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ABSTRACT

Since Berlin and Kay's proposal in 1969 of two universals concerning the meanings of basic color words, three kinds of information have become available to help in understanding the encoding sequence, that is, the temporal order in which the basic color categories are accorded lexical status. McDaniel (1972, 1974, forthcoming) demonstrates that the encoding sequence is best understood not as the successive encoding of foci, but as the successive differentiation of color categories. Field studies of the color systems of a Jivaroan, a Mayan, a Papuan, an Eskimo and three Austronesian languages have been critical in understanding the encoding sequence. Fuzzy set theory as developed by Zadeh (1965, 1971) provides a formalism for expressing the understanding of basic color categories. The present study brings together work by McDaniel, the field studies, and fuzzy set formalism. If a color category is viewed as a fuzzy set, vacillation between focus and boundary is avoided when discussing universal color categories, their interrelations, and the sequence in which they are encoded in evolving color lexicons. (CLK)

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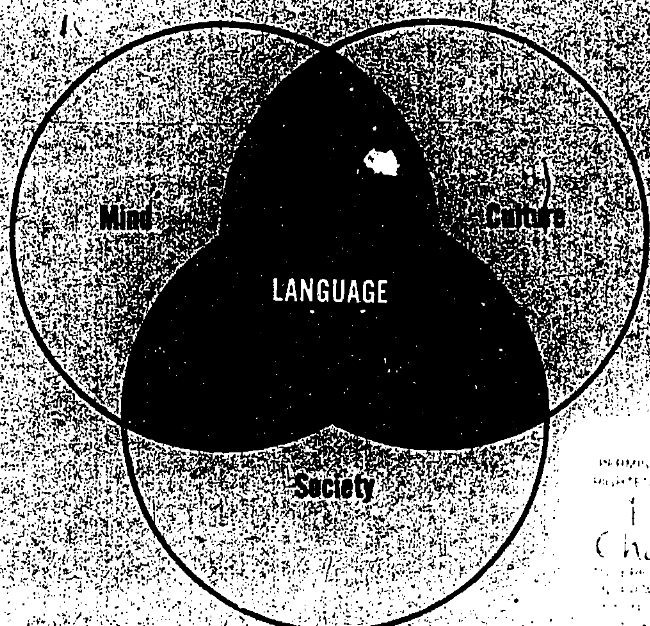

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Paul Kay and Chad K. McDaniel

August 1975

*We have benefited greatly from the ideas of John Atkins, Brent Berlin, Janet Dougherty, Craig Molgaard and Sue Thompson regarding the matters discussed in this paper.

In Basic Color Terms (BCT) Berlin and Kay (1969) proposed two universals regarding the meanings of basic color words:

(1) There is a small, universal set of perceptually definable color categories on which the meanings of basic color words in all languages are based. Hence basic color words are more simply and directly translatable across languages than was thought by linguistic relativists (e.g., Gleason 1961:4; Ray 1952:252; Bohannon 1963:35-7).

(2) There exist strong constraints on the temporal order in which the basic color categories are accorded lexical status[1]. Hence if we know the number of basic color terms in a system, we can predict to a fairly close tolerance just what the meanings of these terms are.

In BCT both universal hypotheses were expressed in terms of color category prototypes rather than in terms of category boundaries.[2] It was found that subjects could more readily identify the focal or ideal stimuli for a color term than the boundary of the category. Subjects could easily pick from an array of color stimuli an ideal or prototypical instance of a color category, whereas questions designed to elicit the boundaries of categories were, to a much greater degree, met with long hesitations, requests for clarification, and instability of response across subjects and across trials. Quantitative data on the extent to which judgements of category foci exceeded judgements of category boundaries in consistency and ease of response were not obtained, in part because the effect was so overwhelming that quantitative documentation seemed gratuitous and in part because it was not envisaged that the contrast would play a formal role in the developing theory.

Thus in 1969 the available data indicated that the foci of categories rather than their boundaries were the appropriate units in which to express the universal hypotheses (1) and (2). This choice, however, lead to equivocation in the expression of the encoding sequence, since while the formal expression of the sequence (cf. Figure 1) spoke only of category foci, the informal discussion explicitly considered category boundaries and dealt specifically with the changes in category boundaries which occur as successive foci are encoded. For example, the category designated 'RED', whose encoding marks developmental Stage II, contains at that stage the warm colors, including yellows. But when yellow is accorded its own term, at Stage IIIb or IV, 'RED' is restricted to reds. Hence the equivocation: 'RED' designates the warm colors at one stage and only reds at a later stage. The same equivocation applies to all the color percepts designated in BCT by capital letters: BLACK, WHITE, RED, GREEN, YELLOW. While the text of BCT speaks of the encoding sequence in terms of the "successive encoding of foci," it also interprets symbols such as 'RED' in terms of boundaries, and, to make matters worse, in terms of different boundaries at different stages. The elements of Figure 1 were interpreted sometimes as foci and sometimes as bounded regions, and in the latter case a given element did not always designate the same region.

$$\begin{tikzcd}[column sep=small]
 & & {[\text{green}]} \arrow[r] & {[\text{yellow}]} \arrow[dr] & & & \\
\begin{bmatrix}\text{white}\\ \\ \text{black}\end{bmatrix} \arrow[r] & {[\text{red}]} \arrow[ur] \arrow[dr] & & & {[\text{blue}]} \arrow[r] & {[\text{brown}]} \arrow[r] & \begin{bmatrix}\text{purple}\\ \text{pink}\\ \text{orange}\\ \text{grey}\end{bmatrix} \\
 & & {[\text{yellow}]} \arrow[r] & {[\text{green}]} \arrow[ur] & & &
\end{tikzcd}$$

(BCT:4)

"...whenever we speak of color categories, <u>we refer to the foci of categories rather than their boundaries or total area</u>, except when specifically stating otherwise" (BCT:13).

The Original Encoding Sequence and Its Interpretation

Figure 1

Since 1969, three kinds of information have become available which permit us to remove the equivocation in the expression and interpretation of the encoding sequence. The first is from the work by McDaniel (1972, 1974, forthcoming) which explains the existence of the universal color categories and their sequential encoding in terms of an opponent processes model of the neural encoding of color sensations. McDanial (1974, forthcoming) shows that color categories as such, and not merely focal colors, are the basic units involved in the encoding sequence. He demonstrates that the encoding sequence is best understood not as the successive encoding of foci, but instead as the successive differentiation of color categories. This interpretation of the encoding sequence is followed in this paper.

The second kind of new information comes from a series of controlled field studies of the color systems of a Jivaroan, a Mayan, a Papuan, an Eskimo and three Austronesian languages (Berlin and Berlin 1975; Harkness 1973; Hage and Hawkes n.d.; Heinrich 1973; Dougherty 1974, 1975; Heider 1972a,b; Kuschel and Monberg 1974). These data have been critical in the development of the new understanding of the encoding sequence and the results of these studies as regards the substance of the encoding sequence are summarized in Berlin and Berlin (1975), Dougherty (1974, 1975), Kay (1975), and McDaniel (1974, forthcoming).

The third development which now allows us to escape the equivocation of the original Berlin and Kay (1969) formulations is an appropriate formalism for expressing our new understanding of basic color categories. Fuzzy set theory as developed by Zadeh (1965, 1971) provides such a formalism (cf. also G. Lakoff 1972). The purpose of the present paper is to bring together the work by McDaniel and the empirical insights of the field linguists cited above with the fuzzy set formalism, which appears ideal to represent basic color categories as they are now conceived. In particular, if we view a color category as a fuzzy set we are saved from having to vacillate between focus and boundary when discussing universal color categories, their interrelations, and the sequence in which they are encoded in evolving color lexicons.

Fuzzy Sets, Membership

This and subsequent sections present, for the convenience of the non-mathematical reader, a non-rigorous description of the basic elements of fuzzy set theory. The mathematical reader is referred to the original works of L.A. Zadeh (1965, 1971). Consider the words Congressman and gourmet. The former seems to denote a set in the accepted sense; in particular something is either a Congressman or it is not--the Congress does not admit of degrees of membership. This is the only sort of set that standard set theories countenance. On the other hand gourmet (like many other words) seems to denote something very like a set, except that individuals appear to have different degrees of membership. Charles may be more of a gourmet than Harry and less of a gourmet than Anne. Zadeh has constructed the notion of fuzzy set to formalize this sort of intuition.

A fuzzy set A is defined by a characteristic function f_A which assigns to every individual x in the domain under consideration a number $f_A(x)$ between 0 and 1 inclusive, which is the degree of membership of x in A.[1] For example, letting 'f_G' symbolize the characteristic function of the fuzzy set gourmet, perhaps $f_G(\text{Harry}) = .4$, $f_G(\text{Charles}) = .7$, and $f_G(\text{Anne}) = .9$. If so the inequalities given above in words are satisfied: $f_G(\text{Charles}) > f_G(\text{Harry})$ and $f_G(\text{Charles}) < f_G(\text{Anne})$.

Color categories are ideally suited for interpretation in terms of this fuzzy set formalism. That degrees of color category membership are commonly recognized is apparent from even casual consideration of the ways colors are talked about. There are good and pure reds and there are poor and off reds. To be a pure red is to be more a member of the class of red things than to be an off red. Blue-greens, which are neither good greens nor good blues, are referred to by a construction which explicity recognizes the intermediate, dual category membership of the colors called by this name. The phrase a slightly purplish-blue is most appropriate for a sensation which is only marginally a member of the category purple while being a nearly perfect example of the category blue.

As these examples illustrate, nearly all of the productive uses of color terminology that can be observed indicate that color percepts continuously intergrade. Thus Gleason (1955), Ray (1952) and others are correct in their assertion that colors form a perceptual continuum. They are wrong in their assertion that this continuum is arbitrarily segmented or categorized. As noted, Berlin and Kay (1969) demonstrate that there are universal focal colors which are prototypical examples of the basic color categories named in all languages. (However, as we shall see, basic color terms do not in all languages map one to one onto the universal color categories.) Treating color categories as fuzzy sets, the regions in the color space where these focal colors are located can be understood as regions where color category membership values universally reach maxima.

