This report summarizes the major findings and recommendations of the 1977 British Columbia Mathematics Assessment, which was designed to inform the public of some of the strengths and weaknesses of the British Columbia school system. Over 100,000 students from Grades 4, 8, and 12 were given tests designed to measure mastery of a limited number of important mathematical skills and concepts. Approximately 3,500 teachers of mathematics at seven different grade levels (1, 3, 4, 7, 8, 10, 12) completed comprehensive questionnaires dealing with numerous aspects of the methods and materials in the teaching of mathematics in the province. A more complete rendering of the results of the Assessment may be found in the three other reports in the series. (MM)
Summary Report

A REPORT TO
THE MINISTRY OF EDUCATION
PROVINCE OF BRITISH COLUMBIA
THE B.C. MATHEMATICS ASSESSMENT

SUMMARY REPORT

This report was prepared for the Learning Assessment Branch of the Ministry of Education by

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This Summary Report contains, in greatly abridged form, the major findings and recommendations of the British Columbia Mathematics Assessment. A more complete rendering of the results of the Assessment may be found in the three other reports in the series: Report Number 1, Test Results; Report Number 2, Teacher Questionnaire; and Report Number 3, Technical Report. Copies of all of these reports are available upon request from the Learning Assessment Branch, Ministry of Education.

Many people contributed to the Assessment in various ways, and their contributions were greatly appreciated. The teachers of mathematics throughout the province and their students completed the Assessment tests and questionnaires. Members of the Mathematics Curriculum Revision Committee, the Review Panels, the Interpretation Panels, and the staffs and students of the pilot schools assisted in the development of the objectives of the Assessment and of the student tests. The staff of the B.C. Research Council provided highly professional and expert technical services to the Contract Team in all phases of the project. The staff of the Learning Assessment Branch provided guidance, encouragement, experience, and enthusiasm. All of the members of the Management Committee gave generously of their time and expertise throughout the almost fifteen month duration of the project. As one of the Management Committee members said, "The pay was short, but the hours were long."

Special thanks and recognition are due to my friends and colleagues, James Sherrill, Heather Kelleher, and John Klassen. As members of the Contract Team which was responsible for the Mathematics Assessment, they made an invaluable contribution. Their willingness to work hard, and their unquestioned expertise in the field of Mathematics Education made the task considerably easier than it might otherwise have been.

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HIGHLIGHTS OF THE MATHEMATICS ASSESSMENT

In the spring of 1977, over one hundred thousand students from Grades 4, 8, and 12 wrote the tests prepared for the 1977 British Columbia Mathematics Assessment. In addition, almost three thousand teachers of mathematics at seven different grade levels (1, 2, 4, 7, 8, 10, 12) completed comprehensive questionnaires dealing with numerous aspects of the teaching and learning of mathematics.

The Assessment Program in general and the Mathematics Assessment in particular are designed to inform the public of some of the strengths and weaknesses of the public school system in this province. The information generated by the Mathematics Assessment will assist school districts in maintaining identified strengths and overcoming weaknesses. It is hoped that curriculum developers and curriculum revision committees will be able to make use of these results in the process of improving curricula and developing suitable resource materials. Such information could be used in the allocation of resources and in the planning of teacher education programs at both the provincial and district levels. Furthermore, the data bank produced by the assessment should be of great value to educational researchers as a source of researchable questions concerning the teaching and learning of mathematics.

The student tests were designed to measure students’ mastery of a limited number of important mathematical skills and concepts at each of three levels: end of primary education (Grade 3), end of elementary education (Grade 7), and end of public schooling (Grade 12). Since mathematics is not a compulsory course beyond Grade 10, the content of the Grade 12 test was restricted to the Grade 10 level or below.

The tests were administered to students enrolled in Grades 4, 8, and 12 regardless of the nature of the mathematics course they were taking, if any. The tests were not tests of any particular course but rather of students’ mastery of a number of essential mathematical skills and concepts. For that reason, the results presented here should not be used to describe the success or failure of a particular course, nor should they be interpreted as being indicative of students’ degree of preparedness for further studies in mathematics at the post-secondary level.

The test results were judged by Interpretation Panels consisting of educators, school trustees, and other members of the general public which were constituted by the Ministry of Education for that purpose.

At the Grade 12 level, the student population was divided into three sub-populations for a number of the analyses because of differences in mathematical background among the students. One group consisted of those students who were taking or had taken their twelfth year of mathematics (Mathematics 12); the second group consisted of those who were taking or had taken their eleventh year of mathematics (Mathematics 11); and the third segment of the Grade 12 population consisted of those students who were taking or had taken their tenth year of mathematics (Mathematics 10).
Among the major findings of the Mathematics Assessment are the following:

* Grade 12 students who have taken twelve years of mathematics did extremely well on all portions of the test. As a group, their scores averaged between seventeen and twenty-five percent better than Grade 12 students with eleven years of mathematics and between thirty-four and thirty-eight percent better than those with ten years of mathematics.

* Grade 12 students who have taken only the minimal requirement of ten years of mathematics (about 15% of the Grade 12 population) generally performed poorly on all portions of the test, including those questions relating to consumer applications of mathematics. This information is noteworthy because over one-third of this group indicated that they plan to enter the labor market upon completion of Grade 12.

**Computational Skills and Knowledge**

* At all three grade levels, most students exhibited a generally satisfactory level of ability to add, subtract, multiply, and divide. Grade/Year 4 students' performance on subtraction of whole numbers was less than completely satisfactory, particularly on items requiring regrouping (borrowing).

* Students at all three levels performed satisfactorily on most items measuring knowledge of mathematical symbols and terminology.

* Grade 8 and 12 students' performance on items requiring them to express a number given as a decimal, fraction or percent in one of the other forms was less than satisfactory.

**Comprehension of Mathematical Concepts**

* Grade/Year 4 students performed poorly on two items assessing comprehension of fraction concepts, and it was concluded that such concepts may be too abstract for many students at this level.

* Some areas of weakness were noted at the Grade 4 and 12 levels in students' familiarity with the metric system of measurement.

* Comprehension of geometric concepts was generally rather weak at Grades 8 and 12.

**Applications**

* Students in Grade 4 and 8 did rather well in problem-solving and this is a commendable result given the difficulty of the topic. The Grade 12 problem-solving results were disappointing on the whole, and seem to indicate that many students are unable to apply the computational skills they have learned to the solution of problems. Problems dealing with geometry and measurement seemed to cause the most difficulty.
A number of the application items on the Grade 12 test concerned consumer mathematics. Results were rated as satisfactory or better on items dealing with buying a car on credit, finding an average, using the concept of discount, interpreting a graph, and interpreting road maps. Results on items dealing with applications of the formula for simple interest, selecting the best purchase, and reading an income tax table were rated marginally satisfactory.

A number of factors are either known or are suspected of being related to students' performance in mathematics. The following findings are based upon trends which are apparent from the data, but should not be interpreted as necessarily implying the existence of cause-and-effect relationships between variables. Follow-up studies designed to explore reasons why the following trends have emerged would be in order.

The Assessment data show that among children born in 1967 who took the Grade/Year 4 test, those born between January and March outperformed other students. A student's age, therefore, may be one of the factors contributing to the range of individual differences confronting teachers of mathematics, particularly at the primary level.

Female students outperformed males on most of the computation objectives on all three tests, while males outperformed females on all of the problem-solving objectives. Most such differences, whether in favour of males or females, were small.

Almost sixty-five percent of the students who took only the minimal requirement of ten years of mathematics are female. On the other hand, about sixty percent of those students taking twelve years of mathematics are male.

Students who were not born in Canada and for whom English is not a first language, did better than Canadian-born students for whom English is the first language on all three sections of the Grade 8 and 12 tests. This was not the case among Grade/Year 4 students, where the Canadian-born, English-speaking group obtained the better result on two of the three sections.

Generally speaking, Grade 12 students' performance on the test was positively correlated with their parents' level of schooling. That is, in general, an increase in student achievement was associated with an increase in the highest level of schooling attained by the parents.

Of six groups established on the basis of students' future plans, the one consisting of students planning to seek employment upon completion of Grade 12 had the weakest results. Students planning to attend university had the highest results.

Grade 8 and 12 students who use hand-held calculators in school, at home, or for homework outperformed those who did not. This result was reversed at Grade/Year 4 for students who used hand-held calculators at school or for homework.
Students in Grade 8 and 12 who spent some time on mathematics homework, but less than thirty minutes per day, obtained higher results than those who spend more time or no time on such assignments.

At the Grade 4 and 8 levels results indicate that an increase in the number of schools attended by a student is associated with a decrease in performance. No consistent trend was evident at the Grade 12 level.

There was a slight but consistent increase in performance on the Grade/Year % test associated with an increase in amount of television watched per day, up to four hours per day. At the Grade 8 and 12 levels, a general decrease in performance was associated with an increase in amount of television watched.

Among the major findings from the Teacher Questionnaire data are the following:

- The average elementary teacher has had four years of professional training and over eight years of teaching experience. Among secondary teachers of mathematics, the corresponding figures are five and nine years respectively.
- About twenty-five percent of teachers of elementary mathematics have taken no post-secondary courses in mathematics, and about fourteen percent have had no training in the teaching of mathematics.
- Grade 8 mathematics classes are larger on the average than at any of the other six levels surveyed and over twenty-five percent of the teachers of mathematics at this level have never taken a course in methods of teaching mathematics. Almost fifty percent of teachers of Grade 8 mathematics did not have mathematics as a major area of study in university.
- The rate of membership in the B.C. Association of Mathematics Teachers is extremely low; about three percent of elementary teachers and twenty-eight percent of teachers of secondary mathematics belong to this group.
- Teachers of elementary mathematics consider mathematics to be one of their favourite and easiest courses to teach but their opinion is less favourable with regard to learning it.
- Insofar as methods used in the teaching of mathematics are concerned, no recent innovations appear to have gained widespread acceptance among teachers.
- Teachers are generally satisfied with the textbooks which they are using, and they were virtually unanimous in expressing a desire to have minimal learning objectives for each grade level specified.
- Teachers of mathematics at all levels were of the opinion that reduction of class size was a high priority need.
- All groups of teachers agreed that elementary students should not be permitted to use hand-held calculators in school and that senior secondary students should be.
Learning Assistance Centres which provide assistance for students experiencing difficulties in mathematics are not available in the majority of elementary schools.

Elementary teachers indicated by a three-to-one margin that a mathematics course should be required in Grade 12. Less than fifty percent of the secondary teachers shared this opinion.
1. INTRODUCTION TO THE MATHEMATICS ASSESSMENT

In the spring of 1977, students enrolled in Grades 4, 8 and 12 in the public schools of the province of British Columbia took part in an assessment of student learnings in Mathematics conducted by the Learning Assessment Branch of the Ministry of Education. During the same period, approximately 3,500 teachers of mathematics at seven different grade levels completed a comprehensive questionnaire dealing with numerous aspects of the methods and materials used in the teaching of mathematics in the province.

The Learning Assessment Program is longitudinal in nature, and the various components of the curriculum are scheduled to be assessed at regular intervals. The 1977 Assessment of Mathematics in the province was intended to collect baseline data against which the performance of students in future assessments could be compared. In this way, it will be possible to compare students' performance on subsequent assessments with the results of this one, thereby obtaining a measure of the change and the direction of change that has taken place in the interim.

Purposes of the Assessment

The major principle underlying the entire Learning Assessment Program is that decisions about education should be based upon knowledge of what and how students are learning. Educational decisions are being made every day, decisions which affect the allocation of resources, in-service education of teachers, teacher training programs, curriculum development, and the adequacy of various programs. The Mathematics Assessment provides decision-makers at all levels with factual and current information concerning the teaching and learning of mathematics upon which to base their decisions.

The Assessment Program in general and the Mathematics Assessment in particular are designed to inform the public of some of the strengths and weaknesses of the public school system in this province. The information generated by the Mathematics Assessment will assist school districts in maintaining identified strengths and overcoming weaknesses. It is hoped that curriculum developers and curriculum revision committees will be able to make use of these results in the process of improving curricula and developing suitable resource materials. Furthermore, such information could be used in the allocation of resources at both the provincial and district levels.

At the university level, the information generated by the Assessment will be useful in indicating directions for change and improvement in teacher education. Finally, the information produced by the Assessment should be of great value to educational researchers both as a data bank and as a source of researchable questions concerning the teaching and learning of mathematics.
Organization of the Assessment

Several groups participated in the organization and implementation of the Mathematics Assessment. These groups included the Learning Assessment Branch of the Ministry of Education, the Contract Team, the Management Committee, the B.C. Research Council and several others with whom consultations were held.

