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AUTHOR Riley, Christine A.; And Others
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ABSTRACT

The purpose of this study was to examine how children represent and compare number symbols. Pre-school, kindergarten, first, third, and fifth grade children and college students judged which of two visually presented digits was numerically larger. Twelve individuals were tested at each grade level. Response latencies were collected. Children at all levels were successful on the task. Results indicated that response latencies were a function of the ratio of the digit sizes; the larger the ratio, the faster the decision could be made. The children's performance demonstrated that: (1) they know the ordered sequence of the digits (the basis of the ordinal property of number); and (2) that numbers later in the sequence represent a greater quantity. This understanding of the quantitative basis of number is present even at an age when children do not understand the cardinal property of number symbols. It is argued that the ordinal number property and the understanding of the quantitative dimension which numbers represent form the psychological basis of the number concept. The results are discussed in terms of their relevance for early mathematics education. (Author/SB)

THE QUANTITATIVE BASIS OF CHILDREN'S NUMBER CONCEPTS

Christine A. Riley

Jing-Mei Hu

and

James V. Hinrichs

University of Iowa

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A number of researchers in the field of cognitive development have recently argued that an understanding of the psychological basis of the development of number concepts is critically important in designing meaningful mathematics instruction. Much of the literature on the development of number concepts has focused on children's understanding of the formal properties of number, such as the cardinal property of number, the ordinal property of number, and conservation of number. Mathematics educators frequently cite these formal properties of number, particularly as studied in the Piagetian tradition, as the psychological foundation on which mathematics education should be based. We wish to suggest, however, that the fundamental psychological basis of number concepts may be the relation of numbers to quantity.

In contrast to the developmental literature, studies of number concepts using college students as subjects have rarely focused on the formal properties of number. Researchers have addressed questions such as "How are numbers represented in memory?" and "How are number symbols compared?" Moyer and Landauer (1967) have argued that numbers are represented and compared in a manner analogous to stimuli from physical continua such as length or brightness. This conclusion was based on the performance of college students on a judgment of numerical inequality task. In this task, subjects were presented a series of slides which contained two single digit numbers and were asked to push a button under the numerically larger of the two digits as quickly as possible.

Educated adults can tell which of two digits is the numerically larger with virtual certainty. When asked to introspect about the processes involved in such a judgment most adults report that they "Just know." However, reliable patterns of response times, from which comparison processes can be inferred, are observed in this task. Figure 1 shows the decision times in the judgment of numerical inequality task as a function of the difference between the digits for college student subjects. Pairs adjacent in the number series have a difference of one, 2 versus 6 has a difference of 4, etc. Note that decision times are inversely related to the numerical difference between the digits. The data presented in Figure 1 are from our replication of Moyer and Landauer's task, but they conform quite closely to what Moyer and Landauer observed.

Moyer and Landauer pointed out that this pattern of data is the same as that observed when subjects compare stimuli which vary on physical continua. Judgments of inequality for line length, for the pitch of tones, and for brightness, for example, require longer times as the differences between the stimuli on the continuum to be judged decrease. In psychophysical judgments, decision times also vary with the ratio of the values of the stimuli. Thus, for a given difference, pairs of stimuli with a larger ratio between their values would be compared more quickly. If the similarity between psychophysical judgments and number judgments hold, then the comparison of 3 vs 4 should be made more quickly than 7 vs 8, and 2 vs 4 should be compared more quickly than 4 vs 6. This relationship was also observed in the judgment of numerical inequality task.

Although the error rate in this task is low, the pattern of errors is similar to the pattern of decision times, with the highest number of errors on pairs with the slowest decision times, those which are adjacent in the number series. This pattern rules out an explanation of the decision times based on a speed-accuracy trade-off.

Moyer and Landauer explored a number of equations for describing these data, and found a reasonable fit of models developed to describe reaction times in psychophysical judgment, such as the Welford equation. Thus, they argued, the comparison process used must be similar to the process used for perceptual comparisons, and furthermore, numbers are represented in memory as analogue magnitudes. Thus, if the number "3" is represented in memory as a length or a weight then "4" is a longer line or a heavier weight. This description is quite unlike one based on the formal properties of number, although presumably these college student subjects had knowledge of the formal properties of number.

