A review of studies comparing self-paced individualized programs with other teaching approaches at the secondary and post-secondary level is presented in this paper. First, the teaching approaches used and the statistical design employed in the studies are described. Then, the studies are classified as secondary or post-secondary, and the results are reported. Finally, an overall interpretation of the results is given. (DT)
INDIVIDUALIZED MATHEMATICS INSTRUCTION:
HOW EFFECTIVE HAS IT BEEN IN
SECONDARY AND POST SECONDARY SCHOOLS?

by
Harold L. Schoen
The University of Iowa
The May, 1972, issue of The Mathematics Teacher contained eight articles dealing with individualizing mathematics instruction. Most of the articles discussed a modular, self-paced approach to individualization. Since that time a substantial amount of research testing the effectiveness of such programs has been reported. This paper is a review of studies comparing self-paced, individualized programs with other teaching approaches at the secondary and post secondary levels. A recent review of similar studies conducted with elementary school students showed that the results are overwhelmingly against individualized instruction as measured by mathematics achievement with some ambiguity on attitude and other affective measures (Schoen, 1975). Do these negative findings continue into secondary school and beyond? Mathematics teachers at these levels will find the results summarized here very informative, especially if they are considering adopting an individualized program.

Teaching Approaches in the Studies

The individualized instruction approaches used in the reviewed studies were, unless otherwise stated, researcher or teacher developed programs based on the Individually Prescribed Instruction (IPI) model. IPI is a program for the elementary schools developed at the Learning Research and Development Center in Pittsburgh. Of course, the individualized approaches differed from study to study but they had several important characteristics in common.

First, they were based on a specific set of behavioral objectives. Second, the mathematics content to be learned was divided into small modules or units. Third, learning packets were written for each unit. The learning packets served as guides for the student to proceed more or less independently through the content at his own pace. Fourth for the most part the students
learned independently from textbooks and worksheets though some programs included other media. Fifth, each packet contained pretests and posttests. The student was required to pass one or both before proceeding to the next unit. The teacher's role was that of manager, record keeper, individual tutor, and sometimes curriculum developer.

In most studies, the individualized treatment was compared to a "traditional" treatment. The researchers did not always describe this method in great detail. With some noted exceptions, the traditional method was teacher centered and teacher paced with common tests given at the same time to all twenty to thirty-five students within a self contained classroom. Again, the traditional approach differed from one study to another but these are common characteristics which mark the contrast to the individualized approach.

General Description of the Studies

Only studies in which the comparison groups were "equivalent" before the treatment are reported. The methods of achieving equivalence were random assignment of students or classes to treatments, or statistical equating by analysis of covariance or analysis of pretest to posttest change scores.

Criterion measures were typically standardized mathematics achievement tests, researcher developed achievement tests, and attitude toward mathematics scales. Analysis of variance and t-tests were the most common statistical techniques used to test differences in group means. Unless otherwise stated, statistically significant differences are reported at the .05 probability level.
Secondary School Studies

Twelve studies which included a statistical comparison of an individualized group and a control group of secondary school students were located. In only one of these did mathematics achievement differences favor the individualized group. Bull (1971) conducted a semester long study involving two individualized geometry classes and two comparable classes taught traditionally. The individualized classes scored significantly higher than the traditional classes on a mid-year geometry posttest.

Significant achievement differences favoring the traditional approach were reported by three researchers. Fisher (1973) found the traditional group's mathematics achievement mean to be significantly higher than that of the individualized group after a full year of geometry. Herceg (1973) and Hirsch (1973) both reported a self-paced approach to be the worst of the three approaches to teaching algebra II.

In Herceg's (1973) study sixteen algebra II classes, three top track and thirteen middle track, were randomly placed in three treatment groups. The treatments were (a) self-paced, (b) traditional with objectives provided to the student, and (c) traditional with no objectives provided. After a unit on complex numbers the traditional with objectives outscored the traditional with no objectives which in turn outscored the self-paced group on a teacher prepared achievement test. These differences occurred with the middle track students only. No significant differences were found with the students in the top track.

Hirsch (1973) also used a unit on complex numbers. His sample consisted of two hundred thirteen algebra II students taught by one of three methods: guided discovery, expository learning packets, and programmed learning packets. The guided discovery group scored significantly higher (p<.01) than the other
two groups on a researcher developed posttest of achievement. Differences in the same direction were found on tests of vertical and horizontal transfer.