The Contract Team was retained by the Learning Assessment Branch to conduct the Mathematics Assessment. The Contract Team's responsibilities included conducting the Goals Assessment and developing the set of objectives to be assessed, constructing the student tests, trying out the tests, and subsequently revising them, constructing the Teacher Questionnaire, and writing the final reports of the Assessment. The Contract Team consisted of two members of the Faculty of Education, University of British Columbia, a primary teacher who was on leave of absence from the New Westminster School District, and a teacher of secondary mathematics from the North Vancouver School District.

The role of the Management Committee was to oversee the operations of the Contract Team, and to provide guidance and suggestions regarding the various phases of the assessment. Members of the Management Committee included two teachers, a supervisor of instruction, a teacher educator, a school trustee, the chairman of the Contract Team, and representatives of the Learning Assessment Branch.

Working under the direction of the Contract Team, the B.C. Research Council was responsible for the majority of the technical and administrative aspects of the Assessment. These responsibilities included overseeing the printing and distribution of the tests, answer cards, and teacher questionnaires, conducting the data processing, and serving as statistical consultants and advisors to the Contract Team and the Management Committee.

Consultative meetings were held with several groups. Representatives of the Contract Team met with the Mathematics Curriculum Revision Committee to discuss aspects of the Assessment. In addition, Review Panels were organized by the Learning Assessment Branch to discuss the objectives to be tested in the Mathematics Assessment. Such panels were intended to be as widely representative as possible of the various groups interested in the mathematics achievement of students. Finally, meetings were held and correspondence exchanged with representatives of other assessment programs in North America, in order that the B.C. Mathematics Assessment could benefit from their experiences.

Components of the Mathematics Assessment

The Mathematics Assessment consisted of four major components: the Goals Assessment, the Student Tests, the Interpretive Analysis, and the survey of Instructional Practices. Each of these is treated in some detail later in this report as well as in the other reports in the series. A few descriptive comments about each are included here.
1. The Goals Assessment

It was not the objective of the Mathematics Assessment to attempt to evaluate students' mastery of any particular mathematics course. Neither was it the objective of the Assessment to obtain information on the achievement of individual students or schools, nor on the performance of individual teachers of mathematics. It was the objective of the Assessment to obtain, and to make widely known, information regarding the present state of mathematics learning on a province-wide basis. In addition, each school district was to be provided with a summary of its own results.

The initial and basic decision regarding the Goals Assessment was to limit the scope of the content areas of mathematics to be assessed to those which most informed observers would agree were among the essential concepts and skills of mathematics at the three levels tested: end of primary education (Grade/Year 4), end of elementary education (Grade 8), and end of public schooling (Grade 12). The Grade 12 test was not a test of any particular course such as Math 10, Math 11 or Math 12 but rather a test of students' mastery of a number of mathematical skills and concepts which, for the most part, all students could be expected to have acquired upon completion of their public schooling. The content of each item on the Grade 12 test was selected from the Grade 10 level or below. Three levels of cognitive behaviour, called domains in the Assessment, each subdivided into a number of objectives, made up the basic framework of the Goals Assessment.

2. The Student Tests

Tests were constructed to measure students' mastery of the objectives identified in the Goals Assessment phase. A separate test was prepared for each of the three levels involved. For each test, a total administration time of ninety minutes was allotted: thirty minutes for instructions, distribution and collection of the test booklets and answer cards, and sixty minutes for completion of the test.

Pilot testing of the assessment instruments was conducted during the late fall of 1976 in several school districts across the province. Approximately 250 students at each of the three grade levels involved wrote the pilot tests; and their results were used in developing the final forms of the tests.

With the exception of a portion of the Grade/Year 4 test, all of the test items were cast in multiple-choice format with four choices (or foils) for each item. In every case, the choices consisted of four possible answers to the item. A fifth choice, the "I don't know" option was used in an attempt to minimize guessing and in order to provide an outlet for students who, for one reason or another, had not been exposed to the material being tested or had forgotten it.
In an effort to assess the development of students' abilities to deal with certain concepts and skills, some items appeared on two or more of the tests. For example, the same five items dealing with knowledge and understanding of the units of the metric system of measurement were used on all three tests. In several of the skill areas, the same item or items appeared on the Grade 4 and 8 tests, or on the Grade 8 and 12 tests. Overall, there were nine items common to the Grade 4 and 8 tests and forty-three items common to the Grade 8 and 12 tests. Of this number, five items were common to all three tests.

The International System of Units (SI) was utilized for all test items involving measurement; no items contained British or Imperial units of measurement. The decision to use the metric system of measurement exclusively did restrict, to some degree, the number and the nature of problem-solving items involving measurement concepts. For example, it was felt that including items dealing with the purchase of consumer goods such as carpeting, or building materials, or the like, in terms of metric units of area or volume would make such items appear overly unrealistic and unfamiliar since these terms and units are not yet in widespread use by consumers in Canadian society. On the other hand, since the curriculum guide does call for implementation of the metric system of measurement in the schools, any reference to the British system was avoided.

3. Interpretive Analysis

Since this was the first major Mathematics Assessment to be conducted by the Learning Assessment Branch, there was no baseline information available for making judgments about the results. To assist in interpreting the results, Interpretation Panels were used to assess the student test data.

Three fifteen-member Interpretation Panels, one for each of the three-grade levels involved, were constituted by the Learning Assessment Branch. Each Panel consisted of seven teachers of mathematics at the particular grade level, two supervisors of instruction, two teacher educators, two school trustees, and two members of the public at large.

Panelists were asked to take the test for their respective grades (4, 8 or 12) and to consider for themselves, without seeing the results, what the student performance on each test item should have been. All three Panels then convened in Vancouver in early June 1977 in order to interpret the test results. In an effort to obtain agreement among the panel members and yet give each panelist every opportunity to influence the final outcome, a four-stage procedure was followed. Each panelist first interpreted the results individually, then they worked in pairs, in groups of five, and, finally, as a Panel of fifteen.
Student performance on each test item was assigned a rating by the Panels. The five rating categories used were:

a) Strength
b) Very satisfactory
c) Satisfactory
d) Marginally satisfactory
e) Weakness

Information gained from the deliberations of the Interpretation Panels was used by the authors of the assessment reports in commenting upon the results of the assessment. Although the procedure used does lack some of the air of precision attributed to strictly numerical comparisons, the wealth of experience which the members of the panels brought to bear upon their examination and interpretation of the results gives their ratings considerable credibility.

4. Instructional Practices

Two questionnaires, one for teachers of elementary school mathematics and the other for teachers of secondary school mathematics, were developed for use in the Mathematics Assessment. The questionnaires, which were completed anonymously, dealt with various aspects of the teachers' backgrounds and training as well as with facets of the methodology of teaching mathematics at different levels and with instructional materials used by teachers of mathematics.

Student Characteristics as Reporting Categories

A number of factors are either known to be or are suspected of being related to students' performance in mathematics. While it would not be possible to identify a causal relationship between a given student characteristic and performance on the assessment test as a part of the Mathematics Assessment, it is possible to identify variables which appear to be related on the basis of the data collected. Relationships so identified may lead to follow-up studies specifically designed to identify cause and effect relationships on the basis of the correlational results discovered in the assessment program.

As a part of each of the three Mathematics Assessment tests, students were asked to report on several aspects of their personal backgrounds. A list of the variables on which data were collected in the Mathematics Assessment is given below. The grade levels at which each item was included are listed parenthetically.

a) Mathematics background (12)
b) Date of birth (4, 8, 12)
c) Sex (4, 8, 12)
d) Number of schools attended (4, 8, 12)
e) Residence in Canada and language spoken (4, 8, 12)
f) Number of hours of television watched (4)
g) Use of hand-held calculators (4, 8, 12)
h) Time spent on homework (8, 12)
i) Semestered versus non-semestered courses (8, 12)
j) Parents' educational backgrounds (12)
k) Future plans (12)
l) Out-of-school work (12)

Assessment tests in Reading were also given at the Grade 8 and 12 levels, and these tests contained similar, and in some cases identical, background information questions. For example, on both the Reading and the Mathematics tests, students were asked their date of birth, sex, and number of schools attended. Because of the common items, it was possible to merge the two sets of data and obtain a new data file containing the information and results obtained on both of the tests. Matches were obtained for 66% of the Grade 8 students and 63% of the Grade 12 students. This new file was used to obtain further information on student background, as well as to correlate some aspects of student performance in reading with the same students' achievement on some of the mathematics objectives.
2. THE GOALS: ASSESSMENT

Mathematics has a large number of subject-matter components. In broad terms, there are areas such as algebra, arithmetic, geometry, and trigonometry which are the major concerns of school mathematics. Each of these broad areas can be broken down into a multitude of sub-divisions, sub-divisions of sub-divisions, and so on. Not all areas or sub-divisions are equally important in school mathematics, and the relative importance attached to various topics in the mathematics curriculum varies according to sociological, psychological, and educational influences of the day.

Accordingly, it was imperative that the goals or objectives to be assessed by the Mathematics Assessment be identified at the outset, and two basic decisions were made in this regard. First, it was decided that the assessment would test students at three levels: end of primary education (Grade/Year 4), end of elementary education (Grade 8), and end of public schooling (Grade 12). Second, it was decided that the topics to be tested would be restricted to what most informed observers would agree were among the essential skills and concepts of mathematics which all students at these three levels should have mastered.

The process of development of the specific content objectives and of the cognitive behaviour levels for the Mathematics Assessment was greatly facilitated by the experiences of others in similar programs. The publications of the National Assessment of Educational Progress in the United States and of various state assessment programs were of notable assistance in this regard.

The Item Specification Model

The end result of the developmental process mentioned above was the Item Specification Model for the Mathematics Assessment which is shown in Figure 2-1. The model portrays, in summary fashion, the three major dimensions of the Mathematics Assessment.

The Item Specification Model for the Mathematics Assessment is a $3 \times 4 \times 3$ "cube": three grades (4, 8, and 12), four major mathematics content areas (Number and Operation, Measurement, Geometry, and Algebraic Concepts), and three domains (Computation and Knowledge, Comprehension, and Applications). The essential idea of the model is that the objectives and test items for the Mathematics Assessment can be classified in three ways: by grade, by content area, and by domain.

The first dimension on the model is grade level. As described earlier, the three points on the K-12 continuum selected for testing were end of primary education, end of elementary education, and end of public schooling. The end-of-primary test was administered at the Grade/Year 4 level, and tested content selected from the Grade 3 level or below. The end-of-elementary test covered content from the Grade 7 level or below,
and was administered to students enrolled in Grade 8. In order to assess students' performance in mathematics at the end of their public schooling, the assessment test was administered to all students in Grade 12. However, since mathematics is not required after Grade 10, the mathematics content on the assessment instrument administered in Grade 12 was restricted to the Grade 10 level at most.

The second dimension of the Item Specification Model is content. Of all the major mathematics content areas that could have been used, the following four were selected: Number and Operation, Measurement, Geometry, and Algebraic Concepts. Topics classified as belonging to the Number and Operation category dealt with the nature and properties of whole numbers, integers, rational numbers, and real numbers as well as with techniques and properties of arithmetic operations. The Measurement category included topics such as selecting the most appropriate unit of measurement, familiarity with metric units of measurement, area, volume, perimeter, angular measurement, and scale drawing. Topics in the Geometry category dealt with the identification of geometric figures, classification of angles and triangles, parts of a circle, and the Theorem of Pythagoras. Topics in the Algebraic Concepts category were concerned with graphs, writing algebraic expressions, simplifying and evaluating polynomials, linear and quadratic equations, slope of a line, and simultaneous equations.
The third dimension of the model consists of three levels of cognitive behaviour, which are referred to as domains. The Computation and Knowledge domain encompasses areas such as knowledge of specific facts, knowledge of terminology, and ability to use algorithms. The Comprehension domain includes knowledge of concepts, knowledge of principles, rules, generalizations, ability to transform problem elements from one mode to another, and ability to read and interpret a problem. Ability to solve routine problems, ability to analyze data, and ability to recognize patterns belong to the Applications domain.

The domain dimension of the model is hierarchical. The Applications domain is cognitively more complex than Comprehension which is in turn more complex than Computation and Knowledge.

The model, as simple as it is, illustrates the complexity of outcomes of mathematics learning. Many important areas of mathematics are not included, and no mention is made of attitudes toward mathematics. In this assessment, concern was limited strictly to cognitive outcomes. It is also true that although the model contains thirty-six cells, some of them are empty. For example, no attempt was made, at the Grade/Year 4 level, to test the area of Geometry, and the Algebraic Concepts domain included only two graphing items.

