ABSTRACT
The lessons and test which comprise this document were used in two experiments concerning the effects of discovery and expository modes of presentation on retention and transfer. The methodology and results of the experiments are briefly summarized. Eight lessons are included: (1) two introductory lessons; (2) three lessons in the expository mode, one on triangles and two on quadrilaterals; and (3) three lessons in the discovery mode, one on triangles and two on quadrilaterals. Also included is a 28-item multiple-choice test dealing with concepts presented in the lessons on quadrilaterals. (Author/DT)
Practical Paper No. 23

LESSONS ON SELECTED GEOMETRY CONCEPTS WRITTEN IN EXPOSITORY AND
DISCOVERY MODES OF PRESENTATION AND A TEST OF CONCEPT MASTERY

By
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Report from the Conditions of Learning
and Instruction Component of Program 1

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September, 1972
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STATEMENT OF FOCUS

Individually Guided Education (IGE) is a new comprehensive system of elementary education. The following components of the IGE system are in varying stages of development and implementation: a new organization for instruction and related administrative arrangements; a model of instructional programming for the individual student; and curriculum components in prereading, reading, mathematics, motivation, and environmental education. The development of other curriculum components, of a system for managing instruction by computer, and of instructional strategies is needed to complete the system. Continuing programmatic research is required to provide a sound knowledge base for the components under development and for improved second generation components. Finally, systematic implementation is essential so that the products will function properly in the IGE schools.

The Center plans and carries out the research, development, and implementation components of its IGE program in this sequence: (1) identify the needs and delimit the component problem area; (2) assess the possible constraints—financial resources and availability of staff; (3) formulate general plans and specific procedures for solving the problems; (4) secure and allocate human and material resources to carry out the plans; (5) provide for effective communication among personnel and efficient management of activities and resources; and (6) evaluate the effectiveness of each activity and its contribution to the total program and correct any difficulties through feedback mechanisms and appropriate management techniques.

A self-renewing system of elementary education is projected in each participating elementary school, i.e., one which is less dependent on external sources for direction and is more responsive to the needs of the children attending each particular school. In the IGE schools, Center-developed and other curriculum products compatible with the Center's instructional programming model will lead to higher morale and job satisfaction among educational personnel. Each developmental product makes its unique contribution to IGE as it is implemented in the schools. The various research components add to the knowledge of Center practitioners, developers, and theorists.
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ABSTRACT

The lessons and test which comprise this paper were used in two experiments which studied the effects of discovery and expository modes of presentation on retention and transfer. The methodology and results of the experiments are briefly summarized. Eight lessons are included: (a) two introductory lessons; (b) three lessons in the expository mode, one on triangles and two on quadrilaterals; and (c) three lessons in the discovery mode, one on triangles and two on quadrilaterals. Also included is a 28-item multiple-choice test dealing with concepts presented in the quadrilateral lessons.
INTRODUCTION

Although various definitions of "discovery" and "expository" methods of teaching have been offered by psychologists (cf., Scott, 1970, Ch. 2), the discovery and expository lessons contained in this paper use methods of presentation which conform to Worthen's (1968) definitions of these terms. Worthen defined discovery and expository teaching in terms of the presentation sequence of stimulus materials. Discovery teaching was defined by him as a method in which verbalization of each concept or generalization is delayed until the end of the instructional sequence by which the concept or generalization is to be taught" (p. 3). Expository teaching was defined as "a method in which the verbalization of the required concept or generalization is the initial step in the instructional sequence by which the concept or generalization is to be taught" (p. 3).

The materials which comprise this paper were used in two experiments which compared the effects of discovery and expository methods of teaching. Advocates of the discovery method have suggested that concepts learned by this method are remembered better than concepts learned by an expository method. Further, they have suggested that students who experience a discovery method of teaching not only learn the material being taught, but acquire a different approach to learning which can transfer to new situations. To test these claims, experiments were carried out which (a) compared the effects of discovery and expository methods on retention, and (b) tested whether teaching one set of concepts by either a discovery or an expository method influenced the learning of a second set of concepts.
To compare the effects of discovery and expository methods of presentation on retention, four "quadrilateral" concepts (quadrilateral, rhombus, parallelogram, and trapezoid) were presented in lessons written in either a discovery or an expository mode. The learning of the quadrilateral concepts required knowledge of prerequisite concepts such as line segment and angle. The prerequisite concepts were presented in two introductory lessons.

The concepts taught in both the introductory and the quadrilateral lessons are frequently introduced in sixth-grade textbooks and were, therefore, thought to be appropriate for the sixth graders who took part in the experiment. Great care was taken to insure that the reading level of the materials was appropriate for the average sixth grader. While students studied the lessons, proctors were available to answer any questions regarding the meaning of difficult words. Few such questions were raised. The reading level was, therefore, believed to be well-chosen.

Knowledge of the concepts was tested by a multiple-choice test which is included in this paper. Since it was of interest to compare the retention of the concepts by the discovery and expository groups, children were tested either 1, 11, or 21 days after they studied the lessons. The multiple-choice test administered was a revision of a test developed by Frayer (1970). A completion-type test was embedded in the lessons themselves to focus the attention of both the expository and discovery groups on the types of questions which would be asked in the multiple-choice test. Production and multiple-choice test items were matched in terms of the number and nature of the questions dealing with each concept.
Results of the study showed that the materials presented in the discovery mode produced the greatest long-term retention, with the test scores of subjects receiving these materials actually increasing over the 21-day period. Test scores of subjects who received the materials presented in the expository mode decreased over the 21-day period.

To determine whether practice in learning from material presented in either the discovery or expository mode would affect new learning in one of these modes, two sets of concepts were presented. First, six "triangle" concepts (triangle, equilateral triangle, isosceles triangle, acute angle triangle, right angle triangle, and obtuse angle triangle) were presented. The triangle and quadrilateral concepts were presented to different groups of children in four lesson sequences: discovery-discovery, expository-expository, discovery-expository, and expository-discovery.

The hypothesis was that students who learned both sets of concepts by the same instructional method would learn the second set of concepts better than students who learned the sets of concepts by different methods. Higher performance would result from practice in learning by the particular instructional method. To test the hypothesis, mastery of the second set of concepts (the quadrilateral concepts) was assessed by administering the multiple-choice test immediately after completion of the lessons. Results showed that knowledge of the quadrilateral concepts did not differ for the various sequences of discovery and expository modes of presentation. Thus, practice in learning by a particular instructional method did not facilitate learning.
The lessons and the tests used in the two experiments which have been described are included in this paper. The lessons are almost entirely self-sufficient. Services of an instructor or proctor are needed only to read the instructions and the word list and to assist students who have difficulty reading particular words or items. The content of the lessons and test are outlined below.

Introductory Lessons

The introductory lessons present concepts which are prerequisite to learning triangle and quadrilateral concepts. The lessons have a modified linear programming format which requires the student to respond to each item. Immediate feedback is provided for each item. The material contained in the introductory lessons is intended to be comprehensive, so that students may be able to understand the lessons even if they have had no prior training in geometry. The concepts presented in each introductory lesson are as follows:

**Introductory Lesson I**—point, line segment, line, ray, angle, right angle, acute angle, and obtuse angle.

**Introductory Lesson II**—open curve, closed curve, simple closed curve, plane, solid, polygon, intersect, parallel, opposite sides, and equal length.

Expository Lessons

In the expository lessons the name of the concept is given, followed by a definition which points out the relevant attributes of the concept. This is followed by a series of figures, each accompanied by a statement indicating whether or not the figure is an example of the concept, why it is or is not, and what the attributes of the figure are. No overt response is required
of the student while studying the lessons. The content of each ex-
pository lesson is as follows:

**Triangle Lesson**—triangle, equilateral triangle, isosceles triangle,
acute angle triangle, right angle triangle and obtuse angle triangle.

**Quadrilateral Lesson I**—quadrilateral, rhombus, and completion-type
tests for quadrilateral and rhombus.

**Quadrilateral Lesson II**—parallelogram, trapezoid, and completion-

---

**Discovery Lessons**

In the discovery method of presentation the concept is introduced
by a general statement giving the concept name and telling the student
that the following figures will help him to learn the concept. This
statement is followed by a series of figures. The student is given
instructions to examine these figures carefully, noting their likenesses
and differences. The figures are accompanied by questions which encourage
the student to focus on certain attributes of the figures. The series
of figures is followed by questions which ask the student to identify
the figures which are alike, and how those figures are alike. Feedback
is provided for these questions. Finally the concept name is stated
again, followed by a definition which points out the relevant attributes
of the concept.

The examples used in the expository and the discovery lessons are
precisely the same. Thus, the number, sequence, and ratio of positive
to negative examples of the concept are the same for both methods of
presentation. The content of each discovery lesson is as follows:
Triangle Lesson—triangle, equilateral triangle, isosceles triangle, acute angle triangle, right angle triangle, and obtuse angle triangle.

Quadrilateral Lesson I—quadrilateral, rhombus, and completion-type tests for quadrilateral and rhombus.

Quadrilateral Lesson II—parallelogram, trapezoid, and completion-type tests for parallelogram and trapezoid.

Tests of Geometry Knowledge

The items for both the completion-type test and the multiple-choice test were taken from tests developed by Frayer (1970). Five types of items were used, testing knowledge of attribute examples, concept examples, concept nonexamples, relevant attributes, and concept definitions. The two tests were parallel in content, item types, and number of items, but the completion-type test required production of answers while the multiple-choice test required recognition of answers.

Completion-Type Test—The items of this test are contained in Quadrilateral Lessons I and II, both expository and discovery modes. These items may be used to determine how well students know each concept immediately after learning.

Multiple-Choice Test—This test is entitled "Tests of Geometry Knowledge: Form JS" and is included as the final component of this paper. The test may be given at some time interval after completion of the lessons in order to determine retention of learning.
REFERENCES


Word List

Here is a list of words that are used in this lesson. Some of the words may be new ones for you.

Look carefully at each word as I read it to you. Then say the word aloud with me when I repeat it.

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1. A point is a location in space. To represent points, we use dots. A dot is not really a point, but it stands for a point.

In the box below, how many points are represented by dots? ________

2. We name a point by placing a capital letter near the dot which represents it.

How many points are pictured? ________

What are the names of the points? ________

A, B, and C
3. A path connecting one point to another point is called a curve.

The curve shown below connects points _______ and _______.

4. Curves may be given the name of the points which they connect. For example, the curve shown below may be called FG or GF.

Now look at the curve below.
It may be called _______ or _______.  
CD or DC
5. A line segment is the shortest curve that can be drawn between two points. It is straight and does not bend. This might sound strange at first, but this is so. A line segment is straight but it is only a special kind of a curve.

Look at the drawing below. Curve 1 and curve 2 both connect points X and Y.

Which curve is shorter, curve 1 or curve 2?

Are the three figures shown below curves?

Yes

Which of the curves is a line segment? MN (or NM)
6. The line segment shown below ends with points C and D. C and D are called endpoints of line segment CD.

What are the endpoints of the line segment shown below? __________

What are the endpoints of the line segment shown below? __________

R and S. The endpoints need not be represented by dots. In fact, line segments are usually drawn without dots.
7. Look at the line segment CD shown below. It is made up of the endpoints C and D, and all points on the line segment between C and D. There are more points on the line segment CD than you could count.

Below is an example of just a few of the points on a line segment. There is a letter above each one so that you can name it.

There are many, many points on the line segment shown below, but only points W and X have been named. Mark two other points on the line segment with dots, and name them Y and Z.

or any other two points between W and X.
8. A short way of writing "line segment DE" is DE.

What is a short way of writing "line segment PQ"?

What is a short way of writing "line segment RS"?

9. XY is a short way of saying "line segment XY." When you see XY you read it by saying "line segment XY."

What would you read when you see BC?

What would you read when you see RT?
10. A line segment has endpoints, but a line has no endpoints. A line is a set of points which goes on and on in both directions. Arrowheads on the drawing of a line are used to show that the line goes on and on and on in both directions, forever.

The drawing below shows a picture of line segment RS (RS).

Does this line segment have endpoints? ___

Does the line segment go all the way to the sides of the page? ___

Could you make the line segment longer by using your ruler and making it go on until it reached both sides of the page? ___

If the page were wider, could you make the line segment even longer? ___

Is all of the line shown in this drawing? ___

Yes (Remember you do not need dots to represent the endpoints.)