In the bulk of the secondary studies no significant differences in mathematics achievement among the treatment groups were reported. This was the finding with algebra I students (Englert, 1972; Ludeman and others, 1973; Taylor, 1972), with geometry students (Hanneman, 1972), with both algebra I and geometry students (Schoen and Todd, 1974), with algebra II students (Thomas, 1972) and with trigonometry students (Penner, 1972). Some of these researchers compared the groups on criteria other than achievement, too. Their findings are summarized in Table 2.

One researcher took a different approach to testing the effectiveness of individualized programs. Walters (1973) identified high schools in Memphis, Tennessee, which had been using individualized programs for four years. She then matched each of these schools with another similar in location, socio-economic level of the student body, and curricular objectives. Using IQ and several achievement measures as covariates, she compared the first and second semester college grade point averages of a class of graduates from the individualized schools with the same year's graduates from the traditional schools. Only those students who attended Memphis State University were considered. While recognizing several limitations to this procedure, Walters reported the traditional group's grade point averages to be higher, but not significantly higher, than the individualized group.

Table 1 presents a summary of the rather ambiguous results using achievement as the criterion measure. These results will be discussed further in a later section.
## TABLE I

<table>
<thead>
<tr>
<th>Achievement*</th>
<th>Achievement Greater in Individualized</th>
<th>Achievement Greater in Control</th>
<th>N. S. D.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Algebra I</td>
<td></td>
<td>Taylor, 1972</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ludeman and others, 1973</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Englert, 1972</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Schoen and Todd, 1974</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Schoen and Todd, 1974</td>
<td></td>
</tr>
<tr>
<td>Algebra II</td>
<td></td>
<td>Herceg (1973)</td>
<td>Thomas, 1972</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hirsch (1973)</td>
<td></td>
</tr>
<tr>
<td>Trigonometry</td>
<td></td>
<td></td>
<td>Penner, 1972</td>
</tr>
</tbody>
</table>

*Some researchers compared adjusted posttest achievement means using one or more covariates. No distinction is made in this table.

Several researchers also compared the treatment groups on criteria other than mathematics achievement, but with no significant differences reported in nearly every case. The results are summarized in Table 2.
TABLE 2

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Favored Individualized</th>
<th>Favored Control</th>
<th>N. S. D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude Toward Mathematics</td>
<td>Englert, 1972</td>
<td>Thomas, 1972</td>
<td></td>
</tr>
<tr>
<td>Vertical Transfer</td>
<td>Hirsch, 1973</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Transfer</td>
<td>Hirsch, 1973</td>
<td></td>
<td></td>
</tr>
<tr>
<td>College Grade</td>
<td>Walters, 1973</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point Average</td>
<td></td>
<td>Hirsch, 1973</td>
<td></td>
</tr>
<tr>
<td>Retention</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Post Secondary Studies

A large number of articles reporting individualized mathematics programs in community colleges, colleges and universities have appeared in the last four or five years. Few of them report an objective evaluation of the program's effectiveness. Good sources for these articles are the Mathematical Education section of The American Mathematical Monthly, The Two year College Mathematics Journal, and The MATYC Journal.

Five relatively objective studies were found in this area, although the general quality of the experimental design and control is below that of the studies reported with elementary and secondary students. In one of the five
studies, mathematics achievement results favored the individualized group (Emery, 1971). Her sample consisted of seventy seven prospective elementary school teachers in a junior college. The researcher developed achievement measure was administered at the end of a one semester mathematics course.

Pond (1973) found that senior elementary education majors taught by an individualized approach exhibited better attitudes toward individualized instruction (as measured by a researcher developed instrument) after a one semester mathematics course than a comparable teacher centered group. No achievement results were reported.

Two researchers found no significant difference in mathematics achievement means when an individualized group was compared to a traditional one. Morman's (1973) sample was eight classes of remedial college algebra while Bázik (1973) used thirty six prospective elementary teachers in a mathematics course. Bázik also found no significant difference in attitude toward mathematics.

Corn and Behr (1975) reported mixed results when they studied over two hundred students who enrolled in remedial freshman mathematics at City University of New York. The students chose between a conventional course, a self-paced tutorial course or a lecture-discussion course divided into short modules. Their study was greatly limited by the fact that the students chose their own method. However, they did find that students choosing the conventional remedial course did better in the future credit mathematics courses. The authors speculated that since the credit courses were taught conventionally, too, the students who have been trained in that manner of learning have an advantage.

