The first part of this publication is a study of students' math skills and logical abilities and their success in an introductory college chemistry course. A 29-item instrument was developed and administered to 343 students. Test scores for students who received either A or B in the course were compared with scores for those who received either D or F. Statistically significant differences, beyond the .01 level, were found between the two groups. A 27-item revision of the test was given to 371 students in the same course. Analysis of variance and regression analysis were performed on the data. It was found that 85% of the factors explaining a student's success in introductory chemistry were different than the math and logic factors found in the test itself. The authors suggest that such a test be used to help identify students who do not function at a formal level in science courses. Part two of this paper presents a model of 14 logic-related components directed towards concepts and problems typically found in natural science courses. This model, it is suggested, could serve to assist faculty in making changes in course content and approach that would maximize the likelihood of furthering formal thought in students. (MR)
A MODEL FOR FACILITATING FORMAL THOUGHT
IN THE COLLEGE SCIENCE STUDENT

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Ed Mellon, Chemistry, FSU
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Introduction

It is becoming increasingly well established that many college students fail to use formal thinking patterns when confronted with content and problems of an abstract, formal nature. While subject matter of a formal nature is not limited to the "natural" sciences, it is apparent that much of the content of a college chemistry course, for example, does require what Piaget has termed, formal operational thinking (Inhelder and Piaget, 1958).

A recent paper by Haley and Good (1976) summarizes a number of the studies which report percentages of college students who "operate" (intellectually) at a formal level. Although the percentages vary considerably, probably due to sampling and testing variations, it seems reasonably clear that about one-half of the students in an introductory college science course do not use formal logic when attempting to solve problems of a formal nature. These "preformal" students are (apparently) unable to organize data systematically, isolate and control variables, identify hypothetical possibilities before experimentation, or use proportions, correlations and other logical operations characteristic of formal thought. The general problem-solving orientation of the formal student is from hypothetical possibility to empirical proof, while the preformal (concrete operational) student proceeds in the opposite direction, from concrete experimentation (incomplete and only partially systematic) toward attempts at hypothesizing. Since much of the content of college-level science (Herron, 1975) and even high school-level science (Hartford and Good, 1976) includes abstract concepts and a formal approach to solving problems, the preformal student is left to "muddle and memorize" as best she or he can.

Solutions to the problem include: 1) flunking the preformal students, 2) "lowering standards" so that all who put forth the effort will pass, or 3) attempting to help preformal college students make the transition from concrete to formal thinking. The authors of this paper believe that the third possibility offers the greatest potential for improving undergraduate science education in the long run, even though the task appears to be formidable. A recent study by Lawson and Wollman (1976) provides a somewhat optimistic outlook regarding the likelihood of helping students make a transition from concrete thought toward a more formal type of functioning.

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Working on the assumption that appropriate instruction can help college students progress toward formal thought, the authors of this paper have been involved in trying to answer the following questions:

1. What is the relationship between selected math skills and logical abilities and success in introductory college science courses.

2. What components of formal thought should be incorporated into the curriculum "received" by college students?

Part One of this paper describes some attempts to answer the former question and Part Two contains a "model" that might be used to answer the latter question.

Part One: Students' Abilities With Math and Logic Problems and Their Success in Introductory College Chemistry

The authors of this paper have been involved in the improvement of undergraduate science education programs at Florida State University for several years and decided to investigate the relationship between success in beginning chemistry with certain math skills and selected logical abilities. If the lack of formal thinking abilities was found to be a major cause of student failure or near failure in such a course, then perhaps a remedial program in thinking could be developed and implemented to help those students achieve greater success in their major fields such as medicine, etc. The first step, of course, was to determine what relationship, if any, existed between math and logic skills and success in a course which could be assumed to require formal thought. The whole project came to be known as Practice in Thinking, the title of an interesting little book written some years ago by Professor Jay Young, who also happened to be one of the persons involved during the early stages of this research and development project.

The following section describes the first phase of project PIT, the development of an instrument which could be easily administered to identify students who would very likely have difficulties with Chemistry 101 at FSU.