Developments of Objectives and Test Items

Specific objectives corresponding to each domain and sample test items were developed for the Mathematics Assessment. The single resource which provided the most guidance and direction in this task was the recently revised Curriculum Guide for Mathematics issued by the Ministry of Education.

Four Review Panels were organized in different parts of the province to give people who were not involved in the creation of the objectives an opportunity to examine the proposed objectives and suggest alterations before the student tests were developed. These panels, consisting of teachers, school trustees and lay persons, met with representatives of the assessment program to discuss the objectives and to seek to improve them.

To obtain more feedback on the proposed set of objectives and sample items, representatives of the Contract Team met with the Mathematics Curriculum Revision Committee. In addition, the objectives were published in Vector, the journal of the B.C. Association of Mathematics Teachers, and sessions dealing with aspects of the Mathematics Assessment were held at several conferences.

The final version of the objectives organized by domain within each grade level are presented in Tables 2-1 through 2-3. The right hand column in each of the three tables lists the number of items on the assessment instrument used to measure mastery of the accompanying objective.
Table 2-1
Grade/Year 4 Mathematics Assessment Objectives

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domain 1: Computation and Knowledge</strong></td>
<td></td>
</tr>
<tr>
<td>1.1 Mastery of Number Facts</td>
<td>24</td>
</tr>
<tr>
<td>1.2 Addition of Whole Numbers</td>
<td>5</td>
</tr>
<tr>
<td>1.3 Subtraction of Whole Numbers</td>
<td>5</td>
</tr>
<tr>
<td>1.4 Knowledge of Notation and Terminology</td>
<td>6</td>
</tr>
<tr>
<td><strong>Domain 2: Comprehension</strong></td>
<td></td>
</tr>
<tr>
<td>2.1 Place Value Concepts</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Number Properties</td>
<td>4</td>
</tr>
<tr>
<td>2.3 Measurement Concepts</td>
<td>5</td>
</tr>
<tr>
<td>2.4 Fraction Concepts</td>
<td>2</td>
</tr>
<tr>
<td><strong>Domain 3: Applications</strong></td>
<td></td>
</tr>
<tr>
<td>3.1 Solution of Practical Problems</td>
<td>6</td>
</tr>
<tr>
<td>3.2 Solution of Computational Problems</td>
<td>6</td>
</tr>
</tbody>
</table>

The Grade/Year 4 Mathematics Assessment instrument contained sixty-nine items measuring mastery of ten objectives. The data in Table 2-1 show that the major emphasis on the Grade/Year 4 test was on the Computation and Knowledge domain. Of the forty items in this domain, twenty-four were used to assess the Mastery of Number Facts objective. These items took the form of six number facts for each of addition, subtraction, multiplication, and division. In addition to the number fact items, there were ten addition and subtraction exercises which required use of the algorithms.

In the Comprehension domain, the emphasis was placed upon understanding of place value concepts and number properties. There were five items dealing with measurement and two with fraction concepts. The Applications domain was evenly divided between computational problems and problems termed practical, e.g., working with time, money, and graphs.

Several cells in the Item Specification Model were not tested at the Grade/Year 4 level, and such exclusions were made for two major reasons. In some cases, the content was not part of the K-3 curriculum, and in others it was felt that the material could not be adequately tested by means of a paper and pencil test. An example of the former would be the algorithms for multiplication and division of whole numbers which are not developed to any great degree in the primary grades. An example of the latter would
be the area of geometry which, in the primary grades should involve the manipulation of three dimensional objects and is, as a result, not amenable to paper and pencil testing.

Table 2-2 lists the domains, objectives, and number of items per objective for the Grade 8 Mathematics Assessment instrument. The test contained sixty items measuring acquisition of twelve objectives.

Table 2-2
Grade 8 Mathematics Assessment Objectives

<table>
<thead>
<tr>
<th>Domain 1: Computation and Knowledge</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Computation with Whole Numbers</td>
<td>5</td>
</tr>
<tr>
<td>1.2 Computation with Fractions</td>
<td>4</td>
</tr>
<tr>
<td>1.3 Computation with Decimals</td>
<td>5</td>
</tr>
<tr>
<td>1.4 Knowledge of Notation and Terminology</td>
<td>9</td>
</tr>
<tr>
<td>1.5 Knowledge of Geometric Facts</td>
<td>4</td>
</tr>
<tr>
<td>1.6 Equivalent Forms of Rational Numbers</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain 2: Comprehension</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Number Concepts</td>
</tr>
<tr>
<td>2.2 Measurement Concepts</td>
</tr>
<tr>
<td>2.3 Geometric Concepts</td>
</tr>
<tr>
<td>2.4 Algebraic Concepts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain 3: Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Solution of Problems involving Operations with Whole Numbers, Fractions, Decimals, and Percent</td>
</tr>
<tr>
<td>3.2 Solution of Problems involving Geometry and Measurement</td>
</tr>
</tbody>
</table>

The emphasis in the Computation and Knowledge domain for Grade 8 was on computation with different sets of numbers. Assessment of knowledge of geometric facts was also included in this level of the assessment. Geometry, in fact, was a factor in each domain for the Grade 8 assessment. All four content areas were assessed in the Comprehension domain, but with varying numbers of items. The emphasis in the Applications domain was on solving problems using different sets of numbers and numbers in different forms.
A comparison of Tables 2-1 and 2-2 shows that the major differences between the Grade 4 and 8 levels were the inclusion of geometric concepts and the shift in emphasis in the Computation and Knowledge domain on the Grade 8 test. By Grade 8, computation includes all four basic operations with whole numbers, fractions, and decimals.

Table 2-3 lists the domains, objectives, and the number of items per objective for the Grade 12 Mathematics Assessment. The test contained seventy-two items measuring acquisition of eleven objectives.

### Table 2-3

<table>
<thead>
<tr>
<th>Grade 12 Mathematics Assessment Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
</tr>
<tr>
<td><strong>Domain 1</strong>: Computation and Knowledge</td>
</tr>
<tr>
<td>1.1 Computation with Fractions</td>
</tr>
<tr>
<td>1.2 Computation with Decimals</td>
</tr>
<tr>
<td>1.3 Knowledge of Notation and Terminology</td>
</tr>
<tr>
<td>1.4 Knowledge of Other Algorithms</td>
</tr>
<tr>
<td><strong>Domain 2</strong>: Comprehension</td>
</tr>
<tr>
<td>2.1 Number Concepts</td>
</tr>
<tr>
<td>2.2 Measurement Concepts</td>
</tr>
<tr>
<td>2.3 Geometry Concepts</td>
</tr>
<tr>
<td>2.4 Algebraic Concepts</td>
</tr>
<tr>
<td><strong>Domain 3</strong>: Applications</td>
</tr>
<tr>
<td>3.1 Solution of Problems involving</td>
</tr>
<tr>
<td>Operations with Whole Numbers, Fractions,</td>
</tr>
<tr>
<td>Decimals, and Percent</td>
</tr>
<tr>
<td>3.2 Solution of Problems involving</td>
</tr>
<tr>
<td>Geometry and Measurement</td>
</tr>
<tr>
<td>3.3 Solution of Algebraic Problems</td>
</tr>
</tbody>
</table>

In the Computation and Knowledge domain, the Knowledge of Notation and Terminology objective involved several more items than were required for the assessment at Grade 8. Computation with fractions and decimals was included, and Knowledge of Other Algorithms was added. The Comprehension domain for Grade 12 was very similar to that for Grade 8 except for Comprehension of Algebraic Concepts which assumed a more important position in Grade 12 than it had had in Grade 8. The Applications domain was most comprehensive for Grade 12, involving eighteen items.
The Grade 8 and 12 objectives have many similarities. In fact, forty-three of the items on the Grade 8 test were repeated on the Grade 12 test.

The distribution of test items organized by grade and by content area is presented in Table 2-4 below. Since the tests had different numbers of items, the data presented in the table are percentages. The fact that some of the rows in the table do not have a total of 100% is due to the effect of rounding each percent to the nearest whole number.

Table 2-4
Percent of Items in Each Content Area by Grade

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number and Operation</th>
<th>Measurement</th>
<th>Geometry</th>
<th>Algebraic Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>78</td>
<td>17</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>63</td>
<td>17</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>50</td>
<td>15</td>
<td>15</td>
<td>19</td>
</tr>
</tbody>
</table>

The data in Table 2-4 show a decreasing emphasis on the Number and Operations content area as grade level increases, and an increasing emphasis on the Algebraic Concepts content area. The Measurement content area has a very consistent emphasis, as does Geometry in Grades 8 and 12.

All three tests were designed to assess students' grasp of a number of important mathematical skills and concepts, and the Review Panels and pilot testing served as indicators of the validity of the test items. In addition, the items were examined statistically to determine their reliability, and a discussion of the test reliabilities is contained in the Technical Report.

The approach to test construction described here should not be confused with that used in the construction of typical (norm-referenced) achievement tests which are designed to rank individual students. For tests of that type, items are usually designed so that approximately fifty percent of the population will be able to correctly answer each item. On a group test such as one used in the B.C. Assessment, it is possible, and in some cases desirable, that virtually the entire population be able to respond correctly to a given item. Whether or not an item was included in the final version of a test depended upon its being considered a valid item to assess a given objective at one or more of the grade levels involved and not upon its power of ranking individuals.
3. THE STUDENT POPULATION

Slightly more than one hundred thousand students at three grade levels completed the Mathematics Assessment tests. As the data in Table 3-1 show, the rate of response varied from a low of seventy-one percent at the Grade 12 level to ninety percent at Grade 8, and ninety-six percent at Grade/Year 4.

Table 3-1
Rate of Completion of Mathematics Assessment Tests

<table>
<thead>
<tr>
<th>Grade/Year</th>
<th>Enrollment as of February 1977*</th>
<th>Number of Completed Tests</th>
<th>Percent Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>36 540</td>
<td>35 277</td>
<td>96.5</td>
</tr>
<tr>
<td>8</td>
<td>66 808</td>
<td>42 250</td>
<td>90.1</td>
</tr>
<tr>
<td>12</td>
<td>32 532</td>
<td>23 136</td>
<td>71.1</td>
</tr>
<tr>
<td>Overall</td>
<td>115 880</td>
<td>100 663</td>
<td>86.9</td>
</tr>
</tbody>
</table>

* Data supplied by the B.C. Ministry of Education

Of the three grade level groups, Grade 12 was the only one which had a lower return rate than might have been attributable to normal absenteeism. In addition, some concern was expressed to the effect that senior secondary students might not have taken the test seriously and responded frivolously.

The National Assessment of Educational Progress (NAEP) program in the United States has encountered similar difficulties of non-participation. In their first analysis they assumed that the non-participating group was similar in composition to the entire population, and that those individuals who failed to participate would not affect the overall results. Subsequent studies have shown that the non-participating group was not exactly similar to the rest of the population and that their lack of participation could result in artificially inflated results. NAEP has found that the extent of this inflation is almost certainly not great enough to affect decision-making. For example, a success rate of 67% achieved by those responding to a given test item might represent a true success rate of 64% for the entire population.

To check for the extent of frivolous response on the Grade 12 test, two steps were taken. Each of the computer cards which were completed by the students was hand-checked for completeness and for obvious patterns of frivolous response, such as the constant use of a single response category or the repetition of a series of responses such as ABC ABC ABC ... Thirty-two such instances (0.1% of the total) were found. Secondly, a computer analysis was undertaken to identify those students who had, in all likelihood, responded by guessing or by selecting answers at random. Two hundred and eight such cases were found, less than one percent of the total.
In summary, the best data available at this time lead to the conclusion that, despite the fact that a sizable proportion of the Grade 12 population failed to take the Mathematics Assessment test, the overall results obtained by those who did are an accurate representation of the total population. Moreover, analysis of individual students' response patterns has failed to show any evidence of widespread lack of due care and attention in completing the test.

Student Characteristics

On each test, students were asked to answer a number of background information questions. Each such question dealt with a variable which had been shown in previous research to be linked to achievement in school. Some of the more interesting findings about these reporting categories are discussed in Section 5 of this report. In the present section, some of the data are used to describe the population of students who took the Mathematics Assessment tests.

1. Sex

Approximately 2,500 more male than female students took the Mathematics Assessment tests. In terms of percentages, fifty-one percent of the respondents were male and forty-nine percent were female.

Combining the information on sex of Grade 12 respondents with their mathematics backgrounds yielded disquieting results. Only forty-two percent of the Mathematics 12 students were female, while sixty-four percent of those whose last mathematics course was Mathematics 10 were female. In other words, a disproportionate number of female students have decided not to study any mathematics in secondary school beyond the last course required, Mathematics 10.

