The purpose of this research is to show that the logical formula for equivalence, (A=A; but is not A'), originates in the early manual actions of the young child. Having two hands that are bilaterally opposed and initially function in bilaterally symmetrical action might serve to structure the form of early sensori-motor experiences and this structured experience serves as the base of the logical operation of equivalence. Subjects were 66 children ranging in age from 7 to 32 months. Each child was presented six different clusters of five blocks each. Children were filmed at block play. Methodology and data obtained are discussed at length. In general the films revealed the gradual atemporalization of successive states and the dissociation of the logical from the contingent. The films also suggest that the physical constraints of material determine the early block structures, but block structures gradually are constrained more by rules of order than by rules of physical fit or physical balance. Appendices include the notation system used for rating the films, tables of frequency of unique action units and a frequency count of productions of the children. (MS)
Transformations in the Manipulations and Productions Performed with Geometric Objects:
An Early System of Logic in Young Children

George E. Forman
in collaboration with
David Kuschner
Jean Dempsey

Center for Early Childhood Education
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University of Massachusetts, Amherst

July 1973 - July 1975
FINAL REPORT

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# Table of Contents

Introduction ........................................... 1

Predicted Sequence and Rationale ....................... 5

Methods .................................................. 8

Subjects .................................................. 8

Procedures .............................................. 9

Materials ................................................ 11

Coding ................................................... 12

Notation System ........................................ 12

Results and Discussion ................................ 14

Position, Plane, and Interval in Space ............... 22

From Repetition to Reiteration ....................... 26

Reiteration of Substraction .......................... 28

From Symmetrical Action to Symmetrical Production . 32

Unpredicted Findings .................................. 43

Transformations in Dynamic Contact ................. 43

The Decentration of Effect ........................... 44

Denotation of Products ................................ 45

The Primordial Sharing Situation .................... 46

Sensitivity to Orientation of Blocks ................ 47

Final Comment .......................................... 51

Bibliography ............................................ 52

Appendix A .............................................. A1

Appendix B .............................................. B1

Appendix C .............................................. C1
If, as Piaget suggests (Piaget, 1970) knowledge is an act of self-regulated construction and granting that this construction begins during early infancy, it stands to reason that our research of this process requires a special methodology. For one, we need an observable response which does not depend on language for either its elicitation or its mode of expression. For another, we need a rich response domain, a response domain that can display structure and pattern, a response domain of sufficient complexity that we can see the system within a stage and how that system evolves through self-regulation. Thirdly, we need an experimental setting that allows the child to control his own sequence of responses. If we are ever to appreciate the constructionistic epistemology, we must give control of trial $n + 1$ to the child, not to an experimenter baiting food wells behind a one-way vision screen.

It was this rather general curiosity about how the child constructs systems of relations that made it reasonable to film young children spontaneously playing with small geometric blocks. Block play seems ideal for the purposes of studying the structuration of thought. It is overt, fluid, intrinsically interesting to a wide age range of children, and it is almost universal. The block itself represents a cultural tool, which, as Bruner repeatedly points out (Bruner, 1974), influences the evolution of intelligence within the culture using the tool.

In the act of placing, removing, releasing, and rearranging blocks, the child is constructing spatial relations. The child is both expressing his knowledge of objects in space and inventing new relations as he turns his thoughts to what he has done. Spatial relations are, according to Cassirer (1957), of fundamental ontological significance for logic. A set of blocks provides a tangible medium for the child to express these relations, to represent some simple relation such as "on top of" so that he might progress to a more complex set of relations, like "on top of and next to" which subsumes the easier relation. These small shapes help the child to scale down the environment through which he moves, so that he might more systematically position objects and construct a more coherent system of spatial relations (Stea and Blaut, 1973). He deliberately constructs products which are balanced physically against gravity and later are balanced visually for aesthetics. These constructions are rule governed. It is in the development of these rules, and their transformations across age, that block play can be studied as an early system of logic.
Logical thought aids the problem-solver in making proper inferences, systematizing information in order to keep track of solutions attempted, distinguishing between apparent differences and real differences, and otherwise going beyond the immediate given. American education has recognized the value of studying the structure of logical thought, as evinced in the current shift to teach the process of thinking rather than content (Bruner, 1960). New math, modern linguistics, and history as a dialectical process are cases in point.

One of the most elementary rules of logic, probably the foundation of all higher forms of conceptual organization, is the rule that two elements are the same type, but physically separate one from the other, the concept of "equivalence." What are its origins? Vygotsky (1962) assumes that the concept of "same yet different" results from the acquisition and use of language. Words serve to mediate the similarity and difference between objects classified together. The Gestalt psychologists (Koffka, 1924) assume that the perception of equivalence is innate, independent of experience, a fact of neurological organization. But Piagetians believe that these elementary concepts are the result of a gradual organization of action patterns into patterns of thought. Patterns of object manipulation in early sensorimotor development contribute to the structure of thought patterns later used in manipulating the world symbolically (e.g., mathematics and logic). Hand actions are precursory determinants of thought patterns. To quote Wartofsky:

"But what underlies this linguistic use (of same), developed as one among the earliest of our language habits? It would be odd to claim that language is somehow "given" in this form, that this is the a priori "nature" of language and that therefore this ultimately explains how the concept of "same" arises. Rather, it seems more reasonable to suppose that the patterns of experience which occur at the prelinguistic level of perceptual and motor activity already prefigure this linguistic concept." Wartofsky, p. 54 (1970)

And in the same vein a quote from Lunzer (1964)

"Logic is not a mode of organization forced on us by the world as experienced. It is one that we construct by co-ordinating our own actions and abstracting relations between them."

While the above writers may differ on the degree to which logical thought is prefigured, they both, in agreement with Piaget, look to early sensorimotor behavior as its origin. The purpose of the research herein is to show that the logical formula for equivalence, \((A = A', \text{ but is not } A')\), originates
in the early manual actions of the young child. Having two hands that are bilaterally opposed and initially function in bilaterally symmetrical action might serve to structure the form of our early sensori-motor experiences, and this structured experience serves as the base of the logical operation of equivalence. Humans may have created, thereby, an organizational system, a logic, which is as completely anthropomorphic and geocentric as was pre-Copernican cosmology.

If we understand the genesis of logic, the stages of its development, and mechanisms that effectuate the growth of logical thought, we then look at our curricula to check for "violations" of the epistemic sequence. Only recently have we learned to use teaching techniques for the young student which are qualitatively different from those used with older students. While older students can learn from abstract, expository presentations, younger students learn better with concrete, discovery experiences. The results of the research herein should yield insights into teaching techniques for even the very young child, techniques founded on understanding the grammar of action. Thus the epistemological continuum, which will direct the pedagogic continuum, runs from action (patterns of manual movement) to product (the patterns of spatial arrangements children create) to logic (the rules of abstract classification). This research is an attempt to trace the continuity in the development of logical thought from its earliest form in infancy to later forms which have been more thoroughly researched.

In their work on the early growth of logic, Inhelder and Piaget (1964) administered classification tasks to children from ages three through twelve. The child was asked to sort geometric tile of various shapes, colors, and sizes into sub-groups: "place those which are the same, which look alike, together." They discovered that the child of three to four added blocks to a group in order to make some spatial whole, like the configuration of a house, or a symmetrical arrangement. These younger children were using spatial configuration to determine placement, while older children used abstracted dimensions such as all red blocks here, all green blocks there. The spatial configuration of the green blocks (or red blocks) was irrelevant to the criterion of placement. Inhelder and Piaget attribute considerable significance to this shift from spatial criteria to formal, abstracted criteria. To them it indicates that class and sub-class concepts originate in the initial work with an element conceived as a physical part of a spatial whole. That is, the skills involved in learning that this object is both a fish and an animal (sub-class is a part of a superclass) is "practiced" in
the earlier learning that this object is a fin which is a physical part of
the spatial configuration fish. Part-whole relations in the sensori-motor
world underpin class-subclass inclusion in the conceptual domain.

The three-year old considers an element a part of something if that
element completes a spatial configuration created by several elements. But
are there levels of part-whole awareness which occur at ages younger than
three years? Consider the child who lifts a single semi-circle and comments
"it broken" (meaning that the circle has broken). Next this child searches
for another semi-circle and upon discovery joins the two together to create
a circle, and then smiles at his accomplishment. Two-and-a-half-year-olds
exhibit this behavior (Forman, 1972a). The two year old, upon finding the
first semi-circle is equally likely to search for another, but upon dis-
covery, he superimposes the two semi-circles instead of completing a circle.
Thus, the notion of equivalence (A=A') seems to precede the notion of part
to whole (A+A' = B), with the former contributing to the development of the
latter. The sequence from equivalence, to part/whole, to class membership
has both a compelling logic and some preliminary empirical support (Forman,
1972a).

Now one might ask, what are the origins of the equivalence concept,
with full cognizant that one is also asking what are the origins of formal
logic. A close look at the child's superimposing equivalent blocks gives some
tentative answers. First, the child generally picks up one object with his
right hand, transfers it to his left, and then searches for the equivalent
object with his right hand. Upon grasping the second object, he bangs the
two together at the midline (age 1 1/2 to 2 years old). Of thirty cases
observed of the midline banging (Forman, 1973), twenty-nine involved two
equivalent blocks (same form and size) even though many other non-equivalent
blocks were available! Equivalence was expressed by fitting A in one hand,
A' in the other hand, and then alternately touching and separating them at
the midline. This alternating motion, done with close visual and tactile
inspection, seemed expressive of an awareness that the two objects, say two
same sized cubes, were in one sense the same (when superimposed) but also
different (separate objects). Given two hands, the child can successively,

1 Please distinguish between the identity concept and the equivalence
concept. Identity describes "A is physically the same object as A," like
John today is the same person as John tomorrow. Equivalence refers to "A
is the same type of element as A', but is not the same physical embodiment
of A' " (see Elkind, 1965).
in a quick alternation, focus on the similarities and differences of physical objects. The child lifts one element capriciously but then searches with the opposing hand for another object like the first. One could reason that the child is almost compelled to establish a balance between the two hands, to wit, what one hand holds in form and size, the other hand must also hold.

The point being developed here is this. The sensitivity we humans have toward equivalences between objects may be prefigured in the bilateral symmetry of our anatomy, prefigured in the sense that particular types of object manipulation are more likely to occur as a result of our having hands bilaterally opposed, and certain types of feedback may be more pleasing to us, such as a type of "haptic equivalence" that occurs between the hands when each holds an identical object.

The notion that the hands work in a balanced motion has been well documented (Gesell, 1936). Up to around 18 months the hands even mirror each other. When the right hand grasps an object, the left hand, even though vacant, palpates in imitation of the occupied right hand. These observations are highly suggestive of neurological symmetry between the hands at an early age. Thus bilateral symmetry occurs both at the level of structure (gross anatomy) and function (neurophysiology). It is but a short extension of reasoning to posit a bilateral symmetry descriptive of many thought patterns (see DeSoto, London, & Handel, 1965; Olson, 1975). Certainly the relation between ambidexterity and perceptual reversals, a common correlation among poor readers (Belmont & Birch, 1965), indicates a tie-in between manual systems and systems deemed more cognitive. The main distinction of the hypothesis under investigation is that ambidexterity, as opposed to unilaterality, serves a positive function, i.e., a sensori-motor foundation for the concept of equivalence.

**Predicted Sequence and Rationale**

At an early age, the child moves both hands in concert. What one hand does, the other follows (Gesell, 1940). But seldom do both hands reach for an object that is not on the midline. The reach to the side is probably an outgrowth of the tonic-neck-reflex which is asymmetrical in form. However, an object placed in the midline elicits two handed grasping with both hands on opposite sides of the object. In this manner, the child receives identical palmar sensations, assuming that the toy is symmetrical, which is usually the case.
As the grasping response becomes more differentiated from the tonic-neck-reflex and the bilateral trapping-type grasp, the child will pick up two objects, one in each hand. However, since this is reminiscent of the tactile experience of palpating a single object at the midline, the child draws his hands together. That is, feeling two objects, one in each hand is quite similar to feeling the opposite sides of a single object. He then bangs the two objects together.

**Predicted sequence:** Two hands grasping one object at the midline will emerge prior to two hands each grasping an object and brought to the midline.

The order of these two units implies that it is more difficult to produce equivalent feelings between the two hands using two blocks, than to produce equivalent feelings between the two hands centered on one block. This difficulty rests in the number of movements required as much as load on memory and sensorimotor skill. However, the fact that these patterns do emerge in this order, for whatever reason, gives the infant certain expectancies in reference to objects. The child in grasping two blocks, is surprised by their separateness because of the more usual encounter with a single object grasped between the hands. The child's attention is thereby directed to the similarity between the blocks (feel alike) but also to their difference (they separate).

As the child continues to have commerce with objects, he will move from a distinction between holding one block and holding two blocks to a distinction between two different-shaped blocks and two same-shaped blocks. This latter difference is more than just existential separateness as in the former, but is also a reaction of form per object.

**Predicted sequence:** Two hands banging identical blocks at the midline will emerge subsequent to two hands banging any two blocks at the midline.

The form of an object is processed by visual and tactual scanning. Physical separateness (one as opposed to two) requires little scanning, only the continuation of the tactual input regardless of the distance between the hands (two blocks). Spreading two blocks apart is the elementary form of difference from which the child can learn; bringing two blocks together that are the same form is an elementary form of similarity from which the child can learn. This does not mean that the child is incapable of recognizing similarity prior to these physical manipulations (see Lewis, 1971); but it does mean that this is an early form of similarity production that can, by its nature, be articulated with the concept of difference as well.
The movements so far discussed have been closely related to the on-going action of the hands. Banging a block on the table and even superimposing blocks in mid-air requires the physical support of the hands. The predicted units to this point has not been dissociated from the hands, as a place-and-release arrangement on the table would be. The expression of similarity has yet to gain objectification to the point that the expression remains independent of on-going hand action. Superimposing two same-size same-shape blocks in mid-air is still "attached" to the body reference, more so than superimposing one block on top of another block resting on the table top. In the latter case, the expression remains even after the grasp has been released. The stacking response is one of the first definite attempts to create a static expression of equivalence. Here again we see a developmental shift from a more action-based expression to a more action-dissociated expression. The child is now more oriented toward the product of his action than toward the production per se.

**Predicted Sequence:** Placing one block superimposed on an identical block resting on the table will emerge subsequent to superimposing two identical blocks at the midline.

The movements to this point have all been performed at the same site, either the same site on the table or two blocks together. Stacking two blocks, one on top of the other, is essentially placing one block in the same site as the other. However, placing one block to the side of another, as in a two block horizontal alignment, is more a shift to a new site. This assumes that the focal locus is determined by the downward direction of the hand. Placing down on top versus placing down next to is then representative of a shift from same site to different (but contiguous) site. Here we see an instance where difference is expressed in terms of physical position, a static property of difference.

**Predicted Sequence:** Placing two identical blocks side-by-side in a horizontal alignment will emerge subsequent to placing two identical blocks in a vertical stack.

The units to this point have all been performed at either identical or contiguous sites. The spatial separateness between two blocks is not used as an element in block arrangements until later. But the emergence of "near-but-apart" seems to be crucial to the formation of the equivalence concept. In other words, if the child conserves equivalence he will know that A is equivalent to A' even though the two blocks are not physically touching. It seems that the child passes through a stage where the need
to produce physical feedback (flushness of one against the other) dominates the A to A' relation (Forman, 1972a). Both the stacking of two blocks and the horizontal alignment are therefore more elementary than placing two identical blocks near-but-apart. The child in the latter case has developed further in an apprehension of equivalence above and beyond the case where equivalence needs to be confirmed in physical superimposition of flushness. He has begun to make a great step toward seeing equivalence as something not bound to the physical object, but as a relation per se. He has begun to abstract equivalence rather than express it only in its more concrete form of superimposition or flushness.

Predicted Sequence: Placing two identical blocks near-but-apart will emerge subsequent to placing two identical blocks side-by-side in a horizontal alignment.

The last several units represent a developmental sequence whereby equivalence moves from an expression in action (banging at midline), to an expression in continuity of boundaries in action (holding in superimposed position), to an expression in continuity of boundaries in product (stacking), to expression in contiguity without continuity (horizontal alignment), to an expression in proximity (separated placement). This continuum can be summarized as increasing differentiation of similarity and difference concurrent with an increasing articulation of these two relations. The child uses sameness and difference in both time and space to express equivalence at each stage of his apprehension and it is the expression of this apprehension which leads him to make discoveries that advances him to the next stage of understanding. The interesting point of consistency throughout these changes is that the anatomy of the hands, the constraints this puts on timing and direction, maximizes both the differentiation and articulation of the similar and different relation.

Methods

Subjects

Subjects were selected from the Amherst Town Hall birth records. Approximately one hundred families were called which eventually lead to a research sample of seventy families. The parents assured the calling researcher that their child would be available for testing in October, February, and June of the academic year 1973-74. These families were mostly middle-class professional people, with about thirty percent blue-collar families. The parents were given two incentives to continue the study, one was a nominal three dol-
lars per visit, the other, more significant incentive was our promise to
give them a private showing of their child on film, spliced over the three
testings spanning the whole year. Out of seventy children, we lost only
four from the study.

The total sample consisted of five age ranges as indicated in Table 1.
Children were scheduled to come as close to the middle of their age group
as possible. Each age group was defined by a 60 day span, e.g., Group 1,
the 7 and 8 month group, ran from 210 days after birth to 270 days after
birth. The subsequent testings of any child were done not sooner than 120
days later and not more than 134 days later than the most recent testing.
There were fourteen children in each age group, half males and half females.