No.

Yes

Yes

No (The line goes on and on in both directions.)
11. Find the dot named A on the line shown below. This dot represents one point on the line. It is called point A. There are so many points on the line that we cannot mark them all. Only point A is marked.

What are the names of the points marked on the line shown below? R, S, and T

12. We saw that a line segment was named by the two letters which name its endpoints. A line has no endpoints so we have to name it by picking any two points on the line. For example, the line shown below could be called line EF, line FG, or line EG.

Or, the line shown below could be called XZ or ZX or XO.

What are three names that could be given to the line drawn below? line XY (or YX), line YZ (or ZY), and line XZ (or ZX)
13. Sometimes we are interested in only part of a line. On the line shown below, find line segment XY (XY).

\[\text{X} \quad \text{Y} \quad \text{Z}\]

Is line segment XY a part of line XZ? Yes

Is line segment YZ a part of line XZ? Yes

Is line segment XZ a part of line XZ? Yes
14. In question 13 you saw that a line segment is part of a line. Now we will look at another part of a line. It is called a ray. It begins at a point and goes on and on in one direction without end.

Look at the drawing of a line below.

M

N

Trace with your pencil on the line shown above. Begin at point M and go on and on to the right past point N. The part of the line you traced is called a ray.

Look at the drawing of a ray shown below.

V

W

It has one endpoint. The other end has an arrow to show that it really goes on and on. The endpoint of this ray is V. Do not be confused by the arrowhead. Remember that the arrow does not represent a point but instead shows that the ray goes on and on in the direction it shows.

Does the ray have an endpoint? ________ Yes

What is the name of the endpoint? ________ Y

Does the ray go on and on in one direction without end? ________ Yes

Does the ray go on and on in both directions without end? ________ 25 No
15. When we want to give a ray a name, we use two letters. The letter of the endpoint comes first. For example, the endpoint of the ray shown below is P, so we will call the ray PQ.

When you see the name of a ray you know that the first letter is always the endpoint.

Now look at the ray shown below.

What is the endpoint of this ray? __________

What is the name of this ray? (Remember that the first letter is always the endpoint) __________

What is the name of the endpoint of a ray named GT? __________
16. We saw that a line goes on and on in two directions and has no endpoints. A ray goes on and on in one direction and has one endpoint. Now look at the figure below.

Does this figure go on and on in both directions or in only one direction? 

How many endpoints does it have? 

Does the figure represent a line or a ray? 

17. Look at the drawing below.

Does the drawing represent a set of points that go on and on in both directions or in only one direction? 

How many endpoints does it have? 

Does the drawing represent a line or a ray?
18. Sometimes two rays have the same endpoint. When two rays begin at the same point, they form an angle.

Look at the figure drawn below.

What is the name of the endpoint of ray FC? ________ F
What is the name of the endpoint of ray FL? ________ F
Do rays FC and FL have the same endpoint? ________ Yes (F)

Now look at the next figure.

How many rays are pictured? ________ 2
Do the rays have the same endpoint? ________ Yes
19. In the drawings below, angles formed by rays beginning at the same point are marked with an X.

On the drawings below, put an X by each angle.

Which of the following figures shows an angle? ________
20. Sometimes in pictures of angles, we use line segments instead of rays. For example, the letter N is made up of line segments which form two angles. Both of these angles are marked with an X on the large letter N shown below.

![Letter N with X marks](image)

Put an X in the three angles formed by the letter M shown below.

![Letter M with X marks](image)

Put an X in each angle formed by the line segments in the following figures.

![Rectangles and triangle with X marks](image)
21. Look at the angle made by rays XY and XZ in the figure below.

![Diagram of an angle with rays XY and XZ]

This is a picture of a special kind of angle called a **right angle**. This kind of angle may also be called a "square corner."

In some of the lessons which you will be doing you will be asked if an angle is a right angle. You can check to find out whether or not an angle is a right angle by placing your ruler in the corner like this:

![Diagram of a ruler in a corner]

Put the corner of the ruler at X and let the bottom of the ruler lie along the bottom ray XZ. Be sure that the corner of the ruler is touching the corner at X. Then look at ray XY. If ray XY lies along the end of the ruler, as is shown here, then you can say that this is a right angle.
22. Now look at the angles drawn below. Use your ruler to find out which one is a right angle. Which angle is a right angle? ____________

a.  

b.  

23. Now look at the drawings below. Which one looks like a right angle? ____________

a.  

b.  

Use your ruler to check your answer.

24. Use your ruler to find the right angles in the figure below. Put an X in the right angles.

a.  

b.
25. Look at this angle. It is not a right angle.

Use your ruler to be sure.

You will notice that this angle is "smaller" than a right angle. If you wish to think of it this way you might say that the space between the rays is smaller. This is called an acute angle.

You might also think of it in terms of the hands on the clock. At a quarter till twelve or a quarter past twelve the hands would make a right angle. At less than a quarter till or a quarter past twelve, for example ten till or five after, the hands make acute angles.

This is another acute angle.

Use your ruler to be sure of this.
26. Look at this figure.

Is it a right angle? _____________  No
Is it an acute angle? _____________  Yes
Use your ruler.

27. Look at this figure:

Is it a right angle? _____________  Yes
Is it an acute angle? _____________  No
28. Now look at this angle. It is not a right angle and it is not an acute angle either. Use your ruler to check.

It is "bigger" than a right angle, or if you care to think of it this way, the space between the rays in this angle is greater than the space between the rays of a right angle. This is called an obtuse angle.

Use your ruler and check the angle. Is it "bigger" than a right angle? Yes

If you wish to think of the clock again, you might say that when it is more than a quarter before or after twelve, for example at twenty till or twenty-five after, the hands make an obtuse angle.

Now look at the two words carefully. Do not confuse them as you will need to be able to tell acute and obtuse and right angles apart many times in the lessons which follow.

ACUTE. Repeat it to yourself a few times, then think of an acute angle or find an example. Now write the word here __________.

OBTUSE. Repeat this to yourself a few times and think of an obtuse angle or find an example. Now write the word here __________.

Compare the words and remember how they are different.
29. Now look at this angle.

Is this a right angle? 

Is this an acute angle? 

Is this an obtuse angle? 

Use your ruler to check. Remember the difference between acute and obtuse angles.

Write the difference here ____________________________

No

No

Yes

An acute angle is smaller than a right angle. An obtuse angle is bigger than a right angle.
30. In the figure below, use your ruler to find the acute angle or angles. Put an A in each acute angle.

31. In the figure below, use your ruler to test the angles. Put an R in each right angle, an O in each obtuse angle, and an A in each acute angle.
INTRODUCTORY LESSON II
Word List

Here is a list of words that are used in this lesson. Some of the words may be new ones for you.

Look carefully at each word as I read it to you. Then say the word aloud with me when I repeat it.

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<th>8. common</th>
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<tr>
<td>2. simple</td>
<td>9. intersect</td>
</tr>
<tr>
<td>3. non-simple</td>
<td>10. parallel</td>
</tr>
<tr>
<td>4. imagine</td>
<td>11. distance</td>
</tr>
<tr>
<td>5. surface</td>
<td>12. opposite</td>
</tr>
<tr>
<td>6. geometric</td>
<td>13. length</td>
</tr>
<tr>
<td>7. polygon</td>
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</table>
1. A curve which has endpoints is called an open curve.

Using your pencil, trace the drawing shown below.

Did you come back to where you started? ________

Does the drawing have endpoints? ________
Remember that endpoints need not have dots.

Which of the drawings below shows an open curve; that is, which one does not come back to where it started? ________

a.  

b.  

2. Which of these drawings is an open curve? ________

a.  

b.  

b.  

a.
3. A figure that is drawn by returning to the starting place has no endpoints and is called a **closed** curve. Trace the next drawing with your pencil.

Did you come back to where you started? __________      Yes
Does the drawing have endpoints? __________     No
Which of the following figures has no endpoints and so could be called a closed curve? __________

\[ a. \quad \quad b. \]

4. Which of these figures is a closed curve? __________      a

\[ a. \quad \quad b. \]

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5. A closed curve which can be drawn without crossing itself at any point is called a simple closed curve. Look at the drawing below.

![Simple Closed Curve](image)

Does the drawing cross itself when you draw it? _____ No
Is it a simple closed curve? _____ Yes


![Diagram](image)

Which is a simple closed curve? _____ a
Which of the figures below is a simple closed curve? _____ a
7. The drawing shown below crosses itself at one point. It is **not** a simple closed curve. We will call this kind of curve a **non-simple** closed curve.

![Non-simple closed curve diagram]

Which of the drawings below is a non-simple closed curve? __________

a. 

b. 

Which of the drawings below is a non-simple closed curve? __________

a. 

b. 
8. Think of the top of a table. Now imagine that the table is beginning to grow longer and longer, and wider and wider, getting bigger without end. A flat surface like this that goes in all directions without end is called a plane. The top of your desk is flat and is something like a plane. Is your desk like a whole plane or only part of a plane?

9. Figures which are made up of points all in the same plane are called plane figures. Look at the figure drawn below. All of its points are in one plane, so it is called a plane figure.

Now look at the next picture. It is a picture of a block. Some of the points which make up the block are in different planes. Since its points are not in one plane, the block is not a plane figure. We call figures like this that have points in different planes solid figures. The dotted lines show the parts of the figure that we can't see.
9. (continued)

Which of the following figures represents a plane figure? ________

a.  

b.  

Which of the following figures represents a plane figure? ________

a.  

b.  

Which of the following drawings represents a solid figure? ________

a.  

b.  

45
9. (continued)
Which of the following drawings represents a solid figure? _______

a.  

b.  

10. Connect the points below with line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$.

A

B

C

Is your figure a simple closed curve? _______
Is your figure a plane figure? _______

Yes  
Yes
11. We give a special name to a plane simple closed curve which is made by joining line segments. We call it a **polygon**. Look at the figure below.

```
Is it a simple closed curve?    _____________  Yes
Is it a drawing of a plane figure? ___________  Yes
Is it made of line segments?    ___________  Yes
Is it a polygon?               ___________  Yes
```

12. Now look at this figure.

```
Is it a drawing of a plane figure? _____________  Yes (To be a plane figure, figure need not be a closed figure.)
Is it made up of line segments?    ___________  Yes
Is it a simple closed curve?      ___________  No
Is it a polygon?                 ___________  No
```
13. Now look at the drawing below.

Is it a drawing of a plane figure?  Yes
Is it a simple closed curve?  Yes
Is it made up of line segments?  No
Is it a polygon?  No

14. Two lines are drawn below. These two lines go through the same point P. We call P a common point of the lines. When two lines have a common point, we say the lines intersect.

Do the lines have a common point?  Yes
Do the lines intersect?  Yes
14. (continued)

Remember that the picture of a line shows only part of a line. Do the lines shown below intersect even though the picture doesn't show a common point? (Remember that both lines go on and on.)

Yes

15. When two lines in the same plane do not intersect, we say that the lines are parallel.

Look at the following figure.

Do the lines intersect? __________ No
Are the lines parallel? __________ Yes
There is one easy test you can use to find out whether or not two lines are parallel. Use your ruler as follows:

Place the end of your ruler flat against the top line and draw a line along one side like this.

The angle marked X is a right angle, as we have seen.

Now turn the ruler around and check like this:

If the angle marked Y is also a right angle then you can say that the lines are parallel.

So in the rest of these lessons whenever you are asked if lines are parallel, or when you are asked if you notice something about a pair of lines, draw a line and make the test as you have been shown above. Be sure to draw the line whenever you want to make this test.
15. (continued)

Which of the pairs of lines below is parallel? 

Use your ruler to check your answer.

\[ \text{a.} \quad \text{b.} \]

Which of the following pairs of lines is parallel? (Remember to draw the line and use your ruler.)

\[ \text{a.} \quad \text{b.} \]


Is MN parallel to PO? 

Is MP parallel to NO? 

Remember to use your ruler.

How many pairs of parallel sides does this figure have? 

46

No

0 pairs
17. Look at this figure.

Is $EF$ parallel to $HG$? ______ Yes
Is $EH$ parallel to $FG$? ______ Yes
How many pairs of parallel sides does this figure have? ______ 2 pairs
Use your ruler to check your answer.