The Instrument

Since no appropriate instrument could be found which tested both math skills and general ability in logic, a test was developed during the summer of 1975. The resulting 29-item test consisted of 19 math skills questions and 10 logic questions. Math skills problems included calculation of significant figures, fractions, exponents, and simple equations with one unknown. The logic items included simple deduction, possible combinations, perimeter-area relations, equilibrium in a beam balance, displacement volume, probability, and correlation. Six of the ten logic items were a version of Piaget-type interview tasks mentioned earlier and four of the questions were adapted from a BSCS test of logic.

One hundred and twenty-two students in Chemistry 101, a course for science or science-related majors, were given the test during the Fall of 1975, near the end of the quarter. Table 1 shows the descriptive statistics of the instrument.
Table 1. Descriptive Statistics for the Instrument (Sample 1)

<table>
<thead>
<tr>
<th>Mean</th>
<th>S. D.</th>
<th>Reliability</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.03</td>
<td>4.20</td>
<td>0.75</td>
<td>2.09</td>
</tr>
</tbody>
</table>

The logic questions proved to be quite difficult for most students. On the average, students missed 5 of the 10 items (see items 18-27 in the revised test at the end of Part One).

Difficulties in the math skills part of the test involved word problems or solving equations for unknowns.

The Analysis Procedure

Students' scores on the test were then compared with their grade in the chemistry course to determine the nature of the relationship between the two. Those of us involved with project PIT were primarily interested in whether the test would discriminate between the A-B student and the D-F student, so the procedure for analyzing the data was designed accordingly. Test scores for all students receiving either an A or B were compared with scores for all students receiving a D or F, using the SPSS version 6.0 analysis of variance for a one-way design. Group 1 (all A-B students) was compared with group 2 (all D-F students) on 1) each item, 2) total on math skills, 3) total on logic, 4) total score, and 5) percentage of omits.

The Second Sample

A second group of students taking the same course the following quarter was administered the same test. The second group consisted of 221 students and they were given the test at the beginning of the quarter rather than at the end. Table 2 shows the descriptive statistics for this second, larger sample.

Table 2. Descriptive Statistics for the Instrument (Sample 2)

<table>
<thead>
<tr>
<th>Mean</th>
<th>S. D.</th>
<th>Reliability</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.68</td>
<td>4.56</td>
<td>0.78</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Results

The data from the first sample of 170 college students taking the Chem 101 course were analyzed in the manner just described. Table 3 shows the nature of the relationship between test scores for Group 1 students (A or B in course) and Group 2 students (D or F in course).
Table 3: Analysis of Variance of Test Scores for A-B Students with D-F Students (Sample 1, n=70)

<table>
<thead>
<tr>
<th></th>
<th>F-Ratio</th>
<th>F-Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I: Math</td>
<td>13.159</td>
<td>.001</td>
</tr>
<tr>
<td>Part II: Logic</td>
<td>6.927</td>
<td>.010</td>
</tr>
<tr>
<td>Total</td>
<td>17.634</td>
<td>.000</td>
</tr>
<tr>
<td>Omits</td>
<td>7.953</td>
<td>.006</td>
</tr>
</tbody>
</table>

The A-B students scored significantly higher on both parts of the test and omitted significantly fewer items than the D-F students.

When the instrument was again administered in the Chem 101 course the following quarter to 221 students, the results shown in Table 4 were obtained.

Table 4. Analysis of Variance of Test Scores for A-B Students with D-F Students (Sample 2, N=121)

<table>
<thead>
<tr>
<th></th>
<th>F-Ratio</th>
<th>F-Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I: Math</td>
<td>32.661</td>
<td>.000</td>
</tr>
<tr>
<td>Part II: Logic</td>
<td>24.514</td>
<td>.000</td>
</tr>
<tr>
<td>Total</td>
<td>41.839</td>
<td>.000</td>
</tr>
<tr>
<td>Omits</td>
<td>5.531</td>
<td>.020</td>
</tr>
</tbody>
</table>

The sizes of the samples indicated in Tables 3 and 4 are smaller than the total N to which the test was administered because all students with a grade of C in the course were omitted. Also, for sample 2, since the test was administered at the beginning of the course, some students dropped out for one reason or another.