<table>
<thead>
<tr>
<th></th>
<th>October</th>
<th>February</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>7-8 months</td>
<td>11-12 months</td>
<td>15-16 months</td>
</tr>
<tr>
<td>Group 2</td>
<td>11-12 months</td>
<td>15-16 months</td>
<td>19-20 months</td>
</tr>
<tr>
<td>Group 3</td>
<td>15-16 months</td>
<td>19-20 months</td>
<td>23-24 months</td>
</tr>
<tr>
<td>Group 4</td>
<td>19-20 months</td>
<td>23-24 months</td>
<td>27-28 months</td>
</tr>
<tr>
<td>Group 5</td>
<td>23-24 months</td>
<td>27-28 months</td>
<td>31-32 months</td>
</tr>
</tbody>
</table>

Table 1. Age in months for each testing during the 1973-74 year.

Procedures

Each child was brought into a small research room by his or her mother.
The small children sat in their mother's lap, larger and more independent
children sat at the table alone with their mother sitting alone to their
rear left. All children were in front of a large four by six foot white
formica table that contained a one and a half by two foot notched recess for
the child's chair. This recess, by enveloping the child, reduced the number
of times blocks were spilled accidentally to the floor. Each child spent
approximately five minutes playing with a warm up toy, which was a little
wooden bear with hinged arms and legs. While the child manipulated this
toy, the experimenter chatted lightly with the mother. This casual chatting
did much to relax the child in this new setting.

Before the blocks were presented to the child, the researcher, Jean
Dempsey, asked the mother not to intervene in any way while the child was
playing. The mother was told that she may replace on the table blocks that
had fallen on the floor, but she was asked not to stack, align, place on
edge, or any other variation of placement when she did so. She was shown a
list of words that she should not say while in the research room, not even casually said to Jean. These words were such words as "stack, pile, roll, circle, square, red, green, build, do better, make a train, etc." Jean explained that the goal of the study was to see what the child thought to do on his own.

Once the child was relaxed, Jean brought out the first can of blocks. At the same time that Jean placed the five blocks from Can I on the table, David Kuschner activated the camera filming at six frames per second. David was filming the children from an adjacent room through a clear glass window camouflaged from the child by a curtain which was draped over David's side of the window. This filming arrangement was effective in both eliminating the sound of the shutter from the experimental room and in hiding the filming equipment. Not a single child commented on the lens protruding through the hole in the curtain.

As soon as the blocks touched the table, Jean activated a concealed stop watch and timed forty-five seconds. At the end of forty-five seconds, Jean presented the empty can to the child and asked him to put all the blocks in the can. The use of the can was an effective way of getting the child to comply with instructions to stop his play. Pilot research had shown that forty-five seconds was a good time range for both the seven month and thirty-one month children. If the child did not volunteer to put the blocks in the can, Jean began doing so, asking the child to assist. Eventually, either Jean or Jean and the child would clear the five blocks away. At the presentation of the can, David stopped the camera. David's camera and Jean's stop watch were begun again at the placement of the second can of blocks to the table.

For all block presentations, Jean made a deliberate attempt not to place similar blocks in homologous positions, like a bunch of three dissimilar blocks flanked right and left by two cylinders. Nor did she ever leave blocks resting in physical contact, nor did she ever leave any blocks standing up, i.e., resting on its more narrow face. Once the blocks were placed, Jean said, "What can you do with these blocks. Show me what you can do." If the child exhibited long pauses, Jean would say, "What else can you do, show me what else you can do."
Materials

Each child was presented six different clusters of five blocks each. Each cluster contained at least two identical blocks but never more than three identical blocks. All blocks within a given cluster were the same color, but color always changed from one cluster presentation to the next. The clusters were composed of a systematic distribution of the following four shapes all cut from the same one and one half inch walnut board (see Figure 1); a one and one half inch diameter cylinder, a two and one half inch diameter circle, a one and one half inch cube, and a two and one half inch square. All blocks were one and one half inches tall.

![Figure 1. The four shapes used to create the five block clusters; cylinder (y), circle (C), cube (u), and square (S).](image)

These four shapes were counterbalanced according to the design shown in Table 2. For all children, the first cluster presented was red, the second yellow, the third green, the fourth yellow, the fifth red, and the sixth green. The color variation was used to create enough novelty at the presentation of a new cluster to sustain interest. Table 2 shows the four different series created from the four shapes seen in Figure 1.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Series A</th>
<th>Series B</th>
<th>Series C</th>
<th>Series D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 red</td>
<td>yyyuu</td>
<td>uuuyy</td>
<td>CCCuu</td>
<td>SSSyy</td>
</tr>
<tr>
<td>2 yellow</td>
<td>Succi</td>
<td>CyySS</td>
<td>Suuyy</td>
<td>Cyuuu</td>
</tr>
<tr>
<td>3 green</td>
<td>CySS</td>
<td>SsuCC</td>
<td>yyySS</td>
<td>uuuCC</td>
</tr>
<tr>
<td>4 yellow</td>
<td>SSSCy</td>
<td>CCCSu</td>
<td>SSSCC</td>
<td>CCCCSS</td>
</tr>
<tr>
<td>5 red</td>
<td>CySu</td>
<td>Succi</td>
<td>CCySu</td>
<td>Ssucy</td>
</tr>
<tr>
<td>6 green</td>
<td>Ssuuy</td>
<td>CCyyu</td>
<td>uuucy</td>
<td>yyySu</td>
</tr>
</tbody>
</table>

Table 2. Four series of cluster sets. The letters represent small cylinder (y), large circle (C), small cube (u), and large square (S).
These four series of cluster sets were then distributed among the five age groups. Since each age group contained seven boys and seven girls, the series distribution could not be even according to sex, nor could it be perfectly even within an age group. The following system was used: Age Group 1 received series A,B,C,D,A,B,C (for males); D,A,B,C,D,A,B,C (for females). Group 2 received B,C,D,A,B,C,D (for males); A,B,C,D,A,B,C (for females). Group 3 received C,D,A, and so forth.

Coding

Films were coded on a stop and reverse action Ektographic Kodak Super-8 projector. Each coder sat in front of a 2' by 3' screen with a remote switch in the left hand and a pencil in the right hand. The coder could single frame action by pressing a remote button for each frame advance whenever the subtlety of the action demanded closer inspection. Coding reliability was quality controlled throughout the year by two independent codings of every sixth film. Coding agreement varied somewhat according to the momentariness of the action. Movements with a clear terminus, like stacking (moves 12 and 14 in Appendix A), banging (moves 1 and 2), and placing one block next to another (moves 21 and 22) had high interrater reliability indices of .87 to .95. Interrater reliability was calculated as:

\[
1.00 - \frac{\text{number of disagreements}}{\text{number of disagreements} + \text{agreements}}
\]

Movements that were more momentary, like adjusting the position of one block on top of another (move 95 in Appendix A), rotating a block in midair for visual inspection (move 5), were less reliably coded with a range from .75 to .87 interrater reliability index. Those moves which had an index between .75 and .80 have been referenced in Appendix A with an asterisk. None of the interrater reliability scores were deemed sufficiently low to warrant a dismissal of that datum. Due to precision possible with the stop action and single frame advance projection system reliability coefficients were higher than in studies using coding procedures in vivo, and probably higher than in studies using the rather poor resolution power of video taping action. All of our children were shot with Kodak Ektachrome color film.

Notation System

The coders used a notation system consisting of thirty-eight short hand symbols that could be combined to record a complete action unit. These short hand symbols indicated movements like /up/ , /down/ , /slide/ ; block
relations like /on top/, /adjacent/, /spaced apart/; syntactic statements like /which is/, /such that/; and summary statements of a product like /stack/, /bridge/, /row/. A complete action unit was defined as beginning with the grasp or touch of a block and ending with a release or a contact of that block to another surface. For example in the action sequence (X)↑↓Y↑↓T: * X/X' we see two separate action units. The first action unit begins with grasping X designated as (X) and ends with the contact of X to Y designated ↓Y. The second action unit begins with the lift of X from Y designated Y↑ and ends with X being released on the table, designated T', such that X is adjacent to X' designated * X/X' (see Appendix A).

The most useful feature of our notation system is that it is generative. It can handle novel patterns of action that we had never seen previously. After all 210 films were coded using this thirty eight symbol alphabet, we then scanned all the protocols for unique action units. We discovered that there were 149 unique action units. These action units, themselves different combinations of the thirty-eight symbols, were each given a numerical code (see Appendix A). These numerical codes were key punched and used in the computer analysis.

Understand that a given action unit does not constrain the type of product produced by that action. For example, releasing one block on top of another may produce a two block stack or it may produce a five block arrangement with elegant symmetry. Or, placing one block adjacent to another might produce a simple two block horizontal alignment or the same action might produce a two by two matrix of blocks. Protocols were scanned a second time in order to extract all unique products. The productions were classified according to their type of transformation. The general transformational formula was: \( P_1 \rightarrow O \rightarrow P_2 \), that is, Product 1 is transformed via Operator \( O \) into Product 2. \( X \rightarrow S_1 \) means that block X was transformed to a stack by adding one block. The transformation \( S_1 \rightarrow X \) means that Stack 1 was transformed to a single block X by successively removing the top blocks in turn. The transformation \( S_A^e \rightarrow X \rightarrow S_A^e \) means that an exhausted set (5 blocks) in a Stack/Alignment arrangement was transformed to another \( S_A^e \), not by adding other blocks, but rather by repositioning one block in the original Stack/Alignment. (A Stack/Alignment contains blocks placed both on top and next to other blocks). In reviewing the code protocols for age Group 1, Group 3, and Group 5 we discovered 204 unique transformations. Some of these transformations occurred too infrequently to list separately. Appendix C presents the transformations and frequencies of those transformations with any appreciable occurrence.
As was suggested by a statistical consultant, we decided to analyze data from Groups 1, 3, and 5 first. Then, as a second stage of the research, Groups 2 and 4 would be analyzed as if this were a replication. Of course the replication in this case also would be a slight generalization of the original data, since Groups 2 and 4 fall in between Groups 1, 3, and 5. It has taken us a full year to code all five groups, and to do a computer analysis on Groups 1, 3, and 5. This report will not present data analysis on Groups 2 and 4. These protocols will be scanned for the action units and products during the year 1975-76. All subsequent discussion of data will come from the sample indicated in Table 3 below. We lost one female from Group 1 (N = 13), one male from Group 3 (N = 13) and had an additional female in Group 5 (N = 15).

<table>
<thead>
<tr>
<th>October</th>
<th>February</th>
<th>June</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>7-8 months</td>
<td>11-12 months</td>
<td>13</td>
</tr>
<tr>
<td>Group 3</td>
<td>15-16 months</td>
<td>19-20 months</td>
<td>13</td>
</tr>
<tr>
<td>Group 5</td>
<td>23-24 months</td>
<td>27-28 months</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 3. Sample of children used in the data analyses cited in this report.

Results and Discussion

The first set of results will refer to the predicted sequence of transformations mentioned in the first section of this report. These transformations are seen as an increase in the subject/object differentiation and an increase in the coordination of similarity with difference. In the predicted sequence emphasis was given to the qualitative similarity between two forms e.g. two cylinders placed together rather than two different forms. The data indicate that there was an increase across age in the attention children give to the identity of two blocks brought into coupling both in the action mode and in the placement mode. These data also substantiate the predicted order of occurrence of the various modes of coupling. Table 4 presents these data.
Taking Table 4 row by row we see that Bilateral Grasping (BilGrsp), two hands on the same block, begins high, drops abruptly, and thereafter stabilizes at 2 or 3 occurrences per child. The ratio of BilGrsp with Single Grasp (SngGrsp) will be presented in Table 5, as will all such ratios. Bilaterally banging two identical blocks to the midline (BiBaSame) begins low and peaks during the early testing of Group 3, around 16 months, and then declines. Stacking one block on top of an identical block (StkSame) increases almost monotonically from the youngest to the oldest child. Aligning one block next to another identical block (AlgSame) increases monotonically with the first appreciable frequencies occurring around 27 or 28 months (Feb, Group 5). Spacing two identical blocks apart (SpcSame) does not occur with even marginal significance until Group 5.

In order to interpret the developmental level of these response categories several assumptions are necessary. One, a response category showing a decrease is in an older phase of development than one showing an increase and then a decrease. Two, a response category showing an increase and then a decrease is in an older phase of development than one showing only an increase. The rationale for these assumptions results from accepting that development along any dimension, if measured along a wide enough age span, would show an initial increase of occurrence (young phase) followed by a subsequent decrease (older phase), as other, more sophisticated response forms required the child's attention. Figure 2 expresses this rationale pictorially.

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th></th>
<th></th>
<th>Group 3</th>
<th></th>
<th></th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oct</td>
<td>Feb</td>
<td>Jun</td>
<td>Oct</td>
<td>Feb</td>
<td>Jun</td>
<td>Oct</td>
</tr>
<tr>
<td>1.</td>
<td>BilGrsp</td>
<td>6.4</td>
<td>2.4</td>
<td>1.8</td>
<td>1.4</td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>SngGrsp</td>
<td>87.0</td>
<td>105.0</td>
<td>78.0</td>
<td>75.0</td>
<td>98.0</td>
<td>81.0</td>
</tr>
<tr>
<td>2.</td>
<td>BiBaSame</td>
<td>0.7</td>
<td>1.6</td>
<td>1.6</td>
<td>2.6</td>
<td>2.1</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>BiBaDiff</td>
<td>1.7</td>
<td>4.6</td>
<td>2.5</td>
<td>1.6</td>
<td>2.6</td>
<td>1.9</td>
</tr>
<tr>
<td>3.</td>
<td>StkSame</td>
<td>0.2</td>
<td>1.0</td>
<td>1.4</td>
<td>1.6</td>
<td>5.1</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>StkDiff</td>
<td>1.2</td>
<td>2.2</td>
<td>4.3</td>
<td>4.3</td>
<td>9.9</td>
<td>11.0</td>
</tr>
<tr>
<td>4.</td>
<td>AlgSame</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
<td>0.1</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>AlgDiff</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>5.</td>
<td>SpcSame</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>SpcDiff</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 4. Mean frequency for five coupling modes comparing expressions using identical blocks versus expressions using different blocks.
Using this hypothetical growth curve as a basic assumption, the developmental order of a set of curves can be determined by looking at three successive points to establish the shape of a particular curve. Three points ABC increasing monotonically indicate a growth dimension developmentally prior to a growth curve showing three points ABC with an initial rise followed by a fall, etc. In this way, the absolute frequencies are unimportant. Often a rather late occurring response, like building a bridge with three blocks, occurs with low frequency throughout the child's history, while an early occurring response, like stacking, occurs with a high frequency. Of course, if some response type does not occur at all in the first three or four age points, while another response does occur in those initial age points, then the former obviously develops after the latter. For example, in Table 4, row 3 (StkSame) and row 4 (AlgSame) both increase fairly monotonically, but row 4 does not even begin to occur until some 20 months after the first appreciable occurrence of row 3. In cases where a curve had initial zero frequencies, this curve was considered developmentally later than a non-zero cell curve, even though both show monotonic increases. The age point at which a growth curve peaks is another way of deciding which of two curves is in a more advanced stage. A growth curve with low frequencies throughout, but one that peaks at 15 months could be developmentally older than a high frequency curve that does not peak until 24 months.
In Figure 3 above, Curve A (declining) is the oldest curve, Curve B (peaking early) is the next oldest, Curve C (peaking late) is the third oldest, and Curve D (inclining) is the youngest curve. Throughout this report these three factors, curve shape, on-set of peak, and on-set of first appreciable frequency, were used to order curves developmentally. A frequency was considered appreciable if the cell showed at least a mean frequency of one response per child. Appendix B shows the mean frequency counts for action patterns plus the number of children per testing who exhibit each pattern. Appendix C shows the mean frequency count for productions plus the number of children per testing who constructed each production.

Table 4 was presented in order to discuss a developmental change in the mode by which children couple identical blocks. However, before these data can be taken as evidence that the child was attentive to identity, we must show some non-random use of identical blocks. This does not necessarily mean that the child placed identical blocks together more often than non-identical blocks. A child placing identical blocks together only one out of every third move could be expressing his awareness of identity. Inspection of Table 2 indicates that of the ten unique block pairs possible for each cluster of five, random coupling would put identical blocks together, a total of two out of every ten couplings per cluster. This would yield a two to
eight ratio (or .25) between identical blocks coupled vs. non-identical blocks coupled. A ratio of .25 was considered an index of random pairing. Table 5 does show an increasing ratio of the number of times a mode was expressed with identical blocks compared to that expression with different blocks. To simplify these trends the ratio was calculated as a group ratio, rather than presenting three different ratios for each age group. These ratios were calculated by dividing BilGrsp by SngGrsp, BiBaSame by BiBaDiff, etc., with testings within age groups collapsed. The ratios increase with age, which indicates that children are using identical blocks more. When the ratio exceeds 1.00, the children are using identical blocks more often than non-identical blocks.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grasping</td>
<td>.04</td>
<td>.02</td>
<td>.03</td>
</tr>
<tr>
<td>Bilateral Banging</td>
<td>.46</td>
<td>.94</td>
<td>1.50</td>
</tr>
<tr>
<td>Stacking</td>
<td>.32</td>
<td>.45</td>
<td>.52</td>
</tr>
<tr>
<td>Aligning</td>
<td>.16</td>
<td>.30</td>
<td>.76</td>
</tr>
<tr>
<td>Spacing</td>
<td>.16</td>
<td>.26</td>
<td>2.30</td>
</tr>
</tbody>
</table>

Table 5. The ratio of identical blocks to different blocks for five modes of coupling.