18. Now look at this figure.

Is $ST$ parallel to $VU$? ______ No
Is $SV$ parallel to $TU$? ______ Yes
How many pairs of parallel sides does this figure have? ______ 1 pair
($SV$ is parallel to $TU$)
Use your ruler to check your answer.
19. Look at the polygon shown below. \( \overline{WX} \) and \( \overline{ZY} \) are opposite sides. \( \overline{WZ} \) and \( \overline{XY} \) are opposite sides.

\[ \begin{array}{c}
W \\
\hline
\hspace{2cm} \hspace{2cm} X \\
\hspace{2cm} \hspace{2cm} \\
Z \\
\hline
\hspace{2cm} \hspace{2cm} Y \\
\end{array} \]

Now look at the next figure.

\[ \begin{array}{c}
J \\
\hline
M \\
\hline
K \\
\hline
L \\
\end{array} \]

What side is opposite from side \( \overline{JK} \)？

ML (or LM)

What side is opposite from side \( \overline{KL} \)？

JM (or MJ)
20. Look at the next figure.

![Image of a quadrilateral with vertices L, M, O, and N]

Are $\overline{LM}$ and $\overline{MN}$ opposite sides?  

Yes

Are $\overline{LM}$ and $\overline{ON}$ opposite sides?  

Yes

Are $\overline{LO}$ and $\overline{MN}$ opposite sides?  

Yes

21. Now look at the next figure.

![Image of a rectangle with vertices A, B, C, and D]

Does $\overline{AB}$ seem to be the same length as $\overline{DC}$?  

Yes

Does $\overline{AB}$ seem to be the same length as $\overline{AD}$?  

No
TRIANGLE LESSON: EXPOSITORY MODE
Word List

Look carefully at each word as I read it to you. Then say the word aloud with me when I repeat it.

1. triangle 5. obtuse angle triangle
2. polygon 6. equilateral triangle
3. right angle triangle 7. isosceles triangle
4. acute angle triangle
Hello,

You are now ready to start your third lesson. This lesson is not quite like the last two lessons. You have been told about the difference in the form of these lessons. Now we want to point out some things to keep in mind as you go through this and the following lessons.

You may have noticed that the last two lessons were about lines and parts of lines (that is, line segments and rays) and about angles. You learned how to tell whether or not two lines are parallel. And you learned how to tell a right angle, an acute angle, and an obtuse angle apart. As you study the figures in this and the following lessons you will use what you learned.

To learn about the figures in the next lesson you will only have to learn about the line segments (or sides, as we will call them) and the angles. When you are learning about a figure look at the sides, find out if they are the same length, and find out whether they are parallel. Look at each angle and check whether it is a right angle or an acute angle or an obtuse angle. You do not have to think of anything else like whether it is big or small, or thin or fat, or straight or turned around. Just think about the sides and the angles and you will learn about the figures.
1. A special kind of polygon made up of three line segments is called a triangle. The following six figures will help you learn how to tell whether or not a figure is a triangle. Look at them carefully and notice how they are alike and how they are different. The polygon drawn below is a triangle.

It has three sides.

2. Here is another figure.

This is not a triangle. It is made up of more than three sides.
3. Look at the next drawing. This is a triangle.

\[ \begin{array}{c}
\text{A} \\
\text{C} \\
\text{B}
\end{array} \]

It has three sides.

4. Now look at this figure.

\[ \begin{array}{c}
\text{A} \\
\text{E} \\
\text{D} \\
\text{C}
\end{array} \]

This is not a triangle. It has more than three sides.
5. This drawing is a triangle. Count the line segments.

It has three sides.

6. And this drawing is a triangle because it has three sides.

To find out whether or not a polygon is a triangle you have to do only one thing; and that is to make sure that the figures have three and only three sides. Check each example.
7. In the rest of this lesson you will be studying about special kinds of triangles. One special kind of triangle is called an equilateral triangle. In an equilateral triangle the three sides are equal in length. The next four figures will help you to learn how to tell whether or not a triangle is equilateral. Look carefully at the figures and notice how they are alike and how they are different. The figure below is an equilateral triangle.

Side AB is 1-5/8 inches long.
Side AC is 1-5/8 inches long.
Side CB is 1-5/8 inches long.

Check these measures with your ruler. Notice that the three sides are equal in length.
8. Now look at this figure. It is not an equilateral triangle. The three sides are not equal in length.

Check the measures of the sides.

- Side AB is $1\frac{3}{4}$ inches long.
- Side AC is $1\frac{3}{4}$ inches long.
- Side CB is $2\frac{1}{2}$ inches long.

\[ AC = AB \] but \[ CB \neq AB \] (\# means "not equal to")

9. Now look at this drawing. It is an equilateral triangle.

Check the measures of the sides and you will see:

- AB is $1\frac{1}{4}$ inches long.
- AC is $1\frac{1}{4}$ inches long.
- CB is $1\frac{1}{4}$ inches long.

The sides are of equal length.
10. The next figure is not an equilateral triangle.

Check the measures of the sides.

\( \overline{AB} \) is 2 inches long.

\( \overline{AC} \) is 1-1/2 inches long.

\( \overline{CB} \) is 1 inch long.

The sides are not equal in length.

There is only one thing you must know if you want to know whether or not a triangle is equilateral. You must find out if the three sides are equal in length. Check each of the examples.
11. Another kind of triangle is an **isosceles triangle**. It has two sides which are equal in length. The following six figures will help you learn whether or not a figure is an isosceles triangle. Look at them carefully and notice how they are alike and how they are different. This is an isosceles triangle.

![Isosceles Triangle](image)

Check the measures of the sides.

\[ \overline{AB} \text{ is } 1-1/2 \text{ inches long.} \]
\[ \overline{AC} \text{ is } 1-1/2 \text{ inches long.} \]
\[ \overline{CB} \text{ is } 1 \text{ inch long.} \]

Two of the sides are equal in length.

12. Look at the drawing below. It is **not** an isosceles triangle. It does not have two sides equal in length.

![Non-Isosceles Triangle](image)

Check the measures of the sides.

\[ \overline{AB} \text{ is } 1 \text{ inch long.} \]
\[ \overline{AC} \text{ is } 1-1/2 \text{ inches long.} \]
\[ \overline{CB} \text{ is } 1-3/4 \text{ inches long.} \]
13. Here is another figure. This is an isosceles triangle.

\[ \text{A} \quad \text{C} \quad \text{B} \]

Check the measures of the sides.

- \(\overline{AB}\) is 2-1/2 inches long.
- \(\overline{AC}\) is 1-3/4 inches long.
- \(\overline{CB}\) is 1-3/4 inches long.

Two of the sides are equal in length.

14. Here is another figure. This is not an isosceles triangle. No two sides are equal in length.

\[ \text{A} \quad \text{C} \quad \text{B} \]

Check the measures of the sides.

- \(\overline{AB}\) is 1-3/4 inches long.
- \(\overline{AC}\) is 1 inch long.
- \(\overline{CB}\) is 1-1/2 inches long.
15. The drawing below is an isosceles triangle.

Check the measures of the sides.

- \( \overline{AB} \) is 1-1/2 inches long.
- \( \overline{AC} \) is 1-1/2 inches long.
- \( \overline{CB} \) is 1-1/2 inches long.

At least two of the sides are equal in length.

16. And this is an isosceles triangle.

Check the measures of the sides.

- \( \overline{AB} \) is 1-1/2 inches long.
- \( \overline{AC} \) is 2-1/2 inches long.
- \( \overline{BC} \) is 1-1/2 inches long.

Two of the lines are equal in length.

To find out whether or not a triangle is an isosceles triangle there is one thing you must find out, and that is whether or not at least two of the lines are equal in length. Check the examples.
17. Another special kind of triangle is called an acute angle triangle. This figure is an acute angle triangle. In acute angle triangles all angles are acute. The next six figures will help you learn how to tell whether or not a figure is an acute angle triangle. Look at them carefully and note how they are alike and how they are different. This is an acute angle triangle.

Check the angles.

The angle at A is acute.
The angle at B is acute.
The angle at C is acute.

All the angles are acute.

18. Now look at this figure. It is not an acute angle triangle. Not all of the angles are acute angles.

Check the angles.

The angle at A is a right angle.
The angle at B is an acute angle.
The angle at C is an acute angle.
19. The next figure is an acute angle triangle. Look at it carefully and you will see that each angle is an acute angle.

Check the angles.

The angle at A is acute.
The angle at B is acute.
The angle at C is acute.

20. Here is another figure.

This is not an acute angle triangle.

Check the angles.

The angle at A is acute.
The angle at B is acute.
The angle at C is obtuse.

Not all of the angles are acute angles. One is obtuse. Therefore, the triangle is not an acute angle triangle.
21. Now look at this figure. This is an acute angle triangle.

Check the angles.

The angle at A is acute.
The angle at B is acute.
The angle at C is acute.

22. And this triangle is an acute angle triangle.

Check the angles.

The angle at A is acute.
The angle at B is acute.
The angle at C is acute.

When you know by looking at the figure and counting the line segments that a figure is a triangle, there is only one other thing you have to find out to tell whether or not it is an acute angle triangle. You must find out whether or not all angles are acute angles.
23. Still another special kind of triangle is called a right angle triangle. A right angle triangle is a triangle that has one right angle. The next six figures will help you learn how to tell whether or not a triangle is a right angle triangle. Look at them carefully and notice how they are alike and how they are different. This figure is a right angle triangle.

Look at it. Use your ruler as you learned in the first lesson to help you check the three angles in this figure.

The angle at A is an acute angle.
The angle at B is an acute angle.
The angle at C is a right angle.

Notice that the angle at C is a right angle and the angles at A and B are acute.
24. Now look at this figure. It is not a right angle triangle because it has no right angle.

Look at it. Use your ruler to help you check the three angles.

The angle at A is acute.
The angle at B is acute.
The angle at C is acute.

None of the angles is a right angle so it is not a right angle triangle.

25. Here is another figure. One of the angles is a right angle.

This is a right angle triangle. Use your ruler to check the angles.

The angle at A is an acute angle.
The angle at B is a right angle.
The angle at C is an acute angle.

Note that the angle at B is a right angle.
26. Look at this figure. No angle is a right angle. 

This is not a right angle triangle.

The angle at A is acute.
The angle at B is acute.
The angle at C is obtuse.

27. This figure is a right angle triangle. Check the angles and you will see that it has one right angle.

The angle at A is a right angle.
The angle at B is an acute angle.
The angle at C is an acute angle.
28. And this figure is a right angle triangle. When you check the angles you should expect to find one right angle.

Check the angles.

The angle at A is an acute angle.
The angle at B is an acute angle.
The angle at C is a right angle.

When you know that a figure is a triangle, there is only one thing you must find out in order to know whether or not it is a right angle triangle: you must find out if one of the angles is a right angle.
29. The last kind of triangle we will study is called an **obtuse angle triangle**. One of the angles of an obtuse angle triangle is always obtuse. The following six figures will help you learn how to tell whether or not a triangle is an obtuse angle triangle. Look at them carefully and notice how they are alike and how they are different. This is an obtuse angle triangle.

![Diagram of an obtuse angle triangle]

Use your ruler to check the angles. One is obtuse.

- The angle at A is acute.
- The angle at B is acute.
- The angle at C is obtuse.

30. Look at this figure. It does not have an obtuse angle.

![Diagram of a right triangle]

This is not an obtuse angle triangle.

Check the angles.

- The angle at A is an acute angle.
- The angle at B is an acute angle.
- The angle at C is a right angle.
31. Now look at this drawing.

This is an obtuse angle triangle.

Check the angles.

The angle at A is obtuse.
The angle at B is acute.
The angle at C is acute.

One of the angles is an obtuse angle.

32. Here is another drawing. It does not have an obtuse angle and so it is not an obtuse angle triangle.

Check the angles.

The angle at A is acute.
The angle at B is acute.
The angle at C is acute.
33. This figure is an obtuse angle triangle.

Check the angles and you will see that one is an obtuse angle.

The angle at A is acute.
The angle at B is acute.
The angle at C is obtuse.

34. And this is an obtuse angle triangle.

Check the angles and notice that one is an obtuse angle.