The major purpose of our study was to examine how children represent and compare number symbols. In order to address this question, we replicated Moyer and Landauer's procedures with college students, children who had recently completed kindergarten, first grade, third grade and fifth grade, as well as a group of pre-school children who would enter kindergarten in the upcoming school year. Twelve individuals were tested at each grade level. The subjects sat in front of a panel which contained two response buttons and a rear-projection screen. The task consisted of 144 trials. Each of the 36 possible nonrepeating pairs of digits from the series 1 to 9 was tested four times. On two of the trials for each pair the larger digit was on the left-hand side, on the other trials the larger digit was on the right-hand side. Subjects were instructed to press the button under the numerically larger digit as quickly as possible.

In order to be successful on this task, a child must know the ordered sequence of digits and understand that numbers which occur later in the sequence have larger quantities as their referents. Children in all of the age groups tested were able to perform the task successfully. Older children made fewer errors, as shown in Table 1. However, performance was significantly above chance for each age level for comparisons of each of the 36 pairs of digits. Additionally, the relative distribution of errors across the 36 pairs was similar at all age levels.

Figure 2 shows the pattern of decision times observed for each age group as a function of the numerical difference between the two digits to be compared. (The adult data shown in this Figure are the same data that are presented in Figure 1. Note that although the response times are slower for younger children, the basic pattern of decreasing decision times with increasing numerical difference is present at all age levels. These data provide support for our argument that young children represent and compare digits in manner qualitatively similar to adults. That is, digits are represented in memory as quantities or magnitudes which are analogues of physical stimuli, and that these quantity analogues are compared by a process which Moyer (1973) has termed an "internal psychophysics."

Further support for this argument is provided when the comparison times at each age level are correlated with the predictions of Welford's model for psychophysical comparisons, as shown in Table 2. The log of L/(L-S), the ratio of the larger digit in the pair to be compared on a given trial to the difference between the larger and smaller digits, takes into account both the ratio between the digit sizes and the difference between the digit sizes. On the top half of the slide the relation between Moyer and Landauer's original data to the predictions from the

Welford equation is shown, as well as a computation of the same relation for the data from Parkman's replication of Moyer and Landauer. On the lower half of the slide the same correlation is computed for each of six groups from our study. For each age level the correlation is similar to that observed in the previous studies with college student subjects.

A further illustration of the similarity between the judgment times for the youngest and oldest subjects in our study is shown in Figure 3. For each age group the mean decision time for each of the 36 pairs of digits was transformed to a Z-score using the overall mean and standard deviation for that age group. In Figure 3, the decision time for each of the 36 pairs of digits obtained from adult subjects is plotted against those obtained from the pre-school subjects. The observed correlation is approximately .8.

We do not wish to argue that there are NO developmental differences in number concepts across the age range four years to twenty-two years. However, there is a striking similarity in the data from the judgment of numerical inequality task across this age range. This task taps knowledge of the quantitative basis of numbers. Even our youngest subjects knew the ordered sequence of digits (the basis of the ordinal property of numbers) and they understood that numbers later in the series represent larger quantities. These children are at an age where they are generally unsuccessful on tasks requiring knowlege of the formal properties of number. However, these children appear to have a concept of number which includes representations of digits as analogue magnitudes, similar to adults. Furthermore, these children are able to compare the magnitudes successfully using a process qualitatively similar to the comparison process used by adults which is very similar to a perceptual comparison process. The decreasing mean and slope of the decision time function as age increases may indicate that the analogue representations of number become more discriminable with increasing age. Alternatively this developmental difference may have nothing to do with number concepts per se, but rather indicate that comparison processes in general are slower for younger children. In order to evaluate these two alternatives a developmental analysis of perceptual comparison processes is necessary. This investigation is currently under way in our laboratory.

We feel that a consideration of quantity concepts and the quantitative referent of number concepts is relevant for mathematics educators who seek the psychological basis of number concept.

REFERENCES

Moyer, R. S. Comparing objects in memory: Evidence suggesting an internal psychophysics. Perception & Psychophysics, 1973, 13, 180-184.

Moyer, R. S., & Landauer, T. K. Time required for judgments of numerical inequality. Nature, 1967, 215, 1519-1520.

Parkman, J. M. Temporal aspects of digit and letter inequality judgments. Journal of Experimental Psychology, 1971, 91, 191-205.

Welford, A. T. The measurement of sensory-motor performance: Survey and reappraisal of twelve years' progress. Ergonomics, 1960, 3, 189-230.