Note that in the first row, grasping, the ratio is between two handed grasping and one handed grasping of a single block. Therefore, this row is somewhat different from the other four which compare identical versus different blocks in block coupling. The fact that the two-handed ratio decreases slightly and then increases slightly caused us to inspect the actual move types which lead to these trends. It was discovered that two-handed actions in Group 1 were such things as bringing block to mouth (move 8), lowering block to table surface (move 9), lifting block for visual inspection (move 5 and 100); while two-handed actions in Group 3 and 5 were such things as lowering block to stack (moves 12 and 14), rotating block to stand on its more narrow edge (move 39), and two-handed grasping of the bottom block of a five block stack prior to toppling that stack (moves 78, 79, 80). Clearly, two-handed action in the older groups was engaged in situations where greater precision or balance was needed, while in the younger group two-handed action was less tied to specific needs for precision. Two-handed grasping as a somewhat automatic response decreases with age.
For all other modes of coupling, the ratio of same blocks to different blocks increases. This increase is particular noteworthy in Bilateral Banging and Spacing. If we combine the data of Table 4 and Table 5, the conclusions read as follows: Children engage in bilateral banging less and less with age, but when they do engage it in the older ages, they use identical blocks. Children engage in deliberate spacing more and more with age and when they do engage it, they use identical blocks. These conclusions come from fairly sparse data in regard to deliberate spacing, but the frequency counts are sufficient to merit some conviction about changes in bilateral banging. The trend suggested is that children are first intrigued with the continuity (bang together) versus discontinuity (draw apart) of the physical contact of the two blocks. Only later do they refine this contrast to one between superimposing identical forms and withdrawing identical forms. These age trends are to be expected, considering the ease of discerning physical contact and the more perceptual scanning required to match forms.

If the means seen in Table 4 are collapsed according to age of testing, Table 6 results. That is, the June testing of Group 1 is combined with the October testing of Group 3 since all of these children were between 15 and 16 months of age. The same procedure can be followed with Group 3 June and Group 5 October since all of these children were between 23 and 24 months of age. This procedure in effect smooths the growth curve somewhat, since practice effects are balanced, at least at two points on the curve. While we grant that this is a composite of both within subject and between subject averaging, this type of averaging is one way to take advantage of the staggered longitudinal/cross sectional design employed.

<table>
<thead>
<tr>
<th></th>
<th>7-8 months</th>
<th>11-12 months</th>
<th>15-16 months</th>
<th>19-20 months</th>
<th>23-24 months</th>
<th>27-28 months</th>
<th>31-32 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>BilGrsp</td>
<td>6.4</td>
<td>2.4</td>
<td>1.6</td>
<td>2.3</td>
<td>2.9</td>
<td>3.7</td>
<td>2.5</td>
</tr>
<tr>
<td>BiBaSame</td>
<td>.7</td>
<td>1.6</td>
<td>2.1</td>
<td>2.1</td>
<td>.8</td>
<td>.6</td>
<td>.5</td>
</tr>
<tr>
<td>StkSame</td>
<td>.2</td>
<td>1.0</td>
<td>1.5</td>
<td>5.1</td>
<td>8.3</td>
<td>9.3</td>
<td>9.3</td>
</tr>
<tr>
<td>AlgSame</td>
<td>.1</td>
<td>.0</td>
<td>.2</td>
<td>.4</td>
<td>.2</td>
<td>1.1</td>
<td>.7</td>
</tr>
<tr>
<td>SpcSame</td>
<td>.0</td>
<td>.1</td>
<td>.0</td>
<td>.1</td>
<td>.3</td>
<td>.4</td>
<td>.3</td>
</tr>
</tbody>
</table>

Table 6. Mean frequency for five coupling modes using identical blocks, practice effects collapsed at 15-16 months and at 23-24 months.

---

1 This procedure of averaging together Group 1 June with Group 3 October and Group 3 June with Group 5 October will be followed in all subsequent tables. The individual group means can be found in Appendices B and C.
The curves indicated in Table 6 show greater monotonicity in StkSame and in SpcSame than does Table 4. In summary, coupling seems to progress through these five stages. The first form occurs when the child experiences similar input between the two hands on a single block. The contrast between similarity (two hands filled) and difference (one hand filled the other vacant) is successive. The next stage of coupling is bilateral banging two blocks. Similarity is expressed by bringing the two impenetrable surfaces together; the difference is expressed by separating the two blocks. Here again the contrast between similarity and difference is successive, but with the added factor that the two hands remained filled during both types of contrast. It must be somewhat paradoxical to the infant holding "an object" to draw his hands apart and still have tactual input continue to both hands. In this sense of a continued tactual input to both hands mediates the physical separation. In this manner, similarity is coordinated with difference. As we saw in Table 5, however, the bilateral banging is not an attempt to relate block form until 23-24 months. This more refined mode of coupling comes after the child has had sufficient experience coupling and uncoupling physical substance.

The child later begins to release one block on another. In this act, two blocks remain coupled independent of the child's continued grasp. In this sense, position at a definite, tangible point has been expressed. The similarity expressed is one of two blocks being able to share the same position. The discontinuity, i.e., difference, is expressed by knocking over the stack or removing the top block. But just after the moment of releasing one block on top of another, the child has brought the continuity/discontinuity contrast into a more compressed temporal frame. The resting block is different from the foundation block in that it can be knocked off; but it is also similar to the foundation block because of the apparent wholeness established in making the stack. The two poles of this contrast are less successive than the now together/now apart poles in banging. In the static stack, the coming together and the drawing apart are both potential actions represented by the product standing. This shift from actual to potential is only possible when some product remains frozen in time, a product that can be reflected upon as either the consequence of making blocks continuous or the antecedent to making blocks discontinuous. The presence of the stack serves as an index of both actions, and makes it possible for the child to eventually coordinate mentally both actions at once. (See Piaget, 1970, for his discussion of "reflective abstraction.")
Aligning blocks together suggests the formation of two positions rather than the one position used in stacking. Given that position is a definite point on the table, the alignment of two blocks side by side is described as having two positions. In essence, the child has expressed discontinuity between coupled blocks by placing each in a different position. The continuity is also expressed in this arrangement via static physical contact. Here continuity and discontinuity each have a static index at the same time. Discontinuity has been dissociated somewhat from continuity, not in action, but in placement. The two poles of the similarity (continuity)/difference (discontinuity) contrast have been further atemporalized in the horizontal alignment.

Spacing two blocks apart represents further progress on dissociating discontinuity from continuity. An actual vacant interval is created between the coupled blocks. The child has made a product which expresses similarity (blocks near) as well as dissimilarity (not touching). This near, but not touching relation is an instance of expanding the differences across which similarity can be expressed. More precisely stated, the deliberate spacing indicates that the child can coordinate continuity across a more exaggerated form of discontinuity. This suggests that continuity has become somewhat more formal, and less a matter of blocks being physically flush. This same progression from physical continuity to formal continuity was seen in the strategies young children use to solve jigsaw puzzles (Forman, Laughlin, and Sweeney, 1971). This separation of formal relations from physical relations is the hallmark of the symbolization process (see Inhelder and Piaget, 1964, particularly their discussion of the transition from graphic collections to true classification). This is not to suggest that a deliberate spacing is a logical relation, as opposed to a spatial relation. However, it is the first type of spatial relation that uses non-substance, the vacant interval, as part of the production. The entry of vacant space in the child's productions is a giant step forward toward what Piaget terms the separation of form from content (Piaget, 1971, p. 152). And it makes possible the coordination of similarity across a wider range of dissimilarity.

These coupling modes seen in Table 6 were used to discuss the progressive atemporalization of the two poles in the similar/different contrast. Similar response modes can be used to describe the type of phenomenal space within which the child is probably working. The next section discusses these trends.
Position, Plane, and Interval in Space

It is deductively clear that a motionless child would not develop a coordinated view of space. Movement of self and movement of objects are fundamental to the development of spatial concepts (Poincare, 1946). Through movement the child discovers the discontinuities and continuities of space. A smooth forward motion of the hand is made discontinuous by contact with a solid surface like the table top. Objects can be made continuous one to the other by placing them in contact. Many of the explorations of objects by the child can be described as an interplay of continuity and discontinuity. The child less than one year old frequently takes a block in each hand, bangs them together, and draws them apart. The child slightly older stacks one block on top of another thereby creating continuity between two elements, only to explore its discontinuity by knocking the two blocks over. This exploration of continuity-discontinuity seems to pass through definite developmental stages. The stages are defined by the extension of the space within which the relations are made. The child can express continuity across blocks in four types of spatial extension: a homogeneous field, a horizontal plane, a particular position, and a momentary contact. Our films suggest that these types of spatial extension appear at different developmental stages in the inverted order mentioned.

Stage I is characterized by momentary contact of object to object. The moment of contact establishes a continuity between filled spaces (solid space) and a discontinuity in empty space (free movement). The withdrawal of contact establishes the reverse, a discontinuity between filled space and a continuity within empty space. The child bangs a block to the table, lifts, and bangs again. The child bangs one block onto another. However, objects do not yet "occupy" a position. Position implies that there exists a "there" prior to placement and somewhat independent of the presence of a tangible object. The highest concept of position is an empty space at rest and indexed by neighboring solid space. While banging one block onto a block held in the left hand expresses a definite figure-ground distinction, the moving target block in the left hand is not at rest nor indexed by neighboring solid space.

Stage II is characterized by a definite placement of one block to the surface of a resting second block followed by a release of the first. In stacking one block on top of another the child has "positioned" an object. Position implies more than momentary contact. Position requires that one object
be fixed in space by reference to some object other than itself. In the case of stacking X on top of Y, however, the place in space is not itself empty but is rather the full bodied block which serves as the foundation of the stack. In more advanced concepts of position, position is not dependent upon contact with a tangible object, but is a relation, e.g. the intercept of two Euclidean axes. In the Stage II concept of position, it is simply an expression of continuity, a physical continuity between two objects, not the placement of a single object at a point fixed in empty space.

By Stage III children are not only placing blocks on top of other blocks, but are also placing blocks on the table next to other blocks. Whereas children in Stage II will only reiterate placements at the same position (repeated stacking), the Stage III child will place one, and then place another at a new position (repeated aligning). By creating a continuous line of blocks the child constructs space extended in a horizontal plane. Until the child departs from stacking all five blocks at the same site we have no way of knowing whether the child conceives of space as containing a variety of possible positions. The concept of a plane is just that, the extension of positions within a two-dimensional limit. The child's construction of a horizontal alignment at a minimum expresses the multiple-positionness of space. This could be the beginning of the concept plane. But just as position in Stage II is embodied in the contact with a tangible object, so are the successive positions in Stage III. The reiteration of placements are each determined by physical contact with a previously placed block. Yet Stage III is an advance over Stage II in the fact that Stage II is a Shift Block, Stay Position reiteration while Stage III is a Shift Block, Shift Position reiteration.

In Stage IV the child not only shifts position, but he also creates a vacant interval between two adjacent blocks. The vacant interval is first created as a spatial gap in a three block bridge. Somewhat later in development two lone blocks are deliberately spaced and released with the child taking particular note of the interval he created. The position of block X is defined by a certain proximity to block Y, but in this case position is constructed mentally rather than constrained physically. In the Stage III alignment, block X is slid forward until it makes contact with block Y. In the Stage IV spaced alignment, block X is slid forward but the child himself stops its forward movement when a certain interval obtains between X and Y. This production is the beginning of the concept interval and is itself a synthesis of continuity across discontinuity.
The field of space within which the Stage IV child works is different from the field of space used by the Stage III child. In stage III the horizontal extension of space is inextricable from the physical contact of adjacent blocks. The horizontal field is not everywhere the same in terms of the possible placement of additional blocks. Placement is still constrained by physical contact and in that sense space is still heterogeneous. By Stage IV space becomes more dissociated from physical contact, even though it is still partially constrained by a rather narrow proximity factor, the interval. This interval is seldom larger than the width of the blocks, but interval is the first type of nonsubstantial extension. The fact that this space, the interval, is both created by the child and is nonsubstantial indicates that space is becoming more of a homogeneous field, rather than an extension of physical adjacencies.

In summary, space as constructed in Stage I is simply a space which limits physical movement. Objects make contact, but do not have position. Objects have substance, can be grasped, can be separated from the table, do not penetrate other solid surfaces, but still do not have position given that position is a resting relation between one object and other objects or surfaces. In stage II space has acquired position, but this position is embodied as a resting block. Position, in this stage, is not any point so designated in a homogeneous plane, but rather position is inextricable with a tangible object, i.e. the foundation block is manifest position. In Stage III space becomes more of a supporting surface or plane, to which several objects can be related simultaneously. In placing one block next to another the child is, in essence, establishing a dual relation: block X to block Y while at the same time block Y to the table. The child understands that the table will support the X to Y relation. However, in stacking, the X to Y relation may be just the reiteration of the scheme of placing objects to a single point. Stage IV is significant in that space has become both continuous and discontinuous. The filled space of the blocks is "coordinated" with the empty space of the interval between blocks. In a spontaneously created bridge of three blocks the vacant space under the bridging block is purely relational, not tangible. We have seen younger children, who accidentally create vacant spaces, search for the hole under the bridge once the bridge was knocked down. In one particular film it

1The phrase "space as constructed" should be noted. Throughout this discussion, we are referring to space as the result of an active organization of object relations, not a result of visual detection of physical features. To avoid the common tendency to oversimplify this distinction, read Piaget's discussion of representational space (Piaget, 1948).
is obvious that the child thought that the hole through which she stuck her finger was there somewhere among the blocks in disarray. The Stage IV child makes and unmakes vacant spaces deliberately. We concluded he does understand that vacant space is the result of relating blocks, rather than being a part of a single, napkin ring type block.

<table>
<thead>
<tr>
<th></th>
<th>7-8</th>
<th>11-12</th>
<th>15-16</th>
<th>19-20</th>
<th>23-24</th>
<th>27-28</th>
<th>31-32</th>
</tr>
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<tr>
<td>Physical Resistance</td>
<td>42.5</td>
<td>29.8</td>
<td>31.0</td>
<td>23.9</td>
<td>14.9</td>
<td>9.7</td>
<td>6.7</td>
</tr>
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<td>3.4</td>
<td>6.6</td>
<td>16.2</td>
<td>27.0</td>
<td>29.1</td>
<td>39.9</td>
</tr>
<tr>
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<td>.7</td>
<td>.4</td>
<td>1.5</td>
<td>.8</td>
<td>4.5</td>
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</tr>
<tr>
<td>IV Multiple Positions, Homogeneous</td>
<td>.0</td>
<td>.3</td>
<td>.0</td>
<td>.3</td>
<td>.6</td>
<td>.5</td>
<td>.9</td>
</tr>
</tbody>
</table>

Table 7. Mean frequency of four response patterns which suggests that phenomenal space progresses from point, to plane, to interval.

The responses which exemplify Stage I and tabulated in Table 7 were: mouthing a block (move 8), banging blocks together (moves 1 and 2), banging blocks to the table (move 9), touching blocks to other blocks without release (moves 11 and 13), touching a block to any surface on the body (move 7), touching block to hand (moves 92.5 and 93.5). Stage II responses included any release of one block on top of another (moves 12, 14, and others). Stage III responses included either sliding one block next to another and release (moves 21 and 22), or sliding two blocks together, one in each hand, and releasing both (moves 3.5 and 4.5). Stage IV was any deliberate spacing of two blocks (moves 35-38.5), either by sliding and releasing one near another, or by sliding two near each other releasing each at the same time, or by sliding two apart as was often done in the process of creating a three block bridge. The developmental trends are clear. Stage I responses decreases with age. Stage II responses increase across age, indicating that this stage comes later than Stage I. Stage III increases with age, but does not begin to show any consistent appearances until 19-20 months. Stage IV responses increase with age, but do not begin to show any appreciable appearances until 31-32 months.
From Repetition to Reiteration

Reiteration is important in both number and measurement. The number two represents one element reiterated beyond the class one. The number three represents one element reiterated beyond the class two. In number and in measurement, each subsequent cardinal value subsumes all preceding classes. The basic operation of reiteration requires that a new element be added to the preceding elements. Repetition, however, need not result in an accumulation. The child may repeat an action which does not result in some growing product. A class of action elements does not show accretion, even though the basic rule /Do Again/ is activated.

In block play, the child engages several versions of the /Do Again/ rule. These versions fall into a developmental sequence according to the number of new rules added to the /Do Again/ rule. These stages can be viewed as precursors to the more sophisticated form of reiteration used in number and measurement.

When the child lifts a block, places it to the table, lifts it and places it to the table again, we can describe the rules of this sequence as: /Place/Do Again/. When the child lifts one block, touches it to resting block, lifts, and contacts the resting block again, this sequence is described as: /Place/There/Do Again/. A child stacking several blocks at the same site is described: Place/There/Release/Do Again/. A child who continues stacking until all five blocks are used is described: Place/There/Release/Do Again/Exhaust/. A child who makes a stack of two or three and then begins a new stack at a second site leaving the first stack intact is described as: Place/There/Release/Shift Position/Do Again. The /Shift Position/ rule refers to that move made just after the last block has been released on the first stack. These stages seem to form a developmental sequence, as summarized below.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Rule Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>/Place/Do Again/</td>
</tr>
<tr>
<td>II</td>
<td>/Place/There/Do Again/</td>
</tr>
<tr>
<td>III</td>
<td>/Place/There/Release/Do Again/</td>
</tr>
<tr>
<td>IV</td>
<td>/Place/There/Release/Do Again/Exhaust</td>
</tr>
<tr>
<td>V</td>
<td>/Place/There/Release/Shift Position/Do Again</td>
</tr>
</tbody>
</table>

Stages I and II describe a /Do Again/ rule using the same block as subject of the /Do Again/ rule. Stages III, IV, and V describe a /Do Again/ rule using a different block as the subject of the /Do Again/ rule. Stage IV adds the /Exhaust/ rule which indicates the child's sense of completeness. Stage V not only describes a new block as subject of the /Do Again/ rule, but also a new position as the object of the /Do Again/ rule. Table 8 presents the mean number of responses which exemplify each of the five stages. (These data come from Appendix C).
Table 8. Mean frequency of response patterns exemplifying five stages in an expression of the /Do Again/ rule.