The angle at A is acute.
The angle at B is obtuse.
The angle at C is acute.

When you know that a figure is a triangle there is one other thing you must find out in order to tell whether or not it is an obtuse angle triangle: You must find out whether or not one of the angles is obtuse.
QUADRILATERAL LESSON I: EXPOSITORY MODE
Word List

Look carefully at each word as I read it to you. Then say the word aloud with me when I repeat it.

1. quadrilateral
2. trapezoid
3. parallelogram
4. rhombus
Hello,

This lesson is not quite like the last lessons. You have been told about the difference in the form of these lessons. Now we want to point out some things to keep in mind as you go through this and the following lessons.

You may have noticed that the last two lessons were about lines and parts of lines (that is, line segments and rays). You learned how to tell whether or not two lines are parallel. As you study the figures in this and the following lessons, you will use what you learned.

To learn about the figures in the next lesson you will only have to learn about the line segments, or sides as we will call them. When you are learning about a figure, look at the sides. Find out if they are the same length or not and whether they are parallel or not. You do not have to think of anything else about the figure, such as whether it is big or small, or thin or fat, or straight or turned around. Just think about the sides. That is how you will learn about the figure.
1. All polygons which are made up of four line segments are called quadrilaterals. The polygon drawn below is a quadrilateral.

Look at it. It has four sides.

2. Here is another figure. It is not made from four sides.

This is not a quadrilateral. It does not have only four sides. It has five sides.
3. Look at the next drawing. It has four sides, so it is a quadrilateral.

4. Look at the next drawing. It does not have four sides. This is not a quadrilateral. It has only three sides.
5. This drawing is a quadrilateral.

It has four and only four sides.

You know without counting that if it is a quadrilateral it can only have four sides.

6. And this drawing is a quadrilateral. It also has just four sides.

To find out whether or not a polygon is a quadrilateral, you have to do only one thing. You have to count the number of sides and make sure that there are four, and only four.
7. One special kind of quadrilateral is a rhombus. Any figure which has all sides of equal length is called a rhombus. The following six figures will help you tell whether or not a figure is a rhombus. Look at them carefully and notice how they are alike and how they are different. This figure is a rhombus.

![Rhombus Diagram]

The measures of the lengths of the sides are written below. Check them with your ruler.

- \( \overline{AB} \) is 1-1/4 inches long.
- \( \overline{AD} \) is 1-1/4 inches long.
- \( \overline{BC} \) is 1-1/4 inches long.
- \( \overline{DC} \) is 1-1/4 inches long.

Notice that \( \overline{AB} = \overline{AD} = \overline{BC} = \overline{DC} \)
8. Now look at the drawing below. It is not a rhombus.

The measures of the length of the sides are written below.

- **AB** is 2-1/2 inches long.
- **DC** is 2-1/2 inches long.
- **AD** is 1-1/4 inches long.
- **BC** is 1-1/4 inches long.

The four sides are not equal in length and so it is not a rhombus.
9. Here is another figure. This is a rhombus.

Here are the lengths of the sides.

- AB is 1 inch long.
- AD is 1 inch long.
- BC is 1 inch long.
- DC is 1 inch long.

All the sides in this figure are of equal length.
10. Now look at this figure. It is not a rhombus because all sides are not of equal length.

The measures of the sides are written below:

AB is \(\frac{3}{4}\) inch long.
AD is \(\frac{3}{4}\) inch long.
BC is 1 inch long.
DC is 1 inch long.

All the sides in this figure are not of equal length. Check with your ruler if you wish.
11. This figure is a rhombus. The sides are equal in length. Notice the measures below.

\[ AB \text{ is } 1-1/2 \text{ inches long.} \]
\[ AD \text{ is } 1-1/2 \text{ inches long.} \]
\[ DC \text{ is } 1-1/2 \text{ inches long.} \]

\[ AB = AD = BC = DC \]
12. And this figure is a rhombus.

Here are the measures of the sides.

- \( \overline{AB} \) is 1-1/2 inches long.
- \( \overline{AD} \) is 1-1/2 inches long.
- \( \overline{BC} \) is 1-1/2 inches long.
- \( \overline{DC} \) is 1-1/2 inches long.

\( \overline{AB} = \overline{AD} = \overline{BC} = \overline{DC} \).

So now you know that when a figure is a quadrilateral there is only one other thing you have to find out to tell whether or not it is a rhombus. You need only find out whether or not all sides are equal in length. Check the examples.
13. The first six figures you studied (questions 1 to 6) were to help you learn how to tell whether or not a figure was a quadrilateral. In case you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a quadrilateral. First we must be sure that you can remember the word quadrilateral itself. Repeat it several times to yourself: quadrilateral, quadrilateral, quadrilateral. Now write the word quadrilateral here. Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else. Now look carefully at the next six figures. Notice which are alike and how they are alike.

14. Here is the first figure.

![Quadrilateral Figure]

Look at it. It has four sides.

15. Here is another figure.

![Hexagon Figure]

This is not a quadrilateral. It does not have only four sides. It has six sides.

16. Look at the next figure. It has four sides, so it is a quadrilateral.

![Rectangle Figure]
17. Look at this drawing. It does not have four sides.

This is not a quadrilateral. It has only three sides.

18. This drawing is a quadrilateral.

It has four and only four sides.

19. And this drawing is a quadrilateral. It also has just four sides.

To find out whether or not a polygon is a quadrilateral, you have to do only one thing. You have to count the number of sides and make sure that there are four, and only four.
20. Using a ruler, connect as many points as you need to close the figure so that it is not a quadrilateral.

21. Using a ruler, connect as many points as you need to finish the figure so it is a quadrilateral.

22. How many sides do quadrilaterals have?

23. Using a ruler, connect as many points as needed to close the figure so it has only 4 sides.
24. List all that is needed to completely describe quadrilateral.

25. Using a ruler, complete this figure so that it is not a quadrilateral.

26. Using a ruler, complete this figure so that it is a quadrilateral.
27. In questions 7 to 12 there were six figures which you studied to help you learn to tell whether or not a figure is a rhombus. In case you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a rhombus. First we must be sure you can remember the word itself. Repeat it several times to yourself: rhombus, rhombus, rhombus. Now write the word 'rhombus' here.

Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else. Now look carefully at the next six figures. Notice which are alike and how they are alike.

28. Here is the first figure.

The measures of the lengths of the sides are written below. Check them with your ruler.

\[ \overline{AB} \text{ is } 3/4 \text{ inch long} \]
\[ \overline{AD} \text{ is } 3/4 \text{ inch long} \]
\[ \overline{BC} \text{ is } 3/4 \text{ inch long} \]
\[ \overline{DC} \text{ is } 3/4 \text{ inch long} \]

Notice that \( \overline{AB} = \overline{AD} = \overline{BC} = \overline{DC} \)
29. Now look at the drawing below. It is not a rhombus.

The measures of the lengths of the sides are written below.

- $AB$ is 2 inches long
- $DC$ is 2 inches long
- $BC$ is 1 inch long
- $AD$ is 1 inch long

The four lines are not equal in length and so it is not a rhombus.

30. Here is another figure. This is a rhombus.

The measures of the lengths of the sides are written below.

- $AB$ is 1 inch long
- $AD$ is 1 inch long
- $DC$ is 1 inch long
- $BC$ is 1 inch long

All the lines in this figure are of equal length.
31. Now look at this figure. It is not a rhombus because all sides are not equal.

```
A

D       B

C
```

The measures of the length of the sides are written below.

- \( \overline{AB} \) is 1-1/4 inches long
- \( \overline{AD} \) is 1-1/4 inches long
- \( \overline{BC} \) is 1/2 inch long
- \( \overline{DC} \) is 1/2 inch long

All the sides in the figure are not of equal length. Check with your ruler if you wish.

32. This figure is a rhombus. The sides are equal in length. Notice the measures below.

```
A

D

C
```

- \( \overline{AB} \) is 1-1/2 inches long
- \( \overline{AD} \) is 1-1/2 inches long
- \( \overline{BC} \) is 1-1/2 inches long
- \( \overline{DC} \) is 1-1/2 inches long

\( \overline{AB} = \overline{AD} = \overline{BC} = \overline{DC} \)
33. And this figure is a rhombus. The sides are equal in length. Notice the measures below.

\[ \overline{AB} \text{ is 2 inches long} \]
\[ \overline{AD} \text{ is 2 inches long} \]
\[ \overline{BC} \text{ is 2 inches long} \]
\[ \overline{DC} \text{ is 2 inches long} \]
\[ \overline{AB} = \overline{AD} = \overline{BC} = \overline{DC} \]

So now you know that when a figure is a quadrilateral there is only one other thing you have to find out to tell whether or not it is a rhombus. You need only find out whether all sides are equal in length. Check the examples you just studied.
34. Using a ruler, connect as many points as you need to close the figure so that all sides are of equal length.

35. Using a ruler, connect as many points as you need to finish the figure to make a quadrilateral that is not a rhombus.

36. Using a ruler, connect as many points as you need to finish the figure so it is a rhombus.

37. All rhombuses have something special that not all quadrilaterals have. Rhombuses have _______ sides that are _________.

38. List all that is needed to completely describe rhombus.
39. Using your ruler, complete this figure so that it is not a rhombus.

40. Using your ruler, complete this figure so that it is a rhombus.
QUADRILATERAL LESSON II: EXPOSITORY MODE
Word List

Some of the words we studied in yesterday's word list were hard ones. Let's look at them again to be sure we know them.

1. quadrilateral
2. trapezoid
3. parallelogram
4. rhombus
Hello,

Yesterday you learned how to tell whether or not a figure is a quadrilateral. You learned that any figure which has four and only four sides is called a quadrilateral. You also learned how to tell whether or not a quadrilateral is a rhombus. You learned that a quadrilateral which has four sides all equal in length is called a rhombus. Today you will learn about two more special kinds of quadrilaterals, parallelograms and trapezoids.

Remember that when learning about these figures you need only look at the sides. Find out if they are the same length or not and whether they are parallel or not. You do not have to think about anything else; like whether the figure is big or small, or thin or fat, or straight or turned around. Just think about the sides. In that way you will learn about the figures.
The next special kind of quadrilateral we will study is called a parallelogram. This figure is a parallelogram. A parallelogram has two pairs of parallel sides.

If you use your ruler you will find that sides AB and DC are parallel. You will also find that sides AD and BC are parallel. Opposite sides of a parallelogram are also equal in length. The measures of the line segments in this figure are given below. Use your ruler if you wish to check them.

\[
\begin{align*}
\overline{AB} & \text{ is 2 inches long.} \\
\overline{DC} & \text{ is 2 inches long.} \\
\overline{AD} & \text{ is 1-1/2 inches long.} \\
\overline{BC} & \text{ is 1-1/2 inches long.}
\end{align*}
\]

Look carefully at the following figures and see how they are alike and how they are different.
2. Look at this figure.

\[ A \quad D \quad B \quad C \]

This is not a parallelogram. It does not have 2 pairs of parallel sides. \( \overline{AB} \) is not parallel to \( \overline{DC} \) and \( \overline{AD} \) is not parallel to \( \overline{BC} \). Opposite sides are not equal. Check the measures below.

- \( \overline{AD} \) is 1 inch long.
- \( \overline{AB} \) is 1 inch long.
- \( \overline{BC} \) is 1-1/2 inches long.
- \( \overline{DC} \) is 1-1/2 inches long.
- \( \overline{AB} \neq \overline{DC}; \overline{AD} \neq \overline{BC} \). (* means not equal to)

3. Now look at this drawing.

\[ A \quad D \quad B \quad C \]

This is a parallelogram. It has two pairs of parallel sides. \( \overline{AB} \) is parallel to \( \overline{DC} \) and \( \overline{AD} \) is parallel to \( \overline{BC} \). Check with your ruler if you wish. Opposite sides are also equal. The measures of the sides are given below. Check them if you wish.

- \( \overline{AB} \) is 1-1/4 inches long.
- \( \overline{DC} \) is 1-1/4 inches long.
- \( \overline{AD} \) is 1-1/4 inches long.
- \( \overline{BC} \) is 1-1/4 inches long.
- \( \overline{AB} = \overline{DC} \) and \( \overline{AD} = \overline{BC} \).
4. Here is another drawing.

