrand (1953) described this curve for many innovations in a rural society. Yet, there are exceptions. Rogers et al. (1972) suggest that this traditional diffusion curve does not appear when the innovation involves a taboo topic (methods of birth control, for example). It is possible, however, that S-curves do not appear with such phenomena simply because the processes involved take so long in getting started.

There have been a number of attempts to describe this S-curve mathematically. Pemberton proposes the binomial formula, $X = (a-b)^n$, where $a$ and $b$, the probabilities of acceptance and rejection, equal 1/2, and $n$ is the growth exponent. This formula describes a normal frequency distribution which, when accumulated, becomes a normal ogive. He reasons that because the time of adoption is complex, factors operating to cause adoption prior to the average time may be regarded as equal to and counter-balancing the factors causing adoption at a later than average time. The time of trait acceptance in any given case is determined by the chance combination of factors for and against adoption (550).
If multiple causes exist which produce chance adoption before and after the mean, however, than \( p = 1/2 \) is wrong because probabilities are not equal. This suggests the polynomial formula

\[
\left(1/m_1 + 1/m_2 + \ldots + 1/m_n\right)^n.
\]

Yet, if probabilities do not equal 1/2, then the curve is assymetrical and does not resemble the S-curve. Pemberton also makes an additional assumption which limits the usefulness of his model: the population must be homogeneous with respect to adoption. This never is the case. It is the characteristics of a heterogeneous society—position in the social structure (Rogers & Shoemaker, 1971; Katz et al, 1972), physical proximity, and information distribution (or, mass media and network integration)—which are the most useful in analyzing diffusion of innovation.

Dodd (1950, 1953, 1955) presents a mathematical model of logistic diffusion. It begins with the differential equation, \( \frac{dp}{dt} = kpq \), where \( p \) = proportion of knowers or adopters in the population, \( q = 1-p \), and \( k \) = a proportionality constant (which describes the probability that an interaction between a knower and nonknower will result in adoption). Dodd's formula is integrated to produce \( P_t = \frac{1}{1+(q/p)e^{kt}} \), the accumulated logistic equation. As with Pemberton, Dodd makes an assumption which invalidates his model as a descriptor of cultural change. He assumes that diffusion populations can be divided into two groups, knowers and nonknowers. Yet, the rate of cultural change derives from continuous movement in the spatial manifold, not from a dichotomous decision to adopt or not. The process is a continuous one and it is impossible to categorize the population into any two groups because there are no criteria for such a decision.
Coleman (1964) proposes two diffusion models. One follows from Dodd but assumes that persuasion becomes increasingly redundant over time. People who already have adopted are the ones who persuade others to adopt at a constant rate, and the diffusion rate is constantly proportional to their number. The differential equation which summarizes this relation is \( \frac{dA}{dt} = kA(N-A) \) where, \( A \) = number of adopters, \( N \) = population size, \( k \) = a change constant. Integrated, the predictive equation becomes \( A = \frac{N^kt}{N-1+ekt} \). Again, this model is flawed by a dichotomous parameter, adoption or non-adoption.

Coleman's second model describes diffusion as a decaying exponential process. In this model, diffusion occurs within a limited population; information proceeds from a constant source (such as mass media) which is independent of the number of adopters. Thus, in this case, the number of adopters at each point in time is proportional to the number of those who have not adopted the innovation. The accumulated number of adoptions increases as a decaying exponential function of time. The differential equation describing the process is \( \frac{dA}{dt} = k(n-A) \). At \( t=0 \) and \( A=0 \) the predictive equation becomes \( A = be^{-kt} \). And again, the assumption of a dichotomous dependent variable renders the model useless for the measurement of conceptual change.

Problems caused by the lack of patterned interpersonal networks and the assumption of equality in mass media usage raise additional doubts about this model. Also, at the lower limits, the process is not described accurately. Here a diffusion curve should reflect exponential growth. Coleman's does not. His second curve always is a decaying exponential function. A final problem here (see figure 1) is that the predictive equation becomes asymptotic at zero rather than at the number
of potential adopters. The integrated equation for correcting this
problem is \( A = b d^{-kt} + KN \).\(^5\)

Figure 1 about here

For Hamblin et al, (1973), diffusion resembles exponential growth
of the type suggested in this differential equation: \( \frac{dA}{dt} = kA \) where,
\( A \) = the quantity of the attribute and \( k \) = the rate of growth. Inte-
grated, the predictive equation is \( A = be^{kt} \). While this model does not
assume binary adoption, it is not without its difficulties (see figure 2).
Because it assumes an infinite population, it does not reflect the
diffusion process accurately at the upper end.