2. Number of Schools Attended

The Assessment results indicate that over fifty percent of the students have changed schools for reasons other than change of level, such as from elementary to junior secondary. One quarter of the Grade/Year 4 students had attended three or more schools. Forty percent of the Grade 8 students and sixty percent of those at Grade 12 had attended at least four schools.

3. National Origin and Languages Spoken

About seventy-five percent of the Grade/Year 4 students who took the test were born in Canada, and eighty-four percent had English as their first language. Although such questions were not part of the Grade 8 and 12 Mathematics Assessments, they were asked on the Reading tests and it was possible to obtain information on the national origin of those students who wrote both tests. Between eighty and eighty-five percent of the Grade 8 and 12 students were born in Canada, and about eighty percent had English as their first language.
4. **Television Viewing Patterns**

Just over thirty percent of Grade/Year 4 students said they watched at least five hours of television per day. In other words, they spend at least as much time watching television as they spend in school, and they may be spending more time watching television than they do on any other activity.

Data from the Reading Assessment test indicate that at the Grade 8 level the percent of students who watch five or more hours per day is also very high, about twenty-five percent. At the Grade 12 level, the comparable figure is only about five percent. About forty-five percent of students at this level say they watch between one and two hours of television daily.

5. **Mathematics Background of Grade 12 Students**

Students are not required to take any course in mathematics beyond Math 10. Data from the assessment reveal that only fifteen percent of Grade 12 students have taken no more than this minimum requirement, and that over thirty-five percent have taken some form of Math 12.

On the basis of their backgrounds in mathematics, Grade 12 respondents were divided into three categories. Those who were taking or who had taken some form of Math 12 were designated the Math 12 group. Those who were taking or who had taken no mathematics courses beyond some form of Math 11 were called the Math 11 group. The remainder, those who were taking some form of Math 10 or for whom such a course was the highest level of mathematics successfully completed, were called the Math 10 group.

6. **Hand-Held Calculators**

The proportion of students who have used a hand-held calculator in school varies from three percent at Grade/Year 4 to ten percent at Grade 8, and to fifty-one percent at Grade 12. This last figure, when broken down by mathematics background, reveals that seventy-two percent of the Math 12 students have used a calculator in school, but only about thirty percent of the Math 10 students have done so. The vast majority of students in Grade 4 and 8 say they have not used a hand-held calculator for homework, while about half the Grade 12 students say they have done so.

7. **Mathematics Assignments**

Fifteen percent of Grade 8 students who are taking a mathematics course and about the same proportion of Grade 12 students said they spend no time at all on out-of-class assignments in mathematics. About eighty percent said they spend some time on such assignments, but usually less than one hour per day.
8. Part-Time Employment of Grade 12 Students

Over fifty percent of Grade 12 students have part-time jobs, and most of them are employed both during the week and on weekends. On average, these students spend between ten and fifteen hours per week at their jobs.

9. Future Plans of Grade 12 Students

About twenty percent of Grade 12 students plan to enter the labour market upon completion of secondary school, and an additional thirteen percent are undecided. Thirty percent are planning to enroll in university or in pre-university programs at community colleges. Among those students taking Math 12, about sixty percent plan to attend university or community college and about seven percent intend to enter the labour market. Among the Math 10 group, about fifteen percent plan to attend university or community college, and thirty-five percent plan to seek employment.
4. TEST RESULTS

The student tests at all three grade levels were machine-scored, and the percent of students who obtained the correct answer was computed for each item. For each multiple-choice item, the percent selecting each incorrect choice was also calculated. Using these results, the mean percent correct for each objective and domain of the three tests was obtained. On the basis of the item results, the Interpretation Panels judged the students' performance on each item, and assigned that performance one of the five ratings which are listed below along with their abbreviated designations:

- Strength (ST)
- Very satisfactory (VS)
- Satisfactory (S)
- Marginally satisfactory (MS)
- Weakness (W)

The Panels' ratings were based not only on the percent of students who had responded correctly, but also on the degree of difficulty of the item, the range of students' abilities within each grade level, and the fact that there is variation in instructional goals and methods throughout the province.

In the sections which follow, summaries of the results of the three tests are presented. At this point, it seems appropriate to make three cautionary remarks. First, since these are summaries, much has been left unsaid. A more thorough discussion of the results and their implications may be found in the Test Results volume which is available from the Learning Assessment Branch. Secondly, any comments which are made about the acceptability of the performance on any given item or group of items are based upon provincial results, and may not be applicable to individual students. Thirdly, comparisons between students' performances on any two items or objectives should not be made by merely comparing percentages. For example, eighty-one percent of Grade/Year 4 students obtained the correct response to Item 21:

\[ 5 \rightarrow 45 \]

and this result was rated satisfactory while a seventy-six percent correct result on a later (and more difficult) item, Item 48 (below), was rated very satisfactory.*

Which is true?  
\[ 35 > 45 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 

* Test items have been photographically reduced for presentation in this report.
The test contained sixty-nine items designed to assess students' mastery of ten objectives grouped into three domains. Although the test was administered in Grade/Year 4, the content of each item was chosen from at most the Grade 3 level; the end of primary education. Student performance on each item was assigned one of the five ratings by the Interpretation Panel. Table 4-1 summarizes, for each objective, the number of items where performance was judged as Weak, the number where performance was judged as Marginally Satisfactory and so on. For example, for Objective 1.2 five items were used to measure students' ability on the addition of whole numbers. Overall student performance was rated as Very Satisfactory for four of these test items and as Satisfactory on the other.

Table 4-1
Panel Ratings of Grade/Year 4 Mathematics Results

<table>
<thead>
<tr>
<th>Objective</th>
<th>Number of Items</th>
<th>Number of Items by Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W</td>
<td>MS</td>
</tr>
<tr>
<td>Domain 1: Computation and Knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1 Mastery of Number Facts</td>
<td>24</td>
<td>-</td>
</tr>
<tr>
<td>1.2 Addition of Whole Numbers</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>1.3 Subtraction of Whole Numbers</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1.4 Knowledge of Notation and Terminology</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>Domain 2: Comprehension</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1 Place Value Concepts</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>2.2 Number Properties</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>2.3 Measurement Concepts</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>2.4 Fraction Concepts</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Domain 3: Applications</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1 Solution of Practical Problems</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>3.2 Solution of Computational Problems</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>TOTALS:</td>
<td>69</td>
<td>4</td>
</tr>
</tbody>
</table>
The Grade/Year 4 results, overall, are encouraging. They indicate that most children at this level are learning many of the mathematical skills expected of them. On six of the ten objectives, students' performance was particularly good. These were Mastery of Number Facts, Addition of Whole Numbers, Knowledge of Notation and Terminology, Comprehension of Number Properties, Solution of Practical Problems, and Solution of Computational Problems. On the other hand, performance on Subtraction of Whole Numbers, Comprehension of Measurement Concepts, and Comprehension of Fraction Concepts was less satisfactory.

The Interpretation Panel judged students' performance on the number fact items for addition and subtraction to be a strength. They were very satisfied with the achievement rate on the multiplication facts, and satisfied with the performance on division facts. Since the number fact section of the test was timed, it was not possible to be certain whether the relatively high rates of missing responses on the last several number fact items (which were also the division fact items) were due to students not knowing those facts or to their having run out of time.

In Domain 1, a weakness was noted on Item 36 for Objective 1.3, Subtraction of Whole Numbers. Fifty-six percent of the students obtained the correct answer to that item, \[ \frac{1054}{1054} - \frac{865}{865} \]

The exercise was difficult since it required multiple regrouping with a zero in the minuend, and students' ability to carry out such a process correctly is dependent upon their understanding of our decimal numeration system. None of the subtraction with regrouping item results was rated any higher than marginally satisfactory, and this suggests that more attention should be paid to developing students' understanding of place value concepts, of techniques for regrouping, and of applying these concepts to the operation of subtraction.

The results for the objectives from Domain 2 were the least satisfactory. While performance on Understanding of Number Properties was very good, performance on the other three objectives was not. Two item results from Objective 2.3, Comprehension of Measurement Concepts, were rated as weaknesses. The items, which are shown in Figure 4-1, measured students' familiarity with temperature measured in degrees Celsius and weight (mass) in kilograms. In Item 64, 25% of the students correctly answered the question, and in number 65, 32% of students responded correctly.

The word "mass" was not used on the tests, although it is the correct term. It was deemed better to use the familiar term "weight".
(64) A ten-year-old boy is likely to weigh:

<table>
<thead>
<tr>
<th>Weight</th>
<th>% of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 grams</td>
<td>9</td>
</tr>
<tr>
<td>75 grams</td>
<td>22</td>
</tr>
<tr>
<td>35 kilograms</td>
<td>25</td>
</tr>
<tr>
<td>75 kilograms</td>
<td>33</td>
</tr>
<tr>
<td>I don't know</td>
<td>8</td>
</tr>
</tbody>
</table>

(65) The temperature on a sunny summer day would most likely be:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>% of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>5° Celsius</td>
<td>5</td>
</tr>
<tr>
<td>25° Celsius</td>
<td>32</td>
</tr>
<tr>
<td>55° Celsius</td>
<td>27</td>
</tr>
<tr>
<td>85° Celsius</td>
<td>29</td>
</tr>
<tr>
<td>I don't know</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 4-1: Grade/Year 4 - Items 64 and 65*

On the whole, the Interpretation Panel found students' performance on the items dealing with the metric system of measurement to be disappointing. The Panel recommended that further steps be taken to provide teachers with materials and in-service training for the teaching of measurement.

Performance on Comprehension of Fraction Concepts was also rather weak. On Item 58, where 60% of students responded correctly, performance was marginally satisfactory and on Item 60, where 54% answered correctly, it was weak.

(58) Which group of dots is one-half \( \frac{1}{2} \) shaded?

- 21%  
- 60%  
- 8%  
- 2%  
- I don't know 7%

(60) Which box is one-fifth \( \frac{1}{5} \) shaded?

- 7%  
- 54%  
- 15%  
- 17%  
- I don't know 5%

Figure 4-2: Grade/Year 4 - Items 58 and 60*

* The correct response has been underlined.
Fraction concepts are a part of the standard curriculum at the primary level, and students are expected to learn about fractions both as parts of wholes and as parts of sets. On the other hand, there is some evidence that such concepts may be too sophisticated for children of this age. In other words, the developmental level of many of these children may not permit them to grasp such concepts. Persons who are involved in the design and implementation of curricular materials for mathematics, and educational researchers should give the question of when to introduce fraction concepts to children their immediate attention.

The results on the items from Domain 3, which concerned problem-solving, were commendable. Problem-solving is the most difficult topic in the mathematics curriculum to teach, and it is at least as difficult to learn. In spite of this, no item results from Domain 3 were rated as weaknesses, and one was rated as strength. This latter item dealt with students' ability to read information from a bar graph.

Grade 8

The test administered to Grade 8 students contained sixty items dealing with some of the essential skills and concepts of mathematics for the intermediate grades (4-7). The items were grouped under twelve objectives which were themselves grouped into three domains.

Student performance on each item was rated by the Grade 8 Interpretation Panel. Table 4-2 summarizes, for each objective, the number of items where performance was judged as Weak, the number where performance was judged as Marginally Satisfactory and so on. For example, five items were used to measure Objective 1.1. Performance on one item was judged as Marginally Satisfactory, another item Satisfactory, and three items Very Satisfactory. The abbreviated designations for the various rating categories which were described earlier have been used in this table as well.

The proportion of Grade 8 item results which were rated by the Grade 8 Panel as being very satisfactory or strengths was lower than the corresponding proportions of ratings given by the other two Panels at their respective grade levels. Correspondingly, a higher proportion of Grade 8 results were either weak or marginally satisfactory although the difference between Grade 8 and 12 was slight. The overall picture, while not being one of weakness, seems to show performance at this level to be the least satisfactory of the three levels tested.

Thirty-nine of the sixty test item results were rated as satisfactory or better and six were seen as weaknesses. While it may be concluded from these results that the majority of students have acquired many of the essential skills and understanding which are expected of them, there are also fairly large numbers who have not done so, and these students need additional assistance. Moreover, there are a few areas, notably those of rational number concepts, geometry, and measurement which are in need of substantial improvement and immediate attention.
Table 4-2
Panel Ratings of Grade 8 Mathematics Results

<table>
<thead>
<tr>
<th>Objective</th>
<th>Number of Items</th>
<th>Number of Items by Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>W</td>
</tr>
<tr>
<td>Domain 1: Computation and Knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1 Computation with Whole Numbers</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Computation with Fractions</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Computation with Decimals</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>1.4 Knowledge of Notation and Terminology</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>1.5 Knowledge of Geometric Facts</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>1.6 Equivalent Forms of Rational Numbers</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Domain 2: Comprehension</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1 Number Concepts</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>2.2 Measurement Concepts</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2.3 Geometric Concepts</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>2.4 Algebraic Concepts</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Domain 3: Applications</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1 Solve Computational Problems</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>3.2 Solve Geometry and Measurement Problems</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>TOTALS:</td>
<td>60</td>
<td>6</td>
</tr>
</tbody>
</table>

The strongest areas in Domain 1 were Computation with Whole Numbers, and Knowledge of Notation and Terminology. The areas of Computation with Fractions and Decimals, Knowledge of Geometric Facts, and Equivalent Forms of Rational Numbers were weaker in varying degrees, with the last-named being the weakest of all.