Stages I and II are on the decrease, while Stages III, IV, and V are on the increase making the former two older than the latter three. The developmental age of Stages I and II is more difficult to differentiate. While Stage I and Stage II both show an overall trend to decrease, Stage I shows an increase in one cell, 15-16 months. Since there is no increase from 7-8 to 11-12 we might interpret the increase from 11-12 to 15-16 as spurious. The fact that Stage I starts higher and reaches zero occurrence before Stage II reaches zero suggests that Stage I is slightly older than Stage II. The most conservative statement is that these two stages are concurrent in their developmental age, and both are older than all following stages. Judging by the on-set of an appreciable frequency, Stages III, IV, and V are increasingly younger curves in that order. The age point at which frequencies become appreciable are, as indicated by an asterisk, 11-12 months for Stage II, 19-20 months for Stage IV, and 27-28 months for Stage V.

When a five or six year old child conserves number by making a one-to-one correspondence between two rows of elements, this one-to-one correspondence contains several component rules, some of which are contained in the versions of the /Do Again/ rules mentioned above. When the child places element $A'_2$ underneath element $A_2$ in a top row of counters, after placing $A'_1$ underneath $A_1$, he has in fact followed the rule /Place/There/Release/Shift Position/Do Again/. If he makes the mistake of placing both $A'_2$ and $A'_1$ underneath $A_1$, he has used the rule /Place/There/Release/Do Again/, i.e., a Stage 4. rule sequence. If the five or six year old does not reiterate the one-to-one correspondence for each element in the top row $A_1 \ldots A_n$, he has failed to exhaust the series. If the child does not release $A'_2$ at the time it makes contact with $A_2$, the correspondence will not attain equivalent cardinal values between the two sets. It can then be reasoned that the apparently
simple motoric ability to make a one-to-one correspondence between two equal sets of objects has component rules which enter the repertoire of competencies at different developmental stages, at least entry as seen in spontaneous, uninstructed block play.

The child learns to repeat an action at no particular site (Stage I) before he learns to repeat an action at a given site (Stage II). The child learns to repeat an action at a given site with the same block (Stage II, repetition) before he learns to repeat that action at the same site with a different block (Stage III, reiteration). The child learns to exhaust the reiteration of adding blocks to a given site (Stage IV) somewhat before he reiterates addition at a new site (Stage V). This last stage is the first example of duplicating a product. It is quite possible that by Stage V, the child is creating a second product. This rule is much more sophisticated than Stages III and IV, which call for the reiteration of an action. The developmental difference between Stage V and the two previous stages seems to represent a shift toward reiterating products. Second refers to product; twice refers to actions. As we shall see, this developmental shift from action to production pervades many of the dimensions of development that we will discuss. In summary, these five stages suggest that as position and release are added to the competency base, the /Do Again/ rule is transformed across age from Repeat Action to Reiterate Element to Duplicate Product.

Reiteration of Substraction

The converse of addition is subtraction. The reverse of adding one block to a product is to remove a block from that product. In the world of tangible products the cumulated structure, the stack or alignment, is not necessarily seen by the child as several elements at rest in contact with each other. The child may just as easily treat the cumulated structure as a solid mass, ignoring the potential discontinuity of the elements. However, when we see the child carefully grasp the top block and with pincher grip remove that block, it is somewhat more obvious that the child has "conserved" the separateness of the elements in the total structure. Yet a younger child may do no more than knock the stack over, or grasp the entire structure in the middle with a lifting motion that indicates the stack was viewed as a solid structure. It could be that substraction lags behind the development of addition.

The developmental lag of substraction is not a matter of the physical necessity for addition to precede substraction. Granted, before the child can remove blocks from a stack or an alignment, he has to first add the
blocks to create the product. However, the films suggest that the on-set of the /Take Away/ rule comes some months after the on-set of the /Add To/ rule. Furthermore, single subtraction precedes reiterated subtraction. This developmental difference appears, even when the two types of subtraction are balanced according to the opportunity for each to occur. In like manner, the double move of subtracting a block from one product while at the same time adding it to a second product, passes through two stages. The child will subtract one block from Stack A and add that same block to Stack B, then shift to another response type, rather than reiterate this subtraction/addition. The reiteration of subtraction/addition occurs later. Later the child will take one block from a four block stack, place it on a singleton elsewhere, then continue removing the remaining blocks on the initial stack, while the second stack continues to grow. The most advanced stage of the reiterated subtraction/addition is to see a child take a five block stack and by successive subtraction/addition create a second five block stack the perfect inverse of the first. Interestingly enough, children will pass through a stage in which four blocks are removed from a five block stack with each block placed directly to the table, not to the base of a growing stack. Later children engage the reiterated subtraction/addition rule. Apparently reiterated subtraction alone precedes the reiterated subtraction/addition. This developmental order might be another example of the child's growing competence to process dual relations (see Pasqual-Leon and Smith, 1969). Moving blocks /to/ or moving blocks /from/ preceeds moving blocks /from and to/ simultaneously. We are reasoning that moving a block /from/ Stack 1 /to/ the table is not an application of the /from and to/ rule, since the movement /to/ is not directed to a particular site, i.e., another block.

The growth from single subtraction to single subtraction/addition to reiterated subtraction to reiterated subtraction/addition is presented in Table 9. The first row gives the mean number of products present which allow for single subtraction to occur. The fourth row gives the mean number of products present which allow for reiterated subtraction to occur. The last four rows compute the ratio between the number of each category type versus the number of opportunities for that category to occur. This ratio thereby gives a picture of what the child did, compared to what he could have done.
Table 9. From Single Subtraction (SinSub) to Reiterated Subtraction/Addition (ReiSub/Add). Mean frequency and ratio of actual to available.

Table 9 suggests that subtraction rules are elaborated in the following developmental sequence. (These data come from Appendix C.) First the /Take Away/ rule develops (SinSub), occurring with an appreciable frequency at 15-16 months. Then the rule is expanded to /Take Away/Add To/ beginning at 23-24 months (SinSub/Add). In SinSub/Add, the child is removing a block from one product while at the same time placing that block to a second product. The developmental age of ReiSub appears younger than SinSub/Add considering that the initial zero cells persist longer in ReiSub. However, ReiSub and SinSub/Add reach an appreciable frequency at the same age, 31-32 months. These two curves are most probably developmental contemporaries, but there are reasons to consider SinSub/Add older than ReiSub. The /Take Away/Add To/ rule might be older than the /Take Away/Release/Do Again/ expansion. ReiSub/Add, which involves the most complicated expansion /Take Away/Add To/Release/Do Again/ is only beginning to occur by 32 months.

Two interesting facts are indicated by these data. For one, the expansion of the kernel /Take Away/ to the form /Take Away/Add To/ is easier than its expansion to /Take Away/Release/Do Again/. This suggests that, at least in the domain of physical movement, a dual and reciprocal relation of leaving A while approaching B is simpler than reiterating a unidirectional relation. This could be because of the difficulty of maintaining the goal set in Rei...
iterated Subtraction after the /Release/ rule has been activated. For another interesting fact, the Reiterated Subtraction occurs in slight advance of the Reiterated Subtraction/Addition. This means that the expansion of /Take Away/ to the form /Take Away/Release/Do Again/ is easier than the expansion /Take Away/Add To/Release/Do Again/. The difference in these two expansions rests in the addition of the /Add To/ rule between the /Take Away/ and /Release/ rules. Here we possibly see an even greater heirarchicalization of thought. In ReiSub/Add, the child is maintaining his set to remove elements across several moves, but he is also engaging, at the same time, a goal statement to construct a new product. The younger child, who only exhibits the ReiSub, sets for himself and maintains the single goal statement of removing blocks.

Notice that it is only by maintaining both goal statements, remove and build, that a new product can be constructed that is the perfect inverse of the first product. By maintaining both goal statements, the child can actually construct a physical representation of reciprocity. The five block tower EDCBA is the reciprocal of the original tower ABCDE. Yet this static representation of reciprocal order is preceded by a dynamic form of reciprocity, that is, movement toward B is reciprocal to movement from A. The activation of the dynamic form of reciprocity does not lead automatically to the construction of the static representation of reciprocity. The rule /Release/ and the rule /Do Again/ must be added to the /Take Away/Add To/ statement before the dynamic expression can be transformed into a static representation of this dynamic relation. We are developing the thesis here that children create products which are the static representations of action. The action is, so to speak, frozen or accumulated in the production. This allows the child to study the relations more leisurely and with less load on memory. Action is compressed into a production and is thereby transformed from a temporal expression to an atemporal expression. One might reason that instead of action becoming internalized as Piaget mentions (Piaget, 1970), action is externalized. More precisely, in order for the internalization of action to occur, the child develops means to make static representations of those actions. The external product, then, becomes an index of all those actions that were successively used to create that production. The opportunity to construct products with elements that can be reiterated might be quite important for the early growth of logic and the atemporalization of actions into more nearly simultaneous relations (see Forman, 1973).
From Symmetrical Action to Symmetrical Production

Symmetry has been used by biologists as a means to distinguish living from non-living systems (Gardner, 1964), by mathematicians as the foundation of geometry and algebra (Weyl, 1952), and by artists as a point and counterpoint for the aesthetically pleasant (Birkhoff, 1933). Froebel and Montessori alike noticed that young children spontaneously produce symmetrical constructions of elegant order. Both of these educators attributed great significance to this spontaneous expression of balance. Froebel suggests that the same laws that operated to produce any well functioning biological system are manifestly observable in the young child's work with blocks and other small objects. In this section an attempt will be made to abstract what some of these laws might be.

We have seen various stages of symmetrical action and symmetrical constructions which indicate that the child is exploring several types of contrasts. The exploration between the poles of each set of contrast leads the child further and further in his ability to construct elegant arrangements. The following is a list of contrasts that will be mentioned as this discussion of symmetry progresses:

- filled space vs. vacant space
- continuity vs. discontinuity
- content vs. form
- production vs. action
- position vs. direction
- static vs. dynamic
- simultaneous vs. successive
- two vs. twice
- contained vs. container
- here vs. there

One of the child's first method of creating symmetry is to clap his hands together. The right palm crashes into the left palm. The effect on the left of the midline is equal to the effect on the right of the midline. The child may simultaneously slap hands to the table top, palms down. Here the effect right and left is the same, but the site of one is different from the site of the other, rather than two inextricable sites as in clapping. These early forms of symmetry are dependent on the timing of action and are not dissociated from the body itself.

A slightly later symmetrical action develops at the time the child begins lifting objects. Since so many of the toys and small objects given to the twentieth century child are themselves symmetrically designed, the child has ample opportunity to grasp an object on right and left and receive
symmetrical tactual input. In fact, symmetrical protrusions make this sort of symmetrical input more likely, given the child's need to fill both hands with something. In passing the object from one hand to another, the child automatically experiences a contrast between symmetrical and asymmetrical inputs. The input is symmetrical when the object is held in both hands, asymmetrical when held in only one hand or the other. It is quite likely that the asymmetry of holding the object in one hand is eventually associated with the counterpoint of recently holding that same object in both hands. The symmetry/asymmetry at this stage is less a matter of object form, and more a matter of object substance. Two hands filled at the same time by some substance is contrasted to one hand filled and the other hand empty. These are early experiences with the difference between two once and one twice, i.e., two hands on one object at once versus one object here once and there once (twice). Symmetry vs. asymmetry is an interplay between time and space, between action and object.

Our films show that the seven and eight month olds spend most of their explorations on a single block. When they do lift a second block with the unoccupied hand, this block is generally held in suspended animation. While the right hand is banging its block to the table or taking its block to the mouth, the left hand merely grips its block firmly. The only time that the left hand moves its block is when the left hand mirrors the right. This most often takes the form of banging the two blocks together at the midline, or taking bot' blocks to the mouth at the same time. It is not until 12 or 16 months that our films begin to show the left hand engage in an action scheme that is topographically and purposively subordinate to the right hand's action scheme. But the child's initial form of two handed action is by and large symmetrical. The asymmetry seems to evolve from the symmetry.

We have repeatedly seen transitional stages, where, for example, the child will attempt to place two small blocks on a resting small block, at the same time. These schemes are not the result of inattentiveness, because the children perform this scheme with great concentration and deliberate movements. They even release the two top blocks in apparent anticipation that both will be accommodated by the small foundation block. The symmetrical position of two blocks side by side at the same time evidently competes with the need to make the two placements successive, first one, then the other. Again, the symmetry of two things at the same place at the same time is developmentally antecedent to two things at the same place (same stack) at different times. The child must learn to dissociate time from space.
The symmetrical action of banging of two blocks together occurs after bilateral grasping of one block, as was seen in Table 4. At first the child is intrigued with the paradox. There is no target object out there which to bang, yet the child can move two active blocks toward a common point and meet with physical resistance. The paradox is the fact that each block is both an agent and a recipient of action. Block A imparts force to block B, but is also the recipient of force from block B. Children repeatedly bang blocks together as if captivated by some contradiction.

So far the forms of symmetry discussed have been dependent on action. As stated earlier, in order for the child to more closely reflect on the relations he creates in action, he must create some static representation of these relations, which become that static embodiment of dynamic relations. The static production most derivative of symmetrical action is simply two blocks brought together side by side on the table top and released. The release of the blocks together is no less than the frozen time frame of the bilateral bang at the time of impact. As will be pointed out in Table 10, this bilateral release of two blocks side by side does not occur very often in any age range we studied. Apparently by the time the children were at the stage of placing blocks side by side, lateral dominance had been so well established that pairs of adjacent blocks were created by one hand sliding a single block next to an isolated, resting block; rather than two hands bringing in one block each and releasing together.

Another type of symmetrical production, more frequent in its expression, is a response we labeled bisection symmetry. Shortly after the child develops sufficient hand-eye coordination to create a stable two block stack, he begins to improve upon the vertical alignment of blocks. He begins to notice whether or not blocks have their vertical sides physically flush. Later he begins to notice a type of balance more aesthetic than functional. He notices the amount of vacant space on either side of a small block resting on a larger block. At this point, the child will make deliberate attempts to center the small top block in the middle of the larger foundation block. He, in effect, is bisecting the foundation block.

![Figure 4. Bisection Symmetry](image)
The rules required to produce bisection symmetry call for some interesting additions in competencies. The /Place/There/Release/ sequence is expanded to /Place/There/Release/Center/Release/. More advanced children can abbreviate the expansion to /Place/There/Center/Release/. The /Center/ rule itself is comprised of the child making minor left to right adjustments until the interval $C_1D_1$ is equal to the interval $D_2C_2$. Bisection symmetry is one of the first products in which the child constructs an equivalence. The duplication of a product, mentioned in the preceeding pages, comes later in development. The construction of equivalence in bisection symmetry is a static representation of equivalence, but the elements that are equivalent are non-substantial. The equivalent elements are vacant spaces, $C_1D_1 = D_2C_2$. The child can express this form of equivalence without using the /Release/ or the /Do Again/ rule. This fact no doubt explains why bisection symmetry can occur so early in development.

In bisection symmetry, the product is expressed by the movement of a single object at a single time. We see here that the symmetrical construction has been dissociated from the action of the two hands working symmetrically. Bisection symmetry is an advance over bilateral banging of two objects and over bilateral placement of two objects because the form of the action is less isomorphic with the form of the product. That is, the child has dissociated the production of the movements from the construction of the product. Another mark of the greater sophistication of bisection symmetry over earlier forms of symmetry is that the symmetrical halves of the equation are more like relations than like discrete objects. The axis of symmetry in the bisected construction is more imaginary, or more the result of a mental construction, than is the axis of symmetry in bilateral banging and bilateral placements. The axis in banging and placing is a physical obstruction. The axis in bisection is purely a matter of how the child superimposes order onto otherwise homogeneous surfaces. Another way of stating these relations is to say that the axis of symmetry in bisection has a fixed position. It is the midline of the foundation block. In earlier forms of symmetry, the axis of symmetry is one and the same as the placement of the two blocks, either in midair or released anywhere on the table. Of course it could be that the body midline is a positional frame of reference to these earlier forms of symmetry, but if position is defined as a frame of reference external to the body, bilateral banging and bilateral placements have no fixed axis of symmetry. In bisection symmetry, the axis is more objectified from the body. Once again we see how position is added to a goal statement and in consequence allows for more advanced schemes to develop.
If the static form of bisection symmetry is less isomorphic to the action used to create it, does that imply that bisection symmetry is less a representation of dynamic qualities? Probably not. Bisection symmetry most likely evolves from the need to physically balance vertical structures against the tangible pull of gravity. Somewhat prior to the first creation of bisection symmetry children can be seen making steady a poorly aligned stack. Centering a small block in the middle of a larger foundation block might well be a case of overgeneralizing this rule. New rules in many domains of the child's development are overgeneralized (see E. Clark, 1973). It is not implausible that the child centers the small block in Figure 4 because he anticipates its fall, or because the activation of the /Place/There/Release/ rules in any situation, through the very momentum of being activated, calls forth the /Center/ rule, usually appropriate with blocks resting more precariously. This suggests that the /Center/ rule would eventually be suppressed, once the child made the basic distinction between small block on large vs. large block on small. However, it seems that the overgeneralization of the /Center/ rule creates a product that the child notices retrospective to making it. The balance contained in bisection symmetry shifts from being elicited by anticipation of physical consequences (falling) to being elicited by formal elegance (order). It is in cases like these that causality, a type of physical knowledge, and equivalence, a type of logico-mathematical knowledge, interact to expand cognitive development. The logico-mathematical relations take on an appeal independent of their physical consequences. They are embellished, and then no doubt lead to the discovery of more complicated forms of physical knowledge. Piaget and Garcia (1974) have written an interesting and difficult book about this interchange between causality and logic. The contingent makes a shift to the purely relational. This is what I have been describing heretofore as the atemporalization of action schemes through static representations, and in part what Piaget calls the internalization of action. Ontologically, bisection symmetry is most likely a static representation of dynamic qualities, even though the symmetrical structure eventually acquires autonomy from the dynamics. Long after children can discern the stability of small on large, they continue to center the small on the large.