This is not a parallelogram. It does not have two pairs of parallel sides. You will notice that you have one pair of parallel sides but not two pairs.

\( \overline{AB} \) and \( \overline{DC} \) are parallel.
\( \overline{AD} \) and \( \overline{BC} \) are not parallel.

Notice also that \( \overline{AD} \) is longer than \( \overline{BC} \).

5. This figure is a parallelogram.

Look at these measures of the sides and you will see.

\( \overline{AB} \) is 1-1/4 inches long.
\( \overline{DC} \) is 1-1/4 inches long.
\( \overline{BC} \) is 1-1/4 inches long.
\( \overline{AD} \) is 1-1/4 inches long.

Notice that \( \overline{AB} = \overline{DC} \) and \( \overline{BC} = \overline{AD} \).

This figure must have two pairs of parallel sides. If you use your ruler to check you will find that \( \overline{AB} \) is parallel to \( \overline{DC} \) and \( \overline{BC} \) is parallel to \( \overline{AD} \).
6. And this is a parallelogram. It has two pairs of parallel sides. Opposite sides are also equal.

Check the measures of the sides written below.

- $\overline{AB}$ is $\frac{1}{2}$ inch long.
- $\overline{DC}$ is $\frac{1}{2}$ inch long.
- $\overline{BC}$ is 2 inches long.
- $\overline{AD}$ is 2 inches long.

If you use your ruler you will find that $\overline{AB}$ is parallel to $\overline{DC}$ and $\overline{AD}$ is parallel to $\overline{BC}$.

You know that all the figures in questions 1, 2, 3, 4, 5, and 6 are quadrilaterals. There are two ways you can tell whether or not they are parallelograms:

1. If there are two pairs of parallel sides; and
2. If opposite sides are equal in length.
7. The last special kind of quadrilateral we will study is a trapezoid. This figure is a trapezoid. One pair of sides is parallel. All trapezoids have one pair and only one pair of parallel sides.

Look at it carefully and use your ruler to check that sides AB and DC are parallel. AD and BC are not parallel.

8. Look at this figure now. It is not a trapezoid. Side AB is not parallel to side DC and side AD is not parallel to side BC.

Use your ruler to check that sides AB and DC in this problem are not parallel. AD and BC are not parallel either. Check if you wish.
9. The next figure is a trapezoid. Look at it carefully and see how it is like the figure in question 7 and different from the figure in question 8.

Use your ruler again and you will find out that sides AB and DC in this problem are parallel, like sides AB and DC in question 7. They are different from sides AB and DC in question 8, since those sides were not parallel. You will also find that AD and BC are not parallel.

10. Here is another figure.

This is not a trapezoid. If you check the sides you will find that side AB is parallel to side DC. You will find also that side AD is parallel to side BC. The figure has two pairs of parallel sides. We said that a trapezoid has one pair and only one pair of parallel sides. This figure has two pairs of parallel sides so it is not a trapezoid.
11. Now look at this figure. This is a trapezoid.

![Diagram of a trapezoid]

Are you surprised? Remember that in a trapezoid only one pair of sides is parallel. AD is parallel to BC and AB is not parallel to DC. Use your ruler again to check this.

12. This figure is a trapezoid, too.

![Diagram of another trapezoid]

Would you have known? Remember that trapezoids have one and only one pair of parallel sides. In this figure AD is parallel to BC and AB is not parallel to DC.

You could tell that the figures in questions 7, 8, 9, 10, 11 and 12 are quadrilaterals because they have four lines. The other thing you have to know to tell whether or not they are trapezoids is that one pair of sides, and only one pair, is parallel.
13. The first six figures you studied (questions 1 to 6) were to help you learn how to tell whether or not a figure was a parallelogram. In case you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a parallelogram. First, we must be sure you can remember the word parallelogram itself. Repeat it several times to yourself: parallelogram, parallelogram, parallelogram. Now write the word parallelogram here.

Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else. Now look carefully at the next six figures. Notice which are alike and how they are alike.

14. Here is the first figure. It is a parallelogram.

\[ \text{A} \quad \text{B} \quad \text{D} \quad \text{C} \]

If you use your ruler you will find that sides AB and DC are parallel. You will also find that sides AD and BC are parallel. Opposite sides of a parallelogram are also equal. The measures of the sides in this figure are given below. Use your ruler if you wish to check them.

- \( AB \) is 2-1/2 inches long.
- \( DC \) is 2-1/2 inches long
- \( AD \) is 1 inch long.
- \( BC \) is 1 inch long.

15. Look at this figure.

\[ \text{A} \quad \text{B} \quad \text{D} \quad \text{C} \]

This is not a parallelogram. It does not have 2 pairs of parallel sides. \( AB \) is not parallel to \( DC \) and \( AD \) is not parallel to \( BC \). Opposite sides are not equal. Check the measures below.

- \( AB \) is 1-1/2 inches long.
- \( DC \) is 1/2 inch long.
- \( AD \) is 1-1/2 inches long.
- \( BC \) is 1/2 inch long.
- \( AB \neq DC, AD \neq BC \)
16. Now look at this drawing.

This is a parallelogram. It has two pairs of parallel sides. AB is parallel to DC and AD is parallel to BC. Check with your ruler if you wish. Opposite sides are also equal. The measures of the sides are given below. Check them if you wish.

\[
\begin{align*}
AB & \text{ is 1 inch.} \\
DC & \text{ is 1 inch.} \\
AD & \text{ is 1 inch.} \\
BC & \text{ is 1 inch.}
\end{align*}
\]

17. Here is another drawing.

This is not a parallelogram. It does not have two pairs of parallel sides. Notice that it has one pair of parallel sides but not two pairs.

\[
\begin{align*}
AB & \text{ is parallel to DC.} \\
AD & \text{ is not parallel to BC.}
\end{align*}
\]
18. And now look at this figure.

This figure is a parallelogram. Look at these measures of the sides and you will see.

- AB is 2 inches long.
- DC is 2 inches long.
- AD is 2 inches long.
- BC is 2 inches long.

19. And this is a parallelogram. It has two pairs of parallel sides. Opposite sides are also equal.

Check the measures of the sides written below.

- AB is 2 1/2 inches long.
- DC is 2 1/2 inches long.
- AD is 1/2 inch long.
- BC is 1/2 inch long.

If you use your ruler you will find that AB is parallel to DC and AD is parallel to BC.

You know that the last six figures are all quadrilaterals. There are two ways you can tell whether or not they are parallelograms.

1. If there are two pairs of parallel sides; or
2. If opposite sides are equal in length.
20. Using a ruler, connect as many points as you need to finish the figure to make a quadrilateral that is not a parallelogram.

21. Using a ruler, connect as many points as you need to finish the figure so it is a parallelogram.

22. All parallelograms have something special that not all quadrilaterals have. Parallelograms have pair(s) of ________ sides.

23. List all that is needed to completely describe parallelogram.
24. Using a ruler, connect as many points as needed to close the figure so it has 2 pairs of parallel sides.

25. Using a ruler, complete this figure so that it is a quadrilateral but not a parallelogram.

26. Using your ruler, complete this figure so that it is a parallelogram.
27. In questions 7 to 12 there were six figures which you studied to help you learn to tell whether or not a figure is a trapezoid. In case you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a trapezoid. First we must be sure you can remember the word trapezoid itself. Repeat it several times to yourself: trapezoid, trapezoid, trapezoid. Now write the word trapezoid here.

Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else. Now look carefully at the next six figures. Notice which are alike and also how they are alike.

28. Here is the first figure. This is a trapezoid.

```
A

D    B

C
```

Look at it carefully and use your ruler to check the sides.

\[ \overline{AB} \text{ is parallel to } \overline{DC} \]
\[ \overline{AD} \text{ is not parallel to } \overline{BC} \]

29. Look at this figure now. It is not a trapezoid.

```
A

D    B

C
```

Use your ruler if you wish to check that sides \( \overline{AB} \) and \( \overline{DC} \) are not parallel. \( \overline{AD} \) and \( \overline{BC} \) are not parallel either.
30. The next figure is a trapezoid.

![Trapezoid Diagram]

Use your ruler again and you will find that sides AB and DC are parallel. Sides AD and BC are not parallel.

31. Here is another figure.

![Another Trapezoid Diagram]

This is not a trapezoid. If you check the sides with your ruler you will find that AB is parallel to DC and that AD is parallel to BC. The figure has two pairs of parallel sides. We said that a trapezoid has one pair and only one pair of parallel sides. This figure has two pairs of parallel sides so it is not a trapezoid.
32. Now look at this figure. This is a trapezoid.

Remember that in a trapezoid only one pair of sides are parallel. AD is parallel to BC and AB is not parallel to DC. Use your ruler again to check this.

33. And now look at this figure.

This is a trapezoid. AB is parallel to DC but BC is not parallel to AD.

In the last six figures you knew that all of them were quadrilaterals. To tell whether or not a quadrilateral is a trapezoid all you have to find out is whether the figure has one and only one pair of parallel sides.
34. Using a ruler, connect as many points as you need to finish the figure so it is a trapezoid.

35. All trapezoids have something special that not all quadrilaterals have. Trapezoids have _______ pair(s) of _________ sides.

36. List all that is needed to completely describe trapezoid.

37. Using a ruler, connect as many points as needed to close the figure so it has only 1 pair of sides parallel.

38. Using a ruler, connect as many points as you need to finish the figure to make a quadrilateral that is not a trapezoid.
39. Using a ruler, complete this figure so that it is a quadrilateral but not a trapezoid.

40. Using a ruler, complete this figure so that it is a trapezoid.
TRIANGLE LESSON: DISCOVERY MODE
Word List

Look carefully at each word as I read it to you. Then say the word aloud with me when I repeat it.

1. triangle
2. polygon
3. right angle triangle
4. acute angle triangle
5. obtuse angle triangle
6. equilateral triangle
7. isosceles triangle
Hello,

You are now ready to start your third lesson. This lesson is not quite like the last two lessons. You have been told about the difference in the form of these lessons. Now we want to point out some things to keep in mind as you go through this and the following lessons.

You may have noticed that the last two lessons were about lines and parts of lines (that is, line segments and rays) and about angles. You learned how to tell whether or not two lines are parallel. And you learned how to tell a right angle, an acute angle and an obtuse angle apart. As you study the figures in this and the following lessons you will use what you learned.

To learn about the figures in the next lesson you will only have to learn about the line segments (or sides, as we will call them) and the angles. When you are learning about a figure look at the sides, find out if they are the same length, and find out whether they are parallel. Look at each angle and check whether it is a right angle or an acute angle or an obtuse angle. You do not have to think of anything else like whether it is big or small, or thin or fat, or straight or turned around. Just think about the sides and the angles and you will learn about the figures.
1. Triangles are special kinds of polygons. The next six figures will help you to tell whether or not a polygon is a triangle. Look at them carefully and notice how the figures are alike and how they are different. Here is the first figure.

Look at it. Describe it by saying how many sides it has. How many sides does it have?

2. Here is another figure.

How many sides does it have?
3. Look at the next drawing. How many sides does it have?

4. Now look at this figure.

How many sides does it have?

123
5. Now look at this drawing.

How many sides does it have?

6. Here is another figure.

How many sides does it have?

120
7. In the last six examples, four were alike and two were different. What are the numbers of the four which were alike? ____________________________ 

8. Remember what we said at the beginning of the lesson about describing figures by saying something about either the sides or angles or both. Say how you think the four figures you named in the last question were alike. 

All had three sides. Figures which have three and only three sides are called triangles.
9. In the rest of this lesson you will be studying about special kinds of triangles. One special kind of triangle is called an equilateral triangle. The four figures following will help you to tell whether or not a figure is an equilateral triangle. Look at them carefully and note how they are alike and how they are different. Here is the first figure.

Measure the sides with your ruler and write what you find here.

The side AB is ______ inches long.
The side AC is ______ inches long.
The side CB is ______ inches long.

What do you notice about the lengths of the sides? _____
10. Now look at this figure.

Measure the sides.
Side AB is _______ inches long.
Side AC is _______ inches long.
Side CB is _______ inches long.
What did you notice about the lengths of the sides? _______

11. Here is another drawing.

Measure the sides.
AB is _______ inches long.
AC is _______ inches long.
CB is _______ inches long.
What do you notice about the sides? ____________
12. Look at this figure.