McDaniel (1972, 1974, forthcoming) shows that there are also universal constraints on the spectral locations of color category boundaries. In the fuzzy set framework these may be understood as points in the color space where certain category membership values always reach zero. As one moves from the boundary of a color category toward its focus, the membership values increase gradually from zero to unity. The intergrading of color categories is captured in this model by assigning many colors positive degrees of membership in more than one color category. For example a blueish green is a color that has greater membership in green than in blue. The difficulties subjects experience making boundary judgements simply reflect the fuzzy nature of color category membership in these non-focal regions of the color space, where the colors perceived are examples, that is members, of several basic color categories.

Color thus presents itself as a perceptually continuous domain which the speakers of all languages fuzzy-categorically segment in accord with a universal set of fuzzy color categorization principles. McDaniel (1974, forthcoming) notes that this understanding of basic color perception and categorization parallels the treatment of color categorization implicit in the opponent process model of human color vision. The opponent process model, for which DeValois, and his co-workers have recently provided direct neurophysiological evidence (DeValois, Abramov, and Jacobs 1966; DeValois and Jacobs 1968), postulates that all colors are perceived as either pure, unique instances or as intergrading mixtures of six discrete primary color sensations--white, black, red, yellow, green and blue.

Psychophysically determined wavelength distributions of the four chromatic fundamental color responses are shown in Figure 2a. The points marked R, Y, G, and B represent points along the spectrum where unique, pure instances of the four fundamental hues are perceived. All other hue perceptions are seen as combinations in varying degrees of pairs of these four fundamental hues. A quantity often used by psychologists working with this model to indicate perceived hue is the hue coefficient, which specifies at each wavelength what percent of the associated hue sensation subjects report as being contributed by each of the four fundamental hues. A typical hue coefficient diagram, derived from Figure 2a, is shown in Figure 2b. The diagram indicates that subjects report the hue associated with light of 475 nm is perceived as 100% or uniquely blue, while light of 650 nm is perceived as 50% red and 50% yellow.

McDaniel (1972, 1974, forthcoming) has shown that if we take this opponent processes model and the hue responses and hue response distributions it embodies as givens and simply assume that basic color terms are named for these neural events and combinations of these neural events, the universality of basic color category foci and boundaries and their sequential encoding can be understood as the direct reflect in language of the neural processes underlying the human perception of color.

Following this analysis, the formal expression of the understanding of basic color categories we present here begins with the assumption that the psychologists' hue coefficient measures can be taken as characterizations of four primitive color category membership functions. In Figure 2c, four primitive fuzzy sets-blue, green, red and yellow-are depicted in a universe where, for simplicity of presentation, saturation and brightness are held constant and variation in hue is characterized in terms of the corresponding physical measure, wavelength. The figure shows for each wavelength the degree to which a saturated color of that dominant wavelength is a member of the universal perceptual categories blue, green, yellow or red. The points of maximum membership indicate the spectral locations of focal blue,

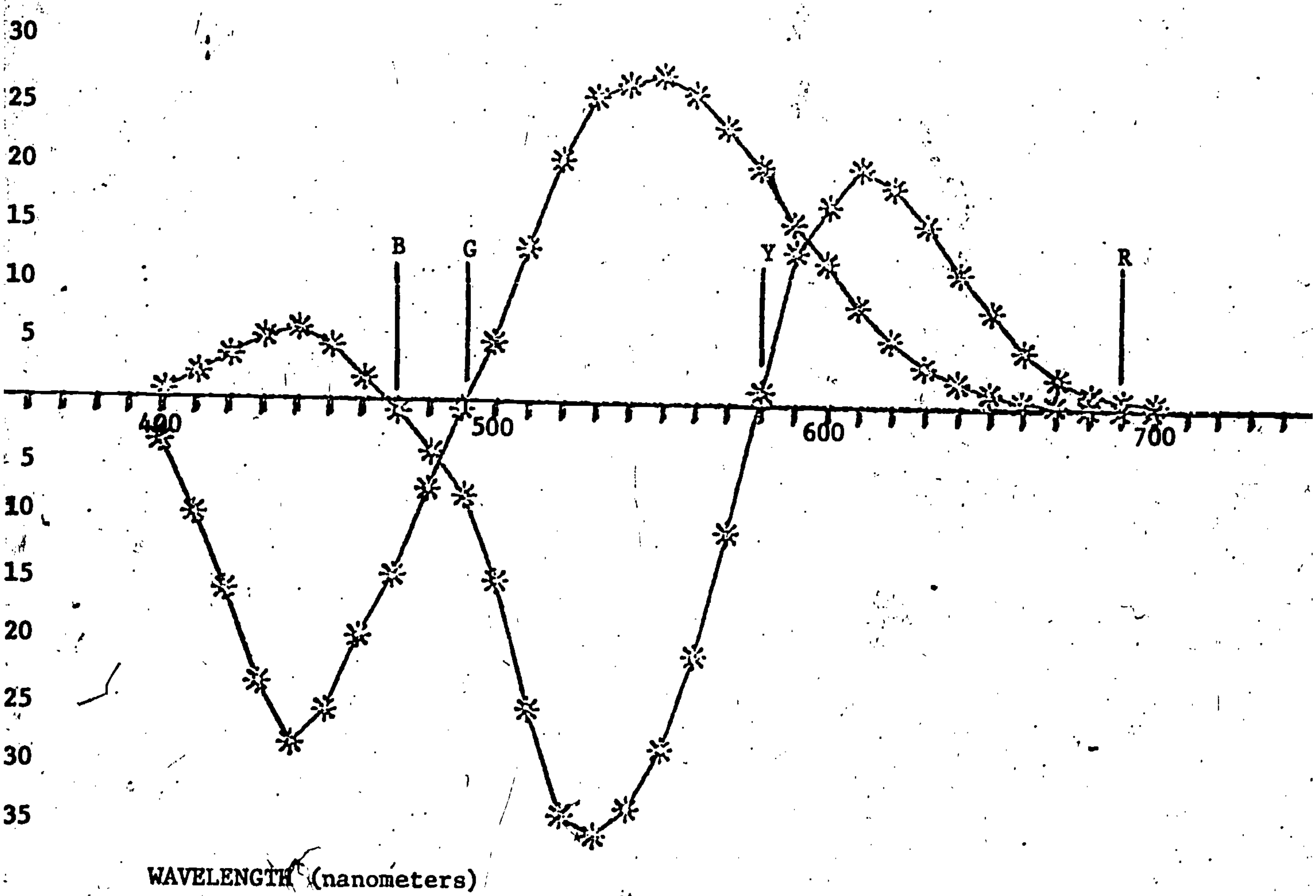


Figure 2a. Chromatic valences of the opponent process mechanisms by wavelegth. Values are theoretical values determined by Wooten (1970) using methods outlined in Hurvich and Jameson (1968). Extra-spectral values (points below 400 nm and above 700nm on the x-axis) can be estimated, but are here, as in Wooten, left undefined.

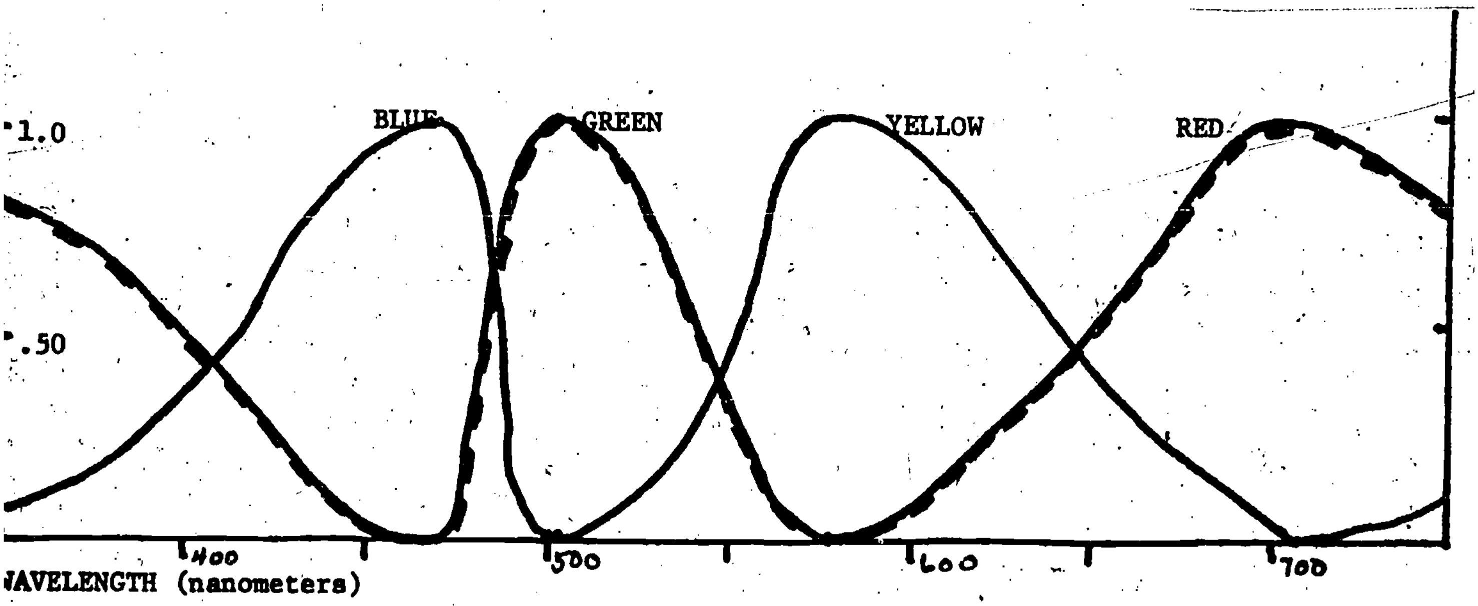


Figure 2c. Membership functions for the primary basic categories red, yellow, green and blue. The points of maximum and minimum membership are the same as those determined by the hue coefficient calculations summarized in Figure 2b.