Students' ability to write a number expressed as a fraction, decimal, or percent in either of the other forms was less than satisfactory. For example, only 38% of students correctly answered Item 31 on the student test (Figure 4-3), a result judged as a weakness. More emphasis on this topic would appear to be warranted by results such as this.
31. Written as a decimal, \( \frac{1}{8} = \) A) 0.12  \( \frac{6}{5} \)  
B) 0.8  \( \frac{41}{5} \)  
C) 0.125  \( \frac{38}{5} \)  
D) 0.18  \( \frac{8}{5} \)  
E) I don’t know  \( \frac{5}{5} \)  

Figure 4-3: Grade 8 - Item 31

Computation with decimals and fractions seems to be satisfactory, but not overly so. Operations with decimals in particular should receive more attention than in the past because of the introduction of the metric system which emphasizes decimal notation over fraction notation.

Strengths were outnumbered by weaknesses in Domain 2. Performance was quite good on Comprehension of Measurement Concepts, but somewhat weaker on Comprehension of Geometric Concepts and of Number Concepts.

Two of the item results rated as weaknesses in Domain 2 dealt with fraction concepts. One of the items (see Figure 4-4) required students to use the basic concept of what a fraction is, and the other required them to order rational numbers. Performance of Grade 12 students on these same two items was judged as weak. Grade 12 results are presented here to illustrate differences between the two levels.

18. There are 13 boys and 15 girls in a group. What fraction of the group is boys? 

<table>
<thead>
<tr>
<th></th>
<th>% of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 8</td>
<td>Grade 12</td>
</tr>
<tr>
<td>A) 15/28</td>
<td>5</td>
</tr>
<tr>
<td>B) 13/15</td>
<td>53</td>
</tr>
<tr>
<td>C) 15/13</td>
<td>0</td>
</tr>
<tr>
<td>D) 13/28</td>
<td>32</td>
</tr>
<tr>
<td>E) I don’t know</td>
<td>2</td>
</tr>
</tbody>
</table>

47. Which number is largest? 

<table>
<thead>
<tr>
<th></th>
<th>% of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 8</td>
<td>Grade 12</td>
</tr>
<tr>
<td>A) 2/3</td>
<td>38</td>
</tr>
<tr>
<td>B) 4/5</td>
<td>29</td>
</tr>
<tr>
<td>C) 3/4</td>
<td>16</td>
</tr>
<tr>
<td>D) 5/8</td>
<td>12</td>
</tr>
<tr>
<td>E) I don’t know</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 4-4: Grade 8 - Items 18 and 47
The weak performance on these items indicates an area of the curriculum where improvement is needed. Fraction or rational number concepts are difficult, and teachers of mathematics at all levels must have available to them the best methods and materials for developing these concepts with their students.

Performance on the Comprehension of Geometric Concepts items was disappointing, with none of the four results being rated higher than satisfactory and two being rated below satisfactory. The poorest performance was registered on an item in which students were asked to find the area of right triangle with legs 6 and 14 units long respectively. Almost twice as many students chose 84 as the response than chose the correct answer, 42 (see Figure 4-5). It may be that insufficient emphasis is being given to geometry and measurement in the intermediate grades and, if this is the case, some improvement in that situation would appear to be warranted by these results.

52. Find the area of this right triangle:

\[ \text{A)} \quad 42 \quad \frac{24}{24} \]
\[ \text{B)} \quad 20 \quad 18 \]
\[ \text{C)} \quad 84 \quad 42 \]
\[ \text{D)} \quad 21 \quad 4 \]
\[ \text{E)} \quad \text{I don't know} \quad 11 \]

Figure 4-5: Grade 8 - Item 52

One item, number 48,

\[ \text{Simplify: } 30 - 4(8 - 2) \]

from Comprehension of Algebraic Concepts, dealt with order of operations. Only eighteen percent of Grade 8 students obtained the correct response and this performance was rated a weakness. The importance of this topic at this level is questionable, but it is part of the mathematics curriculum at the Grade 7 level and virtually everyone who participated in the review process during the development of the objectives for the assessment recommended that such an item be included on the test. If the topic is to remain as part of the elementary mathematics curriculum, then teachers should place more emphasis upon it.

Results from Domain 3, which concerned problem-solving, were satisfactory overall. As with Grade/Year 4, this is a commendable performance because of the difficulty of teaching and learning problem-solving skills. Results on Item 24, a one-step problem involving multiplication of whole numbers, indicated strength while those on Item 37, a multi-step problem involving the concept of area, indicated weakness. The two items are presented in Figure 4-6.
24. There are 25 members in the volleyball club. If the cost for each uniform is $24, how much would it cost to buy new uniforms for all the club members?

<table>
<thead>
<tr>
<th>Cost (in dollars)</th>
<th>% of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>$49</td>
<td>2</td>
</tr>
<tr>
<td>$6000</td>
<td>3</td>
</tr>
<tr>
<td>$600</td>
<td>91</td>
</tr>
<tr>
<td>$96</td>
<td>2</td>
</tr>
<tr>
<td>I don't know</td>
<td>2</td>
</tr>
</tbody>
</table>

E) I don't know

37. What is the area of the shaded portion of this figure?

<table>
<thead>
<tr>
<th>Area (in square units)</th>
<th>% of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>27</td>
</tr>
<tr>
<td>96</td>
<td>28</td>
</tr>
<tr>
<td>120</td>
<td>11</td>
</tr>
<tr>
<td>60</td>
<td>11</td>
</tr>
<tr>
<td>I don't know</td>
<td>21</td>
</tr>
</tbody>
</table>

E) I don't know

Figure 4-6: Grade 8 - Items 24 and 37

Grade 12

The Grade 12 test was administered to all students enrolled in Grade 12 whether or not they were currently taking a mathematics course. It was not a test of any particular course, such as Math 10, Math 11, or Math 12, but rather a test of students' mastery of a number of essential mathematical skills and concepts which, for the most part, students could be expected to have acquired upon completion of secondary school. The content of each item was from the Grade 10 level or below. Accordingly, the results summarized here should not be used to characterize the success of any particular course, nor should they be interpreted as indicators of students' preparedness or lack of preparedness for post secondary courses in mathematics.

The seventy-two test items were grouped under eleven objectives which were themselves grouped into three domains. Student performance on each item was rated by the Grade 12 Interpretation Panel. Table 4-3 summarizes, for each objective, the number of items where performance was judged as Weak, the number where performance was judged as Marginally Satisfactory and so on. For example, four items were used to measure Objective 1.1. Student performance on two of these test items was judged as Satisfactory, and on the other two, performance was judged as Very Satisfactory. As with Tables 4-1 and 4-2, the abbreviated designations for the Panel rating categories have been used.

On the basis of the ratings assigned by the Interpretation Panel, it would appear that the Grade 12 results, overall, are slightly better than the Grade 8 ones and not as good as the Grade/Year 4 results. Only three of the item results were considered strengths, while seven were rated as weaknesses. Of most concern is the fact that five of the seven weaknesses were in the Applications Domain, that is, in problem-solving.
Table 4-3
Panel Ratings of Grade 12 Mathematics Results

<table>
<thead>
<tr>
<th>Objective</th>
<th>Number of Items</th>
<th>Number of Items by Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>W</td>
</tr>
<tr>
<td>Domain 1: Computation and Knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1 Computation with Fractions</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>1.2 Computation with Decimals</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>1.3 Knowledge of Notation and Terminology</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>1.4 Knowledge of Other Algorithms</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>Domain 2: Comprehension</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1 Number Concepts</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2.2 Measurement Concepts</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2.3 Geometric Concepts</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>2.4 Algebraic Concepts</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>Domain 3: Applications</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1 Solve Computational Problems</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>3.2 Solve Geometry and Measurement Problems</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>3.3 Solve Algebraic Problems</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>TOTALS:</td>
<td>72</td>
<td>7</td>
</tr>
</tbody>
</table>

* The Interpretation Panel declined to rate performance on one item in this objective.

All three item results, which were rated strengths, occurred in Domain 1. Two of these items concerned knowledge of the terms "factor" and "reciprocal," while the third dealt with reducing a fraction to lowest terms. The students' performance on the items in this domain was quite satisfactory, and is indicative of the fact that students completing secondary school are fairly competent in performing the four basic operations of addition, subtraction, multiplication, and division.

One of the item results rated as a weakness in Domain 2 for Grade 12 was also rated a weakness among Grade 8 students. This was Item 18 (see Figure 4-4) from Objective 2.1, Comprehension of Number Concepts, and it concerned basic fraction concepts. Only 51% of Grade 12 students correctly answered the question. The only other weakness noted in this domain occurred on an item dealing with familiarity with the metric unit for mass (weight). Generally speaking, the students seemed to be familiar with some of the basic metric concepts although some areas leave room for improvement. Secondary schools should implement programs to familiarize all of their students and especially those at the senior levels, with the metric system of measurement.
The results on five of the eighteen problem-solving items from the Applications domain were rated as weaknesses, and four more as marginally satisfactory. None were rated as strengths. Four of the weak results occurred on geometry and measurement problems, one of these being Item 47 which was discussed earlier as having resulted in a weak performance at the Grade 8 level. The fifth area of weakness was noted on an item dealing with the use of the simple interest formula, \( i = prt \), which was given along with the item.

69. Find the principal, if the interest received after two years at an annual rate of 6% is $60. 

<table>
<thead>
<tr>
<th>Option</th>
<th>Percentage of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) $2000</td>
<td>9</td>
</tr>
<tr>
<td>B) $5000</td>
<td>7</td>
</tr>
<tr>
<td>C) $500</td>
<td>48</td>
</tr>
<tr>
<td>D) $720</td>
<td>17</td>
</tr>
<tr>
<td>E) I don't know</td>
<td>16</td>
</tr>
</tbody>
</table>

Figure 4-7: Grade 12 - Item 69

The results for the Applications domain level are disappointing. They indicate that many students are unable to apply the computational skills they have learned to certain types of problems. This seems to be especially true in the area of geometry and measurement. Teachers and teacher educators need to stress the overriding importance of problem-solving in mathematics, and their students need to learn strategies to use in attempting to solve problems in mathematics.

One of the major factors to be borne in mind in analyzing the Grade 12 results is that students at this level vary considerably in the nature and extent of their mathematics backgrounds. Almost fifteen percent of these students had taken no courses in mathematics beyond Math 10, the last compulsory course. Just over thirty-five percent had taken or were taking some form of Math 12 while the remainder had taken no courses beyond Math 11.

On the average, the Math 12 group's results were approximately twenty percent higher than those of the Math 11 group which were in turn about fifteen percent higher than the Math 10 group's. As a matter of fact, the Math 10 group achieved at about the same level as Grade 8 students on those items which were common to both tests. A comparison of the performance on the three domains by the Math 10, Math 11, and Math 12 groups is shown in Figure 4-8. For Item 69, displayed on this page, 73% of the Math 12 group correctly answered the question, compared to 36% of the Math 11 group and 26% of the Math 10 group.

The low performance by the Math 10 group may be due partly to their having forgotten much of the material tested, as well as to the fact that many of these students are among the least capable mathematically. To the extent that these factors are important in determining the overall result, it might be advisable to consider requiring all students to take more mathematics or to consider postponing the taking of Math 10 by such students until Grade 11 or 12. It goes without saying that the more mathematics referred to means more consumer-related mathematics and not more academic mathematics.
A number of the items on the Grade 12 test dealt with consumer mathematics, that is, with skills and concepts which students will have to use in their day-to-day lives. The items dealt with topics such as percent, discount, commission, interest, selecting the best price for an article, use of formulae, and reading graphs, road maps, and tax tables. Of thirteen such item results, seven were rated satisfactory or above while five were rated marginally satisfactory and one as a weakness. The item result rated a weakness was shown in Figure 4-7.