Measure the sides.

AB is ______ inches long.

AC is ______ inches long.

CB is ______ inches long.

How are this figure and the figure in question 10 different from the figures in questions 9 and 11? _________________
13. Two of the last four figures were alike and two were not. What are the numbers of the two which were alike?

9, 11

14. Remember what we said at the beginning of the lesson about describing figures by saying something about either the sides or angles or both. Say how you think that the two figures you named in the last question were alike.

The three sides were equal in length. Triangles whose three sides are equal in length are called equilateral triangles.
15. Another kind of triangle is an **isosceles triangle**. The following six figures will help you learn how to tell whether or not a figure is an isosceles triangle. Look carefully at them and notice how they are alike and how they are different. Here is the first figure.

![Isosceles Triangle Diagram]

Measure the sides.

- **AB** is _______ inches long.
- **AC** is _______ inches long.
- **BC** is _______ inches long.

What do you notice about the lengths of the sides? _______

16. Look at the drawing below.

![Isosceles Triangle Diagram]

Measure the sides.

- **AB** is _______ inches long.
- **AC** is _______ inches long.
- **BC** is _______ inches long.

What do you notice about the sides in this figure? _______

How are the sides in this figure different from the sides in the last figure? _______
17. Here is another figure.

Measure the sides.

\[ \overline{AB} \] is \underline{______} inches long.

\[ \overline{AC} \] is \underline{______} inches long.

\[ \overline{CB} \] is \underline{______} inches long.

What do you notice about the lengths of the sides? \underline{______}

How are the sides in this figure like the sides in the figure in question 15? \underline{______}

18. And here is another figure.

Measure the sides.

\[ \overline{AB} \] is \underline{______} inches long.

\[ \overline{AC} \] is \underline{______} inches long.

\[ \overline{CB} \] is \underline{______} inches long.

What do you notice about these sides? \underline{______}
19. Look at this drawing.

\[ \triangle ABC \]

Measure the sides.

- \( \overline{AB} \) is ______ inches long.
- \( \overline{AC} \) is ______ inches long.
- \( \overline{CB} \) is ______ inches long.

What do you notice about the sides?

20. And look at this figure.

\[ \triangle ABC \]

Measure the sides.

- \( \overline{AB} \) is ______ inches long.
- \( \overline{AC} \) is ______ inches long.
- \( \overline{BC} \) is ______ inches long.

What do you notice about the sides?

How are the sides in this figure like the sides in the figures in questions 15 and 17?
21. Four of the last six figures were alike and two were not. What are the numbers of figures which were alike? ____________________________

22. Remember what we said at the beginning of the lesson about describing figures by saying something about either the sides or the angles or both. Say how you think that the four figures you named in the last question were alike. ____________________________

______________________________

______________________________

______________________________

Two of the sides were equal in length. Triangles which have two sides equal in length are called isosceles triangles.
23. Another special kind of triangle is called an **acute angle triangle**. The following six figures will help you to learn how to tell whether or not a triangle is an acute angle triangle. Look carefully at the figures and see how they are alike and how they are different.

Name the angles.

The angle at A is

The angle at B is

The angle at C is

24. Now look at this figure.

Name the angles.

The angle at A is

The angle at B is

The angle at C is

What did you notice about the angles? ____________________________

How are they different from the angles in the figure in question 23? ____________

134
25. Here is another figure. Look at it carefully and see how it is like the figure in question 23 and different from the figure in question 24.

\[ \text{\begin{center} \begin{tikzpicture}
\def\figwidth{4cm}
\begin{scope}[scale=0.5,transform shape]
\coordinate (A) at (0,0);
\coordinate (B) at (2,4);
\coordinate (C) at (-2,4);
\draw (A) -- (B);
\draw (A) -- (C);
\draw (B) -- (C);
\end{scope}
\end{tikzpicture}
\end{center}} \]

Name the angles.

The angle at A is ________________________________

The angle at B is ________________________________

The angle at C is ________________________________

How are the angles in this figure like those in the figure in question 23?

26. Here is still another figure.

\[ \text{\begin{center} \begin{tikzpicture}
\def\figwidth{4cm}
\begin{scope}[scale=0.5,transform shape]
\coordinate (A) at (0,0);
\coordinate (B) at (2,4);
\coordinate (C) at (-2,4);
\draw (A) -- (B);
\draw (A) -- (C);
\draw (B) -- (C);
\end{scope}
\end{tikzpicture}
\end{center}} \]

Name the angles.

The angle at A is ________________________________

The angle at B is ________________________________

The angle at C is ________________________________

How are the angles in this figure different from the angles in the figure in the last question?
27. Now look at this figure.

Name the angles.
The angle at A is
The angle at B is
The angle at C is
Are the angles in this figure like the angles in the figure in the last question? 

28. And look at this figure.

Name the angles.
The angle at A is
The angle at B is
The angle at C is
How are the angles in this figure like those in the last figure? 

29. Four of the last six figures were alike and two were not. What are the numbers of the four figures which were alike?  

23, 25, 27, 28

30. Remember what we said at the beginning of the lesson about describing figures by saying something about either the sides or the angles or both. Say how you think that the four figures you named in the last question were alike. 

All had three acute angles. Triangles with three acute angles are called acute angle triangles.
31. Still another special kind of triangle is called a **right angle triangle**. The next six figures will help you learn whether or not a figure is a right angle triangle. Look at them carefully and note how they are alike and how they are different. Here is the first figure.

![Diagram of a right triangle]

Look at it. Use your ruler as you learned in the first lesson to help you name the three angles in this figure. Give a name to the three angles.

- The angle at A is 
- The angle at B is 
- The angle at C is 

32. Now look at this figure.

![Diagram of another triangle]

Look at it. Use your ruler to help you name the three angles. Write the names of the angles here.

- The angle at A is 
- The angle at B is 
- The angle at C is 

How is this figure different from the figure in the last question? Look carefully at the angles.
33. Here is another figure.

Use your ruler to check the angles.

The angle at A is ______________________
The angle at B is ______________________
The angle at C is ______________________

How is this figure like the figure in Question 31? ______________________

34. Look at this figure.

Name the angles.

The angle at A is ______________________
The angle at B is ______________________
The angle at C is ______________________

How is this figure different from the figure in the last question? ______________________
35. Now look at this figure.

Name the angles. Use your ruler to help you.

The angle at A is ________________________________

The angle at B is ________________________________

The angle at C is ________________________________

How is this figure like the figure in Question 33? ________________________________

36. And here is another figure.

Name the angles.

The angle at A is ________________________________

The angle at B is ________________________________

The angle at C is ________________________________

140
37. Four of the last six figures were alike and two were not. What are the numbers of the four figures which were alike? ________________

31, 33, 35, 36

38. Remember what we said at the beginning of the lesson about describing figures by saying something about either the sides or the angles or both. Say how you think that the four figures you named in the last question were alike. ________________

They had one right angle. Triangles which have one right angle are called right angle triangles.
39. The last kind of triangle we will study is called an **obtuse angle triangle**. The next six figures should help you to tell whether or not a figure is an obtuse angle triangle. Look carefully at them and note how they are alike and how they are different.

Use your ruler to name the angles.

The angle at A is ________________________________________________

The angle at B is ________________________________________________

The angle at C is ________________________________________________

40. Look at this figure.

Name the angles. Use your ruler if you wish.

The angle at A is ________________________________________________

The angle at B is ________________________________________________

The angle at C is ________________________________________________

How are the angles in this figure different from those in the figure in question 39? ________________________________________________
41. Now look at this drawing.

Name the angles.

The angle at A is ____________________________

The angle at B is ____________________________

The angle at C is ____________________________

How is this figure like the figure in question 39? __________

42. Here is another drawing.

Name the angles.

The angle at A is ____________________________

The angle at B is ____________________________

The angle at C is ____________________________

Can you see how it is different from the figure in question 41?

Use your ruler to check if you wish and then write how you think this figure is different from the figure in question 41.
43. Look at this figure.

Name the angles.
The angle at A is ______________________
The angle at B is ______________________
The angle at C is ______________________
Check. Write what you find about the angles here. ______________________________________

44. And here is another figure.

Check the angles.
The angle at A is ______________________
The angle at B is ______________________
The angle at C is ______________________
How are the angles in this triangle like the angles in the last question? ______________________
45. Four of the last six figures were alike and two were not. What are the numbers of the four figures which are alike? ________________________________

39, 41, 43, 44

46. Remember what we said at the beginning of the lesson about describing figures by saying something about either the sides or the angles or both. Say how you think that the four figures you named in the last question were alike. ________________________________

They had one obtuse angle. Triangles which have one obtuse angle are called obtuse angle triangles.
QUADRILATERAL LESSON I: DISCOVERY MODE
Word List

Look carefully at each word as I read it to you. Then say the word aloud with me when I repeat it.

1. quadrilateral
2. trapezoid
3. parallelogram
4. rhombus
Hello,

This lesson is not quite like the last lessons. You have been told about the difference in the form of these lessons. Now we want to point out some things to keep in mind as you go through this and the following lessons.

You may have noticed that the last two lessons were about lines and parts of lines (that is, line segments and rays). You learned how to tell whether or not two lines are parallel. As you study the figures in this and the following lessons, you will use what you learned.

To learn about the figures in the next lesson you will only have to learn about the line segments, or sides as we will call them. When you are learning about a figure, look at the sides. Find out if they are the same length or not and if they are parallel or not. You do not have to think of anything else about the figure, such as whether it is big or small, or thin or fat, or straight or turned around. Just think about the sides. That is how you will learn about the figure.
1. This section has six figures in it. You will learn from studying these figures how to tell whether a polygon is a quadrilateral or not. Look carefully at the polygons shown in the following questions. Notice how they are alike and how they are different. Some of these are quadrilaterals. Others are not. Here is the first figure.

Look at it. Describe it by giving the number of sides. How many sides does it have?

2. Here is another figure.

How many sides does it have?
3. Look at the next drawing. How many sides does it have? [Diagram of a quadrilateral]

4. Look at this drawing. How many sides does it have? [Diagram of a triangle]
5. Here is another drawing. Count the sides.

How many sides does it have?

6. Now look at this figure.

How many sides does it have?
7. In the last six examples, four were alike and two were different. What are the numbers of the four which were alike?

1, 3, 5, 6

8. Remember what we said at the beginning of the lesson about describing figures by saying something about the sides. Say how you think that the four figures you named in the last question were alike.

All had four sides.

The four figures which are alike are quadrilaterals.
9. One special kind of quadrilateral is a rhombus. The following six figures will help you tell whether or not a quadrilateral is also a rhombus. Look at them carefully and note how they are alike and how they are different. Here is the first figure.

![Diagram of a rhombus](image)

Use your ruler to measure the lengths of the sides and write the lengths here.

- AB is ____ inches long.
- AD is ____ inches long.
- BC is ____ inches long.
- DC is ____ inches long.

What did you notice about the length of the sides? ___
10. Now look at this figure.

Measure lengths of the sides and write them here.

\[ \text{AB is } \_\_\_\_\_ \text{ inches long.} \]
\[ \text{DC is } \_\_\_\_\_ \text{ inches long.} \]
\[ \text{AD is } \_\_\_\_\_ \text{ inches long.} \]
\[ \text{BC is } \_\_\_\_\_ \text{ inches long.} \]

You can see how this figure is different from the figure in the last question. Write how it is different. ___
11. Here is another figure.

Write the lengths of the sides here.

\[ \overline{AB} \text{ is } \underline{______} \text{ inches long.} \]
\[ \overline{AD} \text{ is } \underline{______} \text{ inches long.} \]
\[ \overline{BC} \text{ is } \underline{______} \text{ inches long.} \]
\[ \overline{DC} \text{ is } \underline{______} \text{ inches long.} \]

How is this figure like the one in question 9 and different from the one in question 10? ____________________
12. Now look at this figure.

Write the lengths of the sides here.

- $AB$ is ______ inches long.
- $DC$ is ______ inches long.
- $AD$ is ______ inches long.
- $BC$ is ______ inches long.

How is this figure different from the figure in the last question?
13. Here is another figure.

Measure the sides.

- \( AB \) is _____ inches long.
- \( AD \) is _____ inches long.
- \( BC \) is _____ inches long.
- \( DC \) is _____ inches long.