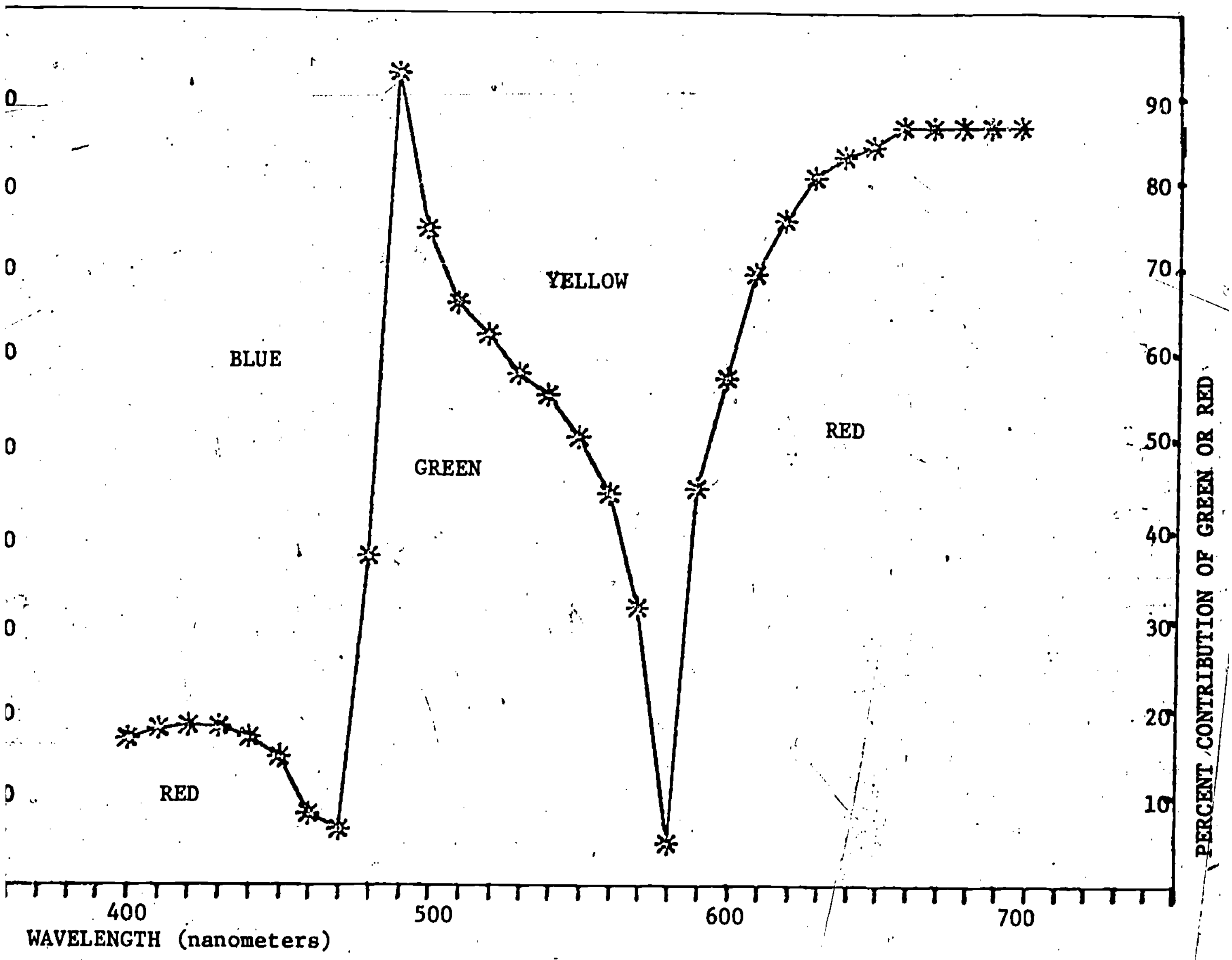


Figure 2b. Hue coefficients calculated from opponent response chromatic valence values given in Figure 2a by the equations $HC_{rg} = |RG| / |RG| + |YB|$ and $HC_{yb} = |YB| / |RG| + |YB|$. (It should be noted that $HC_{rg} = 1 - HC_{yb}$.) Point shown at 400nm indicates that when light of this wavelength is seen subjects report the percieved hue as roughly 15% red and 85% blue.

green, yellow and red. The membership maxima (focal points) for these four fundamental hues correspond in wavelength to the points where membership value reach zero for the adjacent fundamental categories. These four focal points correspond logically and in physical measure to the physiologically unique hue points posited by the opponent process model (McDaniel 1972, forthcoming). Hence each of these curves, like the hue coefficients, has a single maximum at its own unique hue point and meets the abcissa at the neighboring unique hue points.[2]

Taking these membership functions as primitives, we can show that all basic color categories at all stages of basic color lexicon development can be treated formally as functions defined on these primitive membership functions. Simplest are Stage V color term systems, with six basic color terms. These systems associate a single basic color term with each of the six fundamental color responses. The membership functions for the terms in any Stage V language are therefore identical with the primitive membership functions shown in Figure 2c plus those for black and white. Categories like these, whose membership functions are the same as one of the primitive membership functions, will be referred to as primary categories. Other basic color categories are treated below as more complex functions defined on these primitive membership functions.

When we cast color categories in this mold, several heretofore informal observations regarding the synchronic and diachronic relations among color categories can also be given direct formal expression. Consequently it is convenient to use color itself as a domain in which to provide illustration of some additional basic concepts of fuzzy set theory.

Containment

In ordinary set theory a set A is contained in a set B (equivalently, A is a subset of B) just in case everything that is a member of A is also a member of B. If there is also at least one member of B which is not a member of A, we say that A is a proper subset of B. In fuzzy set theory, a set A is contained in a set B just in case everything that is a member of A is a member of B to an equal or greater degree. Consider the sets designated by the English word blue and the Tzeltal word yaš 'green or blue, grue'. The former set is contained in the latter in that every color percept that is to any degree a member of English blue is an equally good representative of Tzeltal yaš. Furthermore there are many percepts that have zero degree of membership in blue that have non-zero degree of membership in yaš, that is, all the greens that have no blue in them. Hence, in an obvious sense, blue is also a fuzzy proper subset of yaš. Consider now a bluish-green (i.e. a color containing more green than blue) in the range of colors designated in English by a word such as turquoise. Such a color has a non-zero but quite low degree of membership in blue when compared to, for example, the color of the sky on a clear day. Informal observations indicate that such colors are better examples of the category yaš (green or blue) than they are of blue and this may be expressed in fuzzy set theory, as we shall see, although facts of this kind are unrepresentable in ordinary set theory. It is hard to imagine the kind of experiment that could establish this observation as a fact unless we were to find a population of Tzeltal-English bilinguals, but Tzeltal-Spanish bilinguals exist and relevant experiments could be performed on such subjects substituting Spanish azul for English blue. Closer to home we can imagine experiments--of the sort performed by Rosch (1973) to determine the degree of membership of various stimuli in categories--which would employ color categories such as scarlet, crimson and red. Stimuli that occur on the perceptual boundary of scarlet and crimson might well receive lower degrees of membership in both scarlet and crimson than they would in red. The fuzzy set formalism seems to capture neatly our intuitions that scarlet and crimson are both reds and that a color which is scarlet to some slight degree and crimson to some slight degree may be red to a greater degree than it is either scarlet or crimson.

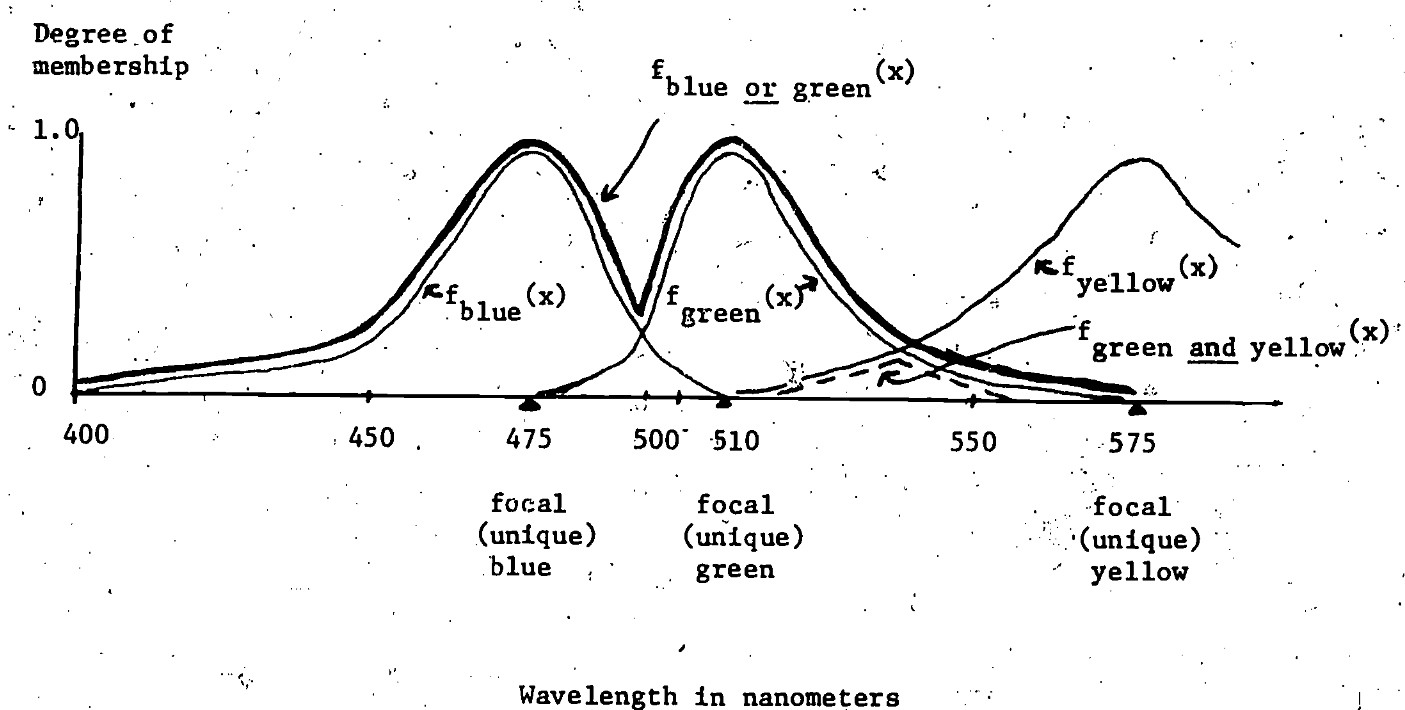
Union

In standard set theory, the union of two sets A, B is the set that contains everything that is either in A or in B or in both. Thus, if the set of people eligible for cheap tickets is the union of the set of registered students and the set of people under twelve years old, people who are either registered students or under twelve or both are eligible. The union of the fuzzy sets A, B, which we will denote 'A or B' is defined by a function which assigns to each individual x the larger of the two values $f_A(x)$, $f_B(x)$. In symbols, we define the union of two fuzzy sets A, B by the equation.