Consumer mathematics skills are of great importance, and these results indicate that many students are completing school without having mastered the skills required to solve such problems. Some initial steps to correct this situation have already been taken (e.g., the introduction of an elective course in Consumer Mathematics at the senior secondary level, and the inclusion of a unit on consumer mathematics in the Math 10 course), and such initiatives should be endorsed by all teachers. All students should have been taught the major concepts of consumer mathematics before completing secondary school and, preferably, at the senior secondary level where such material is more likely to be of interest to them.
Achievement in mathematics is the end result of the coalescing of a great number of student-related factors, both extrinsic and intrinsic. Attributes inherent in the student, programmatic and curricular variables, as well as the effect of environmental variables such as teacher differences, all contribute in varying and largely unknown degree's to a given student's performance. Of the fairly large number of such variables which the conventional wisdom, current educational practice, and educational research have identified as being linked to achievement in mathematics, several were examined in the Mathematics Assessment.

There is a danger that reporting category data may be misinterpreted as implying the existence of cause-and-effect relationships between two variables when, in fact, no such conclusions are warranted. The data may show that two variables are linked and seem to be related, but they cannot be used to conclude that changes in one variable will cause changes in the other. For example, the data may show that Grade/Year 4 students who watch four hours of television per day obtained higher scores on the test than those who watch less than that amount. This does not imply that students who watch little television would improve their scores by watching more television. It may be the case that the changes in performance are due to the influence of factors other than those discussed here or to some combination of such factors. In other words, while the assessment data may show that certain variables appear to be related to achievement in mathematics, this should not be interpreted to mean that one causes the other.

Age (Grades 4, 8, and 12)

At all three levels, the younger students outperformed their older counterparts. In Grade/Year 4 the nine-year-olds' achievement surpassed that of the ten-year-olds' who in turn outscored the eleven-year-olds. This is not a surprising result since many of the older students at each level have been progressing through the system at a slower than average pace because they are less capable academically.

Of greater interest is the finding that, among Grade/Year 4 students born in 1967, there is a relationship between date of birth and achievement. Children born in the first quarter of 1967, that is between January and March, did better on all Grade/Year 4 test objectives than those born between April and December of the same year. In other words, among nine-year-olds, the older children outperformed the younger ones. The group differences were greatest on the objectives dealing with Knowledge of Notation and Terminology, Understanding of Measurement Concepts, Understanding of Fraction Concepts, and Problem-Solving. These findings, organized by domain are summarized in Figure 5-1.
Previous research in the field of Mathematics Education supports the finding that, at least in the early grades, children born in the first part of the year have an academic advantage over the others. It is important to note the continuance and the consistency of this trend across domains among children completing Grade/Year 4, and the size of the group differences particularly in those areas of the mathematics curriculum which are among the most abstract such as understanding of fraction and measurement concepts.

**Sex Differences (Grades 4, 8, and 12)**

A comparison of the results obtained on each domain by males and females is presented in Figure 5-2. At the Grade 4 and 8 levels, boys outperformed girls on as many objectives as girls outperformed boys. Girls had the advantage on some objectives from Domains 1 and 2, while the boys' results were higher on other objectives from Domains 2 and 3. In general, the girls were superior on objectives dealing with skills and concepts which were lower on the scale of cognitive behaviours than those on which the boys obtained higher results. Most differences were not large whether in favour of the girls or of the boys.
When sex comparisons were made with the mathematics background variable controlled, the same general picture emerged. The males continued to obtain higher results, but the differences between the two Math 12 groups were small. The differences were somewhat larger among the Math 11 group, and larger still among the Math 10's as is shown in Figure 5-3.
Number of Schools Attended (Grades 4, 8 and 12)

As was mentioned earlier, the data on number of schools attended indicate that over fifty percent of students have changed schools for reasons other than change of level such as from elementary to secondary. Twenty-five percent of the Grade/Year 4 students had attended at least three schools. Forty percent of Grade 8 and sixty percent of Grade 12 students had attended four or more schools.

One of the findings of the 1976 assessment in Reading was that an increase in the number of schools attended was accompanied by a decrease in performance in reading at the Grade/Year 4 level. The same trend was observed with the Mathematics Assessment results at both the Grade 4 and 8 levels. At the Grade 12 level, the results do not indicate any relationship between achievement and number of schools attended except that those students who had attended eight or more schools scored lower than all of the other groups.

Amount of Television Watched (Grade/Year 4)

Many students spend considerable amounts of time each day watching television. Over forty percent of Grade 4 and 8 students stated they watched at least four hours of television per day during the week, but only eighteen percent of Grade 12 students watch that much television.

Grade/Year 4 students who reported watching about four hours of television per day performed better on the Mathematics Assessment test than other children. There was a fairly consistent but slight increase in performance associated with an increase in television watching up to a maximum of four hours per day. Students who said they watched no television and those who watched five or more hours per day performed at about the same overall level. The poorest performance was registered by the group that watched some television, but usually less than one hour per day.

Among students in Grades 8 and 12, students who watch one hour or less of television per day obtained the highest results. Generally speaking, an increase in amount of television watched was accompanied at these levels with a decrease in performance.

National Origin and First Language (Grades 4, 8, and 12)

Students were asked three questions concerning whether or not they were born in Canada, and whether or not they spoke languages other than English. On the basis of their responses to these questions, they were categorized as belonging to one of five groups, and the performances of these groups on the three tests were then compared.
The definitions of the groups are somewhat complex, and there is not sufficient space to discuss them and all of the results here. A more thorough discussion may be found in the Test Results report.

One of the five groups, Canadian, English-speaking, consisted of students born in Canada and for whom English was their first language. Students who were not born in Canada and who usually spoke a language other than English before entering Grade 1 were classified as non-Canadian, non-English-speaking. The relative performances of these two groups are portrayed in Figure 5-4.

At the Grade/Year 4 level, the Canadian, English-speaking students outperformed all other students, including the non-Canadian, non-English-speaking ones on all three domains. Their advantage was greatest in Domains 2 and 3, as might have been expected because of the increased importance of reading skills in those domains.

A reversal in this trend was noted at the Grade 8 and 12 levels. In both cases, the non-Canadian, non-English-speaking group outperformed the Canadian, English-speaking group on all three domains.

These results should be interpreted cautiously because this is a complex issue. For example, students who were born in Canada and who spoke a language other than English in their homes before starting Grade 1 did not do as well as the non-Canadian, non-English students although they did outperform the Canadian, English-speaking students in Grades 8 and 12.
Use of Hand-Held Calculators (Grades 4, 8, and 12)

At all three levels, students who said they sometimes used a calculator at home obtained higher scores than those who did not. At the Grade 12 level those who sometimes used a calculator for homework and those who sometimes used a calculator in school outperformed those who did not. These differences were reversed at the Grade/Year 4 level, with the advantage being held by those who had not used a calculator for homework or in school. Among Grade 8 students there was less consistency in the results with the calculator group having the advantage on some objectives, and the non-calculator group on others.

Time Spent on Assignments (Grades 8 and 12)

Students who spend some time on out-of-class assignments in mathematics but who spend less than thirty minutes per day on such work obtained higher results on the assessment test than those who spent no time on such assignments or more than half an hour per day. Of the three groups, the results for those who spend no time on assignments were substantially lower than the other two on just about every objective. Results for the other two were usually about the same with several of the differences being less than one percentage point.

Parental Educational Level (Grade 12)

The Mathematics Assessment test results showed a general pattern of increase (see Figure 5-5) in student achievement with an increase in the highest educational level attained by the father or guardian. Two questions were asked concerning the highest level of education attained by the father (or guardian) and the mother (or guardian). However, because the results were similar whether the father's or the mother's educational level was used, only the relationship between student performance and father's educational level is mentioned here.
Future Plans (Grade 12)

About twenty percent of the Grade 12 students plan to enter the labor market upon completion of Grade 12, and the vast majority of such students are from the Math 10 and Math 11 groups. Of the almost forty percent of Grade 12 students who plan to attend community college or university, the majority are from the Math 12 group. About fifteen percent of the Math 10 students intend to continue their education at a community college or university.

In each Domain, those students who said they intended to continue their education at a university achieved the highest results. The poorest performance on this test of essential skills was registered by those students who indicated they would seek full-time employment upon completion of secondary school. The results obtained in this reporting category are summarized in Figure 5-6.
6. THE TEACHERS OF MATHEMATICS

Data obtained from a number of the items on the teacher questionnaire can be used to sketch the broad outlines of the nature and extent of the training of teachers of mathematics, their involvement in professional development activities, their membership in professional associations, and their opinions about the subjects they teach. Such outlines should be interpreted cautiously, and the findings should not be applied to any individual teacher.

Teachers of mathematics at each of Grades 1, 3, 5, 7, 8, 10, and 12 were systematically selected as potential respondents to the questionnaires which were mailed out shortly after the student tests were administered. Of 3 451 questionnaires sent out, 2 955 were returned completed. Returns at each level were high enough (the overall return rate was 85.6%) so that there is a 95% chance that the results reported are a true reflection of the results which would be obtained from a poll of all of the teachers of mathematics at these levels.

From one point of view, the general picture that emerges at both the elementary and secondary levels is that mathematics is being taught by experienced teachers with fairly extensive backgrounds in professional training. On the other hand, too many of these teachers have had little or no training either in mathematics or in the teaching of mathematics. Moreover, relatively few teachers of mathematics are members of professional associations specializing in the teaching of mathematics.

The situation appears to be particularly acute at the Grade 8 level, as was also found in the Language Arts Assessment conducted in 1976. In the first year of secondary school, where many of the foundations for future work are laid, students need the guidance and direction of the very best-prepared teachers of mathematics that can be provided. While it may be understandable that highly qualified teachers of mathematics would prefer to teach the mathematics content of the senior grades, this must be balanced against the needs of the students. Schools should ensure that at all levels, but particularly in the secondary grades, mathematics is taught only by persons adequately qualified to do so.

Professional Training

The average teacher of elementary mathematics has had just over four years of post-secondary education. In general, the number of years of training increases with grade level taught. About fourteen percent of elementary teachers have had no professional training in the teaching of mathematics, and another 35% have not had such a course in the past ten years.

The average secondary teacher has had slightly more than five years of post-secondary education. As with elementary teachers, an increase in years of post-secondary education accompanies an increase in grade-level taught. Almost 35% of teachers of secondary mathematics did not have mathematics as one of their major subject areas in their undergraduate training. At the Grade 8 level, this figure approaches fifty percent.
Twenty percent of secondary mathematics teachers have had no training in the teaching of mathematics, and another thirty-seven percent took such training more than ten years ago.

Teaching Experience

The results reported on teaching experience are based on grouped data and are therefore approximations of the true figures. All reported averages are conservative estimates, i.e., the true averages are almost certainly somewhat greater than those reported.

The average number of years of teaching experience among the elementary teacher respondents was 8.5 years; for their colleagues at the secondary level the average was 9.2 years. Grade 12 teachers of mathematics had the highest average number of years of teaching experience, 11.1, and Grade 5 teachers the lowest at 8.3 years. Just over one fourth of the elementary teachers and one third of the secondary teachers had more than thirteen years of experience. Less than six percent of either group were in their first year of teaching.

Professional Affiliations

Fifty-six percent of primary teachers belong to the Primary Teachers Association (PTA), twenty-five percent of intermediate teachers to the Provincial Intermediate Teachers Association, and twenty-seven percent of teachers of secondary mathematics to the B.C. Association of Mathematics Teachers (BCAMT). With the exception of the PTA, other associations (no information regarding membership in BCTF was obtained) do not seem to be attracting members in great numbers. Of particular interest is the fact that less than three percent of the elementary teachers said they belonged to BCAMT, the specialists' association for mathematics.

Professional Development

Teachers of mathematics at the primary and at the senior secondary levels are much more likely to have attended a recent conference session or in-service day dealing with mathematics than are teachers at the Grades 5, 7 and 8 levels. The Grade 8 level is also the one with the highest concentration of teachers of secondary mathematics with no university level background in mathematics or the teaching of mathematics. They also have the lowest rate of membership in professional associations of the three secondary groups surveyed.
7. INSTRUCTIONAL PRACTICES

Samples of teachers of mathematics at each of Grades 1, 3, 5, 7, 8, 10, and 12 completed the Teacher Questionnaire. The four major sections of that questionnaire dealt with classroom organization, use of textbooks, classroom instruction, and the importance of certain mathematical content objectives. The results for each of these sections are summarized below.

Classroom Organization

Questionnaire returns indicate that a considerable amount of time is spent in preparing for and teaching mathematics classes, and that the teaching of mathematics is highly traditional in character. Putting together the results of several items shows that the most frequently used teaching techniques are total class instruction and teacher explanation. Among the most commonly used student activities are individual work and textbook exercises. In other words, and particularly at the higher levels, these results indicate that few organizational innovations are being used in the mathematics classes of the province.