Test Revision and Further Testing

The 29-item PIT test was revised to a limited degree by the authors during the summer of 1976 and the result was a 27-item test (see Table 6 at the end of Part One) that would provide (hopefully) more valid data. During the 1976-77 academic year, the revised test was administered to another 371 students in Chemistry 101 at FSU. The results were generally similar with an improvement in the discrimination index for 7 of the 27 items. The relationship between test scores and course grades stayed at about the same level as reported in Tables 4 and 5, using the 29-item instrument.
An interesting set of scores resulted from the use of the revised test with 115 high school chemistry students, mostly juniors. Table 5 shows the comparison of test scores for students in college chemistry and those in high school chemistry.

Table 5. Comparison of Scores for High School Chemistry Students and College Chemistry Students on the Revised PIT Test.

<table>
<thead>
<tr>
<th></th>
<th>Mean Score</th>
<th></th>
<th>Mean Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H. S.</td>
<td>College</td>
<td></td>
</tr>
<tr>
<td>Part I: Math</td>
<td>10</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Part II: Logic</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Omits</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

The only noticeable difference in the results was on the math skills part of the test, with the average favoring college students by two items. Both groups scored a 5 out of a possible 10 for the logic problems. A comparison of the difficulty indexes for the test items showed that both groups found the items to be of similar difficulty. In other words, if item x was difficult for the college students, that item was of nearly equal difficulty for the high school students.

The overall test reliability ranged from .74 for the high school chemistry students to .80 for the same revised form administered to the college chemistry students.

Summary and Discussion

In an attempt to determine the relationship between success in introductory college chemistry and selected math skills and logic abilities, a 29-item test was constructed and administered to 343 students in an introductory chemistry course. Test scores for students who received either an A or B in the course were compared with scores for students who received either a D or F. Statistically significant differences, beyond the .01 level, were found between the two groups.

A revision of the test resulted in a 27-item instrument which was administered to another 371 students in the same chemistry course. The results were very similar to those obtained using the original test and overall discrimination of the items was improved.
Although analysis of variance results showed significant differences between A-B students and D-F students on the test scores in introductory chemistry, regression analysis revealed that, on the average, only about 15% of the variance in course grades could be accounted for by scores on the PIT test. Said another way, 85% of the factors explaining a student's success in introductory chemistry were different than the math and logic factors found in the test itself.

This statistic is a bit puzzling since highly significant differences in test scores were repeatedly found between A-B students and D-F students. One explanation could be the restricted range in mean scores for A-B students and D-F students. The range for mean scores was from about 14-15 for the D-F group to about 20 for the A-B group. Increasing the total number of items would offer a solution but at the same time, it would make the test less desirable in terms of total time for administration.

Another explanation could be the motivation factor, widely assumed to be a significant determinant in successful navigation through college courses. If motivation accounts for a large part of the unexplained 85% variance in course grades, then it might be assumed that quizzes, midterms, and finals in the chemistry course can be passed successfully, even though seemingly important math skills and logical abilities are lacking.

In conclusion, the authors were able to develop a test on math skills and logic abilities that differentiated between successful and unsuccessful students in introductory college chemistry (see Table 6). Since it can be administered quickly (30-40 minutes) and easily to large groups of students, the instrument could be used to help some students early in the course, identify and attempt to remedy specific shortcomings in math and logic.

Table 6. Revised, 27-item instrument.

Please read: This test has been designed to assess certain skills and thinking abilities which are related to success in this course. The results will be used to assist future students in this course who indicate that they might have difficulty with some of the items.

Please make no marks on this test. Use the machine-scored response sheet provided for your answers and the "worksheet" for all calculations, reasons, etc. It is important to show how you arrived at your answer. Remember to put your name on both the machine-scored response sheet and the work sheet. Begin when your instructor gives the indication.
Part I: Math Skills and Problems

Please select the answer which you calculate to be correct, show your choice in pencil on the machine-scored response sheet, and show your work or reasons on the work sheet.