How is this figure like the figures in questions 9 and 11?
14. And here is another figure.

Write the length of the sides here.

- AB is _______ inches long.
- AD is _______ inches long.
- BC is _______ inches long.
- DC is _______ inches long.

What do you notice about the lengths of the sides? ___
15. In the last six figures, four were alike and two were different. What are the numbers of the four which were alike?

9, 11, 13, 16

16. Remember what we said at the beginning of the lesson about describing figures by saying something about the sides. Say how you think that the four figures you named in the last question were alike?

All four sides were equal in length.

Each of the four figures which are alike is called a rhombus.
17. The first six figures you studied (questions 1 to 6) were to help you learn how to tell whether or not a figure was a quadrilateral. In case you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a quadrilateral. First, we must be sure you can remember the word quadrilateral itself. Repeat it several times to yourself: quadrilateral, quadrilateral, quadrilateral. Now write the word quadrilateral here.

Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else. Now look carefully at the next six figures. Notice which are alike and how they are alike.

18. Here is the first figure.

Look at it. Describe it by giving the number of sides it has. How many sides does it have?
19. Here is another figure.

How many sides does it have? ________________________________

How is this figure different from the figure in the last question?
________________________________________________________

20. Look at the next figure.

How many sides does it have? ________________________________

How is it different from the last figure? ______________________
________________________________________________________

How is it like the figure in question 18? ______________________
________________________________________________________
21. Look at this drawing.

[Diagram of a triangle]

How many sides does it have? ________________________________

How is the figure different from the figure in question 20?

[Blank line]

22. Here is another figure. Count the sides.

[Diagram of a quadrilateral]

How many sides does it have? ________________________________

How is this figure like the figures in questions 18 and 20?

[Blank line]

How is this figure different from the figure in question 21?

[Blank line]
23. And look at this figure.

How many sides does it have?

How is this figure like the figure in question 22?
24. In the last six examples, four were alike and two were different. What are the question numbers of the four which were alike?

25. Remember what we said at the beginning of the lesson about describing figures by saying something about the sides. Say how you think that the four figures you named in the last question were alike.

All had four sides. Every figure which has four and only four sides is called a quadrilateral.
26. Using a ruler, connect as many points as you need to close the figure so that it is not a quadrilateral.

27. Using a ruler, connect as many points as you need to finish the figure so it is a quadrilateral.

28. How many sides do quadrilaterals have?

29. Using a ruler, connect as many points as needed to close the figure so it has only 4 sides.
30. List all that is needed to completely describe quadrilateral.

31. Using a ruler, complete this figure so that it is not a quadrilateral.

32. Using a ruler, complete this figure so that it is a quadrilateral.
33. In questions 9 to 14 there were six figures which you studied to help you learn to tell whether or not a figure is a rhombus. If you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a rhombus. First we must be sure you can remember the word rhombus itself. Repeat it several times to yourself: rhombus, rhombus, rhombus. Now write the word rhombus here. 

Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else. Now look carefully at the next six figures. Notice which are alike and how they are alike.

34. Here is the first figure.

A

B

D

C

Use your ruler to measure the length of the sides and write the lengths here.

AB is ______ inches long.

AD is ______ inches long.

BC is ______ inches long.

DC is ______ inches long.

What did you notice about the lengths of the sides?
35. Now look at the drawing below.

Measure the lengths of the sides and write them here.

\( \overline{AB} \) is ________ inches long.

\( \overline{AD} \) is ________ inches long.

\( \overline{BC} \) is ________ inches long.

\( \overline{DC} \) is ________ inches long.

You can see how this figure is different from the figure in the last question. Write how it is different. 

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36. Here is another figure.

Write the lengths of the sides here.

- $\overline{AB}$ is _______ inches long.
- $\overline{AD}$ is _______ inches long.
- $\overline{BC}$ is _______ inches long.
- $\overline{DC}$ is _______ inches long.

How is this figure like the one in question 34 and different from the one in question 35?
37. Now look at this figure.

Write the lengths of the sides here.

\[ \overline{AB} \text{ is } \underline{\phantom{0}} \text{ inches long.} \]
\[ \overline{AD} \text{ is } \underline{\phantom{0}} \text{ inches long.} \]
\[ \overline{BC} \text{ is } \underline{\phantom{0}} \text{ inches long.} \]
\[ \overline{DC} \text{ is } \underline{\phantom{0}} \text{ inches long.} \]

How is this figure different from the figure in the last question?
38. Now look at this figure. Measure the sides and write the lengths here.

\[ \overline{AB} \text{ is } \underline{\text{_____}} \text{ inches long.} \]

\[ \overline{AD} \text{ is } \underline{\text{_____}} \text{ inches long.} \]

\[ \overline{BC} \text{ is } \underline{\text{_____}} \text{ inches long.} \]

\[ \overline{DC} \text{ is } \underline{\text{_____}} \text{ inches long.} \]

How is this figure like the figure in questions 34 and 36? _____

_____
39. And here is another figure.

Write the lengths of the sides here.

\[ \overline{AB} \text{ is } \underline{\text{_______}} \text{ inches long.} \]

\[ \overline{AD} \text{ is } \underline{\text{_______}} \text{ inches long.} \]

\[ \overline{BC} \text{ is } \underline{\text{_______}} \text{ inches long.} \]

\[ \overline{DC} \text{ is } \underline{\text{_______}} \text{ inches long.} \]

What do you notice about the lengths of the sides? 

\[ \underline{\text{_______}} \]
40. In the last six examples, four were alike and two were different. What are the numbers of the four which were alike?

41. Remember what we said at the beginning of the lesson about describing figures by saying something about the sides. Say how you think that the four figures you named in the last question are alike.

The four sides of each of them are equal in length. Every quadrilateral which has four sides equal in length is called a rhombus.
42. Using a ruler, connect as many points as you need to close the figure so that all sides are of equal length.

43. Using a ruler, connect as many points as you need to finish the figure to make a quadrilateral that is not a rhombus.

44. Using a ruler, connect as many points as you need to finish the figure so it is a rhombus.

45. All rhombuses have something special that not all parallelograms have. Rhombuses have ________ sides that are _________.

46. List all that is needed to completely describe rhombus.
47. Using your ruler, complete this figure so that it is not a rhombus.

48. Using your ruler, complete this figure so that it is a rhombus.
QUADRILATERAL LESSON II: DISCOVERY MODE
Word List

Some of the words we studied in yesterday's word list were hard ones. Let's look at them again to be sure we know them:

1. quadrilateral
2. trapezoid
3. parallelogram
4. rhombus
Hello,

Yesterday you learned how to tell whether or not a figure is a quadrilateral. You learned that any figure which has four and only four sides is called a quadrilateral. You also learned how to tell whether or not a quadrilateral is a rhombus. You learned that a quadrilateral which has all four sides equal in length is called a rhombus. Today you will learn about two more special kinds of quadrilaterals, parallelograms and trapezoids.

Remember that when learning about these figures you need only look at the sides. Find out if they are the same length or not and if they are parallel or not. You do not have to think about anything else like whether the figure is big or small, or thin or fat, or straight or turned around. Just think about the sides. In that way you will learn about the figures.
1. The next special kind of quadrilateral we will study is called a parallelogram. The next six problems will help you tell whether or not a figure is a parallelogram. Look at them carefully and notice how the figures are alike and how they are different. Here is the first figure.

Use your ruler to find out some things about sides $AB$ and $DC$. Did you measure them? How long are they?

$AB$ is ______ inches long.
$DC$ is ______ inches long.

What else did you notice about $AB$ and $DC$? Check again and write what you find out here. ________________________________

What about $AD$ and $BC$?

$AD$ is ______ inches long.
$BC$ is ______ inches long.

What else did you notice about $AD$ and $BC$? ________________________________

Is $AD$ parallel to $BC$? ________________________________

Is $AB$ parallel to $DC$? ________________________________
2. Look at this figure.

Measure sides $AB$, $AD$, $BC$, and $DC$.

$\overline{AD}$ is ______ inches long.

$\overline{AB}$ is ______ inches long.

$\overline{BC}$ is ______ inches long.

$\overline{DC}$ is ______ inches long.

Is $\overline{AD}$ parallel to $\overline{BC}$? __________________________

Is $\overline{AB}$ parallel to $\overline{DC}$? __________________________

How is this figure different from the figure in the last question?

_____________________________
3. Now look at this drawing.

Measure the sides.

\[ \overline{AB} \text{ is } \underline{\hspace{2cm}} \text{ inches long.} \]
\[ \overline{DC} \text{ is } \underline{\hspace{2cm}} \text{ inches long.} \]
\[ \overline{AD} \text{ is } \underline{\hspace{2cm}} \text{ inches long.} \]
\[ \overline{BC} \text{ is } \underline{\hspace{2cm}} \text{ inches long.} \]

Is \( \overline{AB} \) parallel to \( \overline{DC} \)?

Is \( \overline{AD} \) parallel to \( \overline{BC} \)?

How is this different from the last figure?

How is it like the figure in Question 1?
4. Here is another drawing.

Measure the sides.

\[AB\] is ______ inches long.
\[DC\] is ______ inches long.
\[AD\] is ______ inches long.
\[BC\] is ______ inches long.

Can you see how this is different from the figure in Question 3?

Use your ruler. Say how you think the figure in Question 3 is different from the figure in this question.

____________________________

Is \(AD\) parallel to \(BC\)?

____________________________

Is \(AB\) parallel to \(DC\)?

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5. And now look at this figure.

Measure the sides.

\[ \overline{AB} \text{ is } \blank \text{ inches long.} \]

\[ \overline{AD} \text{ is } \blank \text{ inches long.} \]

\[ \overline{BC} \text{ is } \blank \text{ inches long.} \]

\[ \overline{DC} \text{ is } \blank \text{ inches long.} \]

What else do you notice about the sides?

\[ \text{Is } \overline{AD} \text{ parallel to } \overline{BC}? \blank \]

\[ \text{Is } \overline{AB} \text{ parallel to } \overline{DC}? \blank \]
6. Here is another figure.

Write the lengths of the sides here.

- \( \overline{AB} \) is ______ inches long.
- \( \overline{DC} \) is ______ inches long.
- \( \overline{BC} \) is ______ inches long.
- \( \overline{AD} \) is ______ inches long.

Use your ruler and say what you note about sides \( \overline{AB} \) and \( \overline{DC} \) and \( \overline{AD} \) and \( \overline{BC} \) besides the lengths. ________________________

Is \( \overline{AD} \) parallel to \( \overline{BC} \)? ________________________

Is \( \overline{AB} \) parallel to \( \overline{DC} \)? ________________________
7. Of the last six figures, four were alike and two were different.
   What are the numbers of the four which were alike?

8. Remember what we said at the beginning of the lesson about describing figures by saying something about sides. Say how you think that the four figures you named in the last question are alike.

Each of the four figures which are alike is called a parallelogram.

9. The next six figures will help you learn about still another special kind of quadrilateral. It is called a trapezoid. Look carefully at the figures so that you can tell how they are alike and how they are different.

Here is the first figure.

Look at it carefully and use your ruler to check the sides, especially sides AB and DC.

What do you find?

Is \( AB \) parallel to \( DC \)?
Is \( AD \) parallel to \( BC \)?
10. Look at this figure now. Can you see how it is different from the figure in the last problem?

Use your ruler to find out how sides AB and DC in this problem are different from sides AB and DC in the last problem.

Are they equal in length? ____________________________
What else did you find? ____________________________
Is \( \overline{AB} \) parallel to \( \overline{DC} \)? ____________________________
Is \( \overline{AD} \) parallel to \( \overline{BC} \)? ____________________________
11. Look at this figure carefully and see how it is like the figure in question 9 and different from the figure in question 10.

Use your ruler again to find out how sides $AB$ and $DC$ in this problem are like sides $AB$ and $DC$ in question 9 and different from sides $AB$ and $DC$ in question 10. Write what you found about sides $AB$ and $DC$ here.

____________________________________________________

____________________________________________________

Is $AB$ parallel to $DC$? __________________________________

Is $AD$ parallel to $BC$? __________________________________

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8.
12. Here is another figure. This is different from the figure in the last problem.

Can you write here how this figure is like the figure in question 11 and how it is different from it?