A final form of symmetry involves, at a minimum, the linear arrangement of at least three blocks. A middle block of form A is flanked right and left by two blocks of form B. The production of this arrangement most commonly begins with the placement of B next to A. At this point the child usually
notices the asymmetry of the BA construction, searches for another block B, identical to B, and then places B' in the space next to A on the opposite side of B (see Figure 5). Form B to the left of A is reiterated with form B' to the right of A. This type of symmetry was therefore labeled reiterative symmetry.

**Figure 5. Reiterative Symmetry**

For convenience, call the vacant space to the right of A space b', and the space to the left of A space b. The blocks themselves occupying these spaces carry the corresponding capital letter designation. One can see that reiterative symmetry involves a special type of position, a special type of space filling. Position b' exists in the child's mind not just as a vacant space next to A, but as an unfilled form as well. This type of space filling is more than the type elicited by the presence of an empty container. Children much younger than the on-set age of reiterative symmetry are compelled to fill an empty cup. But in filling an empty cup, the vacancy is set by the tangible walls of the container. The vacancy in reiterative symmetry is purely the result of constructing the relation of B and A. The vacancy constructed by the child just prior to placing B' is even more advanced than the space filling he uses in placing B to the left of A. In placing B, the emptiness is suggested by the right angle made by the table and the vertical face of A. However, in the process of selecting and placing B', the child is operating under additional constraints. The right angle made by the right vertical wall of A must be filled with the same form as B. This suggests that the child has transcended the immediate constraints of contiguous space, such as matching two triangles together, and is engaging in the transposition of the BA relation. The constraints governing behavior are less physical than they are the result of constructing order among physical materials.

The difference between bisection symmetry and reiterative symmetry is in the addition of the /Do Again/ rule and the /Release/ rule. The basic kernel of making a two block alignment, /Place/Slide Next/Release/, is expanded considerably to /Place/Slide Next/Release/Choose Same/Place Opposite/Slide Next/Release/. The two new rules involve the relational concepts of same and opposite. In this construction we once again see the budding coordination of
similarity with difference. The similarity is shape; the difference is position. However, the position is not just any position, no more than the fact that some element considered different from another is ever randomly different. Difference, by definition, involves a dimension of similarity. For example, boots are different from shoes, but to say that ravens are different from writing desks, is to border on the realms of fantasy. The dimension of similarity operative in reiterative symmetry is the fact that both B and B' are termini of the linear arrangement. Two hands moving onto BAB' would respectively contact B and B' at the same time. Block B' is similar, yet different from block B. Space b' is privileged in reference to and analogous with space b.

Analogy, in its more abstract form, implies a relation between two discrete elements of comparison which are otherwise dissimilar. Using this definition makes the placement of B' at space b' more like an analogy than placing some form B' on top of and superimposed on an identical form B. Children who clearly have progressed into the stacking scheme will often rotate a cube on top of another cube until both cubes form four continuous vertical surfaces. This is a type of perceptual matching. However, in reiterative symmetry, the perceptual match is dislocated from the standard form, that is B' is not physically superimposed on the face of form B. The expression of similarity is non-contiguous, it is transposed. This dissociation of perceptual matching from spatial contiguity suggests the beginning of a continued dissociation that ultimately leads to the dissociation of relations from content. Reiterative symmetry is an expression somewhere between contiguity and pure relation, since reiterative symmetry does call for a particular shape and is thereby not content free.

In the dissociation of formal relations from content, as for example the relation larger than, the child must learn that a particular object is not of itself larger. A two inch cube is larger than a one inch cube, but is smaller than a three inch cube. At first the child tries to associate the concept larger to a particular block, that is, he has not dissociated the formal from the content. In converse manner, the formal is dissociated from the content when the child understands that both a two inch cube and a three inch cube can be larger depending on the comparison object. Or, both the green circle or the red circle can be the third of three circles, depending on the order of counting. Formal relations are separated from particular objects when the child begins to understand the equifinality of different actions (see Piaget, 1970).
This type of dissociation of the formal from content can be practiced as the child makes variations on symmetrical structures. If B is placed next to A's left, then symmetry is restored by placing B' to the right of A. However, if A is placed next to B's left, then A' is needed to restore symmetry. This type of equifinality is what distinguishes perceptual matching, a type of physical knowledge, from reiterative symmetry, a type of logico-mathematical relations have origins in physical consequences, like building a three block bridge requires the reiteration of similar forms to physically support the lintel; but we are hard pressed to see any physical reinforcement of constructing a single plane BAB' reiterative symmetrical structure. Yet children in preschools across the nation spontaneously produce this form of symmetry (Woodcock, 1941). Somewhere in development the contingent is dissociated from the logical, and the formal becomes independent of content.

As a way of summarizing and substantiating this discussion on symmetrical actions and productions, Table 10 presents the age trends for the various response schemes mentioned.

<table>
<thead>
<tr>
<th></th>
<th>7-8 months</th>
<th>11-12 months</th>
<th>15-16 months</th>
<th>19-20 months</th>
<th>23-24 months</th>
<th>27-28 months</th>
<th>31-32 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilateral Grasp</td>
<td>6.4</td>
<td>2.4</td>
<td>1.6</td>
<td>2.3</td>
<td>2.9</td>
<td>3.7</td>
<td>2.5</td>
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<tr>
<td>Bilateral Bang</td>
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<td>6.2</td>
<td>4.2</td>
<td>4.7</td>
<td>2.0</td>
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<td>.5</td>
<td>.1</td>
<td>.7</td>
<td>.2</td>
<td>1.3</td>
<td>.9</td>
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<tr>
<td>Bisection Symmetry</td>
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<td>.2</td>
<td>.2</td>
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<tr>
<td>Reiterative Symmetry</td>
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<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.1</td>
<td>.3</td>
</tr>
</tbody>
</table>

Table 10. From Symmetrical Action to Symmetrical Production. Mean frequency for each age range.

As was discussed in reference to Table 5, the ratio data of identical couplings to different blocks coupled, Bilateral Grasping in the older months was performed less as a general mode of handling a block and more as a special grasp at times when greater precision was needed. In that same discussion of Table 5, it was also said that even though Bilateral Banging decreased with age, the use of identical blocks to execute Bilateral Banging increased. This suggests that the symmetry in the younger months was in terms of action only, but that the equation A = A shifts to include block form as well as action in the older months.

The coupling data in Table 6 included both two handed horizontal alignments and one handed horizontal alignments. The data in the above table, Table 10, includes under Bilateral Place, only two handed horizontal alignments.
any two blocks. The ratio of placing identical blocks adjacent divided by non-identical bilateral placements is shown below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Same/Diff</td>
<td>months</td>
<td>months</td>
<td>months</td>
<td>months</td>
<td>months</td>
<td>months</td>
<td>months</td>
</tr>
<tr>
<td>Bilateral Place</td>
<td>none</td>
<td>all</td>
<td>all</td>
<td>same</td>
<td>same</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The data on Bilateral Place are difficult to interpret. For one, the mean frequency for 27-28 months and for 31-32 months is the contribution of a single subject, RIMO. The other cells do not have appreciable frequencies. For another, as we have discussed elsewhere, horizontal alignments do not appear at all until an age where lateral dominance has been established. Most horizontal alignments were made with one hand moving one block toward a second, passive block alone on the table. The most conservative statement to be made is that the symmetrical bilateral action which occurs in midair does not have an isomorphic, static version later in development. Even though symmetrical alignments are created using identical blocks (see Tables 5 and 6) the action itself is not symmetrical. This suggests that the child has developed to a point beyond where symmetrical action is translated isomorphically into symmetrical product. Interestingly enough, over 90 percent of all deliberate spacings (Table 6) were performed with one hand on each opposite block. Here again bilateral action is activated when precision is needed. This is not the same point as saying that bilateral placements result from a residual scheme from bilateral banging. In the first case, bilateral action is progressive, in the second case, it is regressive.

That horizontal placements did not involve bilateral action might have been pre-empted by the sophistication of a child in general who could effectively use the /Place/Slide Next/Release/ rules. This caused us to look for instances of bilateral action which use only the /Place/Slide Next/rules, without /Release/. These moves, as defined in Appendix A, are moves 3, 4, 9, and 11 when they occur with suffixes EDS, ESS, DDS, or DSS. For example, for move 9Ess, the child took two identical blocks, one in each hand, placed them side by side on the table, and then lifted them without a release. In move 11ES8, the child took two identical blocks, one in each hand, lowered them to the top

1Children could create horizontal alignments using both hands, one on each of two blocks, in several ways. The following numerical codes indicate which moves (see Appendix A) were used to get the totals for Bilateral Place: 3.5, 4.5, 10, 12, 14, 21, 22, 24, 26, 28, 29.5, 33, 34, 51, 97, when any of these occurred with the suffixes DDS, DSS, EDS, or ESS.
surface of a third block, and then lifted the two blocks without release. Moves 3 and 4 were defined as moving two blocks together, sliding them across the table, and lifting them after making contact. The difference between the midair bilateral placements and moves 3, 4, 9, and 11 is that these latter moves involve a third surface, the table. But unlike the moves used to compute Bilateral Place in Table 10, there is no /Release/. The bilateral banging in midair can be described as using the single rule /Next To/, moves 3, 4, 9, and 11 as /Place/Slide Next To/ and the bilateral placement of Table 10 as /Place/Slide Next To/Release/. This means that moves 3, 4, 9, and 11 are intermediate to bilateral banging and bilateral placements. If these moves occur with sufficient frequency, it would suggest that the bilateral action is carried forward in development as the child makes a transition from midair actions to table top placements. Table 11 presents these three bilateral schemes.

<table>
<thead>
<tr>
<th></th>
<th>7-8 months</th>
<th>11-12 months</th>
<th>15-16 months</th>
<th>19-20 months</th>
<th>23-24 months</th>
<th>27-28 months</th>
<th>31-32 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Bilateral Contact in Midair</td>
<td>2.4</td>
<td>6.2</td>
<td>4.2</td>
<td>4.7</td>
<td>2.1</td>
<td>.9</td>
<td>.8</td>
</tr>
<tr>
<td>2. Bilateral Contact on Table Top</td>
<td>.7</td>
<td>1.8</td>
<td>.9</td>
<td>1.2</td>
<td>.5</td>
<td>1.1</td>
<td>.9</td>
</tr>
<tr>
<td>3. Bilateral Release on Table Top</td>
<td>.0</td>
<td>.5</td>
<td>.1</td>
<td>.7</td>
<td>.2</td>
<td>1.3</td>
<td>.9</td>
</tr>
</tbody>
</table>

Ratio: Moves using identical blocks / Moves using different blocks

<table>
<thead>
<tr>
<th></th>
<th>7-8 months</th>
<th>11-12 months</th>
<th>15-16 months</th>
<th>19-20 months</th>
<th>23-24 months</th>
<th>27-28 months</th>
<th>31-32 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Bilateral Contact in Midair</td>
<td>.40</td>
<td>.34</td>
<td>1.10</td>
<td>.80</td>
<td>.70</td>
<td>2.00</td>
<td>1.60</td>
</tr>
<tr>
<td>2. Bilateral Contact on Table Top</td>
<td>D</td>
<td>1.30</td>
<td>1.60</td>
<td>.30</td>
<td>4.00</td>
<td>1.0</td>
<td>.60</td>
</tr>
<tr>
<td>3. Bilateral Release on Table Top</td>
<td>Z</td>
<td>.80</td>
<td>S</td>
<td>.30</td>
<td>S</td>
<td>.80</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Table 11. Mean frequencies showing the transition from bilateral contact in midair to bilateral placement with release. In the table showing the ratios, D means that all moves were done with different blocks, Z means no occurrence, and S means all moves were done with identical blocks.

Table 11 suggests that Bilateral Contact on Table Top, curve 1, does occur with an appreciable frequency. Both curve 1 and curve 2 show an increase from 7-8 months to 11-12 months and then a general decrease thereafter, albeit the decline is less monotonic in curve 2. The fact that Bilateral Contact in Midair reaches an appreciable frequency at 7-8 months, while Bilateral Contact on
Table Top does not reach an appreciable frequency until 11-12 months indicates that curve 2 is younger than curve 1. The implication here is that children generalize their actions performed in midair to a new location, i.e., the surface of the table. What was earlier learned as pure action is generalized to a scheme which is beginning to approximate a placement. However, by the time the child effectively uses the /Place/Slide Next To/Release/ rules this bilateral residual has gone, as indicated by the low frequencies seen in curve 3. Recall that the last two cells of curve 3 were the contributions of a single subject, RIMO. In both curves 1 and 2, the bilateral scheme is increasingly activated by identical blocks as children grow older, even though the total frequency of occurrence declines with age. This suggests that the bilateral actions in the younger months is more of a scheme intrinsic to the motor system, while bilateral actions in the older months is more likely a consequence of external input, the form of the blocks. More conservatively stated, bilateral actions with blocks begin as a general response to physical substance, independent of form, and then become more selective in the later months as attention is directed to form. Of course, there is sufficient evidence in infant development research to show that bilateral action is intrinsic to the motor system, since even in the first weeks of life the arms and hands contract bilaterally without external input from objects (Knoblock and Pasamanick, 1974). The point being made here is that the form of the bilateral action of infants occurs first reflexively. This places constraints on the type of movement performed with objects and the type of products that will be created. The bilateral action in air is transferred to action on a supporting surface. Shortly after this, the form of the action and the form of the product are dissociated, even though the product was "suggested" by the earlier action patterns. This dissociation of form of action from form of product brings us nicely back into a discussion of Table 10 where data on making symmetrical products is presented.

Bisection symmetry and reiterative symmetry are symmetrical products produced by asymmetrical actions. Bisection symmetry shows a clear upward progression across age, with an appreciable frequency occurring at 31 months. Reiterative symmetry is almost negligible across all age groups, with a few cases occurring in 31 months (Table 10). In an earlier study of 19 children filmed across a three year age span from 24 to 60 months (Sigel, Secrist, and Forman, 1973) reiterative symmetry became the most dominant rule for block construction beginning at 36 months. These children in the 1973 study also exhibited a higher form of symmetry we termed radial symmetry, which consisted of the co-
ordination of two reiterative symmetries. After placing B' to the right of the hub block as in Figure 5, these children would place an additional balanced pair, one fore and the other aft of the central hub. It is almost certain that the 38 children in the current study would have begun to create more reiterative symmetries and also radial symmetries had we continued testing beyond June.

Unpredicted Findings

Transformations in Dynamic Contact

If a child both presses a block to the table and at the same time slides this block across this surface he is expressing what we have termed dynamic contact. The contact with this surface is more than physical resistance, it is resistance and movement simultaneously. Contact is continuous across the slide, yet movement is also continuous. This scheme must intrigue the child. Movement to the infant has usually been associated with zero physical resistance. Physical resistance has usually been associated with the cessation of movement. When the child contacts surface A with block X, the free movement of X is obstructed. Yet we see in our films a curious attention to the block as the somewhat older child deliberately slides it across the table's surface. He bends his head down to look at this block more at eye level. He modulates the move as if to get the full effect of contact combined with movement. Once again, it seems that characteristics of his world that had originally been successive are now, by his deliberate efforts, made more simultaneous. Movement then contact is transformed into movement and contact, another example of the atemporalization of contrasts. Table 12 presents data relevant to the transformations from the successive to the simultaneous forms of the contact/movement contrast. To simplify this table, and all subsequent tables, mean frequencies have been calculated for all testings within a give age group.

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Touch X to T, lift</td>
<td>12.0</td>
<td>6.7</td>
<td>2.6</td>
</tr>
<tr>
<td>2. Slide X, lift</td>
<td>5.5</td>
<td>3.7</td>
<td>3.0</td>
</tr>
<tr>
<td>3. Touch X to T, release</td>
<td>6.6</td>
<td>7.3</td>
<td>5.8</td>
</tr>
<tr>
<td>4. Slide X, release</td>
<td>2.5</td>
<td>2.5</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 12. Transformations in the movement/contact contrast from successive to simultaneous expression. Mean frequency of occurrence.
The first row describes all movements which bring a block to the table's surface momentarily followed by a lift. This was move 9 in Appendix A. This scheme begins high and decreases across age. The second row describes all movements in which a block is placed to the table and moved across the table's surface prior to lifting. This scheme was coded as moves 19, 20, 50, and 50.5 in Appendix A. This scheme also shows a decline across age. Curve 2 is probably a close developmental contemporary of curve 1. However, considering the greater ease of banging a block to the table, compared to maintaining contact during a slide, had we tested infants younger than 7-8 months, we might have seen an age in which sliding virtually did not occur, while banging block to table did occur. Unfortunately, we can only speculate about the priority of momentary contact over dynamic contact from the present data.