Calculate the value of the following numbers to three significant figures:

1. \((3.82) \times (0.410) = (A) 1.56, (B) 0.156, (C) 1.57, (D) 1.67, (E) 1.48\)
2. \(2.80 \div 0.105 = (A) 2.67, (B) 267, (C) 26.7, (D) 28.7, (E) 25.7\)
3. \(7.25 - 3.18 = (A) 4.07, (B) 6.93, (C) 7.22, (D) 7.07, (E) 7.568\)
4. Express \(\frac{3}{8}\) as a decimal: \((A) .300, (B) 3.75, (C) .375, (D) .038, (E) .380\)

Find the value of the following and express in the simplest fractional form:

5. \(\frac{2}{3} - \frac{2}{6} + \frac{5}{7} = (A) 1 \frac{1}{18}, (B) 1 \frac{5}{42}, (C) 17/21, (D) \frac{1}{42}, (E) 1.1/21\)
6. \(\frac{2}{7} \div \frac{3}{4} = (A) 8/21, (B) 6/28, (C) 21/8, (D) 2/21, (E) 5/28\)

Calculate the value of the following exact numbers:

7. \(\left(\frac{2^4 \times 2^{-3}}{2} \right)^2 = (A) -2, (B) 2, (C) 4, (D) 8, (E) 16\)
8. \(2^5 \div 2^{-3} = (A) 8, (B) 4, (C) 32, (D) 128, (E) .256\)
9. \((\frac{1}{4})^{\frac{1}{2}} = (A) 1/16, (B) 1/8, (C) 2/8, (D) 1/2, (E) 2/16\)

Calculate the value (in scientific notation) of the following exact numbers:

10. \((3 \times 10^{-3}) - 1 \times 10^4 = (A) (1.5 \times 10^4), (B) (-1.5 \times 10^4), (C) 1.5 \times 10^4, (D) 1.5 \times 10^3, (E) 1.5 \times 10^{-3}\)
11. \((3 \times 10^7) - (8 \times 10^6) = (A) (2.2 \times 10^7), (B) (-5 \times 10^{13}), (C) 5 \times 10^{13}, (D) (1.0 \times 10^6), (E) (-2.4 \times 10^{14})\)

If \(x = -5\), then \(-2x - x^2 = (A) 35, (B) -35, (C) 15, (D) -15, (E) 0\)

13. Solve the following equation for \(x\): \(3x - 1 = 8 - x/3. (A) -1/3, (B) 24, (C) 30, (D) 5 \frac{3}{8}, (E) 2.7/10\)
14. What percentage of the darts thrown hit red balloons?
   (A) 20, (B) 10, (C) 5, (D) 15, (E) need more information to determine

15. What percentage of the darts missed?
   (A) 23, (B) 53, (C) 46, (D) 51, (E) 47

16. If 12 apples at 30 cents apiece and 18 pineapples at 50 cents apiece are purchased, what is the average price per item?
   (A) 45, (B) 40, (C) 42, (D) 35, (E) 38

17. Four persons A, B, C, and D are involved in a "tug-of-war." A and B form one team against the other team, C and D. A pulls only 1/2 as hard as B. If A pulls with a force equal to 100 pounds and C pulls with a force equal to 75 pounds, how hard does D have to pull to keep the battle "even?"

   (A) 25 pounds, (B) 200 pounds, (C) 175 pounds, (D) 275 pounds, (E) 225 pounds

Part II: Logic

Joe is the fastest of four men; Bill, the next fastest; Ken, the next fastest, and Dave: the next fastest. The fastest man has the smallest feet; the next fastest man the next smallest feet, and so on.

* 18. Who has the largest feet?
   (A) Joe, (B) Bill, (C) Ken, (D) Dave, (E) cannot determine.

* 19. Who has the next-to-smallest feet?
   (A) Joe, (B) Bill, (C) Ken, (D) Dave, (E) cannot determine.

* Adapted from a BSCS test of logic by William Gray
All of the following sentences are true:

- The maid likes her job, the wife faints, the cook runs out the door, and the husband lives.
- The wife does not faint, the maid likes her job, the cook does not run out the door, and the husband lives.
- The wife faints, the cook does not run out the door, the maid does not like her job, and the husband does not live.

What must be necessary for the husband to live?

* 20. (A) the maid likes her job, (B) the wife faints, (C) the cook does not run out the door, (D) the wife does not faint, (E) the cook runs out the door.

Four companies (Ford, G.E., IBM, Post) are going to have offices on the first four floors of a new building. Each company may choose any of the floors for its offices. No two companies can be on the same floor.