The average size of a mathematics class at the elementary level is 25.0; at the secondary level it is 29.4. The largest average class size occurs at the Grade 8 level, where it is 30.6. The grade-by-grade averages are as follows: Grade 1 - 22.7, Grade 3 - 23.9, Grade 5 - 25.6, Grade 7 - 27.9, Grade 8 - 30.6, Grade 10 - 28.5, and Grade 12 - 26.4.

Elementary teachers spend an average of fifty-one minutes per day teaching mathematics, and an additional thirty-eight minutes in lesson preparation and grading of mathematics assignments. Secondary teachers of mathematics, who are more highly specialized and who likely have fewer classes to prepare than elementary teachers, spend an average of 176 minutes per day teaching mathematics, fifty-three minutes in class preparation, and thirty-eight minutes grading.

The self-contained classroom is by far the most common teaching situation at all grade levels. About twenty percent of Grade 7 teachers indicated that there was a degree of departmentalization in their classes: i.e., different teachers for different subjects. Open area classes and team teaching have made some inroads at the primary level, but not at the higher grade levels.

A large proportion of teachers at all levels indicated that, in addition to total class instruction, some form of ability grouping, and partially individualized instruction were used in their mathematics classes. The use of total class instruction tended to increase with grade level, while the other two decreased.

The most prevalent classroom activities in elementary mathematics classes are individual work, teacher explanation, oral work, drill on basic facts, and work on textbook exercises. At the secondary level they are individual work, textbook exercises, and teacher explanation. The use of activity centres and creative projects for the teaching of mathematics is very limited at both the elementary and secondary levels.
Use of Textbooks

Almost all teachers of mathematics use one or more textbooks in their teaching. The lowest rates of usage, 90.5% and 91.4%, were found to occur at the Grade 1 and 8 levels respectively. Even considering these two, it is safe to say that the use of textbooks in mathematics classes is virtually universal at all grade levels.

On the whole, teachers are quite satisfied with the textbooks they are using, although many of them seem to be using texts which are no longer on the prescribed list. The highest rating of dissatisfaction with textbooks occurred at the Grade 8 level where thirty-six percent of the teachers expressed negative opinions.

More elementary than secondary teachers prefer to have several prescribed textbooks for a given grade. However, a clear majority of teachers at each level would prefer to have several texts from which to choose rather than just one. Relatively few teachers of mathematics have adopted a multi-text approach to the teaching of mathematics, if by that approach is meant the more or less equal utilization of several texts. A majority of teachers said they prefer to use one text predominantly and others as the need or occasion arises. An overwhelming majority of teachers agree that there should be made available an outline of the minimum learning outcomes for mathematics at each level or grade to guide them in the selection of textbooks, materials, and activities. The September 1977 version of the Mathematics Curriculum Guide has been revised to include such an outline.

Teachers do not require their students to read very extensively from their mathematics textbooks. Among elementary teachers, there is a tendency for those in the higher grades to require more reading than in the lower grades. Among secondary teachers, there is virtually no difference in this respect among teachers at the Grades 8, 10 or 12 levels.

All teachers use their mathematics texts primarily as sources of exercises. They appear to be used less frequently for purposes of reviewing concepts presented in class, and even less frequently to develop new concepts. Teachers at all levels say they do not want textbooks which place greater emphasis upon concepts and principles than upon skills and drill. They particularly want textbooks to provide material for drill and practice.

Teachers' ratings of various characteristics of annotated teacher's editions of mathematics textbooks were positive but tended to decrease as grade level increased. In other words, all teachers seem to appreciate the value of such editions, but they are less important to teachers of the higher grades.

Teachers were asked to rate the prescribed texts with respect to four factors. In general, the two areas which seemed to be a cause for concern were emphasis on problem-solving and on computation. At every grade level, a sizeable proportion of the texts being evaluated were seen to be weak in these two areas.
Classroom Instruction

Of five content areas, drill on number facts and computational skills are the two on which elementary teachers spend the most time. The least time is spent on geometry, which may account for the relatively weak performance students recorded on the geometry items. Secondary mathematics teachers spend the most time on problem-solving and algebraic concepts and the least time on metric measurement. With less than a year left before the metric units are to be the predominant units used in the schools in all instruction, it was found that a majority of teachers are still using both the metric and British units of measurement in their teaching.

All seven groups of teachers surveyed were asked to rate the usefulness of a number of teaching resources. They all agreed that the students' textbook was a useful resource, and elementary teachers felt that the accompanying teacher's guidebook was useful. District mathematics specialists and supervisors were given the lowest ratings of the resources on the list, but this may be due to the fact that such personnel are not available in every district.

In ranking sixteen factors purported to affect mathematics instruction, elementary teachers gave high priority to eight. The eight items can be organized into the following three groups: teaching load -- reduction of class size, reduction of total pupil load, and greater release time for lesson preparation; materials -- more mathematics manipulative materials for individual classrooms, textbooks more suited to instructional needs, curriculum guides that outline content in specific terms, and curriculum guides that offer more assistance in the instructional process; training -- more effective in-service and professional development. Secondary mathematics teachers gave high priority to the following four factors: reduction of class size, textbooks more suited to instructional needs, ability grouping of students for classes, and reduction of total pupil load.

Some interesting trends were identified from the data gathered concerning the frequency of use of selected media, materials, and methods in the teaching of mathematics. Elementary teachers make frequent use of only one medium, the chalkboard, in presenting mathematics lessons. Though the overhead projector is making some inroads, the chalkboard is also the most popular teaching aid among teachers of secondary mathematics. Elementary teachers tend to make frequent use of more different materials than secondary mathematics teachers and teacher-prepared materials head the lists among all groups of teachers surveyed.

Total class instruction and individualized instruction rank first and second in frequency of use of methods for presenting mathematics instruction for all groups. These two methods rank well ahead of any other method listed. Learning centres were used much more often by primary teachers than by any other groups.
All teachers were in substantial agreement that elementary students should not be allowed to use hand-held calculators and that senior secondary students should be. If students are permitted to use hand-held calculators at any level, teachers say that they should not be allowed unrestricted use and they should not be allowed to use hand-held calculators during tests. Teachers whose students do use hand-held calculators report that their students are allowed to use hand-held calculators to check work as well as to shorten computation time and effort in class work and on non-test assignments. Hand-held calculators are also used to offer enrichment experiences. Elementary teachers use hand-held calculators to show students how to use calculators, to shorten computation so that concepts may be covered in greater depth, and to show multiple examples of concepts.

Over seventy percent of the teachers of secondary mathematics reported that computers are not used for instructional purposes in their schools. Another seventeen percent responded that computers are used for instructional purposes in their schools, but that they do not use computers in their mathematics classes. In other words, relatively few teachers of secondary mathematics make use of the computers with their classes.

Teachers were asked to rate eight evaluation techniques, four teacher-prepared ones and four other-prepared, according to the importance they attached to each. Elementary teachers rated each of the four teacher-prepared evaluation techniques well above any of the other-prepared ones. Secondary teachers of mathematics rated teacher-prepared tests far above any other evaluation technique. They also attached considerable importance to evaluating performance on assignments and teacher observation of students' work.

Among the sources of mathematics assistance listed, a majority of elementary teachers reported the availability of only one, the mathematics resource person at the district level. A slight majority of secondary teachers reported the availability of mathematics assistance from Learning Assistance Centres and of mathematics resource personnel at the school level. A substantial proportion of both elementary and secondary teachers said they did not have access to Learning Assistance Centres for mathematics.

About fifteen percent of the elementary teachers reported that they had mathematics programs designed by the teachers of that school to serve as the basis for mathematics instruction. A majority of the secondary mathematics teachers reported the existence of such programs.

Grade 8 and 10 teachers agreed that students should spend less than thirty minutes per day at out-of-class time on mathematics assignments. Grade 12 mathematics teachers felt the students should spend thirty to sixty minutes per day on such work.
Importance of Selected Curriculum Objectives

All but the Grade 12 teachers were presented lists of possible mathematics learning outcomes for their specified grades. They were asked to rank each learning outcome on a scale from 1 (Not Important) to 5 (Very Important). Relative to this scale, only about ten percent of the learning outcomes were given below average rankings. As might have been expected, the computation-oriented learning outcomes were given high ratings. All of the Grade 3 learning outcomes that were ranked greater than 4.0 were computation-oriented. The geometry learning outcomes were given relatively low rankings by all groups of elementary teachers. Order of operations was ranked second out of twenty-one learning outcomes by Grade 7 teachers, but the order of operations item on the Grade 8 Mathematics Assessment Test yielded the lowest performance on the test. Learning outcomes that were common to Grade 7 and 8 were given similar rankings in most cases by both groups of teachers. Grade 10 teachers followed the pattern which had been established by the other groups by giving the computation-oriented learning outcomes high ratings. Grade 10 mathematics teachers also gave high ratings to two geometry and three algebra learning outcomes.

Results concerning minimal mathematics objectives for graduation from secondary school showed very clear patterns. All seven groups of teachers put a high premium on graduates being able to perform the four basic operations with whole numbers, fractions, and decimals. They also felt it was essential that graduates be able to apply their mathematical knowledge in both physical-world and consumer-related situations. All teachers agreed it was essential for graduates to be able to use the metric units of measurement. Differences of opinion between elementary teachers and secondary teachers of mathematics appeared to be over the more technical aspects of mathematics. The two groups disagreed over the relative importance of being able to apply the Pythagorean Theorem, evaluate an algebraic expression, and use basic formulas for area and volume with secondary teachers assigning a higher priority to each.

All teachers surveyed agreed that mathematics courses for Grade 8, 9, and 10 should continue to be required. Elementary teachers reacted slightly more strongly than the secondary mathematics teachers that there should be a required mathematics course for Grade 11. Elementary teachers indicated by a three-to-one margin that a mathematics course should be required in Grade 12. Less than fifty percent of the secondary mathematics teachers shared this opinion.
8. RECOMMENDATIONS

Analysis of the test results and of the information obtained from the questionnaire administered to teachers resulted in the formulation of a number of specific recommendations for change. The recommendations, which were stated in the Test Results and Teacher Questionnaire reports, are presented here as the conclusion of the Mathematics Assessment reports.

The recommendations have been grouped under several headings, with each grouping consisting of recommendations directed at the particular group or institution which was judged to be primarily interested in or responsible for those areas. The notations in parentheses following each recommendation refer to the location of the recommendations in the general reports. For example, TR 3-1 refers to the first recommendation in Chapter 3 of the Test Results report, and TQ 4-2 refers to the second recommendation in Chapter 4 of the Teacher Questionnaire report.

The Ministry of Education:

- should ensure that all persons teaching mathematics at the elementary school level have, as a required part of their training, the equivalent of at least one course in the teaching of mathematics and one course in mathematics for teachers. (TQ 2-1)

- should ensure that all persons teaching mathematics at the secondary school level have mathematics as one of their major areas of undergraduate study, as well as training in methods of teaching mathematics. (TQ 2-2)

The Curriculum Development Branch of the Ministry of Education:

- should undertake the development of a list of mathematical terms which students should learn, as well as a teaching sequence for developing this vocabulary. The list and sequence should take into account the developmental nature of the acquisition of meaningful mathematical vocabulary. (TR 3-3)

- should ensure that materials for teaching the metric system of measurement are available in all schools. (TR 3-5)

- and educational researchers should address the problem of the optimum time for introducing fraction concepts in the mathematics classroom, bearing in mind children's age, their level of development, and the sophistication of the ideas involved. (TR 3-7)

- should consider the impact of the use of hand-held calculators in mathematics classrooms at various levels, and provide guidance to teachers of mathematics regarding the most appropriate uses of these devices in their teaching. (TR 5-3)

- should examine the situation with regard to the teaching of percent and its applications, and give specific suggestions to teachers regarding appropriate materials to be used in teaching these topics. (TR 5-5)
should give immediate and serious consideration to ways and means of ensuring that all students completing Grade 12, have been taught the major topics of consumer mathematics. (TR 5-8)

should reconsider the nature and the scope of the geometry curriculum at the secondary school level. (TR 5-9)

should provide teachers of mathematics with an outline of the minimum learning outcomes at each level or grade to aid them in the selection of textbooks, materials, and activities. (TQ 4-1)

should conduct a study to discover why so many elementary teachers are still using Seeing Through Arithmetic texts even though they are no longer prescribed. (TQ 4-2)

along with school district curriculum specialists and textbooks publishers, should take steps to ensure that mathematics textbooks for all grades are designed to be easily read by the students, inasmuch as it is possible to do so. (TQ 4-4, 4-7)

and persons responsible for the approval and adoption of mathematics textbooks at the Grade 8 level, should take under advisement the concerns expressed by Grade 8 teachers regarding the level of emphasis on computational skills and problem-solving in the prescribed textbooks. (TQ 4-9)

School Districts:

should explore ways-and means of making specialists' services more readily available and of more benefit to teachers of mathematics. (TQ 5-1, 5-2)

should ensure that Learning Assistance Centres which provide remedial services in mathematics are available in all schools. (TQ 5-4)

Schools:

should provide elementary teachers with appropriate manipulative devices for the teaching of place value concepts and of operations on numbers. (TR 3-1)

and school districts should ensure that materials for teaching the metric system of measurement are available in all schools. (TR 3-5)

should encourage more female students to continue their studies in mathematics at the senior secondary level. (TR 5-1)

should implement programs to familiarize all students, but especially those at the senior secondary levels, with the basic concepts and principles of the metric system of measurement. (TR 5-4)

should apply for group membership in various subject-matter specialist associations, thereby making the benefits of membership available to all staff members. (TQ 2-3)

should ensure that all secondary mathematics courses are taught by only those teachers who are qualified to do so. Particular attention in this regard should be paid to the Grade 8 level. (TQ 2-4)
Teachers of mathematics and curriculum developers:

- should encourage the use of appropriate learning aids for the teaching of place value concepts and of operations on numbers. (TR 3-1)

- and teacher educators, should stress the overriding importance of place value concepts and the necessity of developing understanding of place value concepts by using concrete learning aids.