(3) $f_{A\ \underline{or}\ B} =_{df} \text{Max}\ [f_A, f_B]$

Let us suppose that we are for some reason interested in forming the union of the fuzzy sets competent basketball player (B) and competent landscape painter (P). (Perhaps we are composing a guest list for a potentate whose principal avocations are basketball and landscape painting.) Let us further suppose that Kareem Abdul-Jabbar has degree of membership of .99 in B and degree of membership of .02 in P, while Joe Furge, who has played semi-pro basketball and sold a few watercolors, has a degree of membership in these fuzzy sets of .5 and .6 respectively. Intuitively, our potentate will be more interested in meeting Abdul-Jabbar than in meeting Furge, and so we are glad to note that our definition of (B <u>or</u> P) gives a higher degree of membership to Abdul-Jabbar (.99) than to Furge (.6), despite the fact that the sum and product of Furge's degrees of membership both exceed those of Abdul-Jabbar. In standard set theory an individual is in the union of two sets if it is in either set and in fuzzy set theory an individual is in the union of two sets to the greatest degree that it is in either set (not to the degree that it is in both, whatever that might mean).

Several basic color categories found in pre-stage V color systems are formally representable as unions of various of the six primary categories (See Figure 2d).



The Fuzzy Sets Blue, Green and Yellow Showing the Union of Blue and Green and the Intersection of Green and Yellow.

Figure 2d

On the figure the heavy solid line represents $f_{\text{blue } \underline{\text{or}} \text{ green}}$, that is the (fuzzy) union of blue and green, (blue or green) = grue. Like other categories formed from the union of two or more primaries, grue will be referred to as a composite category. Grue is composite in the simple sense of being composed of all greens and all blues. Of the fifty-seven composite categories which might be formed by the union of various primaries, only three in addition to grue (cool) have actually been observed as basic color categories. These are warm (red or yellow), light-warm (white or red or yellow), and dark-cool (black or green or blue) (Berlin & Berlin 1974; Dougherty 1975; Kay 1975; McDaniel 1974, forthcoming). These composite categories are found only in Stage IV and earlier systems.

Referring again to Figure 2d, note that a blue-green stimulus of, say, 495 nm has a lower degree of membership in the grue category than a stimulus near either the blue or green focal point, 475 nm and 510 nm respectively. This corresponds to the claim that in a language such as Tzeltal a stimulus of this color is a poorer example of yaš 'grue' than a stimulus near either the blue or green unique hue points. No direct evidence on this sort of question has been gathered from a language with a basic color term meaning grue. However, the fact that in such languages speakers nearly always select best examples of grue from stimuli at or very near one of the unique hue points (Kay 1975) lends strong, if indirect, support to this claim. As in the case of the containment relation, the union operation on fuzzy sets deals with facts and intuitions not capturable by considering color categories as sets of stimuli in the ordinary sense of "set".

Intersection

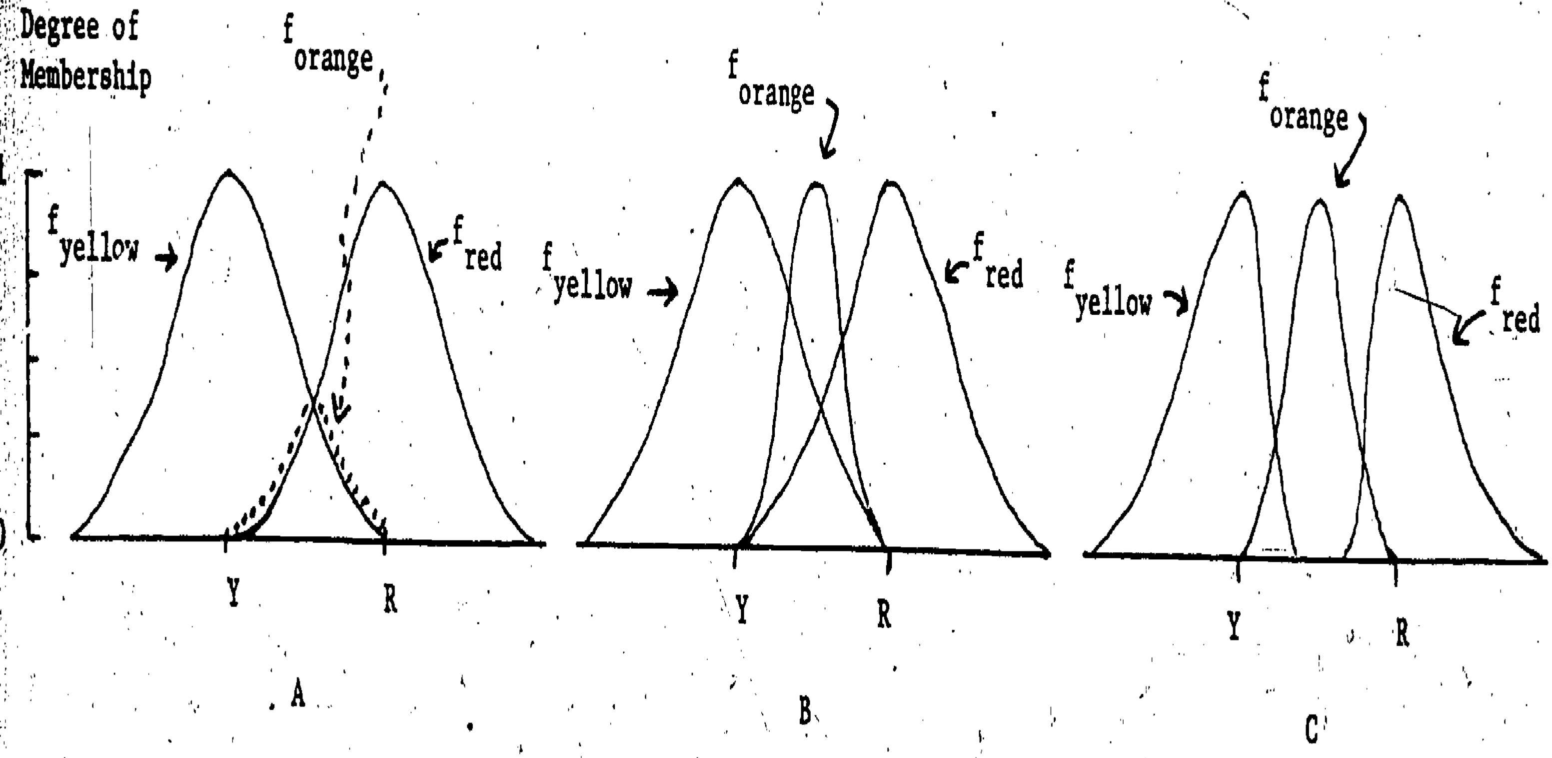
In standard set theory the intersection of two sets A, B is the set that contains just those individuals that are members of A and also members of B. The intersection of fuzzy sets A, B, denoted here 'A and B', is defined by the function that assigns to each individual x the smaller of the two values $f_A(x)$, $f_B(x)$. In symbols, the intersection of the fuzzy sets A, B is defined by the equation

(4) $$f_{A \text{ and } B} =_{df} \mathrm{Min}[f_A, f_B]$$

If the color category corresponding to the fuzzy set green and yellow is called chartreuse, the defining function of chartreuse is indicated by the dotted line in Figure 2d. Colors below 510 nm and above 575 nm have zero degree of membership in chartreuse. As one advances to the right from unique green (510 nm) and to the left from unique yellow (575 nm) one has initially quite poor examples of chartreuse, but as one continues from either end the stimuli exemplify chartreuse increasingly well, reaching a maximum somewhere in between unique green and unique yellow (though not necessarily exactly in the middle, the location of the maximum depending on the exact shape of the $f_{green}(x)$ and $f_{yellow}(x)$ curves as empirically determined. Cf. footnote 2).

Derived Color Categories

It appears that basic color categories like orange, which like non-basic categories such as chartreuse consist psychophysically of mixtures of primaries (Sternheim and Boynton 1966), should be related in some way to the corresponding formal constructs consisting of the fuzzy intersections of primary colors (e.g., yellow and red). The correspondence turns out, however, not to be one of identity.[3] Figure 3 shows three possible models for the semantic category orange in a language which has a basic color term for orange. (In plotting the membership functions of the fuzzy sets yellow, red and orange we have used the subjective hue scale as the abcissa because the perceptually purest red is non-spectral, that is, it corresponds to no monochromatic light. Dimmick and Hubbard 1939).