The third curve adds the /Release/ rule to the procedure chain. This was move 10 as defined in Appendix A. The child makes contact of block to table, and this contact is followed by a release. Curve 4 is a /Place/Slide/Release/ coded as moves 51, 81, 83, and 84 in Appendix A. Curve 3 shows an increase, followed by a decrease; curve 4 shows an increase. These two curves are sequenced developmentally in the order numbered. At least within contact and sliding schemes which involve the /Release/ rule, the momentary form of contact has developmental priority over the continuous form of contact. It was during moves 51, 81, 83, and 84 that eye contact with the block was maintained, more so than in moves 19, 20, 50, and 50.5. If eye contact can be taken as an index of greater concentration and deliberateness then it was during these moves that the child was more likely experimenting with the paradox between physical contact and physical movement.

The Decentration of Effect

Children can make block X move by making hand-to-block-X contact, or by a more indirect method of pushing block Y with block X. In the second case, the action of X is transferred to Y. In order for a child to push block Y, he must keep X to the rear of the line of movement and must maintain contact with block Y. Grasping block X more tightly does not assure the continued movement of Y as it might assure the continued movement of X. Pushing one block with another requires a type of accommodation to the external object, a type of accommodation that requires the child to decenter from schemes appropriate to more proximal, more egocentric objects. Our films show the development of this sort of decentering from the proximal object. This is an early version of the development of tool use, at least, an early development of the use of an
intermediary object. The relevant data came from a computer search for all moves which were followed by the suffix F. This suffix indicated that the child had held one block X, and in the process of moving X, also moved Y without actually making manual contact with Y. This suffix was found primarily as part of pushing a horizontal alignment from one end (move 50) or as part of lifting two blocks or more while only holding the bottom block (moves 78, 79, 80). Table 13 shows how the developing child uses this transfer of effect with increasing frequency.

<table>
<thead>
<tr>
<th>Move two, Grasp one</th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.2</td>
<td>.4</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 13. The decenterion of effect. Mean frequency of occurrence.

Denotation of Products

Werner and Kaplan (1967) describe in interesting detail the developmental shift in the first year of life from physically grasping objects to visual contemplation of objects. This represents a shift of function (op cit., p. 70) from obtaining an object to referencing or denoting an object. This shift is important to the development of a symbol system, since symbols need to be dissociated from particular instrumental acts so that the symbols can "stand for" many objects. The fingers flexed to grasp an object change across age in attitude and begin to point to the object. This primitive gesture at first closely resembles the grasp, but gradually assumes the role of denoting an object under contemplation out there. Moves 72 and 73 describe the child pointing at a single block. Moves 74, 75, and 76 describe the child pointing at a product he has made. At what age do children begin to denote objects? Does the denotation of products parallel the development of the denotation of single blocks? Table 14 presents relevant data.

<table>
<thead>
<tr>
<th>Denote Block X</th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.8</td>
<td>.9</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Denote Stack S</th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.1</td>
<td>.3</td>
<td>1.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stacks Made</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.3</td>
<td>13.3</td>
<td>37.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio of Denotation for Products</th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2/100</td>
<td>2/100</td>
<td>4/100</td>
</tr>
</tbody>
</table>

Table 14. Denotation of products. Mean frequency of occurrence. Ratio = number of stacks denoted over number of stacks made.
The rise in the frequency of pointing to a single block does not seem appreciable. However, we note that the rise in denoting the product Stack does increase appreciably. This increase is not the automatic result of an increase in the number of stacks at which to point. Children in Group 3 made an average of 13 stacks per testing, but only denoted about 2 out of every one hundred. This ratio increased to four in every one hundred by Group 5. The difference between Group 3 and Group 5 is most revealing. Children denote single objects before they denote products, even though those products are available for denotation. This is seen most clearly in Group 3 who have already developed denotation of objects alone, but have not begun to denote the products which they have made. This comes later, in Group 5. If, in Group 3, the created product is still strongly associated with the action schemes instrumental to its construction, the denotation may be less likely to occur. According to Werner and Kaplan (1967) the child must differentiate instrumental schemes from referential schemes in order to eventually form a symbol system. Even though denotation of single objects seems to have developed by Group 3, denoting products is not yet present. This suggests that denotation of products lags behind the denotation of single objects. What the child learns at the level of the single object he gradually applies to the composite product. Considering the instrumental effort children place in making stacks, it is reasonable that denotation, as a shift away from instrumentality, would take a while in regard to product denotation.

The Primordial "Sharing" Situation

Werner and Kaplan (1967, p. 42) advance the hypothesis that the child pointing to or holding up an object to a parent is in a sense asking that parent, as audience, to share in a contemplation of that object. The child's awareness of the audience is essential, in Werner and Kaplan's theory, to the formation of a symbol system. The symbol, in its higher form, is a signal to an assumed audience. We have noticed in our films that many children would lift a single block and hold it high, apparently so that their mother could see the object. This gesture, as independently interpreted by three adult observer's, meant "look at what I have." Werner and Kaplan state that the formation of this primordial sharing is preceded in development by more egocentric forms of contemplation. One such earlier form, a form we coded in our films, is the rather stereotyped response of lifting a single block and rotating it slowly in midair with gaze fixed upon the block. This scheme was found in moves 5 and 100 (Appendix A). The sharing form, or less egocentric
form, was found in move 6. Table 15 shows that this shift from egocentric contemplation to shared contemplation occurs at approximately 20 months, the mean age of Group 3. Then both forms decline during the age of Group 5, mean age of 28 months.

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Egocentric Contemplation</td>
<td>4.4</td>
<td>3.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Shared Contemplation</td>
<td>.9</td>
<td>2.7</td>
<td>.9</td>
</tr>
</tbody>
</table>

Table 15. Egocentric and shared contemplation of single objects. Mean frequency of occurrence.

Sensitivity to Orientation of Blocks

We were struck by the very few times that the younger children would place a square (S), circle (C), or cylinder (y) on its side. For the S and C such a placement would create a taller object than leaving these objects in the orientation presented. For placing y on edge the block would not be taller, but could be rolled in that orientation, as could the circle (C). Of course, at times a block, either dropped from mid air or knocked from a stack, would accidentally land standing on its edge. However, moves 39, 42.5, 43, 88, and 103 were coded only when the child created the "on-edge" orientation with some deliberation. The developmental data suggest that the younger children have a greater tendency to accept as fixed the presented orientation of an object. Even in the process of stacking a large square on the top surface of another large square the younger child's movements assured that the two squares were both in the most horizontal orientation. This matching of orientation was in part due to the younger child's tendency to grasp a block, lift without rotation, and stack. By the law of least effort blocks are more likely to maintain a constant orientation. In other words, we cannot say that maintaining the horizontal orientation was the result of actively inhibiting placing blocks vertically or whether it was the fortuitous result of releasing blocks in the same orientation as grasped.

The older children would make obvious changes in orientation of a block to establish the vertical. One such obvious case of a deliberate change was on moves 42.5 and 43. Here the child would place in the verticle orientation a large Square on top of a resting Square, itself on-edge. The child would begin changing the top Square to the verticle orientation as it approached the resting Square. Other cases of deliberate creation of the vertical were seen as the abortion of a horizontal orientation in deference to creating a vertical orientation when in fact the vertical orientation duplicated the
orientation of an adjacent block (moves 40 and 42) or a nearby block (moves 41 and 43). Considering the size of the blocks used in this study, i.e., a "narrow" edge of 1 1/2 inches, the dexterity required to place a block in its more vertical orientation was nominal. The rules involved are /Grasp/Lift/Rotate/Place/Release/. The /Grasp/Lift/Place/Release/ chain was certainly present during the first testing of Group 1 (see frequencies for Move 10 in Appendix B). The /Grasp/Lift/Rotate/ chain was also present during the testings of Group 1 (see frequencies for moves 5 and 100 in Appendix B). The transformation necessary to place a block in its vertical orientation seemed to involve the deletion of a rule. That is, /Grasp/Lift/Rotate/Place/Release/, which was move 5 or 100 followed by move 10, was transformed to /Grasp/Lift/Rotate/Place/Release/, which was a chain found in moves 39, 40, 40.5, 41, 42, 42.5, 43, 88, and 103 (see Appendix A). The rule deletion was the second /Rotate/ command. The summary of these several points leads us to conclude that establishing the vertical orientation does require an active inhibition of a motor command. To consider development as an increase in inhibitory prowess is consonant with the work of Russian psychologists (see Cole and Maltzman, 1969), the learning theory of Harry Harlow (1959), and the stages of prehension discovered by Halverson (1931). In regard to the prehension, development advances as the child learns to inhibit the reflexive tonicity of closing the digits around an object that stimulates the palm, that is, the child learns to release an object.

The activation of the command to inhibit the second /Rotation/ will enter the procedure chain only when the child structures in advance the goal to place the block in its vertical orientation. Inhibition is a feedforward mechanism that prevents the occurrence of something that has not yet currently, but habitually has historically, occurred. The child must consider the possibility of blocks being in the vertical orientation before he can deliberately execute the procedures which lead to that orientation. In other words, the child must transcend the given orientation (horizontal) in order to create a new orientation (vertical). We have noticed in earlier research that young children have a great tendency not to make changes in the given orientation and position of blocks. This seemed to be a type of positional fixedness. In this earlier research (Forman, Laughlin, and Sweeney, 1971) three year old children would place a jigsaw piece into the formboard such that the piece was positioned in the correct space (all spaces were the same irregular shape) but in the wrong horizontal orientation. These children would continue jabbing a second piece into the adjacent space, made impossible by the misplacement of the first piece or they would remove the first piece altogether. What
they would not do is rotate the first piece in place. Once the piece was placed in the form board they never repositioned it, unless to remove it altogether. This obstacle was so obvious that teachers and researchers alike felt almost irresistible urges to instruct the child to rotate a piece. The placed piece has a certain absoluteness that prevented solution of the puzzle. It seems that the younger children in the current study approximate this same sort of absoluteness in the orientation of a block in free space. All blocks are presented in a horizontal plane and this orientation was seldom violated. Table 16 presents the relevant data.

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 3</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place X Vertical</td>
<td>.9</td>
<td>1.6</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 16. Production of the Vertical Orientation. Mean frequency of occurrence.

One wonders what would have happened if all the blocks had been presented in the vertical orientation, that is, with their longer dimension perpendicular to plane of the table top. Would the age trends show a high frequency of vertical placements in the younger months followed by an increasing proportion of horizontal placements in the older months? Such a trend would be predicted if younger children use a law of least effort and older children use an active inhibition of the second Rotate/ command. If the block were grasped in the vertical orientation, the /Grasp/Lift/Horizontal/Rotation/Rotation/Place/Release/ or the /Grasp/Lift/Horizontal/Place/Release/ would result in a vertical placement. Alternatively, the priority of the horizontal position could have resulted from an active preference (rather than least effort) to place objects in their most stable orientation. A Square placed flat or a Circle placed flat are less likely to fall or roll than these blocks placed vertically.

A test of these two alternative possibilities, least effort or active preference, occurred to us one third of the way through completion of the final testing of our children. At the conclusion of the sixth cluster of blocks, 1

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1Least effort can occur in two ways during these routines. In /Grasp/Lift/Place/Release/ there is less effort than in /Grasp/Lift/Horizontal/Rotation/Rotation/Place/Release/ due to the omission of the rotation command. However, the effort involved in /Grasp/Lift/Horizontal/Rotation/Rotation/Place/Release/, from a kinematic analysis is actually less than that involved in the /Grasp/Lift/Horizontal/Place/Release/ command chain. In the latter, the natural anatomical position of the thumb-forefinger must be actively inhibited during the /Place/ command by sustained lateral pronation (see Plagenhoef, 1971). The second /Horizontal/ command in the former is actually a command to relax thumb-forefinger pronation. It is this command to relax that is inhibited in the more advanced chain /Grasp/Lift/Horizontal/Place/Release/.

53
the remaining two thirds of the children were presented two yellow Squares, one in the vertical orientation, the other in the horizontal orientation to the left of the vertical Square. We then coded whether the child moved to reposition the vertical or the horizontal block, and plotted these moves across age groups. We predicted that the younger groups would reposition the vertical block to the horizontal position, while the older groups would reposition the horizontal block to the vertical position. Our rationale for this prediction was as follows. The younger children would look at the vertical block as a signal for action, i.e. to knock it over or place it on its more stable surface. To the younger child the vertical block was precariously positioned. The realization of this pending fall triggers, in the young child, schemes to produce the anticipated movement. The older children look at objects less in terms of action, more in terms of static states, e.g. orientation per se. The vertical orientation is more novel (of course, since all children had just had six clusters of blocks presented in a horizontal orientation). This more novel orientation intrigues the older child, not because it violates block-action expectations, but because it violates block-appearance expectations. The older child's response to this type of novelty is to duplicate the novel, rather than to re-establish the usual. The older child sees the vertical block and this triggers a motor command to upright the nearby horizontal block and thereby make both blocks rest in the more novel orientation. The younger child sees the vertical block, thinks about it falling, and this triggers the motor command to lay the vertical block flat. The younger children probably did less visual comparison between the two blocks, and were operating more on their anticipations of the single, vertical block. We know from other research (Vurpillot, 1971) that younger children do less visual scanning between two alternatives prior to making a choice. We would need actual visual scanning data to substantiate this corollary prediction, but the corollary is that the younger children were probably comparing the vertical block to an earlier movement of that same block, while the older children were comparing the horizontal block on the left to the vertical block on the right. What is true of the vertical block, can be true of the horizontal block. Children both younger and older see an implication, but this implication is between blocks for the older and within blocks for the younger. The transfer from is to can be, the transfer from actual to potential is a between block transfer in the older children, but it is a within block, between state transfer in the younger children. We reasoned that the anticipation of state within block (changing the vertical to the horizontal) would proceed in development the
anticipation of state between blocks (changing the horizontal to the vertical). Changing one block from state A (vertical) to A' (horizontal) should require less thought than storing state A (vertical) of one block and duplicating that state by changing another block from B' (horizontal) to B (vertical).

Unfortunately our data were inconclusive. Children did more than we had hoped. They lifted both blocks simultaneously, rotated one up while rotating the other down, pushed both to the middle without changing either orientation, and engaged other responses which were not categorizeable according to the two response types in which we were interested. Had we hinged the two Squares to the table surface, this might have set enough constraints on the possible actions for us to see developmental trends. We plan to continue our research along these lines, adding more or less constraints and recording eye movement patterns.

Final Comment

This data bank will reveal many more developmental trends. The two central themes that these films have revealed in general, however, is the gradual atemporalization of successive states and the dissociation of the logical from the contingent. Atemporalization occurs as the child builds structures that remain for close inspection even after the child's action has ceased. Banging a block to the table is in a sense frozen in static relation by releasing a block on the top of another. The child can visually inspect the juncture, anticipate separation, and then activate separation of the stack. Constructing rotating relations allows the child sufficient time to retrieve old schemes, anticipate effects, and study the spatial relations qua relations. What had before been two states separated by time becomes one spatial arrangement which is an index of both of those states at once. As the actual movements submerge and become potential movements, successive states are atemporalized and take on the form of a pure relation.

These films also suggest that the physical constraint of material determine the early block structures, but block structures gradually are constrained more by rules of order than by rules of physical fit or physical balance. The contingent relation of cause and effect at first determines the site of placements, an exchange between structure and function. Later blocks are placed less to serve a particular function, and more to express a particular relation. Once purpose is dissociated from structure the child explores a whole new world of logico-mathematical knowledge. This then in turn can lead to new discoveries in the physical world, due to new inventions in the logico-mathematical implications for early childhood education, as it certainly has had an impact on the progress of science.
Bibliography


Appendix A

Notation System

The following notations were used by the coders as they sat in front of the Super-8 projector coding the films. Each action of the child was represented by a particular symbol. Some of these symbols represented actual hand movements, other represented "syntactical" relations between actual hand movements. Others represented the position established between blocks as the result of a placement. The list below is a partial dictionary of symbols. In addition to the brief definitions given below, the definitions of individual symbols can be more fully understood by reading the second dictionary, which defines how the symbols were combined to make a complete action unit.

DICTIONARY OF SYMBOLS

Movement Symbols:

- Grasps block
- Releases block
→ Lowers block (or hand alone)
→ Lifts block (or hand alone)
→ Throws block
→ Slides block
→ Rolls block
→ Inverts block
→ Uprights block
→ Seats block flat
• Maintains grasp over time
→ Pushes block

Syntactic Symbols:

| Which is... |
| Such that... |
| Such that product one shares relation to product two. |
| And (a relation between two blocks) |
| And (a relation between two actions) |
| This action is embedded as a subgoal to another action |

Position Symbols:

- On top of
= Underneath
/ Adjacent
\ Near, but not touching
→ Deliberately spaced
→ Adjusts block
→ Centers block
→ Aligns parallel

Product Symbols:

S Stack
A Horizontal alignment (blocks in row)
SA Stack and horizontal alignment combined.
LLU Bridge, definite gap.
DICTIONARY OF COMPLETE ACTION UNITS

The symbols were combined to create a complete action unit. An action unit, the basic unit of this research, was defined as beginning with hand contact with block, either a touch or a grasp, or as a beginning with the contact of a block with another surface. The terminus of a complete action unit was either the release of a block, the removal of the fingers after a touch, or the contact of a held block with a surface. For example, grasping a block and releasing it is one unit. Grasping a block and touching it to a second is one unit. Moving the fingers to a block, touching it, and withdrawing fingers from that block is one unit.