TABLE 1

Overall percent errors in the judgment of numerical inequality task (12 subjects x 144 trials = 1728 observations at each grade level)

GRADE	PERCENT ERRORS
Adult	0.8
5th	1.7
3rd	1.1
1st	3.6
Kindergarten	8.0
Pre-school	8.1

TABLE 2

Correlations between RT and log (L/(L-S)*
in judgment of numerical inequality studies

Moyer & Landauer (1967)	
Adults	*r* = .72
Parkman (1971)	
Adults	*r* = .91
Riley, Hu & Hinrichs (1977)	
Adults	*r* = .86
5th Grade	*r* = .81
3rd Grade	*r* = .82
1st Grade	*r* = .78
Kindergarten	*r* = .66
Pre-school	*r* = .82

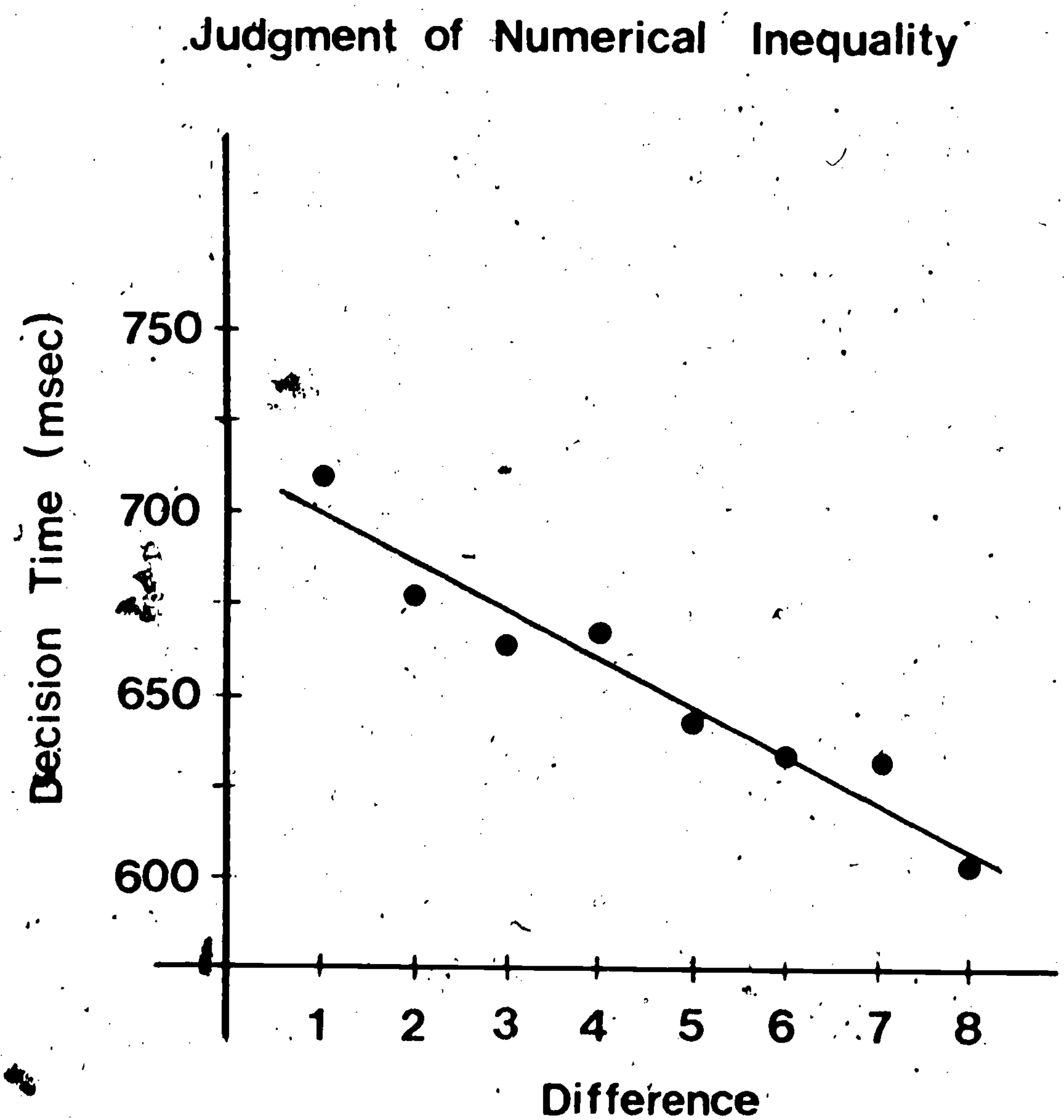
*Welford (1960) equation for psychophysical comparisons: RT = *a* + *k* log (L/(L-S))
L = stimulus with larger value in the pair to be compared, S = stimulus with smaller value