Three Models for Derived Categories

(Abcissa represents subjective hue scale. 'Y' and 'R' designate focal (unique) yellow and red points respectively)

Figure 3

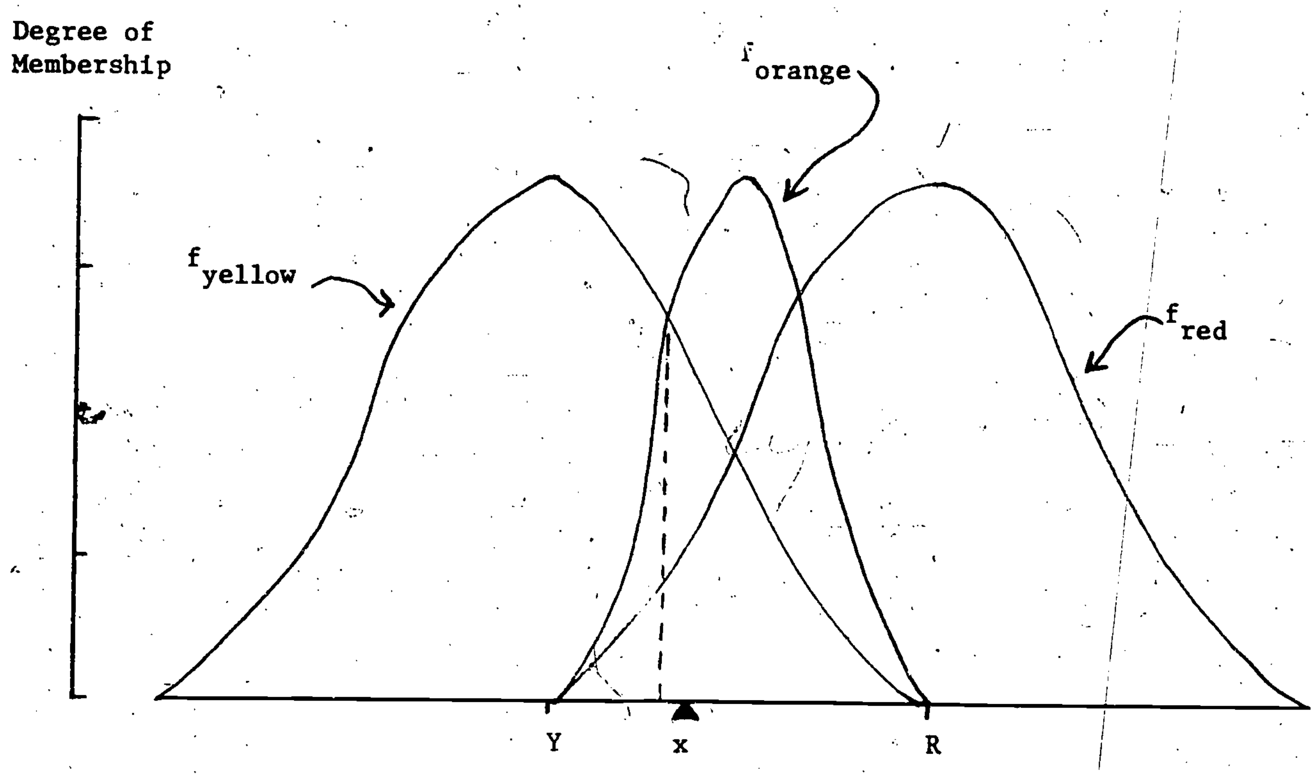
Model A takes the category orange as identical to the fuzzy set yellow _and_ red. There are two kinds of evidence that militate against this model. First, Model A implies that there are no really good examples of a derived category like orange. That is, the maximum of the membership function for orange is well below the maxima for yellow and red in this model. This implication contradicts the generally known fact that subjects are essentially as confident about assigning good examples of orange to that category as they are for the categories yellow and red. Many subjects declare that for them orange is just as fundamental a category--just as unique a color sensation--as either yellow or red. A second counterfactual implication of Model A is that no single hue sensation has a higher degree of membership in orange than it has in either yellow or red. Every hue point under the left half of the orange membership function has a higher degree of membership in yellow than in orange and every point under the right half has a higher degree of membership in red than in orange. This is quite contrary to experience, both experimental and casual. There is unquestionably a range of hues that speakers of English more readily label orange than either yellow or red in both experimental and natural speaking contexts (Sternheim and Boynton 1966).

Hence, we are forced to abandon the simplest model, A, in favor of a model in which orange is defined, not simply as the fuzzy intersection yellow _and_ red, but as an independent category with a separate membership function covering the region from unique yellow to unique red and reaching a maximum at or near unity somewhere in between. Models B and C both have these properties. The difference between Models B and C is that, while C stipulates the retraction of the inner portions of the red and yellow membership curves with the advent of a basic category orange, B maintains that every primarily red sensation on the yellow side of unique red is still yellow to a degree and that every primarily yellow sensation on the red side of unique yellow is still red to a degree. The sort of experiment that could decide between models B and C would be one in which subjects were asked to categorize orange stimuli, that is stimuli whose dominant hue lies between unique red and unique yellow, without using the category orange. If subjects could identify a small red component in stimuli close to unique yellow and a small yellow component in stimuli close to unique red, this issue would be decided in favor of Model B over Model C. Exactly such an experiment has been performed (Sternheim and Boynton 1966) with results clearly supporting Model B. When orange is not allowed as a response category, English speakers produce yellow and red naming responses which extend from one unique hue point to the other, showing a smooth decrease from the "home" unique hue point to the adjacent one. We therefore reject Model C in favor of Model B.[4]

Model B also captures the fact that while certain hue stimuli are categorized primarily by speakers of English as orange they can also be categorized in certain contexts, e.g., when the orange category is deliberately suppressed, as mixtures of yellow and red (Sternheim and Boynton 1966).

Model B also allows us to account for some of the variety of ways an English speaker might ask for a book of, for example, a yellowish orange color, where "yellowish orange color" means in the fuzzy set formalism having a dominant wavelength slightly to the right of the intersection of the membership functions of yellow and orange as shown by 'x' in Figure 4. A speaker might ask for "that yellowish orange book" or "that orange book", but if a book of such a color were on a shelf where all the other books were green, blue or near-focal red, an English speaker might simply ask for the "yellow" book.

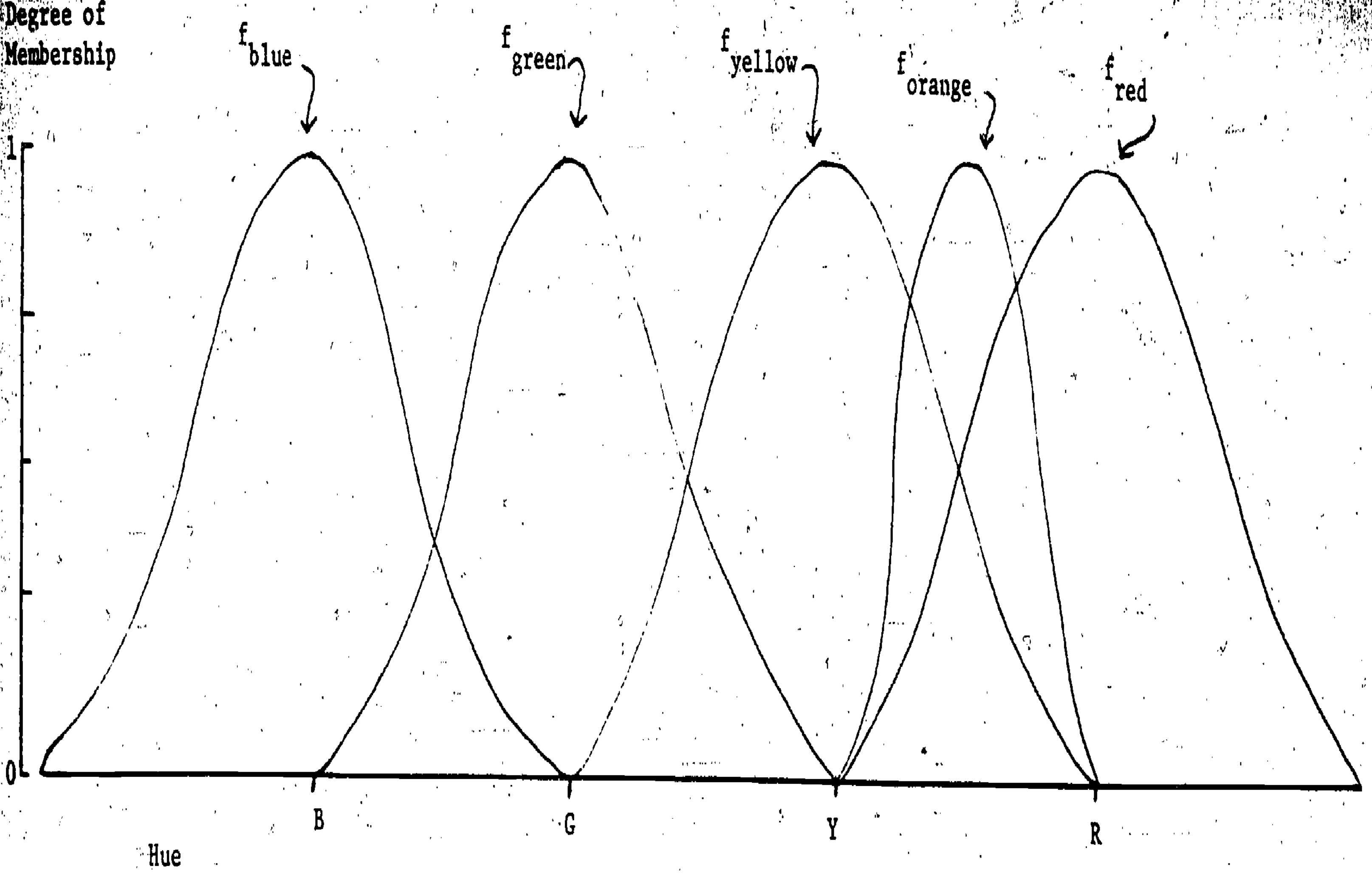
In this context, the relatively high degree of membership of x in yellow permits the use of the primary term yellow rather than the seemingly more appropriate derived term orange (in the sense that $f_{orange}(x) > f_{yellow}(x)$), since the context is such that no confusion with other objects in the stimulus field can arise.



A Yellowish Orange (x)

Figure 4

Derived basic color categories such as orange, pink, purple and grey thus have much in common with the six primaries. They have their own membership functions with fairly well defined maxima at or near unity and tails which approach zero at the neighboring focal points. There is, however, an important difference between derived and primary basic color categories; the membership maxima of the former are not associated with unique hue points in the opponent process model (McDaniel 1972). Figure 5 shows the membership functions for blue, green, yellow, orange and red. Note again that for blue, green, yellow, and red the points at which membership maxima occur correspond to points of zero membership for each other membership function. As mentioned these points correspond to the hues experimentally determined to be physiologically unique in the opponent process model. This, however, is not the case for orange as may be seen in Figure 5. The location of maximal membership, focal orange is not correlated with the existence of an underlying unique hue point. The adjacent categories, red and yellow, do not have membership function values of zero at focal orange. Thus the points of maximal membership in orange, and other derived categories, are not as simply relatable to events in the opponent-process model as is the case for focal red, yellow, green and blue.