* 21. What is the greatest number of ways that the companies' offices could be arranged on the floors? (A) 8, (B) 12, (C) 16, (D) 24, (E) 30

22. Two long pieces of rope, A and B, are the same length. A is shaped into a square, B into a rectangle (as in the figure) and both are placed on a field of grass. Which of the following is true?

(A) More grass inside A, (B) More grass inside B, (C) Same amount of grass in both, (D) Cannot tell without taking measurements, (E) None of the above.

![Diagram of A and B squares and rectangles]

23. What will happen to the balance on the left as these identical, dripless candles burn? The balances on the right are for your information.

(A) Side A will go down, (B) Side B will go down, (C) Neither side will go down, (D) First B will go down but then come back up, (E) First A will go down but then come back up.

* Adapted from a BSCS test of logic by William Gray
24. What will happen to the water level A, if the object (rubber stopper), B, which is resting on the bottom is pulled up off the bottom so that the top of the rubber stopper is level with point C?

(A) A will go up, (B) A will go down, (C) A will not change, (D) A will first rise and then fall, (E) cannot determine

25. If there were 5 cards with blue eyes and brown hair, the chances of getting such a card from the deck of 40 would be:

(A) 1 in 5, (B) 1 in 8, (C) 1 in 10, (D) 5 in 20, (E) cannot determine from the above information.

26. If the deck of 40 cards contains 10 cards with blue eyes and blond hair, 10 cards with blue eyes and brown hair, 10 cards with brown eyes and blond hair, and 10 cards with brown eyes and brown hair, the chances of drawing a face with brown eyes and brown hair from the shuffled deck would be:

(A) 1 in 2, (B) 1 in 4, (C) 1 in 5, (D) 1 in 8, (E) 1 in 10.

27. If the possible combinations of eye and hair color are always equal as in question 26, then the correlation between eye and hair color, for the colors given, will be:

(A) 1, (B) .75, (C) .50, (D) .25, (E) 0.
References: Part One


PART TWO

A MODEL FOR FACILITATING FORMAL THOUGHT IN THE COLLEGE STUDENT

Part One of this paper dealt with the development of a test to assess math and logic skills and the analysis of the relationship between college students' scores on this test and their success in an introductory chemistry course. This part of the paper presents some ideas on characteristics of the content of a course or program designed to facilitate formal thought in college students. A test such as the one described in Part One can help to identify students who do not function at a formal level in science courses, but it says very little about what might be done to help these students become more "formal operational" with regard to problem solving. The components of the model described on the following pages could be used in a variety of ways and some of these applications will be discussed. A model such as the one described on the following pages, could serve to assist faculty in making changes in course content and approach that would maximize the likelihood of furthering formal thought in students.

Model Components

A. Computation and Math Skills
   1. Operations with whole numbers
   2. Fractions and decimals
   3. Exponential numbers
   4. Solving equations

B. General Skills
   1. Measurement involving unit notation
   2. Estimation
   3. Organizing and describing data
   4. Transposing word problems into symbols

C. Logic-Related Skills
   1. Making all possible combinations
   2. Using proportions
   3. Making correlations
   4. Using probability
   5. Analyzing and diagramming physical phenomena
   6. Identifying relevant and irrelevant data
   7. Defining and using concepts in terms of other concepts
   8. Developing hypotheses from data
   9. Identifying and resolving contradictions in data
   10. Making predictions based on given data
   11. Designing controlled experiments
   12. Analyzing and criticizing experimental design
   13. Comparing theories with data
   14. Comparing conflicting theories

Theoretical Basis of the Model

Inhelder and Piaget (1958) described the growth of logical thinking from childhood to adolescence by tracing the development of mental structures from 1) those which explain the logic of simple classes and
relations with concrete subjects, to 2) more advanced structures which allow for hypothetical-deductive logic. It is these more advanced structures which constitute the formal approach required to practice and understand modern science.

Late in their book in a chapter entitled "Concrete and Formal Structures," Inhelder and Piaget (1958) describe eight schemas or special operations which explain many of the specific characteristics of formal thinking, as required in the field of science. These schemas form the basis of the model for facilitating formal thought in college students and a brief description of each of them follows:

1. **Combinations.** A complete combinatorial system is available to the formal thinker which allows for the testing of all factors and their combinations.