By using a total of 53 symbols (previous page is partial list) it was discovered that 151 unique action units expressed all of the actions seen in the films. The following dictionary defines each of these 151 unique action units.

Symmetrical movements of two blocks, one in each hand:

1. \( R(\cdot) \rightarrow Y \rightarrow L \rightarrow L \rightarrow R \rightarrow \) The right and left hand each grasp a block. These blocks are not identical. The two blocks are moved together in midair until they make contact at the midline of the body. They are then moved apart.

2. \( R(X) \rightarrow X' \rightarrow L \rightarrow L \rightarrow R \rightarrow R \rightarrow \) Same move as above, except the two blocks grasped are identical in size and shape.

3. \( R(X) \rightarrow Y \rightarrow L \rightarrow L \rightarrow R \rightarrow \) Two non-identical blocks are grasped and are slid together across the surface of the table. They are not lifted from the table. Then after contact they are lifted from the table.

3.5 \( R(X) \rightarrow Y \rightarrow L \rightarrow 2A \rightarrow L \rightarrow R \rightarrow 2A \rightarrow \) Same move as above, except the two blocks are released upon contact with each other. Release yields a two block horizontal alignment.

4. \( R(X) \rightarrow X' \rightarrow L \rightarrow L \rightarrow R \rightarrow \) Same as move 3. except the two blocks are the same size and shape.

4.5 \( R(X) \rightarrow X' \rightarrow L \rightarrow 2A \rightarrow L \rightarrow 2A \rightarrow \) Same as move 3.5, except the two blocks are the same size and shape.
Body oriented moves:

5. (X) \rightarrow E
Child grasps, lifts, and inverts block making visual contact with its underside.

6. (X) \rightarrow P
Child grasps and lifts block high toward an adult nearby, apparently as some gesture of sharing sight or possession of the object with the adult.

7. (X) \rightarrow B
Block is moved into contact with the body, e.g. the chest, leg, etc.

8. (X) \rightarrow M
Block is moved into contact with the mouth.

Block to table:

9. (X) \uparrow \uparrow \uparrow T
Block grasped, lifted, lowered to table, then lifted again.

10. (X) \uparrow \downarrow \downarrow T
Block grasped, lifted, lowered to table and released on the table surface.

Block to block from above:

11. (X) \uparrow \downarrow \uparrow Y
Block lowered to another block, clear contact made, then block lifted away from resting block. The two blocks are non-identical.

12. (X) \uparrow \downarrow \uparrow Y \cdot *2S_1
Same as move 11 except the moved block is released on top of the resting block, thereby creating a two block stack.

13. (X) \uparrow \downarrow X \cdot *2S_1
Same as move 11, except the two blocks are identical in size and shape.

14. (X) \uparrow \downarrow X' \cdot *2S_1
Same as move 12, except the two blocks are identical in size and shape.

15. (X) \uparrow \downarrow Y \cdot *2S_1
Block X lifted, lowered to top of block Y, an adjustment is made and after the adjustment, block X is released, making a well balanced stack.

16. (X) \uparrow \downarrow X' \cdot *2S_1
Same as move 15, except the two blocks are the identical size and shape.

16.5 X || Y \cdot X \rightarrow
Block X, which is resting off-center on top of a larger block Y, is adjusted in place until X is centered (i.e. bisects) block Y, then X is released.

17. (X) \uparrow \downarrow Y \cdot *2S_1
Block X is lowered to a larger block Y, and then X is centered on top of Y, bisecting Y, and then block X is released, creating a two block stack.

* Interrater reliability between .75 and .80. All others between .80 and .95.
Block X grasped, lifted, lowered to make contact with both blocks Y and X'. X' is next to Y. X is centered on the juncture of blocks Y and X', then X is released, centered on this crack.

Block X is grasped, lifted, and lowered to an identical block X'. X is rotated until the form of X is perfectly superimposed on the form of X', then X is released. This move makes no sense when X and X' are circles, since two circles need not be rotated in a horizontal plane in order to be superimposed on each other.

Block is slid across table without breaking contact with table:

19. Block X is grasped on the table, moved across the table, making contact with block Y, then X is lifted from table.

20. Same as move 19, except the two blocks are identical in size and shape.

21. Block X is grasped, slid into contact with block Y, and then released there, making a two block horizontal alignment.

21.5 Block X is grasped in one hand, slid over to make contact with Y which is held on the table in the other hand, then block X is released next to block Y.

22. Same as move 21, except blocks are identical in size and shape.

22.5 Same as 21.5, except blocks are identical in size and shape.

Block placed down on and next to two surfaces:

23. Block X placed down on table, and at the same time, next to block Y, then X is lifted without a release.

24. Block X is placed down on table, and at the same time next to block Y, and then X is released next to Y, such that a two block horizontal alignment remains.

25. Same as move 23, except the two blocks are identical in size and shape.

26. Same as move 24, except the two blocks are identical in size and shape.
Block removed from stack:

26.5 \((X \in S_1) \uparrow \downarrow T \uparrow\)

Block X, which is on top of stack one, is lifted from stack one and returned to stack one, and then lifted again.

27.

\((X \in S_1) \uparrow \downarrow T \uparrow\)

Block X, which is on top of stack one, is lifted from stack one, lowered to table, and then lifted again.

* 27.5

\((X \in S_1) \uparrow \downarrow \ldots\)

Block X, which is on top of stack one, is lifted from stack one, and then held while some other action is performed with the other hand. This move is used whenever block X is removed as an obstacle for the stacking of another block by the other hand, at the site of stack one.

28.

\((X \in S_1) \uparrow \downarrow T \uparrow\)

Block X, which is on top of stack one, is lifted and lowered to the table where it is released.

29.

\((X \in S_1) \uparrow \downarrow Y \uparrow\)

Block X is lifted from stack one, lowered to block Y resting elsewhere, then X is lifted from the surface of Y without release.

30.

\((X \in S_1) \uparrow \downarrow Y \cdot \ast 2S_2\)

Same as move 29, except X is released on Y forming a new stack with Y as the foundation.

30.5

\((X \in S_1) \uparrow \downarrow S_1\)

Block X is clearly lifted from stack one, and then lowered to the same stack and released.

31.

\((X \in S_1) \uparrow \downarrow X' \uparrow\)

Same as move 29 except the two blocks are identical in size and shape.

32.

\((X \in S_1) \uparrow \downarrow X' \cdot \ast 2S_2\)

Same as move 30 except the two blocks are identical in size and shape.

33.

\((X \in S_1) \uparrow \downarrow T \rightarrow / Y \cdot \ast 2A_1\)

Block X lifted from stack one, lowered to the table, then slid over next to block Y, and released, thereby creating a two block horizontal alignment. That is, the child has transformed the vertical relation of X into a horizontal relation.

34.

\((X \in S_1) \uparrow \downarrow T \rightarrow / X' \cdot \ast 2A_1\)

Same as move 33, except the two blocks left in horizontal alignment are identical in size and shape.

Block removed from an alignment:

(Note, these codes were used only if the child himself had created the alignment from which a block was separated. If the block was contiguous to another as the fortuitous result of some random movement, this contiguity was ignored.)
28.5  
\[(X|A)|\ldots\]  
Block X is removed from an alignment and held while other actions are performed with the other hand.

29.5  
\[(X|A)|\downarrow T\]  
Block X is removed from an alignment and placed elsewhere on the table and released.

31.5  
\[(X|A)|\rightarrow / Y * 2A_2\]  
Block X is slid from next to alignment one to next to block Y elsewhere on the table. Then X is released, creating a new horizontal alignment.

52.5  
\[(X|A)|\rightarrow / X' * 2A_2\]  
Same as move 31.5, except the two blocks adjacent in the new horizontal alignment are identical in size and shape.

33.5  
\[(X|A)|
\rightarrow Y * 2S_1\]  
Block X removed from alignment one, lifted, lowered onto block Y elsewhere, and then X is released on block Y creating a two block stack. The child has transformed X from a horizontal to a vertical relation.

33.6  
\[(X|A)|\leftrightarrow Y\]  
Block X is removed from alignment one, lifted, lowered to block Y elsewhere, but lifted again.

34.5  
\[(X|A)|\leftrightarrow X' * 2S_1\]  
Same as move 33.5, except the two blocks creating the stack are identical in size and shape.

34.6  
\[(X|A)|\leftrightarrow X'\]  
Same as move 33.6, except block X is moved down to an identical block X'.

Blocks moved to create a deliberate, vacant space:

35.  
\[(X|Y)*X|Y|*Y\]  
Block X placed on top of block Y, then X is moved in order to make a definite gap between X and another block Y', which is also on top of block Y.

36.  
\[(X|Y)\rightarrow *X|X'|Y\]  
Same as move 35, except the two blocks flanking the vacant interval are identical in size and shape.

37.  
\[(X|T)\rightarrow *X|Y\]  
Block X placed on table and then slid near block Y and released with a definite space between block X and block Y. While deliberateness was often difficult to discern, deliberateness was assumed whenever the child made certain adjustments of the gap (as if to produce a particular gap) and/or whenever the child exhibited concentrated eye contact to the vacant space between X and Y.

38.  
\[(X|T)\rightarrow *X|X'\]  
Same as move 37, except the two blocks flanking the vacant space are identical in size and shape.
37.5

\((x) \uparrow \downarrow \leftrightarrow x \uparrow y \uparrow\)

Same as move 37, except block X is lifted from the table, rather than released.

38.5

\((x) \uparrow \downarrow \leftrightarrow x \uparrow x' \uparrow\)

Same as move 37.5, except the two blocks flanking the vacant space are identical in size and shape.

Vacant space created by spreading blocks:

37.3

\(R(x1 \uparrow z) \leftarrow \cdot \cdot x \uparrow z\)

\(L(y1 \uparrow z) \leftarrow \cdot \cdot x \uparrow y\)

Block X held in right hand, block Y held in left. X and Y are both under block Z. Handspread X and Y apart slowly so that block Z remains supported by X and Y. Vacant space left under block Z (i.e. a bridge).

38.3

\(R(x1 \uparrow z) \leftarrow \cdot \cdot z\)

\(L(x'1 \uparrow z) \leftarrow \cdot \cdot x \uparrow x'\)

Same as 37.3, except the two supporting blocks are the same size and shape. Block Z may or may not be identical to X and X'.

Blocks changed to an upright orientation:

39.

\((x) \uparrow \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\)

Any block, other than a cube, is rotated on the table and released such that its longest dimension is resting vertically, e.g. child moves a square from laying flat to resting upright. This move is used when child rests a cylinder on its curved side.

39.5

\((x) \uparrow \downarrow 4\)

Same as move 39, except the uprighted block is not released. It is lifted after the change in orientation.

40.

\((x) \uparrow \downarrow \leftrightarrow x \uparrow y \uparrow\)

Block X is uprighted and released such that it was physically contiguous with block Y which was already in an upright position.

40.5

\((x) \uparrow \downarrow \leftrightarrow y \uparrow \cdot \cdot x \uparrow z \uparrow\)

Block X is placed on Y which is already upright. Block X is released such that it too is upright on top of Y.

41.

\((x) \uparrow \downarrow \leftrightarrow x \uparrow l \uparrow / y \uparrow l\)

Block X is placed upright on the table and released such that X is near, but not touching, block Y which is also upright.

42.

\((x) \uparrow \downarrow \leftrightarrow x \uparrow l \uparrow / x' \uparrow l\)

Same as move 40, except the two upright blocks are identical in size and shape.

43.

\((x) \uparrow \downarrow \leftrightarrow x \uparrow l \uparrow / x' \uparrow l\)

Same as move 41, except the two upright blocks are identical in size and shape.

44.

\((x) \uparrow \downarrow \leftrightarrow x \uparrow l \uparrow / y \uparrow l\)

Same as move 41, except block X is lifted instead of released.

45.

\((x) \uparrow \downarrow \leftrightarrow x \uparrow l \uparrow / y \uparrow l\)

Same as move 40, except block X is lifted instead of released.
* 46.  
\[ (x) \rightarrow (x) \uparrow x, x' \uparrow \]  
Same as move 43, except block X is lifted instead of released.

* 47.  
\[ (x) \rightarrow (x) \uparrow x, x' \uparrow \]  
Same as move 42, except block X is lifted instead of released.

Note, if block placement is not designated as an uprightment, it is assumed that the placement is flat, i.e. with longest dimension of the block in a horizontal plane.

Block changed from upright orientation to flat orientation:

48.  
\[ (x) \downarrow \uparrow \]  
Block X, which is in an upright orientation, is placed flat on the table, and then lifted from the table.

49.  
\[ (x) \downarrow \downarrow \]  
Block X, which is in an upright orientation, is placed flat on the table, and then released.

Block is slid across table, without contact with another block:

50.  
\[ (x) \rightarrow \uparrow \]  
Block X is grasped, slid across the table, and then lifted from the table.

50.5  
\[ \downarrow x \rightarrow \uparrow \]  
Block X is touched, but not grasped, then slid across that table. Then the fingers disengage from the block which remains on the table.

51.  
\[ (x) \rightarrow \downarrow \]  
Block X is grasped, slid across the table, and then released.

51.5  
\[ (x) \rightarrow \rightarrow x \sim F \]  
Block X is grasped, slid across the table to the edge of the table and released, whereupon the block falls to the floor.

Block is held in place and then released:

52.  
\[ (x) \cdot \]  
A single block resting on the table is held and then released.

53.  
\[ (x) \rightarrow \rightarrow \rightarrow \]  
A single block is held on the table for an extended period of time.

54.  
\[ (x) \cdot \]  
A block that has been held for an extended period of time is at this point released. An "extended" period of time is defined as holding the block in one hand while the other hand has made at least one move.
55. \[\text{A block that has been held for an extended time is lowered to the table at the same spot from which it was lifted and then held there on the table without release.}\]

56. \[\text{A block X resting on top of block Y is grasped and then released without lifting.}\]

57. \[\text{Same as move 56. except X is resting on top of a stack.}\]

58. \[\text{Block X which is resting under block Y is grasped and then released without lifting and without disturbing the two block stack.}\]

59. \[\text{Same as move 58. except X is resting under a stack.}\]

60. \[\text{Block X which is resting on its most narrow edge is grasped and then released without lifting or without changing the orientation of block X.}\]

61. \[\text{Block X which is resting adjacent to block Y is grasped and then released without lifting or without disturbing the adjacency of X and Y.}\]

Block is released in midair

62. \[\text{Block X is grasped, lifted, released in midair such that block X falls to the floor.}\]

63. \[\text{Same as move 62 except X falls to the table.}\]

64. \[\text{Same as move 63 except X begins to roll when it hits the table.}\]

Block is touched but not grasped

65. \[\text{The hand descends toward a resting block, the fingers make contact with the resting block without grasping it and then the fingers break contact with the block.}\]

65.5 \[\text{The fingers of one hand make contact with a block that is held in the mouth with the other hand, then this contact is broken without a grasp.}\]

66. \[\text{The fingers of one hand make contact with a block already held by the other hand, and this contact is broken.}\]

67. \[\text{The fingers touch a block X which is resting on top of another block Y and then this contact is broken.}\]
67.5
\[ \downarrow X \downarrow \downarrow Y \downarrow \uparrow \]
Child touches block X which is under block Y. Withdraws fingers, does not grasp.

68.
\[ \downarrow T \downarrow \downarrow X \downarrow \uparrow \]
Child touches the table and block X with his fingers, then withdraws his fingers from table and block.

69.
\[ \downarrow X \downarrow \downarrow Y \downarrow \uparrow \]
Child touches block X which is adjacent to block Y, then withdraws his fingers from X.

70.
\[ \downarrow X \downarrow \downarrow \downarrow \]
Child touches block X which is in an upright orientation, then withdraws fingers from X.

Child points to surface with extended finger:

71. \[ ! \uparrow T \]
Child points to a spot on the table surface.

72. \[ ! \uparrow X \]
Child points to a block, making contact with the surface of the block.

72.5 \[ ! \uparrow X \downarrow \downarrow H \]
Child points to block X which is held in the opposite hand.

73. \[ ! \uparrow X \]
Child points to a block resting on table, but extended finger does not make actual physical contact with target block.

74. \[ ! \uparrow X \downarrow \downarrow S_1 \]
Child points to and makes physical contact with block X which is on top of stack one.

75. \[ ! \uparrow X \downarrow \downarrow S_1 \]
Child points to, but does not make physical contact with, block X which is on top of stack one.

76. \[ ! \uparrow X \downarrow \downarrow S_1 \]
Child points to, and makes contact with, block X which is under stack one.

77. \[ ! \uparrow X \downarrow \downarrow \downarrow Y \]
Child points to, and makes contact with, block X which is adjacent to block Y. Move 77 is also used if X is adjacent to X'.

Child lifts an assembled structure:

78. \[ (X \downarrow \downarrow Y) \uparrow \uparrow Y \downarrow \downarrow T \]
Child grasps block X which is under block Y, lifts X such that Y falls to the table.