Membership Functions of the Primary Categories Blue, Green, Yellow, and Red and the Derived Category Orange, Showing that the Latter Lacks a Unique Hue Point

Figure 5

The membership functions of "derived" color categories may be literally derived from the functions of the relevant primaries, at least in general outline. It may be observed in Figure 5 that the orange function varies inversely with the absolute difference of the yellow and red functions. At the yellow and red unique hue points, the value of the absolute difference $\left| f_{yellow}(x) - f_{red}(x) \right|$ is unity and the value of $f_{orange}(x)$ is zero. As one moves to the right from unique yellow and to the left from unique red this difference decreases until it reaches zero at the point where the f_{yellow} and f_{red} functions intersect. Near this point the value $f_{orange}(x)$ approaches unity. The fuzzy set f_{orange} may thus be defined over the domain for which the membership function of (yellow <u>and</u> red) exceeds zero:

$$(5) \qquad f_{orange}(x) =_{df} 1 - \left| f_{yellow}(x) - f_{red}(x) \right|$$

Equation (5) is not a summary of empirical measurements actually made, although experiments could be conducted to assess its accuracy. The purpose of equation (5) in the present context is just to demonstrate at a conceptual level that when we speak of color categories such as orange as "derived" from yellow and red, there seems a fairly literal sense of "derive" in which the term is appropriate.[5]

If equation (5) or something like it corresponds to an actual neural event, it should leave behavioral traces. One possible observable effect of this extra step in the neural processing of derived colors might be that reaction times for recognizing focal examples of derived colors would be longer than for primaries. In a study in which the "focal" examples used were probably not optimal for a proper test of this hypothesis, Heider found nevertheless that "primary focal colors [black, white, blue, green, yellow, red] were named significantly more rapidly than non-primary focal colors [pink, brown, orange, purple]; $\underline{t}(22) = 2.86$, $\underline{p} < .01$ (for correlated measures)" (Heider 1972b:15).[6]

To summarize, we have described three types of basic color categories. There are six primary categories which correspond to the unique color sensations black, white, blue, green, yellow, and red (Hering 1964 [1920]; Jameson and Hurvich 1955; DeValois and Jacobs 1968; McDaniel 1972, 1974, forthcoming). Stage V color systems name these six categories with basic color terms. They may be represented as fuzzy sets whose characteristic functions, in the case of the hue terms blue, green, yellow and red, have a maximum of unity at the corresponding unique hue point and drop off to zero at the neighboring unique hue points. The second group, the composite categories, occur in terminologies prior to Stage V and consist in fuzzy sets which are unions of primaries. The third type, derived categories, are related, but do not correspond precisely, to the intersections of primaries. Although the same (non-fuzzy) set of stimuli have non-zero degrees of membership in the derived category orange as in (yellow <u>and</u> red), the fuzzy sets orange and (yellow <u>and</u> red) have distinct characteristic functions as described above. The characteristic function of a derived category reaches a value of zero at two unique hue points that are adjacent on the hue scale. This contrasts with the structure of the primary categories, which have zero values at unique hue points that have a third unique hue point between them. While a primary category attains its maximum membership at the latter unique hue point, the maximum of the membership function for a derived category does not correspond to a unique hue point. The characteristic function of a derived category may, moreover, be expressed as a function of the characteristic functions of the two constituent primaries, as in (5). The operation performed on two primary categories to obtain a derived category will be denoted with a plus sign. Hence (orange) = (yellow + red) ≠ (yellow <u>and</u> red).

Finally a question arises regarding the status of secondary color categories corresponding to non-basic color terms such as <u>chartreuse</u>. Like derived categories, these categories are psychophysically combinations of primaries. The fuzzy set formalism provides an attractive possibility to model them, but it is not clear whether or not this move is empirically justified. We may take a secondary category like chartreuse as corresponding to the intersection of the fuzzy sets green <u>and</u> yellow (see Figure 2, where this is tacitly assumed). An empirical claim entailed by this formal move is that that speakers with <u>chartreuse</u> in their active vocabularies should judge any stimulus they judge as chartreuse to a positive degree to be an equal or better representative of either green of yellow. This speculation is not immediately rejected by introspection or casual observation, but on the other hand we know of no systematic data that support it. If some subjects were found to react to appropriate stimuli with the labels <u>chartreuse</u>, <u>yellow</u> and <u>green</u> in the manner just suggested and others were to use <u>chartreuse</u> in the way that subjects generally use labels for derived basic categories such as <u>orange</u>, we might have grounds to say that <u>chartreuse</u> has achieved basic term status for the second group of subjects although not for the first.

The Encoding Sequence

The empirical facts relevant to the restatement of the encoding sequence (McDaniel 1974, forthcoming) are summarized in Berlin and Berlin (1974), Dougherty (1974, forthcoming), Kay (1975), and McDaniel (1974, forthcoming). In diagram (6) the arrow represents a binary relation which is irreflexive, antisymmetric, and transitive, and which may be read 'occurs before' or 'precedes in time'. The primary color categories, white, black, red, yellow, green, blue may be thought of as the fuzzy sets named by the English words _white_, _black_, _red_, _yellow_, _green_, _blue_ or the corresponding words in any language of Stage V, VI, or VII.[7] The two composite categories which constitute Stage I, light-warm (white _or_ red _or_ yellow) and dark-cool (black _or_ green _or_ blue), correspond on the one had to the "inherently light" and "inherently dark" groupings of color sensations recognized by Hering (1964 [1920]; see discussion in McDaniel forthcoming) and on the other are equivalent in extension to the two basic color terms in Dani (Heider 1972a, Heider and Olivier 1972). The composite category red _or_ yellow (warm), which occurs at Stages II and IIIa, is a basic color category for Stage II speakers in Bellonese (Kuschel and Monberg 1974) and for Stage IIIa speakers in Aguaruna (Berlin and Berlin 1975) and West Futunese (Dougherty 1974, 1975). The composite category green _or_ blue is the well known grue or cool category reported for many languages of the world including probably the majority of New World languages (Hays et. al. 1972; Bornstein 1973a,b). The derived categories brown, orange, purple, pink and grey are all present in English.

s:	I	II	IIIa	IV	V	VI	VII
	W or R or Y Bk or G or Bu	W R or Y Bk or G or Bu	W R or Y G or Bu Bk	W R Y G or Bu Bk	W R Y G Bu Bk	W R Y G Bu Bk Y+Bk (Brown)	W R Y G Bu Bk Y+Bk (Brown) R+W (Pink) R+Bu (Purple)
			IIIb				
			W R Y G or Bu or Bk				

← - - - - W+Bk(Grey) - - - - - - - - - - - - - - - - - →

The Evolutionary Sequence of Lexical Encoding of Basic Color Categories

A glance at the encoding sequence (6) shows several patterns that are neatly revealed by the fuzzy set formalism. First, we are able using this device to talk in a more general manner about the development of categories (= fuzzy sets) rather than having to shift back and forth between foci and boundaries, because fuzzy set theory is a more natural formalism for speaking about categories with degrees of membership than is the language of "focus" and "boundary". Furthermore the fuzzy set language is more accurate for color categories per se. When we formulate color categories as fuzzy sets we can see that the focus and boundary model says both too much and too little. To talk of boundaries entails--or at least invites the inference--that the edges of color categories are shaprly defined and that all stimuli within the boundary are equally first class citizens of the category. This is not so, since stimuli increasingly far from the focus are increasingly poor members of a category. The way in which the focus and boundary language says too little is closely related. In the focus and boundary terminology we in effect distinguish exactly three degrees of category membership, focal member, non-focal member and non-member, but the great mass of experimental data in color naming and classification as well as common intuition indicate that in fact color categories have continuously graded degrees of membership, rather than some particular small finite number, such as three.

Secondly, we can express formally the fact that progression through the encoding sequence amounts to cutting up the color space into successively smaller pieces (McDaniel 1974, forthcoming). The precise sense in which this is true will become clearer in the next section.

Thirdly, one may note that all the composite categories are dissolved into their constituent primaries, at Stage V, before any of the derived categories are formed, with the exception of the "wild card" category grey. Until Stage V, color lexicons develop by dividing categories composed of unions of primary color sensations into these primary categories. After Stage V, new basic color categories are derived from existing categories (McDaniel 1974; forthcoming) via functions such as (5).

Fourthly, the fact that so far no language investigated contains more than eleven basic color terms, with the possible exception of Russian (see Berlin and Kay 1969:35-6), seems now more an accident of the present moment in world history than a theoretical inevitability. Russian goluboy 'light blue, blue + white' is a potential instance of a twelfth basic color term, as it is a basic term for some Russian speakers, though probably not for all.[8] We cannot be certain that goluboy will not in the future achieve basic term status for all speakers of Russian. Similarly, it is possible that several secondary terms in English derived from pairs of basic categories will also become basic terms, for example, aqua, turquoise 'blue + green', maroon, burgundy 'red + black', chartreuse, lime 'green + yellow'. Some of these may be basic terms for some speakers already. Certain secondary categories in current American English appear to be derived from one primary and one derived category and it is not unthinkable that some of these may also some day become basic categories for a large number of speakers. Examples are beige 'yellow with grey', lavender 'purple with white'.[9] The fuzzy set formalism allows us to see how derived basic categories are produced by combining the primary categories, which denote the fundamental color percepts, and to see with greater clarity the general process involved in evolution of advanced color term systems. There is no apparent reason to believe that that pattern is complete at the present number of eleven basic terms.

Fuzzy Partitions

To continue our discussion of the encoding sequence of basic color categories viewed as fuzzy sets, it is desirable to extend fuzzy set theory in a minor and mathematically trivial way by defining fuzzy partition. In standard set theory a collection of subsets partitions a set A just if (a) everything in A belongs to one of the subsets and (b) nothing in A belongs to more than one of the subsets. The subsets are often called cells or blocks of the partition.[9] We take as our universal set the set of color percepts, and we wish to generalize conditions (a) and (b) so that we may ask and answer questions such as whether a given set of color terms partitions the universal set of color percepts or whether a given pair of color categories such as blue and green partition a third category such as blue or green (grue). Condition (a), the exhaustiveness condition, is taken over directly into the definition of fuzzy partition, translating 'belongs to' as 'having a non-zero degree of membership in'.[10] But condition (b), the mutual exclusion condition, cannot be taken over directly since distinct but adjacent primary color categories overlap a great deal. As we saw above (see Figure 2), every green above 510nm (unique green) is also to some positive degree yellow, and every yellow below 575nm (unique yellow) is to some positive degree green. As Figure 2 shows, all greens except for a point or very narrow band width around 510nm have some component of either blue or yellow in them. These are not suppositions of the formalism but facts for which the formalism must account.