Figure 2 about here

These authors later suggest a model which does. It is a combination
of an exponential growth and decaying exponential function, and can be
described by the differential equation \( \frac{dA}{dt} = kA(N-A) \) where, \( k \) = the
level of reinforcement, \( N \) = the population size, \( A \) = the number of
adopters. Integrated, the predictive equation becomes \( A = N/1-be^{-kt} \). Yet,
again, utility is limited because of the assumption of a dichotomous
dependent variable.

So, while none of these formulae are totally applicable to a
conceptualization of cultural change (as presented here), they do pro-
vide descriptive insight into the diffusion curve. For example, note the
comment of Hamblin et al, (1973:48) on Pemberton's model:

... Pemberton's theory has never gained acceptance,
in part because there is no equation that describes
the normal ogive (we have only the difference equation
for the normal frequency distribution). This means
that no one has ever been able to investigate the
parameters of the ogive to see if they make sense. Thus, rather ironically, the ogive has never been rejected; so far at least, primarily for theoretical reasons. There is no theoretical equation and, therefore, there are no parameters that can be explained.

There is discrepancy on this point. The formula for a curve normally distributed about the mean at zero is $y = (2\pi)^{-1/2} \exp \left(-1/2 x^2\right)$. Integrated, this formula describes the normal ogive which equals

$$
\int_{-\infty}^{\infty} (2\pi)^{1/2} \exp \left(e^{-1/2t^2}\right) \, dt.
$$

The problem with this curve is that it assumes infinity. It starts at negative infinity and approaches the population size after an infinite time period. Finite limits must be placed on the curve if it is to be realistic in describing social phenomena. This is done by limiting observations to include only those persons who adopt within three standard deviations of the mean, thus including more than 99 percent of the population. Problems caused by negative infinity are solved by translating the coordinates from a mean of zero to the observed mean. The general formula for a normal curve which applies in this case is $y = 1/\sigma (2\pi)^{1/2} \exp \left(e^{-1/2(x-\mu)^2/\sigma}\right)$. The predictive equation (see figure 3) becomes

$$
\int_{-6\sigma}^{6\sigma} 1/\sigma \exp \left(e^{-1/2(x-\mu)^2/\sigma}\right) \, dt.
$$

This provides the best predictive model for innovation diffusion in an idealized situation. It assumes only that the population is distributed...
normally with respect to the adoption probabilities.

Figure 3 about here

In the typical situation, one can expect the diffusion curve to reflect exponential growth until an innovation is approximately one-half acculturated. At this point, the curve will become one of decaying exponential growth. Thus, the predictive equations are:

\[ X = be^{kt} \text{ for } 0 < t < 1/2 \text{ and } X = be^{-kt} + kn \text{ for } 1/2 < t < 1. \]

\( X \) becomes asymptotic with the rate of behavior suggested by new information as it is acquired by persons in the receiver system. Such an asymptote applies only to unidimensional cases; for the multidimensional concept presented here, the curve becomes asymptotic with the distance relation advocated by new information.

This model assumes that adoptive behaviors are distributed normally in a population of potential adopters. If this assumption can be met, then the curve may be described as the normal ogive with the modifications suggested above. This model describes the idealized diffusion curve and need not be accurate for each and every case. If rapid adoption were to take place, as when a change in the law requires a change in behavior, the slope would surpass 2.0 at the midpoint of the process. On the other hand, the diffusion of a taboo topic would take place slowly and the slope would not reach 2.0 during the process.

Yet, the variables that Rogers and Shoemaker (1971) discuss are parameters affecting the curve's slope and the length of the process. Structural factors (a well-integrated communication network within the adopting society, for example) shorten the process and increase the slope. And this is typical of modern industrial societies. Yet, in traditional
societies (without well-defined media systems)—such as the ones detailed by Hägerstrand (1953)—associative structures change slowly; the slope of the resultant curves are not nearly as steep. Thus, the curve does provide a model against which the adoption of individual innovations and diffusion campaigns can be compared.

Empirically, the curve is established by plotting actual change rates against time. Then, a line is fitted to a least-squares best fit. Finally, function is defined by a determination of the slope at each point.