Is \( \overline{AB} \) parallel to \( \overline{DC} \)?

Is \( \overline{AD} \) parallel to \( \overline{BC} \)?
13. Now look at this figure.

![Quadrilateral ABCD](image)

Use your ruler again and find out how this figure is like the figures in questions 9 and 11 and different from the figures in questions 10 and 12. Write what you found here.

Is $\overline{AB}$ parallel to $\overline{DC}$? 

Is $\overline{AD}$ parallel to $\overline{BC}$? 

14. And now look at this figure.

![Parallelogram ABCD](image)

Do you think this figure is like any of the others you just looked at?

Use your ruler if you are not sure. Write here how it is like the figure in question 13.

Is $\overline{AB}$ parallel to $\overline{DC}$? 

Is $\overline{AD}$ parallel to $\overline{BC}$?
15. Of the last six figures, four were alike and two were different. What are the numbers of the four which were alike?

16. Remember what we said at the beginning of the lesson about describing figures by saying something about the sides. Say how you think that the four figures you named in the last question were alike.

Each of the four figures which were alike is called a trapezoid.

17. The first six figures you studied (questions 1 to 6) were to help you learn how to tell whether or not a figure was a parallelogram. In case you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a parallelogram. First, we must be sure you can remember the word parallelogram itself. Repeat it several times to yourself: parallelogram, parallelogram, parallelogram. Now write the word parallelogram here.

Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else. Now look carefully at the next six figures. Notice which are alike and how they are alike.
18. Here is the first figure.

Measure the sides $AB$ and $DC$.

$AB$ is _______ inches long.

$DC$ is _______ inches long.

What else did you notice about $AB$ and $DC$? Check again and write what you find out here.

Measure sides $AD$ and $BC$.

$AD$ is _______ inches long.

$BC$ is _______ inches long.

What else did you notice about $AD$ and $BC$?

Is $AD$ parallel to $BC$?

Is $AB$ parallel to $DC$?
19. Look at this figure.

Measure the sides \(AB\), \(AD\), \(BC\), and \(DC\).

- \(AB\) is _____ inches long.
- \(AD\) is _____ inches long.
- \(BC\) is _____ inches long.
- \(DC\) is _____ inches long.

Is \(AD\) parallel to \(BC\)?

Is \(AB\) parallel to \(DC\)?

How is this figure different from the figure in the last question?
20. Now look at this drawing.

Measure the sides.

\[ AB \text{ is } \underline{\phantom{0}} \text{ inches long.} \]
\[ AD \text{ is } \underline{\phantom{0}} \text{ inches long.} \]
\[ BC \text{ is } \underline{\phantom{0}} \text{ inches long.} \]
\[ DC \text{ is } \underline{\phantom{0}} \text{ inches long.} \]

Is \( AB \) parallel to \( DC \)? 

Is \( AD \) parallel to \( BC \)?

How is this figure different from the last figure?

How is this figure like the figure in question 18?
21. Here is another drawing.

![Diagram of a quadrilateral with vertices A, B, C, D]

Measure the sides.

\( \overline{AB} \) is _______ inches long.
\( \overline{DC} \) is _______ inches long.
\( \overline{AD} \) is _______ inches long.
\( \overline{BC} \) is _______ inches long.

Can you see how this figure is different from the figure in question 20? Use your ruler. Say how you think the figure in question 20 is different from the figure in this question.

________________________________________________________________________

________________________________________________________________________

Is \( \overline{AD} \) parallel to \( \overline{EC} \)?

Is \( \overline{AB} \) parallel to \( \overline{DC} \)?
22. And now look at this figure.

Measure the sides.

\[ \overline{AB} \text{ is }\underline{_____} \text{ inches long.} \]

\[ \overline{DC} \text{ is }\underline{_____} \text{ inches long.} \]

\[ \overline{AD} \text{ is }\underline{_____} \text{ inches long.} \]

\[ \overline{BC} \text{ is }\underline{_____} \text{ inches long.} \]

What else did you notice about the sides? ________________

Is \( \overline{AB} \) parallel to \( \overline{DC} \)? ________________

Is \( \overline{AD} \) parallel to \( \overline{BC} \)? ________________
23. And here is another figure.

Write the lengths of the sides here.

\[ \begin{align*}
&AB \text{ is } \underline{\phantom{0000}} \text{ inches long.} \\
&DC \text{ is } \underline{\phantom{0000}} \text{ inches long.} \\
&AD \text{ is } \underline{\phantom{0000}} \text{ inches long.} \\
&BC \text{ is } \underline{\phantom{0000}} \text{ inches long.}
\end{align*} \]

Use your ruler and say what you notice about sides \textit{AB} and \textit{DC} and sides \textit{AD} and \textit{BC} besides the length.
24. In the last six examples, four were alike and two were different. What are the question numbers of the four which were alike.

25. Remember what we said at the beginning of the lesson about describing figures by saying something about the sides. Say how you think that the four figures you named in the last question were alike.

Two pairs of opposite sides were equal in length and two pairs of opposite sides were parallel. Quadrilateral which have two pairs of opposite sides equal in length and two pairs of opposite sides parallel are called parallelograms.
26. Using a ruler, connect as many points as you need to finish the figure to make a quadrilateral that is not a parallelogram.

27. Using a ruler, connect as many points as you need to finish the figure so it is a parallelogram.

28. All parallelograms have something special that not all quadrilaterals have. Parallelograms have _______ pair(s) of _______ sides.

29. List all that is needed to completely describe parallelogram.
30. Using a ruler, connect as many points as needed to close the figure so it has 2 pairs of parallel sides.

31. Using a ruler, complete this figure so that it is a quadrilateral but not a parallelogram.

32. Using your ruler, complete this figure so that it is a parallelogram.
33. In questions 9 to 14 there were six figures which you studied to help you learn to tell whether or not a figure is a trapezoid. In case you are not sure, we are going to look at six more figures. These should help you know for sure whether or not a figure is a trapezoid. First we must be sure you can remember the word trapezoid itself. Repeat it several times to yourself: trapezoid, trapezoid, trapezoid. Now write the word trapezoid here.

Remember what we said at the beginning of the lesson about describing a figure in terms of its sides and nothing else.

Now look carefully at the next six figures. Notice which are alike and how they are alike.

34. Here is the first figure.

```
A

D

B

C
```

Look at it carefully and use your ruler to check the sides, especially sides AB and DC. What did you find?

Is \( \overline{AB} \) parallel to \( \overline{DC} \)?

Is \( \overline{AD} \) parallel to \( \overline{BC} \)?
35. Look at this figure. Can you see how it is different from the figure in the last problem?

![Diagram]

Use your ruler to find out how sides AB and DC in this problem are different from sides AB and DC in the last problem.

Is AB parallel to DC? 
Is AD parallel to BC? 

36. Look at this figure carefully and see how it is like the figure in question 34 and different from the figure in question 35.

![Diagram]

Use your ruler.

Is AB parallel to DC? 
Is AD parallel to BC?
37. Here is another figure.

Is AB parallel to DC? ____________________________

Is AD parallel to BC? ____________________________

Can you write here how this figure is like the figure in the last question and how it is different from it?

38. Now look at this figure.

Use your ruler again and find out how this figure is like the figures in questions 34 and 36 and different from the figures in questions 35 and 37. Write what you found here.

Is AB parallel to DC? ____________________________

Is AD parallel to BC? ____________________________
39. And now look at this figure.

Do you think this figure is like any of the others you just looked at?

Is $AB$ parallel to $DC$?

Is $AD$ parallel to $BC$?

Write here how this figure is like the figure in the last question:
40. Of the last six figures, four were alike and two were different. What are the numbers of the four which were alike?

[34, 36, 38, 39]

41. Remember what we said at the beginning of the lesson about describing figures by saying something about their sides. Say how you think that the four figures you named in the last question were alike.

Each had one pair and only one pair of parallel sides. A quadrilateral with one and only one pair of parallel sides is called a trapezoid.
42. Using a ruler, connect as many points as you need to finish the figure so it is a trapezoid.

43. All trapezoids have something special that not all quadrilaterals have. Trapezoids have _________ pair(s) of _________ sides.

44. List all that is needed to completely describe trapezoid.

45. Using a ruler, connect as many points as needed to close the figure so it has only 1 pair of sides parallel.

46. Using a ruler, connect as many points as you need to finish the figure to make a quadrilateral that is not a trapezoid.
47. Using a ruler, complete this figure so that it is a quadrilateral but not a trapezoid.

48. Using a ruler, complete this figure so that it is a trapezoid.
INSTRUCTIONS: Use only the special pencil.
Make neat, heavy marks.
Keep your place on the answer sheet.
Make only one mark for each question.
If you change your mind about an answer, erase the first mark completely.

Example 1
1. Which of these figures is a circle?

a. b. c. d.

Example 2
2. How many sides does this figure have?

a. 1  b. 2  c. 3  d. 4
1. Which figure is not a quadrilateral?

   a. ▲  b. □  c. △  d. □

2. All rhombuses have:
   a. 4 right angles
   b. only 1 pair of parallel sides
   c. no parallel sides
   d. all sides of equal length

3. Which figure has 2 pairs of parallel sides?

   a.  b.  c.  d.

4. Which figure is a trapezoid?

   a.  b.  c.  d.

5. All quadrilaterals are:
   a. plane closed figures with opposite angles equal
   b. plane closed figures with 5 sides
   c. 4-sided closed plane figures with 4 right angles
   d. 4-sided closed plane figures

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6. Which figure is not a rhombus?

a.  

b.  

c.  

d.  

7. Which figure is a parallelogram?

a.  

b.  

c.  

d.  

8. Which figure is not a trapezoid?

a.  

b.  

c.  

d.  

9. All quadrilaterals have:

- a. no sides of equal length
- b. no sides parallel
- c. closed sides
- d. 2 sides of equal length

10. Which figure has all sides of equal length?

a.  

b.  

c.  

d.  

11. Which figure is a parallelogram?

a. \[ \square \]  
b. \[ \diamond \]  
c. \[ \triangle \]  
d. \[ \triangle \]

12. All trapezoids are:

a. plane open figures with 4 sides  
b. plane closed 4-sided figures with opposite sides of equal length  
c. plane closed 4-sided figures with only 1 pair of parallel sides  
d. plane closed 4-sided figures with 2 pairs of parallel sides

13. Which figure is not a quadrilateral?

a. \[ \triangle \]  
b. \[ \pentagon \]  
c. \[ \square \]  
d. \[ \triangle \]

14. Which figure is a rhombus?

a. \[ \parallel \]  
b. \[ \square \]  
c. \[ \triangle \]  
d. \[ \square \]

15. Which figure is not a parallelogram?

a. \[ \square \]  
b. \[ \square \]  
c. \[ \triangle \]  
d. \[ \square \]
16. All trapezoids have:
   a. 2 sides of equal length
   b. 2 pairs of parallel sides
   c. all sides of equal length
   d. 1 pair of parallel sides

17. Which drawing has 4 sides?

   a.  
   b.  
   c.  
   d.  

18. Which figure is a rhombus?

   a.  
   b.  
   c.  
   d.  

19. All parallelograms are:
   a. quadrilaterals with all sides of equal length
   b. quadrilaterals with 2 pairs of opposite sides of equal length
   c. quadrilaterals with all sides of unequal length
   d. quadrilaterals with only 1 pair of parallel sides

20. Which figure is a quadrilateral?

   a.  
   b.  
   c.  
   d.  
21. Which figure is not a rhombus?

a.  

b.  

c.  

d.  

22. All parallelograms have:

a. all sides of equal length
b. no sides parallel
c. only 1 pair of parallel sides
d. 2 pairs of parallel sides

23. Which figure has only 1 pair of parallel sides?

a.  

b.  

c.  

d.  

24. Which figure is a quadrilateral?

a.  

b.  

c.  

d.  

25. All rhombuses are:

a. quadrilaterals with 4 right angles
b. quadrilaterals with only 1 pair of sides of equal length
c. parallelograms with all sides of equal length
d. parallelograms with all angles equal
26. Which figure is not a parallelogram?
   a. □  b. □  c. □  d. □

27. Which figure is a trapezoid?
   a. □  b. □  c. □  d. □

28. Which figure is not a trapezoid?
   a. □  b. □  c. □  d. □
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