2. **Proportions.** Using ratios between factors to determine other relationships is one of the most common aspects of problem solving in science. The ability to establish logical proportions and find quantitative solutions is characteristic of the formal thinker.

3. **Reference Systems.** Relativity of motion is a good example of this formal "schema" which requires a coordination of one reference system with another. Predictions about the motions within two different reference systems require the combining of both inversions and compensations with respect to each system.

4. **Equilibrium.** A formal concept of equilibrium requires mental actions similar to the previous schema, that is, inversions and compensations. As one factor in a system at equilibrium is changed, one or more other changes might be necessary to maintain the balanced state.

5. **Probabilities.** An accurate notion of the concept of chance occurrence is exceedingly important in modern science and, at the formal level, reflects an ability to reason strictly at the hypothetical level first, before empirical tests are tried.

6. **Correlations.** When causal relationships are not known with accuracy, the confirming cases must be distinguished from the total possible cases. Through this process a correlation is determined and one can see that this concept is closely related to the concepts of proportions and combinations.

7. **Compensations.** An understanding of this concept allows one, for example, to conserve the volume of a liquid even though the dimensions of its container are changed.

8. **Advanced Conservation.** An example of a type of conservation that is difficult to verify in a direct, positive way, is the principle of inertia. As compared to the concepts of number or area or weight, inertia is more abstract and, therefore, the process of verifying the conservation of inertia requires a more formal deductive approach.
These eight formal schemas combined with the author's thoughts about the nature of the scientific process, form the basis of the model for facilitating formal thought in college students, outlined in the previous section.

**Brief Description of Model Components**

The math skills part of the model was designed for an introductory course in college chemistry and could easily be modified to "fit" physics or other subject matter.

General skills included 1) measurement involving unit notation, 2) estimation, 3) organizing and describing data, and 4) transposing word problems to symbols. Like math skills, these activities are not seen as requiring "formal" thought, but they are very important areas in which a student should become fairly proficient.

**General Skills**

1. "Measurement involving unit notation":

Real as well as "mythical" units are a part of the measurements and, again, all exercises and problems are keyed to easily measured objects and events. Measurements of "static" objects precede measurements of dynamic systems. Mythical units (glerk, medrile, etc.) are used to emphasize the importance of keeping track of units as well as numbers in solving equations.

2. "Estimation":

Estimation is a powerful technique to determine whether certain operations yield a "sensible" answer. Simple estimation of one type of unit is followed by more complex types where 5 or 6 different steps are involved.

   **Example:** Estimate the number of grains of sand in a teaspoon. Given: ____ grains/volume

   **Example:** Estimate the amount of water (weight) which would be required to fill the Houston Astrodome.

3. "Organizing and describing data":

It is easier to "see" patterns if data are organized in various ways. Classifying and ordering are two important processes in the early growth of logic and variations on the processes continue to help in solving quantitative problems. The general purpose of this component is to emphasize the importance of organizing data in ways which help people to make more sense out of it. Data from experiments are recorded in a random way and then successive "de-randomizing" actions are taken. At first just simple groupings are made and finally various types of graphing procedures are used.
4. "Transposing word problems to symbols":

Identifying relationships among factors in descriptive "word" problems and quantifying these relationships is a source of great difficulty for many students. This component helps the student to see the economy in allowing symbols to represent various factors in simple and more complex relationships. Starting with concrete situations, students are asked to represent the reality using symbols.

Logic-Related Skills

1. "Logic of Combinations":

Being able to combine the relevant factors in a given problem solving situation in all possible ways is a part of formal thought. Determining the nature of the effects of various factors in a simple pendulum experiment requires a complete combinatorial system to be sure that possibly relevant factors are not overlooked. In this component students are given combinatorial problems which can grow out of actions with objects.

Example: You have one red die and one white die. How many different combinations are possible using both dice at once?

Example: The weight of the object and length of the string can be varied in an experiment to determine what affects the period of a swinging object. How many different ways can the factors (weight, length) be varied (to check on the effects) of the swing?

2. "Logic of Proportions":

Proportional thinking is often involved in solving problems where one is to predict the required change in some factor in a system, given some change in a related factor. If a variable in a system is changed, what corresponding change will restore the initial state of equilibrium? Or given that we have a certain relationship between 2 factors, what is the relationship between other related factors?