Summary and Conclusion

The model advocated here can be used to conceptualize general studies of socio-cultural change as well as specific investigations which focus on changes within cultures as innovations diffuse. The model is posited at the socio-cultural level. Thus, it is designed for the investigation of change within social systems rather than for attitudes held by individuals—although it is consistent with other information processing and attitude change models (Barnett, 1976; Craig, 1976; Woelfel, 1977; Saltiel and Woelfel, 1975; Woelfel and Saltiel, 1974).

This conceptualization examines the innovations within a cultural context. Setting is defined by complex interactions among cultural objects, and between these elements and social change mechanisms (the innovations themselves). The associational structure provides a holistic picture which leads to increased predictability of future behaviors because it takes into account all situation-specific relations affecting the actions of a society. This has been demonstrated within the political context by Barnett et al. (1976), Serota et al. (1977), and Cody (1977). Also, over time measurement increases predictability and provides a dynamic
picture of the change within the social system being investigated.

The model has the additional advantage of allowing the derivation of message strategies to most effectively alter the associational structures with respect to the innovation. And, the obvious corollary to this is that probabilities for adoption of the object, practice, or idea are maximized. Procedures for this involve vector analysis of the multidimensional spaces and some assumptions derived from the Woelfel-Saltiel attitude theory. Equations necessary to perform the analysis are provided by Woelfel et al. (1976) and empirical demonstrations are provided in a political context by Serota et al. (1977), and Cody (1977).

Thus, the model presented here provides a paradigm for future research into diffusion of innovation phenomena. In applying this approach, one requirement is continual measurement from the start of the process. This is necessary if the mathematical model is to be tested realistically. Clearly, the general model should be tested and effects of various strategies measured in terms of the overall curve.

In this way, future diffusion campaigns can be run more effectively.
NOTES

1. Acknowledgment is due to Joseph Woelfel, Everett H. Rogers, Rolf T. Wigand, James A. Danowski, Robert D. McPhee and Craig Harkins without whom this paper could not have been written.

2. The technique proposed here is based on the classical multidimensional scaling model (Torgerson, 1958). Other non-classical multidimensional scaling models are available, but these techniques apply principally to the reduction of matrices which are merely ordinal, and so are not applicable to the continuous, reliable ratio scaled data provided by the measurement system proposed in this paper. While they provide an accurate description of the structure of the data, change in the space over time cannot be observed. (Shepard, '66 '61972).

3. Woelfel (1974:13) has outlined several key advantages to this technique: "First and foremost, no restrictions are placed upon the respondent, who may report any positive real value whatever for any pair. Thus, the scale is unbounded at the high end and continuous across its entire range. Secondly, because the unit of measure is always the same (i.e., the unit is provided by the investigator in the conditional, "If \( x \) and \( y \) are \( u \) units apart," and thus every scale unit is \( 1/u \) units), and because the condition of zero distance represents identity between concepts and is hence a true zero, not at all arbitrary, this scale is what social scientists usually call a ratio scale, which allows the full range of standard arithmetic operations. Third, since the unit of measure is
provided by the experimenter it is possible to maintain the same unit of measure from one measurement to another, both across samples and across time periods, which is crucially important since time is one of the primitive variables of scientific theory. These three characteristics taken together provide the capacity for comparative and time-series analyses at very high levels of precision."

4. This assumes that the communication within the social system is either random or normally distributed. (Solomonoff and Rapoport, 1951).

5. I would like to thank Robert D. McPhee for his help in reformulating Coleman's equations to a non-zero asymptote. The proof of the correct solution is given below:

\[ \text{Problem: } \frac{dA}{dt} = K(N - A) \]

\[ \text{Solution: } A = be^{-Kt} + KN \]

\[ \text{Proof: } \frac{dA}{dt} = -Kbe^{-Kt} \]

\[ = KN - KN - Kbe^{-Kt} \]

\[ = K(N - A) \]

Q.E.D.
Figure 1

Coleman's Model of Delaying Exponential Growth

Number of Adopters

Reformulation to asymptote at N

NOTE: deviation at this point

Coleman's model
FIGURE 2

HAMLIN'S MODEL OF EXPONENTIAL GROWTH

Number of Adopters

Population Size

Time
FIGURE 3
NORMAL OGIVE

Unbounded Case

Bounded Case

$\pm \infty$

$0$

$\pm 3\sigma$

$\mu$
REFERENCES