We will replace condition (b), the mutual exclusion condition, by a statement to the effect that each cell of the partition has at least one member that belongs to no other cell.

It will be noted that in availing ourselves of the convenience of saying that an individual belongs to a fuzzy set A just in case that individual has a non-zero degree of membership in A, we have given tacit recognition to the fact that to every fuzzy set A there corresponds a standard set, which we will denote 'A_s', whose members are the individuals that belong to A. For example, the standard set corresponding to the fuzzy set green is denoted '$green_s$' and consists of those color percepts that are to some positive degree green; that is any color whose dominant wavelength is between 475 nm and 575 nm (see Figure 2). The relevance of this comment is that when we define the notion partition for fuzzy sets we want the set being partitioned to be a standard, not a fuzzy set, although we want the cells of the partition to be fuzzy sets. For example, when we say that the fuzzy sets green and blue form a partition of the set "grue", by "grue" we mean $grue_s$, the standard set whose members are grue to any positive degree. This approach allows us to ask and answer the question whether a full color terminology partitions the universal set of color percepts. The latter is a standard, not a fuzzy set, and so particularly in this case we want a fuzzy partition to take a standard set into a collection of fuzzy sets.

Given a standard set A_s and a collection P of fuzzy sets $P_1, P_2, \ldots, P_n$ the union of whose corresponding standard sets, $p_{1_s} \cup \ldots \cup p_{n_s}$, is A_s, we say that P constitutes a fuzzy partition of A_s just in case (a) for every individual x in A_s there is a p_i in P such that x belongs to p_i and (b) for every p_i in P, there is at least one x in A_s such that (i) x belongs to p_i and (ii) there is no p_j in P ($p_j \neq p_i$) such that x belongs to p_j. In symbols, a collection P of fuzzy sets is a partition of a standard set A_s if and only if

(7) (a) $p_{1_s} \cup \ldots \cup p_{n_s} = A_s$

(b) $(\forall x)(\text{if } x \varepsilon A_s \text{ then } (\exists p_i)(p_i \varepsilon P \text{ and } x \varepsilon p_i))$

(c) $(\forall p_i)(\text{if } p_i \varepsilon P \text{ then } (\exists x)((x \varepsilon A_s \text{ and } x \varepsilon P_i) \text{ and}$

$\sim((\exists p_j)\ p_j \varepsilon P \text{ and } p_j \neq p_i \text{ and } x \varepsilon p_j)))$.

The primary categories, whose one-to-one lexical encoding constitutes a Stage V system, partition the domain of color. That is, (a) subjects given the labels black, white, red, yellow, green, blue can classify any color stimulus and (b) for each of these six categories there exist color stimuli that are classifiable only in that the category and not in any of the other five (Hering 1964 [1920]; Sternheim and Boynton 1966).

From this observation and a fuzzy set interpretation of the encoding sequence (6), it follows that every basic color lexicon contains a partition of the universe of color. Consider first the Stage VI and VII lexicons. Since all six primaries are contained in Stage VI and VII terminologies, these terminologies contain a partition of the color domain, which consists simply of the six primaries. Now consider terminologies of Stages I through IV. Each of these types of terminology consist of composite categories, composed by unions of and exhausting the list of primaries. It is easy to show that if we start with a fuzzy partition P of a standard set A_s and create a new collection of composite sets by taking unions of the members of P in such a way that every member of P occurs in exactly one of the composite sets, the collection of resulting composite sets is also a fuzzy partition of A_s.[12] Hence each terminology of Stages I through IV also constitutes a fuzzy partition of the domain of color percepts. So at any stage, the basic color categories contain a set of categories which partition the entire color space. The basic color term vocabularies at each stage provide terms for all colors.

From Stage I through Stage V, each evolutionary development consists of a refinement of the partition of the color domain, where "refinement of a partition" refers to the creation of a new partition which classifies separately everything classified separately in the old partition and in addition classifies separately at least two individuals classified as the same in the ole partition. The addition of derived basic categories at stages subsequent to V does not refine the partition of the color domain because the derived categories do not satisfy condition (7c) of fuzzy partition. There is no color sensation that belongs to a derived category that does not also belong to a primary category. In fact, every point stimulus in a derived category belongs to both of the categories from which it is derived. This fact supports the observation that derived categories are fundamentally less important than primary categories. Derived categories are in two senses gratuitious: (a) any color sensation can be referred to without using one of them, (b) no color sensation can only be referred to by using one of them (Hering 1964 [1920]; Sternheim and Boynton 1966).

In this context the question may be asked whether derived categories should be considered basic color categories at all in the sense of providing the meanings of basic color terms. In specifying the referential attributes of the definition of basic color terms, Berlin and Kay had in mind tacitly a standard set theory model when they said "[A basic color term's] significance is not included in that of any other color term" (1969:6). We may ask whether a basic but non-primary category like orange satisfies the translation of this criterion into fuzzy set theory. The answer is that it does. The derived category orange is not contained in either yellow or red (or of course any other category). It will be recalled that a fuzzy set, say that defined by $f_{red}(x)$ contains another, say that defined by $f_{orange}(x)$, just in case $f_{red}(x)$ exceeds $f_{orange}(x)$ for all x. But as we have defined the derived category orange this holds for neither yellow and orange nor for red and orange (see Figure 3B). Thus the notion that non-primary, derived categories may nonetheless be basic color categories survives the translation into the fuzzy set model.

Conclusion

It has been shown that it is possible to give natural expression in fuzzy set theory to a large number of empirical observations regarding the extent, nature, and evolutionary relationships among basic color categories. Unlike standard set theory, fuzzy set theory is a continuous, rather than discrete, mathematics. Hence, it appears that the natural mathematics to describe the perceptual/cognitive schema underlying at least one semantic domain is continuous. Evidence is accumulating that the domain of color is not unique in this regard (Kay n.d.), and the sum of this evidence offers additional challenge to traditional, discrete feature theories of semantics (Fillmore 1975).

Footnotes

[1] Actually the range of the function need not be restricted to the real interval [0,1], but we have no need to concern ourselves here with these more general mathematical considerations.

[2] For convenience in this and subsequent figures, characteristic functions are drawn as symmetrical, bell-shaped curves with the unique hue point at the center. This is strictly a graphic convenience in an argument that is not concerned with the details of the shape of these curves. Existing evidence suggests that the real functions are not in fact symmetrical about the unique hue points. Moreover, unique hue points are not equally spaced on either the wavelength or subjective (jnd) scales, as they appear in some subsequent figures. Nothing in the argument of this paper, however, hinges on the over-simplifications employed here in depicting the characteristic function curves.

[3] The question of non-basic categories like chartreuse is taken up again later.

[4] At the moment we have no evidence which would allow us to determine if, with the introduction the derived category orange, the membership functions for the categories red and yellow would remain the same or might be lowered while still remaining positive throughout the region between unique yellow and unique red. Experiments of the type performed by Sternheim and Boynton (1966) comparing speakers of a Stage V language (naturally without orange) to Stage VII speakers prohibited from use of orange might reveal these details of the effect that introducing a derived category has on the primary category membership functions.

[5] As we remarked in note 2, the curves given here are not intended to be taken literally. If the membership curves for yellow and red were bell-shaped as pictured, the derived curve for orange given by (5) would be pointed at the top, i.e., discontinuous, rather than reaching its peak in a smooth, bell-shaped manner. Again we leave to empirical investigation the establishment of the detailed shape of the curve. Perhaps log or power functions will ultimately be found to relate overt color naming behavior to the relevant physical continua, as functions of this general kind are frequently involved in relations between subjective sensations and the underlying physical scales (Stevens 1957).

John Atkins has pointed out (personal communication) that the right side of (5) is equivalent to

$$2\ (\ |f_{yellow}(x)\ \underline{and}\ f_{red}(x)|\ )$$

on the reasonable assumption that at any point on the abcissa all the membership functions for the four primary color categories sum to unity.

[6] The absolute order of reaction times was not in exact agreement with the hypothesis. Whereas orange, purple, pink and brown should have had the four longest response latencies, the actual rank order of latencies was (from shortest to longest) black, yellow, white, <u>purple</u>, blue, red, pink, brown, <u>green</u>, orange (Heider 1972b:15). But there are at least two difficulties with Heider's methodology which suggest caution in interpreting these results. First Heider determined "focal" colors by taking the "geometric centers" of the areas enclosing all the focal choices in the Berlin and Kay (1969:9) data (Heider 1972b:12). Unfortunately, focal greens in Berlin and Kay were overextended toward blue due to a misclassification of Vietnamese <u>xanh</u>, grue, with a near unique blue focus as GREEN. Thus the "focal" green Heider used,

Munsell 7.5G 5/10, is rather distant in both hue and saturation from the 10 GY and 2.5 G hues typically chosen as focal greens. This particular difficulty might help explain the especially low position of green in the response latency rankings.

Second, and more importantly, Berlin and Kay obtained their focal color judgements, which Heider used to select her stimulus materials, under a standard illuminant A while Heider conducted her tests with daylight flourescent illumination. The difficulty here is that object colors can shift radically with changes in illuminant conditions. Thus, Heider's relatively well defined illuminant A focals may have been poor approximations to the focal colors when viewed under her daylight flourescent illuminant conditions. A less than ideally constituted set of stimuli for the given experimental conditions may therefore be in part responsible for the equivocal nature of these results.

[7] The dotted arrow indicates that grey may occur "as a wild card at various points in the sequence" (Berlin and Kay 1969:45). Whereas, Berlin and Kay originally guessed that it might occur, "say at any point after Stage IV" (p. 45), more recent information shows that grey may occur at any stage from IIIa onward, or possibly even earlier (Alpher, personal communication; MacLaury n.d.).

[8] (Cf. Daly n.d.) Other Slavic languages have mono-lexemic terms for 'light blue', but these appear to be basic terms for very few speakers, if any (Daly n.d.). For a discussion of inter-informant variability in color lexicon within a language, see Kay 1975.