Example: A seesaw has 2 persons on it, one is x pounds and y meters from the balance point and the other is x' pounds and y' meters from the balance point. If y changes by 10% (closer to the fulcrum) what must happen on the other side to maintain equilibrium?
3. "Making Correlations":

In rolling dice, there should be no relationship (zero correlation) between the "up" number on one die and the "up" number on the other die. Students start with random events and then begin to look for nonrandomness (patterns) and try to determine the magnitude of the relationship.

Is event y always associated with x or only so often? This notion is obviously closely related to probability.

4. "Using Probability":

This notion is closely related to the logic of combinations since a concept of change depends on an understanding of "what is possible." Knowing the chances of rolling a six on a die requires a knowledge of the total possibilities and what part of the total the six face represents. Students conduct many simple chance experiments and observe and deduce the probability of certain events occurring. Other data on occurrences of various events are also made available and the students determine the probabilities of the events actually happening.

5. "Analyzing and Diagramming Physical Phenomena":

Data which are presented in a "random" manner are sorted out, categorized and represented in tables, diagrams, etc. This is an extension of "Organizing and Describing Data" and is intended to involve actual data collection.

6. "Identifying Relevant and Irrelevant Data":

If a person has a reasonably good grasp of a problem, he/she will be able to sort out variables which are probably relevant from those which are probably irrelevant. Especially if a relationship among the variables is given (\(v = \frac{d}{t}, F = ma\), etc.) one would expect that extraneous data could be discarded and only the relevant data used.

Two different problem "types" are given here as examples of ways to encourage thought and action about relevant and irrelevant data.

**Problem Type 1**: Determine what affects the rate at which spherical objects roll down an incline. Rank each factor from most important (1) to least important ( ).

- factors: size, weight, hardness, smoothness, color, mass distribution (hollow, solid), temperature.

**Problem Type 2**: The average velocity of an object can be found by measuring how long (time) it takes to travel a given distance. Find the average velocity of a spherical object, given the following data:

- weight: 1000 grams
- size: 10 cm (diameter)
- time of travel: 10 sec.
- starting speed: 0 cm/sec.
- top speed: ____ cm/sec.
- distance travelled: 1000 cm
- angle of incline: x degrees
- weight of inclined plane: 10 kg

Velocity as a concept is used in defining acceleration and therefore acceleration can be considered a "second-order" operation. When concrete operations are combined thru implication, conjunction, etc. to form new "propositions," formal thinking is occurring.

This component should consist mostly of combining concrete operations into second-order operations and in reducing second-order operations to the more basic concrete operations. Attaining reversibility in this process should transcend the mechanical manipulation of equations as might be done in "solving equations."

8. "Developing Hypotheses From Data":

Developing hypotheses which are logically related to data requires beginning at a somewhat abstract level and then devising means of testing the hypotheses. Identifying and controlling variables is an obvious factor here. Students would be asked to relate hypotheses from given data as well as generating their own hypotheses from data. An important technique here is to "force" students to make logical links between data and hypotheses rather than links based on experience.

9. "Identifying and Resolving Contradictions in Data":

Searching for patterns typically involves resolving contradictions in some of the data. In "The Growth of Logical Thinking," Inhelder and Piaget identify the ability to eliminate contradictions as one of the characteristics of formal thought. Components 13 and 14 are closely related to this component, but the emphasis here is on data about concrete events rather than data-theory compatibility.

Students can do experiments designed to provide contradictions to a pattern of development or they can consider data presented in tabular or graphic form. In either case the purpose is to identify and resolve the contradictions. The reasons for the apparent contradictions in the data would be generated by students. Also, possible reasons would be offered to the students and they are to decide which possibilities "make sense."

10. "Making Predictions Based on Given Data":

Extrapolating from given data is the focus for this component. Given a certain pattern of data, what predictions can be derived? Problems can become more complex as different types of data are brought into the picture, as in weather predictions. Although a knowledge of more weather-related factors can allow for more accurate predictions, it also becomes more complicated in trying to coordinate all of the factors.