79. \[ (X \downarrow \downarrow S_1) \uparrow \uparrow S_1 \downarrow \downarrow T \]
Child grasps block X, which is the foundation of the stack, lifts such that the top blocks of the stack fall to the table.
80. \((X \Rightarrow S) \wedge *S \bowtie T\)  
Child grasps block \(X\), which is the foundation of the stack, gives \(X\) a clearly defined tilt, thereby making the top blocks deliberately fall to the table.

Block is given a definite push: A push consists of a touch or grasp, followed by an acceleration of the hand, such that the block accelerates beyond the limits of the fingers. The slide is different from a push, in that the child maintains continuous finger contact with the block during the slide, whereas in the push that block makes an abrupt break with the fingers.

81. \(\Rightarrow X\)  
Child pushes a single block away, no particular target.

82. \(\Rightarrow X / \text{Bunch}\)  
Child pushes block \(X\) into an undifferentiated target, such as a bunch of blocks.

83. \(\Rightarrow X \bowtie X \bowtie F\)  
Child pushes \(X\) such that \(X\) falls from the table to the floor.

84. \(\downarrow X \Rightarrow *X \bowtie F\)  
Child touches block \(X\) without grasping it, pushes \(X\) forward such that \(X\) falls from the table to the floor.

85. \(\Rightarrow X \bowtie X / Y\)  
Child pushes block \(X\) into block \(Y\). This is not to be confused with move 21, which is a definite sliding of \(X\) into \(Y\). In move 85 the child releases contact with \(X\) prior to its contact with \(Y\). Move 85 also used when \(X\) is pushed into \(X'\).

86. \(\Rightarrow X \downarrow \text{now-circle}\)  
Child pushes block \(X\), which is in an upright orientation. Block \(X\) is not round.

Child deliberately causes block to roll:

87. \(\Rightarrow X \downarrow \Rightarrow *X \bowtie F\)  
Child pushes block \(X\), which is upright, such that \(X\) begins to roll.

88. \((X \downarrow \uparrow \Rightarrow *X \bowtie F)\)  
Block \(X\) is grasped, slid forward with a clean release at the end of the slide, thereby setting the block rolling.

89: \(\downarrow X \downarrow \uparrow *X \bowtie F\)  
Child puts fingers on the edge of upright block, withdraws fingers, making the block roll in the process.

The child destroys an assembled structure:

90. \(\Rightarrow S \bowtie S \bowtie T\)  
Child pushes a stack over.
91. $S \frac{1}{2} S \sim$

Child swipes at stack and knocks it over. The swipe is more exaggerated than the push. Pushing over is defined by the hand making an initial contact with the stack, pausing briefly, and then pushing the stack. The swipe is one continuous swing that happens to catch the stack in its trajectory.

90.5 $A \frac{1}{2}$

The child swipes at an alignment thereby breaking the arrangement apart.

Child throws a block: The throw is differentiated from the push in that the push has a clear horizontal movement. The push indicates more clearly that the child intends the block to follow the horizontal surface of the table. The throw is an accelerated hand movement with a release at some forward point of the hand movement in midair.

91.5 $(X) \downarrow F$

Child grasps block $X$ and throws it to the floor.

92. $(X) \downarrow T$

Child grasps block $X$ and throws it to the surface of the table.

Transfer of block from one hand to another:

93. $(X) \uparrow \sim R$

Child grasps block $X$ with the left hand, lifts it, and transfers $X$ to the right hand. The right hand grasps $X$ at the same time that the left hand releases $X$.

94. $(X) \uparrow \sim L$

Same as move 93, except the right hand transfers block $X$ to the left hand.

93.5 $(X) \uparrow \sim L \leftarrow$

Block $X$ is lifted by right hand, then moved to the site of the left hand, which touches block $X$, but the transfer from right to left is not made.

92.5 $(X) \uparrow \sim R \leftarrow$

Same as move 94 but transfer is not made. Block remains in the left hand.

Child adjusts a resting block:

* 94.5 $: X \leftarrow S$

Block $X$ is held lightly and adjusted under several blocks above $X$.

* 95. $: X \leftarrow S$

Block $X$, which is on top of a stack, is held slightly and adjusted, then released. Often done to make the block in greater alignment with the lower blocks.

* 95.5 $: X \leftarrow S \leftarrow H$

Child adjusts block $X$ which is under the stack. The stack above $X$ is held in the opposite hand.
96. \( x \vdash -y \vdash -z \)  
Child adjusts block X which is under block Y and on top of block Z, i.e. block X is in the middle of the stack.

96.5 \( x \vdash -x' \)  
Child adjusts into alignment block X which is adjacent to block X'.

Child places one block against the surface of the two other blocks:

97. \( (x) \uparrow \downarrow y \vdash x/z \vdash y \)  
Block X is grasped, lifted from the table, lowered to block Y and released such that block X is adjacent to block Z, which is itself also on top of block Y.

97.5 \( (x) \uparrow \downarrow y \vdash x/z \vdash y \uparrow \)  
Same as move 97.0, except that block X is not released, but rather lifted after X touches Y and Z.

98. \( (x) \uparrow \downarrow y \vdash z \vdash y \vdash \)  
Block X is grasped, lifted, and lowered to the crack made by the adjacency of Y and Z. Then block X is released, making a stack-alignment.

98.5 \( (x) \uparrow \downarrow y \vdash z \vdash y \uparrow \)  
Same as move 98, except that block X is not released, but rather lifted after X touches the crack made by Y and Z.

Child makes bridge:

99. \( (x) \uparrow \downarrow y \vdash z \vdash y \vdash \)  
Child grasps X lifts, lowers X onto top of both Y and Z which are themselves spaced apart. This makes a bridge with a vacant space under X.

99.5 \( (x) \uparrow \downarrow y \vdash z \vdash y \uparrow \)  
Same as move 99, except immediately upon closing the vacant space under X, X is lifted away.

Child manipulates block in midair, usually concomitant with visual inspection:

100. \( (x) \uparrow \odot \)  
Block X is grasped, lifted, and rotated in midair for visual inspection.

Child places block on an uprighted block:

100.5 \( (x) \uparrow \downarrow y \vdash z \vdash *2S \)  
Block X is grasped, lifted, and lowered to the top edge of block Y, itself uprighted. Block X is released there laying flat on the uprighted Y.

101.5 \( (x) \uparrow \downarrow y \vdash \uparrow \)  
Same as move 100.5 except block X is lifted from Y rather than released there.
Child places an uprighted block on a flat block:

103. 

\[ (x) \uparrow \uparrow y \cdot x \cdot x \cdot y \]

Block X is lifted and lowered to the top surface of block Y. Block X is then released in an upright orientation. Block Y is laying flat.

103.5

\[ (x) \uparrow \uparrow y \cdot x \cdot x \cdot y \uparrow \]

Same as move 103, except block X is not released on Y, but is lifted from Y.

Rolling block:

104.

\[ x \mid \infty \]

Child fortuitously makes a block roll. For example, a cylinder falls off a stack and begins to roll.

101.

\[ (x) \uparrow \uparrow y \mid \infty \]

Child visually tracks the rolling block X and then stops the rolling by capturing block X.

Opposite hand assists in stacking:

102.

\[ (x) \uparrow \uparrow y \mid h \cdot s \]

Child grasps block X in one hand, lifts X, lowers X to Y, itself steadied in left hand, and releases both X and Y such that a stack is created.

102.5

\[ (x) \uparrow \uparrow y \mid h \uparrow \]

Same as move 102, except that block X is not released on block Y, rather X is lifted from Y.
DICTIONARY OF SUFFIXES ADDED TO SOME UNITS

Units that were performed with two blocks simultaneously required a special suffix in order to specify the form of the unit. For example, a child may stack one block at a time, or he may stack simultaneously two blocks at two different bases, or he may try to stack two different blocks on the same base simultaneously. The following notations were appended to all units that involved a movement of more than one block at a time. The first letter of a three letter suffix identifies the distribution of the blocks between the two hands (e.g. one in each hand, two in one hand, etc.). The second letter designated whether the two blocks were identical in size and shape or not. The third letter designated whether the two blocks being moved were guided to different sites, sites horizontally adjacent, sites vertically contiguous, or sites that were near but related.

Ass  Two or more blocks in one hand, identical blocks, same site.
Ads  Two or more blocks in one hand, non-identical blocks, same site.
Bss  Two or more blocks in each hand, blocks within right hand are the same, blocks in the left hand are the same, target site the same.
Bds  Same as above except blocks in either hand are non-identical.
C    Two hands moving one block
Dss  One block held in each hand, blocks are already together in a placement, blocks are identical in size and shape, and are at the same general site, not clearly vertically or horizontally adjacent.
Dsv  One block held in each hand, blocks are identical, blocks are already together in a vertical stack.
Dsj  One block held in each hand, blocks identical, blocks already together in a horizontal alignment.
Dds, Ddv, Ddj  Same as corresponding suffix above, but the two blocks are non-identical.

Ess  One block in each hand, identical blocks, hands initially apart, but the two blocks are moved to the same site.

Esj  Same as above, except blocks are moved together into a horizontal adjacency.

Esv  Same as above, except blocks are moved together into a vertical adjacency.

Esn  Same as above, except blocks are moved to different, but near (apparently related) sites.

Esd  Same as above, except blocks are moved to different sites without any apparent concern for the relation of one side to the other site.

Eds, Edj, Edv, Edn, Edd  Same as corresponding suffix above, but the two blocks are non-identical.

Fss  Block in one hand moved such that another block, not grasped, is also moved. Blocks are identical and in the same general site. This suffix is used when the child communicates an effect from one block to another, e.g. rolls one block by pushing it with another.

Fsv  Hand grasps one block, moves it, and another block, itself not grasped, is also moved. This other block is identical to the first and is on top of the first.

Fsj  Same as above, except the second block is resting horizontally contiguous with the first.

Fdv and Fdj  Same as the respective suffixes above except the second block in each case is non-identical to the first.

Gsv, Gsj, Gdv, and Gdj  Same as the F suffixes, except the child grasps two blocks and affects at least two others that are not grasped.

Hss, Hsj, Hsv, Hsn, Hsd  Same as the E suffixes, except the child is holding two blocks in one hand with a third in the other, or two blocks in each hand. The two hands are initially apart, not together as the result of grasping a block structure in place as in the D suffixes.

M  This suffix was used to designate that the action was an embedded subunit of another action, e.g. the child removes cylinder 1 from cube 1 in order to stack cube 2 on top of cube 1.
This suffix was used whenever a transfer of a block from one hand to the other was mediated by using the mouth.

This suffix was used whenever a unit was not clearly ended, but rather held in suspension while other moves were performed with the other hand.

Examples of the use of the suffixes:

10Esd  Child places two identical blocks, one in each hand, on the table and releases each.

51Fsj  The child grasps one of two identical blocks resting side by side and slides it forward, moving the adjacent block forward as well, and then releases his grasp.

12Ads  Child is holding two non-identical blocks in one hand. He lowers these two blocks to the top of a third block, which is itself non-identical to the block in the hand touching it, and releases the two blocks in his hand.

95Dsv  The child grasps the top block of stack one in one hand and the bottom block of stack one in the other hand and makes a slight adjustment in their vertical alignment.

52Bds  Child grasps several blocks in each hand and releases these blocks. The blocks are non-identical within each hand.

90Cdv  Child knocks over tower using both hands. Tower is composed of several blocks that are non-identical.

ADDITION OF PREFIXES TO DESIGNATE ACTUAL BLOCK USED AND ACTUAL HAND USED

Prior to each unit one number and one letter was added to designate the block used and the hand used respectively. Each block within a set was given a specific number, one of five. The right hand was coded R, the left L, and a T was used when both hands executed the unit. Examples of the prefixes:

2R1C  The child places block 2 on the table with the right hand and releases.

2710Esd  The child places two blocks on the table, one of which is block 2 which is in one hand, an identical block is in the other hand, and then releases both blocks at different sites.
Child touches two uprighted blocks resting adjacent to each other. One block is block 5, the other is identical. One block is touched by one hand, the other block by the other hand. Then the two uprighted blocks are simultaneously sat flat on the table. Then both hands are withdrawn from the blocks.
Appendix B

Frequency of Unique Action Units

The next sixteen pages present individual subject data for each of the 151 unique action units defined in Appendix A. The individual subject is designated by a four letter abbreviation across the top row of each page. The unique action units are designated by the appropriate numerical code down the left margin. The three lines for each action unit marked O, F, and J designate the three testings October, February, and June.
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Note: The table above represents data collected from a study on Group 5, detailing the distribution or performance metrics across different age groups for both males and females.
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Appendix C  Productions

The following appendix presents a frequency count of those productions which showed an appreciable occurrence. A total of forty seven productions, coded as transformations from one production to another, evinced significant frequencies. This total, of course, is less that the two hundred and four transformed productions that occurred at least once. The transformation of productions were coded using the general formula: $P_1$ transformed to $P_2$. $P_1$ refers to the product prior to transformation; $P_2$ is the transformed state of $P_1$ after the child activates some operator. Eleven symbols were used to code products; ten symbols were used to code the operators applied to products. These symbols are defined below.

Products

- $X$  
  A single block

- $S_1$  
  A stack of less than five blocks

- $S_1e$  
  A stack of five blocks

- $S_2$  
  A second stack that coexists on the table with a first stack

- $SA_1$  
  A stack with blocks also aligned horizontally side by side, i.e. a Stack/Alignment

- $SA_1e$  
  A Stack/Alignment involving all five blocks

- $A_1$  
  A horizontal alignment of less than five blocks

- $A_1e$  
  A horizontal alignment of all five blocks

- $A_2$  
  A horizontal alignment that coexists with another horizontal alignment elsewhere on the table.

- $X + S_2$  
  This combination is used only as a $P_2$, the result of transforming some $P_1$ into a single block and a new stack, e.g. removing one block from the top of a two block $S_1$ and placing that block on the top of a new $X$, making an $X + S_2$. The $X$ in this combination refers to the foundation block of the original $S$.

- $1/S_2$  
  This code was used only as a $P_2$ to indicate that $S_1'$ had been transformed into its reciprocal. For example, the child, in the process of removing blocks from $S_1$, constructs a new stack $S_2$ that has the order of blocks inverse that of $S_1$. 

Operators

The child deliberately knocks a product apart.

The child, in the process of lifting or moving a product, causes certain blocks to fall away from $P_1$.

The child grasps, lifts, and deliberately tilts $P_1$ so that certain blocks fall away from $P_1$.

The child adds a single X to $P_1$.

The child adds several Xs, held together in one hand, to $P_1$.

The child repeatedly adds blocks to $P_1$, adding each X successively.

The child removes a single X from $P_1$.

The child removes several blocks from $P_1$ by grasping them simultaneously, e.g. removing a stack of two block from $S_1$.

The child removes blocks repeatedly, removing each X successively.

The child repositions block within $P_1$, but does not place it to another independent product.

Explanation of Tables

These 21 symbols were then combined to describe the transformations of $P_1$ to $P_2$. The following table presents the $P_1$ Operator $P_2$ category in the left column and the frequency of occurrence in the same line. The numbers to the left of the slash mark indicates the frequency of occurrence for all subjects of a given sex and age. The number to the right of the slash mark indicates the number of subjects exhibiting that product transformation within the specified sex and age. The three lines within a product transformation category, the lines designated O, F, and J, indicate the three testings October, February, and June respectively.
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<td>Female</td>
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<td>O</td>
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<td>F 19/6 3/2</td>
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<td>( S_1 \times X )</td>
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<td>F 1/1</td>
<td>O</td>
<td>F 1/1 5/3</td>
<td>O</td>
<td>F 15/5 15/4</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>J 2/1</td>
<td>J</td>
<td>J 1/1 6/3</td>
<td>J</td>
<td>J 4/2 8/5</td>
</tr>
<tr>
<td>( S_1 \times S_1 )</td>
<td>O</td>
<td>F 7/2</td>
<td>O</td>
<td>F 3/3 11/4</td>
<td>O</td>
<td>F 1/1 1/1</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>J 3/2</td>
<td>J</td>
<td>J 2/2 1/1</td>
<td>J</td>
<td>J 11/4 1/1</td>
</tr>
<tr>
<td>( S_1 + S_1 )</td>
<td>O</td>
<td>F 2/1</td>
<td>O</td>
<td>F 3/2 2/2</td>
<td>O</td>
<td>F 3/2 2/2</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>J 2/1</td>
<td>J</td>
<td>J 3/2 2/2</td>
<td>J</td>
<td>J 6/5 3/2</td>
</tr>
<tr>
<td>( S_1 + S_1 )</td>
<td>O</td>
<td>F 2/2</td>
<td>O</td>
<td>F 2/2 1/1</td>
<td>O</td>
<td>F 1/1 1/1</td>
</tr>
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<td></td>
<td>J</td>
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<td>J</td>
<td>J 2/1 1/1</td>
<td>J</td>
<td>J 3/2 1/1</td>
</tr>
<tr>
<td>( S_1 + S_1 )</td>
<td>O</td>
<td>F 3/2</td>
<td>O</td>
<td>F 3/3 3/2</td>
<td>O</td>
<td>F 1/1 1/1</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>J 3/2</td>
<td>J</td>
<td>J 1/1 3/2</td>
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<tr>
<td>( S_1 + S_1 )</td>
<td>O</td>
<td>F 7/3</td>
<td>O</td>
<td>F 6/4 2/2</td>
<td>O</td>
<td>F 5/4 2/2</td>
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<td>J</td>
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<td>J</td>
<td>J 11/6 5/4</td>
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<td>O</td>
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<td>O</td>
<td>F 1/1 1/1</td>
<td>O</td>
<td>F 1/1 1/1</td>
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<tr>
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<td>J</td>
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