It is difficult to draw any conclusions about individualized mathematics
programs with post secondary students from these studies. In one study mathematics achievement favored the individualized group while in three studies no significant difference was reported. In one study the individualized group had a more positive attitude toward individualized instruction while another researcher reported no significant difference in attitude. Another researcher provided some evidence that students taught conventionally in a remedial mathematics course tend to do better in future credit courses which are also taught conventionally.

Interpretation

Even people who are strongly committed to this type of individualized instruction admit that it is very expensive to implement and continues to be more expensive in the long run than traditional instruction (e.g. Edmunds, 1971; Johnson, 1972). Many researchers report that much more work is involved for the teachers and/or extra personnel must be available in the classroom (e.g. Frary, 1971; Palow, 1973; Amendola, 1973; Lipson, 1974; Sutton, 1967). The burden of proof that there is some pay off for this extra investment of money and human effort must surely be with the proponents of self-paced, individualized instruction.

The studies summarized here, with all their limitations, do not provide evidence of any such pay off. At the secondary level only one of a dozen studies resulted in improved mathematics achievement while three researchers reported greater achievement in traditional programs. Many proponents of this type of individualization point to other improvements especially in the affective area, but not one of the secondary studies showed this. There were too few studies at the post secondary level to draw any conclusions.

Two possible sources of invalidity in these studies also work in favor of
the individualized programs, namely experimentor bias and the Hawthorne Effect. With one or two exceptions the researcher set out to prove the effectiveness of his individualized approach. Thus, if the bias of the experimentor had any effect on the outcomes it was to make them less damaging to the individualized programs. The Hawthorne Effect, or novelty of being in a new or experimental program, can tend to temporarily inflate results in an experimental group. In nearly all the reported studies, the individualized approach was new for the students. It is impossible to know what, if any effect, this had on the outcomes but it certainly did not work against the individualized approaches.

It is possible, too, that the self-paced, modularized approach to individualization is an effective, even a superior, instructional strategy, but the particular applications of this strategy in the reported studies were inferior. Perhaps the researchers and/or teachers used poor development or management techniques in their applications of this method. No commercially developed or standardized individualized programs were used with which to compare the researcher developed programs at this level. However, research summarized in another paper shows that elementary school students did no better in IPI and other commercially developed programs than they did in teacher and researcher developed individualized approaches. Yet IPI has evolved through over ten years of development, testing, and revision under the guidance of some of the best experts in this field under nearly ideal conditions. In addition though the studies reported here were of relatively short duration, there is evidence that after two or three years in totally individualized programs the mathematics achievement of elementary students falls further behind comparable traditionally taught students (Schoen, 1975).

The findings in the studies summarized here combined with those at the
elementary level clearly lead to several conclusions. While the patterns of the results varied with the grade level of the subjects, improvements in mathematics achievement did not occur under individualization at any level. In fact, at the fifth to eighth grade level the evidence strongly points to decreased achievement. The same was true at the secondary level, with a bit more ambiguity. Student improvement on affective measures was noted with individualized kindergarten to grade four students but was absent at the other levels.

The reasons for this failure to demonstrate effectiveness are no doubt many and varied. Some students, with the right content, the right teachers, etc. might succeed with this approach. It may also be possible to identify and correct the weaknesses in the existing theory and/or programs. Research of the type summarized here is not designed to provide reasons or solutions. However, some light is shed by a fairly large body of recent research concerned with specific aspects of self-paced, individualized mathematics programs. A review of this research is under preparation now.

For the moment, a mathematics teacher or principal who adopts a program such as those described in this paper in its entirety can be sure that (1) it will be more expensive and (2) it will be more work for the teacher. In addition, (3) mathematics achievement is not likely to increase and may very well decrease, and (4) student attitude is not likely to improve except perhaps in the primary grades. In fact, after over fifty studies at all grade levels aimed at showing the effectiveness of this approach there is no consistent objective evidence that there will be student improvement of any sort. The most consistent result is lesser mathematics achievement with an individualized teaching approach.
REFERENCES


Hirsch, C. R. *An Experimental Study Comparing The Effects of Guided Discovery and Individualized Instruction on Initial Learning, Transfer, and Retention of Mathematical Concepts and Generalizations.*