Simple prediction patterns can be based on symbols for objects or data from experiments. Whatever the format, the basic process is the same.
Once factors are grouped as relevant variables, the process of "finding out" can take place only by changing one thing at a time. In the pendulum experiment referred to in component 1, factors which might affect the rate of swing include length of string, weight of the object, size of the object, how the string is attached to the support, height of release, etc. Controlling all variables except one reduces the experiment to essentially one variable.

Students will analyze designs of simple experiments as well as design their own. It is important here to have interaction among students to allow comparison of designs on the same investigation. All experiments should be such that they are a part of the student's environment. Questions such as these could be posed:

1) Can smokers identify their own brand of cigarettes?
2) Can drinkers identify their own brand of bourbon, scotch, beer, etc.?

A lab setting should also be available with equipment for pursuing controlled experiments.

12. "Analyzing and Criticizing Experimental Design":

The nature of experimental "proof" is seldom clear-cut and definite. A good experimental design should consider all possible variables and provide for proper data analysis. A faulty design can lead to misleading data and, therefore, improper conclusions.

13. "Comparing Theories with Data":

Are the available data consistent with a given theory? If not, in what ways do the data seem to be inconsistent with the theory?

14. "Comparing Conflicting Theories":

How do theories, which attempt to explain similar phenomena, differ and what are the strengths and weaknesses of each theory? Do certain data suggest revision of part of a theory?

Discussion

The 14 logic-related components that have been briefly described are largely "content-free" even though the model is directed toward concepts and problems typically found in natural science courses. Just how the various components might be incorporated into a course or program has received little attention up to this point in the paper. There seems to be general agreement, however, that formal thought should be facilitated by teachers:

1. Beginning with familiar, concrete examples.
2. Emphasizing a "noncookbook" laboratory approach to identifying and solving problems.
3. Questioning all answers.
4. Intentionally introducing (apparent) contradictions that cause cognitive conflict.
3. Encouraging student interaction in considering ideas, problems, etc.

6. Removing obstacles to maximum exploration of ideas.

The "subject matter" for such a teaching approach could be specific, such as chemistry, or much more general. An example of a more general program is "Accent on Developing Abstract Processes of Thought (ADAPT)" at the University of Nebraska in Lincoln. Robert G. Fuller, Director, and eight other faculty members have developed and implemented a Piaget-based program which constitutes a student's entire freshman year. Courses in English, history, economics, physics, anthropology, and mathematics are used to assist students in developing formal thought. The primary advantage of such a comprehensive approach is the ability to create a total environment for the students that, hopefully, does, in fact, facilitate progress toward formal, abstract thought. An obvious disadvantage is the difficulty in coordinating such an effort where each faculty member lends his or her unique interpretation of Piaget's theory of cognitive development to classroom practice. The various factors that might affect a student's growth toward formal thought can be better controlled in, say a single chemistry course, but the amount of time a student is involved is drastically reduced.

The authors of this paper had hoped to develop a competency-based program consisting of a set of instructional modules based on the 22 "components" already described. The PLATO computer system at FSU was to be a major tool in this program, but a lack of funds changed the course of events. Since it is very expensive to develop quality instructional modules with a computer-based component, it now appears more realistic to think in terms of an ADAPT-type approach. The model proposed in Part Two of this paper could be used to assist willing faculty members in "adapting" their courses to a common core of components. Self-tests should be developed to allow a student to determine his/her level of competency (a PLATO-type facility would be valuable for this) to deal with the various components identified in the proposed model for facilitating formal thought in college students. While ample time must be provided for students to become better able to deal with problems requiring formal thought, the program should be competency-based rather than time-based. That is, the level of competency of "making combinations" or "using proportions" or "designing controlled experiments," etc. should be concretely defined to allow for variations in the rate at which students progress toward formal reasoning ability. Once the components for a working model of formal thought can be agreed upon, the corresponding "competencies" can be identified for individual courses in chemistry and other areas or for more comprehensive programs such as ADAPT. The "model" outlined in this paper represents one attempt at defining formal thought that is commonly required in modern science courses. Such a model will probably be useful only if the reader already has some understanding of the growth of logical thought as defined by Piaget's work.
Reference


Suggested Readings

ADAPT: A Piagetian-based Program for College Freshmen. A guide available from Robert G. Fuller, Physics Department, University of Nebraska-Lincoln, 1976.