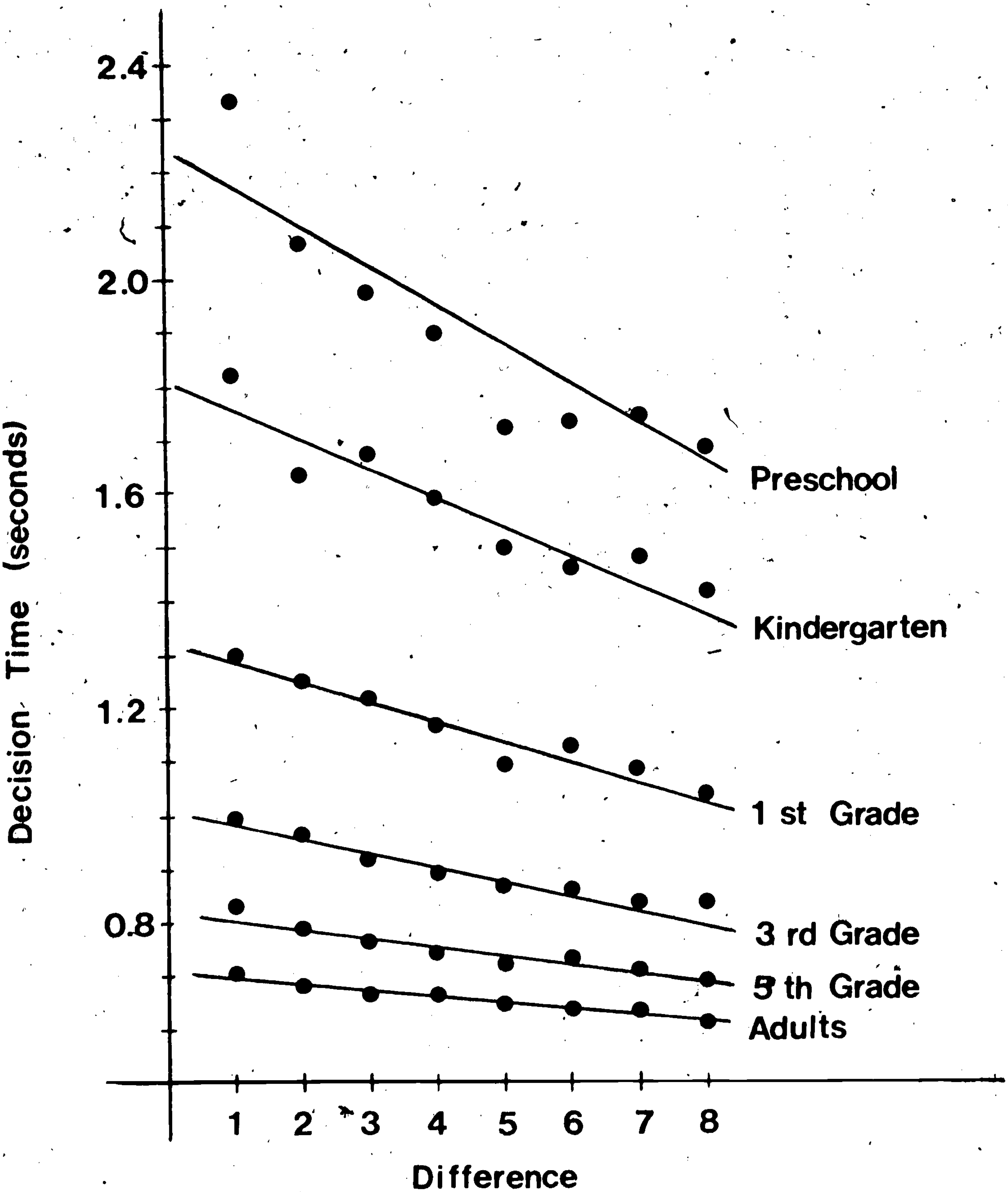
FIGURE CAPTIONS

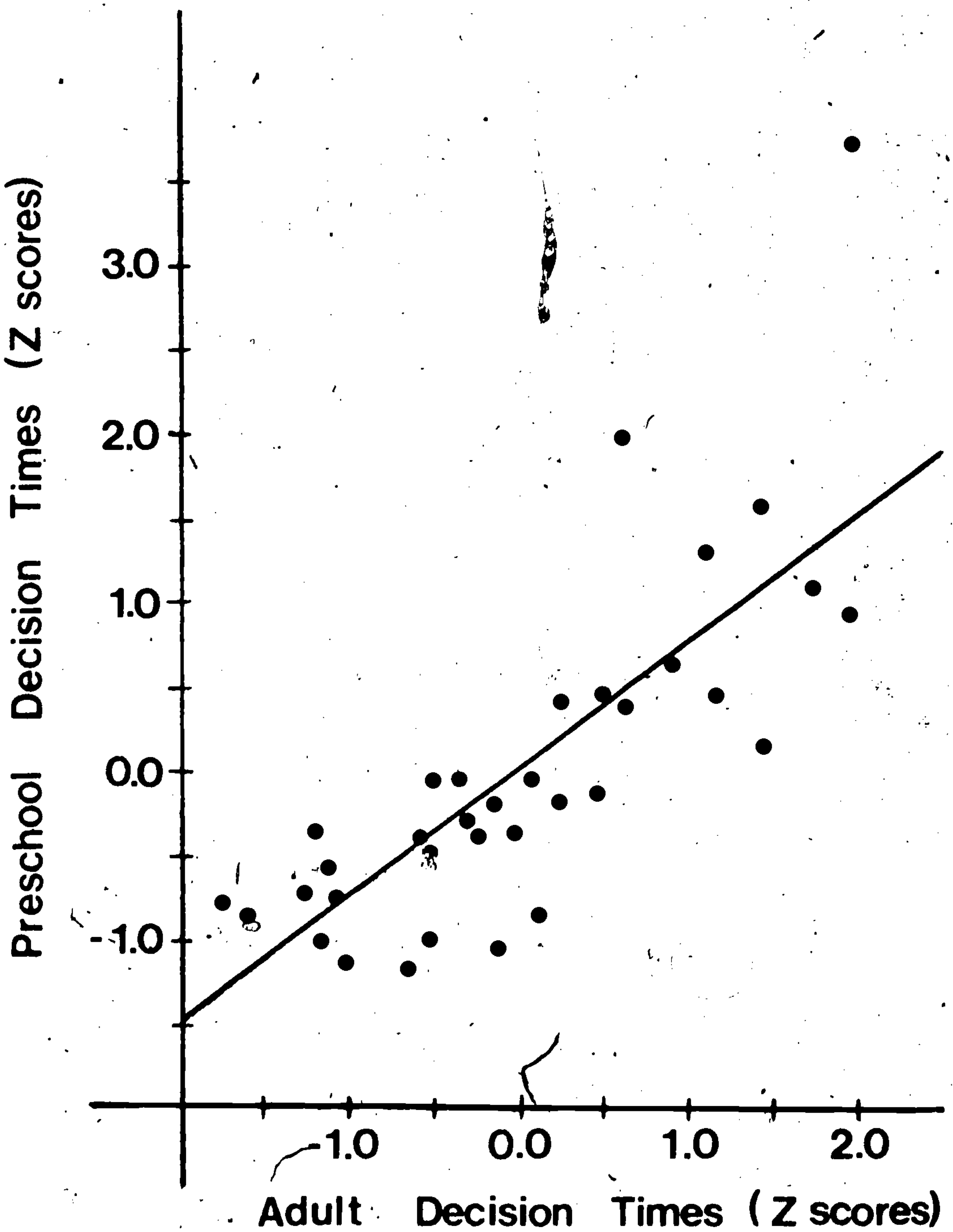
FIGURE 1. Harmonic mean decision times find the larger of two digits as a function of the numerical difference between the two digits for adult subjects.

FIGURE 2. Harmonic mean decision times to find the larger of two digits as a function of the numerical difference between the two digits for subjects at six grade levels.

FIGURE 3. Scatter plot of adult vs pre-school harmonic mean decision times for 36 pairs of digits. Decision times are transformed to Z-scores.


Judgment of Numerical Inequality
750
700
650
600
Decision Time (msec)
1
2
3
4
5
6
7
8
Difference


Decision Time (seconds)
2.4
2.0
1.6
1.2
0.8
Preschool
Kindergarten
1 st Grade
3 rd Grade
5 th Grade
Adults
1
2
3
4
5
6
7
8
Difference


Preschool Decision Times (Z scores)
3.0
2.0
1.0
0.0
-1.0
-1.0
0.0
1.0
2.0
Adult Decision Times (Z scores)