[9] We have written "with" rather than "+" because we have not worked out the mathematics of these categories. It does not appear that a simple

extension of the operation + will work in these cases.

10 For example, the cells of a jail normally provide a partition of the set of prisoners since very prisoner is assigned to a cell and no prisoner is assigned to more than one cell.

11 Zadeh cautions, "the notion of 'belonging,' [membership] which plays a fundamental role in the case of ordinary sets, does not have the same role in the case of fuzzy sets. Thus, it is not meaningful to speak of a point x 'belonging' to a fuzzy set A except in the trivial sense of $f_A(x)$ being positive" (1965:342). We will henceforth use 'belong' in just this sense, since with respect to color the notion is not trivial. Zadeh appears to have in mind applications in which few, if any, individuals in the relevant domain will have zero membership in any of the fuzzy sets under discussion. Such is not the case in color. For example, no color percept that belongs to red, belongs to green, and conversely. The same holds for blue and yellow. In discussing color percepts and categories it is often of interest to know if a given percept belongs (to any positive degree) to a certain category. Similarly, it is often of interest to know whether two categories have members in common, like green and yellow, or are disjoint, like green and red.

12 If

(i) A_s is a standard set,

(ii) $P = \{f_1, f_2, \ldots, f_n\}$ is a fuzzy partition of A_s,

(iii) P' is a collection of fuzzy sets such that (a) each $p_i \varepsilon P'$ is the union of one or more members of P and (b) each $f_i \varepsilon P$ occurs in exactly one member of P';

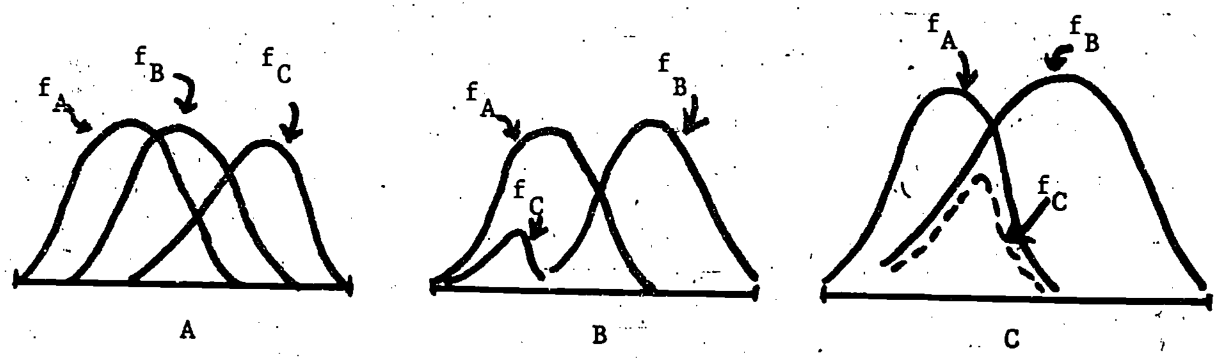
Then P' is a fuzzy partition of A_s.

Proof: (a) Consider an arbitrary element $x \varepsilon A_s$. By hypothesis (ii) there is at least one $f_i \varepsilon P$ such that x belongs to f_i. By hypothesis (iii) there is at least one $p_i \varepsilon P'$ such that x belongs to p_i.

(b) Consider an arbitrary $p_i \varepsilon P'$. By hypothesis (iii) p_i contains at least one $f_i \varepsilon P$. By hypothesis (ii) f_i has a member x which belongs to no other $f_j \varepsilon P$ ($f_j \neq f_i$). Hence x belongs to no other $p_j \varepsilon P'$. So for an arbitrary $p_i \varepsilon P'$ there exists an element $x \varepsilon A_s$ which belongs to p_i and which belongs to no other $p_j \varepsilon P'$ ($p_j \neq p_i$). The proof is complete.

[13] As noted, the way we have defined fuzzy partition excludes derived categories from being possible cells of a partition because they do not meet the mutual exclusion condition (7)c. Fuzzy partition may be alternatively defined with a weakened mutual exclusion condition which is met by derived color categories as well as by the primaries.

Recall that in the original statement of fuzzy mutual exclusion (7)c, a collection of fuzzy sets meets this condition just if for each fuzzy set there is an individual that has positive membership in this set and zero degree of membership in each other set in the collection. This may be weakened by defining a collection of fuzzy sets as mutually exclusive just if for each fuzzy set there is an individual that has a higher degree of membership in this set than in any other set in the collection. In Figure IA fuzzy sets A,B,C partition the domain of individuals, represented by the abcissa, while in Figures IB and IC there is no partition.



An alternate definition of fuzzy mutual exclusic ı
and Fuzzy Partition

Figure I

Note that none of the situations depicted in Figure I conforms to the definition of fuzzy mutual exclusion (and hence fuzzy partition) given in (7)C. If this weakened version of fuzzy mutual exclusion and fuzzy partition is adopted, then addition of derived categories after Stage V does further refine the partition of the color domain. This formulation would appear to characterize the systems of those individuals for whom derived categories such as brown, pink, orange, and grey are on a perceptual/conceptual par with the primaries. Note that in the alternate definition of fuzzy partition categories such as crimson (Cf. Figure IB) and chartreuse (Cf. Figure IC) still do not participate in a refinement of the partition.

References

Berlin, B. and E.A. Berlin
1975 Aguaruna Color Categories. American Ethnologist, 2:61-87.

Berlin, B. and P. Kay
1969 Basic Color Terms: their universality and evolution. Berkeley, University of California Press.

Bohannan, P.
1963 Social Anthropology. New York. Holt, Rinehart and Winston.

Bornstein, M.H.
1973a Color vision and color naming: A psychophysiological hypothesis of cultural difference. Psychological Bulletin, 80:257-285.

Bornstein, M.H.
1973b The psychophysiological component of cultural difference in color naming and illusion susceptability. Behavior Science Notes, 8:41-101.

Daly, T. (n.d.)
Color terms in the Slavic languages. U. of Cal. Berkeley ms.

DeValois, R.L., I. Abramov, and G.H. Jacobs
1966 Analysis of response patterns of LGN cells. Journal of the Optical Society of America, 56:966-977.

DeValois, R.L. and G.H. Jacobs
1968 Primate Color vision. Science, 162:533-540.

Dimmick, F.L. and M.R. Hubbard
1939 The spectral location of psychologically unique yellow, green and blue. American Journal of Psychology, 52:242-254.

Dougherty, J.W.D.
1974 Color categorization in West Futuna: variation and change. Paper presented to American Anthropological Association Annual Meeting, Mexico, D.F.

Dougherty, J.W.D.
1975 A universalist analysis of variation and change in color semantics. Unpublished Ph.D. dissertation, Department of Anthropology, University of California, Berkeley.

Fillmore, C.J.
1975 An alternative to checklist theories of meaning. in C. Cogen et al. (eds) Papers presented to the First Annual Meeting of the Berkeley Linguistics Society. Berkeley, California.

Gleason, H.A.
1961 An introduction to descriptive linguistics. New York. Holt, Rinehart and Winston.

Hage, P. and K. Hawkes (n.d.)
Binumarien color terms. University of Utah, Salt Lake City.

Harkness, Sara
1973 Universal aspects of learning color codes: A study in two cultures. Ethos, 1:175-200.

Hays, D.G., E. Margolis, R. Naroll and D.R. Perkins
1972 Color term salience. American Anthropologist, 74:1107-1121.

Heider, E.R.
1972a Probabilities, sampling, and ethnographic method: The case of Dani colour names. Man (n.s.), 7:448-466.

Heider, E.R.
1972b Universals in color naming and memory. Journal of Experimental Psychology, 93:10-20.

Heider, E.R. and D.C. Olivier
1972 The structure of the color space in naming and memory for two languages. Cognitive Psychology, 3:337-354.

Heinrich, A.C.
1972 A non-European system of color classification. Anthropological Linguistics, 14:220-227.

Hering, Ewald
1964 [1920] Outlines of a theory of the light sense. translated by Leo M. Hurvich and Dorthea Jameson. Harvard University Press, Cambridge, Mass.

Jameson, D. and L.M. Hurvich
1955 Some quantitative aspects of an opponent-colors theory. I. Chromatic responses and spectral saturation. Journal of the Optical Society of America, 45:546-552.

Kay, P.
1975 Synchronic variability and diachronic change in basic color terms. Language in Society, 4:257-270.

Kuschel, R. and T. Monberg
1974 'We don't talk much about colour here': A study of colour semantics on Bellona Island. Man (n.s.), 9:213-242.

Lakoff, G.
1972 Hedges: A study in meaning criteria and the logic of fuzzy concepts. In P.M. Peranteau et al. (eds) Papers from the Eighth Regional Meeting of the Chicago Linguistics Society. Chicago, Illinois.

MacLaury, R. (n.d.)
Reconstructions of the Evolution of some Basic Color Term Lexicons. Ms. 72 pp. University of California, Berkeley, Ms.

McDaniel, C.K.
1972 Hue perception and hue naming. Unpublished A.B. honors thesis. Harvard College.

McDaniel, C.K.
1974 Basic color terms: Their neurophysiological bases. Paper presented to the American Anthropological Association Annual Meeting, Mexico, D.F.

McDaniel, C.K.
Universals in color term semantics and their neuropsychological sources. Forthcoming.

Ray, V.F.
1952 Techniques and problems in the study of human color perception. Southwestern Journal of Anthropology, 8:251-259.

Ray, V.F.
1953 Human color perception and behavioral response. Transactions, New York Academy of Sciences (ser. 2), 16:98-104.

Rosch, E.H.
1973 On the internal structures of perceptual and semantic categories. In T.E. Moore, Ed., Cognitive development and the acquisition of language. Academic Press, New York, New York.

Sternheim, C.D. and R.M. Boynton
1966 Uniqueness of perceived hues investigated with a continuous judgemental technique. Journal of Experimental Psychology, 72:770-776.

Stevens, S.S.
1957 On the Psychophysical Law. Psychological Review. 64:153-181.

Zadeh, L.A.
1965 Fuzzy Sets. Information and Control, 8:338-353.

Zadeh, L.A.
1971 Quantitative fuzzy semantics. Information Sciences, 3:159-176.