- at both the elementary and secondary levels should emphasize classroom, school, and local situations for developing 'real' problem-solving experiences which will be relevant to their students. (TR 3-8, 5-7)

- should take special care to lay the foundations for understanding of the expansion of the numeration system to the decimal form of rational numbers. Understanding of the decimal form of rational numbers should then be used to improve performance with the four basic operations with decimals. (TR 4-1)

- should place more emphasis upon decimals and operations with decimals than upon fractions. (TR 4-2)

- should emphasize the importance of geometry and measurement at both the elementary and secondary levels. (TR 4-3, 4-5)

- should emphasize the topic of equivalent forms of rational numbers. Students need many experiences of starting with a rational number in fraction form, decimal form, or percent form and writing it in the other two forms. (TR 4-4)

- should place greater emphasis upon the topic of order of operations if this topic is to remain a part of the mathematics curriculum of the elementary grades (TR 4-6)

- at all levels should stress the overriding importance of problem-solving in mathematics, and they should attempt to teach their students various strategies to employ in attempting to solve problems in mathematics. (TR 4-7, 5-6)

- at all levels should place more emphasis upon teaching students how to read mathematics texts with understanding. (TQ 4-3, 4-6)

- at all levels should vary their teaching approaches to include such techniques as the use of learning centres and mathematics laboratory activities. (TQ 5-3)

Those involved in the education of teachers:

- whether pre-service or in-service, are urged to emphasize the importance of having students use manipulative devices as models for mathematical concepts and skills at all times, but particularly when such concepts and skills are being introduced for the first time. (TR 3-2)

- should organize workshops and conferences dealing with the metric system as a follow-up to what has already been done. Such workshops and conferences should emphasize the best materials and methods to be used in the teaching of measurement, and they should stress the importance of students obtaining "hands-on experience" in measuring in order to facilitate the development of their ability to "THINK METRIC". (TR 3-6)
- should emphasize the importance of instruction in geometry in the elementary school mathematics curriculum (TR 4-3)

- should encourage their students to develop the skills required to use alternative teaching strategies such as the use of learning centres and mathematics laboratory activities. (TQ 5-3)

Educational researchers:

- should attempt to ascertain why such a high proportion of female students do not continue to study mathematics beyond the level of the last compulsory course, Mathematics 10. Such research should be given high priority by the Ministry of Education. (TR 5-1)

- and supervisors of instruction should investigate the ways in which mathematics textbooks are used at all levels in an attempt to clarify the interaction between teacher-based discussion and textbook-based discussion. (TQ 4-5, 4-8)

- should investigate in greater depth the relationship between achievement in mathematics and such student background variables as age, sex differences, number of schools attended, amount of television watched, national origin and first language, use of hand-held calculators, time spent on assignments, parental educational level, and future plans.
APPENDIX A

Contributors to the Mathematics Assessment
The authors of this report are very grateful to the administrators and staff of the following schools which participated in piloting the student tests in the autumn of 1976.

**Grade 4 Piloting**
- Douglas Road Elementary, Burnaby School District
- Hillcrest Elementary, Coquitlam School District
- King George V Elementary, Prince George School District
- Lakeview Elementary, Burnaby School District
- MacDonald Elementary, Vancouver School District
- Muriel Baxter Elementary, Cranbrook School District
- Sir William Van Horne Elementary, Vancouver School District

**Grade 8 Piloting**
- Alpena Secondary, Burnaby School District
- Connaught Junior Secondary, Prince George School District
- Gladstone Secondary, Vancouver School District
- Handsworth Secondary, North Vancouver School District
- Kitsilano Secondary, Vancouver School District
- Laurie Junior Secondary, Cranbrook School District
- Mary Hill Junior Secondary, Coquitlam School District

**Grade 12 Piloting**
- Alpha Secondary, Burnaby School District
- Gladstone Secondary, Vancouver School District
- Handsworth Secondary, North Vancouver School District
- Kelly Road Secondary, Prince George School District
- Kitsilano Secondary, Vancouver School District
- Mount Baker Secondary, Cranbrook School District
- Port Moody Senior Secondary, Coquitlam School District
Review panels comprised of educators and members of the lay public were organized in the autumn of 1976 at four provincial centres to examine and amend the proposed objectives of the mathematics assessment before the student tests were developed.

CASTLEGAR REVIEW PANEL

Mr. Jack Allen, Supervisor
Cranbrook School District

Mr. Larry Cerny, Teacher
Fernie School District

Ms. Sheila Crane, Teacher
Arrow Lakes School District

Mr. Jack Edson, Teacher
Nelson School District

Mr. Dale Fike, Personnel Officer
Cominco, Trail

Mr. Bruce Gerrard, Teacher
Castlegar School District

Mr. Tom Googeon, Teacher
Castlegar School District

Mr. Tom Johnson, Teacher
Nelson School District

Ms. Joan Knowles, Teacher
Castlegar School District

Mr. Peter Makiev, Teacher
Nelson School District

Mr. Gary Mitchell, Teacher
Cranbrook School District

Mr. Bruce Morrison, Teacher
Arrow Lakes School District

Mr. SebastiaM Nutini, Supervisor
Trail School District

Mr. Frank Perehudoff, Teacher
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Mrs. Jean Ryley, Primary Co-Ordinator
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Mr. Dan Shimizu, Teacher
Trail School District

Mr. Mac Sinclair, Selkirk Community College, Castlegar

Mr. Satoshi Uchida, Teacher
Castlegar School District

Mrs. Adele Yule, Homemaker
Castlegar

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Mr. Don Heise, Teacher
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Mr. Henry Janzen, Teacher
Delta School District

Mr. Ted Kagetsu, Teacher
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Mr. Garry Phillips, Teacher
New Westminster School District

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Mr. Dave Rivers, Education Services Officer, British Columbia School Trustees Association, Vancouver

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Mrs. Mary Ammerlaan, School Aide
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Mr. J. Allistair Brown, Chartered
Accountant, Maple Ridge

Mrs. Helen Casher, School Board
Member, Maple Ridge School District

Mr. Mike Cianci, Teacher
Kamloops School District

Mr. Richard Collins, Teacher
Coquitlam School District

Mr. James Connor, Supervisor
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Mr. Neville Cox, School Board
Member, Mission School District

Mr. Alan Davies, Teacher
Coquitlam School District

Mrs. Grace Dilley, Curriculum
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Mr. George Eldridge, Teacher
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Mr. Len Fowles, Principal
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Mr. Roger Freschi, Teacher
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Mr. Ralph Gardner, Supervisor
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Langley School District

Mrs. Lynda Haylow, Homemaker
Maple Ridge

Mr. Peter Koropatnick, Teacher
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Mr. Roy Kurita, Teacher
Surrey School District

Mrs. Ozan McSweeney, Teacher
Chilliwack School District

Mrs. Marion Mussalem
HOMEMAKER, MAPLE RIDGE

Mrs. Mary Wright, Teacher
Langley School District

VICTORIA REVIEW PANEL

Mr. George Atamanenko, Town
Planner, Victoria

Mrs. Jean Barnes, Teacher
Gulf Islands School District

Dr. William Bloomberg, Forest
Chemist, Victoria

Mr. Geoff Booth, Teacher
Nanaimo School District

Mrs. Kirsten Cox, Teacher
Qualicum School District

Mr. William Dale, Teacher
Qualicum School District

Mr. John Epp, Teacher
Sooke School District

Mr. David Harris, Teacher
Victoria School District

Dr. Harold Knight, School Board
Member, Victoria School District

Mrs. Helga Lenke, School Board
Member, Lake Cowichan School District

Mrs. Rosemarie Lowe, Teacher
Sooke School District

Mr. Daryl McIntyre, Principal
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Mrs. Betty Morphet, Teacher
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Mrs. Margaret Nelson, Homemaker
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Mr. Arthur Olson, Principal
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Mrs. Margaret Strongitharm, Teacher
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Mr. Brian Tetlow, Teacher
Victoria School District

Dr. James Vance, Faculty of Education
University of Victoria
In October of 1976, the Contract Team met with the following members of the Mathematics Revision Committee to obtain their opinions on the proposed design of the mathematics assessment:

Mr. James Bourdon, Supervisor, North Vancouver School District
Mr. Ronald Edmonds, Teacher, Victoria School District
Mr. Earl Johns, Teacher, Vancouver School District
Mr. Stan Heal, Teacher, Courtenay School District
Dr. Elizabeth Kennedy, Faculty of Arts & Science, University of Victoria
Mr. William Kokoskin, Teacher, North Vancouver School District
Mr. George Nachtigal, Teacher, Abbotsford School District
Mr. Willard Dunlop, Consultant, Curriculum Development Branch, Ministry of Education
The following three panels contributed to the interpretation of test results by rating the pupil performance on each item.

Grade 4 Test Interpretation Panel

Mr. Jack Allen, Supervisor, Cranbrook School District
Mr. James Bourdon, Supervisor North Vancouver School District
Mrs. Jacque Boyer, School Board Member, Coquitlam School District
Mrs. Grace Dilley, Curriculum Advisor, Surrey School District
Miss Evelyn Grimston, Teacher, Burnaby School District
Mrs. Jean Hall, Homemaker, Vancouver
Mrs. Helen MacDonald, School Board Member, Mission School District
Miss Pat Montgomery, Teacher, Vancouver School District
Miss Pat Pender, Teacher, Vancouver School District
Mr. Ed Richmond, Faculty of Education, University of Victoria
Mrs. Anne Robarts, Teacher, Vancouver School District
Mrs. Shirley Rudolph, Teacher, Vancouver School District
Ms. Pat Takasaki, Teacher, Richmond School District
Dr. John Trivett, Faculty of Education, Simon Fraser University
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Grade 8 Test Interpretation Panel

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Mr. Robert Campbell, Teacher, Richmond School District
Mr. Richard Collins, Teacher, Coquitlam School District
Mrs. Ishbel Elliott, School Board Member, Richmond School District
Mrs. Barbara Girling, School Board Member, Surrey-School District
Mr. Don Heise, Teacher, Burnaby School District
Mr. Henry Janzen, Teacher, Delta School District
Mr. William Kokoskin, Teacher, North Vancouver School District
Mrs. M. Mussalem, Homemaker, Maple Ridge
Mr. Tomo Naka, Principal, Nelson School District
Mr. Sebastian Nutini, Supervisor, Trail School District
Dr. Douglas Owens, Faculty of Education, University of British Columbia
Mr. Thomas Poulton, Teacher, Delta School District
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Grade 12 Test Interpretation Panel

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Mr. Peter Benson, Director of Education, Institute of Chartered Accountants, North Vancouver
Mr. Neville Cox, School Board Member, Mission School District
Mr. Michael Downing, Supervisor, West Vancouver School District
Mr. John Epp, Teacher, Sooke School District
Mr. Ian Hooper, Teacher, Vancouver School District
Dr. Ted Horne, Faculty of Education, University of Victoria
Mrs. Diane McKendrick, School Board Member, Powell River School District
Mr. Frank Perehudoff, Teacher, Castlegar School District
Mr. Bernie Pregler, Continuing Education Administrator, Coquitlam School District
Mr. Mel Richards, Principal, Richmond School District
Mrs. Ona Mae Ray, President, B. C. Home & School Federation, Port Moody
Mr. Alan Taylor, Teacher, Coquitlam School District
Mr. R. Bruce Wood, Teacher, Vancouver School District